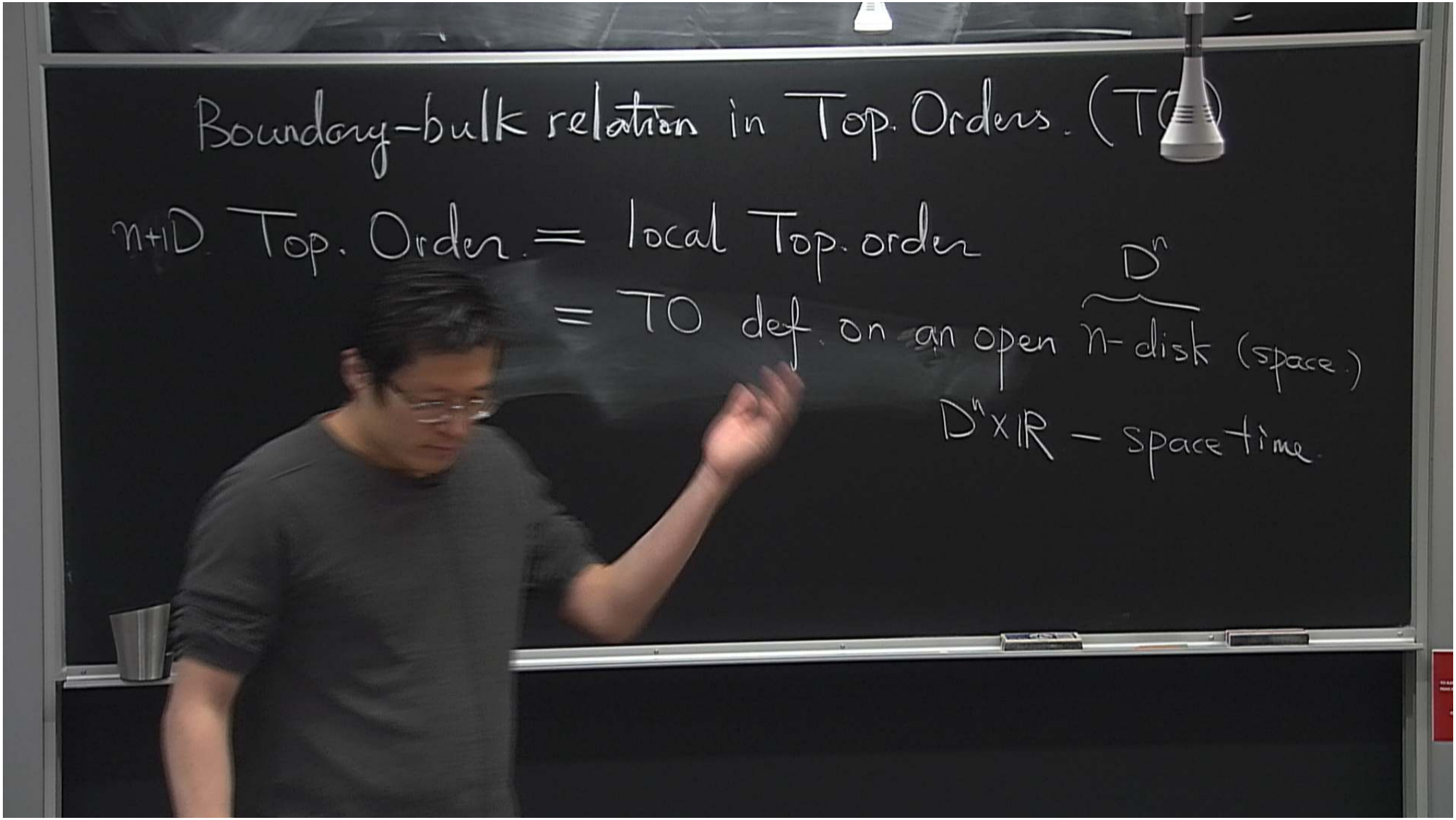
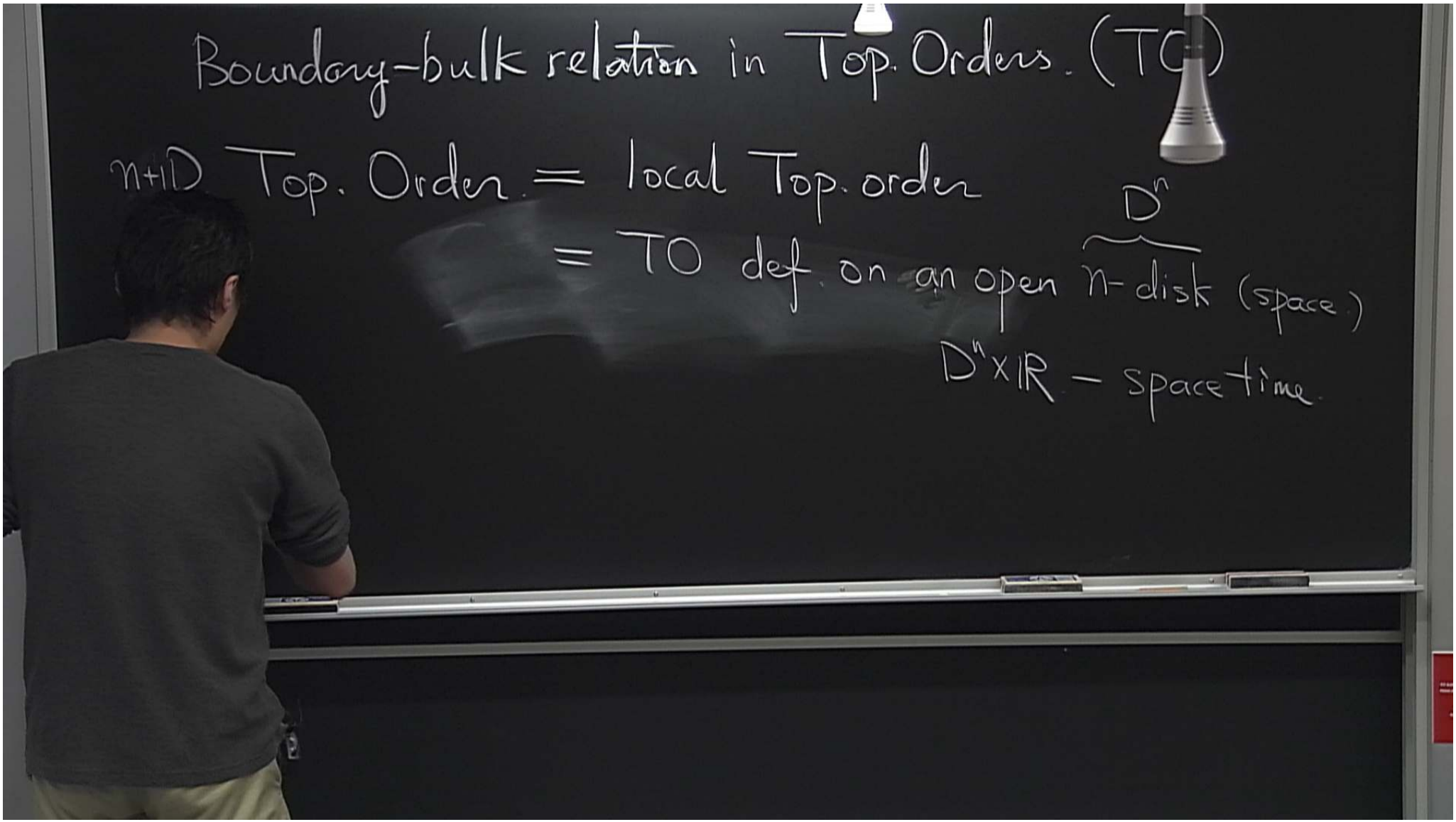
Title: TBA

Date: Mar 10, 2016 03:30 PM

URL: http://pirsa.org/16030126

Abstract:


Boundary-bulk relation in Top. Orders (TO)
n+1D Top. Order = local Top. order
= TO def. on an open $\underbrace{n\text{-disk}}_{D^n}$ (space)
$D^n \times \mathbb{R}$ – spacetime


Boundary-bulk relation in Top. Orders. (TO)
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= TO def. on an open $\overbrace{n\text{-disk}}^{D^n}$ (space)
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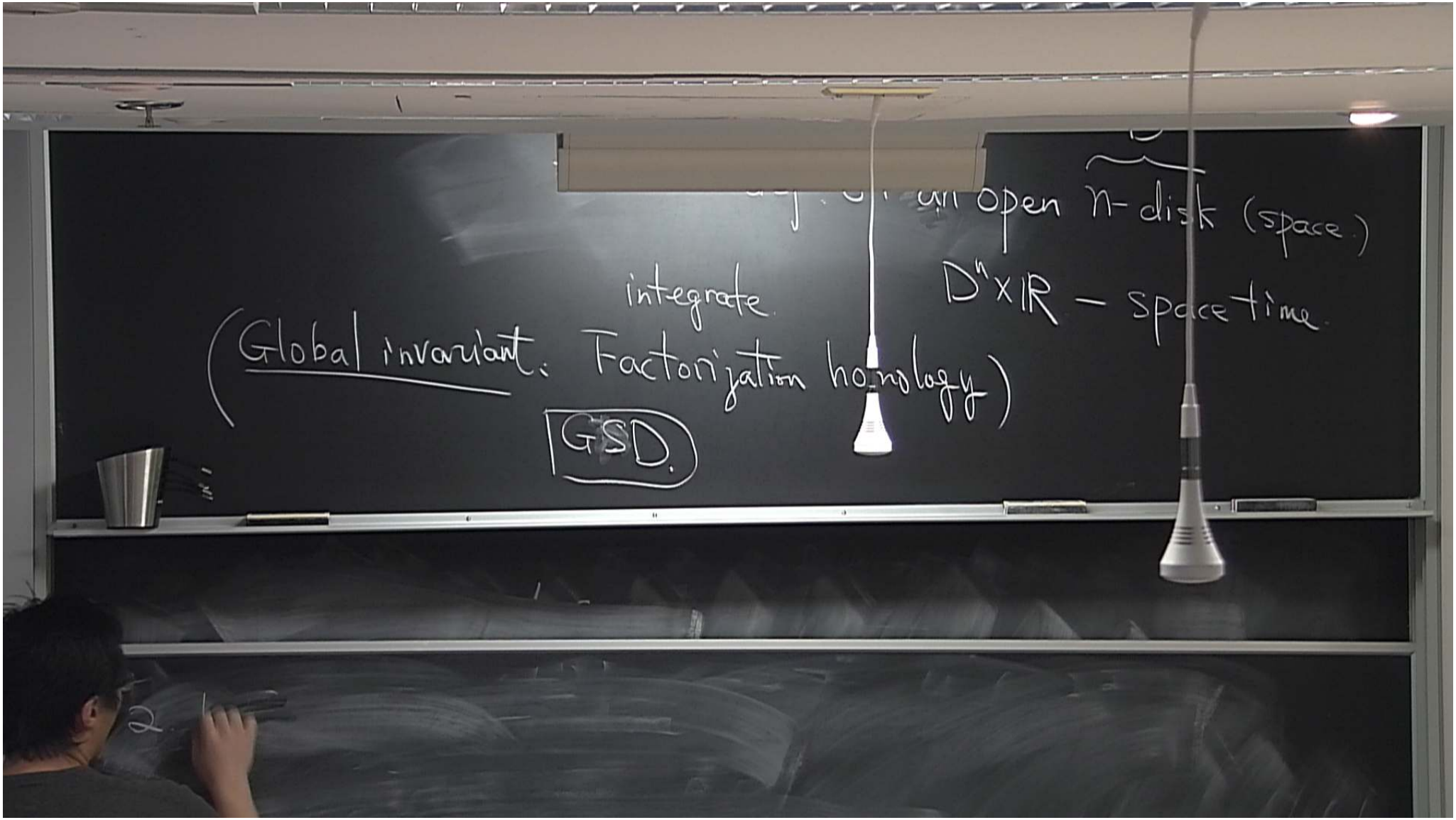
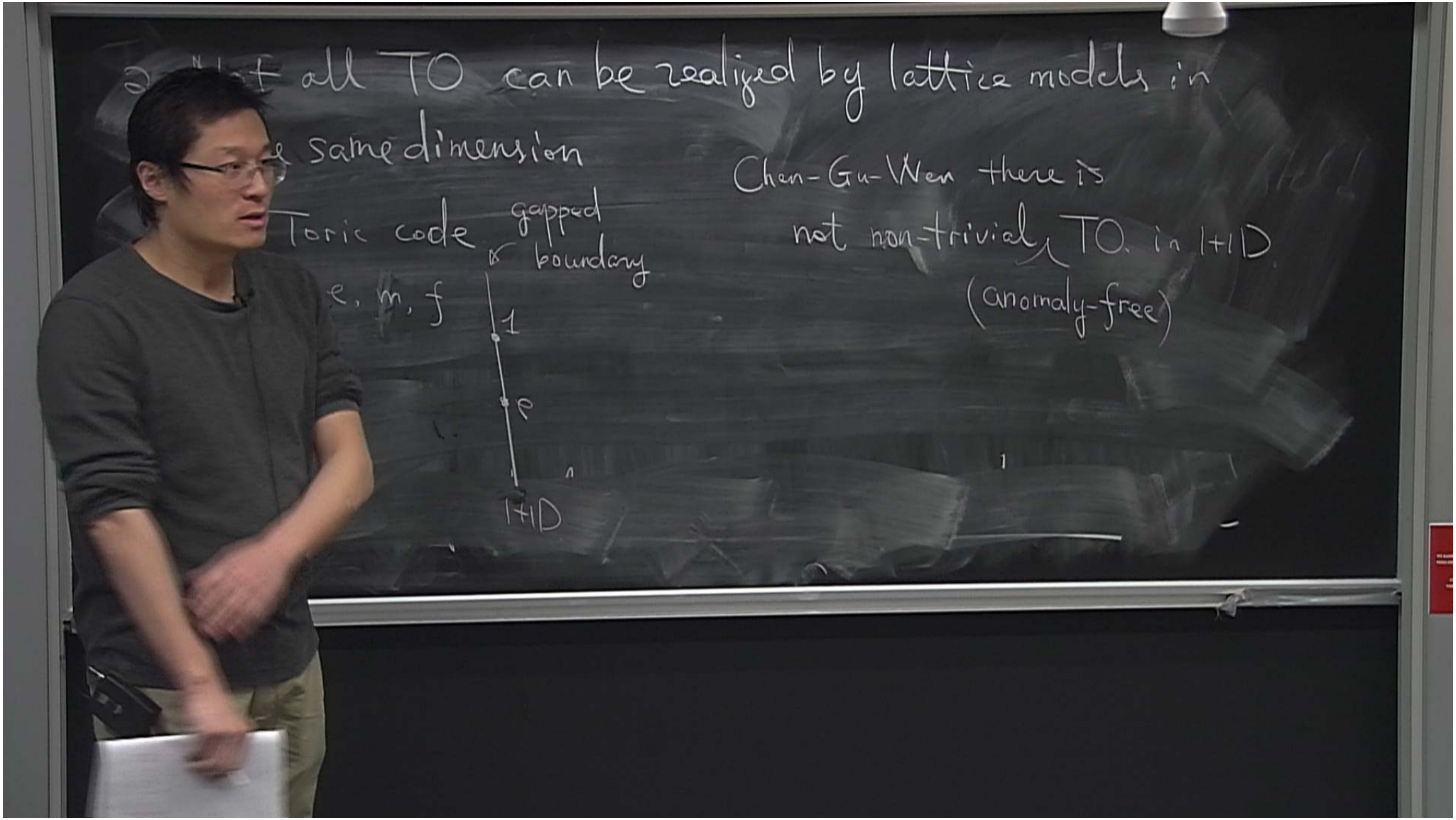

an open n-disk (space)
integrate
$D^n \times \mathbb{R}$ – spacetime
(Global invariant: Factorization homology)
GSD.


Not all TO can be realized by lattice models in
same dimension
Toric code
gapped
boundary
e, m, f
1
e
1+1D
Chen-Gu-Wen there is
not non-trivial TO. in 1+1D
(anomaly-free)
1

2. Not all TO can be realized by lattice models in the same dimension

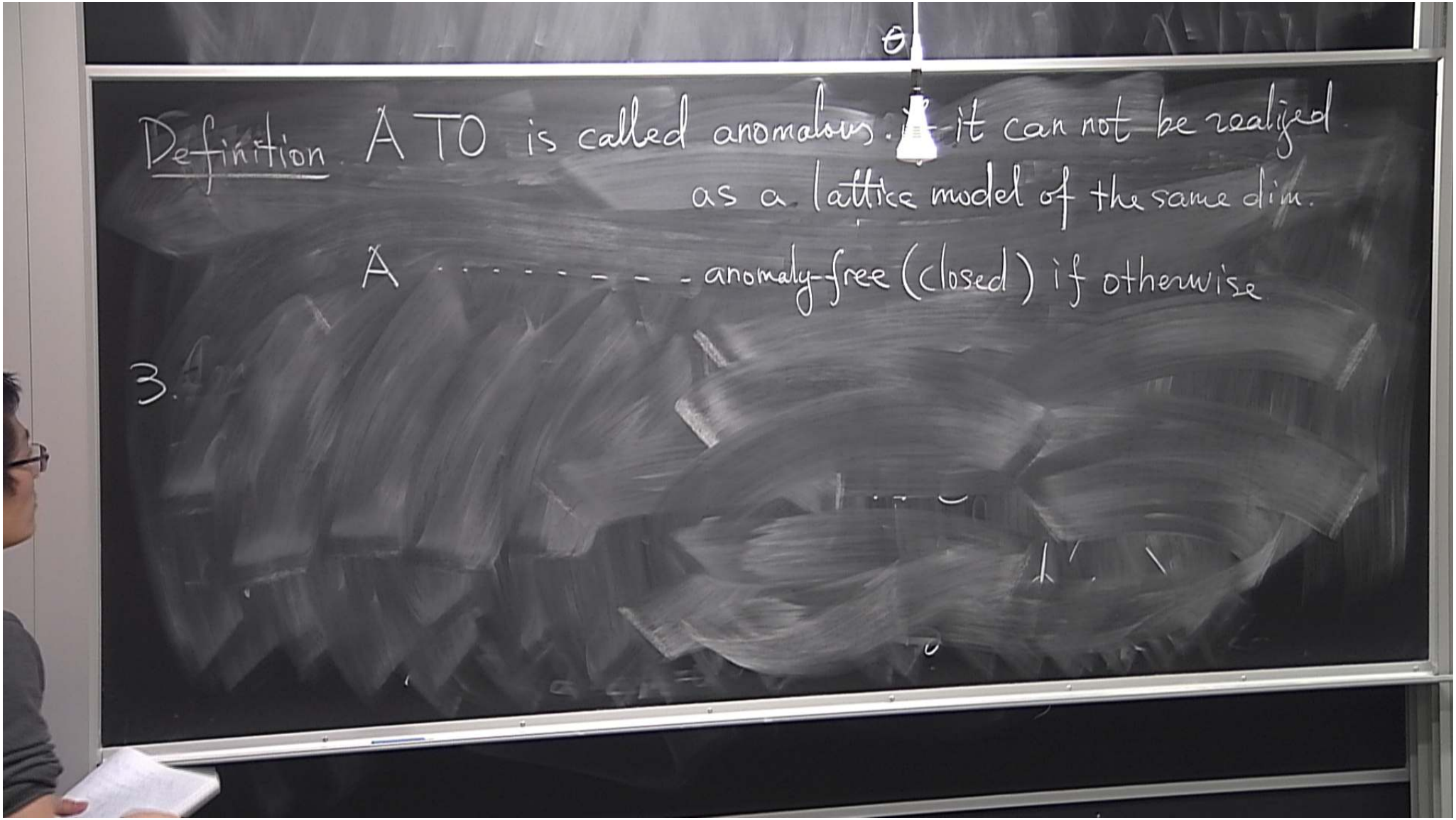
Toric code

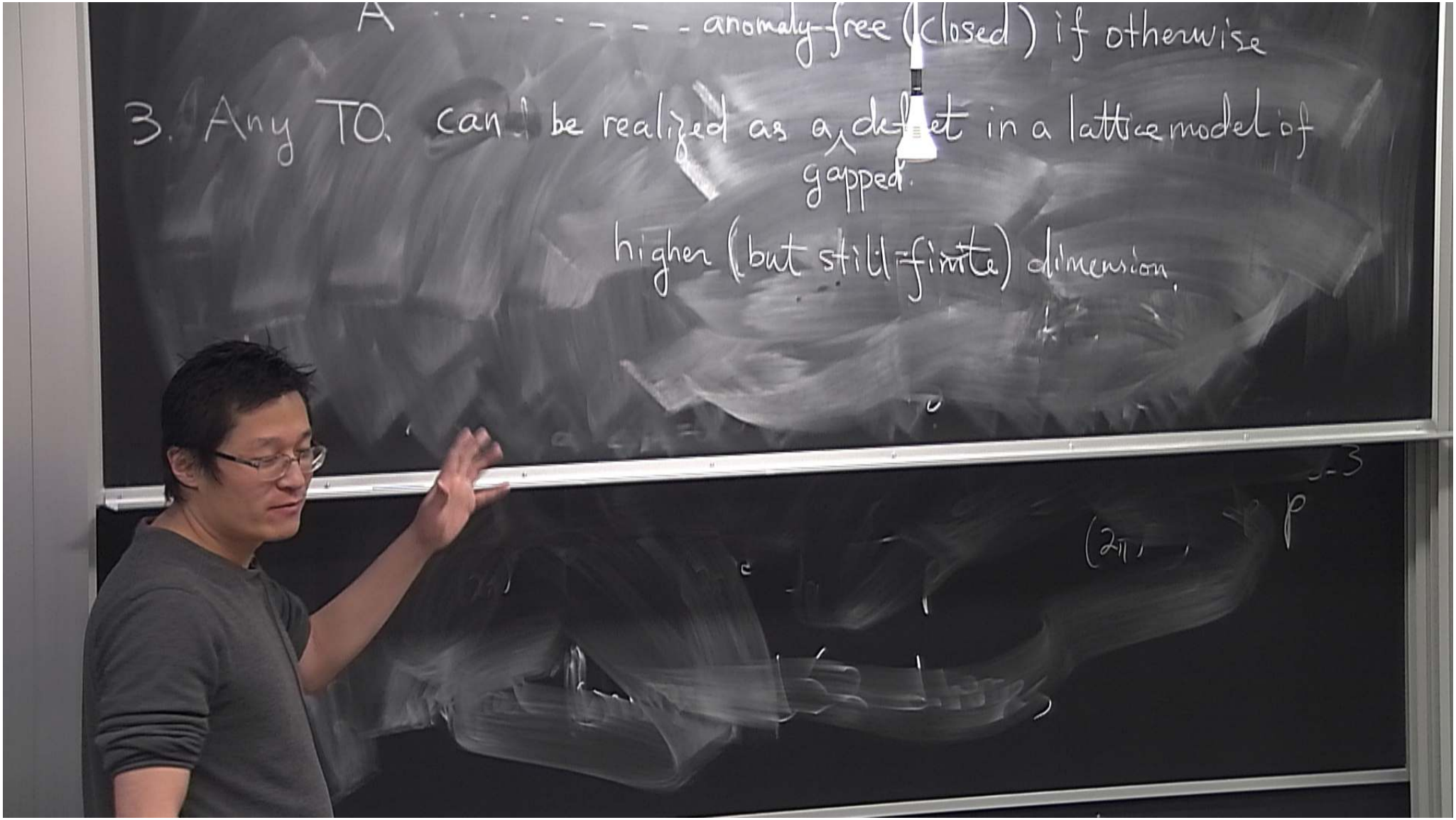
gapped boundary

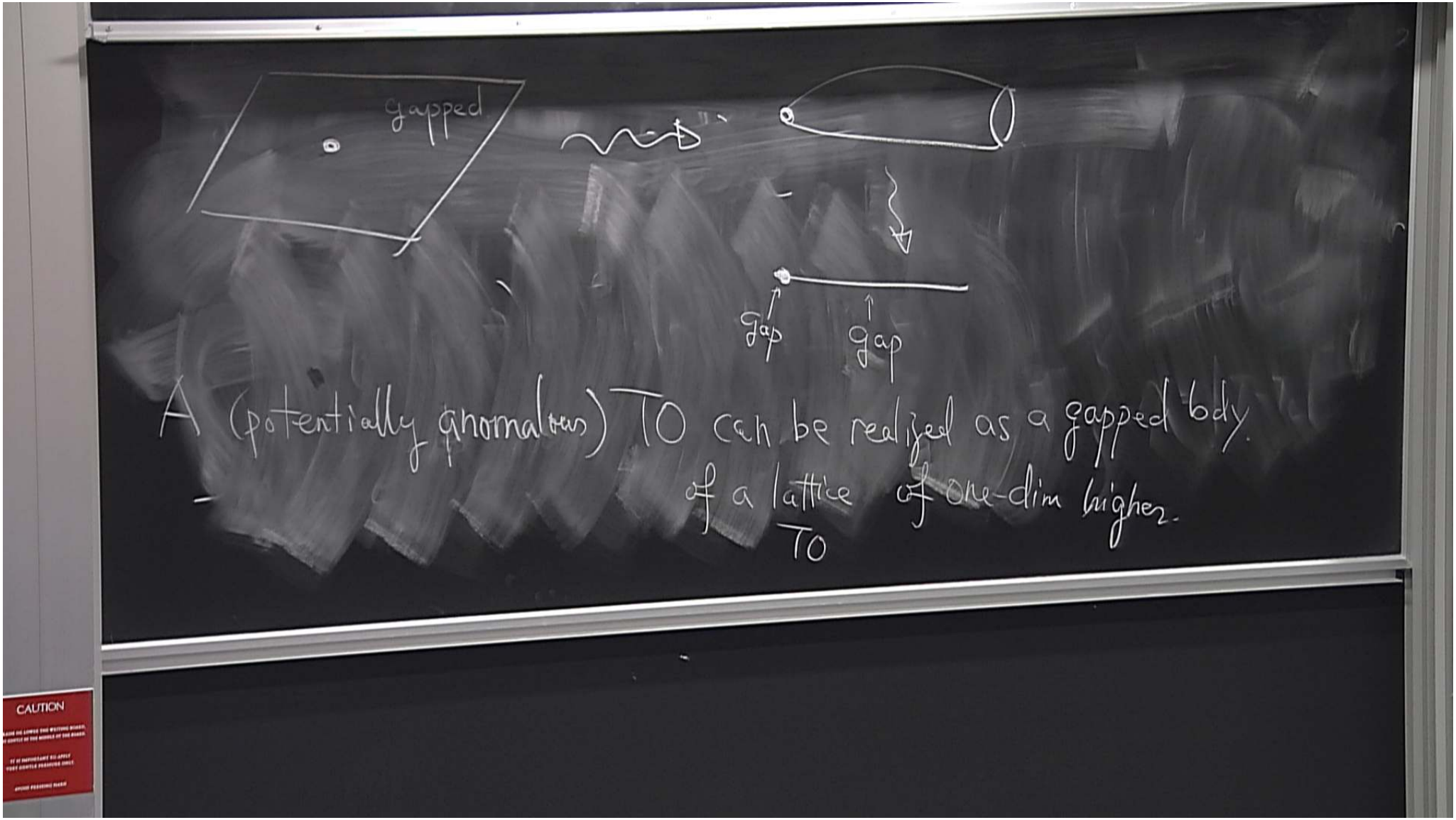
1, e, m, f

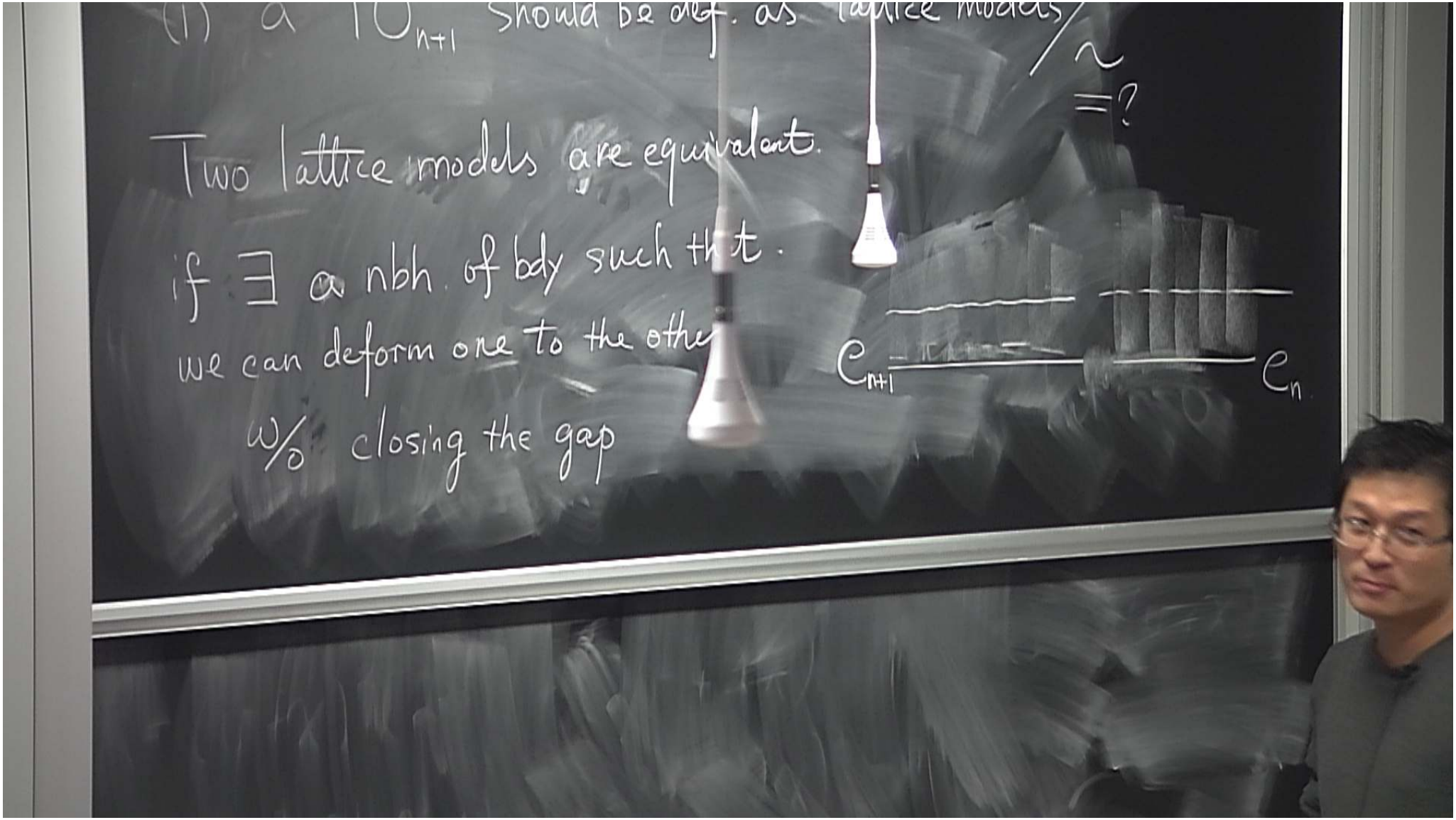
...en there is ...on-trivial TO. in 1+1D.

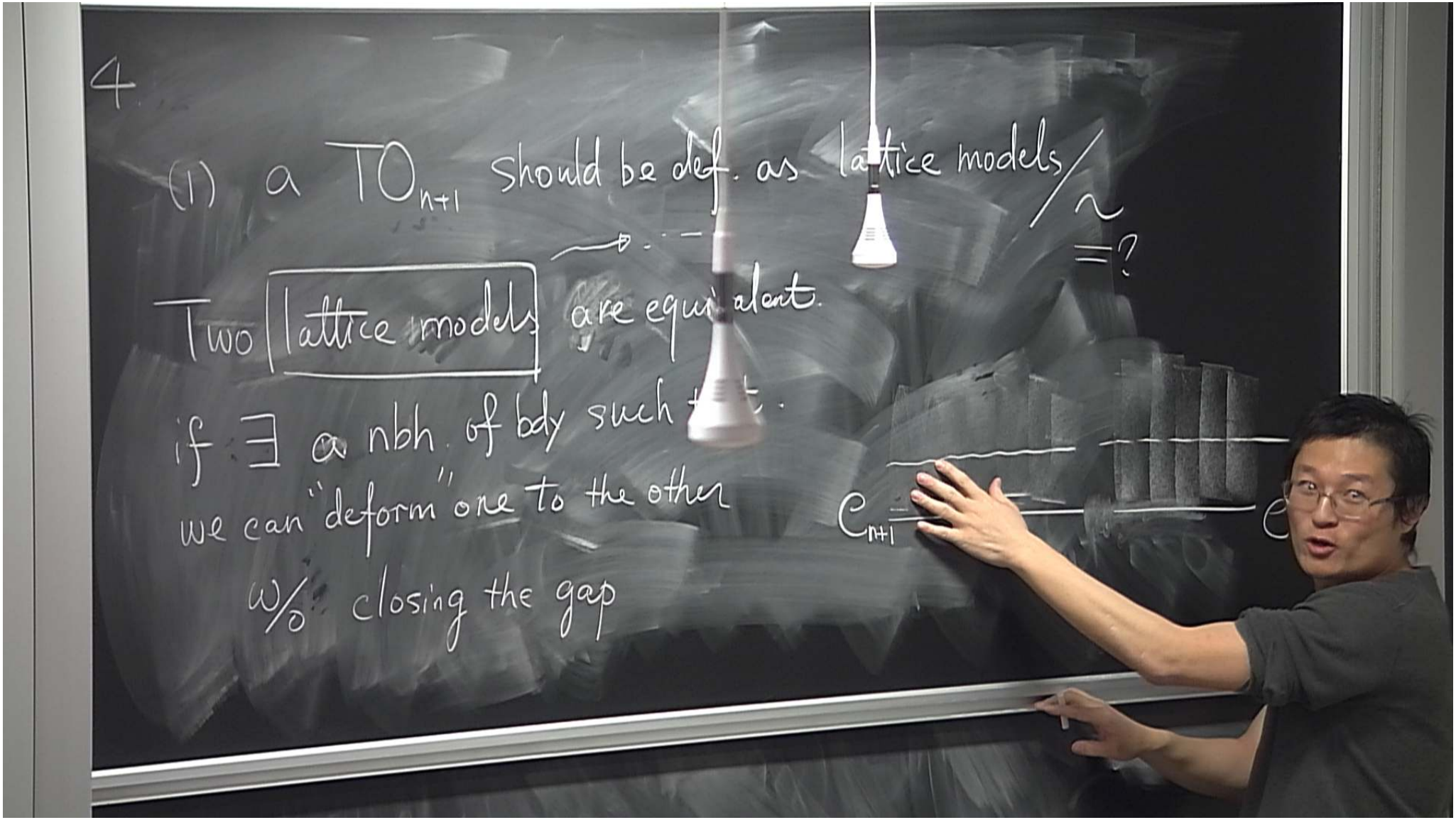
(anomaly-free)

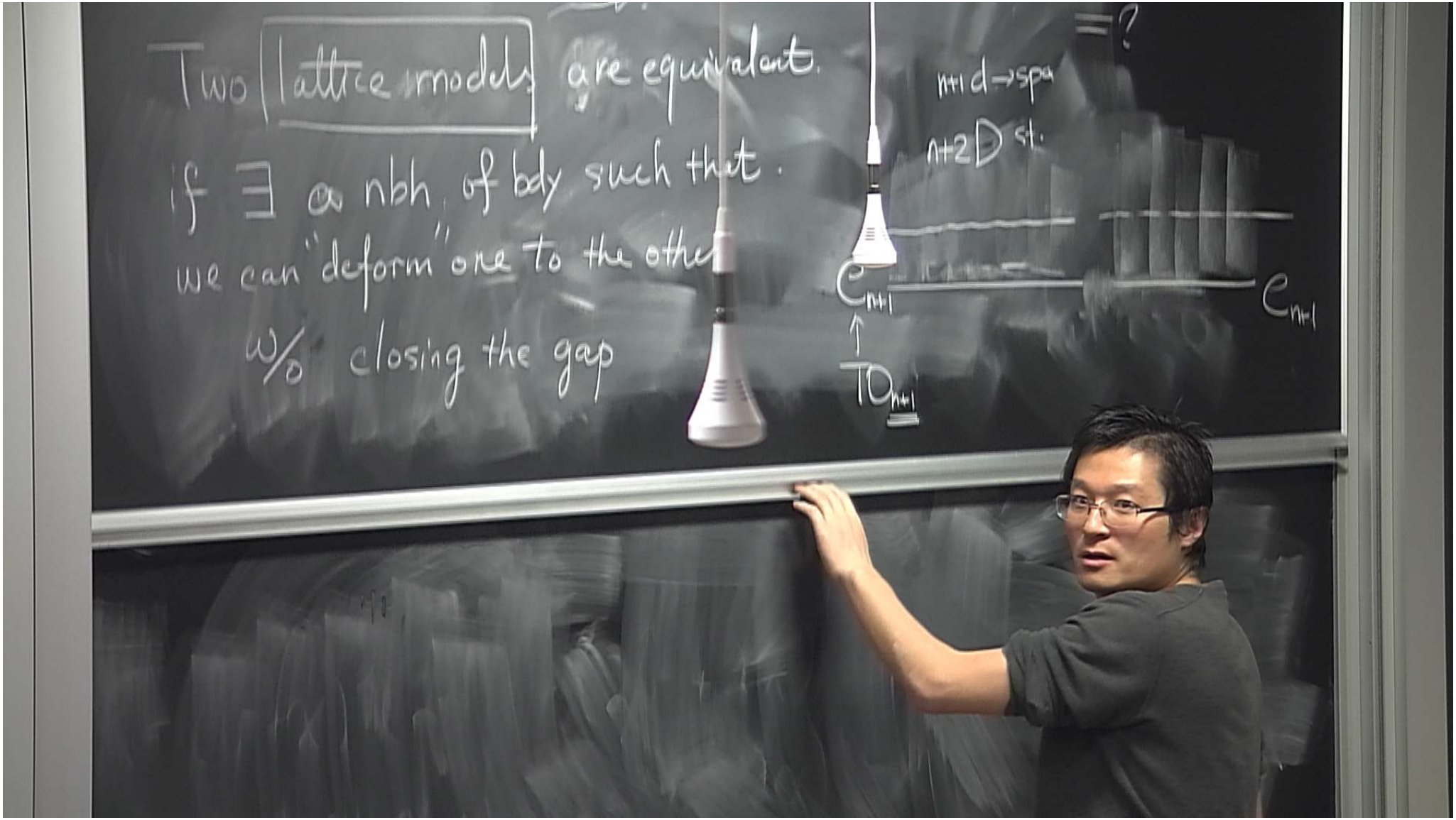

Definition A TO is called anomalous - it can not be realized as a lattice model of the same dim.
A ..... - anomaly-free (closed) if otherwise
3.

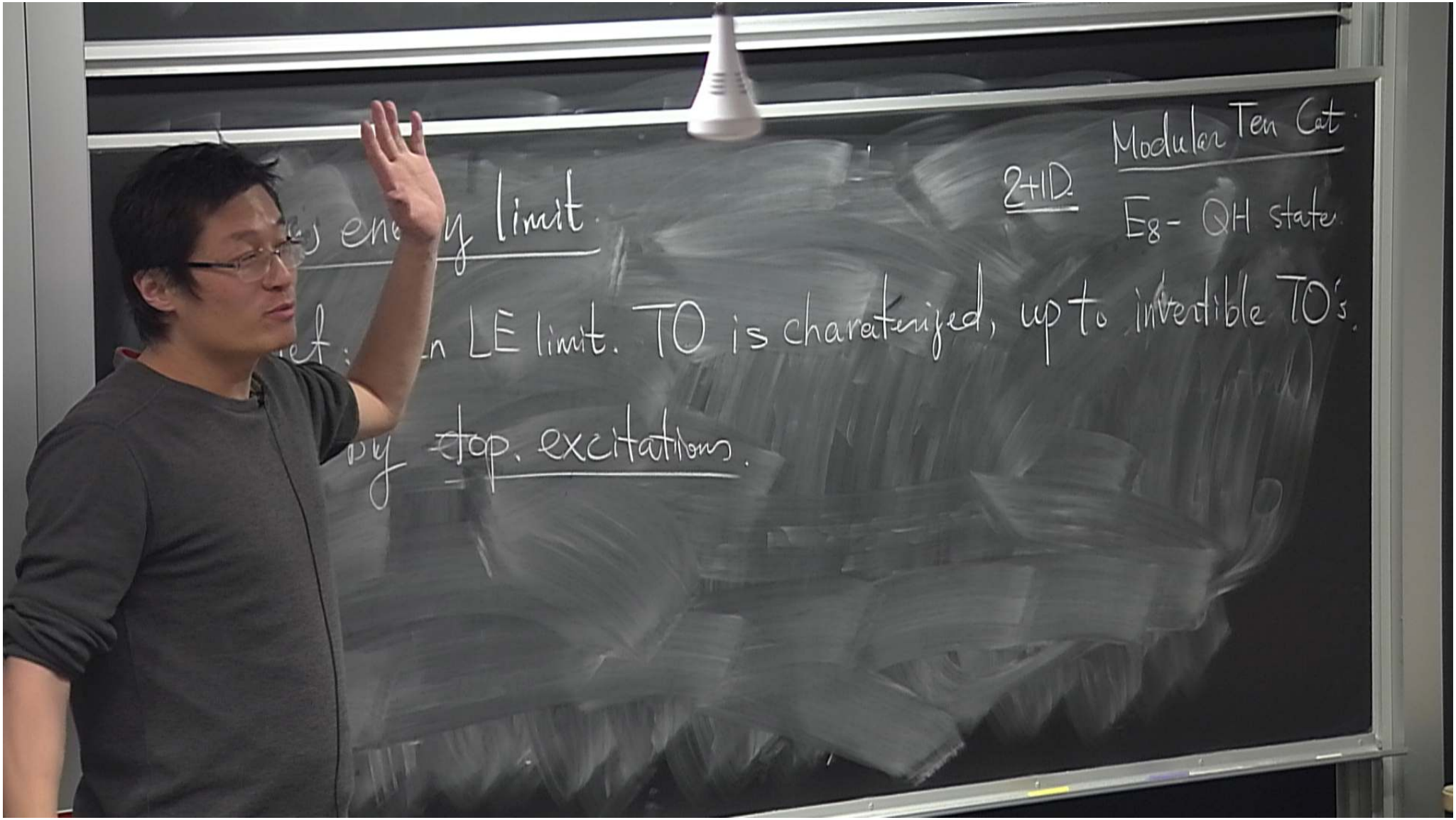

A ..... - anomaly-free (closed) if otherwise
3. Any TO can be realized as a defect in a lattice model of gapped
higher (but still finite) dimension.

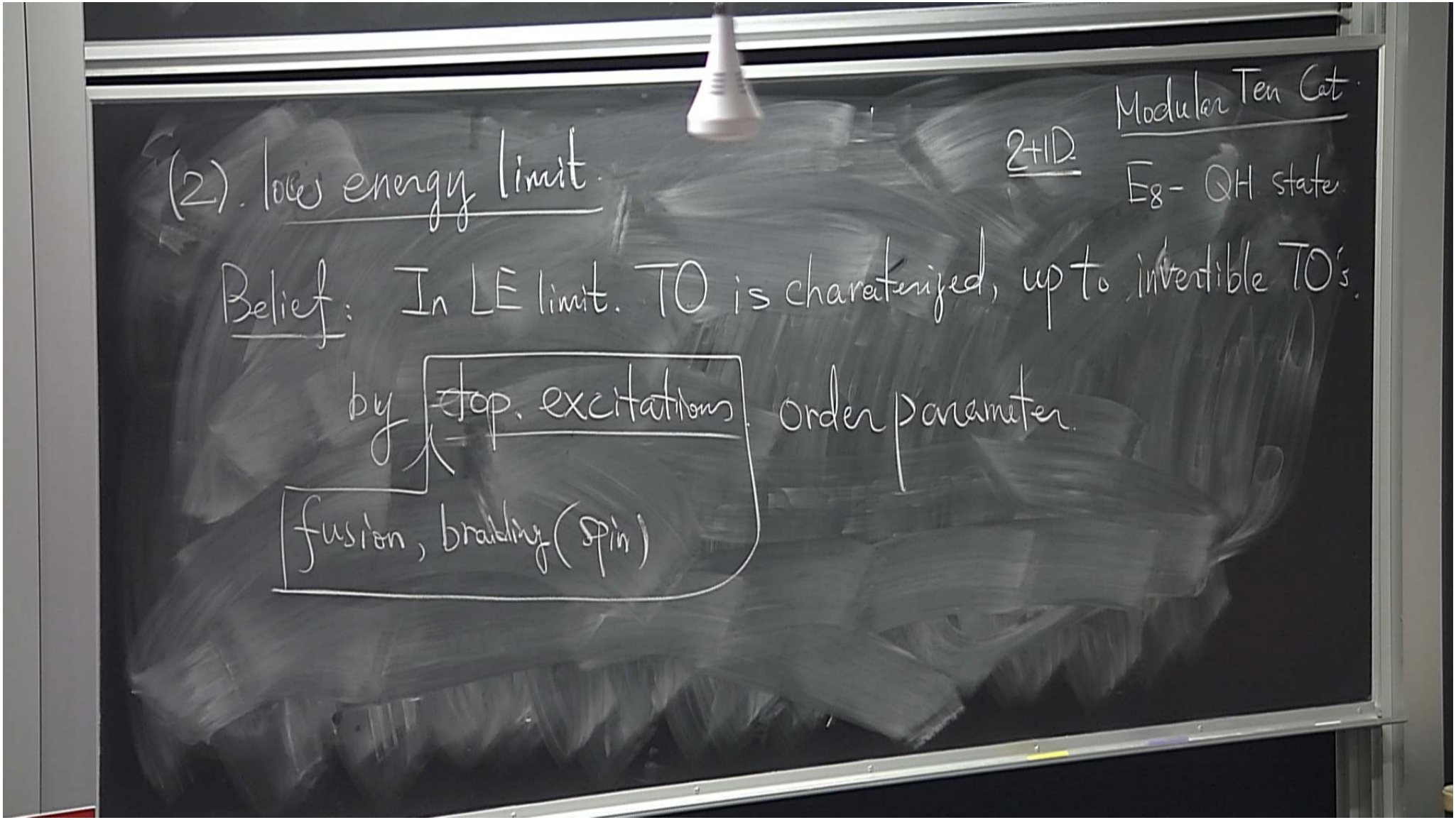

gapped
gap
gap
A (potentially anomalous) TO can be realized as a gapped bdy.
of a lattice of one-dim higher.
TO

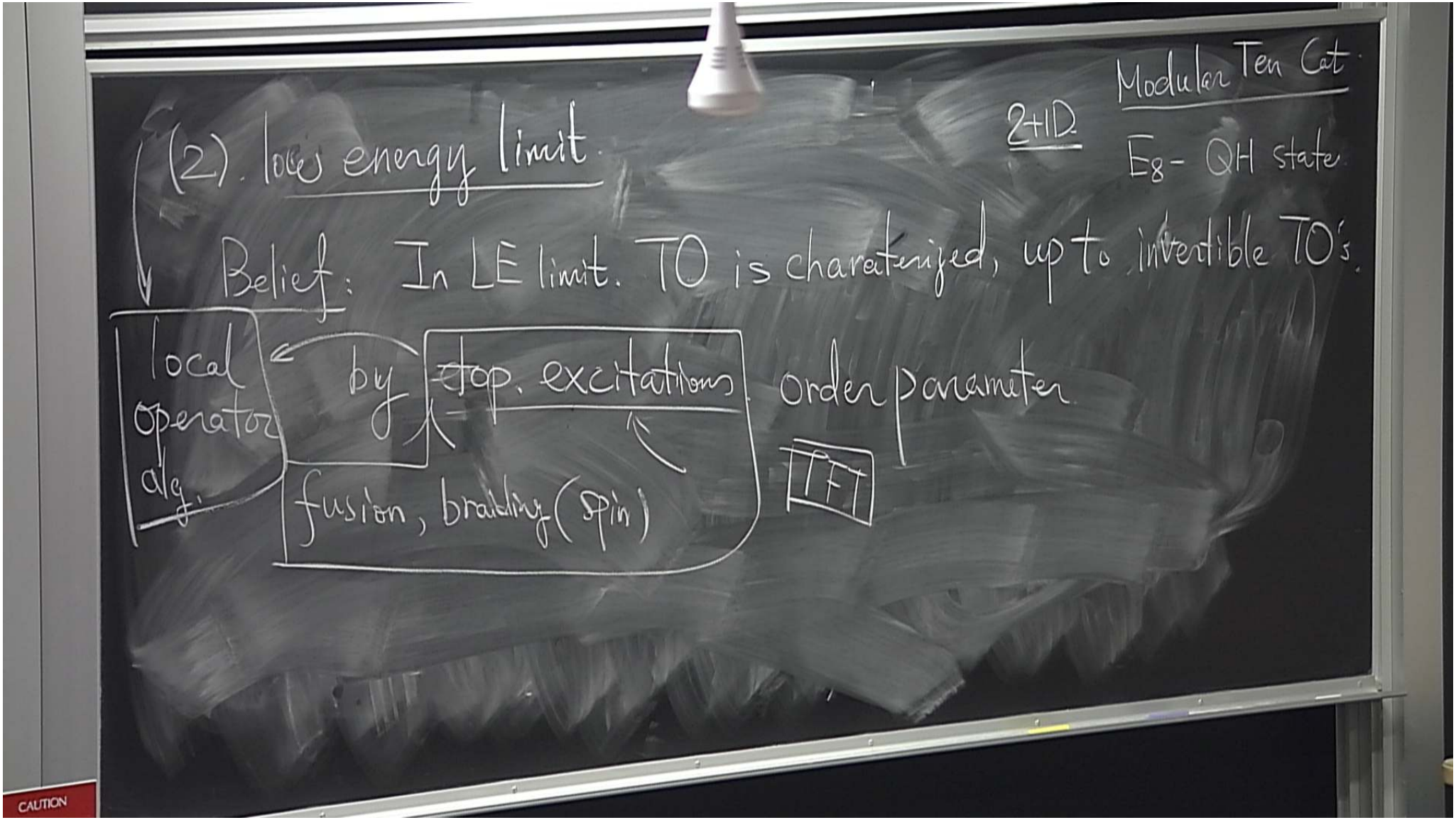

$\mathcal{O}_{n+1}$ should be def. as lattice models
$\sim$
$=?$
Two lattice models are equivalent.
if $\exists$ a nbh. of bdy such that.
we can deform one to the other
w/o closing the gap
e_{n+1}
e_n

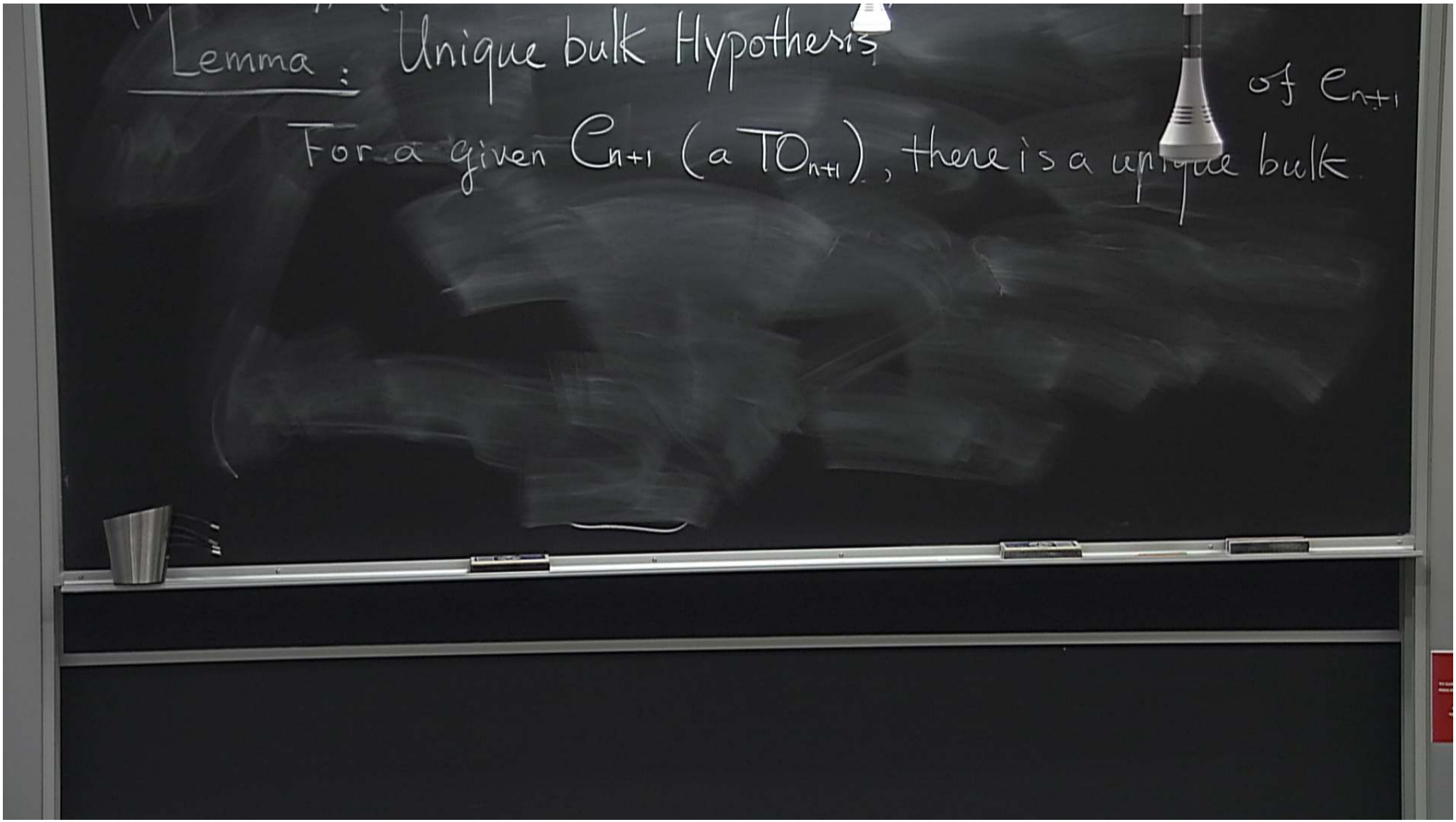

4
(1) a TO_{n+1} should be def. as lattice models / $\sim$
$=?$
Two lattice models are equivalent.
if $\exists$ a nbh. of bdy such that.
we can "deform" one to the other
w/o closing the gap
C_{n+1}

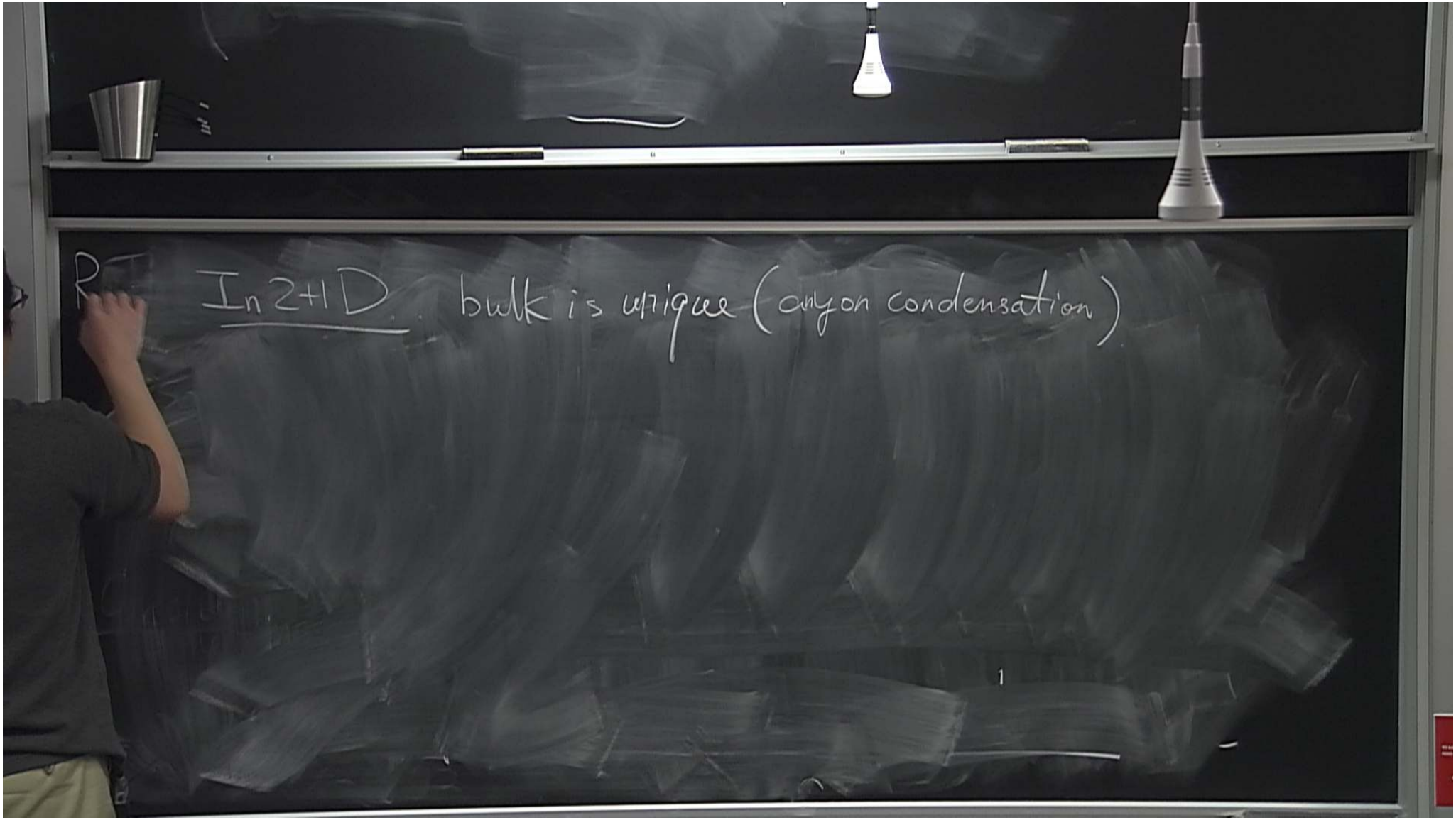

Two lattice models are equivalent.
if ∃ a nbh of bdy such that.
we can "deform" one to the other
w/o closing the gap
n+1 d → spt
n+2 D st.
e_{n+1}
TO_{n+1}
e_{n+1}

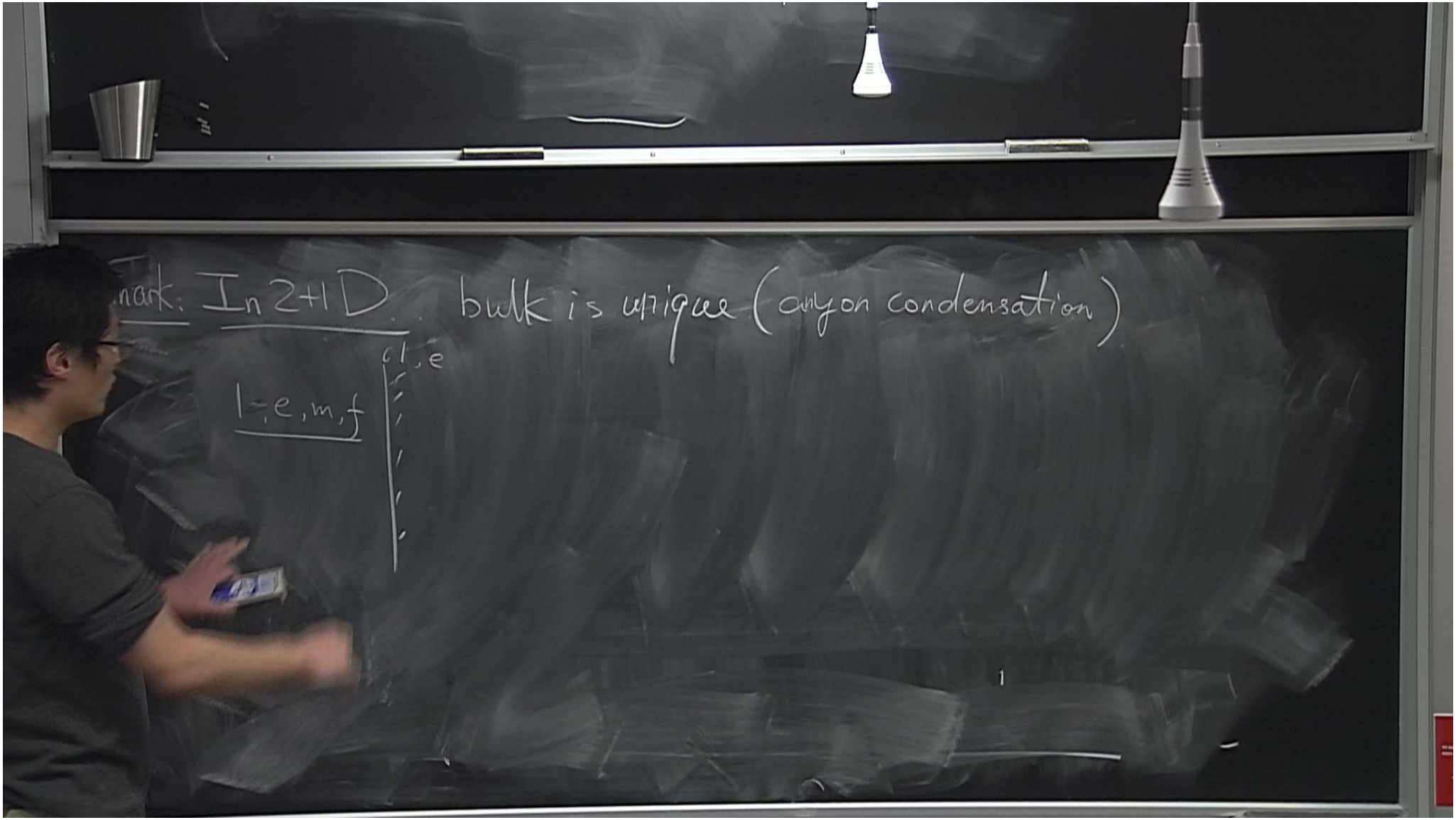

energy limit.
2+1D.
Modular Ten Cat.
E_8 - QH state.
LE limit. TO is characterized, up to invertible TO's.
by top. excitations.

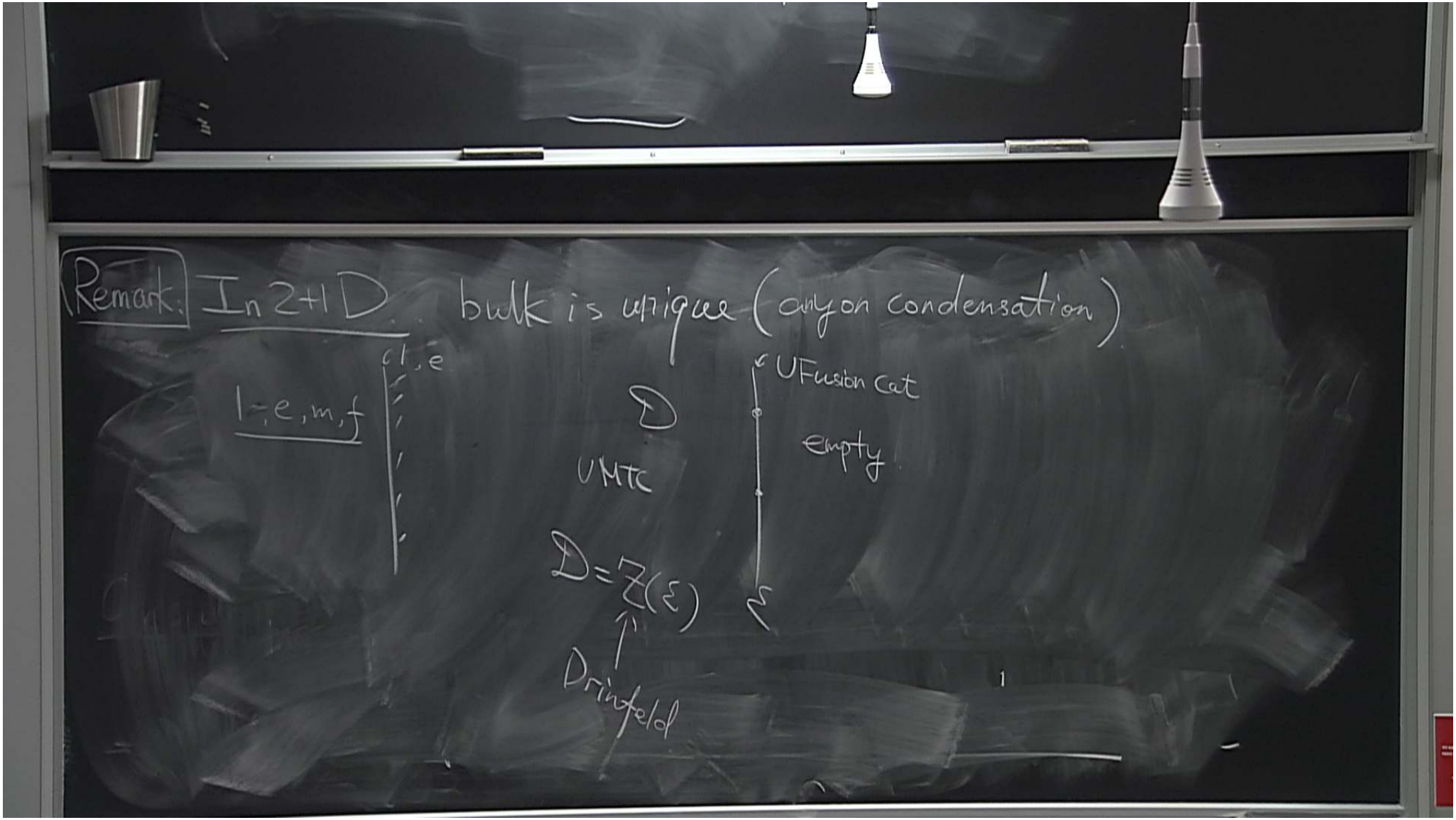

(2). low energy limit.
2+1D.
Modular Ten Cat.
E8 - QH state.
Belief: In LE limit. TO is charaterized, up to invertible TO's.
by top. excitations. order parameter.
fusion, braiding (spin)

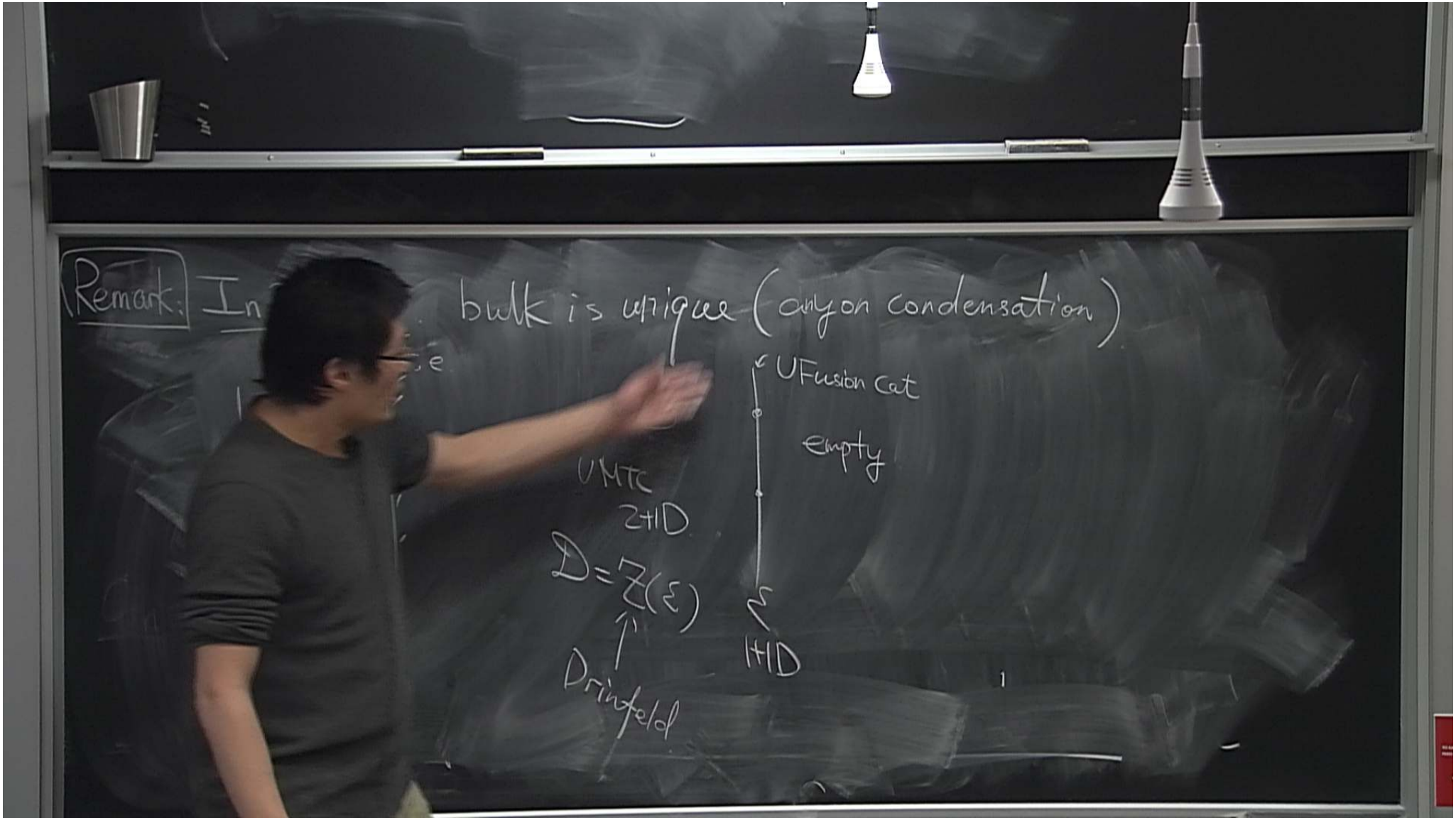

(2). low energy limit
2+1D
Modular Ten Cat
E8 - QH state
Belief: In LE limit. TO is characterized, up to invertible TO's.
local operator alg.
by
top. excitations
order parameter
fusion, braiding (spin)
FT


Lemma: Unique bulk Hypothesis
For a given C_{n+1} (a TO_{n+1}), there is a unique bulk
of C_{n+1}


RT
In 2+1D bulk is unique (anyon condensation)
1


nark: In 2+1D bulk is unique (anyon condensation)
1, e, m, f
1, e


Remark: In 2+1D bulk is unique (anyon condensation)
1, e
1, e, m, f
D
UMTC
D = Z(C)
Drinfeld
UFusion Cat
Empty


Remark:
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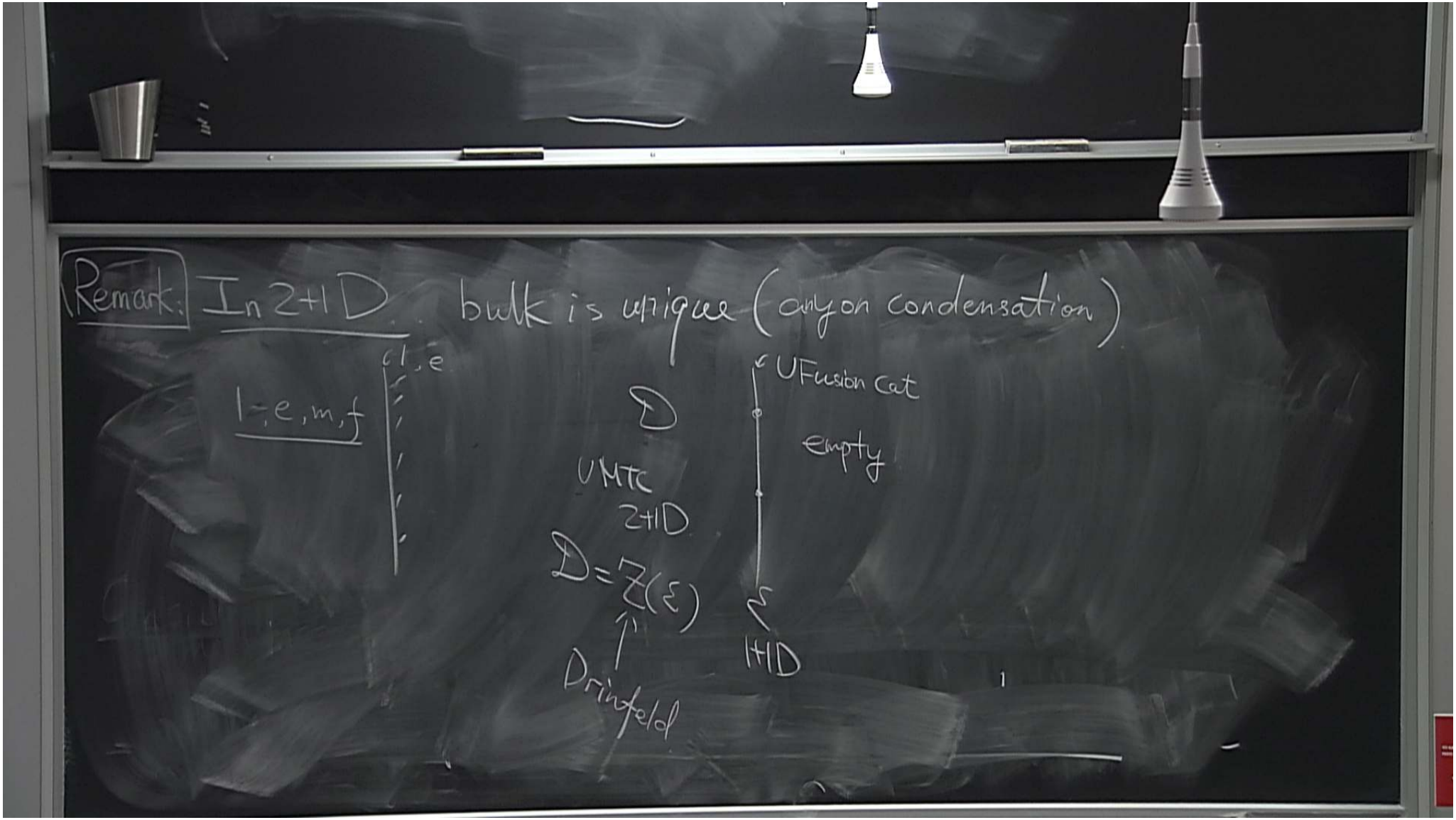
UMTC
2+1D
D=Z(ℰ)
Drinfeld
1+1D


Remark: In 2+1D bulk is unique (anyon condensation)
1, e
1, e, m, f
D
UMTC
2+1D
D = Z(ℰ)
Drinfeld
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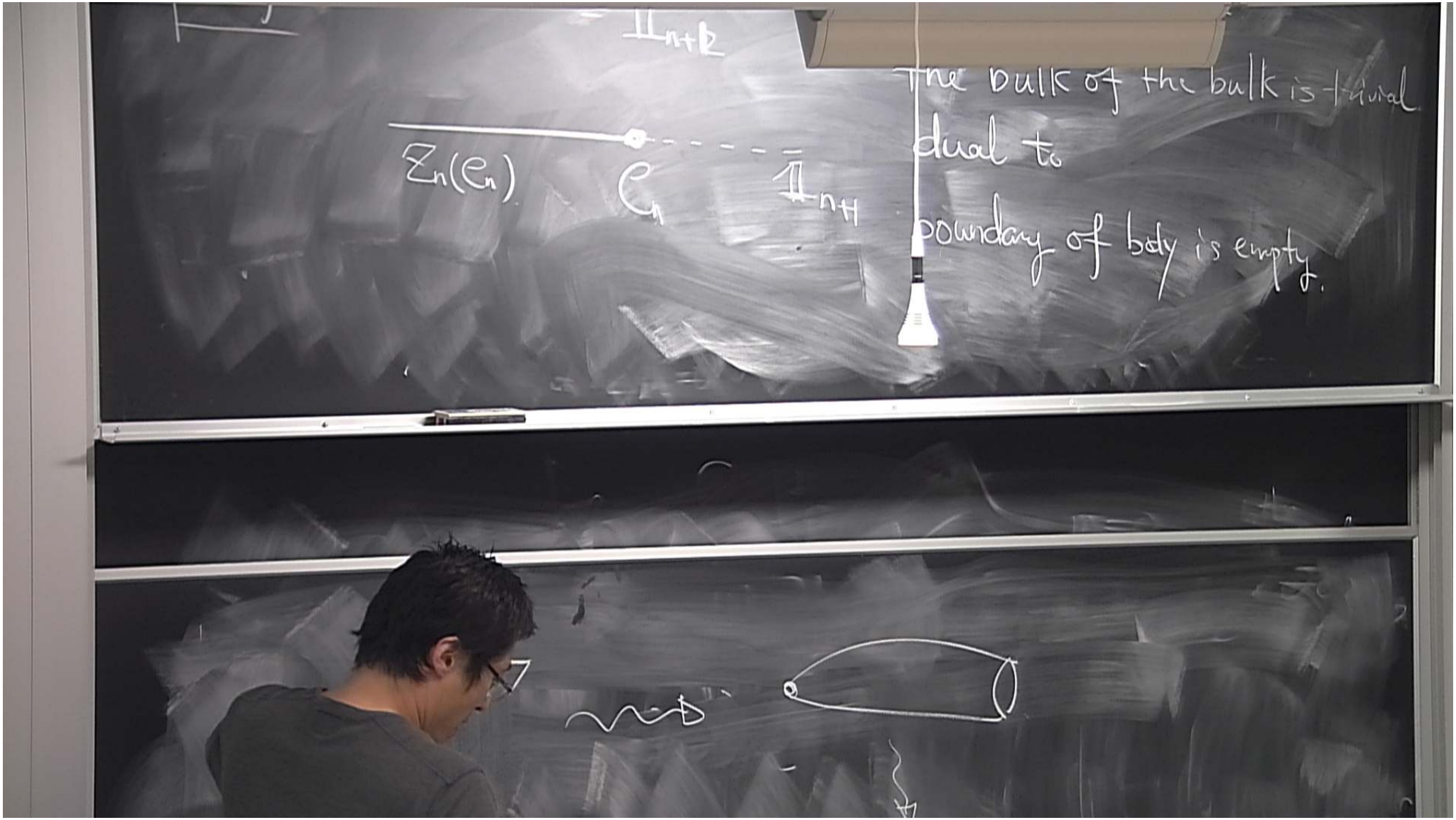
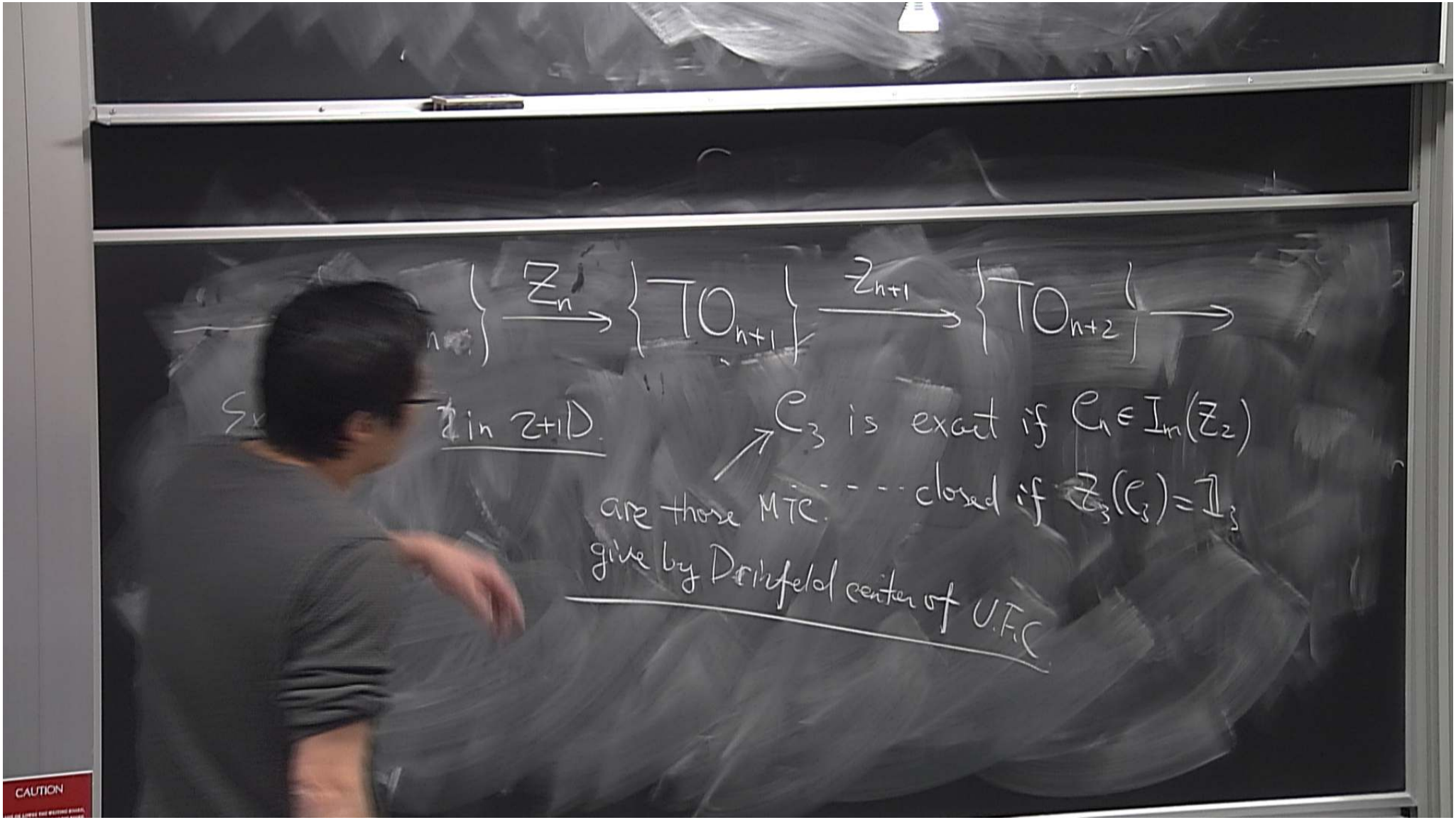
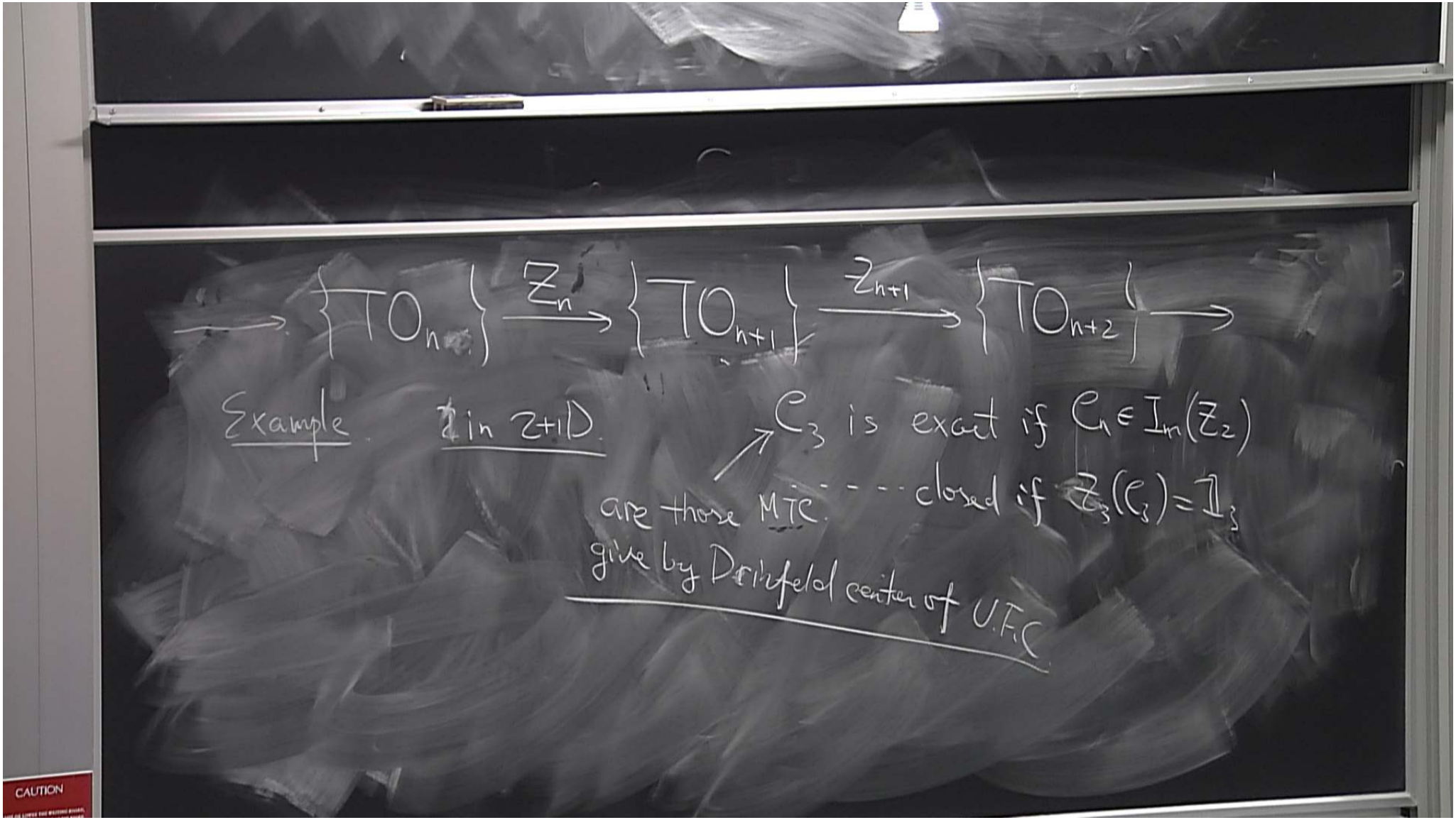
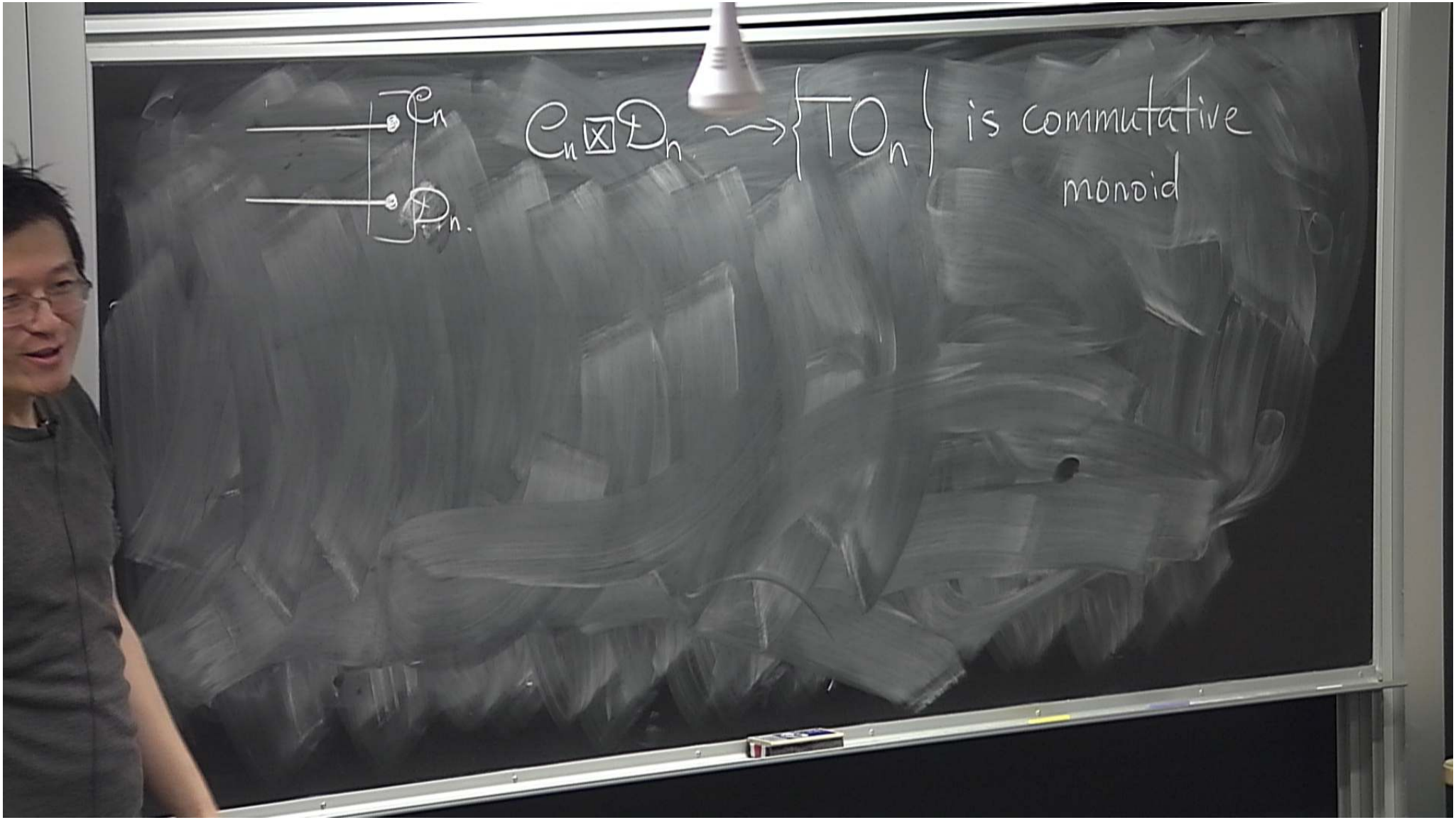
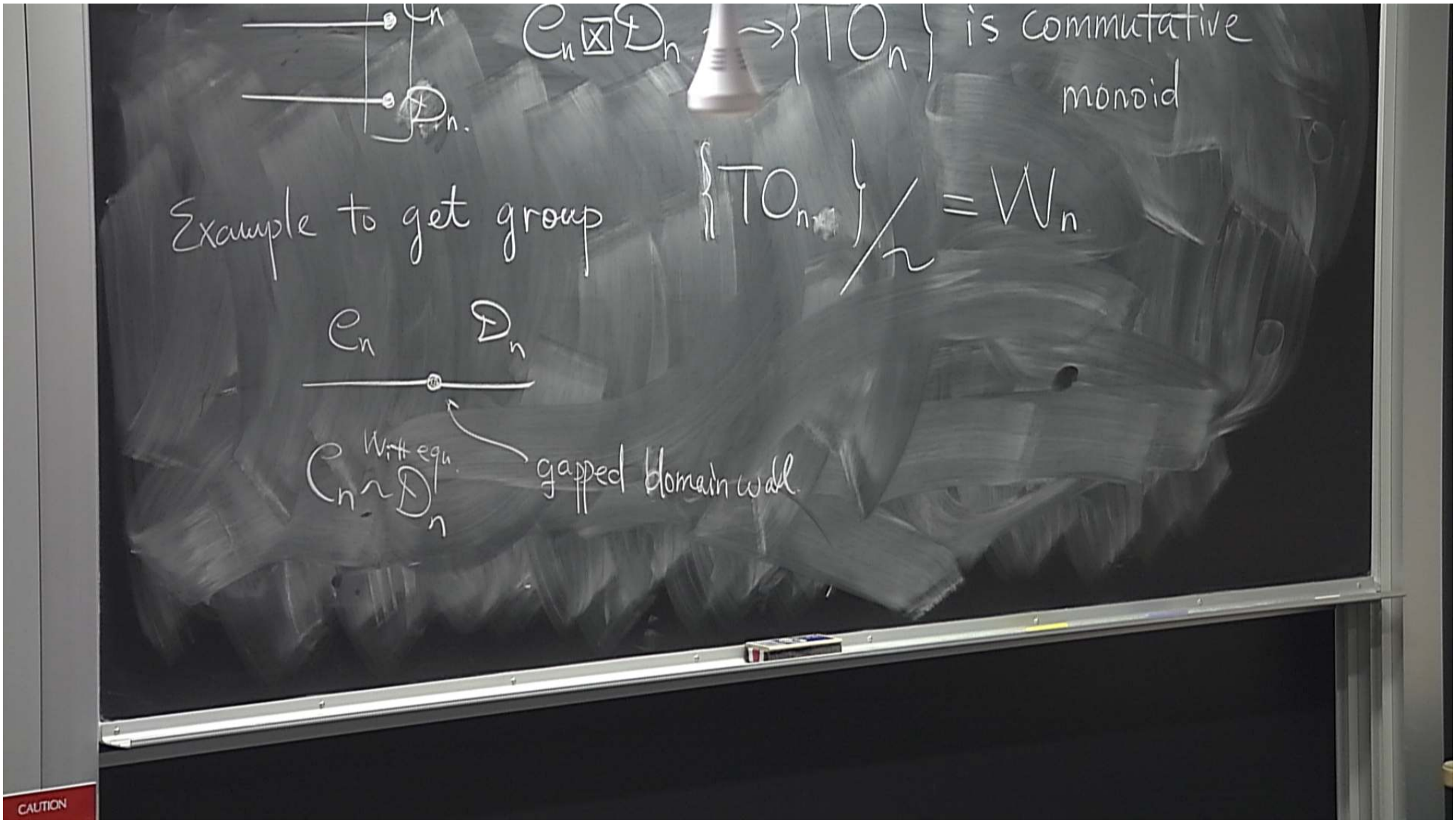
1+1D

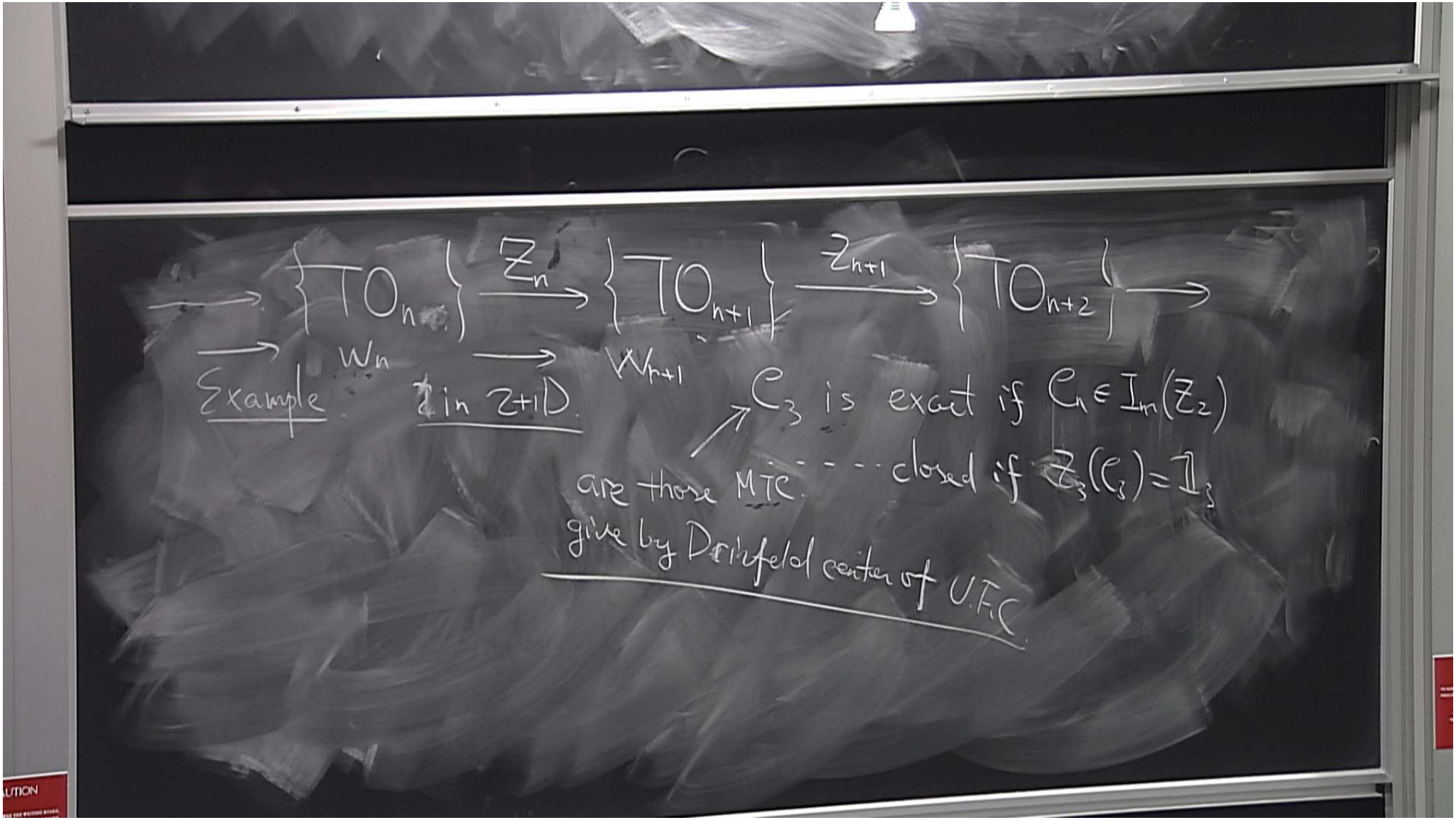

Π_{n+k}
$Z_n(C_n)$
C_n
Π_{n+1}
The bulk of the bulk is trivial
dual to
boundary of bdy is empty.

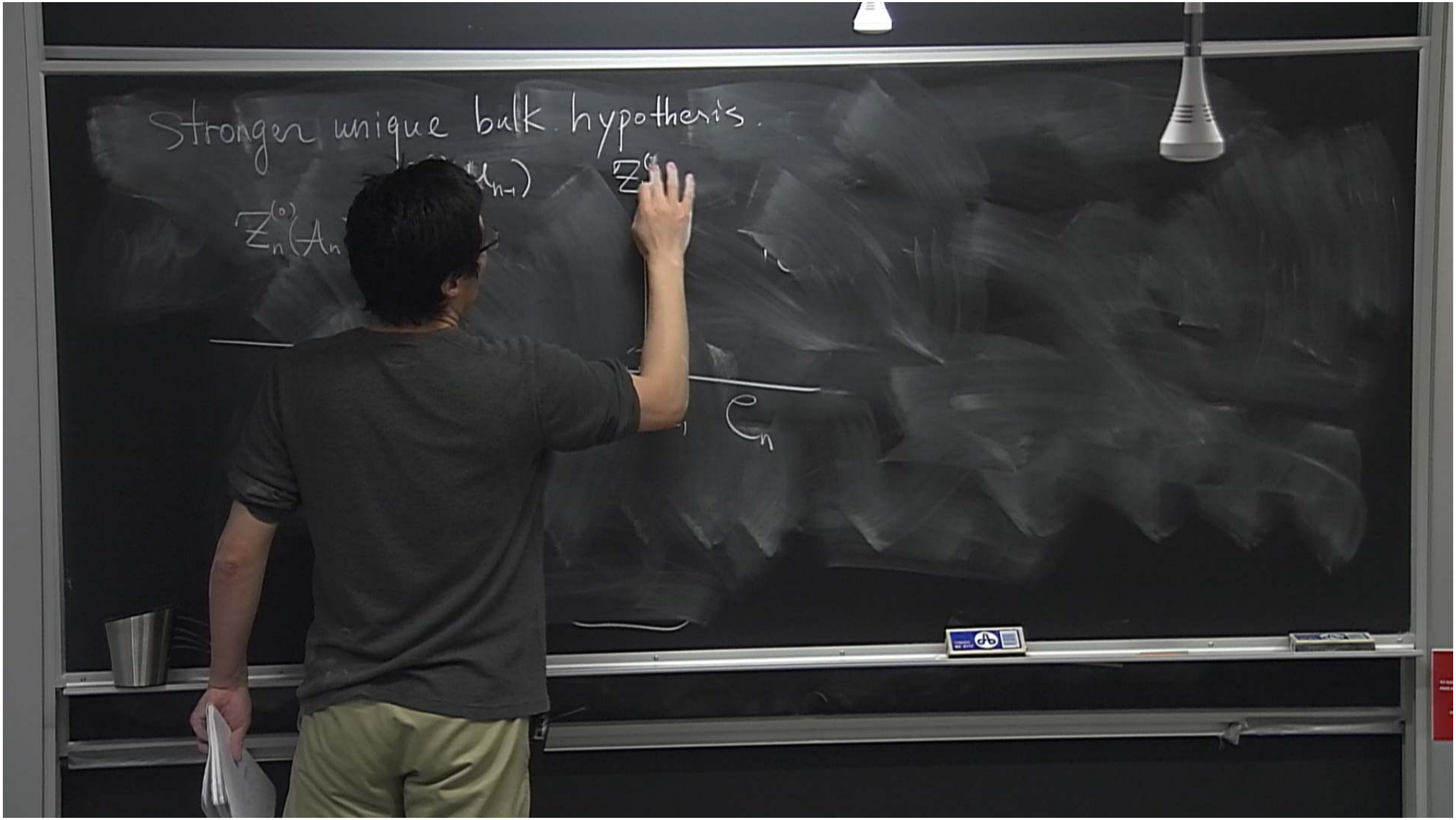

$\{Z_n\}$ $\{TO_{n+1}\}$ $\{Z_{n+1}\}$ $\{TO_{n+2}\}$
$\mathcal{C}_3$ is exact if $\mathcal{C}_4 \in Im(Z_2)$
closed if $Z_3(\mathcal{C}_3) = \mathbb{1}_3$
are those MTC
give by Drinfeld center of U.F.C
CAUTION

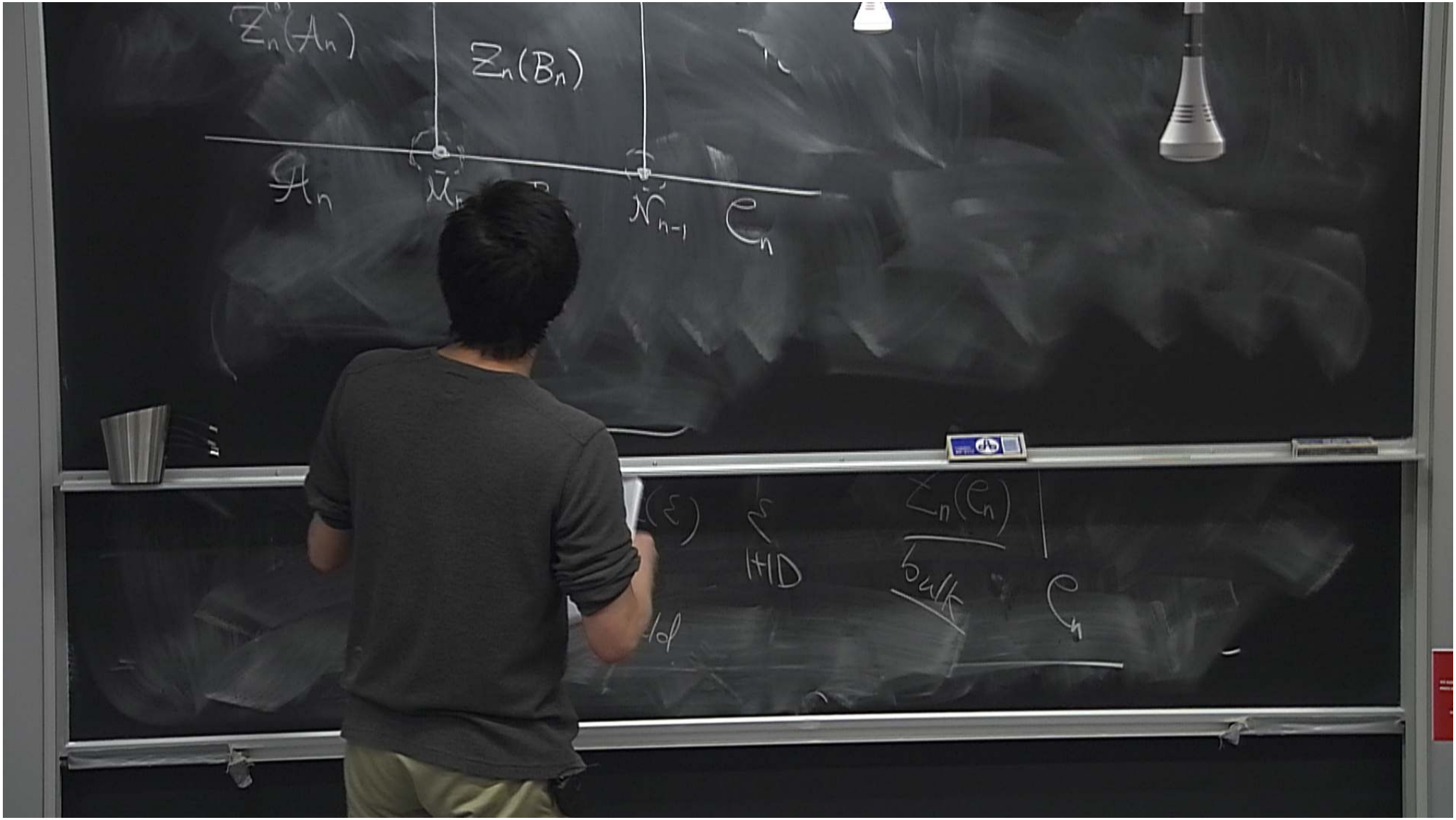

$\{TO_n\} \xrightarrow{Z_n} \{TO_{n+1}\} \xrightarrow{Z_{n+1}} \{TO_{n+2}\} \longrightarrow$
Example
$\mathbb{Z}_2$ in 2+1D.
C_3 is exact if $C_3 \in Im(Z_2)$
closed if $Z_3(C_3) = \mathbb{1}_3$
are those MTC.
give by Drinfeld center of U.F.C.
CAUTION

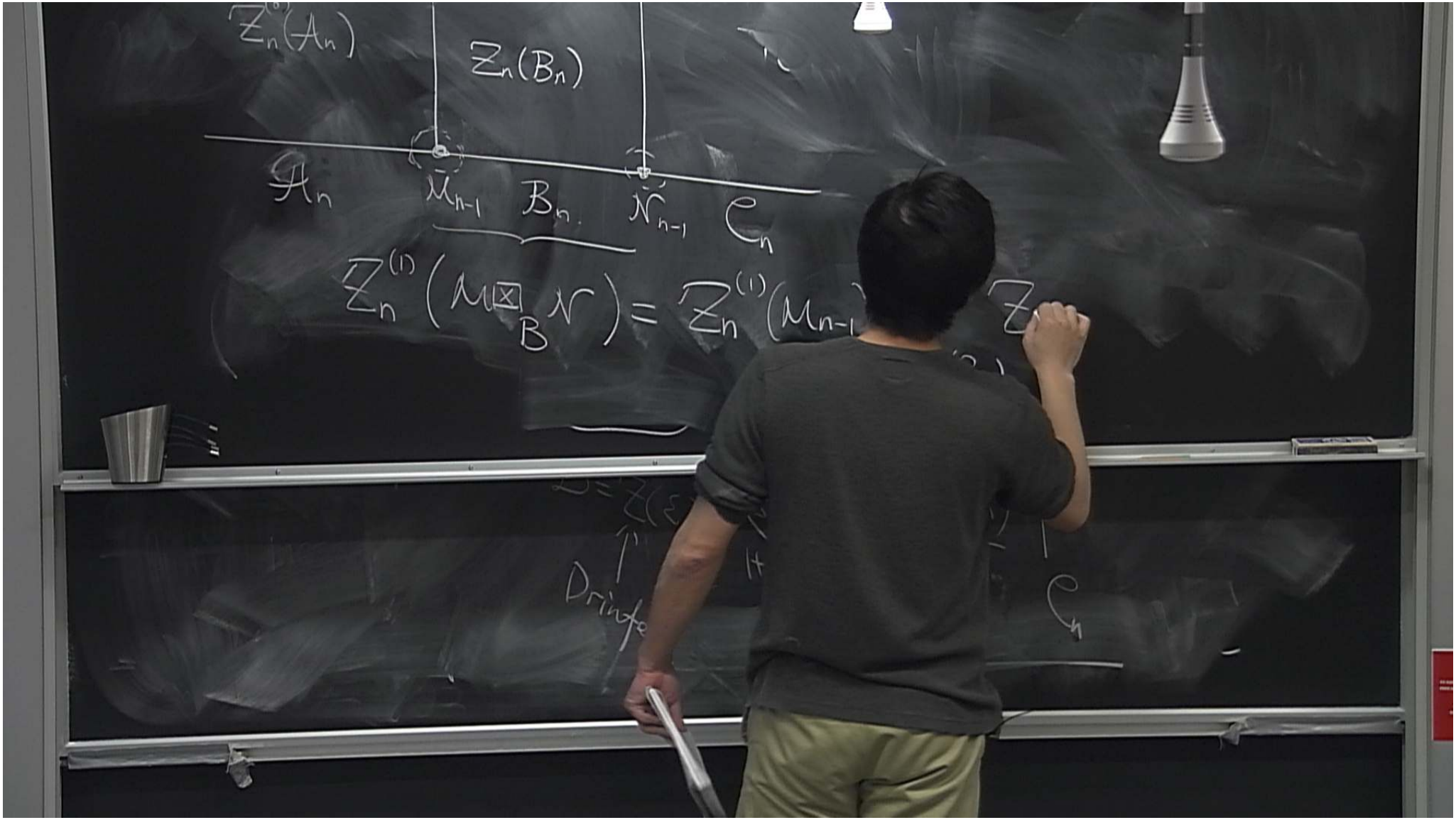

$C_n \boxtimes D_n \rightsquigarrow \{TO_n\}$ is commutative monoid

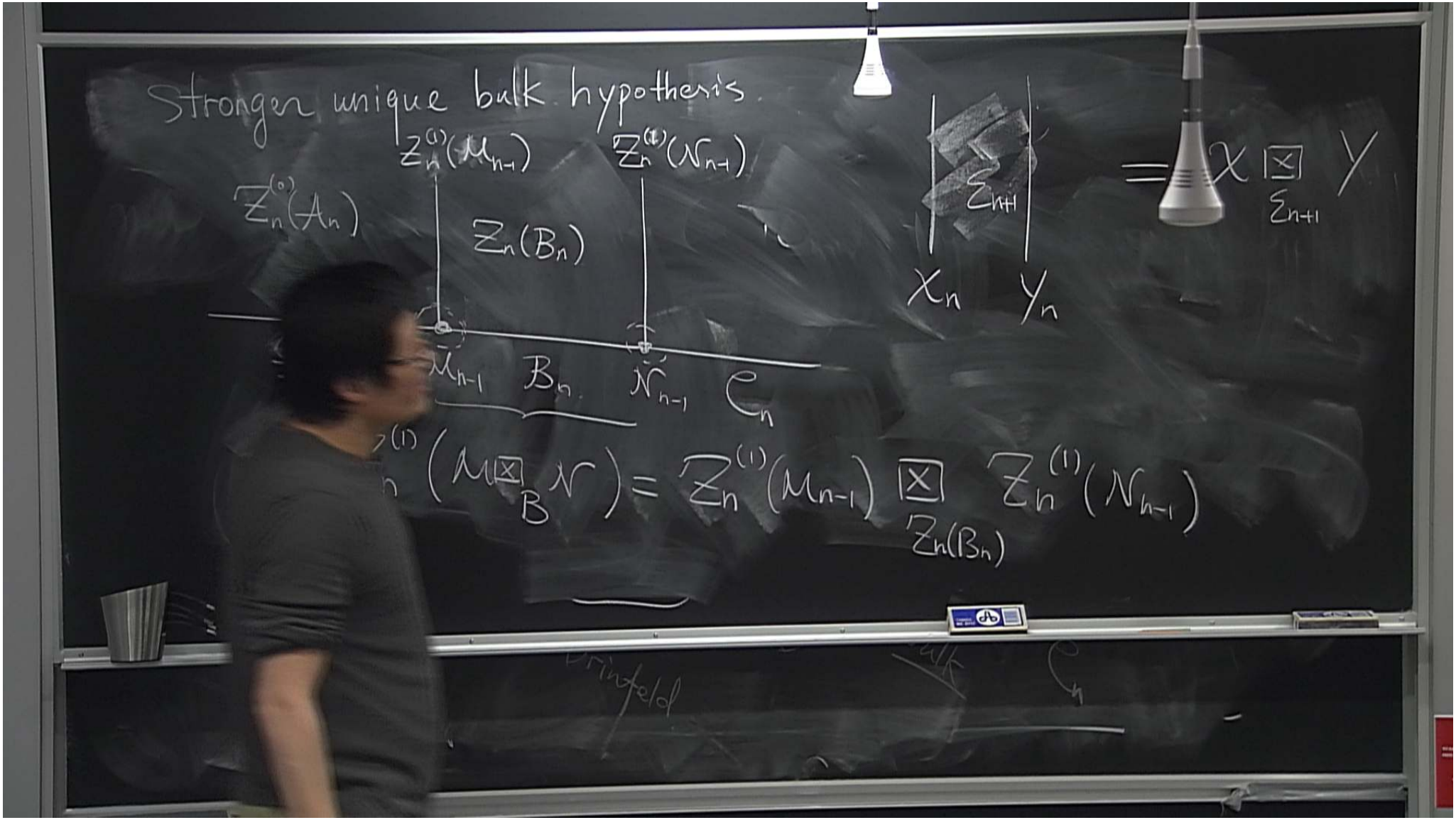

$C_n \boxtimes D_n \to \{TO_n\}$ is commutative monoid
Example to get group
$\{TO_n\}/\sim = W_n$
C_n
D_n
With equ.
$C_n \sim D_n$
gapped domain wall.

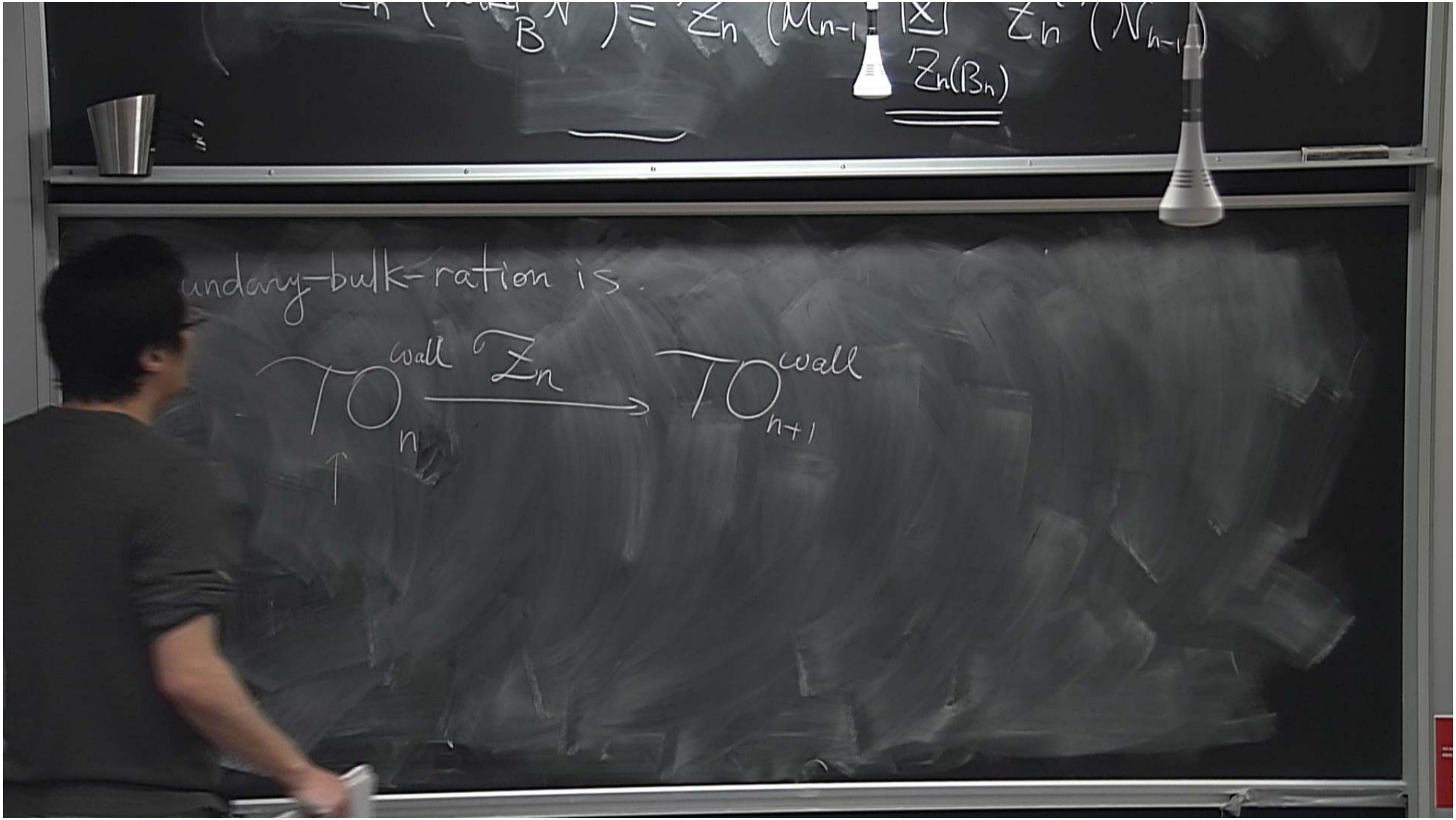

$\{TO_n\}$ Z_n $\{TO_{n+1}\}$ Z_{n+1} $\{TO_{n+2}\}$
Example W_n W_{n+1}
Z in $2+1D$.
C_3 is exact if $C_1 \in Im(Z_2)$
closed if $Z_3(C_3) = 1_3$
are those MTC
give by Drinfeld center of UFC

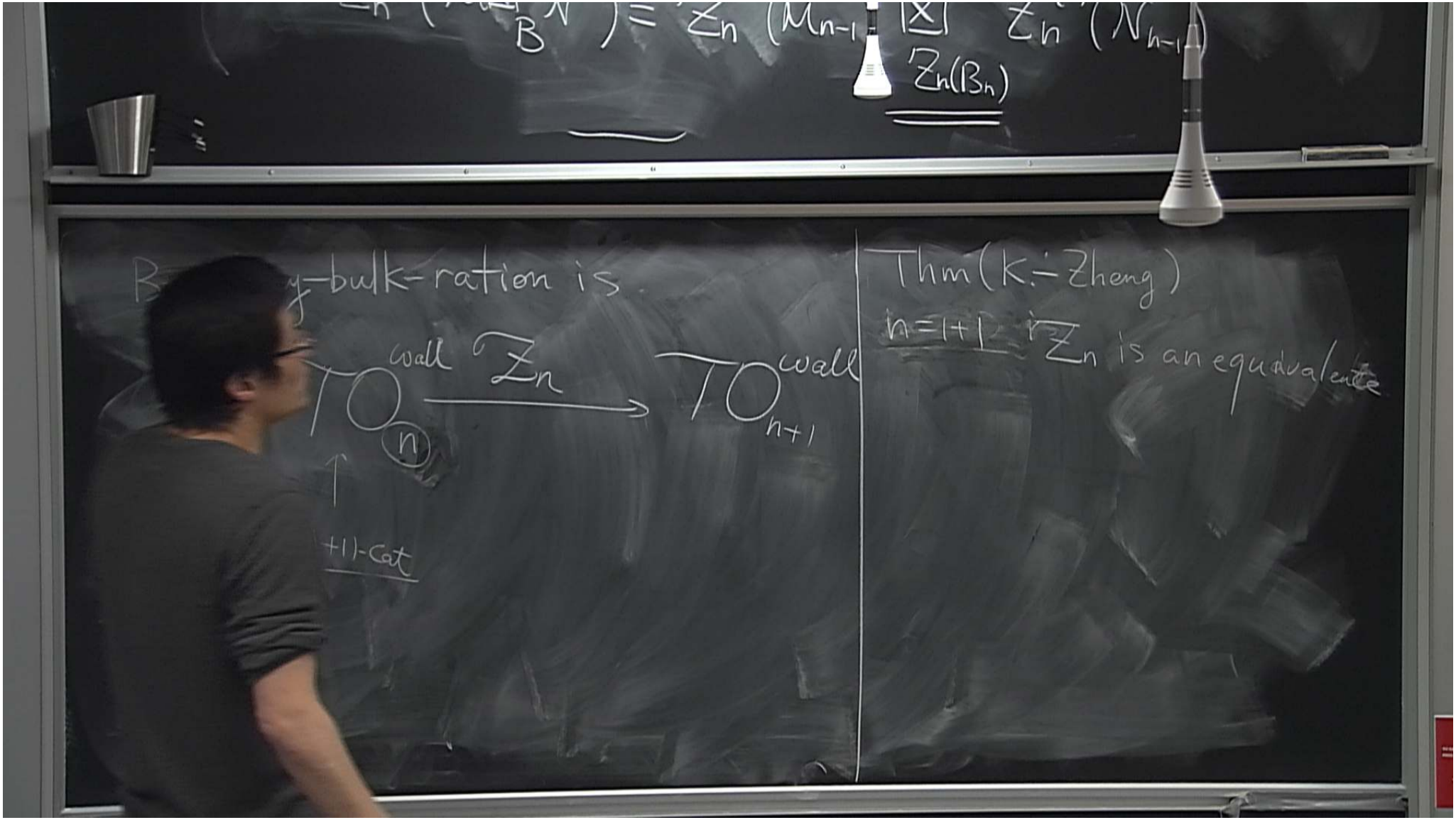

Stronger unique bulk hypothesis.

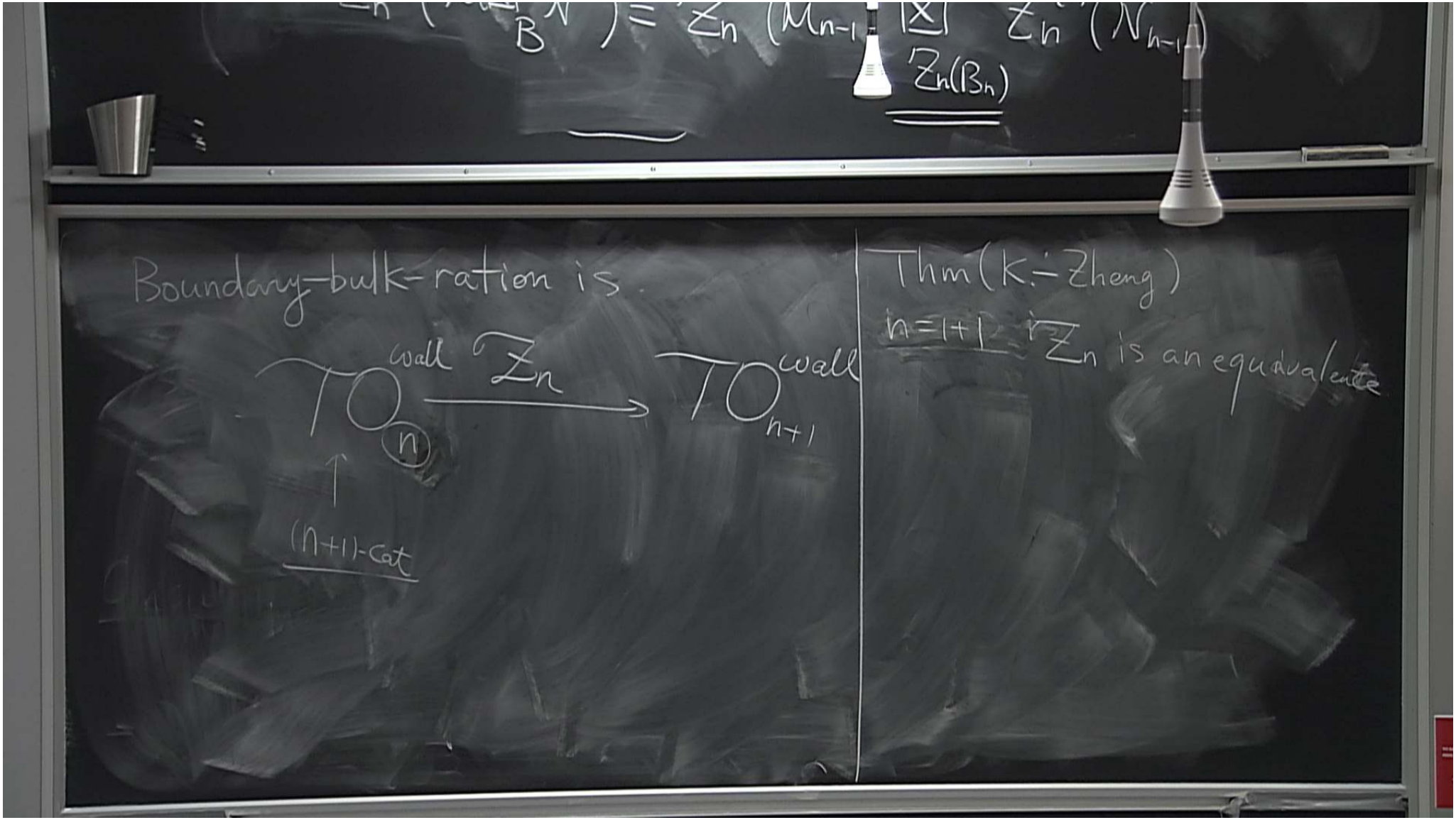

$Z_n(A_n)$
$Z_n(B_n)$
$\mathcal{A}_n$
$\mathcal{C}_n$
HID
bulk

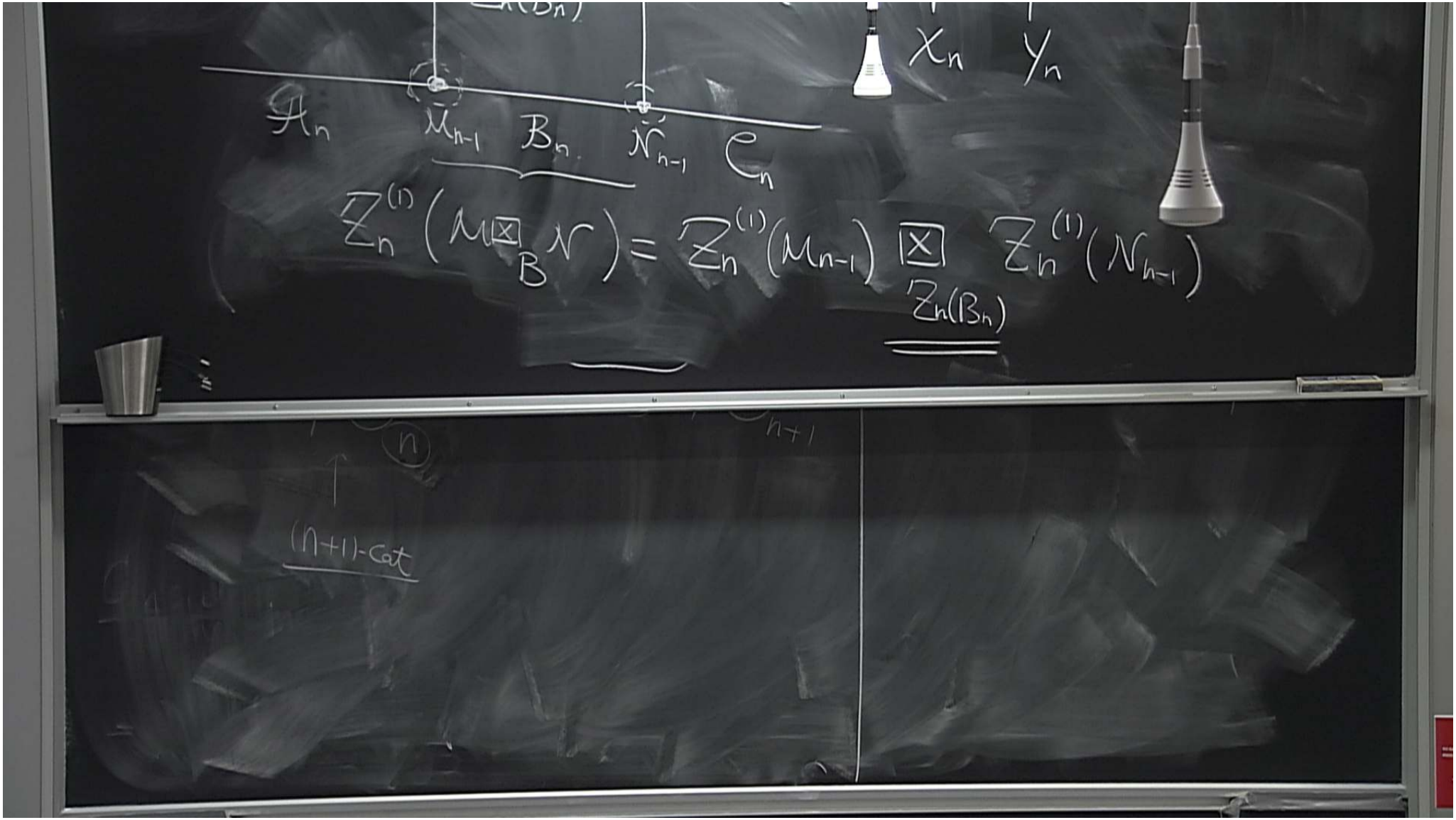

$Z_n(A_n)$
$Z_n(B_n)$
$\mathcal{A}_n$
$\mathcal{M}_{n-1}$
$\mathcal{B}_n$
$\mathcal{N}_{n-1}$
$\mathcal{C}_n$
$Z_n^{(1)}(\mathcal{M} \boxtimes_B \mathcal{N}) = Z_n^{(1)}(\mathcal{M}_{n-1}$

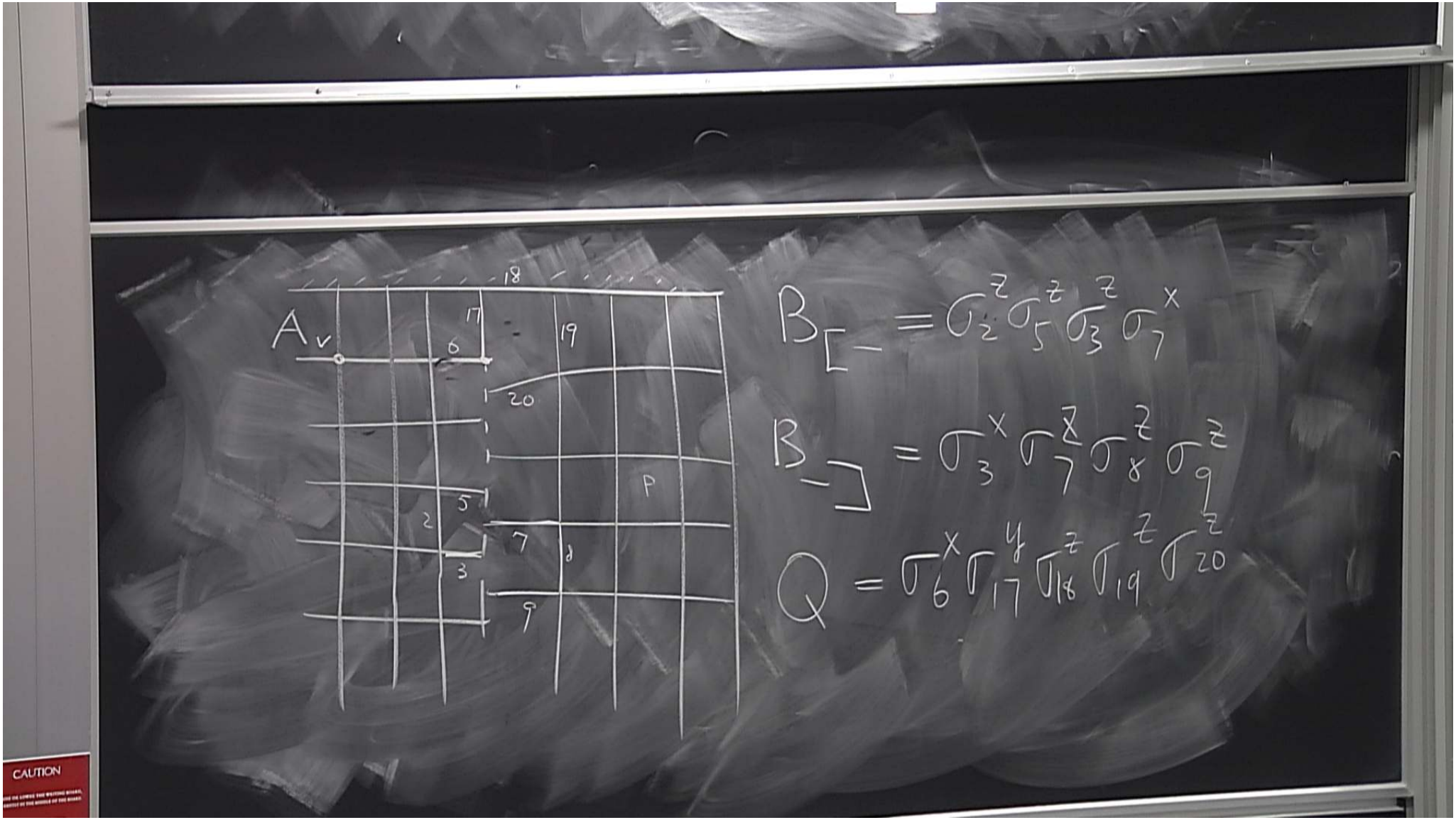

Stronger unique bulk hypothesis.
$Z_n^{(1)}(M_{n-1})$
$Z_n^{(1)}(N_{n-1})$
$Z_n^{(0)}(A_n)$
$Z_n(B_n)$
M_{n-1} B_n N_{n-1} C_n
Σ_{n+1}
X_n Y_n
$= X \boxtimes_{\Sigma_{n+1}} Y$
$Z_n^{(1)}(M \boxtimes_B N) = Z_n^{(1)}(M_{n-1}) \boxtimes_{Z_n(B_n)} Z_n^{(1)}(N_{n-1})$

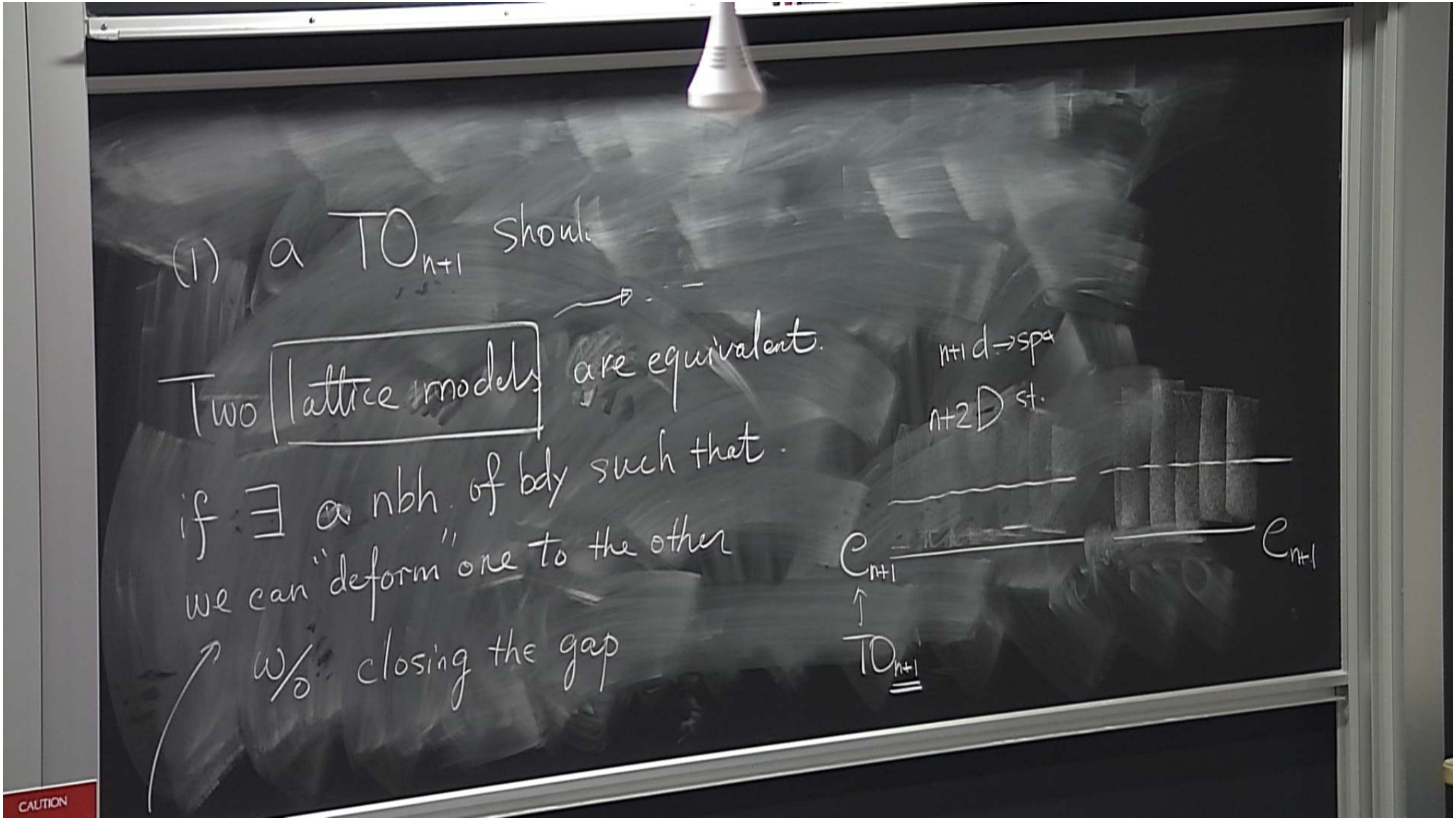

$Z_n(B_n)$
undary-bulk-ration is.
TO_n^{wall}
Z_n
TO_{n+1}^{wall}

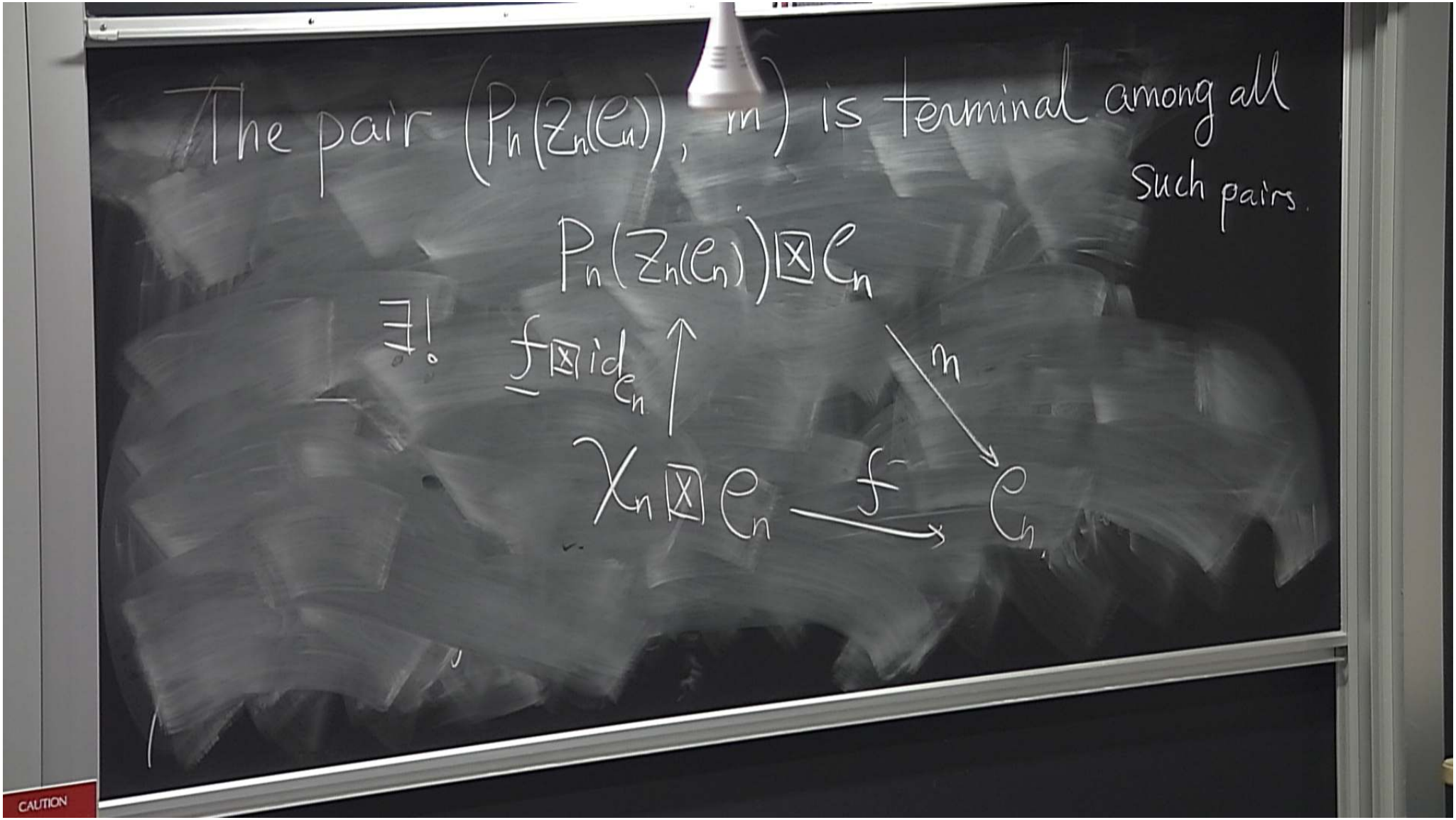

$Z_n(B_n)$
-bulk-ration is
wall $\mathcal{Z}_n$
TO_n
TO_{n+1}^{wall}
n
(n+1)-Cat
Thm (K.-Zheng)
n=1+1 $\mathcal{Z}_n$ is an equivalent

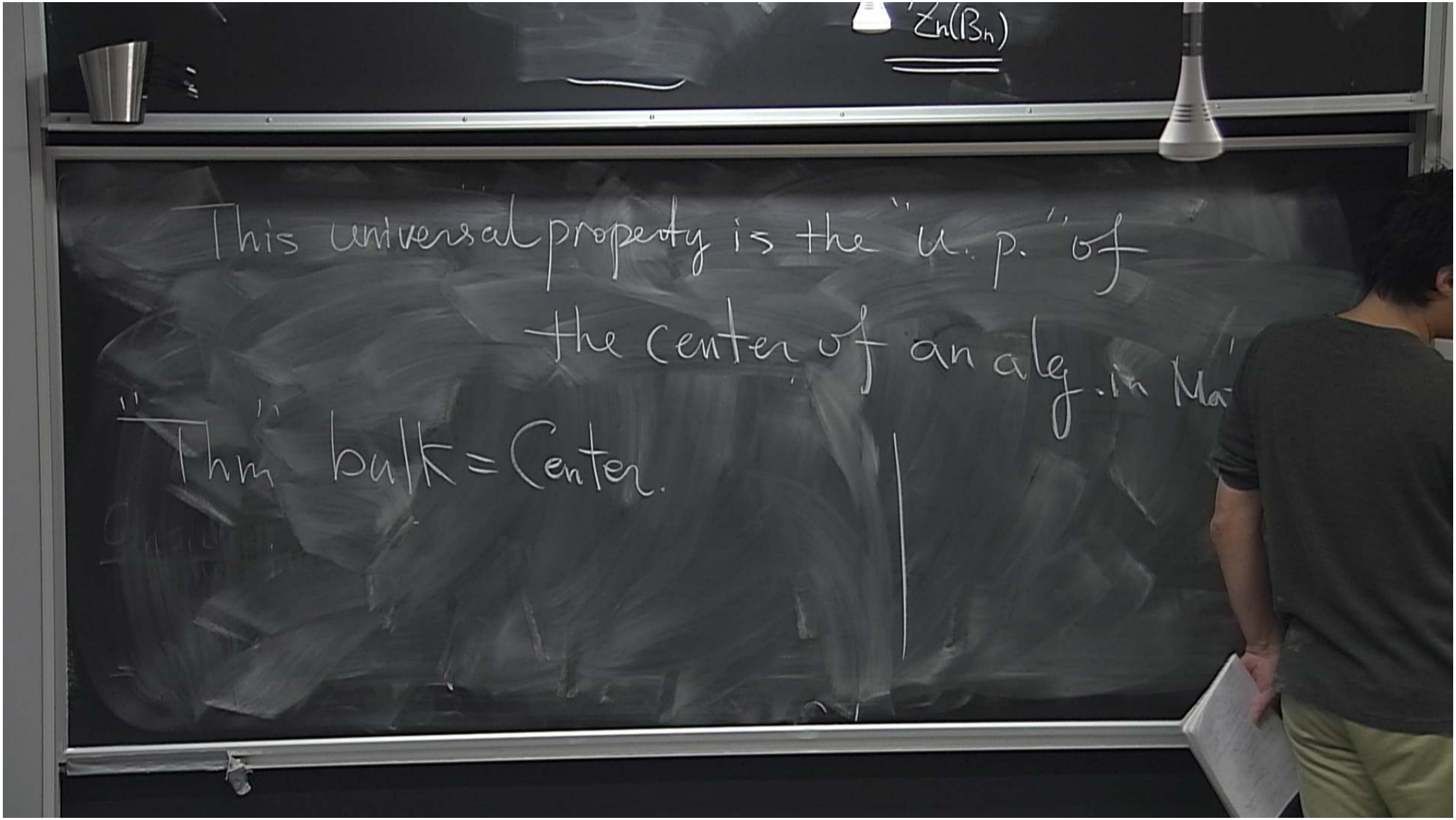

$Z_n(B_n)$
Boundary-bulk-ration is.
wall Z_n
TO_n
TO_{n+1}^{wall}
(n+1)-cat
Thm (K.-Zheng)
n=1+1 Z_n is an equivalent

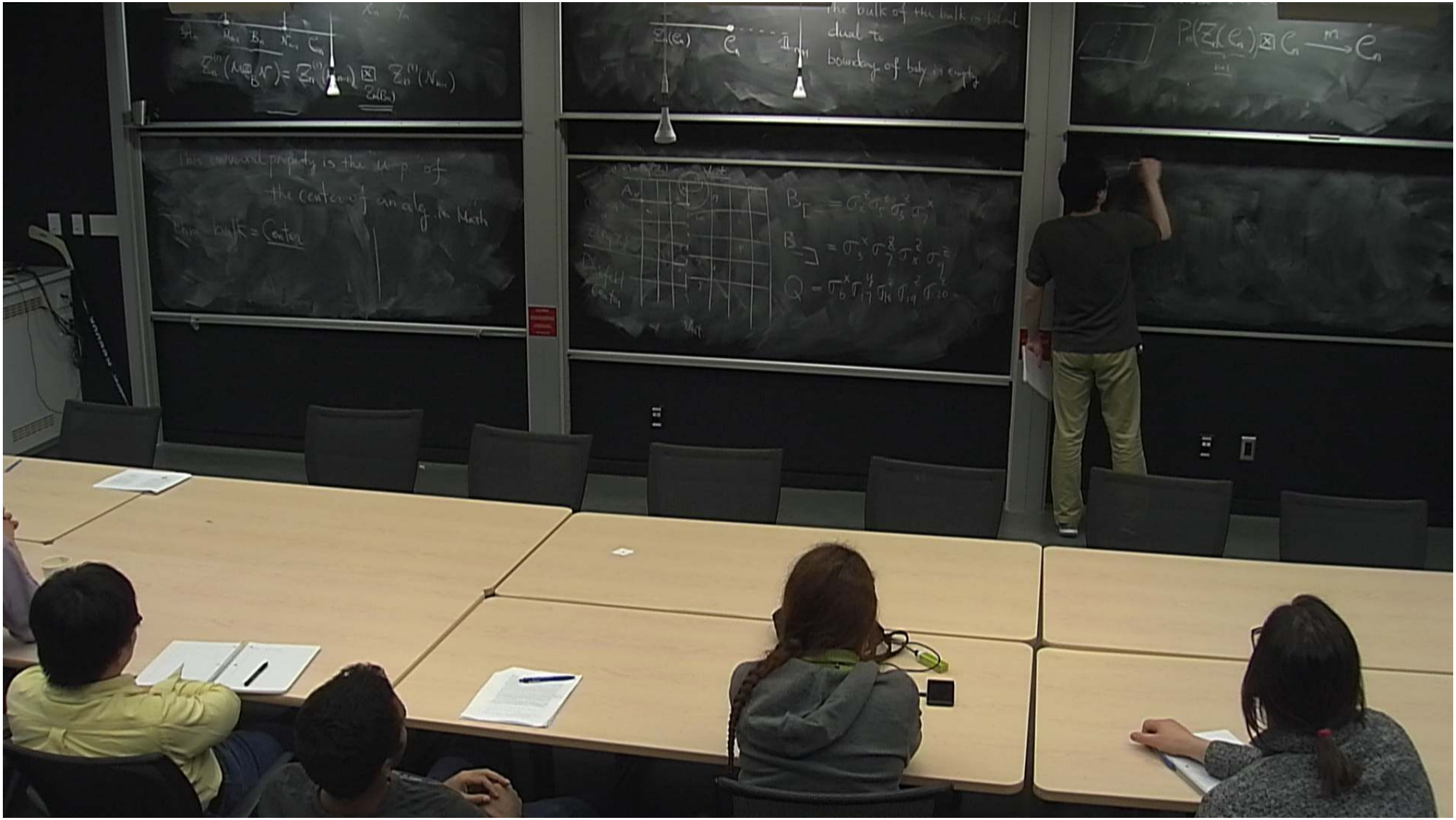

$\mathcal{A}_n$
$\mathcal{M}_{n-1}$
$\mathcal{B}_n$
$\mathcal{N}_{n-1}$
$\mathcal{C}_n$
X_n
Y_n
$Z_n^{(1)}(\mathcal{M} \boxtimes_B \mathcal{N}) = Z_n^{(1)}(\mathcal{M}_{n-1}) \boxtimes_{Z_n(\mathcal{B}_n)} Z_n^{(1)}(\mathcal{N}_{n-1})$
n
n+1
(n+1)-Cat

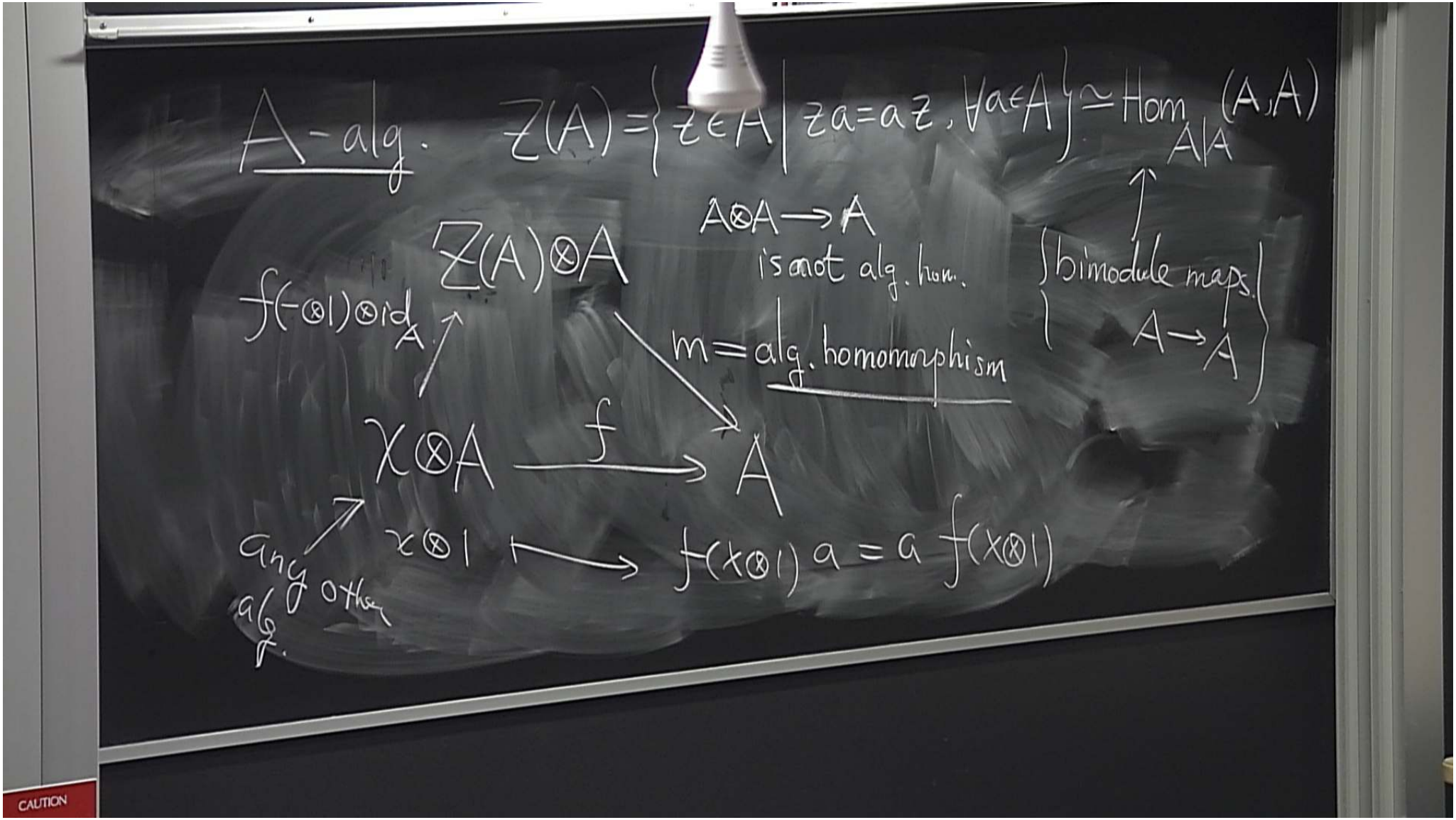

A_v
$B_{\square} = \sigma_2^z \sigma_5^z \sigma_3^z \sigma_7^x$
$B_{\square} = \sigma_3^x \sigma_7^x \sigma_8^z \sigma_9^z$
$Q = \sigma_6^x \sigma_{17}^y \sigma_{18}^z \sigma_{19}^z \sigma_{20}^z$
CAUTION


(1) a TO_{n+1} should
Two lattice models are equivalent.
if ∃ a nbh of bdy such that
we can "deform" one to the other
w/o closing the gap
n+1 d → spt
n+2 D st.
C_{n+1}
C_{n+1}
TO_{n+1}


The pair $(P_n(Z_n(C_n)), m)$ is terminal among all such pairs.
$P_n(Z_n(C_n)) \boxtimes C_n$
$\exists!$
$f \boxtimes id_{C_n}$
m
$\chi_n \boxtimes C_n$
f
C_n
CAUTION


$Z_n(B_n)$
This universal property is the "u. p." of
the center of an alg. in Ma
"Thm" bulk = Center.


the bulk of the bulk is dual to
boundary of bulk is empty
Center
Unit


A-alg.
$Z(A)=\{z\in A \mid za=az,\ \forall a\in A\}\simeq \mathrm{Hom}_{A|A}(A,A)$
$A\otimes A\to A$ is not alg. hom.
{bimodule maps $A\to A$}
$Z(A)\otimes A$
$f(-\otimes 1)\otimes id_A$
m = alg. homomorphism
$X\otimes A \xrightarrow{f} A$
any other alg.
$x\otimes 1 \longmapsto f(x\otimes 1)\,a = a\,f(x\otimes 1)$
CAUTION