Title: Nonlinear backreaction in cosmology

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Abstract: In this talk I discuss the effects of nonlinear backreaction of small scale density inhomogeneities in general relativistic cosmology. It has been proposed that in an inhomogeneous universe, nonlinear terms in the Einstein equation could, if properly averaged and taken into account, affect the large scale Friedmannian evolution of the universe. In particular, it was hoped that these terms might mimic a cosmological constant and eliminate the need for dark energy. After reviewing some of these approaches, and some of their flaws, I will describe a perturbative framework (developed with R. Wald) designed to properly take into account these effects. In our framework, we assume that the spacetime metric is "close"---within 1 part in 10^4, except near strong field objects---to a background metric of FLRW symmetry, but we do not assume that the background metric satisfies the Friedmann equation. We also do not require that spacetime derivatives of the metric be close to derivatives of the background metric. This allows for significant deviations in geodesics, and very large curvature inhomogeneities. A priori, this framework also allows for significant backreaction, which would take the form of new effective matter sources in the Friedmann equation. Nevertheless, we prove that if the matter stress-energy tensor satisfies the weak energy condition, then large matter inhomogeneities on small scales cannot produce significant backreaction effects on large scales, and in particular cannot account for dark energy. As I will also review here, with a suitable â€˜dictionary,â€™ Newtonian cosmologies provide excellent approximations to cosmological solutions to EinsteinÊ¼s equation (with dust and a cosmological constant) on all scales. Our results thereby provide strong justification for the mathematical consistency and validity of the LCDM model within the context of general relativistic cosmology. While our rigorous framework makes use of 1-parameter families and weak limits, in this talk I will provide a simple heuristic discussion that places emphasis on the manner in which "averaging" is done, and the fact that one is solving the Einstein equation.

Historical introduction

- In cosmology, the universe is homogeneous and isotropic at large scales. Perturbations behave linearly.

- At small scales, density fluctuations are large, $|\delta\rho/\rho_0| \gg 1$, and nonlinear dynamics important (e.g., bound systems).

- Issue: Because of nonlinear terms, computing the Einstein tensor of a metric, and averaging, do not commute, so

$$G_{ab}\left(\langle g\rangle\right) \neq \langle G_{ab}(g)\rangle = \langle 8\pi T_{ab}\rangle$$

Historical introduction

- Large scale metric satisfies an Einstein equation with new effective stress-energy tensor

$$G_{ab}\left(g^{(0)}\right) + \Lambda g^{(0)}_{ab} = 8\pi\left(T^{(0)}_{ab} + t^{(0)}_{ab}\right)$$

"Backreaction" arises from averages of small scale nonlinear terms

- Idea revived after discovery of accelerated expansion: What if $t^{(0)}_{ab} = -Cg^{(0)}_{ab}$?

Historical introduction

- Arguments against significant backreaction

Can the acceleration of our universe be explained by the effects of inhomogeneities?

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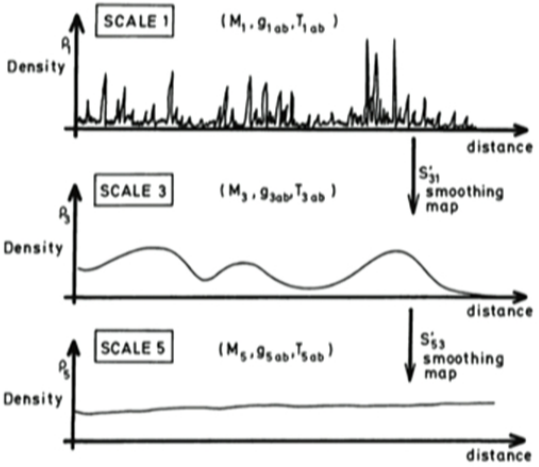
Abstract
No, it is simply not plausible that cosmic acceleration could arise within the context of general relativity from a back-reaction effect of inhomogeneities in our universe, without the presence of a cosmological constant or 'dark energy'.

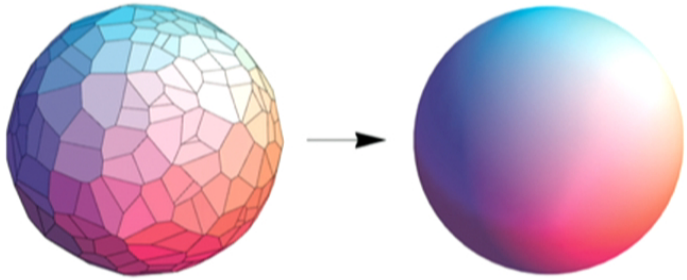
Motivation

- **Observation 1**: *While density inhomogeneities are large at small scales, metric inhomogeneities are small, except near strong field objects (BHs, NSs).*

"Fitting problem" for the matter



"Fitting problem" for the metric



- Intuition: Poisson equation $\nabla^2\phi = 4\pi\rho$

Motivation

- **Observation 2**: *Even with small metric perturbations, significant backreaction is possible if derivatives are large.*

E.g., vacuum universe filled with gravitational waves. $g_{ab} = g^{(0)}_{ab} + \gamma_{ab}$

FLRW symmetry

$\gamma \sim \mathcal{A}\sin(x/l)$

Motivation

- **Observation 2**: *Even with small metric perturbations, significant backreaction is possible if derivatives are large.*

  E.g., vacuum universe filled with gravitational waves. $g_{ab} = g^{(0)}_{ab} + \gamma_{ab}$

  FLRW symmetry

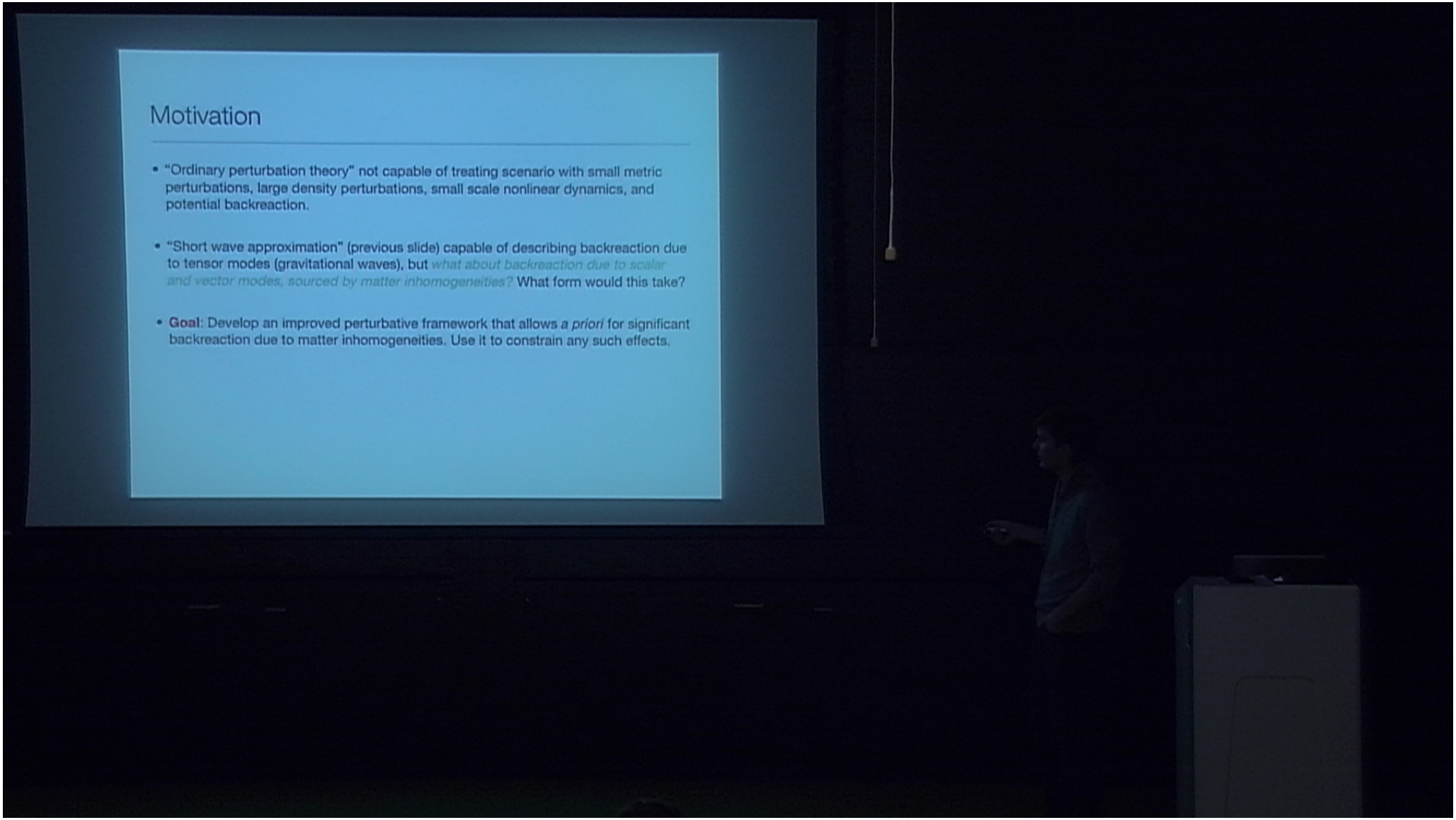
  $\gamma \sim \mathcal{A}\sin(x/l)$

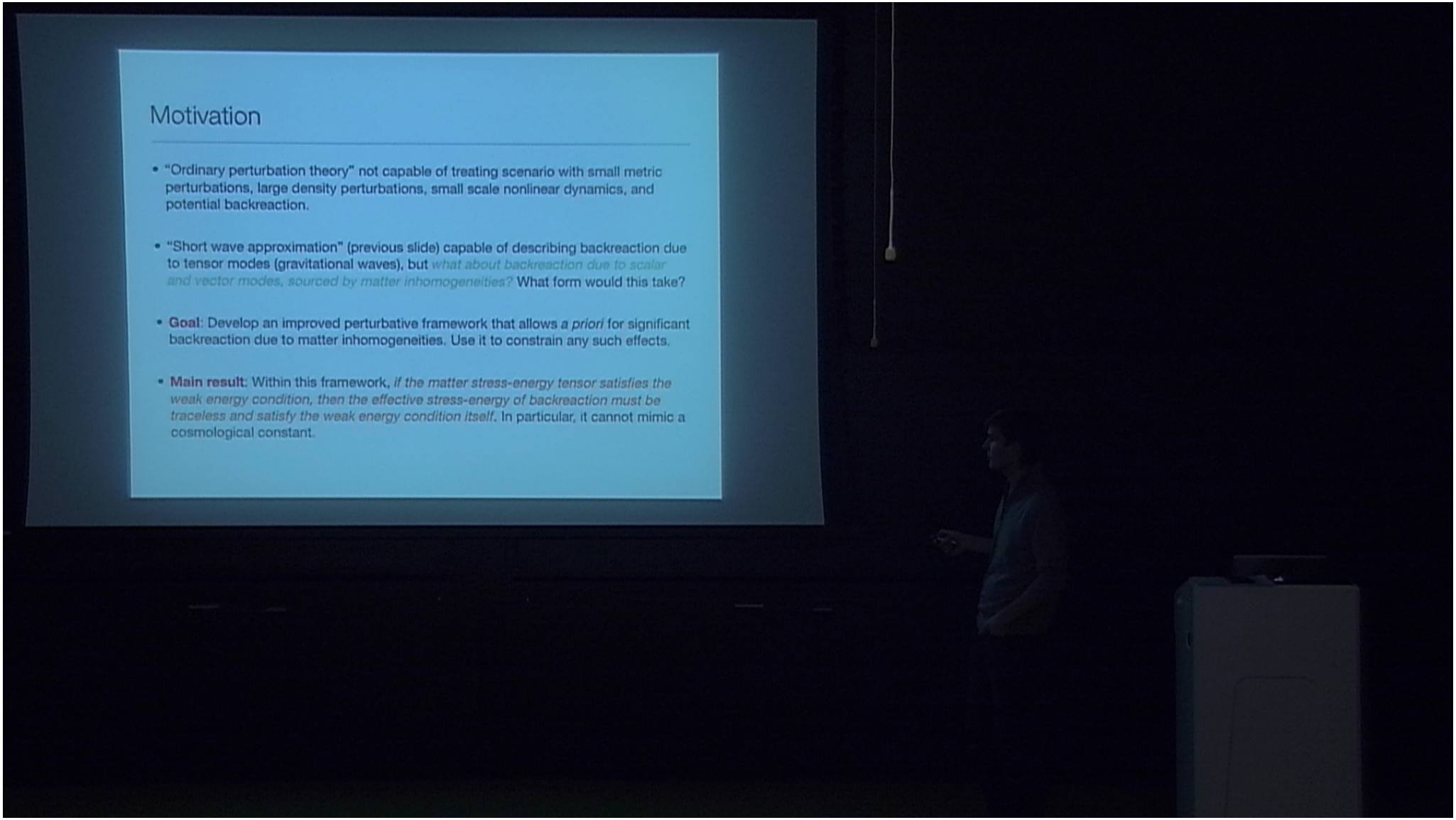
- Expand Einstein equation:

$$0 = G_{ab}(g^{(0)} + \gamma)$$
$$\approx \underbrace{G_{ab}(g^{(0)})}_{1/R^2} + \underbrace{G^{(1)}_{ab}(g^{(0)}, \gamma)}_{\mathcal{A}/l^2} + \underbrace{G^{(2)}_{ab}(g^{(0)}, \gamma)}_{\mathcal{A}^2/l^2} + \cdots$$

next order | leading order | next order

take $\mathcal{A}R/l \sim 1$


Motivation
• "Ordinary perturbation theory" not capable of treating scenario with small metric perturbations, large density perturbations, small scale nonlinear dynamics, and potential backreaction.
• "Short wave approximation" (previous slide) capable of describing backreaction due to tensor modes (gravitational waves), but what about backreaction due to scalar and vector modes, sourced by matter inhomogeneities? What form would this take?
• Goal: Develop an improved perturbative framework that allows a priori for significant backreaction due to matter inhomogeneities. Use it to constrain any such effects.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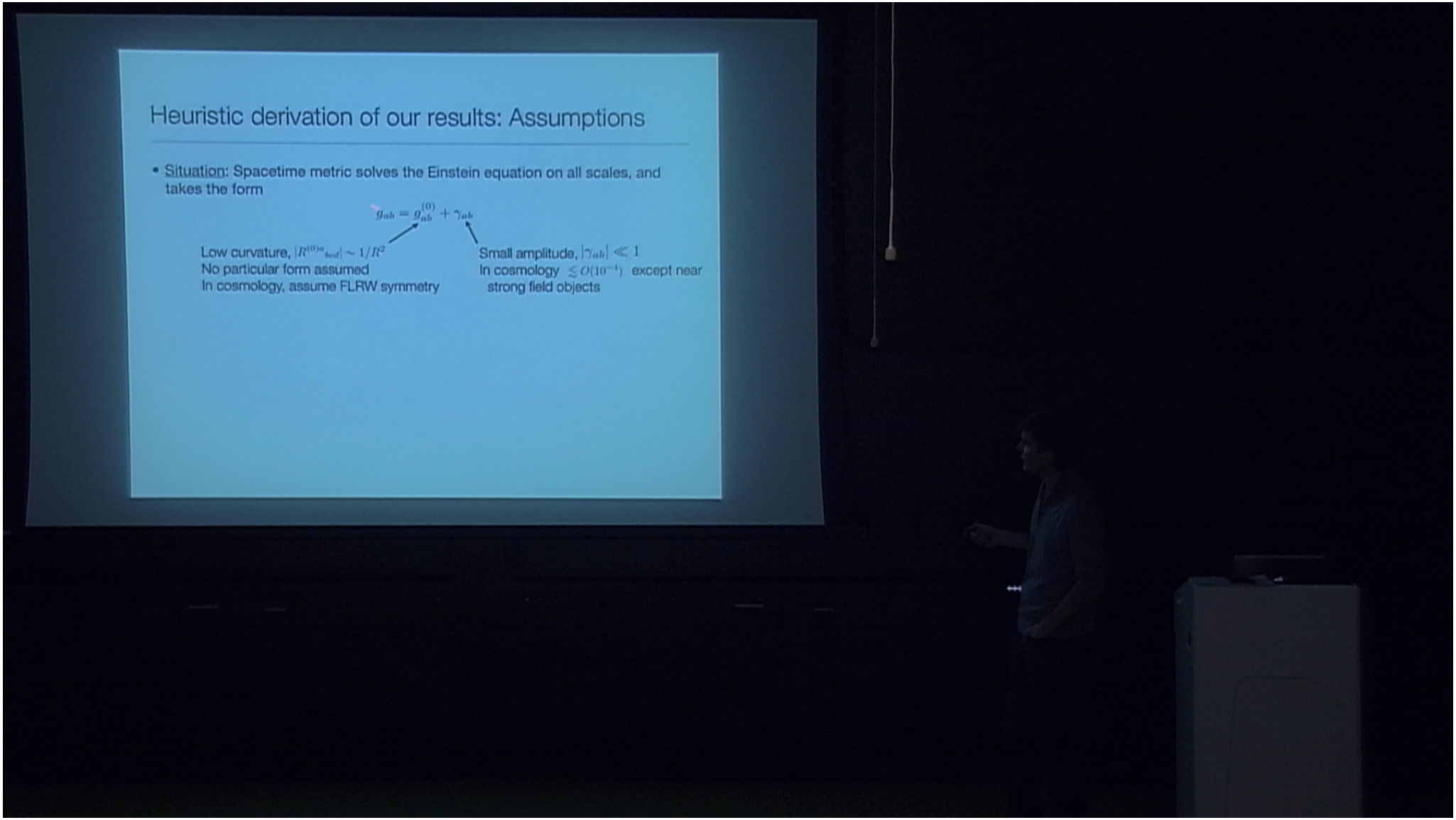
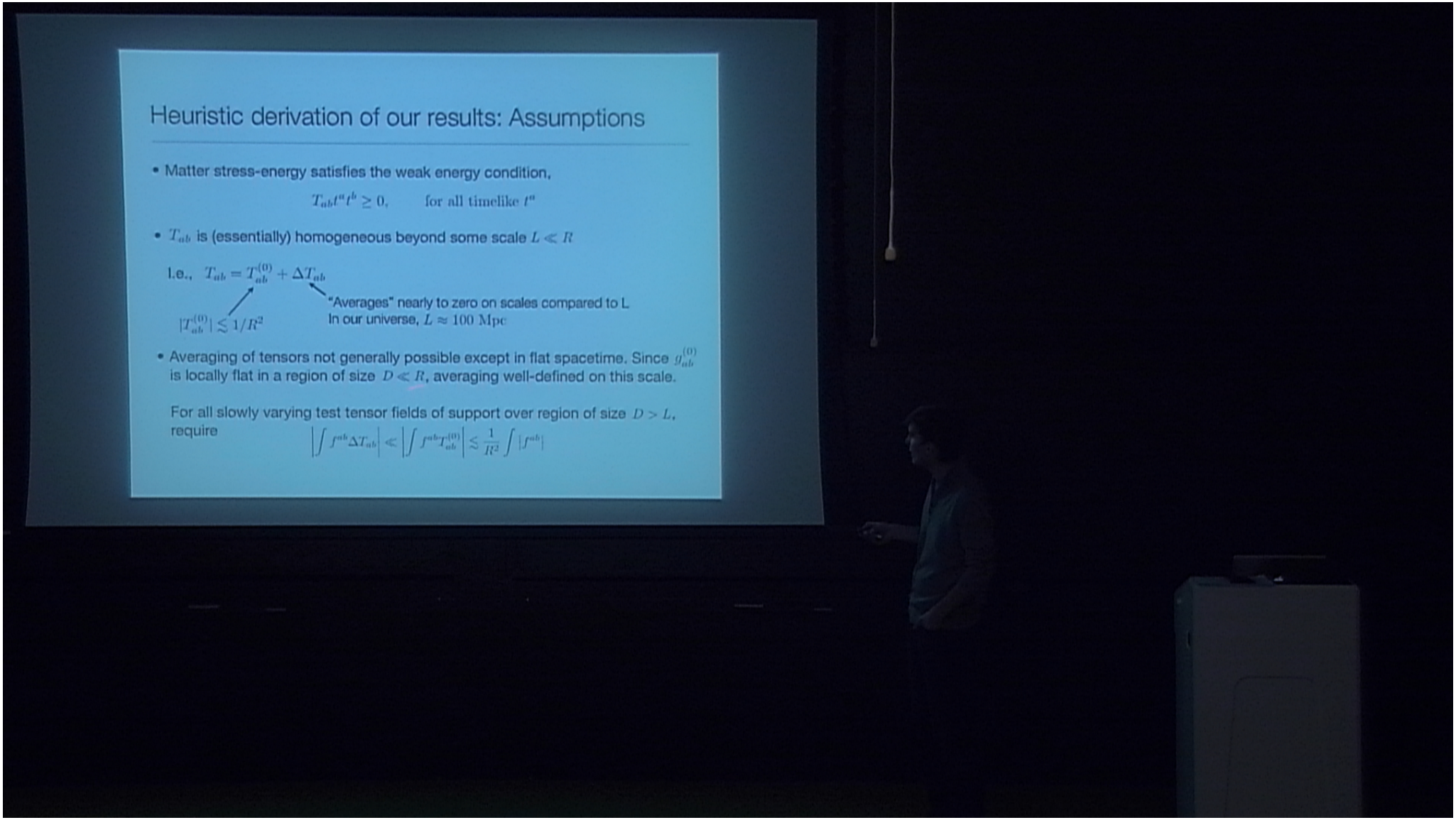
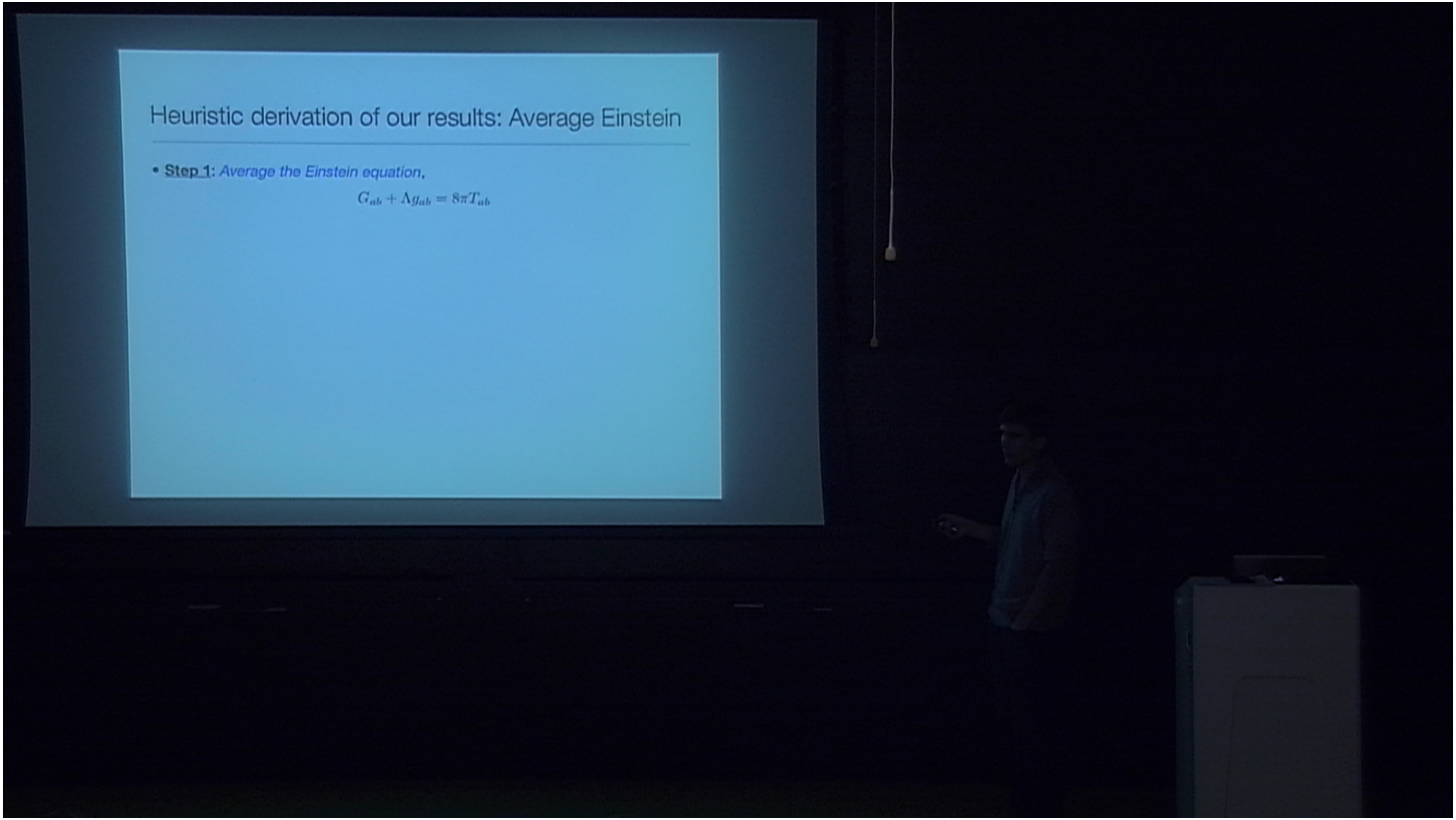
• Goal: Develop an improved perturbative framework that allows a priori for significant backreaction due to matter inhomogeneities. Use it to constrain any such effects.
• Main result: Within this framework, if the matter stress-energy tensor satisfies the weak energy condition, then the effective stress-energy of backreaction must be traceless and satisfy the weak energy condition itself. In particular, it cannot mimic a cosmological constant.


Heuristic derivation of our results: Assumptions
• Situation: Spacetime metric solves the Einstein equation on all scales, and takes the form
$g_{ab} = g^{(0)}_{ab} + \gamma_{ab}$
Low curvature, $|R^{(0)a}{}_{bcd}| \sim 1/R^2$
No particular form assumed
In cosmology, assume FLRW symmetry
Small amplitude, $|\gamma_{ab}| \ll 1$
In cosmology $\lesssim O(10^{-4})$ except near strong field objects


Heuristic derivation of our results: Assumptions
• Matter stress-energy satisfies the weak energy condition,
$T_{ab}t^a t^b \geq 0, \qquad$ for all timelike t^a
• T_{ab} is (essentially) homogeneous beyond some scale $L \ll R$
I.e., $T_{ab} = T^{(0)}_{ab} + \Delta T_{ab}$
$|T^{(0)}_{ab}| \lesssim 1/R^2$
"Averages" nearly to zero on scales compared to L
In our universe, $L \approx 100$ Mpc
• Averaging of tensors not generally possible except in flat spacetime. Since $g^{(0)}_{ab}$ is locally flat in a region of size $D \ll R$, averaging well-defined on this scale.
For all slowly varying test tensor fields of support over region of size $D > L$, require
$\left|\int f^{ab}\Delta T_{ab}\right| \ll \left|\int f^{ab}T^{(0)}_{ab}\right| \lesssim \frac{1}{R^2}\int |f^{ab}|$


Heuristic derivation of our results: Average Einstein
• Step 1: Average the Einstein equation,
$G_{ab} + \Lambda g_{ab} = 8\pi T_{ab}$

Heuristic derivation of our results: Average Einstein

- **Step 1**: *Average the Einstein equation*,

$$G_{ab} + \Lambda g_{ab} = 8\pi T_{ab}$$

- First, re-write as

Expansion in γ_{ab}

$$G^{(0)}_{ab} + \Lambda g^{(0)}_{ab} - 8\pi T^{(0)}_{ab} = 8\pi \Delta T_{ab} - \Lambda \gamma_{ab} - G^{(1)}_{ab} - G^{(2)}_{ab} - G^{(3+)}_{ab}$$

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Average

LHS unchanged under averaging

$$G^{(0)}_{ab} + \Lambda g^{(0)}_{ab} - 8\pi T^{(0)}_{ab} \equiv 8\pi t^{(0)}_{ab}$$

Define average of RHS to be effective stress-energy

- Consider each RHS term separately.

Heuristic derivation of our results: Constrain $t^{(0)}_{ab}$

- **Step 2**: *Prove tracelessness of effective stress-energy*

- Need control on γ_{ab}:
  Would like to impose linearized Einstein equation, $G^{(1)}_{ab} = 8\pi\Delta T_{ab}$, but that would constitute a new unjustified assumption.

  Instead, multiply full Einstein equation by γ_{ab}, and average.

  Obtain $\int f^{cdab}\gamma_{cd}G^{(1)}_{ab} = 8\pi \int f^{cdab}\gamma_{cd}\Delta T_{ab}$

- Weak energy condition implies RHS is negligible. Choose $f^{cdab} = f^{cd}t^a t^b$

  Then $\int f^{cd}\gamma_{cd}t^a t^b \Delta T_{ab} = \int f^{cd}\gamma_{cd}[\rho - \rho^{(0)}]$

Heuristic derivation of our results: Constrain $t^{(0)}_{ab}$

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  negligible

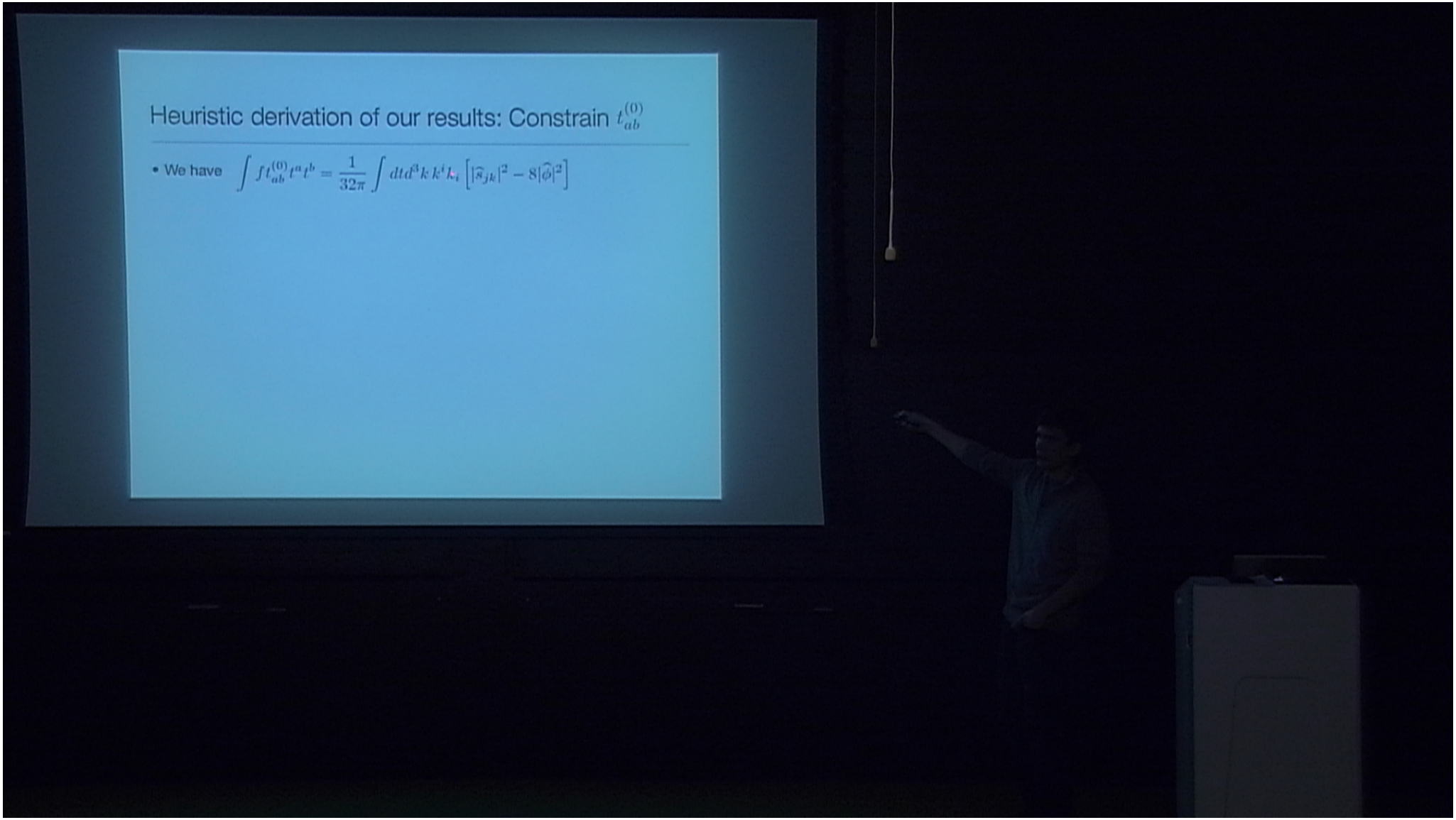
Heuristic derivation of our results: Constrain $t^{(0)}_{ab}$

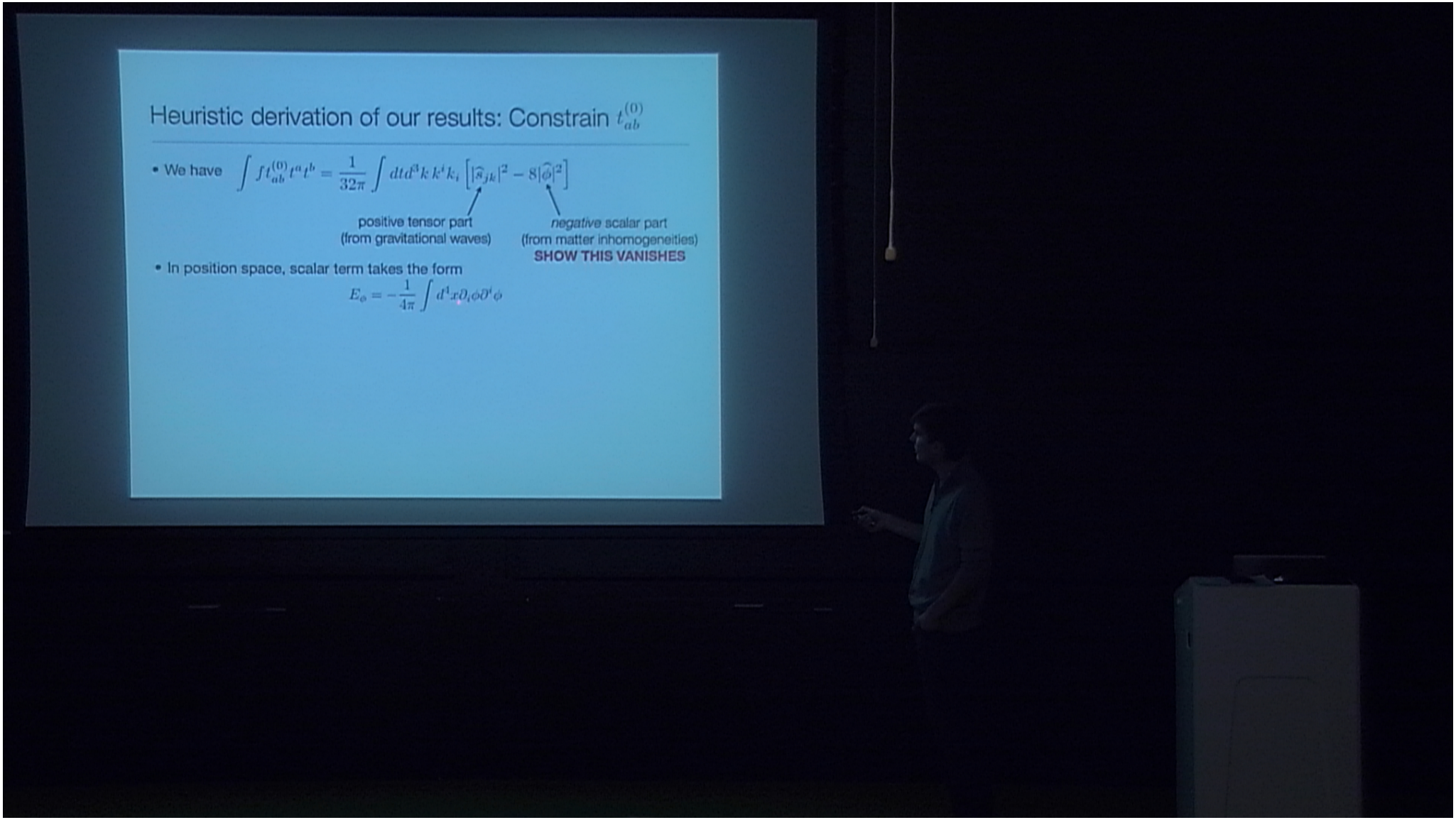
- By the weak energy condition $\rho \geq 0$, so

$$\left| \int f^{cd} \gamma_{cd} \rho \right| \leq \max |\gamma_{cd}| \int |f^{cd}| \rho \sim \max |\gamma_{cd}| \int |f^{cd}| \rho^{(0)}$$

$$\sim \frac{\max |\gamma_{cd}|}{R^2} \int |f^{cd}| \ll \frac{1}{R^2} \int |f^{cd}|$$

- But any f^{cdab} can be approximated by linear combination of $f^{cd} t^a t^b$ terms, so, to our level of approximation, $\int f^{cdab} \gamma_{cd} G^{(1)}_{ab} = 0$


Heuristic derivation of our results: Constrain $t^{(0)}_{ab}$
• We have $\int f t^{(0)}_{ab} t^a t^b = \frac{1}{32\pi} \int dt d^3k\, k^i k_i \left[|\widehat{s}_{jk}|^2 - 8|\widehat{\phi}|^2\right]$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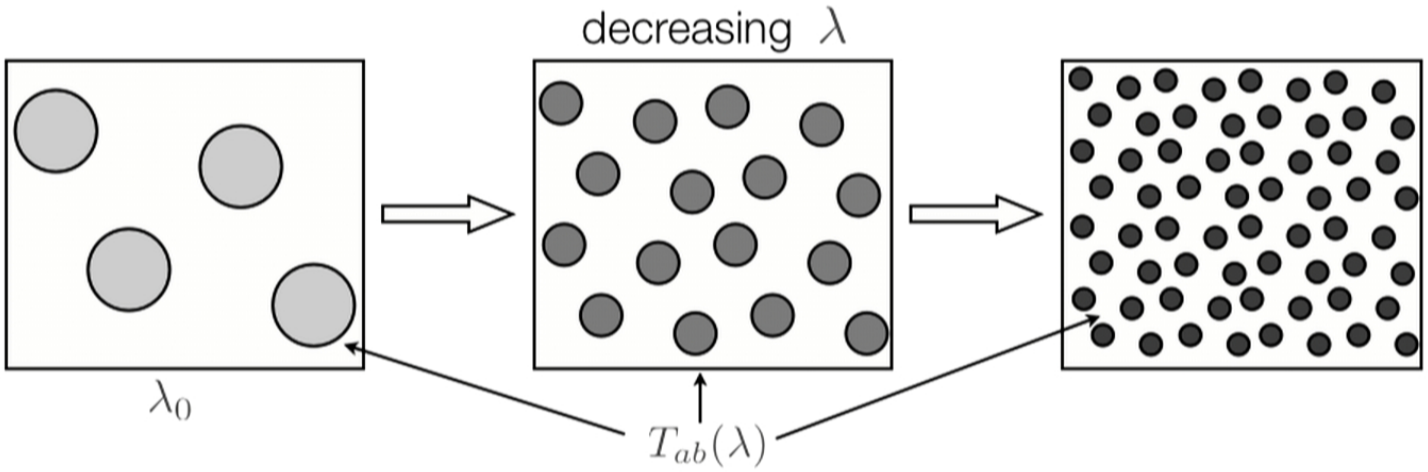
positive tensor part
(from gravitational waves)
negative scalar part
(from matter inhomogeneities)
SHOW THIS VANISHES
• In position space, scalar term takes the form
$E_\phi = -\frac{1}{4\pi} \int d^4x \partial_i \phi \partial^i \phi$

One-parameter family

- Spacetime of interest exists at some fixed λ_0, to which we attach a one-parameter family. Since metric converges uniformly but first derivatives are only bounded, the spatial scale of perturbations in general decreases as $\lambda \to 0$.

- Assumptions say nothing about the magnitude of $\nabla_c \nabla_d \left(g_{ab}(\lambda) - g_{ab}(0) \right)$, so it can be unbounded as $\lambda \to 0$. Thus $T_{ab}(\lambda)$ unbounded.



One-parameter family

- Assumption 4: $\nabla_a \left(g_{cd}(\lambda) - g_{cd}(0)\right) \nabla_b \left(g_{ef}(\lambda) - g_{ef}(0)\right) \to \mu_{abcdef}$ weakly.

- Recall definition of weak limit: $\alpha_a(\lambda) \to \alpha_a(0)$ weakly iff for all compact support test fields t^a,

$$\lim_{\lambda \to 0} \int_M \left(\alpha_a(\lambda) - \alpha_a(0)\right) t^a \, \mathrm{d}^4 x = 0$$

- Examples of weak convergence:

$$\sin\left(\frac{x}{\lambda}\right) \to 0$$

$$\sin^2\left(\frac{x}{\lambda}\right) \to \frac{1}{2}$$

- Taking the weak limit corresponds to averaging at small λ. This assumption thus assumes that necessary averages exist.

Results

THEOREM: Let $g_{ab}(\lambda)$ be a one-parameter family of metrics satisfying assumptions (1) through (4). Then the background metric $g_{ab}(0)$ is a solution of the *effective Einstein equation*,

$$G_{ab}(g(0)) + \Lambda g_{ab}(0) = 8\pi \left(T^{(0)}_{ab} + t^{(0)}_{ab} \right)$$

The effective stress energy tensor $t^{(0)}_{ab}$ is:

a. traceless, $g^{ab}(0)t^{(0)}_{ab} = 0$, and

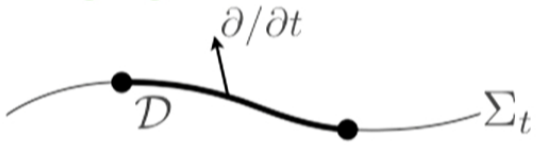
b. satisfies the weak energy condition.

- Results apply to our universe as long as we are sufficiently close to the limiting case in which it applies exactly.

Alternative approach: Buchert averaging

- Main alternative approach to averaging in the literature.
- Put the exact metric into a comoving, synchronous gauge:

$$ds^2 = -dt^2 + q_{ij}(t,x)dx^i dx^j$$



- Define spatial averages of scalars: $\langle \Psi \rangle_{\mathcal{D}} \equiv \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}} \Psi d\Sigma$
- Averaged "scale factor" defined as $a_{\mathcal{D}} \equiv (V_{\mathcal{D}})^{1/3}$
- Equations for $a_{\mathcal{D}}$ can be derived by averaging the scalar parts of the Einstein equation: the Hamiltonian constraint and the Raychaudhuri equation.

Alternative approach: Buchert averaging

- "Modified Friedmann equations":

spatial Ricci scalar

$$3\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = 8\pi\langle\rho\rangle_{\mathcal{D}} - \frac{1}{2}\langle\mathcal{R}\rangle_{\mathcal{D}} - \frac{1}{2}\mathcal{Q}_{\mathcal{D}}$$

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -4\pi\langle\rho\rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}}$$

"backreaction" scalar:

$$\mathcal{Q}_{\mathcal{D}} \equiv \frac{2}{3}\langle(\theta - \langle\theta\rangle_{\mathcal{D}})^2\rangle_{\mathcal{D}} - \langle\sigma_{ij}\sigma^{ij}\rangle_{\mathcal{D}}$$

There is also an integrability constraint:

$$\left(a_{\mathcal{D}}^6\,\mathcal{Q}_{\mathcal{D}}\right)^{\cdot} + a_{\mathcal{D}}^4\left(a_{\mathcal{D}}^2\langle\mathcal{R}\rangle_{\mathcal{D}}\right)^{\cdot} = 0$$

- Notice that if $\mathcal{Q}_{\mathcal{D}}$ is sufficiently large an acceleration of $a_{\mathcal{D}}$ can occur!

Advantages over Buchert

1. When the deviations of the metric from exact FLRW are not small, it is not clear how to interpret averaged quantities, such as $a_{\mathcal{D}}$.

   - We obtain an approximation $g^{(0)}_{ab}$ to the true metric g_{ab}. Measurable quantities can thus be calculated unambiguously.

2. Only the scalar parts of the Einstein equation are averaged.

   - *Weak limits* are used instead of averages in proving general theorems, thereby avoiding the difficulties involved in averaging. Averages are well-defined since the *metric* perturbation is small.

3. They work in the comoving, synchronous gauge, where late-time perturbations to the metric are in general not small.

   - We would use the *longitudinal gauge* in cosmology, instead of the synchronous gauge.

4. The Buchert equations are only a partial set of equations: the evolution of $\mathcal{Q}_{\mathcal{D}}$ is not determined.

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Generalized perturbation theory

- Instead of taking ordinary limits, one can assume existence of an analogous weak limit:

$$\gamma_{ab}^{(L)} = \text{w-}\lim_{\lambda \to 0} \frac{g_{ab}(\lambda) - g_{ab}(0)}{\lambda}$$

Equation for $\gamma_{ab}^{(L)}$

- By analogy with ordinary perturbation theory, derive an equation for $\gamma_{ab}^{(L)}$ by subtracting the background Einstein equation from the exact Einstein equation, dividing by λ, and taking a weak limit.

- **Ordinary perturbation theory:**

$$\left.\frac{\partial E_{ab}(\lambda)}{\partial \lambda}\right|_{\lambda=0} = \lim_{\lambda\to 0} \frac{E_{ab}(\lambda) - E_{ab}^{(0)}}{\lambda} = \text{linearized Einstein equation}$$

- **Generalized perturbation theory:**

$$\text{w-}\lim_{\lambda\to 0} \frac{E_{ab}(\lambda) - E_{ab}^{(0)}}{\lambda} = \text{generalized linearized Einstein equation}$$

- Not entirely straightforward to compute, since non-linear terms in Einstein tensor can in general have non-zero weak limit, as in background case.

Equation for $\gamma^{(L)}_{ab}$

$$
\begin{aligned}
&\nabla^c\nabla_{(a}\gamma^{(L)}_{b)c} - \frac{1}{2}\nabla^c\nabla_c\gamma^{(L)}_{ab} - \frac{1}{2}\nabla_a\nabla_b\gamma^{(L)c}{}_{c} - \frac{1}{2}g^{(0)}_{ab}\left(\nabla^c\nabla^d\gamma^{(L)}_{cd} - \nabla^c\nabla_c\gamma^{(L)d}{}_{d}\right)\\
&+\frac{1}{2}g^{(0)}_{ab}R^{cd}(g^{(0)})\gamma^{(L)}_{cd} - \frac{1}{2}R(g^{(0)})\gamma^{(L)}_{ab} + \Lambda\gamma^{(L)}_{ab} + \frac{1}{8}\gamma^{(L)}_{ab}\left(\mu^{c\ d\ e}_{c\ d\ e} + \mu^{c\ de}_{c\ \ de} - 2\mu^{cd\ e}_{c\ de}\right)\\
&+\gamma^{(L)cd}\left(\frac{1}{2}\mu_{abc}{}^{e}{}_{de} - \frac{1}{2}\mu_{c(ab)d}{}^{e}{}_{e} - \frac{1}{2}\mu^{e}{}_{(ab)ecd} + \frac{3}{4}\mu_{cdab}{}^{e}{}_{e} - \frac{1}{2}\mu_{cda}{}^{e}{}_{be} - \mu_{c}{}^{e}{}_{abde} + \mu_{c}{}^{e}{}_{e(ab)d}\right.\\
&\left.+\frac{3}{4}\mu^{e}{}_{eabcd} - \frac{1}{2}\mu^{e}{}_{eacbd} + \frac{1}{8}g^{(0)}_{ab}\left\{-\mu_{cd}{}^{e}{}_{e}{}^{f}{}_{f} - \mu_{cd}{}^{ef}{}_{ef} + 4\mu_{c}{}^{e}{}_{d}{}^{f}{}_{ef} - 2\mu^{e}{}_{ecd}{}^{f}{}_{f} + 2\mu^{ef}{}_{cedf}\right\}\right)\\
=&\,8\pi T^{(1)}_{ab} + \frac{1}{8}g^{(0)}_{ab}\left\{-\mu^{(1)c\ de}_{c\ \ de} - \mu^{(1)c\ d\ e}_{c\ d\ e} + 2\mu^{(1)cd\ e}_{c\ de}\right\} + \frac{1}{2}\mu^{(1)cd}{}_{acbd}\\
&-\frac{1}{2}\mu^{(1)c}{}_{ca}{}^{d}{}_{bd} + \frac{1}{4}\mu^{(1)}{}_{ab}{}^{cd}{}_{cd} - \frac{1}{2}\mu^{(1)c}{}_{(ab)c}{}^{d}{}_{d} + \frac{3}{4}\mu^{(1)c}{}_{cab}{}^{d}{}_{d} - \frac{1}{2}\mu^{(1)cd}{}_{abcd}\\
&+\frac{1}{8}g^{(0)}_{ab}\left\{2\omega^{(1)c\ de\ f}_{c\ \ de\ f} + 2\omega^{(1)c\ de\ f}_{c\ \ d\ ef} + \omega^{(1)cd\ e\ f}_{cd\ e\ f} + \omega^{(1)cd\ \ ef}_{cd\ \ ef} - 4\omega^{(1)cd\ e\ f}_{c\ d\ ef} - 2\omega^{(1)cdef}{}_{decf}\right\}\\
&-\frac{1}{2}\omega^{(1)}{}_{ab}{}^{cd\ e}{}_{c\ de} + \frac{1}{2}\omega^{(1)}{}_{(a\ |c|\ b)d}{}^{c\ \ d\ \ e}{}_{e} + \frac{1}{2}\omega^{(1)}{}_{(a\ \ b)cde}{}^{cde} - \frac{1}{8}\omega^{(1)c}{}_{cab}{}^{d\ e}{}_{d\ e} - \frac{1}{8}\omega^{(1)c}{}_{cab}{}^{de}{}_{de} - \frac{3}{4}\omega^{(1)c\ de}{}_{c\ \ abde}\\
&+\frac{1}{2}\omega^{(1)c\ de}{}_{c\ \ adbe} + \frac{1}{4}\omega^{(1)cd}{}_{abd}{}^{e}{}_{ce} - \frac{3}{4}\omega^{(1)cd}{}_{cdab}{}^{e}{}_{e} + \frac{1}{2}\omega^{(1)cd}{}_{cda}{}^{e}{}_{be} + \frac{1}{2}\omega^{(1)cd\ e}{}_{c\ abde} - \frac{1}{2}\omega^{(1)cd\ e}{}_{c\ adbe}\\
&+\frac{1}{2}\omega^{(1)cd\ e}{}_{d\ abce} - \frac{1}{2}\omega^{(1)cd\ e}{}_{d\ aebc} + \frac{1}{4}g^{(0)}_{ab}\left\{2\nabla_d\nu^{(1)c\ de}_{c\ \ e} + \nabla_e\nu^{(1)c\ d\ e}_{c\ d}\right\} - \frac{1}{4}\nabla_{(a}\nu^{(1)c}{}_{b)c}{}^{d}{}_{d}\\
&+\frac{1}{4}\nabla_c\nu^{(1)}{}_{(ab)}{}^{cd}{}_{d} - \frac{1}{2}\nabla_c\nu^{(1)c}{}_{ab}{}^{d}{}_{d} - \nabla_d\nu^{(1)}{}_{(ab)}{}^{c\ d}{}_{c} + \nabla_d\nu^{(1)c}{}_{abc}{}^{d} + \frac{1}{2}\nabla_d\nu^{(1)c}{}_{c(ab)}{}^{d}
\end{aligned}
$$

Conclusions

- The only effect of small scale inhomogeneities on the leading order dynamics of the universe is via a $P = \frac{1}{3}\rho$ fluid, corresponding to gravitational radiation.

  The weak energy condition on matter plays an essential role in this proof.

- More generally, no matter what happens at small scales, $t^{(0)}_{ab}$ has positive energy properties. Implications for stability? What about alternative theories?

- We have split the deviation of the metric from the background into short and long wavelength parts. The long wavelength part, $\gamma^{(L)}_{ab}$, obeys a linear equation, as in ordinary perturbation theory, but with additional terms arising from small scale inhomogeneities. The short wavelength part, $\gamma^{(S)}_{ab}$, is allowed to evolve non-linearly, which is essential to describe gravitational dynamics at small scales.

- *Non mentioned:* This work also motivated an analysis of Newtonian cosmology. We showed that with a suitable dictionary, Newtonian cosmologies provide excellent approximations to Einstein solutions *on all scales*.