

Title: PSI 2015/2016 Explorations in Quantum Information - Agata Branczyk - 10

Date: Apr 04, 2016 09:00 AM

URL: <http://pirsa.org/16040001>

Abstract:

Week 3

Mon (Lec) - Brief intro to Q. Optics

- Wigner functions

- Nonlinear optics

- Nonclassical state generation (SPDC)

Mon (Tut) - Multimode HOM interference

Tues (Lec) - Multimode SPDC

↳ spectral correlations/purity } Grice, PRA 56, 1627 (1997)

Wed (Disc) - Open

Thu (Lec) - Coherent state LOQC
 $|\psi\rangle = \mu|\alpha\rangle + \nu|-\alpha\rangle$
cat states } Ralph, PRA 68, 042319 (2003)

Fri (Lec) - Cluster State Q.C. } Raussendorf, PRL 86, 5188 (2001)
Nielsen quant-ph/0504097

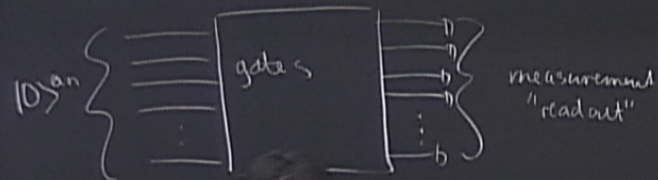
Fri (Tut) - GHZ states



Mon (Tut) - Nonclassical state generation (SPDC)
 - Multimode HOM interference

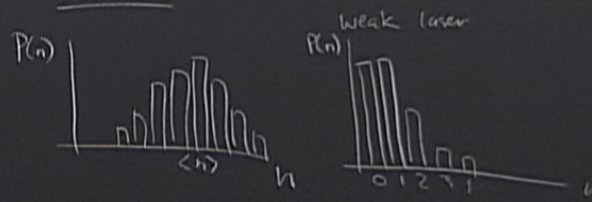
Tues (Lec) - Multimode SPDC
 ↳ spectral correlations/purity } Grice, PRA 56, 1627 (1997)

Fri (Tut) - GHZ states



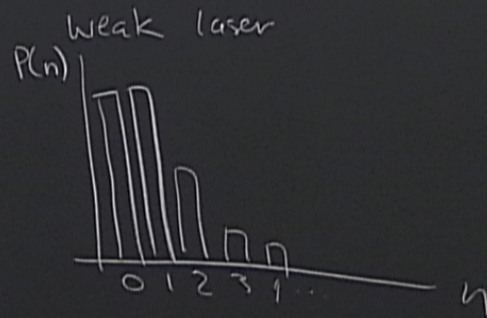
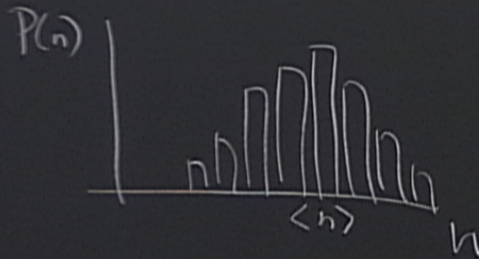
Single photon $|1\rangle$

Laser?



Single photon $|1\rangle$

Laser?



Quantum state

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Mean photon # $\langle n \rangle = |\alpha|^2$
 $\arg(\alpha)$ is phase

What are $|n\rangle$?

3)

Eigen states of EM Hamiltonian.

4)

Recipe for quantising the EM field.

5)

1) Start with Maxwell's eqn.

2) Choose Coulomb Gauge $\nabla \cdot \vec{A} = 0$

6)

3) Write down wave eqn.
for vector potential $\vec{A}$

4) Solve wave eqn in Volume V

5) Write corresponding solns for E & B

$$E(r,t) = i \sum_k \sqrt{\frac{\hbar \omega_k}{2\epsilon_0}} a_k u_k(r) e^{-i\omega_k t} + h.c.$$

6) Choose $a_k, a_k^\dagger$ to be mutually adjoint
operators $[a_k, a_{k'}] = [a_k^+, a_{k'}^+] = 0, [a_k, a_k^\dagger] = \delta_{kk'}$

The Fock states $|n\rangle$ are eigenstates of H

$$H|n_k\rangle = \hbar\omega_k \left(\underset{\substack{\uparrow \\ \text{photon number}}}{n_k + \frac{1}{2}} \right) |n_k\rangle$$

Also.

$$\hat{n}_k |n_k\rangle = n_k |n_k\rangle$$

Useful to know

$$a_k |n_k\rangle = \sqrt{n_k} |n_k - 1\rangle$$

$$a_k^\dagger |n_k\rangle = \sqrt{n_k + 1} |n_k + 1\rangle$$

Side note.

Completely analogous to:

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$$

$$\text{if } x = \sqrt{\frac{\hbar}{2m\omega}} (a^\dagger + a)$$

$$p = i\sqrt{\frac{\hbar}{2m\omega}} (a^\dagger - a)$$

$$\text{get } \Rightarrow H = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right)$$



angular is phase

$$\frac{|a|^{2n} e^{-|a|^2}}{n!}$$

3) Write down wave eqn
for vector potential $\vec{A}$

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Classical Hamiltonian for EM field

$$H = \frac{1}{2} \int (\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0} \vec{B}^2) dV$$

Substitute E and B

$$H = \sum_k \hbar \omega_k \underbrace{(a_k^\dagger a_k + \frac{1}{2})}_{\hat{n}_k}$$

$$a_k^\dagger |n_k\rangle = \sqrt{n_k+1} |n_k+1\rangle$$

Similarly, in Q.O. useful to define:

$$q = \frac{1}{\sqrt{2}}(a^\dagger + a)$$

$$p = \frac{i}{\sqrt{2}}(a^\dagger - a)$$

can define quasi probability
function.

Why Wigner function?

- * Another way to compute dynamics.
- * helps to build intuition
- * help communication between physicists.

function.

+ beautiful

Fourier Transform of characteristic function

$$\chi[\eta] = \text{Tr}[p e^{i\eta^* a - i\eta a}] ; \eta \in \mathbb{C}$$

Let $\beta = q + ip$

$$W(\beta) = \frac{1}{\pi^2} \int d^2\eta \exp(\eta^* \beta - \eta \beta^*) \chi[\eta]$$

Properties

- can be negative
- $\int dq \int dp W(q,p) = 1$

Marginals:

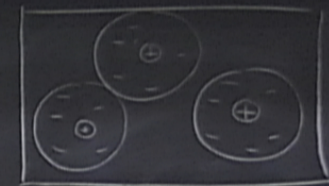
$$pr(q) = \int dp W(q,p)$$

$$pr(p) = \int dq W(q,p)$$



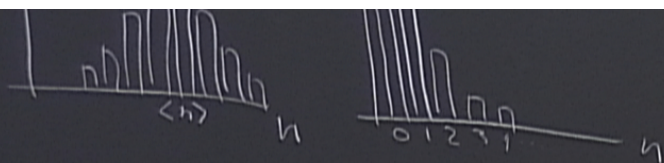
Typical nonlinear material: Dielectric

- type of insulator
- electrons don't move freely
- but can move a little in response to an applied field



Apply a field:
→ E





angular is phase

$$= \frac{|a|^{2n} e^{-|a|^2}}{n!}$$

Dielectric



field



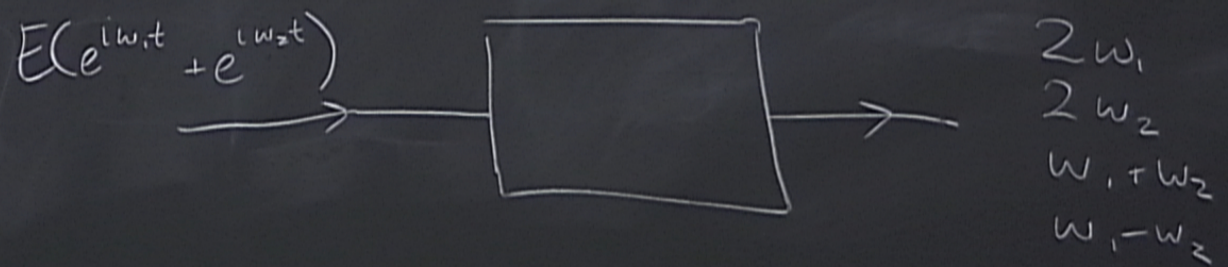
Polarization density

$$P(t) = \epsilon_0 \left(\chi^{(1)} E(t) + \underbrace{\chi^{(2)} E^2(t) + \chi^{(3)} E^3(t) + \dots}_{P^{NL}} \right)$$

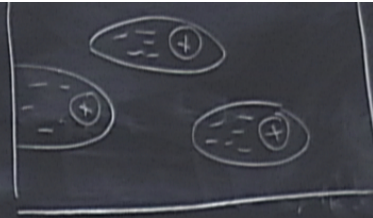
Source in wave equation

$$\nabla^2 E - \frac{n^2}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2}{\partial t^2} P^{NL}$$

For example $\chi^{(2)}$ medium.



response to an applied field



$$\nabla \cdot \mathbf{E} - \epsilon^2 \nabla^2 \mathbf{E} = \epsilon_0 \nabla \cdot \mathbf{P}$$

ample $\chi^{(2)}$ medium.



$$\begin{aligned} 2\omega_1 \\ 2\omega_2 \\ \omega_1 + \omega_2 \\ \omega_1 - \omega_2 \end{aligned}$$

Spontaneous Parametric Down Conversion $\chi^{(2)}$

- single mode
- classical pump

Type 1 $\rightarrow$ output photons same polarization

$$H_I = -i\hbar \frac{\chi}{2} (aa + a^\dagger a^\dagger)$$