

Title: TBA

Date: Mar 10, 2016 03:30 PM

URL: <http://pirsa.org/16030126>

Abstract:

# Boundary-bulk relation in Top. Orders. (TO)

$n+1$ D Top. Order. = local Top. order  
= TO def. on an open  $\overbrace{D^n}$ -disk (space.)  
 $D^n \times \mathbb{R}$  - spacetime



# Boundary-bulk relation in Top. Orders. (TO)

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= TO def on an open  $\overbrace{D^n}$ -disk (space)

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integrate  
(Global invariant: Factorization homology)  
GSD.

an open  $n$ -disk (space)  
 $D^n \times \mathbb{R}$  - spacetime



2. If all TO can be realized by lattice models in the same dimension

Toric code  $\leftarrow$  gapped boundary  
 $e, m, f$

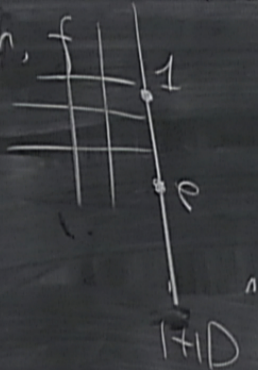


Chen-Gu-Wen there is not non-trivial TO in 1+1D (anomaly-free)



2. Not all TO can be realized by lattice models in the same dimension

Toric code  $\times$  gapped boundary  
i.e. m, f



Often there is non-trivial TO in 1+1D (anomaly-free)



Definition A TO is called anomalous if it can not be realized as a lattice model of the same dim.

A ..... anomaly-free (closed) if otherwise

3.



A ..... - - - anomaly-free (closed) if otherwise

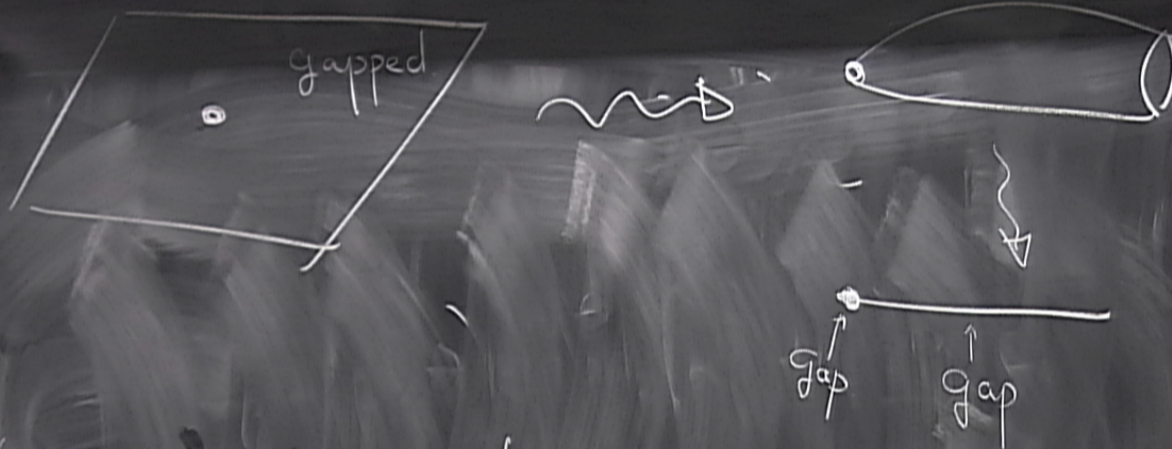
3. Any TO. can be realized as a defect in a lattice model of gapped.

higher (but still-finite) dimension.



$(2\pi)$   $p$   $-3$





A (potentially anomalous)  $TO$  can be realized as a gapped body of a lattice of one-dim higher.

CAUTION

PLEASE DO NOT WRITE ON THIS BOARD.  
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BE CAREFUL NOT TO TOUCH THE BOARD.  
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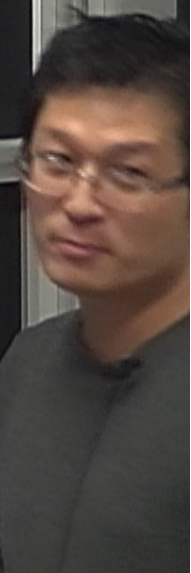
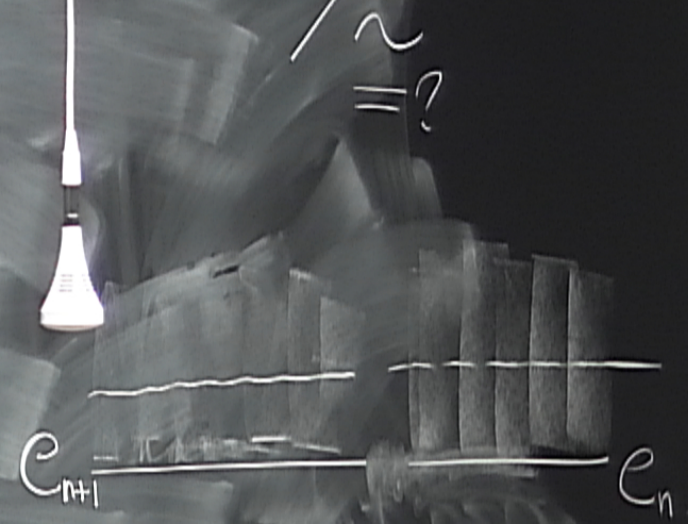
(1) a  $U_{n+1}$  should be def. as lattice models

Two lattice models are equivalent.

if  $\exists$  a nbh. of bdy such that.

we can deform one to the other

w/o closing the gap





4

(1) a  $TO_{n+1}$  should be def. as lattice models  $\sim$

Two lattice models are equivalent.

if  $\exists$  a nbh. of bdy such that  
we can "deform" one to the other

w/o closing the gap

$C_{n+1}$



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$n+1 \rightarrow \text{spa}$

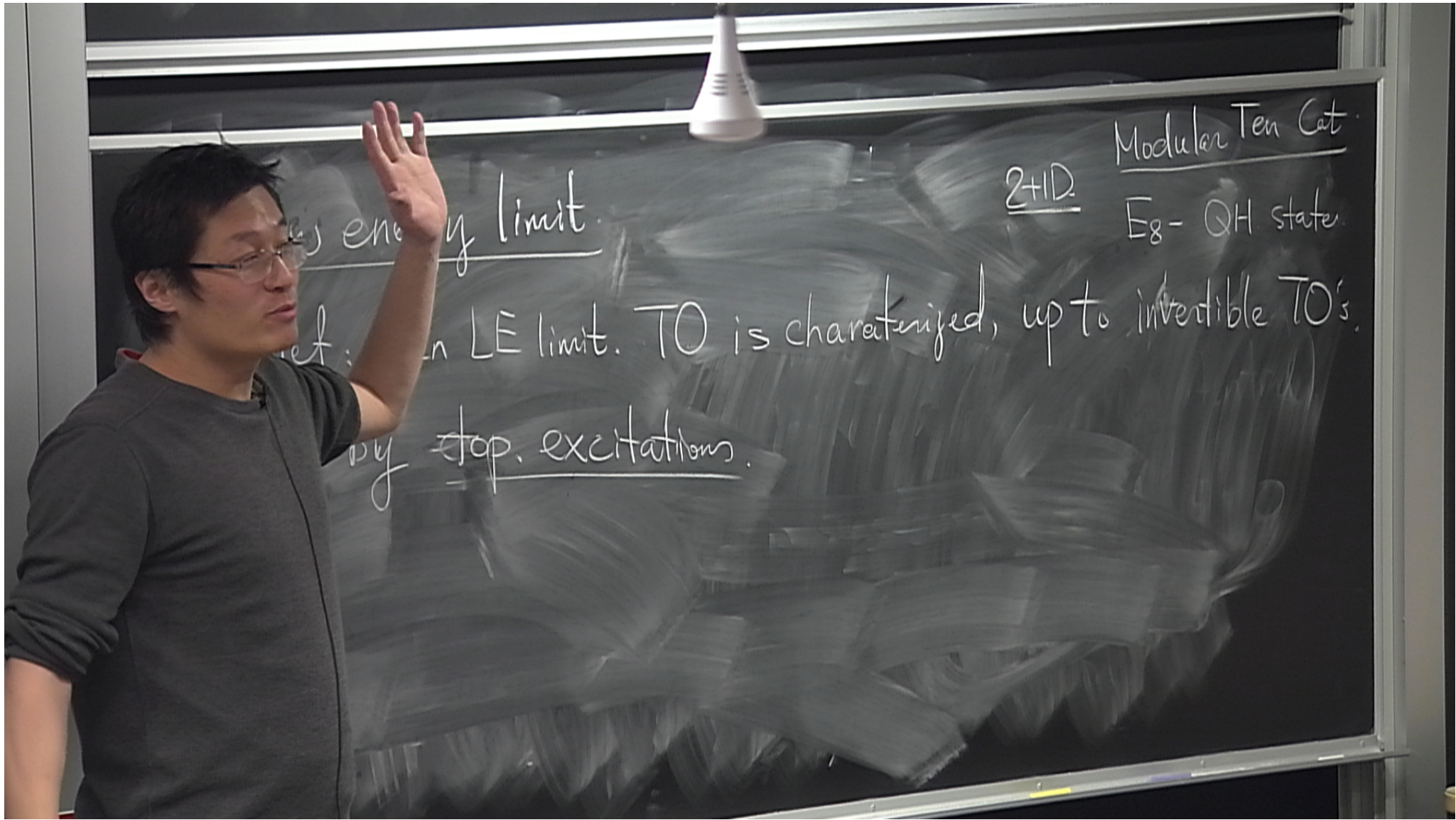
$n+2 \text{D st.}$

$C_{n+1}$

$\uparrow$   
 $TD_{n+1}$

$C_{n+1}$







(2) low energy limit

2+1D

Modular Tensor Cat.

Eg - QH state

Belief: In LE limit, TO is characterized, up to invertible TO's

by top. excitations order parameter.

fusion, braiding (spin)



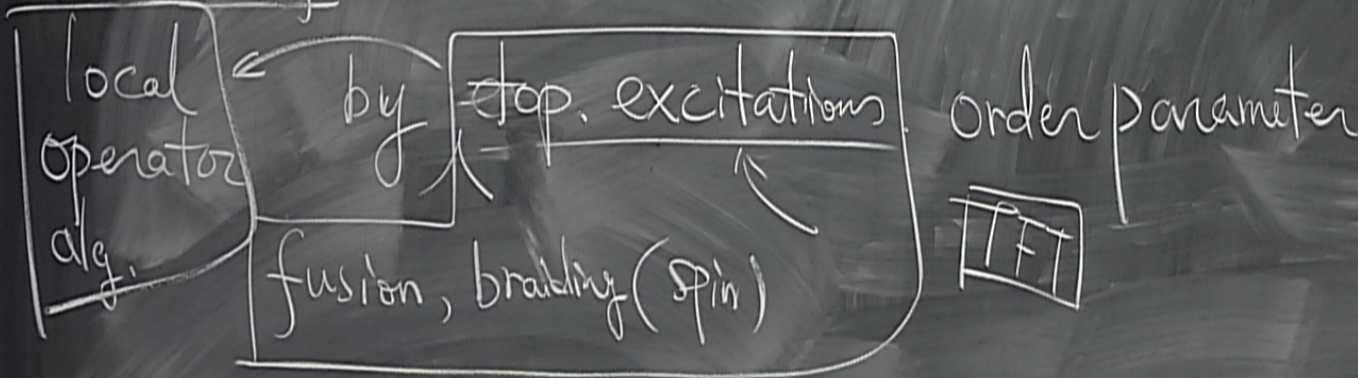
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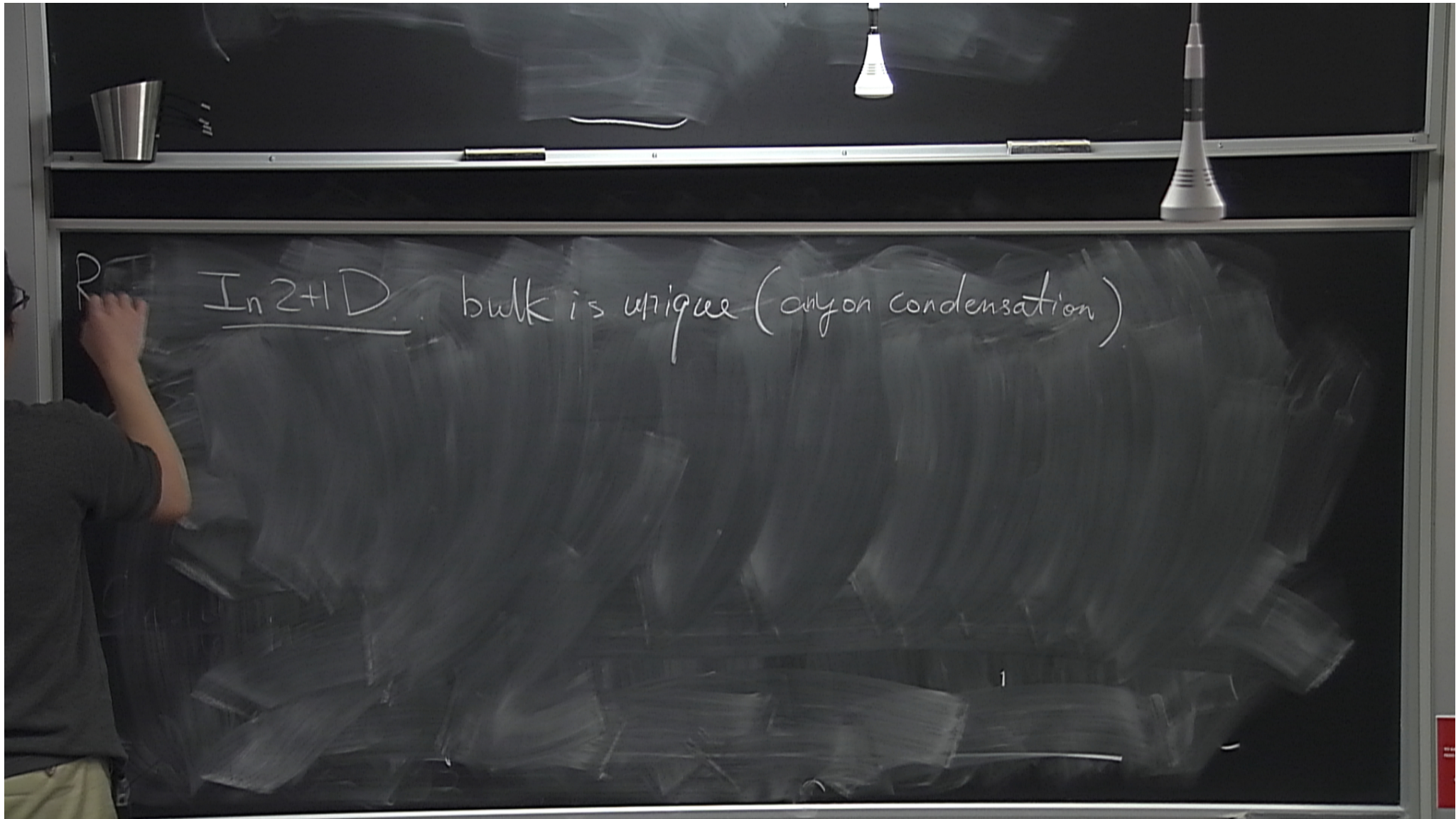




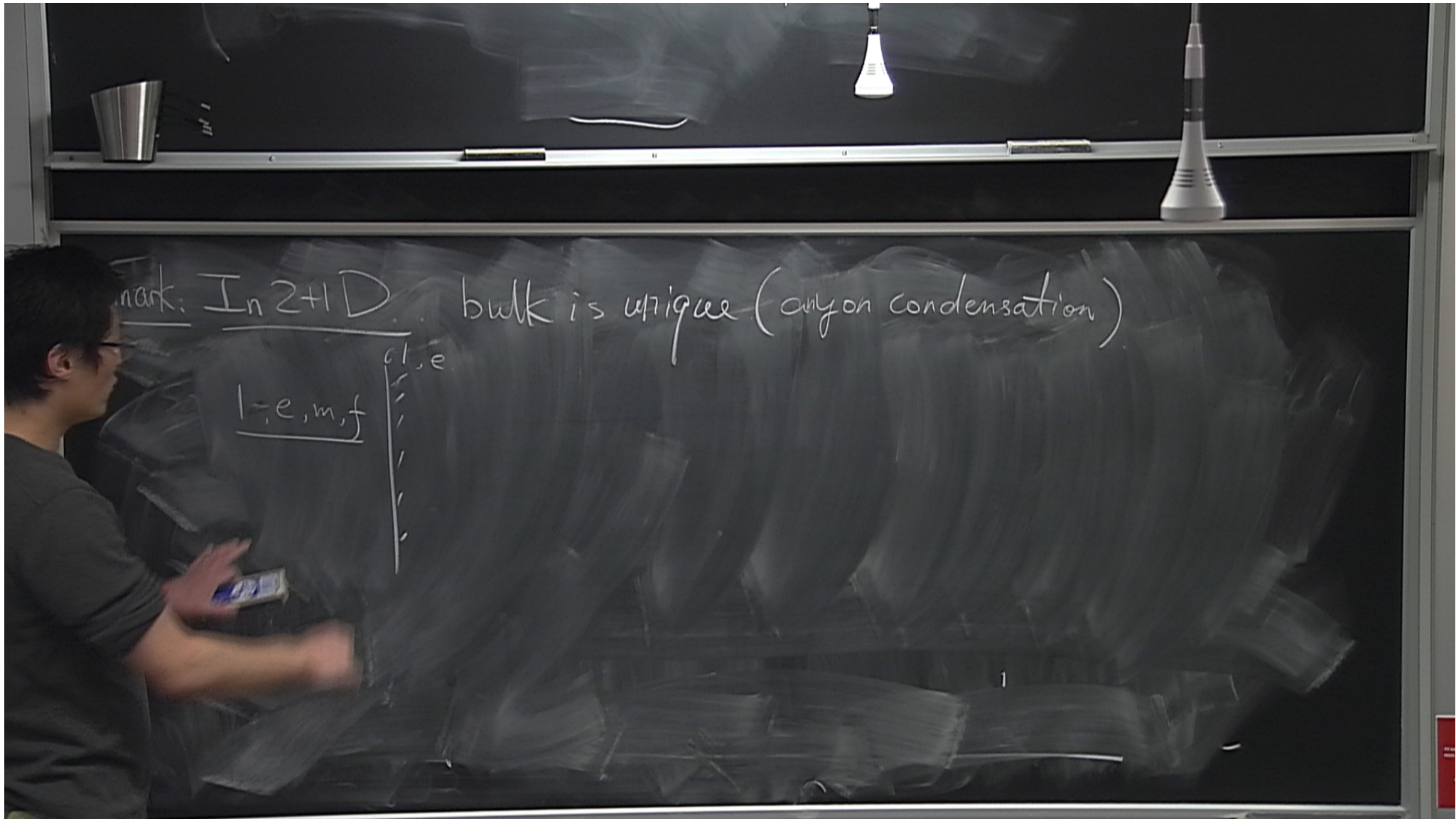
Lemma: Unique bulk Hypothesis

of  $C_{n+1}$   
For a given  $C_{n+1}$  (a  $TO_{n+1}$ ), there is a unique bulk.





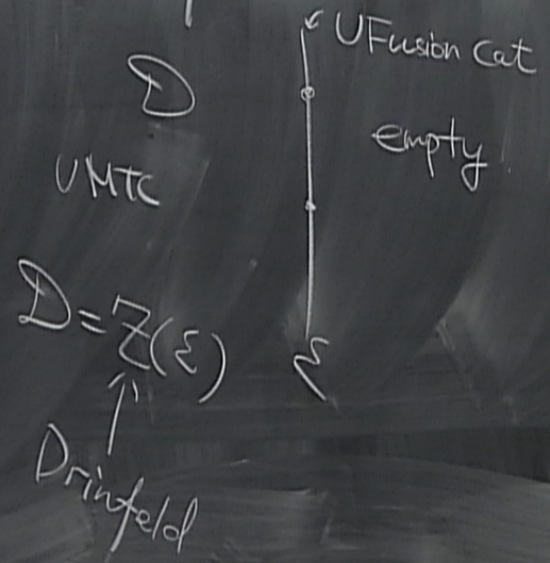






Remark: In 2+1 D bulk is unique (anyon condensation)

l = e, m, f  
cl, e

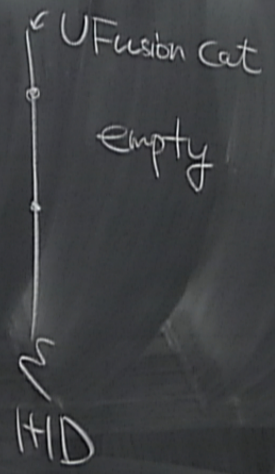




Remark: In

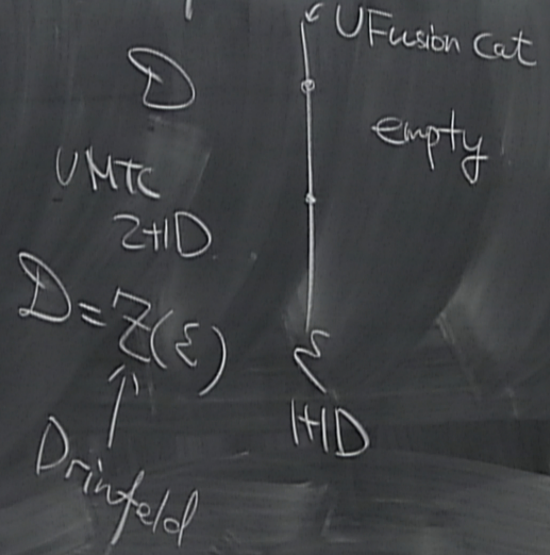
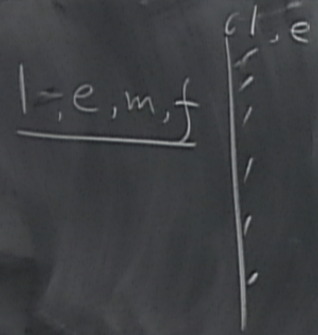
bulk is unique (anyon condensation)

UMTC  
 $Z \# D$   
 $D = Z(\mathcal{E})$   
↑  
Drinfeld

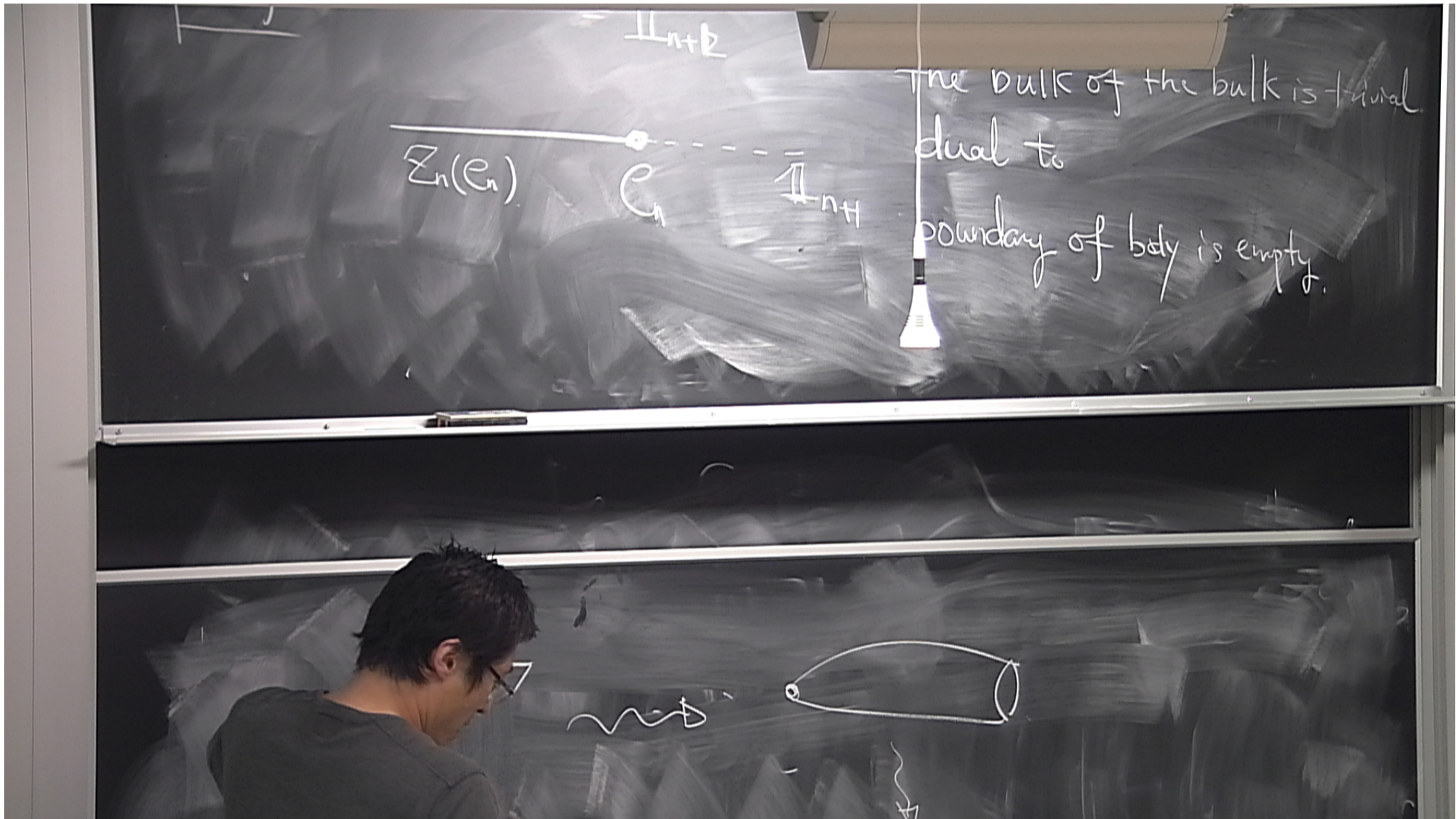




Remark: In 2+1 D bulk is unique (anyon condensation)







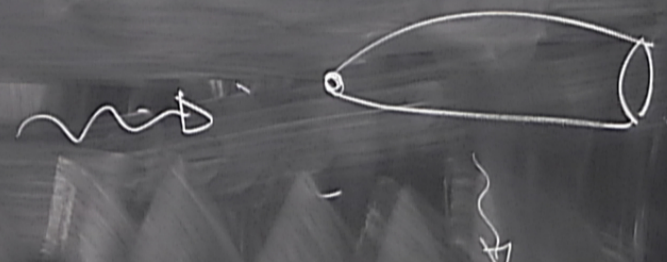
$Z_n(E_n)$

$I_{n+k}$

$E_n$

$I_{n+1}$

The bulk of the bulk is trivial dual to boundary of body is empty.





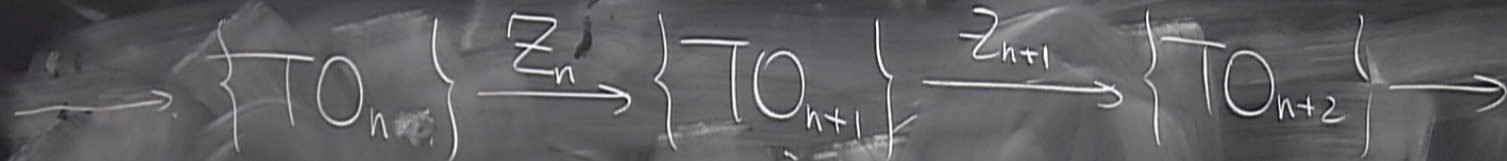


$\Sigma$  in  $Z+D$ .

$C_3$  is exact if  $C_1 \in \text{Im}(Z_2)$   
 are those MTC. --- closed if  $Z_3(C_3) = \mathbb{1}_3$   
 give by Drinfel'd center of U.F.C.

CAUTION





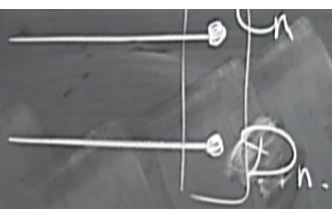
Example  $\mathbb{Z}$  in  $\mathbb{Z}+i\mathbb{D}$ .

$\mathcal{C}_3$  is exact if  $\mathcal{C}_1 \in \text{Im}(Z_2)$   
 are those MTC. --- closed if  $Z_3(\mathcal{C}_3) = \mathbb{I}_3$   
 give by Drinfel'd center of U.F.C.



$$\begin{array}{c}
 \text{---} \circ C_n \\
 | \\
 \text{---} \circ D_n
 \end{array}
 \quad
 C_n \boxtimes D_n \rightsquigarrow \{TO_n\} \text{ is commutative monoid}$$

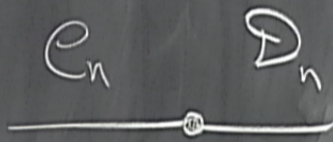




$C_n \boxtimes D_n \rightarrow \{TO_n\}$  is commutative monoid

Example to get group

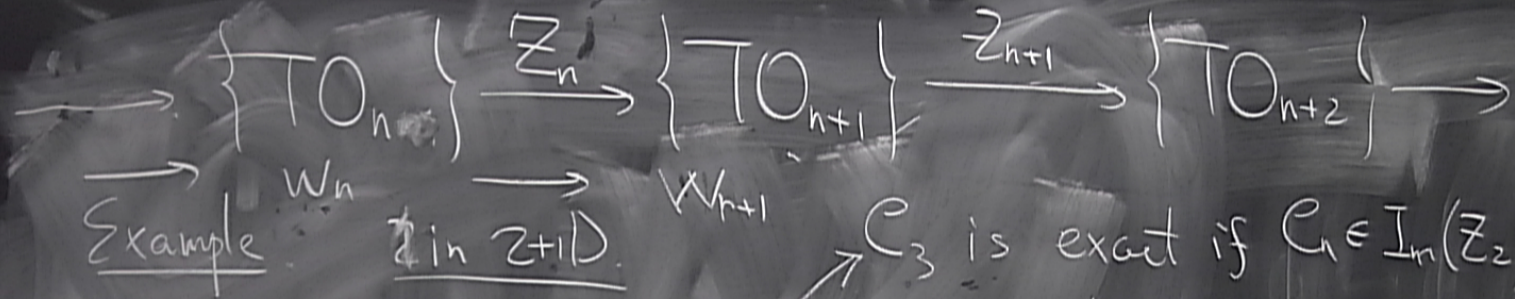
$$\{TO_n\} / \sim = W_n$$



With eqn  
 $C_n \sim D_n$

gapped domain wall.





Example

$Z$  in  $Z+1D$

$C_3$  is exact if  $C_1 \in I_m(Z_2)$

are those MTC. --- closed if  $Z_3(C_3) = I_3$   
 give by Drinfel'd center of U.F.C.

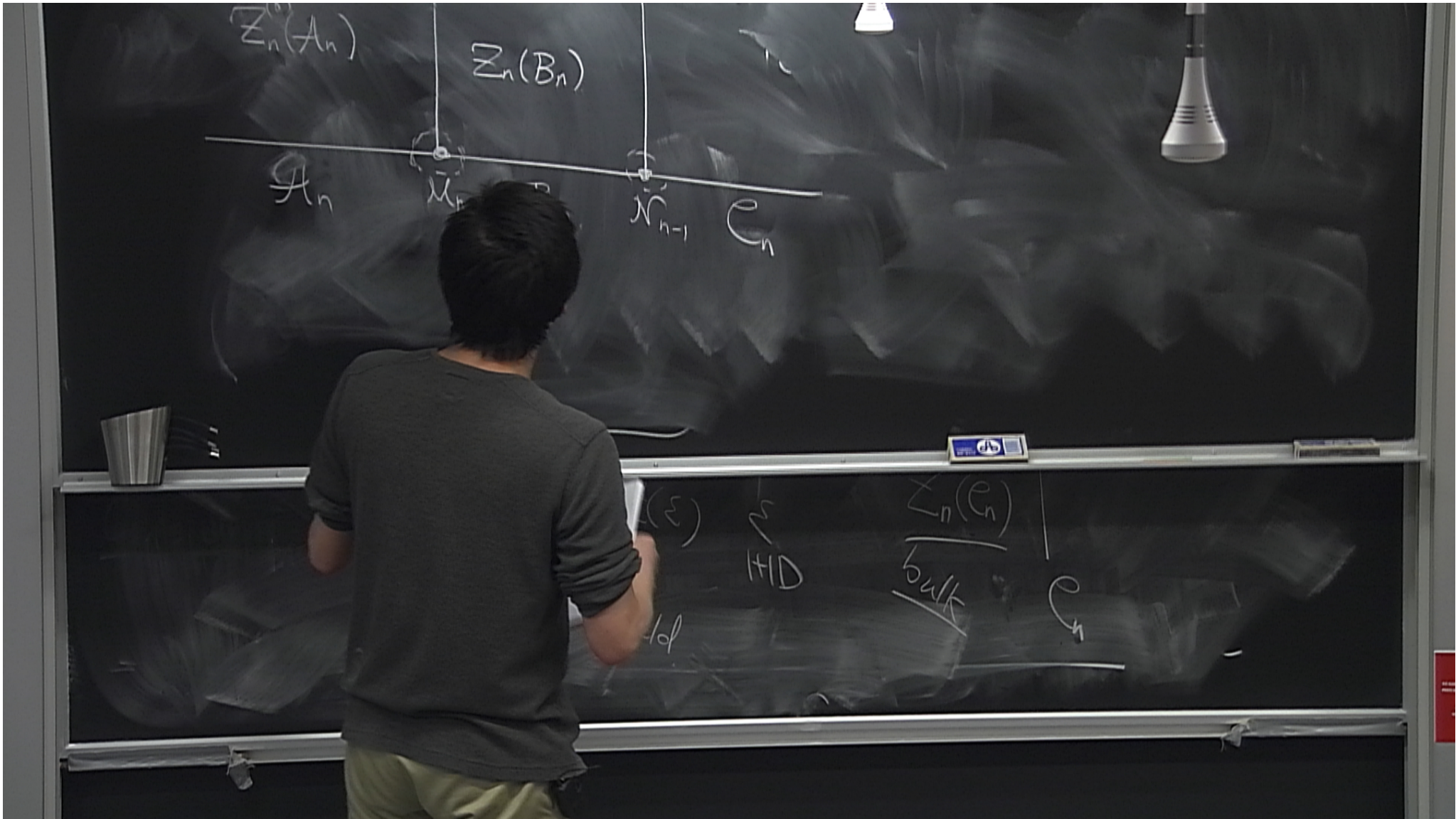


Stronger unique bulk hypothesis.

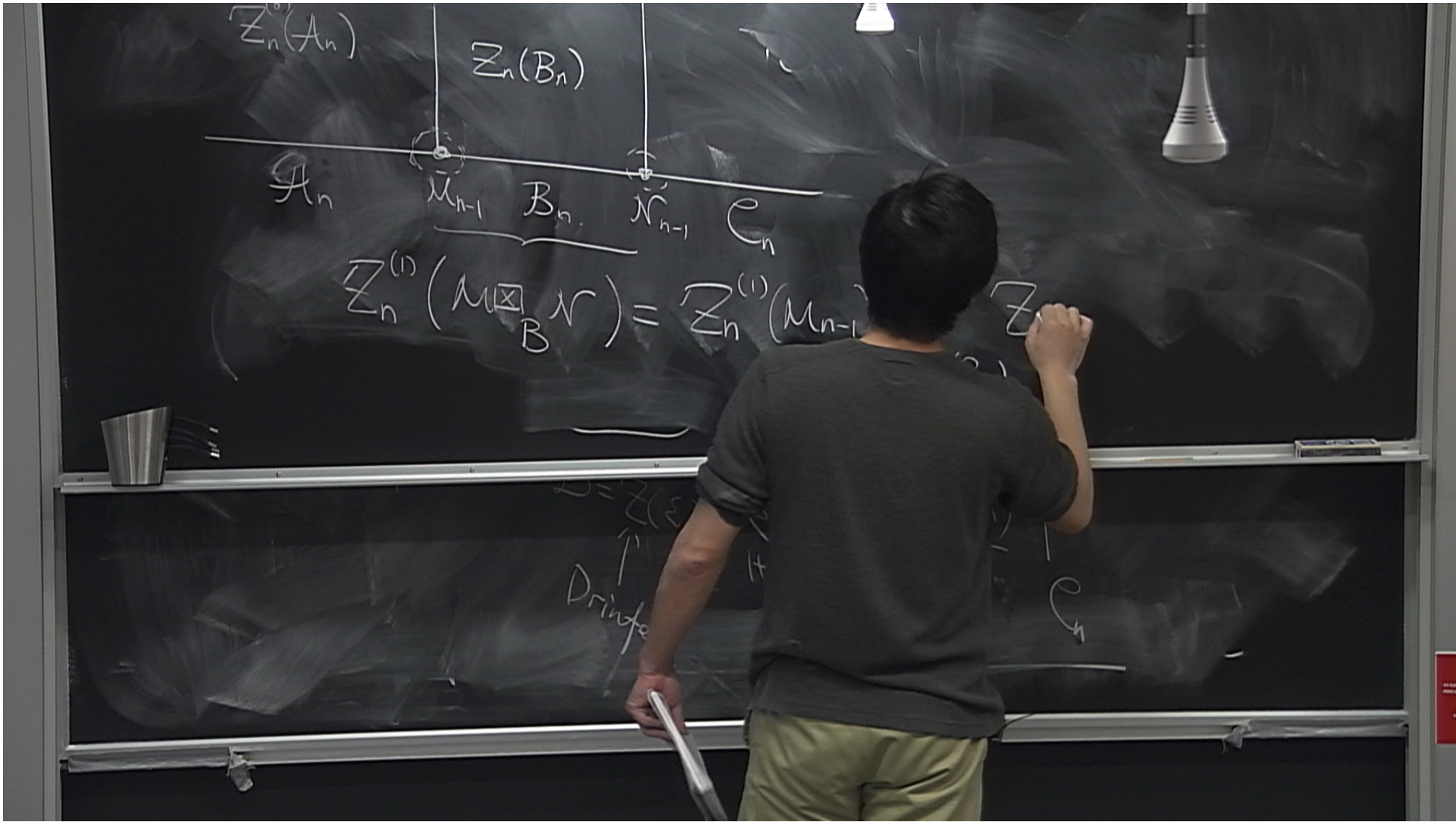
$$\mathbb{Z}_n^{(0)}(A_n) \quad \mathbb{Z}_n^{(1)}(A_{n-1})$$

$$c_n$$



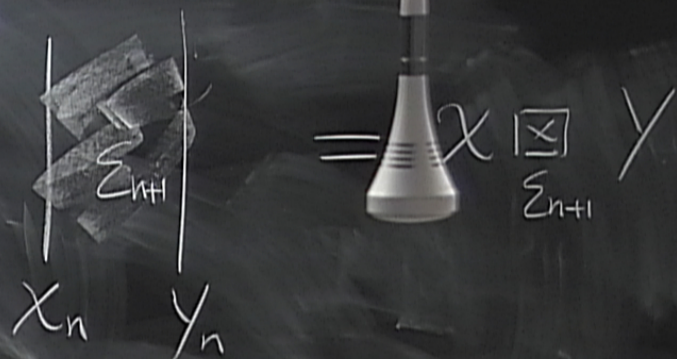
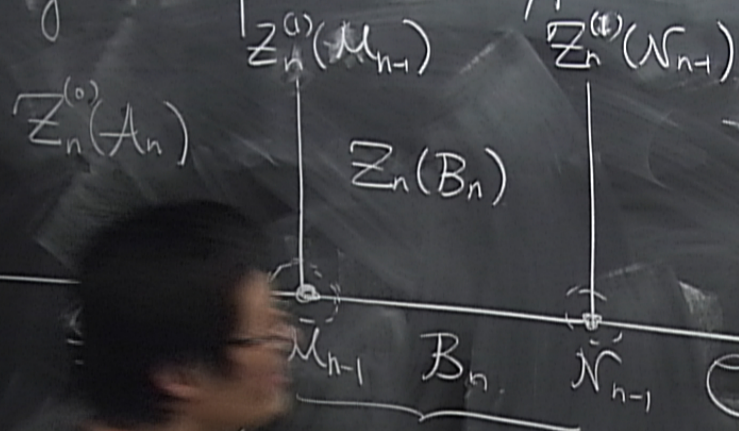








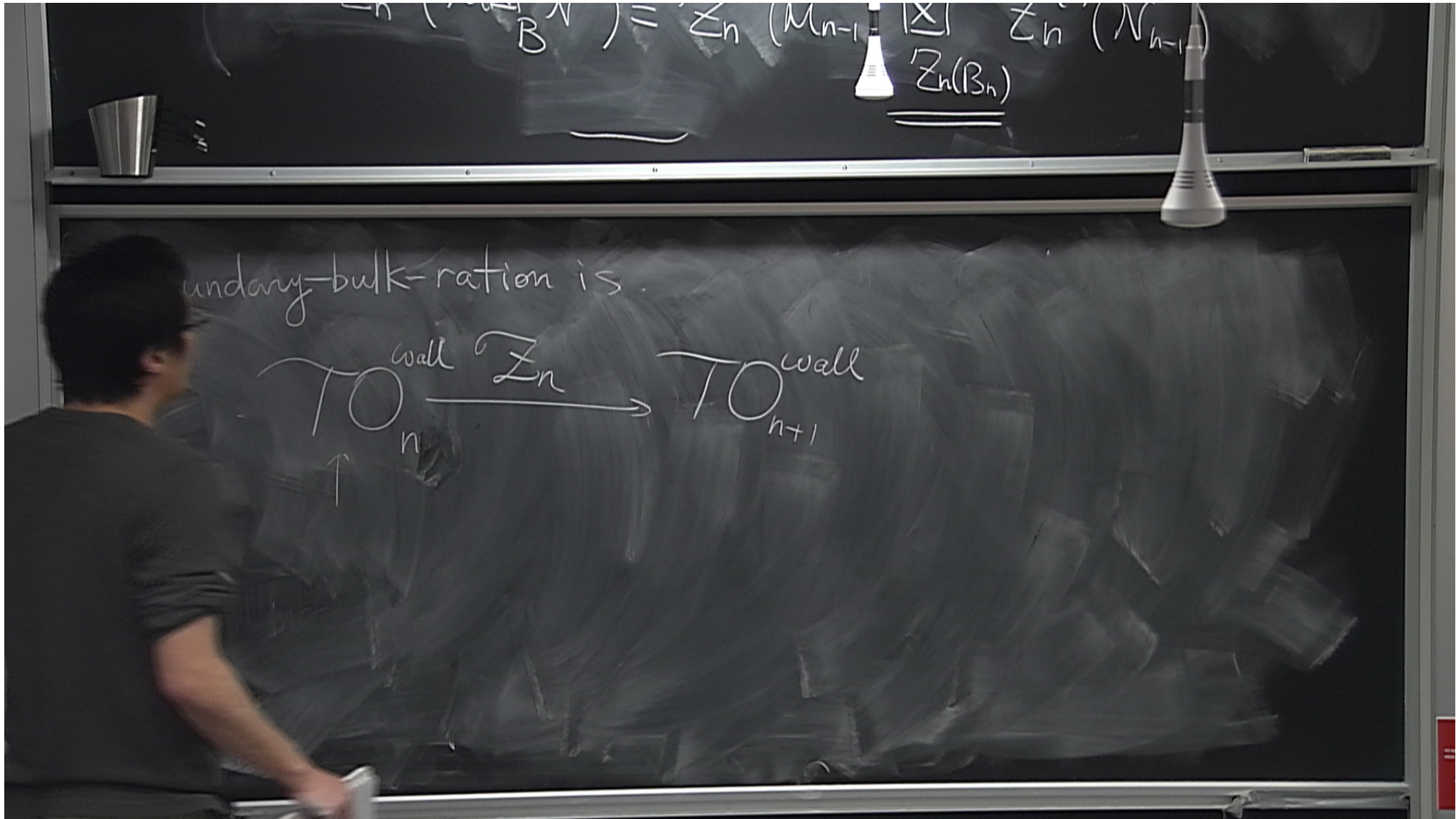
Stronger unique bulk hypothesis



$$Z_n^{(1)}(M \boxtimes_B N) = Z_n^{(1)}(M_{n-1}) \boxtimes_{Z_n(B_n)} Z_n^{(1)}(N_{n-1})$$

Drinfeld



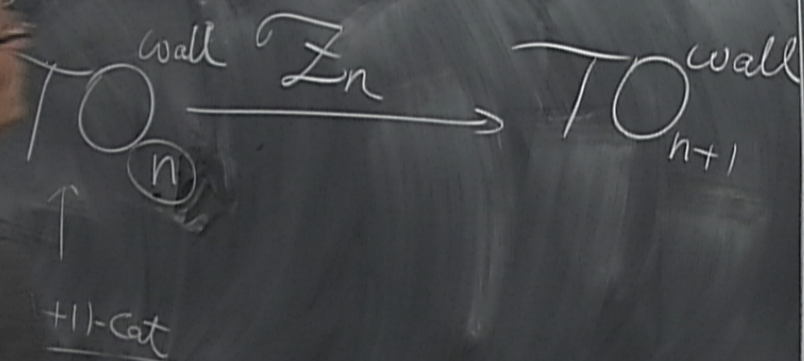




$$Z_n(B^N) = Z_n(M_{n-1}) \times Z_n(N_{n-1})$$

$$\underline{\underline{Z_n(B_n)}}$$

Bulk-ratation is



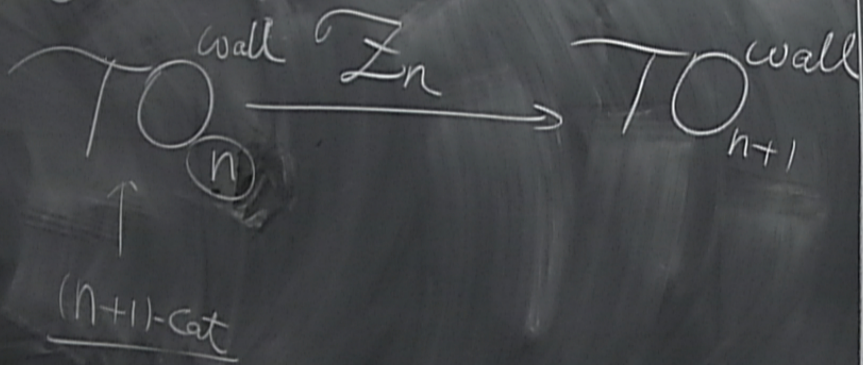
Thm (K.-Zheng)  
 $n=1+1$   $Z_n$  is an equivalence



$$Z_n(B^N) = Z_n(M_{n-1}) \times Z_n(N_{n-1})$$

$$\underline{\underline{Z_n(B_n)}}$$

Boundary-bulk-ration is



Thm (K.-Zheng)

$n=1+1$   $Z_n$  is an equivalence



$X_n \quad Y_n$

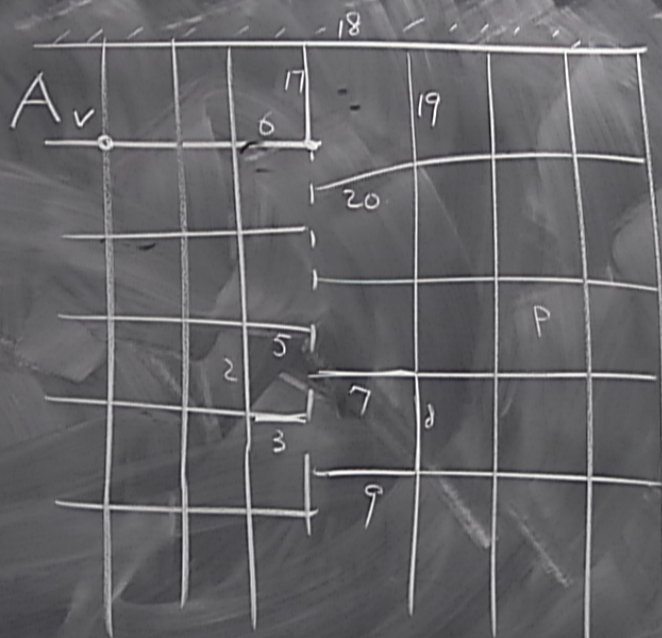
$\mathcal{A}_n \quad \mathcal{M}_{n-1} \quad \mathcal{B}_n \quad \mathcal{N}_{n-1} \quad \mathcal{C}_n$

$$\mathbb{Z}_n^{(n)}(\mathcal{M} \boxtimes_{\mathcal{B}} \mathcal{N}) = \mathbb{Z}_n^{(1)}(\mathcal{M}_{n-1}) \boxtimes_{\mathbb{Z}_n(\mathcal{B}_n)} \mathbb{Z}_n^{(1)}(\mathcal{N}_{n-1})$$

$(n+1)$

$(n+1)\text{-cat}$





$$B_{[-]} = \sigma_2^z \sigma_5^z \sigma_3^z \sigma_7^x$$

$$B_{-]} = \sigma_3^x \sigma_7^z \sigma_8^z \sigma_9^z$$

$$Q = \sigma_6^x \sigma_{17}^y \sigma_{18}^z \sigma_{19}^z \sigma_{20}^z$$

CAUTION



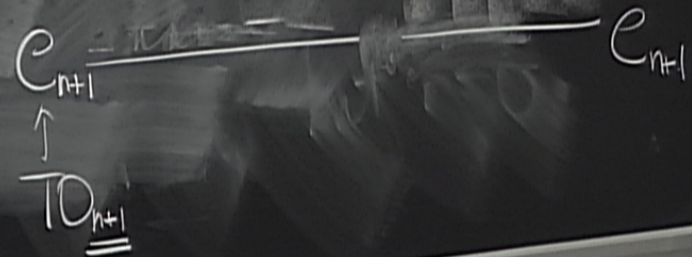
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Two lattice models are equivalent.

if  $\exists$  a nbh. of bdy such that  
we can "deform" one to the other

w/o closing the gap

$n+1 d \rightarrow spa$   
 $n+2 D$  st.





The pair  $(P_n(\mathbb{Z}_n(\mathcal{C}_n)), m)$  is terminal among all such pairs.

$$\begin{array}{ccc}
 & P_n(\mathbb{Z}_n(\mathcal{C}_n)) \boxtimes \mathcal{C}_n & \\
 \exists! \downarrow f \boxtimes \text{id}_{\mathcal{C}_n} \uparrow & & \searrow m \\
 X_n \boxtimes \mathcal{C}_n & \xrightarrow{f} & \mathcal{C}_n
 \end{array}$$

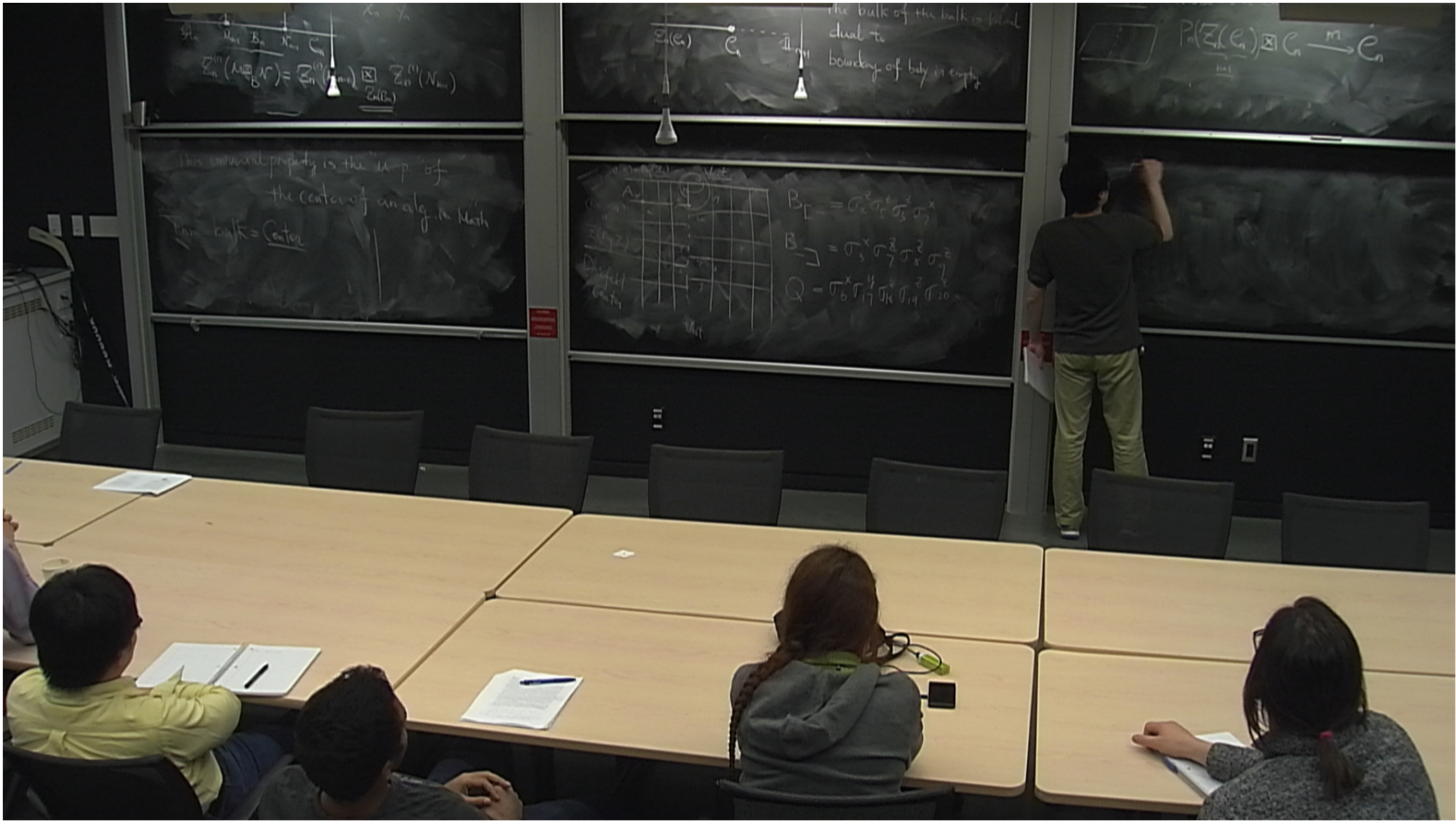


$Z_n(B_n)$

This universal property is the "u. p." of  
the center of an alg. in  $\mathcal{M}_n$

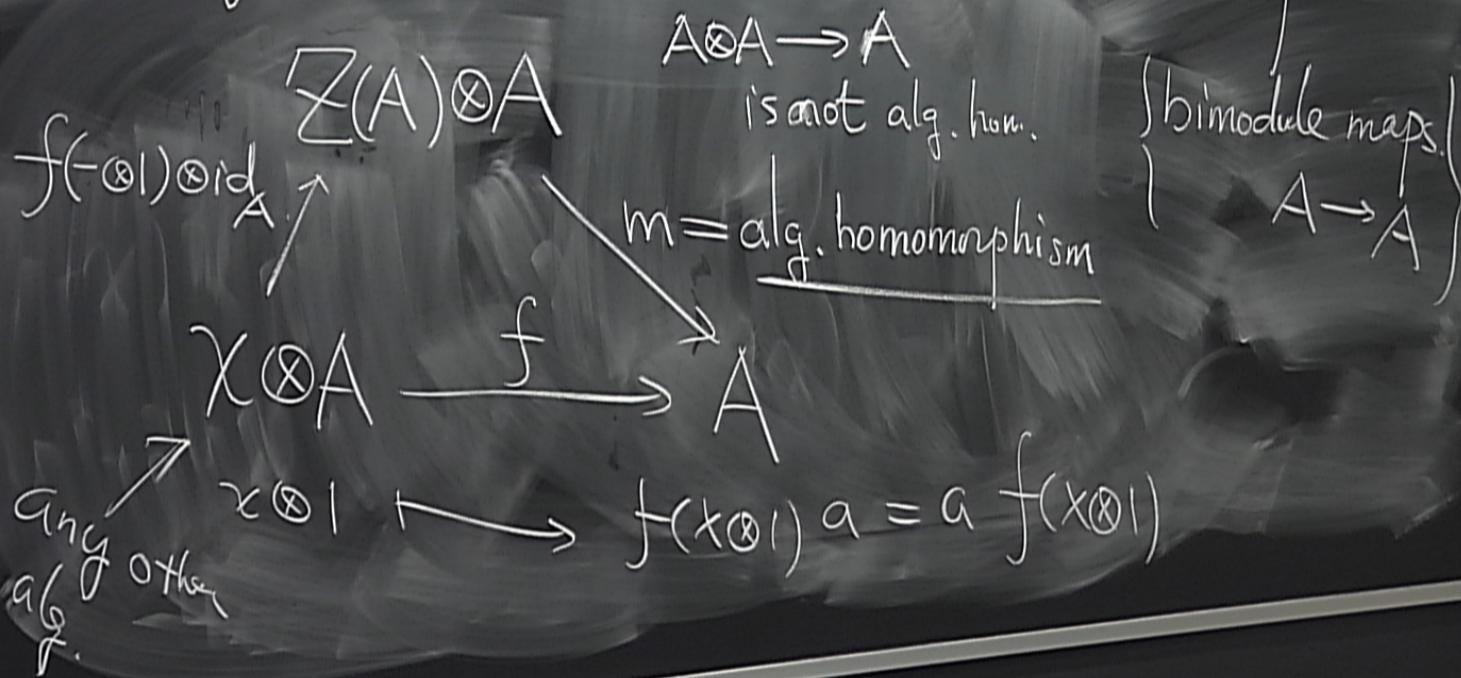
"Thm"  $\text{bulk} = \text{Center}$ .







A-alg.  $Z(A) = \{z \in A \mid za = az, \forall a \in A\} \simeq \text{Hom}_{A/A}(A, A)$



CAUTION