Title: From locally covariant quantum fields to effective quantum gravity

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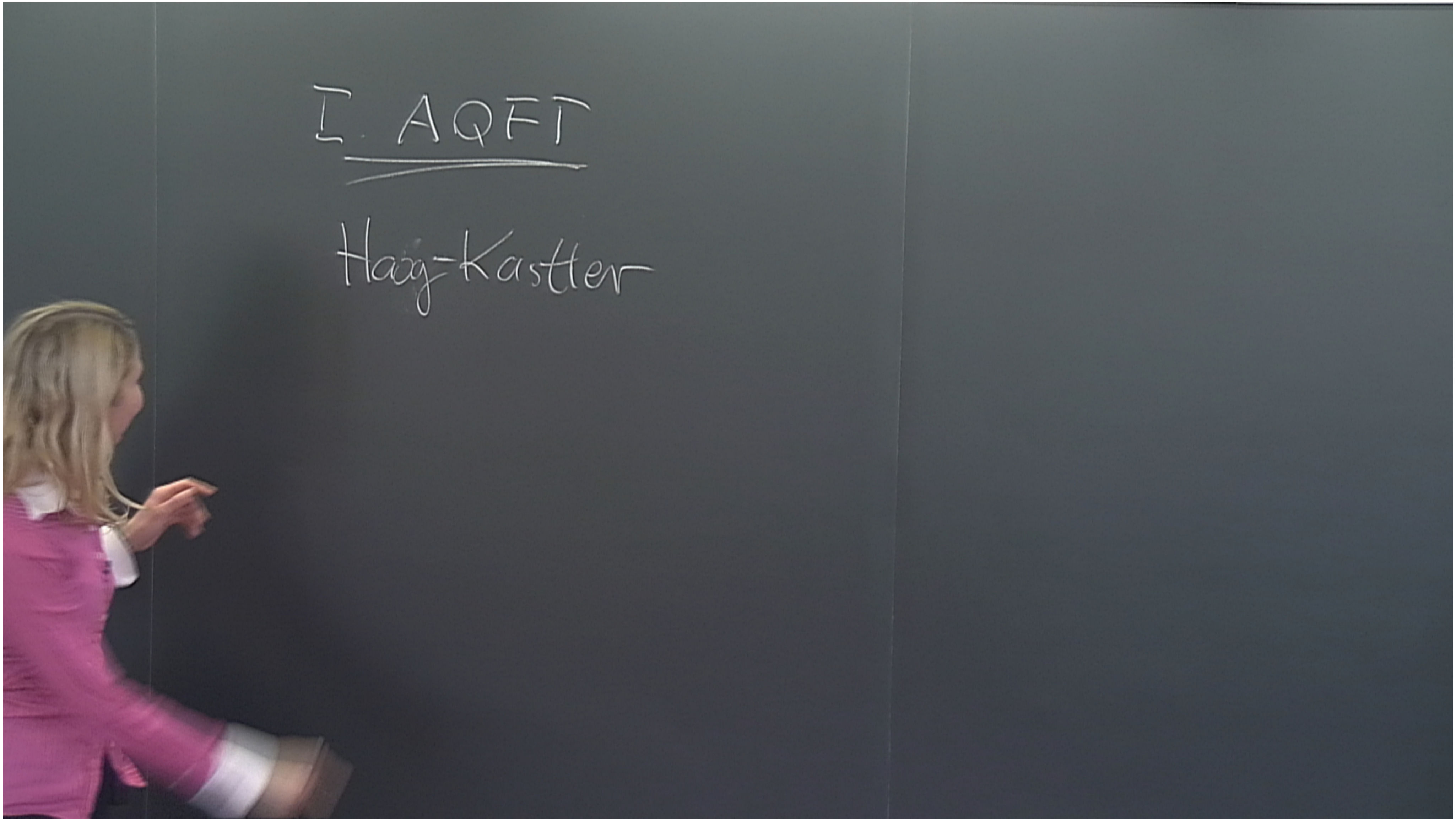
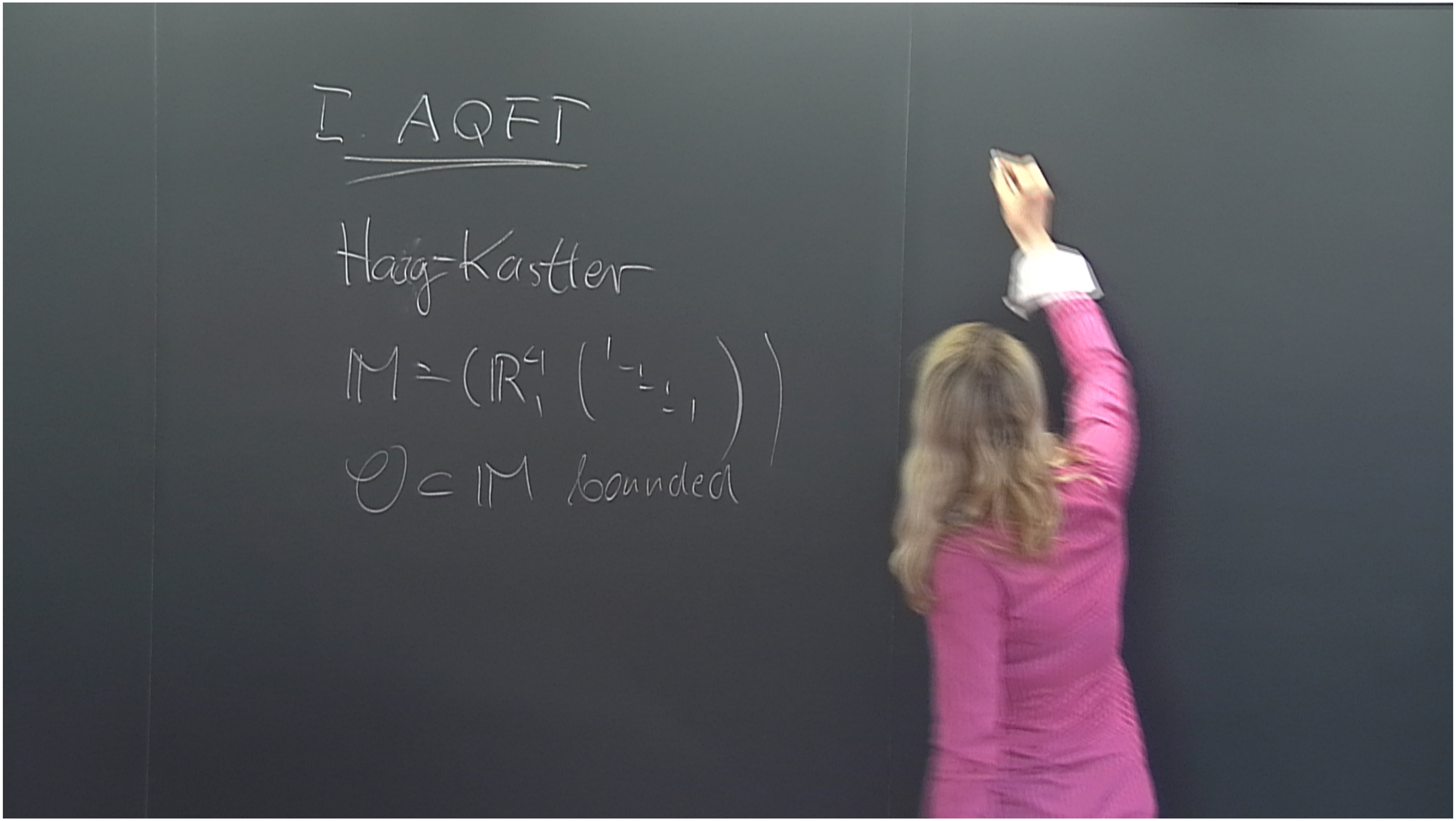
Abstract: <p>I will present recent result on constructing effective quantum gravity theory as a locally covariant QFT. The approach I advocate uses the BV formalism for dealing with the gauge freedom and Epstein-Glaser renormalization to control the UV divergences. I will show how gauge invariant observables that satisfy a weak notion of locality can be constructed in this framework and I will sketch the argument for perturbative background independence. Recently these ideas were applied to models relevant in cosmology.</p>


I. AQFT
Haag-Kastler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$\mathcal{O} \subset \mathbb{M}$ bounded

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$\mathcal{O} \subset \mathbb{M}$ bounded

$\mathcal{O} \longmapsto \mathfrak{A}(\mathcal{O})$ algebra of observables

1) $\mathcal{O}_1 \subset \mathcal{O}_2$ then

$\mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$

Isotony.

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2) Causality

$\mathcal{O}_1 \times \mathcal{O}_2$ spacelike regions

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$)$.

2) Causality

$\mathcal{O}_1 \times \mathcal{O}_2$ spacelike regions.

then $[A, B] = 0, \ \forall \ A \in \mathfrak{A}(\mathcal{O}_1), \ B \in \mathfrak{A}(\mathcal{O}_2)$

3)

QFT

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Define:

2) Ca

$\mathcal{O}_1$

then

spacelike
regions.

$, \forall$

$(\mathcal{O}_1)$

$(\mathcal{O}_2)$

3) Time-slice axiom

$\Sigma \subset M$ Cauchy surface

$\mathcal{N}$ neigh. of Σ.

$\mathfrak{A}(\mathcal{N}) \cong \mathfrak{A}$

...ion

...auchy surface

Σ.

$\mathcal{O}$

...: QFT

$\mathcal{M}, \ddot{\mathcal{M}}$ globally hyperbolic spacetimes

(has a Cauchy surface $\mathcal{M} = \Sigma \times \mathbb{R}$)

oriented, time-oriented.

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QFT

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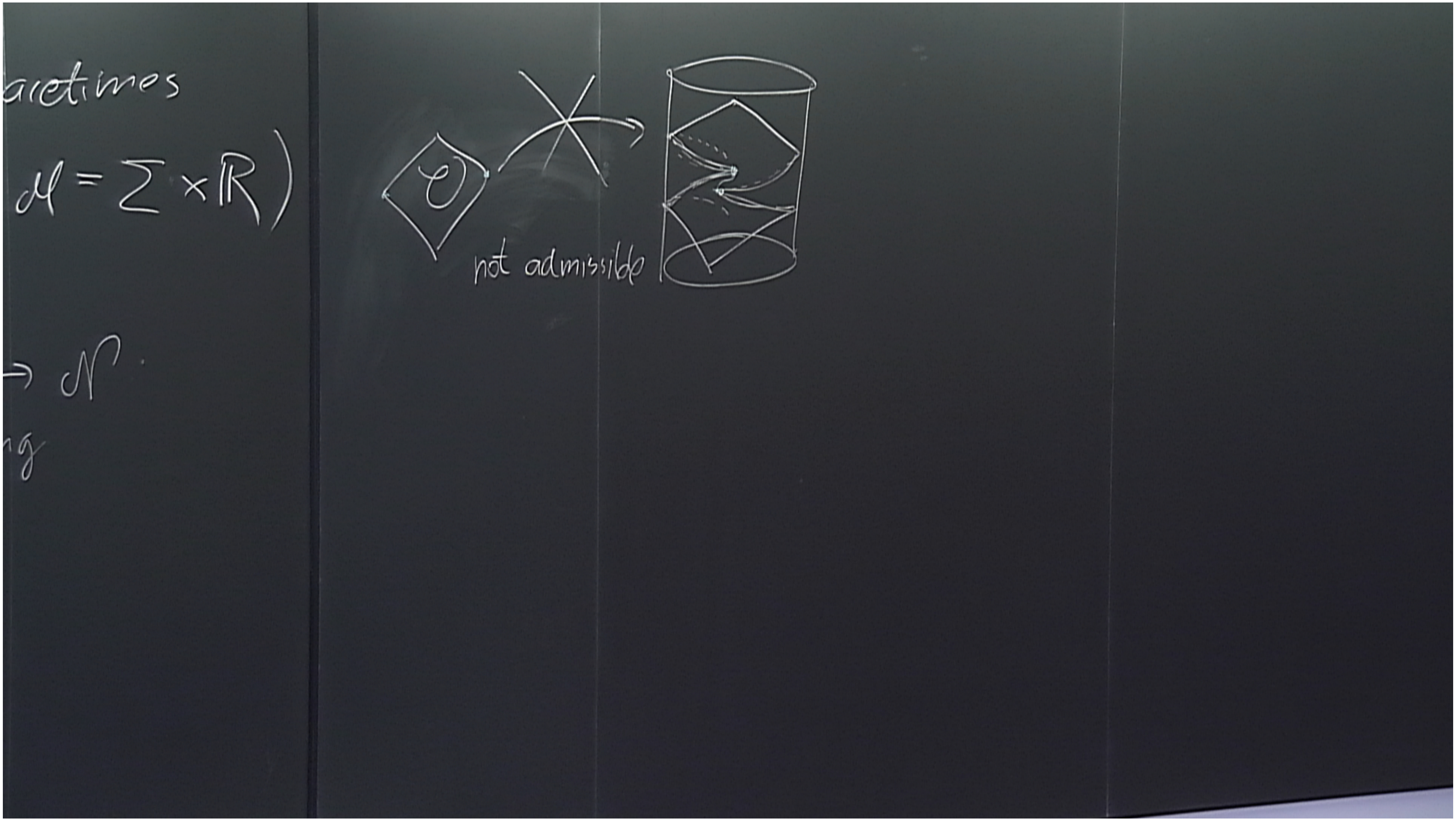
Admissible embeddings $\chi : \mathcal{M} \longrightarrow \mathcal{N}$.

1) isometries, orientations preserving
2) "causality preserving".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M = Σ × R)
→ N
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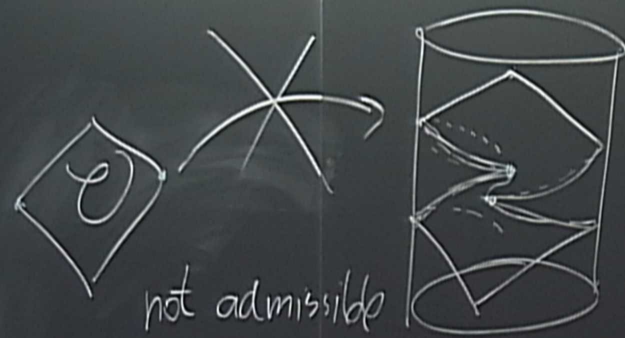

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LCQFT: Assignment $\mathcal{M} \mapsto \mathfrak{A}(\mathcal{M})$

$\chi: \mathcal{M} \to \mathcal{M} \mapsto \alpha_\chi : \mathfrak{A}(\mathcal{M}) \to \mathfrak{A}(\mathcal{N})$

covariance: $\alpha_{\chi_1 \circ \chi_2} = \alpha_{\chi_1} \circ \alpha_{\chi_2}$

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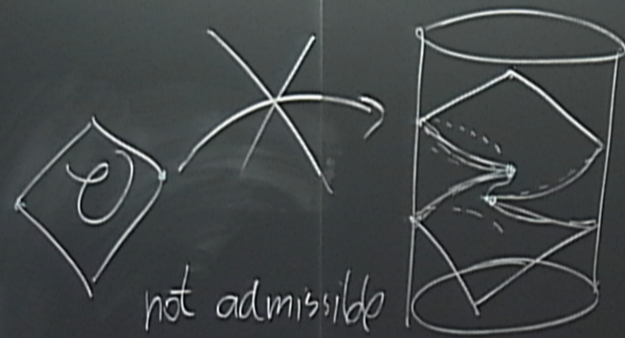

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$\mathfrak{A}$ is a functor from <u>Loc</u> to algebras ($*$-algebras, Poisson, ...)

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$= \alpha_{\chi_1} \circ \alpha_{\chi_2}$

$\chi_1 \circ \chi_2$

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<u>Def</u>: Locally cov. quantum field

is a family of maps labelled by spacetimes

$$\Phi_{\mathcal{M}} : \mathcal{D}(\mathcal{M}) \to \mathfrak{A}(\mathcal{M})$$

$$\mathcal{D}(\mathcal{M}) = \mathcal{C}_c^\infty(\mathcal{M}, \mathbb{R})$$

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...son, ...

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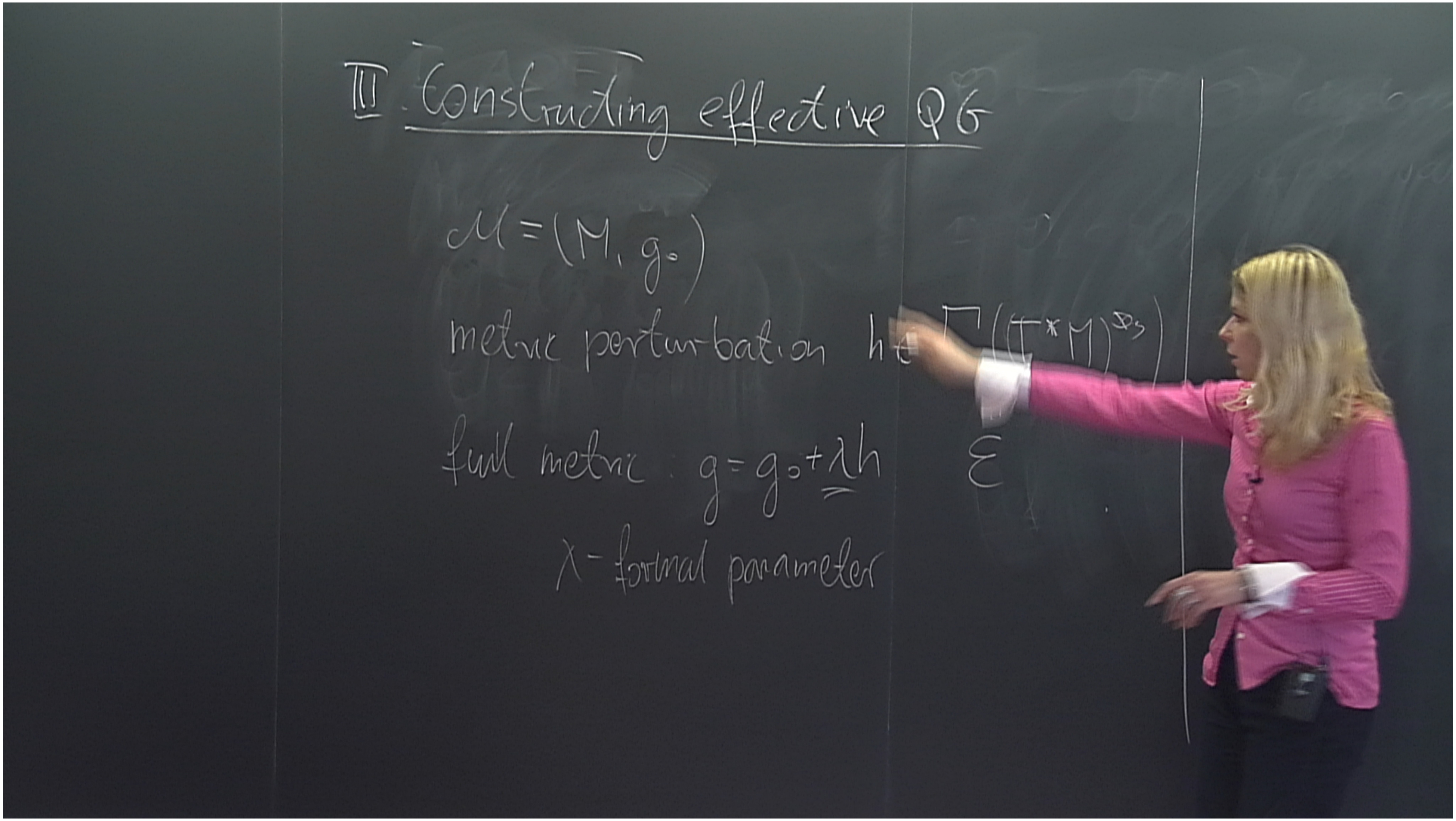

III. Constructing effective QG
$\mathcal{M} = (M, g_0)$
metric perturbation $h \in \Gamma((T^*M)^{\otimes_S})$
full metric: $g = g_0 + \lambda h$
ε
λ - formal parameter

Functionals on $\mathcal{E}$

$\mathcal{F}_{loc}$ --- local functionals are of the form:

$$F(\varphi) = \int_M \omega(j^k(\varphi)),$$ for given φ, k is finite.

ω — density on M

$\mathcal{F}$ is the space of products of local nite.

$\mathcal{F}$ is the space of sums of products of local functionals.

$\mathcal{F}$ - multilocal.

Take $\Phi_{(M,g_0)}(f)(h) = \int R[g_0+\lambda h] f \, d\mu_{g_0+\lambda h}$

$\xi \in \Gamma(TM)$, for $F \in \mathcal{F}_{loc}$ $(\rho(\xi)F)(h) = \left\langle \frac{\delta F}{\delta h}, \mathcal{L}_\xi h \right\rangle$

Let (M, g_0) be non-generic

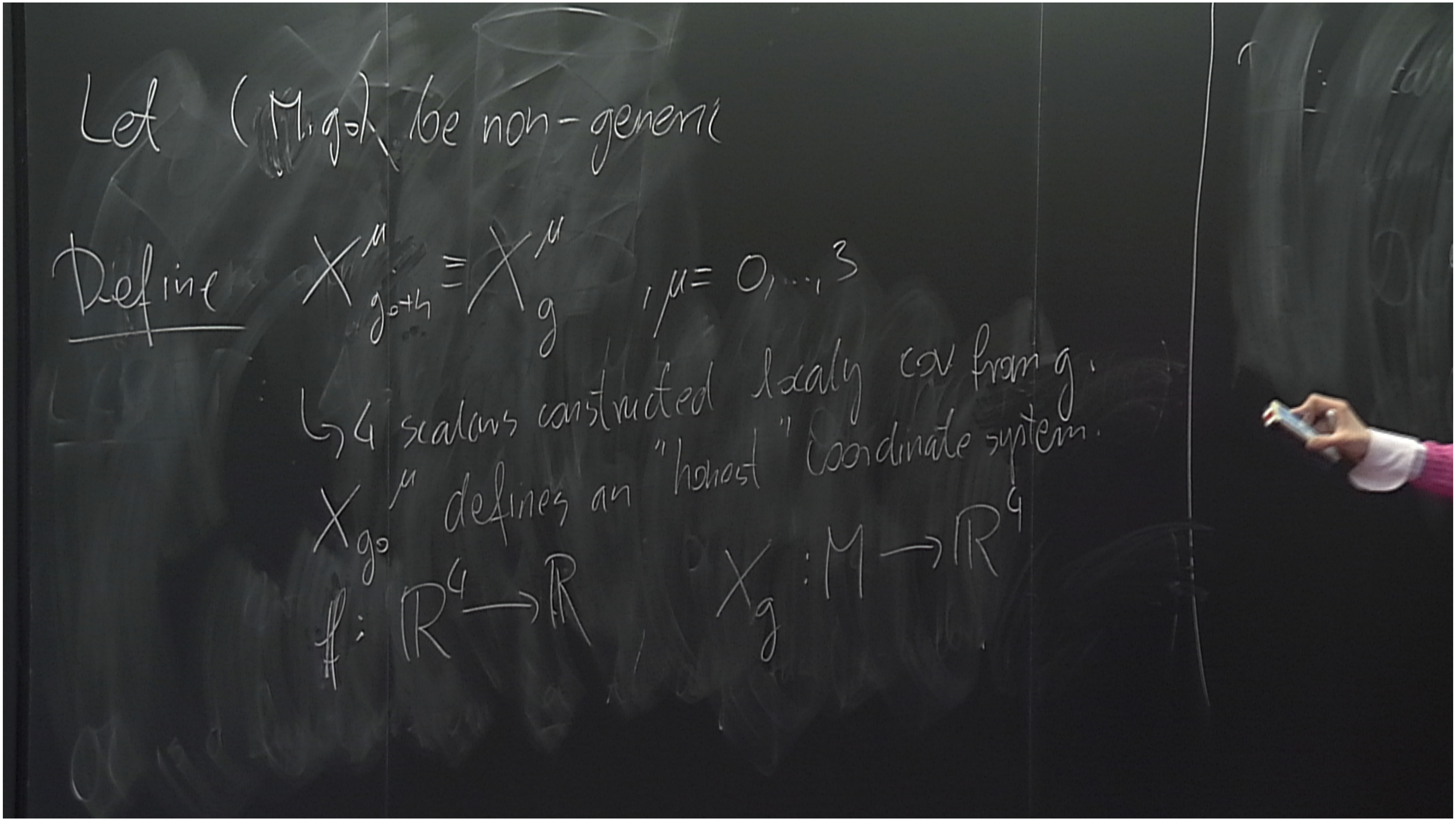
Define $X^{\mu}_{g_0+h} \equiv X^{\mu}_{g}$, $\mu = 0, \dots, 3$

↳ 4 scalars constructed localy cov from g.

Def:

is a

Φ_{α} :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Define $X^{\mu}_{g_0+h} \equiv X^{\mu}_{g}$, $\mu = 0, \ldots, 3$
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$X_{g_0}^{\mu}$ defines an "honest" coordinate system.
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$X_g : M \to \mathbb{R}^4$

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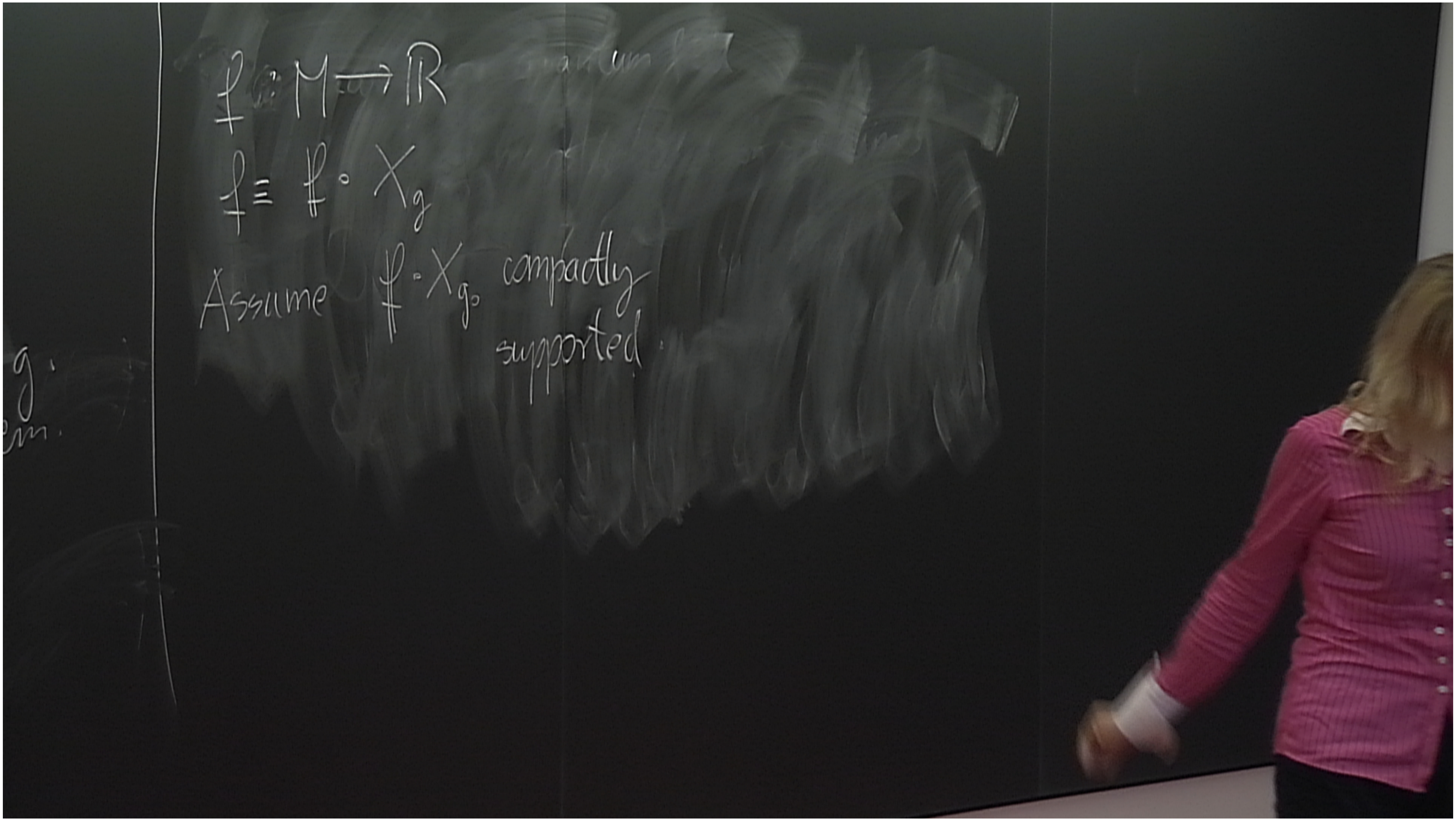
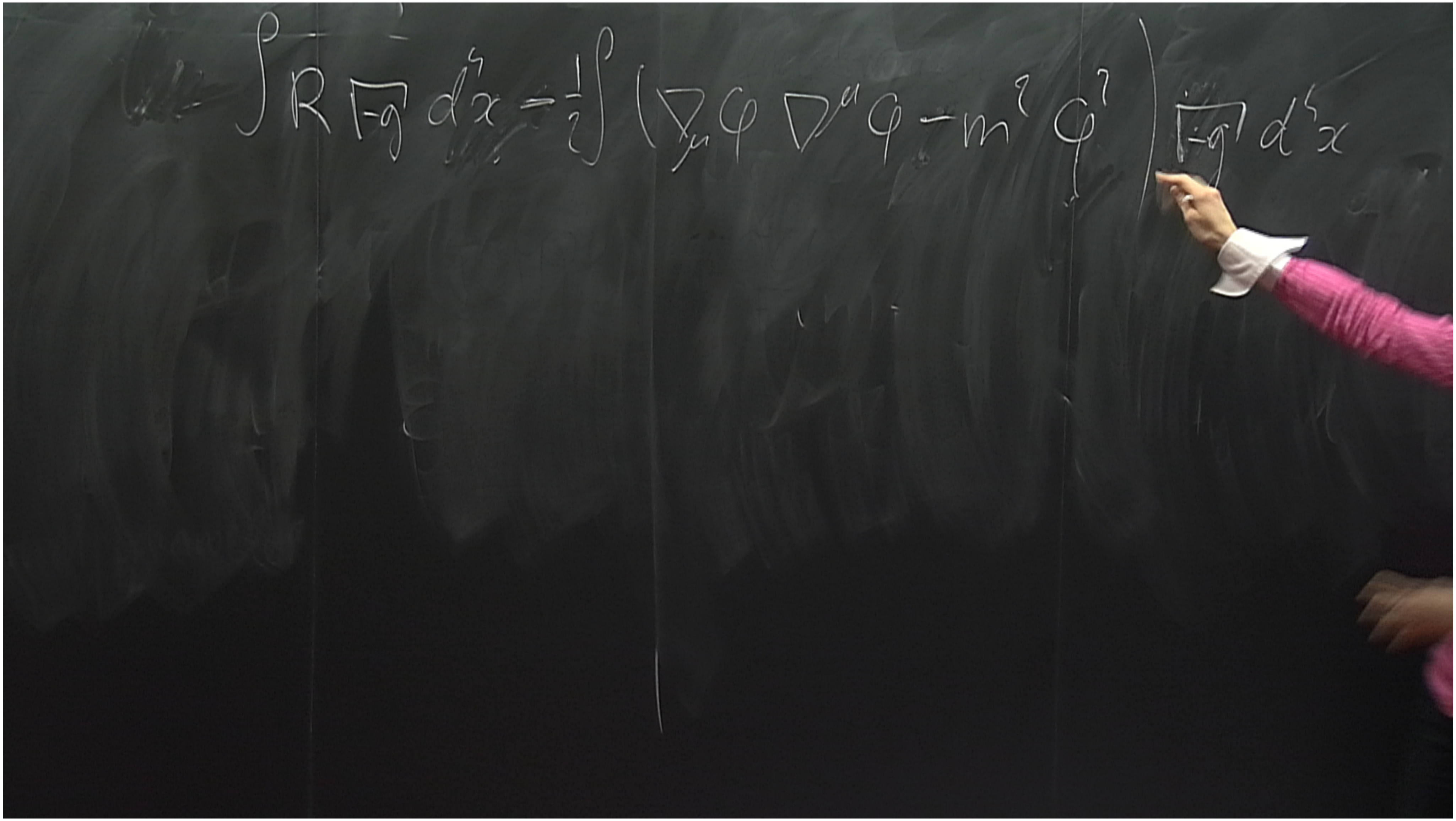

$f : M \to \mathbb{R}$
$f \equiv f \circ X_g$
Assume $f \circ X_{g_0}$ compactly supported.


$\int R\sqrt{-g}\,d^4x = \frac{1}{2}\int \left(\nabla_\mu\varphi\,\nabla^\mu\varphi - m^2\varphi^2\right)\sqrt{-g}\,d^4x$

$$\int \left(\nabla_\mu \varphi \nabla^\mu \varphi - m^2 \varphi^2\right) \sqrt{-g}\, d^4x = L_0 + L_I$$

φ, h

linearized theor

→ scalar field

→ linearized gravity.

f is t

product

$\bar{f}$ - m

Take

$\xi \in \Gamma(T$

1) Deformation quantization
for the linearized theory.

2) Epstein-Glaser renormalization
↳ in position space.

$f : M \to \mathbb{R}$

$f \equiv \mathfrak{f} \circ X_g$

Assume $\mathfrak{f} \circ X_g$

for the linearized theory.

2) Epstein-Glaser renormalization

↳ in position space, for effective theories

3) BV quantization

$\mathcal{F} =$

Assume