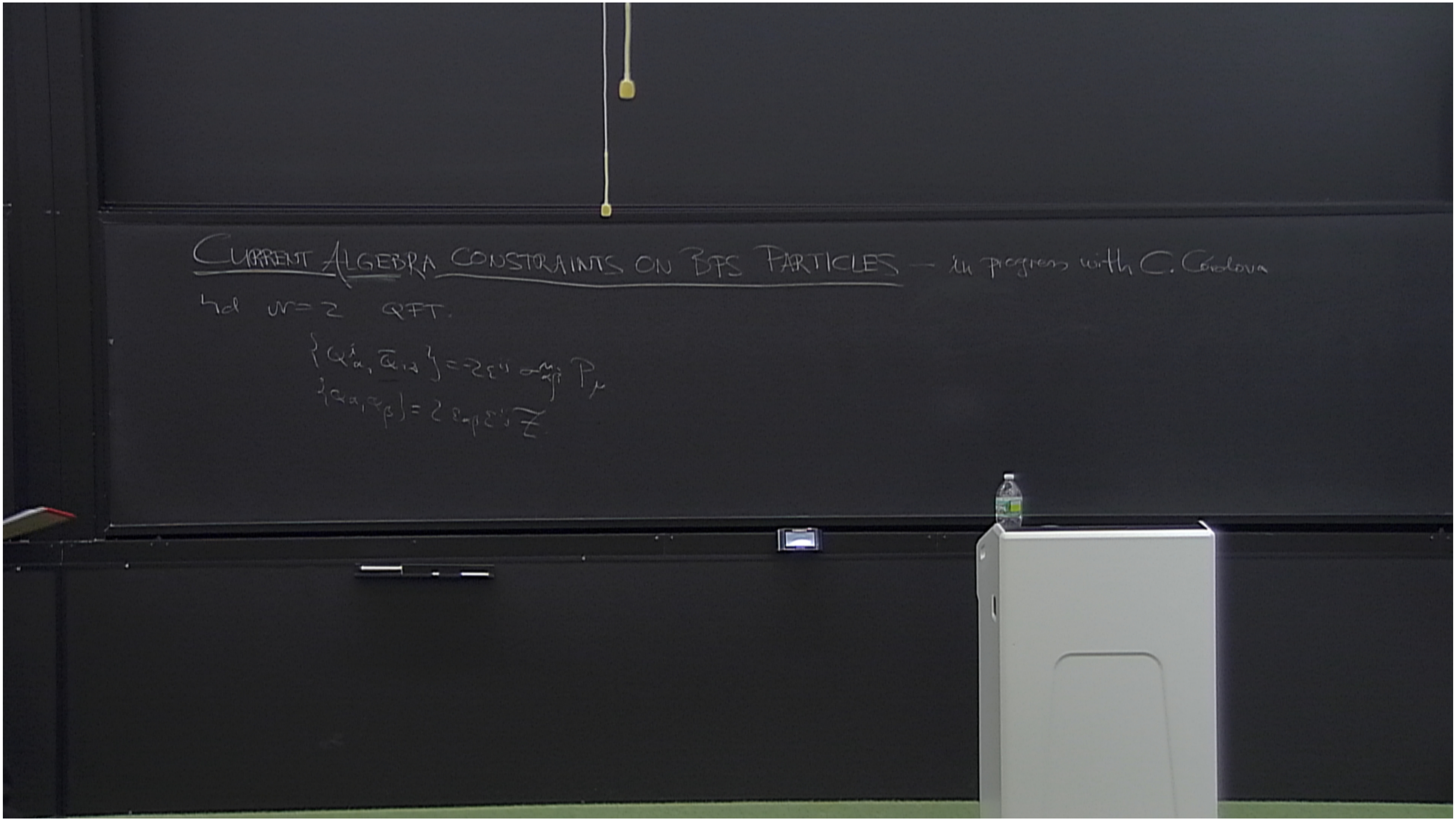
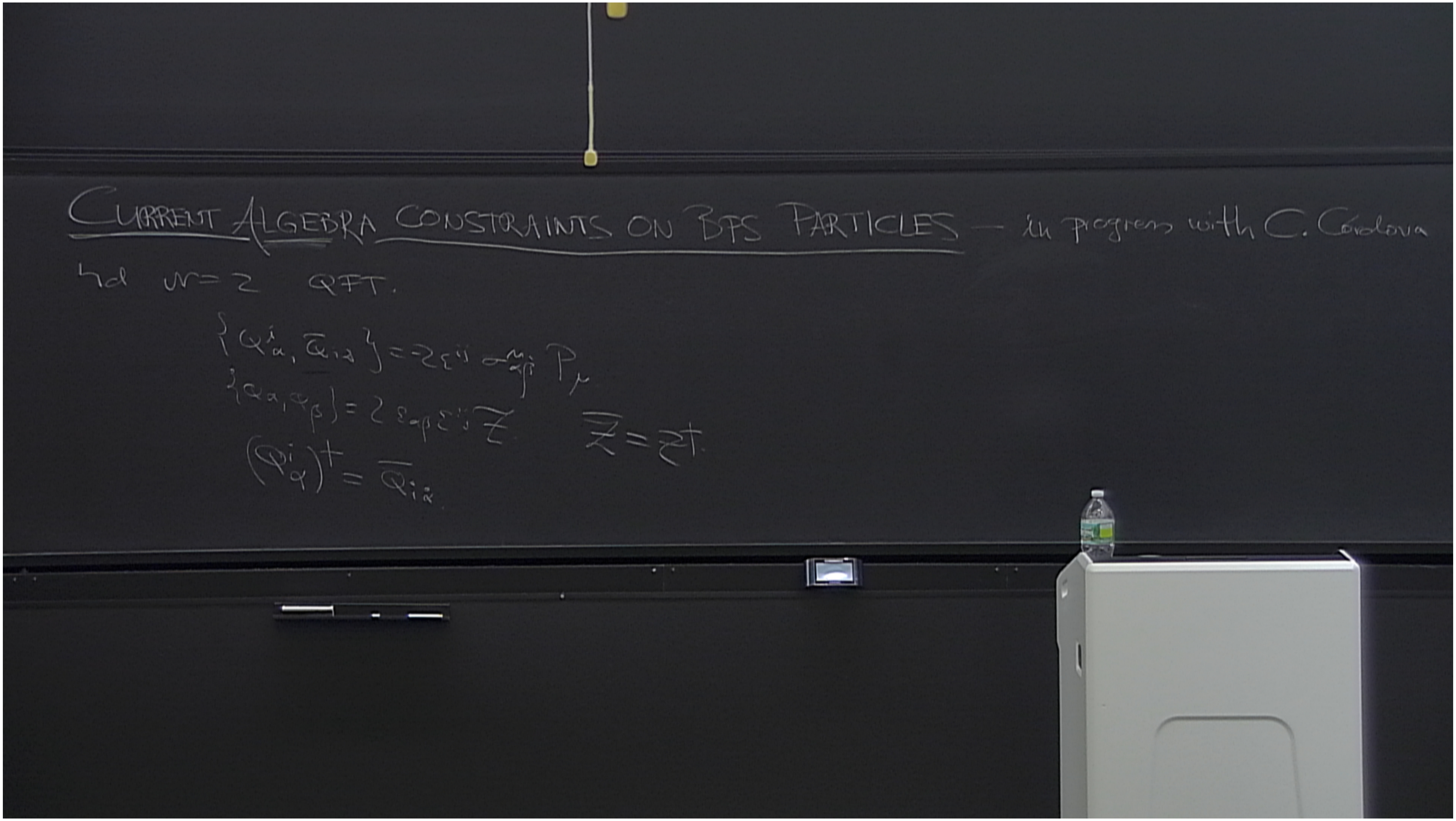
Title: Current Algebra Constraints on BPS Particles

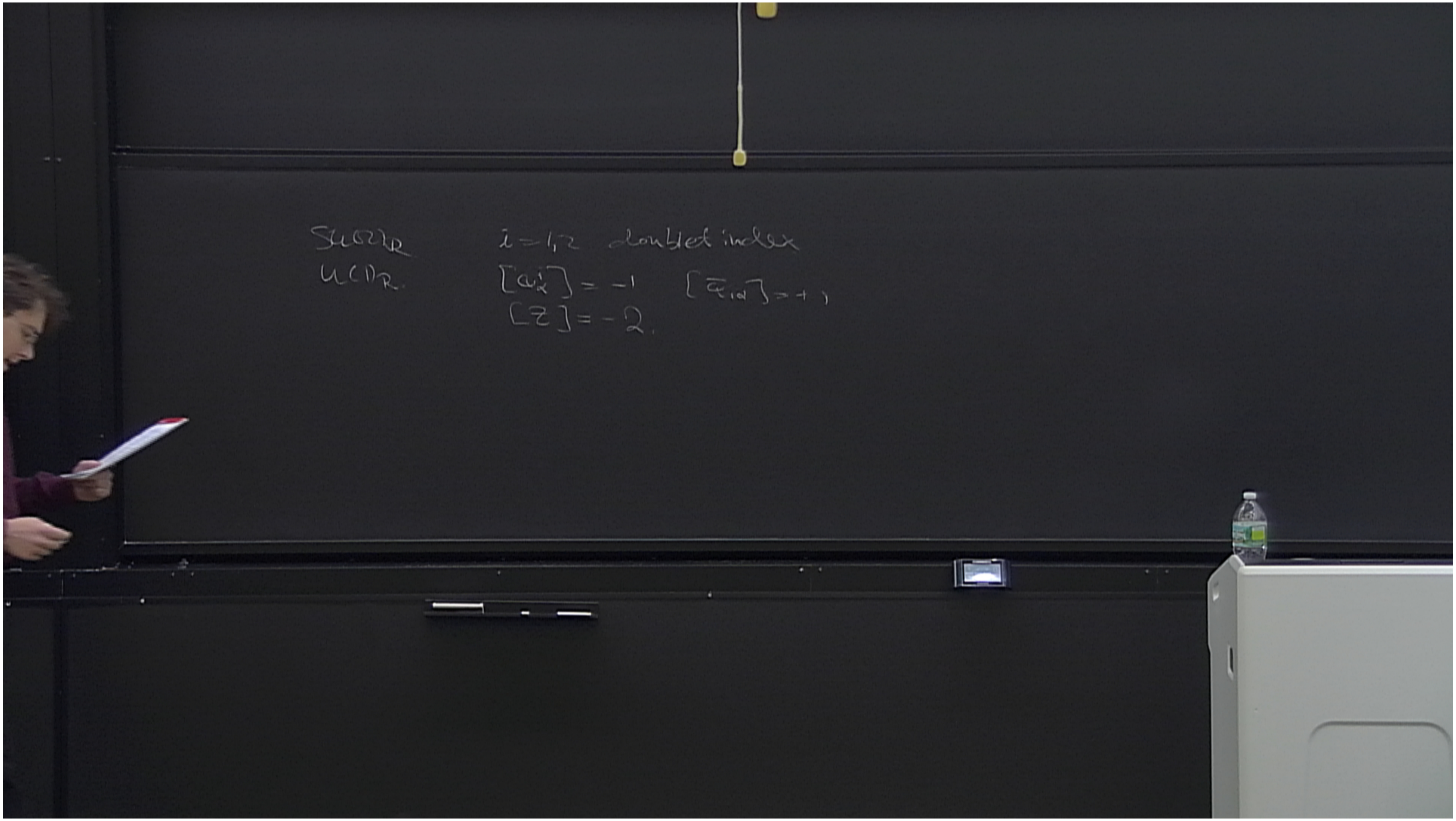
Date: Mar 29, 2016 02:00 PM

URL: http://pirsa.org/16030131

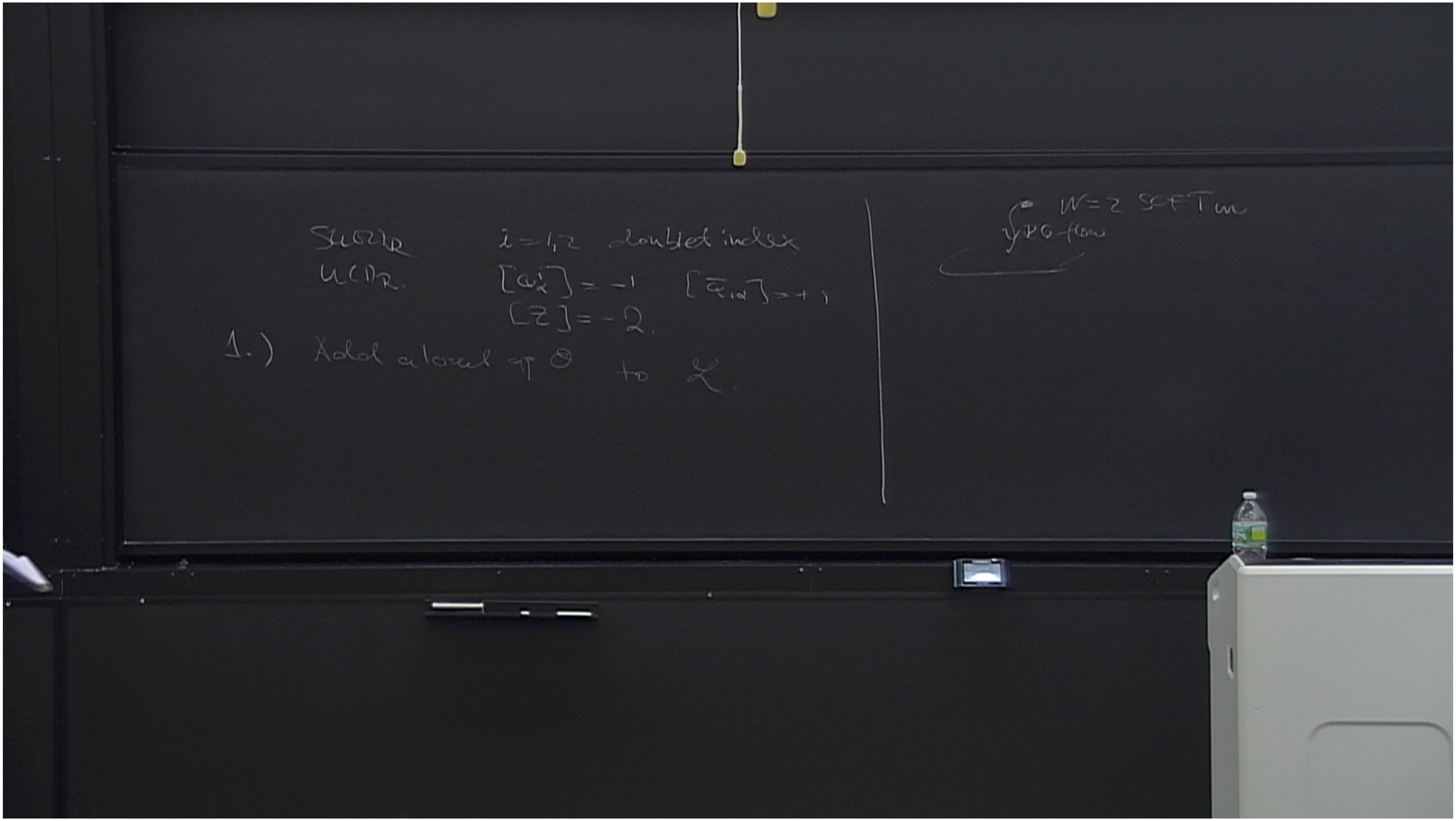
Abstract:


CURRENT ALGEBRA CONSTRAINTS ON BPS PARTICLES — in progress with C. Cordova
4d $\mathcal{N}=2$ QFT.
$\{Q^i_\alpha, \bar{Q}_{j\dot{\alpha}}\} = 2\delta^i_j \sigma^\mu_{\alpha\dot{\beta}} P_\mu$
$\{Q_\alpha, Q_\beta\} = 2\varepsilon_{\alpha\beta}\varepsilon^{ij} Z$


CURRENT ALGEBRA CONSTRAINTS ON BPS PARTICLES — in progress with C. Cordova
4d $\mathcal{N}=2$ QFT.
$\{Q^i_\alpha, \bar{Q}_{i\dot\beta}\} = 2\varepsilon^{ij}\sigma^\mu_{\alpha\dot\beta} P_\mu$
$\{Q_\alpha, Q_\beta\} = 2\varepsilon_{\alpha\beta}\varepsilon^{ij} Z$
$\bar{Z} = Z^\dagger$.
$(Q^i_\alpha)^\dagger = \bar{Q}_{i\dot\alpha}$



$SU(2)_R$ $i = 1,2$ doublet index
$U(1)_R$ $[a^i_\alpha] = -1$ $[\bar{a}_{i\dot\alpha}] = +1$
$[z] = -2$.


SU(2)_R i = 1,2 doublet index
U(1)_R [a^i_α] = -1 [ā_{iα}] = +1
[Z] = -2.
1.) Add a local op O to L.
N=2 SCFT UV
RG-flow

$SU(2)_R$ $\quad i = 1,2$ doublet index

$U(1)_R$ $\quad [Q^i_\alpha] = -1 \quad [\bar{Q}_{i\dot\alpha}] = +1$

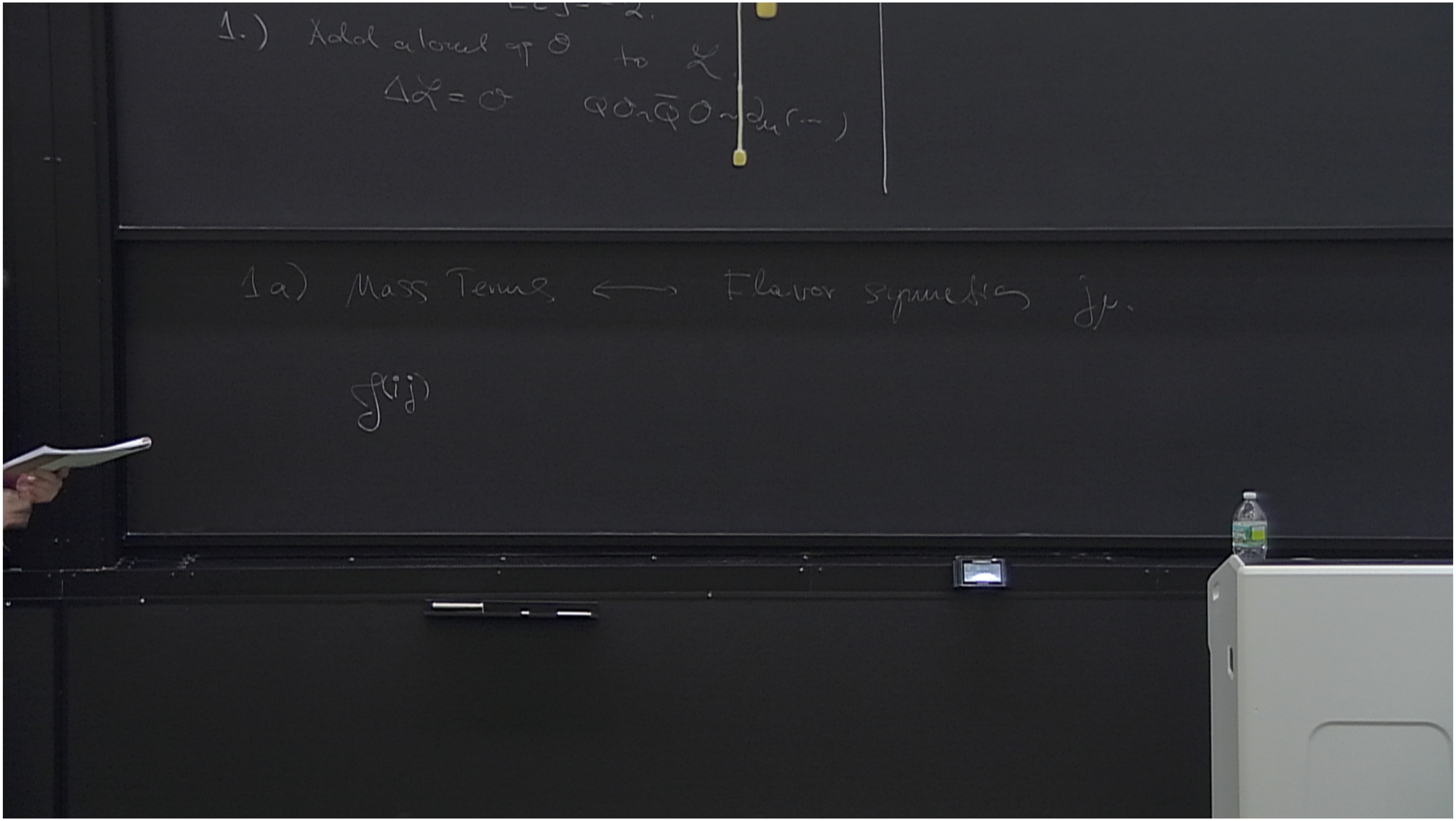
$[Z] = -2$

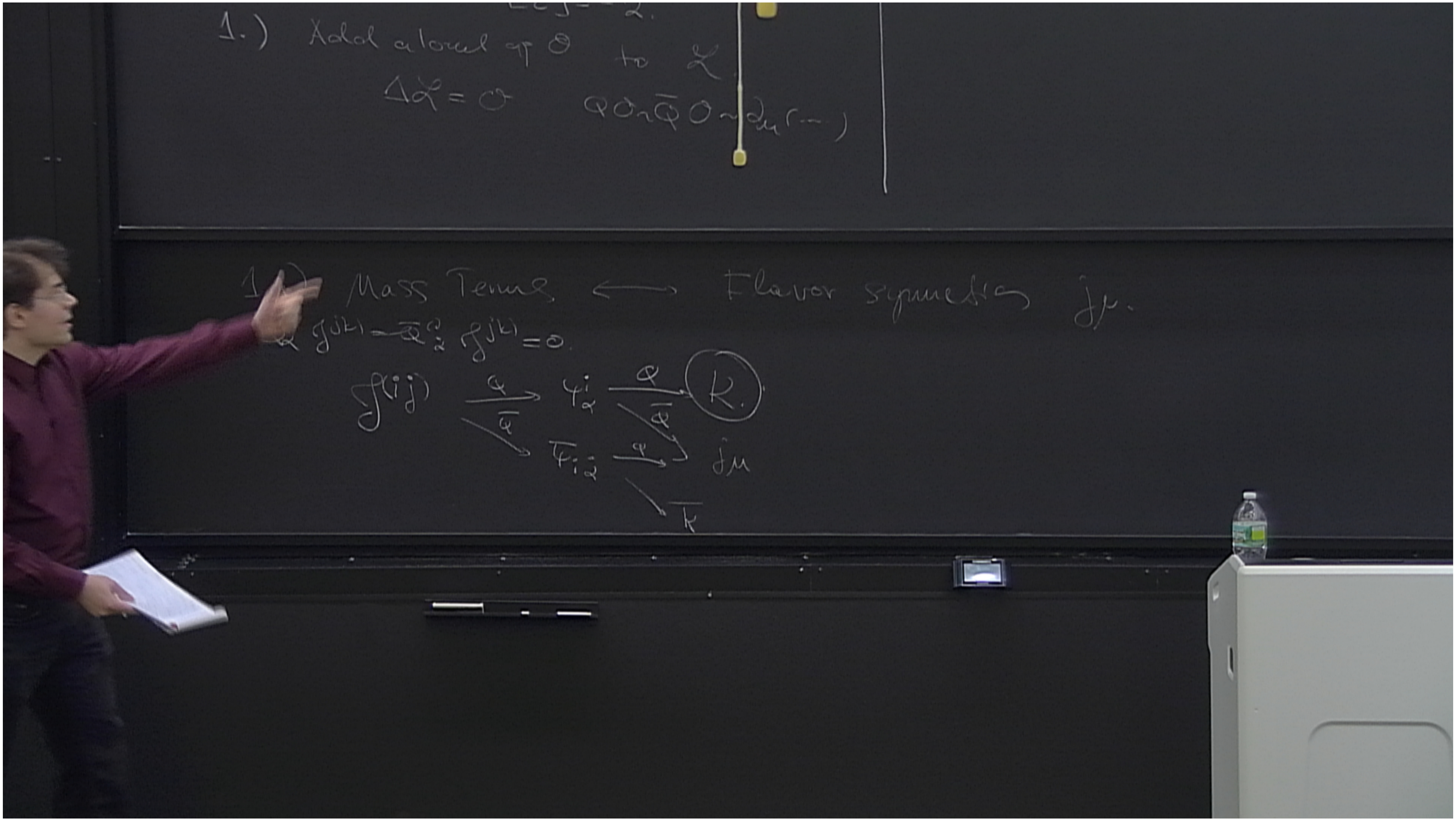
1.) Add a local op $\mathcal{O}$ to $\mathcal{L}$.

$$\Delta\mathcal{L} = \mathcal{O} \qquad Q\mathcal{O}, \bar{Q}\mathcal{O} \sim \partial_\mu(\dots)$$

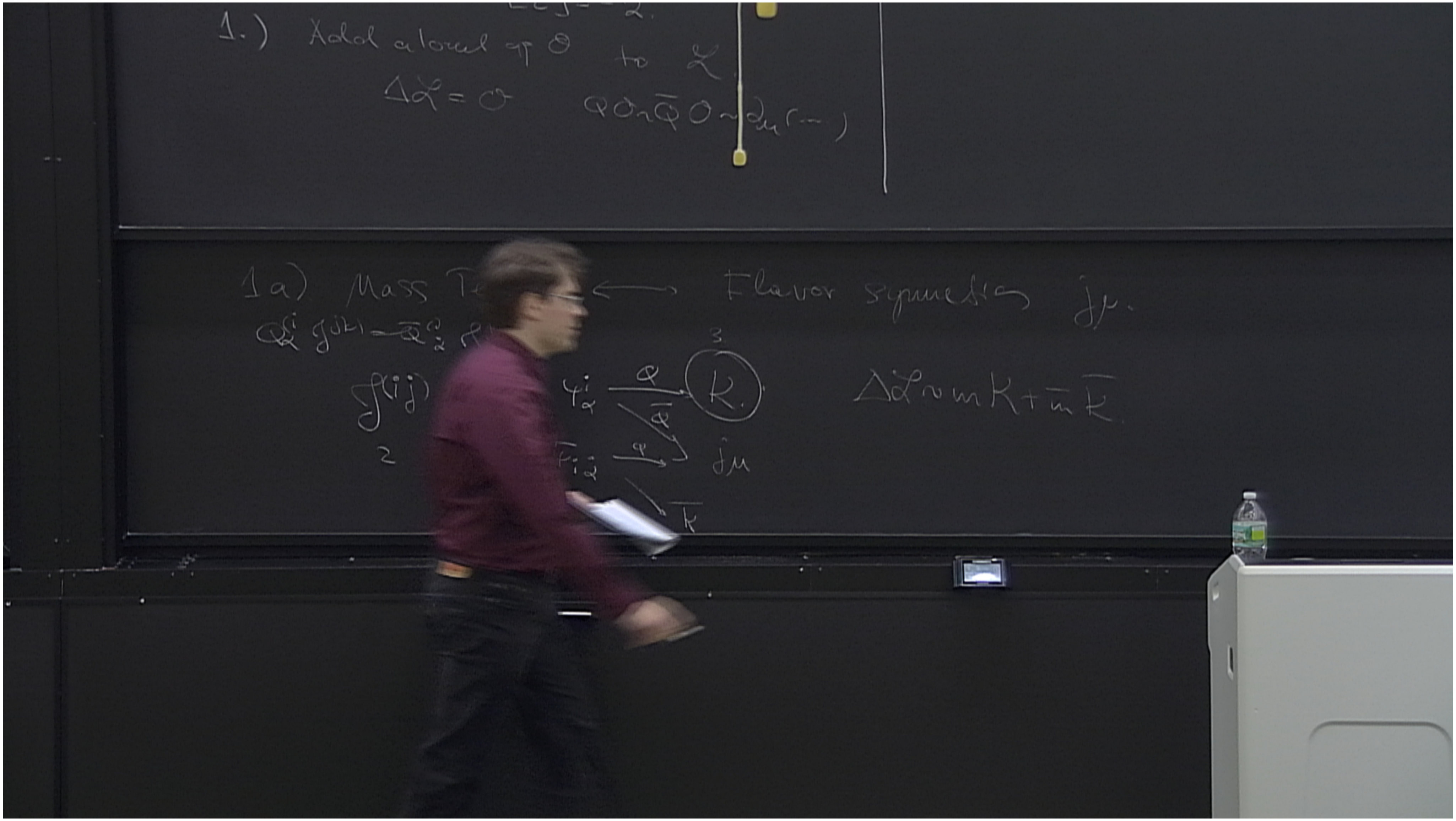
$\mathcal{N}=2$ SCFT on

RG-flow


1.) Add a local op O to L.
ΔL = O
1a) Mass Terms ⟷ Flavor symmetries jμ.



1.)
to
Mass Term
Flavor symmetry


1.) Add a local op O to L
ΔL = O
1a) Mass
Flavor symmetry jμ
ΔL ~ mK + m̄K̄
jμ

1.) Add a local op $\mathcal{O}$ to $\mathcal{L}$

$$\Delta\mathcal{L} = \mathcal{O} \qquad Q\mathcal{O} \sim \bar{Q}\mathcal{O} \sim \partial_\mu(\cdots)$$

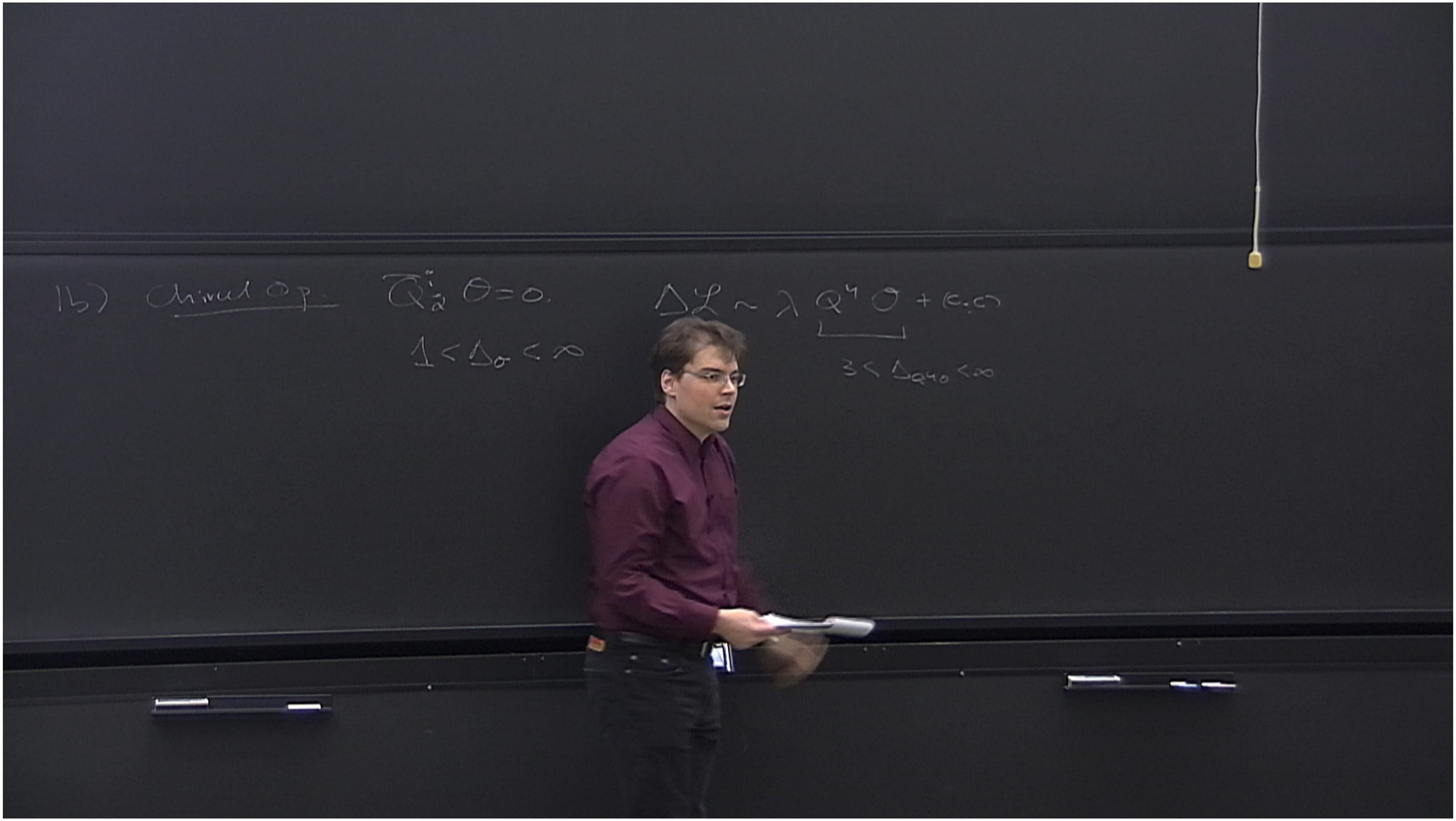
1a) Mass Terms $\longleftrightarrow$ Flavor symmetries $j_\mu$.

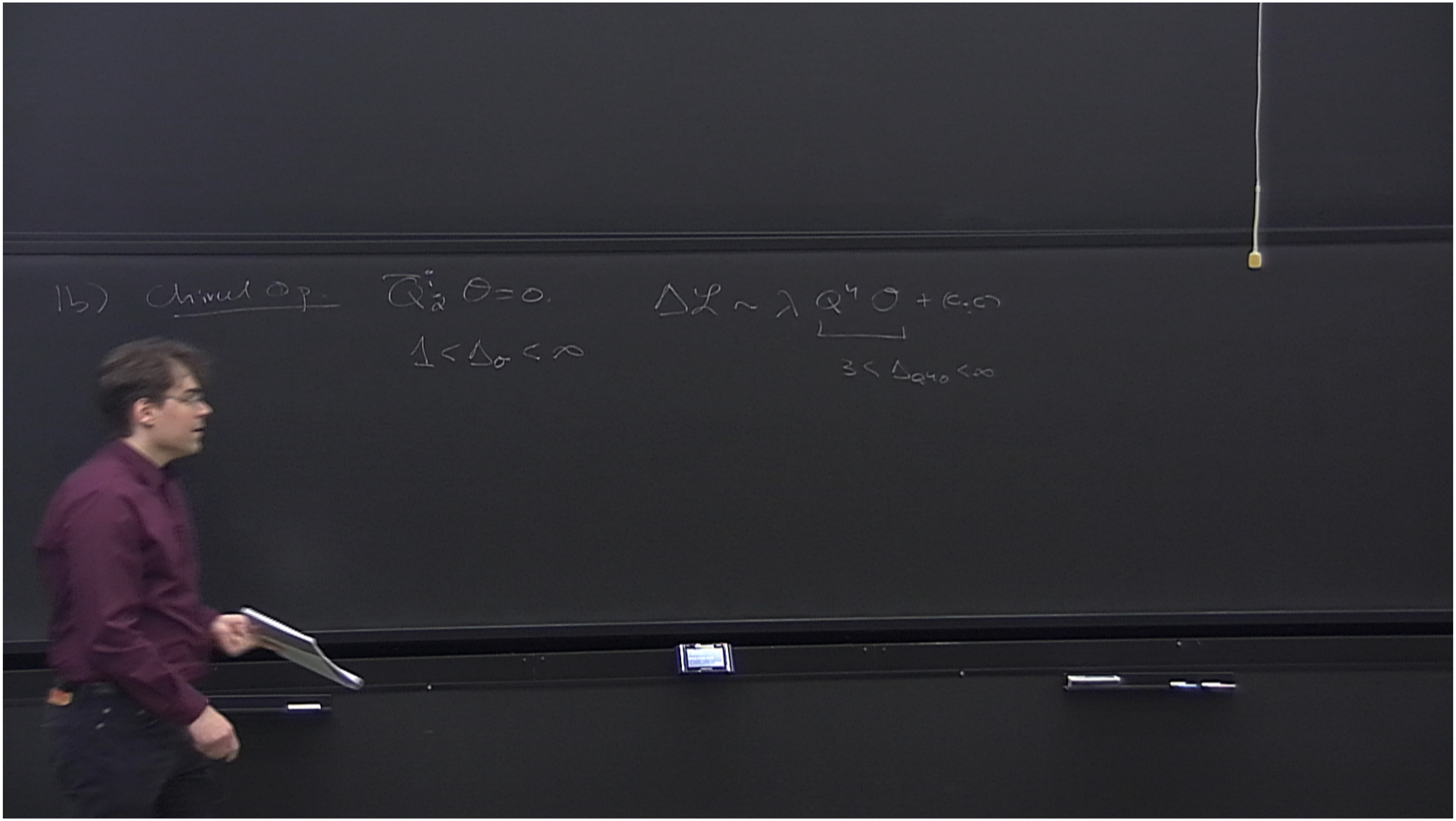
$$Q^i_\alpha\, \mathcal{J}^{(jk)} = \bar{Q}^i_{\dot\alpha}\, \mathcal{J}^{(jk)} = 0.$$

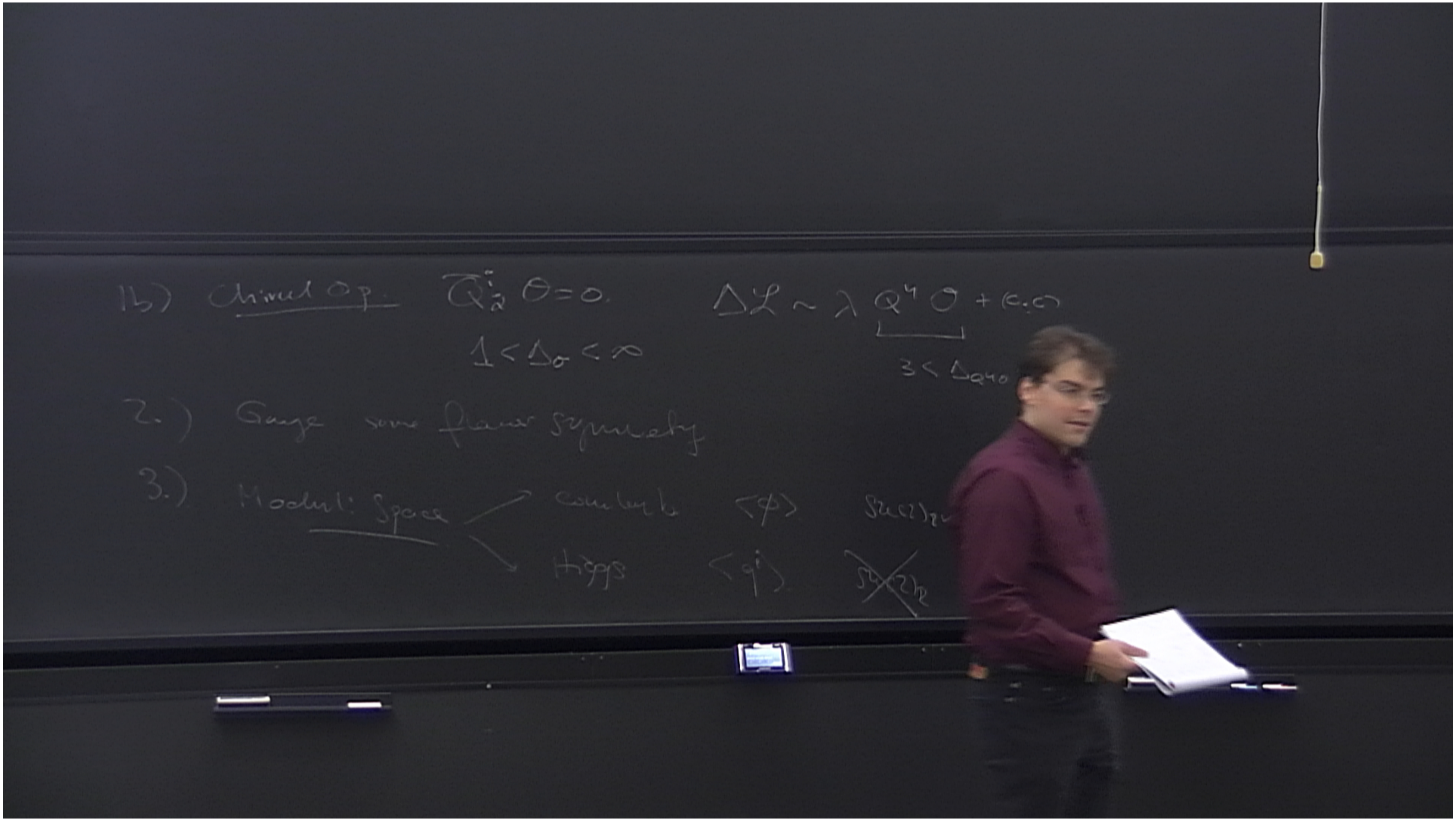
$$(\mathcal{J}^{(ij)})^\dagger = \bar{\mathcal{J}}_{(ij)}, \quad \underset{2}{\mathcal{J}^{(ij)}} \xrightarrow{Q} \psi^i_\alpha \xrightarrow{Q} \overset{3}{K}$$

$$\mathcal{J}^{(ij)} \xrightarrow{\bar{Q}} \bar{\psi}_{i\dot\alpha} \xrightarrow{Q} j_\mu, \qquad \psi^i_\alpha \xrightarrow{\bar{Q}} j_\mu, \qquad \bar{\psi}_{i\dot\alpha} \xrightarrow{\bar{Q}} \bar{K}$$

$$\Delta\mathcal{L} \sim m K + \bar{m}\bar{K} + \mathcal{O}(|m|^2)$$


1b) Chiral Op. $\bar{Q}^{i}_{\dot{\alpha}}\,\mathcal{O} = 0.$
$1 < \Delta_{\mathcal{O}} < \infty$
$\Delta\mathcal{L} \sim \lambda\, Q^4 \mathcal{O} + (c.c)$
$3 < \Delta_{Q^4\mathcal{O}} < \infty$


1b) Chiral Op: $\bar{Q}_{\dot\alpha}\,\mathcal{O}=0.$
$1<\Delta_{\mathcal{O}}<\infty$
$\Delta\mathcal{L}\sim\lambda\,Q^4\mathcal{O}+(c.c)$
$3<\Delta_{Q^4\mathcal{O}}<\infty$


1b) Chiral Op.
$\bar{Q}^i_{\dot\alpha} \mathcal{O} = 0$.
$1 < \Delta_{\mathcal{O}} < \infty$
$\Delta\mathcal{L} \sim \lambda Q^4 \mathcal{O} + (c.c)$
$3 < \Delta_{Q^4\mathcal{O}}$
2.) Gauge some flavor symmetry
3.) Moduli Space
coulomb $\langle\phi\rangle$
Higgs $\langle q^i \rangle$

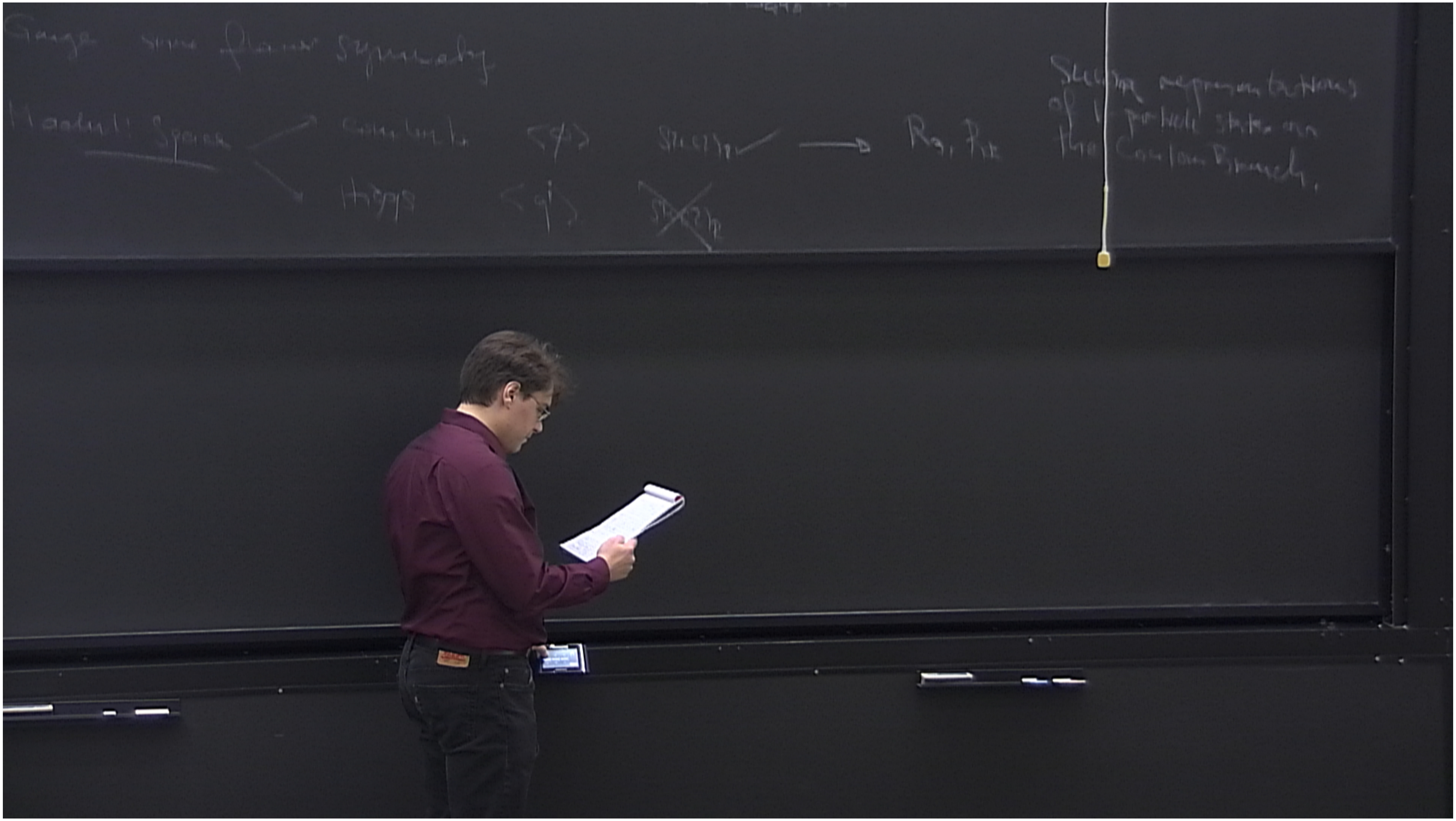
) Chiral Op. $\bar{Q}^{i}_{\dot\alpha}\, \mathcal{O} = 0.$ $\Delta\mathcal{L} \sim \lambda\, \underbrace{Q^4 \mathcal{O}} + (\text{c.c})$

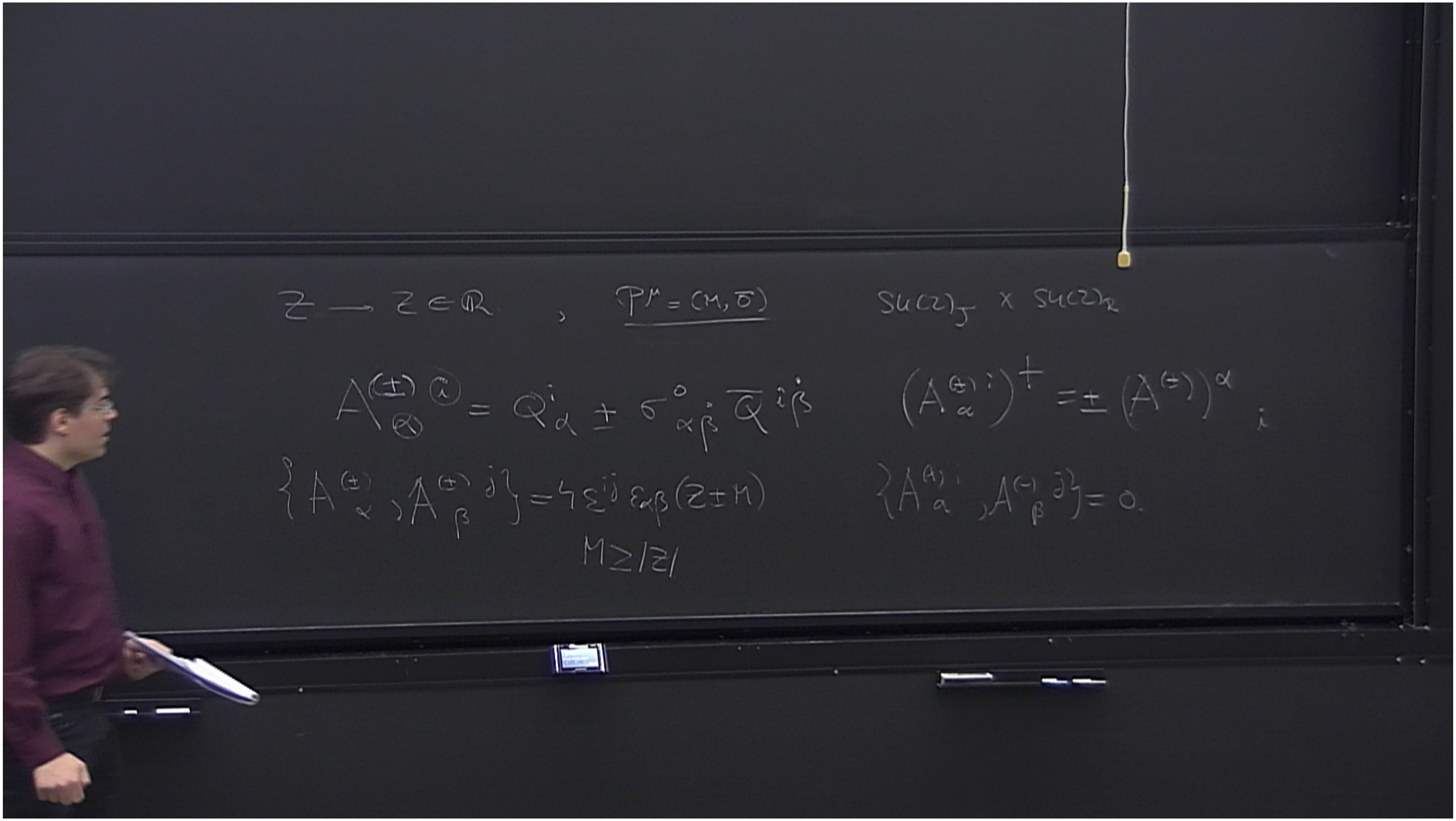
$1 < \Delta_{\mathcal{O}} < \infty$ $3 < \Delta_{Q^4\mathcal{O}} < \infty$

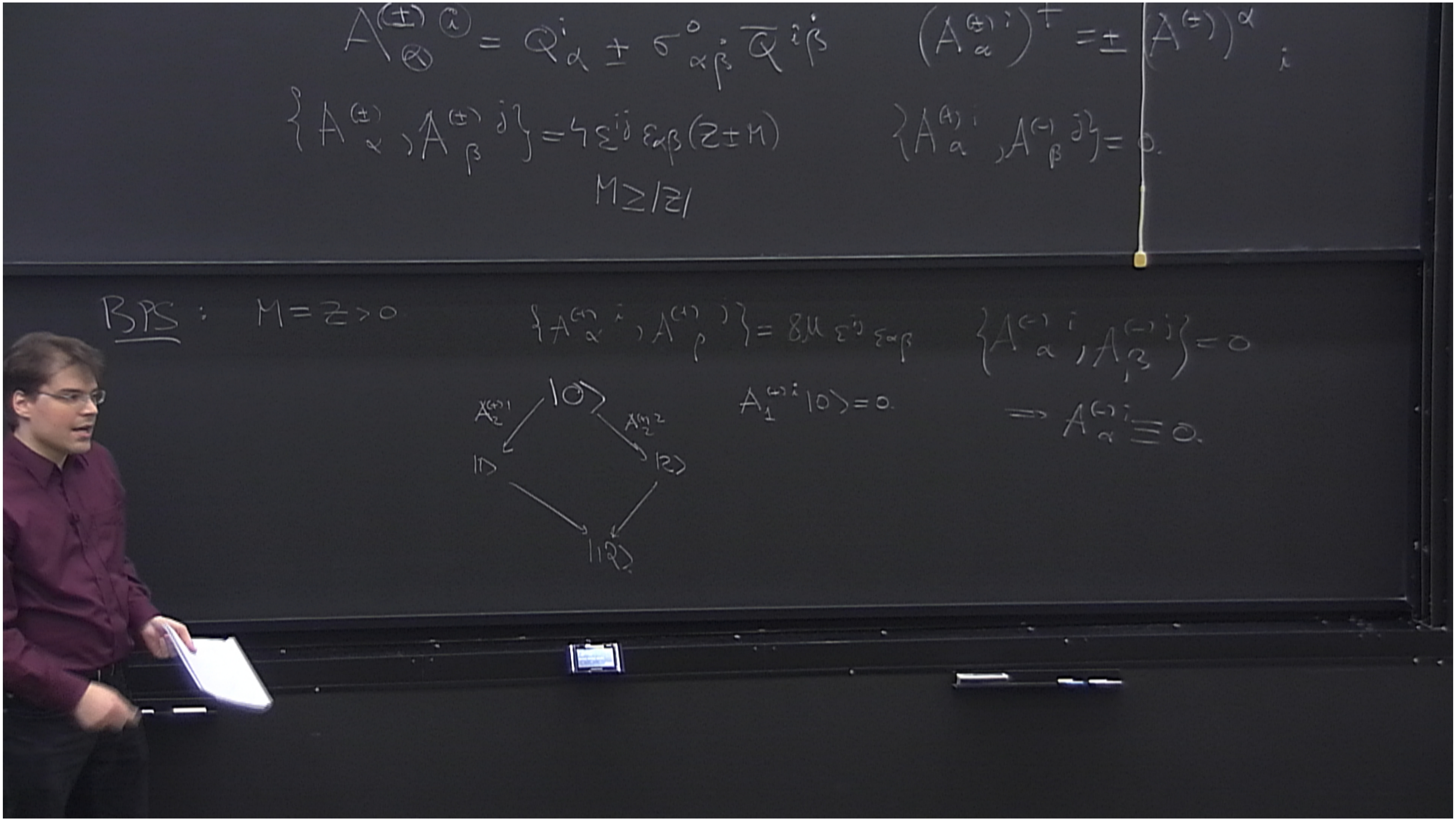
) Gauge ... symmetry

.) Modu... → coulomb. $\langle \phi \rangle$ $SU(2)_R$ ✓ ⟶ $R_3, R_t$

→ Higgs $\langle q^i \rangle$ $\cancel{SU(2)_R}$




Z → Z ∈ ℝ , P^μ = (M, 0̄)
SU(2)_J × SU(2)_R
A^{(±)i}_α = Q^i_α ± σ^0_{αβ̇} Q̄^{iβ̇}
(A^{(±)i}_α)^† = ± (A^{(±)})^α_i
{A^{(±)i}_α , A^{(±)j}_β} = 4 ε^{ij} ε_{αβ} (Z ± M)
{A^{(+)i}_α , A^{(−)j}_β} = 0.
M ≥ |Z|



$A^{(\pm)}{}^{i}_{\alpha} = Q^{i}_{\alpha} \pm \sigma^{0}_{\alpha\dot\beta}\bar{Q}^{i\dot\beta}$
$(A^{(\pm)i}_{\alpha})^{\dagger} = \pm (A^{(\pm)})^{\alpha}{}_{i}$
$\{A^{(\pm)}{}^{i}_{\alpha}, A^{(\pm)}{}^{j}_{\beta}\} = 4\varepsilon^{ij}\varepsilon_{\alpha\beta}(Z \pm M)$
$\{A^{(+)i}_{\alpha}, A^{(-)j}_{\beta}\} = 0.$
$M \geq |Z|$
BPS: $M = Z > 0$
$\{A^{(+)i}_{\alpha}, A^{(+)j}_{\beta}\} = 8M\varepsilon^{ij}\varepsilon_{\alpha\beta}$
$\{A^{(-)i}_{\alpha}, A^{(-)j}_{\beta}\} = 0$
$|0\rangle$
$A^{(+)1}_{2}$
$A^{(+)2}_{2}$
$|1\rangle$
$|2\rangle$
$|12\rangle$
$A^{(+)i}_{1}|0\rangle = 0.$
$\Rightarrow A^{(-)i}_{\alpha} \equiv 0.$

$$\{A^{(\pm)\,i}_{\alpha}, A^{(\pm)\,j}_{\beta}\} = 4\,\varepsilon^{ij}\,\varepsilon_{\alpha\beta}\,(Z \pm M) \qquad \{A^{(+)\,i}_{\alpha}, A^{(-)\,j}_{\beta}\} = 0.$$

$$M \geq |Z|$$

<u>BPS</u>: $M = Z > 0$

$$\{A^{(+)\,i}_{\alpha}, A^{(+)\,j}_{\beta}\} = 8M\,\varepsilon^{ij}\,\varepsilon_{\alpha\beta} \qquad \{A^{(-)\,i}_{\alpha}, A^{(-)\,j}_{\beta}\} = 0$$

<u>free hyper</u>

$|1\rangle \longleftrightarrow q^1$

$|2\rangle \longleftrightarrow q^2$

$|0\rangle \quad |12\rangle \sim \psi_\pm$

$$\begin{tikzcd} & {|0\rangle} \arrow[dl, "A^{(+)1}_2"'] \arrow[dr, "A^{(+)2}_2"] & \\ {|1\rangle} \arrow[dr] & & {|2\rangle} \arrow[dl] \\ & {|12\rangle} & \end{tikzcd}$$

$A^{(-)\,i}_1 |0\rangle = 0.$

$\Longrightarrow A^{(-)\,i}_{\alpha} \equiv 0.$

1.) Add a local op $\mathcal{O}$ to $\mathcal{L}$

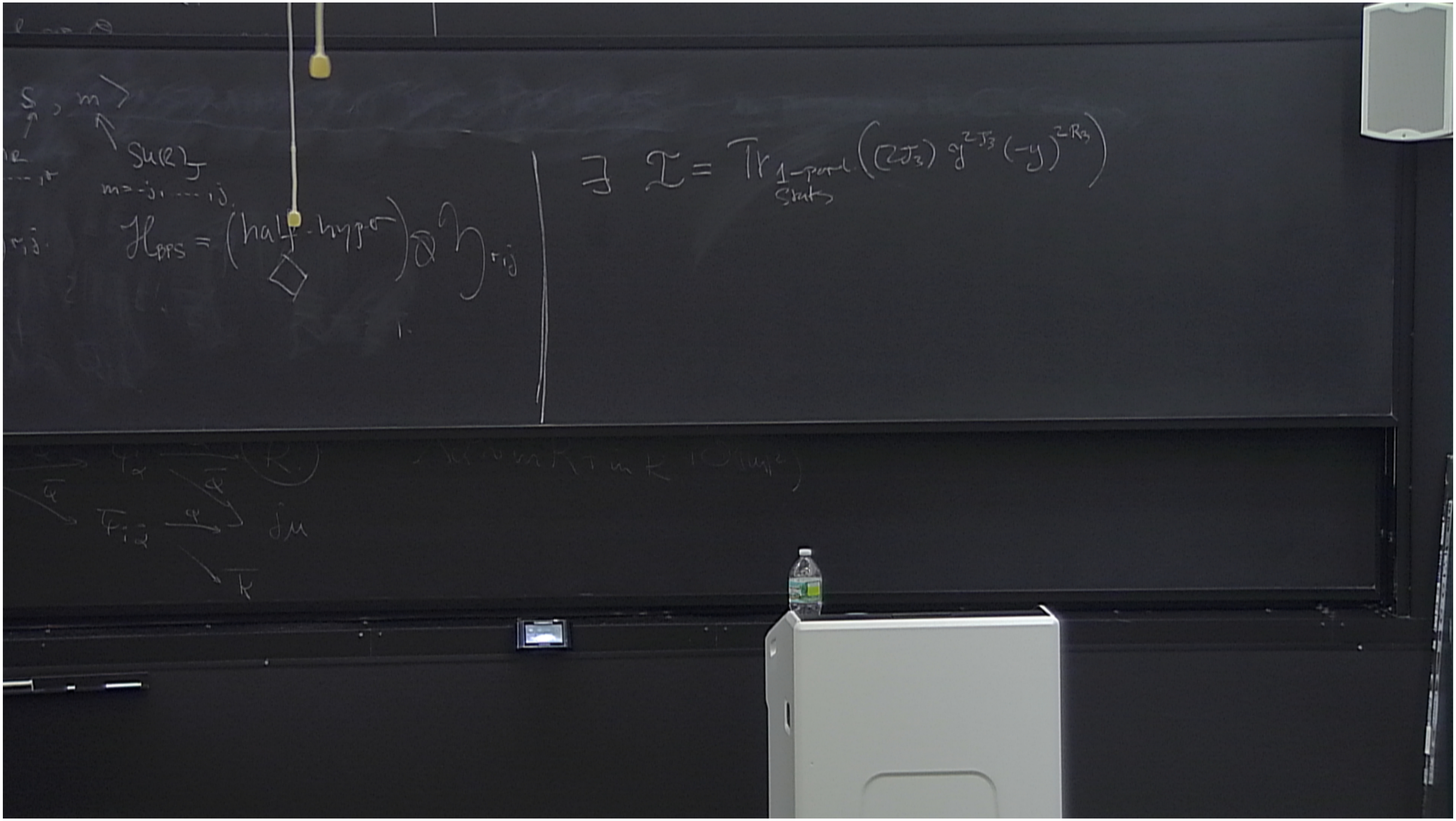
$$\Delta\mathcal{L} = \mathcal{O} \qquad Q\mathcal{O} \sim \bar{Q}\mathcal{O} \sim \partial_\mu(\cdots)$$

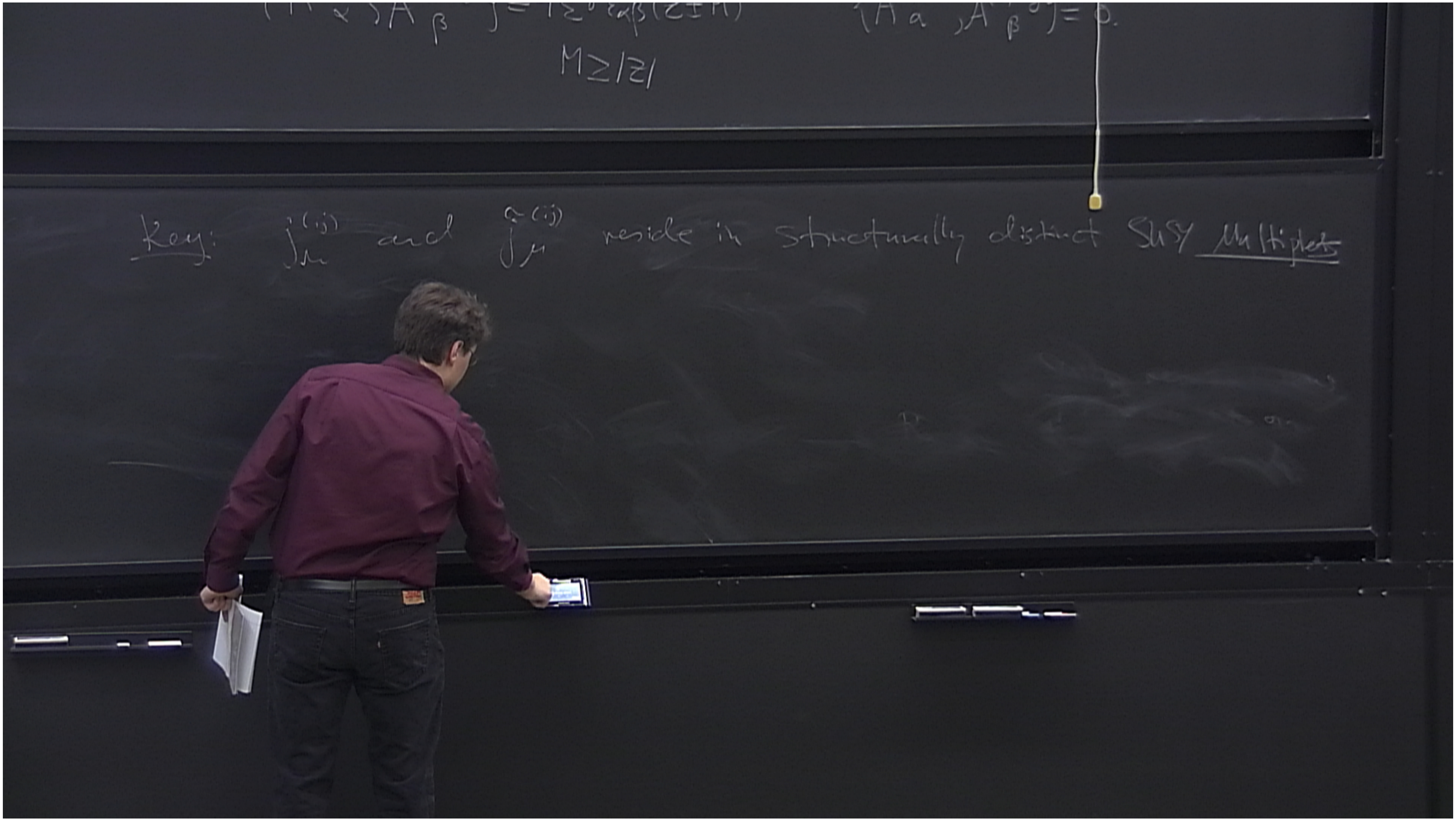
$$|0\rangle \longrightarrow |0; s, m\rangle$$

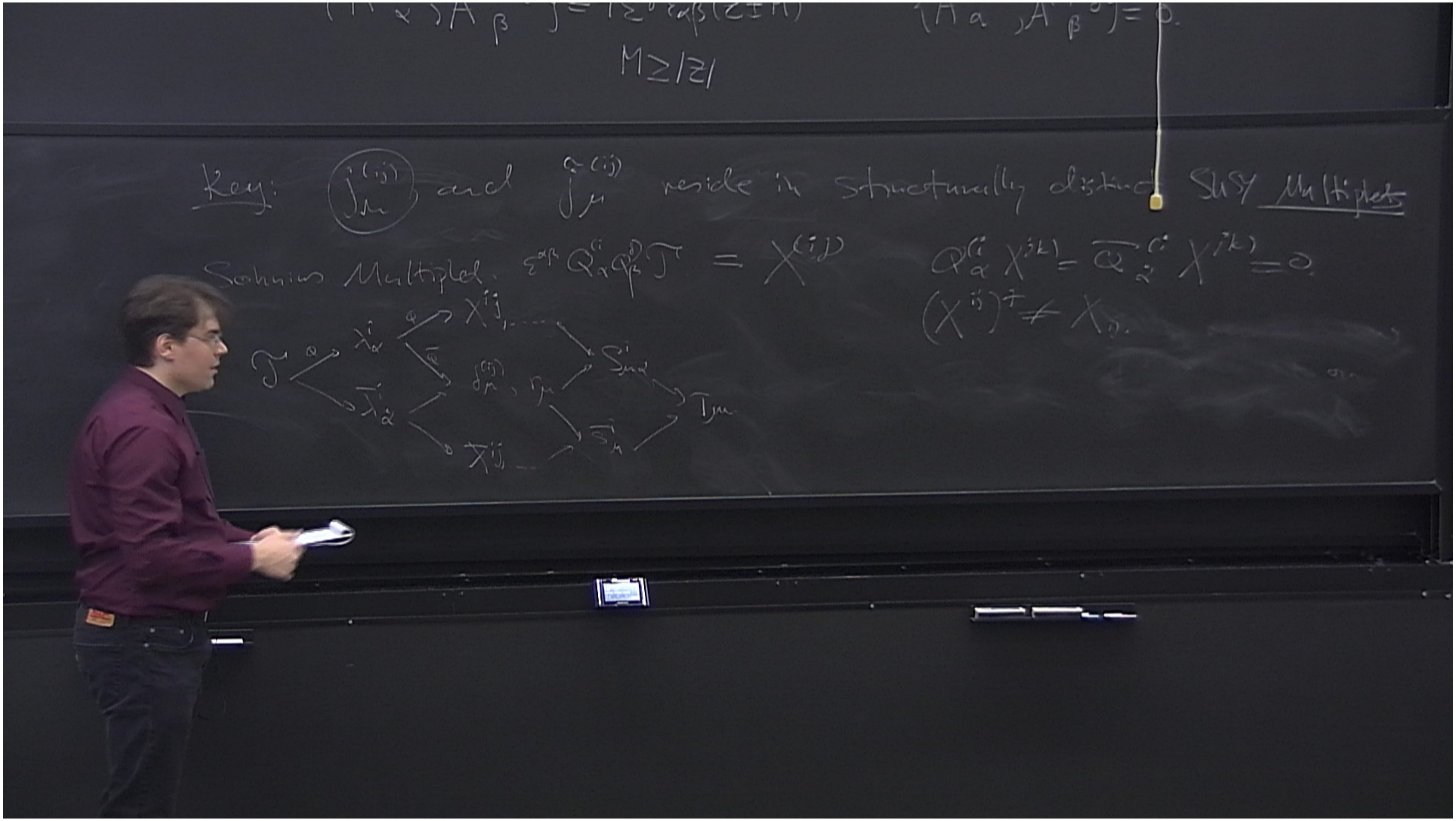
$s$: $SU(2)_R$, $s=-r,\dots,r$

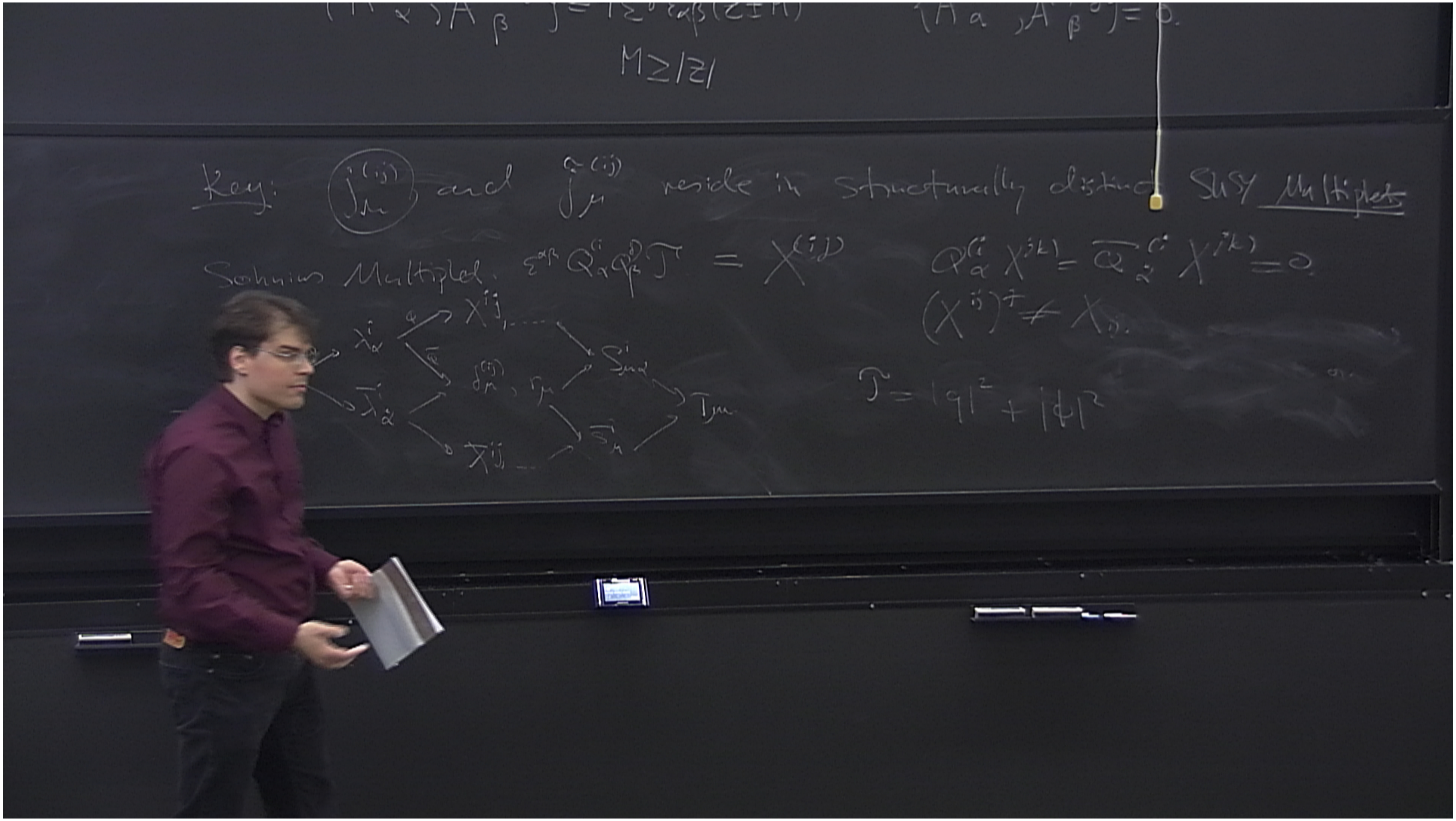
$m$: $SU(2)_J$, $m=-j,\dots,j$

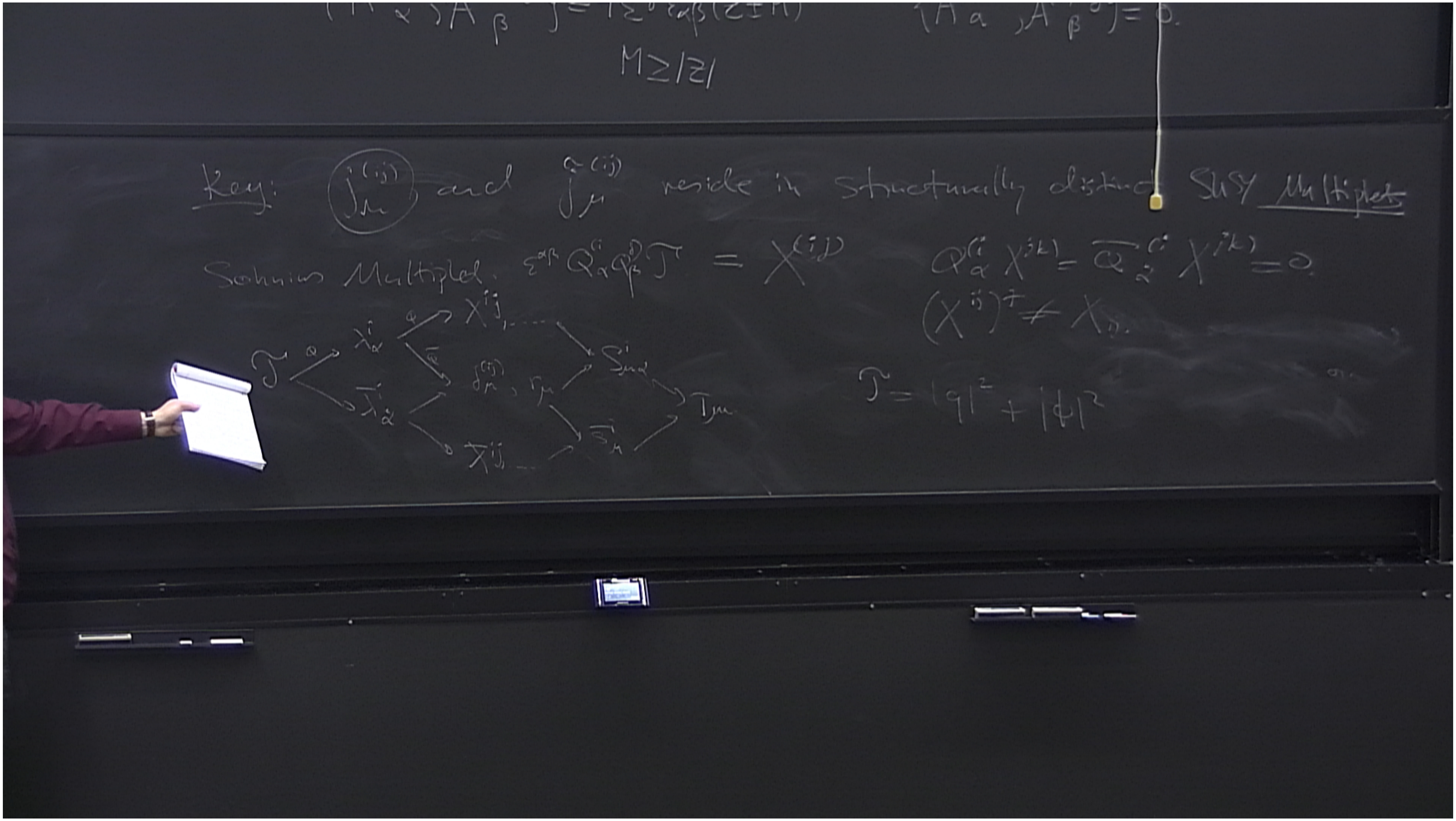
$\mathcal{D}_{r,j}$. $\qquad \mathcal{H}_{BPS} = (\text{half-hyper}) \otimes \mathcal{D}_{r,j}$

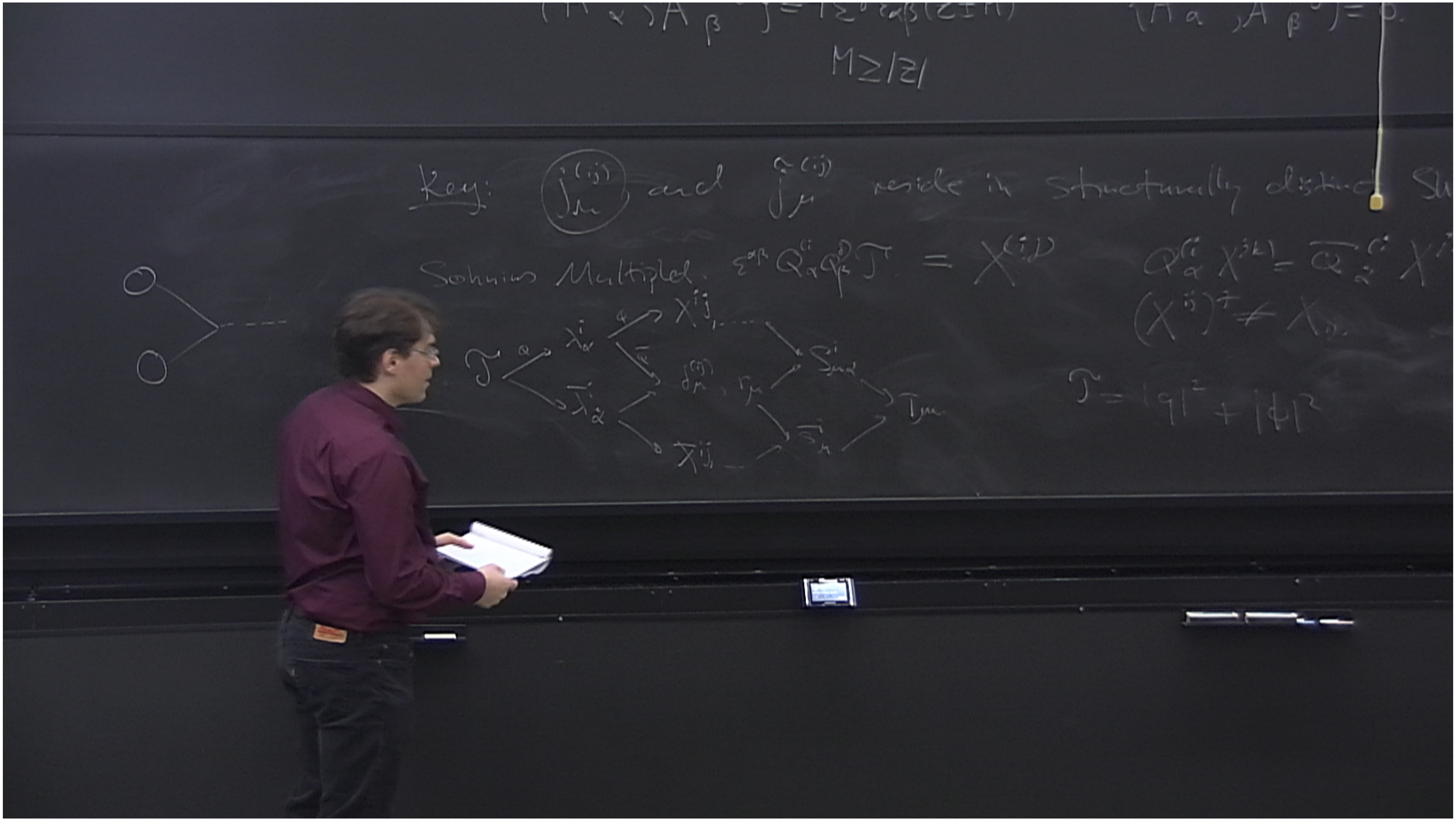

SU(2)_J
m = -j, ..., j
H_BPS = (half-hyper) ⊗ h_{r,j}
∃ I = Tr_{1-part. states} ((2J_3) y^{2J_3} (-y)^{2R_3})


M≥|Z|
Key: $j_\mu^{(ij)}$ and $\tilde{j}_\mu^{(ij)}$ reside in structurally distinct SUSY Multiplets


M ≥ |Z|
Key: j_μ^(ij) and j̃_μ^(ij) reside in structurally distinct SUSY Multiplets
Sohnius Multiplet. ε^αβ Q^(i)_α Q^j)_β J = X^(ij)
Q^(i_α X^jk) = Q̄^(i_α̇ X^jk) = 0.
(X^ij)^† ≠ X_ij.
J
λ^i_α
λ̄^i_α̇
X^ij
X̄^ij
j_μ^(ij), r_μ
S^i_μα
S̄^i_μ
T_μν


M ≥ |Z|
Key: and reside in structurally distinct SUSY Multiplets
Sohnius Multiplet.


M ≥ |Z|
Key: j_μ^(ij) and j̃_μ^(ij) reside in structurally distinct SUSY Multiplets
Sohnius Multiplet. ε^αβ Q_α^i Q_β^j J = X^(ij)
Q_α^(i X^jk) = Q̄_α̇^(i X^jk) = 0.
(X^ij)^† ≠ X_ij.
J = |q|^2 + |φ|^2


Key:
reside in structurally distinct
Sohnius Multiplet