

Title: recent advances in Particle-In-Cell simulations of relativistic plasmas

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Abstract:

FEEDBACK OVER 44 ORDERS OF MAGNITUDE: FROM GAMMA-RAYS TO THE UNIVERSE

March 14-16, 2016 - Toronto, Canada

Recent advances in Particle-in-Cell simulations of relativistic plasmas

J.-L. Vay¹, R. Lehe¹, H. Vincenti^{1,2}, B. Godfrey^{1,3}, P. Lee⁴, I. Haber³,
C. Geddes¹, E. Esarey¹, C. Schroeder¹, W. Leemans¹

¹Lawrence Berkeley National Laboratory, California, USA

²Commissariat à l'Energie Atomique, Saclay, France

³University of Maryland, Maryland, USA

⁴University of Paris-Sud, Orsay, France



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Outline

- **Physics problems: astrophysical shocks, plasma accel.**
 - choice of Lorentz frame
- **Particle-in-Cell method basics**
- **Standard Boris & Lorentz invariant particle pusher**
- **Standard Yee & NSFDTD, high-order, spectral Maxwell solvers**
- **Current deposition, field gather, smoothing**
- **Numerical Cherenkov instability**
- **Exascale supercomputing & common tools**
- **Conclusion**



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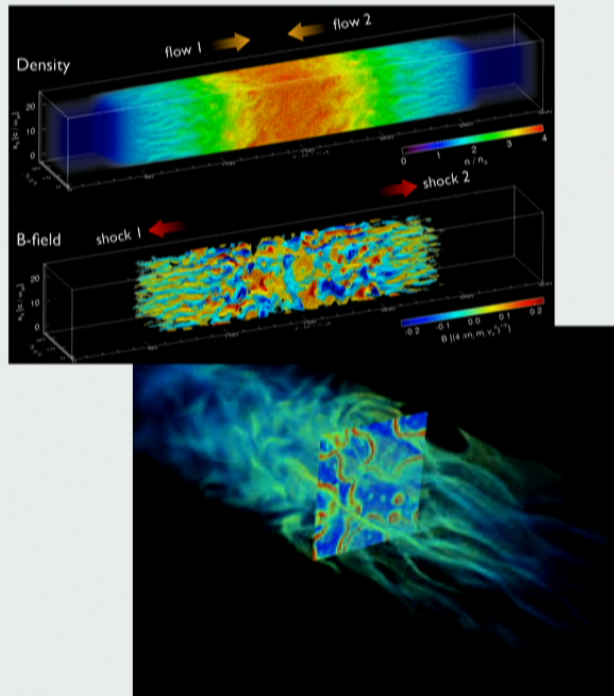
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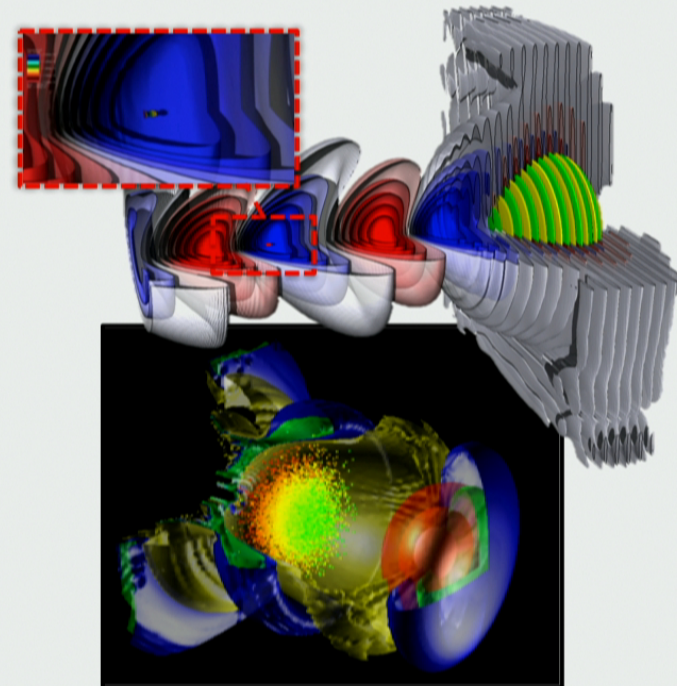
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Examples of relativistic plasmas

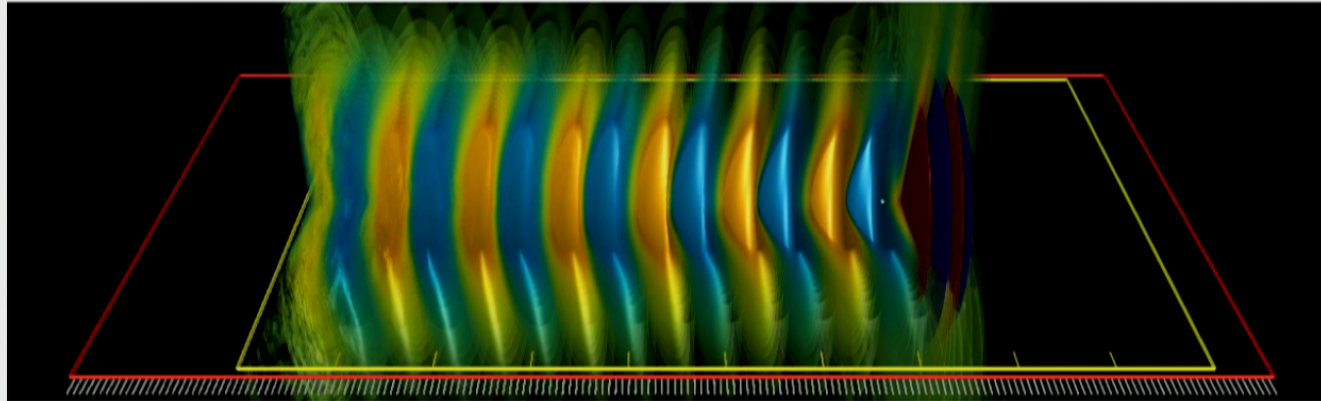
Astro shocks



Plasma-based accelerators



Modeling from first principle is very challenging



For a 10 GeV scale stage:

$\sim 1\mu\text{m}$ wavelength laser propagates into $\sim 1\text{m}$ plasma

→ millions of time steps needed



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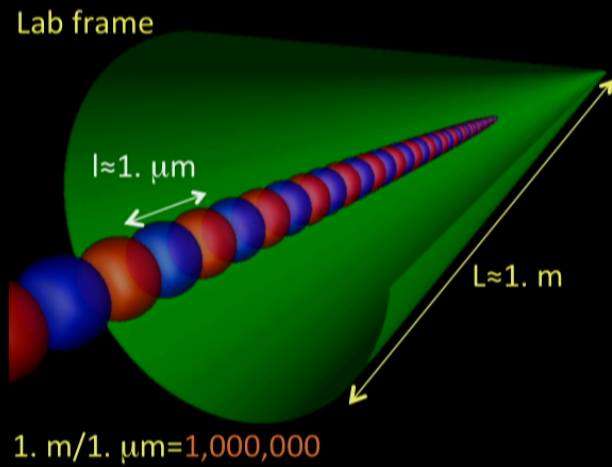
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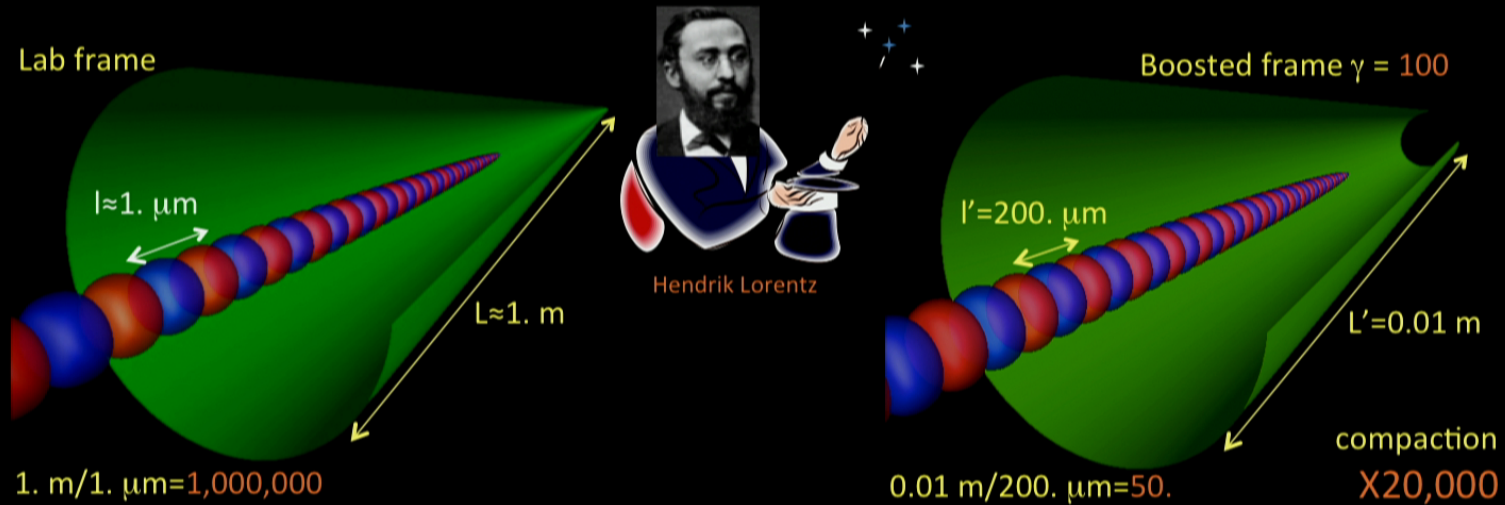


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Solution: use frame moving close to speed of light*



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*J.-L. Vay, *Phys. Rev. Lett.* **98**, 130405 (2007)



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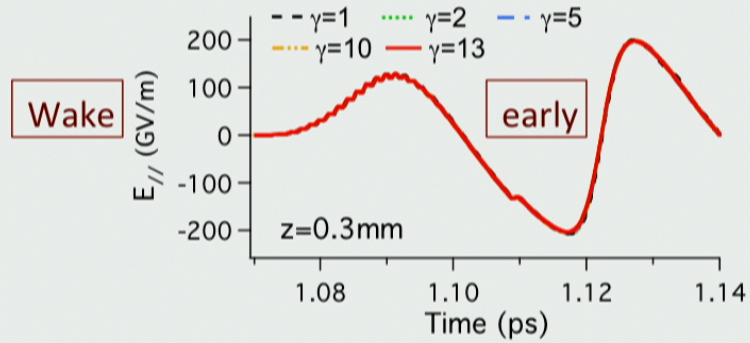


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LBF method carefully validated in deeply depleted beam loaded stages

-- Excellent agreement between runs at various γ boost

Warp-3D – $a_0=1$, $n_0=10^{19}\text{cm}^{-3}$ (~ 100 MeV) scaled to 10^{17}cm^{-3} (~ 10 GeV)



*J.-L. Vay, et al., *Phys. Plasmas* **18**, 123103 (2011)

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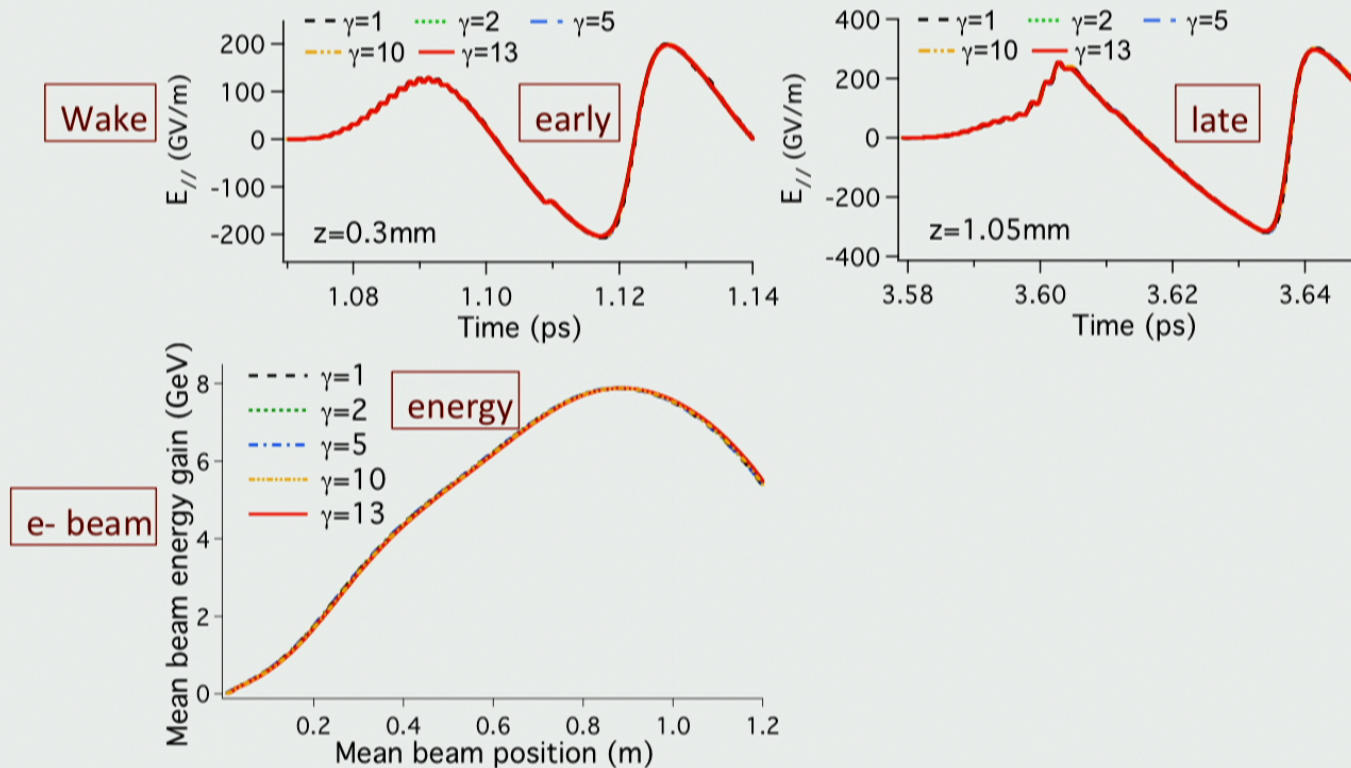
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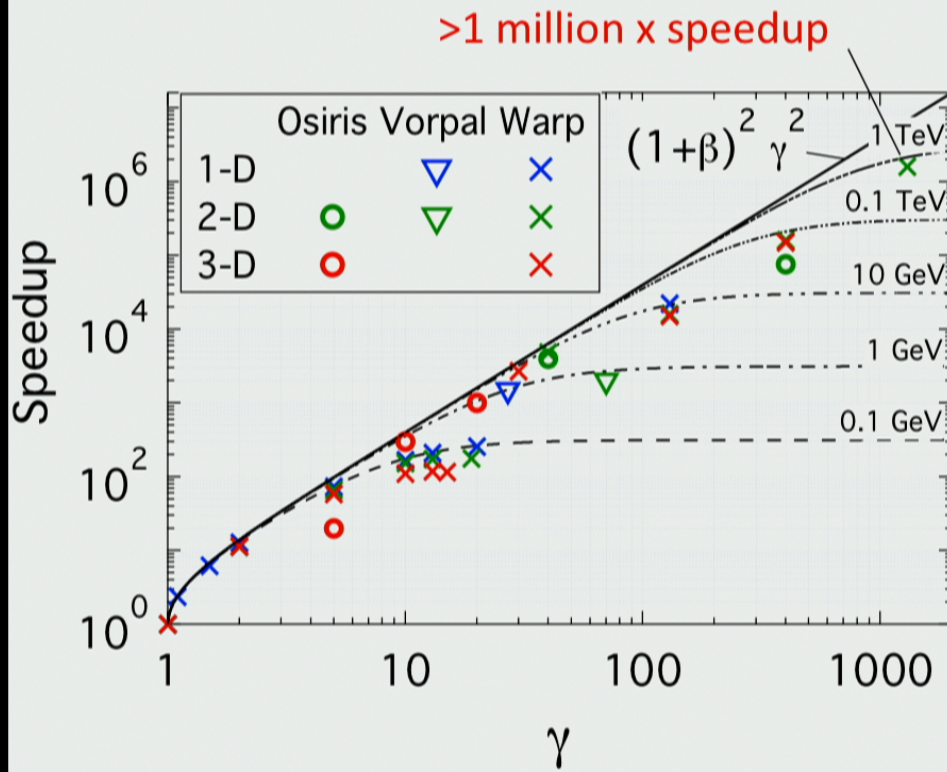


*J.-L. Vay, et al, *Phys. Plasmas* 18, 123103 (2011)

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Speedup verified by us and others to over a million



Warp:

1. J.-L. Vay, et al., *Phys. Plasmas* **18** 123103 (2011)
2. J.-L. Vay, et al., *Phys. Plasmas (letter)* **18** 030701 (2011)
3. J.-L. Vay, et al., *J. Comput. Phys.* **230** 5908 (2011)
4. J.-L. Vay et al, PAC Proc. (2009)

Osiris:

1. S. Martins, et al., *Nat. Phys.* **6** 311 (2010)
2. S. Martins, et al., *Comput. Phys. Comm.* **181** 869 (2010)
3. S. Martins, et al., *Phys. Plasmas* **17** 056705 (2010)
4. S. Martins et al, PAC Proc. (2009)

Vorpals:

1. D. Bruhwiler, et al., *AIP Conf. Proc* **1086** 29 (2009)



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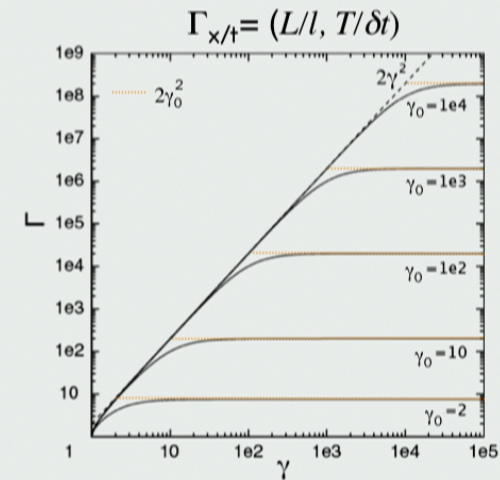
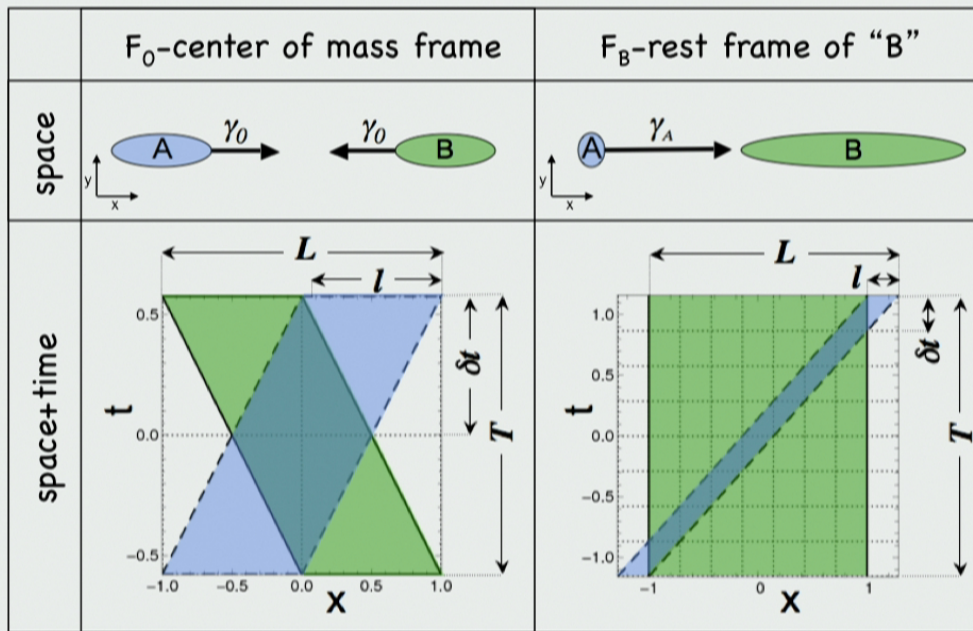
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Formulation is very general and applies to crossing relativistic objects

crossing of 2 relativistic objects



Γ is **not invariant** under a Lorentz transformation:
 $\Gamma_{x/t} \propto \gamma^2$.

γ^2 speedup demonstrated for 2-stream insta., plasma accelerators, FEL, ...

Can it apply to 2-stream insta of relevance to blazar studies?

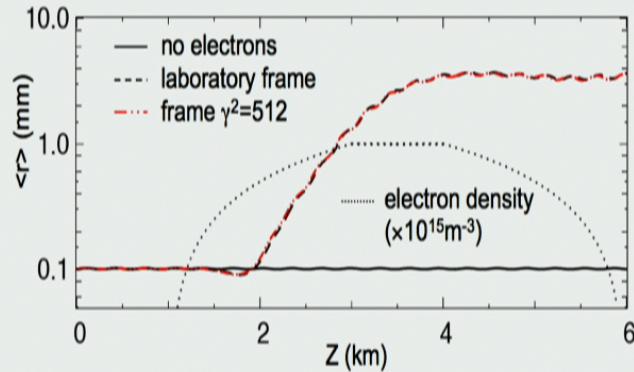
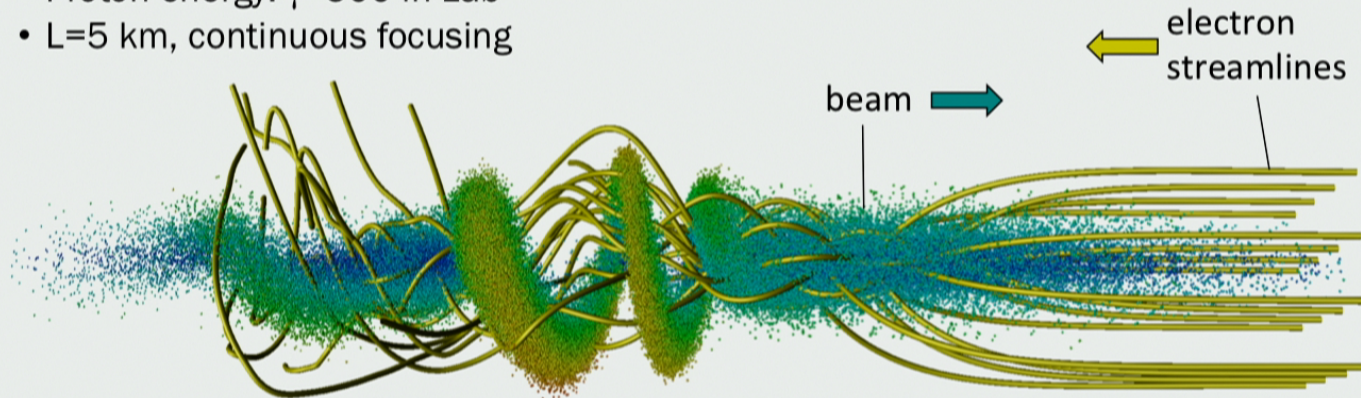
*J.-L. Vay, *Phys. Rev. Lett.* **98**, 130405 (2007)



Example of application to 2-stream instability

Calculation of e-cloud induced instability of a proton bunch*

- Proton energy: $\gamma=500$ in Lab
- $L=5$ km, continuous focusing



CPU time (2 quad-core procs):

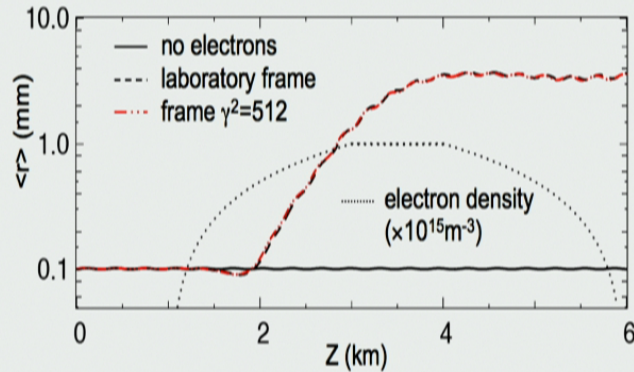
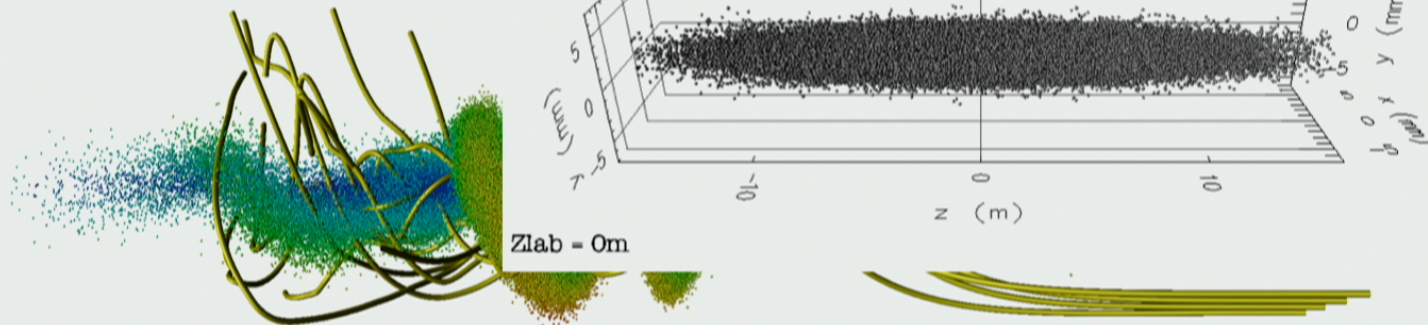
- lab frame: **>2 weeks**
- frame with $\gamma^2=512$: **<30 min**

Speedup x1000

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Test beam-plasma simulation in various frames

frame boost $\gamma=1$

$\gamma=1.5$

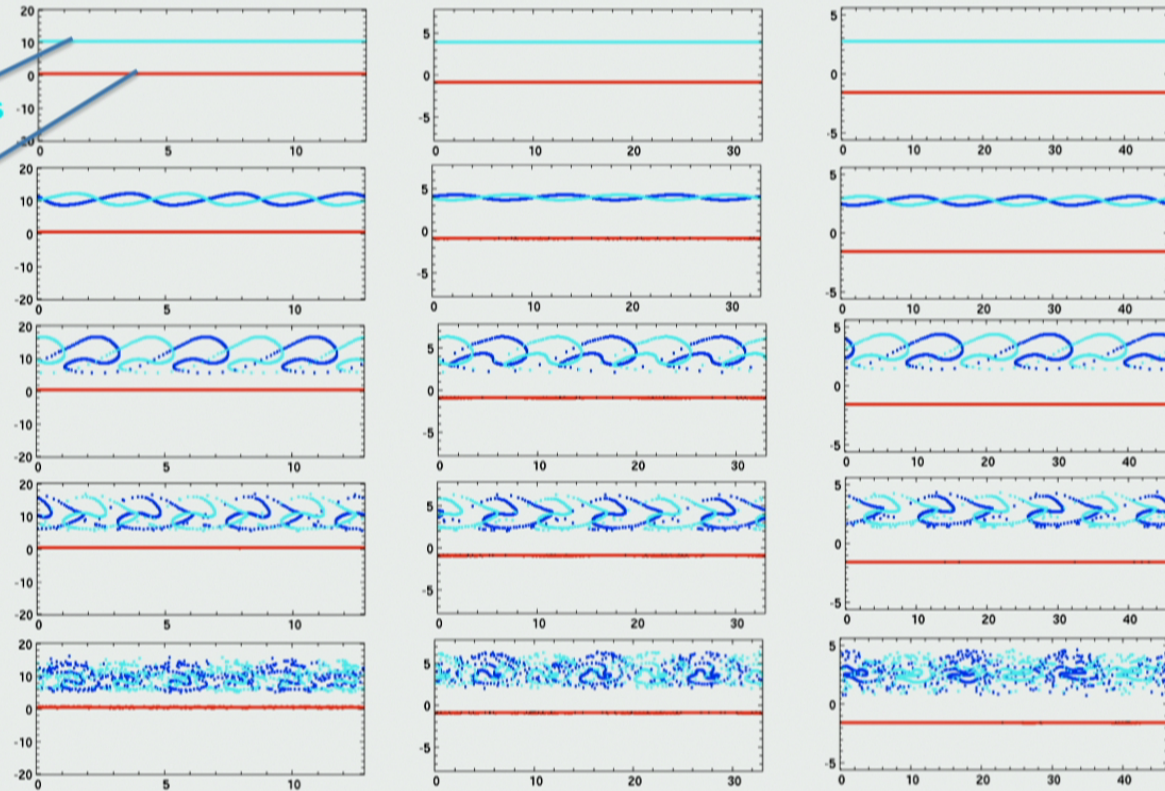
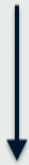
$\gamma=2$

Beam: $\gamma_0=10$
positrons+electrons

Plasma: protons+electrons
 $V_{th_0}=1.e-3c$

$n_{\text{plasma}}=1000n_{\text{beam}}$

time



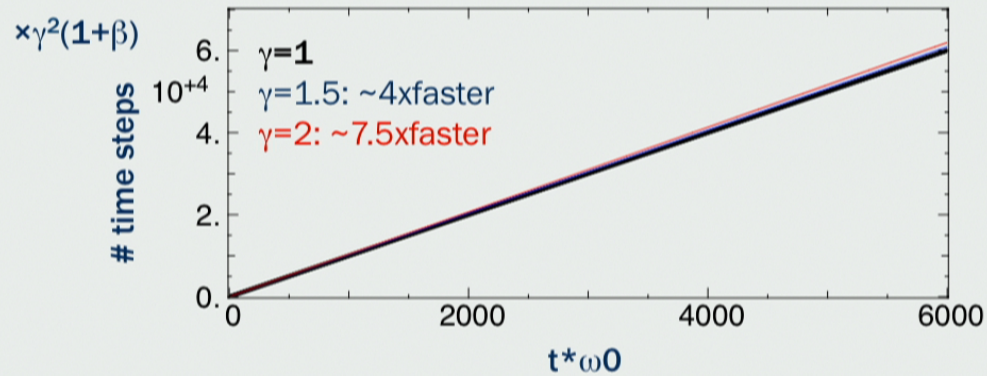
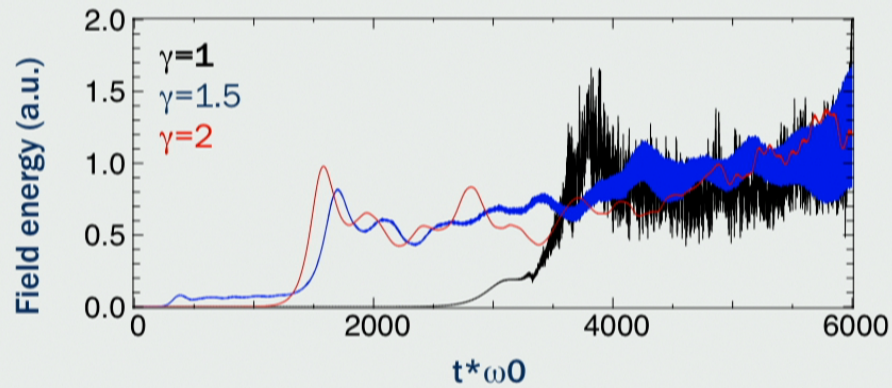
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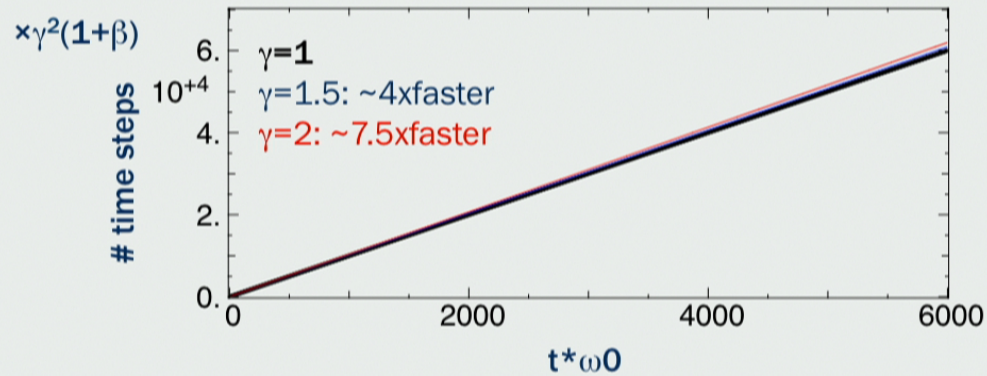
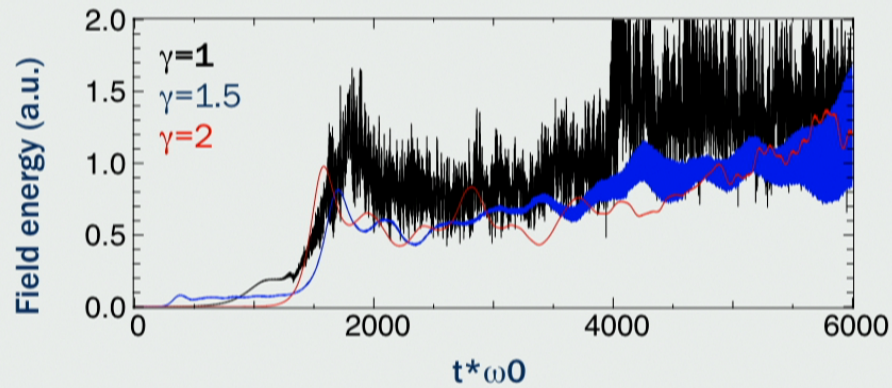
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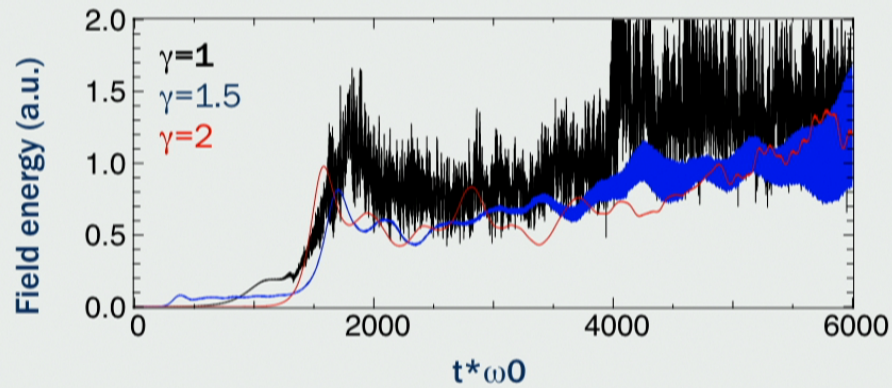
Test beam-plasma simulation in various frames



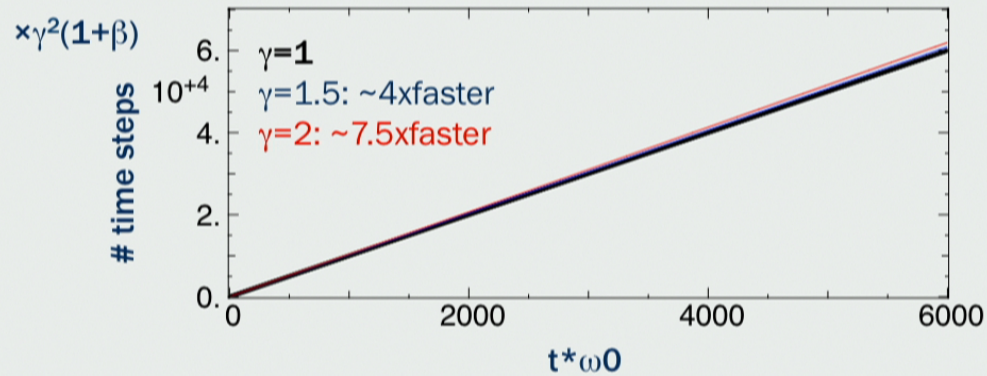
Test beam-plasma simulation in various frames



Test beam-plasma simulation in various frames



Also less noisy at higher γ .



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 - choice of Lorentz frame for modeling of plasma accelerators
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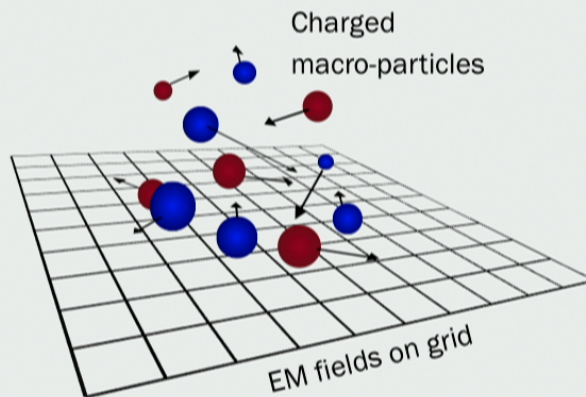
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Particle-In-Cell widely used for modeling plasmas

Particle-In-Cell



- based on first principles:
 - includes nonlinear, 3D, kinetic effects,
- particle push and EM solver are local:
 - scales well to >1M cores.

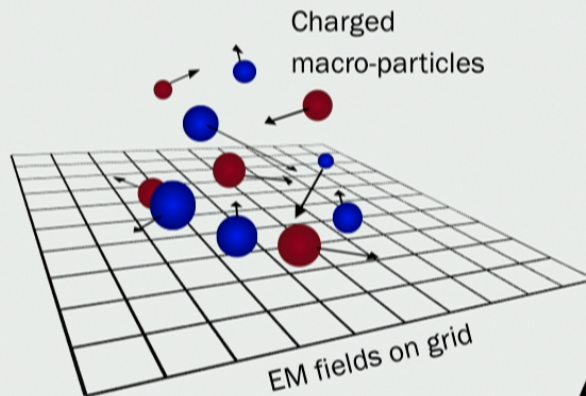


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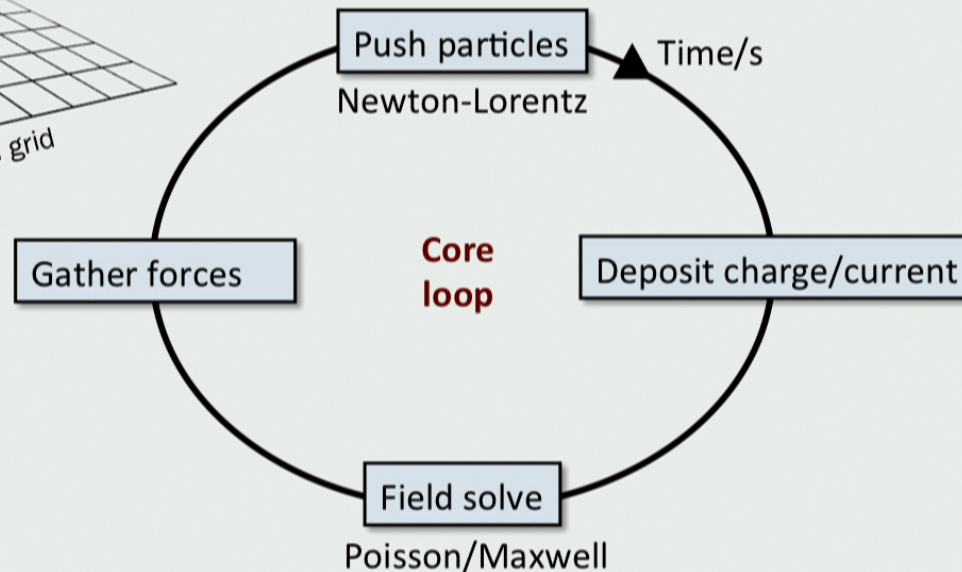
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Berkeley Lab Accelerator Simulation Toolkit

Warp – open source PIC+accelerator framework

<http://blast.lbl.gov>



Warp

- 3-D, Circ, RZ, 2-D on structured meshes
- **Ref. frames** – lab, window, boost
- **Poisson/Ampere** – FFT, multigrid; AMR; implicit; arbitrary conductors (cut-cell)
- **Maxwell** – Yee mesh/node centered, arbitrary order, Yee/CK/Lehe, pseudo-spectral, PML, MR, arbitrary conductors/dielectric (grid-cell)
- **Particle pushers** – Boris/Cohen/Vay/Maps
- **Accelerator** – dipoles, quads, sextupoles, solenoids, acceleration gaps, arbitrary fields (gridded/functions), linear maps, ...
- **Emission** – particles: space charge limited, thermionic, prescribed, secondary; laser
- **Monte Carlo collisions** – ionization, capture, charge exchange
- **Programming** – Python+FORTRAN, parallel MPI



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Particle pusher



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Boris pusher is most widely used

- Boris pusher: leapfrog second order in time

- Position push: $x^{n+1/2} = x^{n-1/2} + v^n \Delta t$
- Velocity push: $u^{n+1} = u^n + \frac{q\Delta t}{m} \left(E^{n+1/2} + \frac{u^{n+1} + u^n}{2\gamma^{n+1/2}} \times B^{n+1/2} \right) \quad u = \gamma v$

Solved as follows:

$$u^- = u^n + \frac{\Delta t}{2} \frac{q}{m} E^{n+1/2} \quad \leftarrow \text{1/2 electric field push}$$

$$u^+ = u^- + \frac{q\Delta t}{m} \left(\frac{u^+ + u^-}{2\gamma^{n+1/2}} \times B^{n+1/2} \right) \quad \leftarrow \text{1 magnetic field rotation}$$

$$u^{n+1} = u^+ + \frac{\Delta t}{2} \frac{q}{m} E^{n+1/2} \quad \leftarrow \text{1/2 electric field push}$$

*J. P. Boris, *Proc. Fourth Conf. on Num. Sim. Plasmas* 3-67 (1970)



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Recent developments in particle pusher

Problem with Boris pusher: not Lorentz invariant



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Lorentz invariant particle pusher

- Boris pusher introduces error in cancellation of self E and $\mathbf{v} \times \mathbf{B}$

- Velocity push:
$$\mathbf{u}^{n+1} = \mathbf{u}^n + \frac{q\Delta t}{m} \left(\mathbf{E}^{n+1/2} + \frac{\mathbf{u}^{n+1} + \mathbf{u}^n}{2\gamma^{n+1/2}} \times \mathbf{B}^{n+1/2} \right) \quad \mathbf{u} = \gamma \mathbf{v}$$

issue: $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$ implies $\mathbf{E} = \mathbf{B} = 0 \Rightarrow$ large errors when $\mathbf{E} + \mathbf{v} \times \mathbf{B} \neq 0$ (e.g. relativistic beams).

- Solution

- Velocity push:
$$\mathbf{u}^{n+1} = \mathbf{u}^n + \frac{q\Delta t}{m} \left(\mathbf{E}^{n+1/2} + \frac{\mathbf{v}^{n+1} + \mathbf{v}^n}{2} \times \mathbf{B}^{n+1/2} \right) \quad \mathbf{u} = \gamma \mathbf{v}$$

- Looks implicit but solvable analytically*

$$\left\{ \begin{array}{l} \gamma^{i+1} = \sqrt{\frac{\sigma + \sqrt{\sigma^2 + 4(\tau^2 + u^{*2})}}{2}} \\ \mathbf{u}^{i+1} = [\mathbf{u}' + (\mathbf{u}' \cdot \mathbf{t})\mathbf{t} + \mathbf{u}' \times \mathbf{t}] / (1 + t^2) \end{array} \right. \quad \left(\text{with } \mathbf{u} = \gamma \mathbf{v}, \quad \mathbf{u}' = \mathbf{u}^i + \frac{q\Delta t}{m} \left(\mathbf{E}^{i+1/2} + \frac{\mathbf{v}^i}{2} \times \mathbf{B}^{i+1/2} \right), \quad \tau = (q\Delta t / 2m) \mathbf{B}^{i+1/2}, \right.$$

$$\left. \mathbf{u}^* = \mathbf{u}' \cdot \boldsymbol{\tau} / c, \quad \sigma = \gamma'^2 - \tau^2, \quad \gamma' = \sqrt{1 + u'^2 / c^2}, \quad \mathbf{t} = \boldsymbol{\tau} / \gamma^{i+1} \right).$$

*J.-L. Vay, *Phys. Plasmas* 15, 056701 (2008)



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*J.-L. Vay, *Phys. Plasmas* 15, 056701 (2008)



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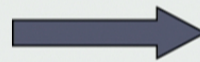
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Single particle test of Lorentz invariant pusher

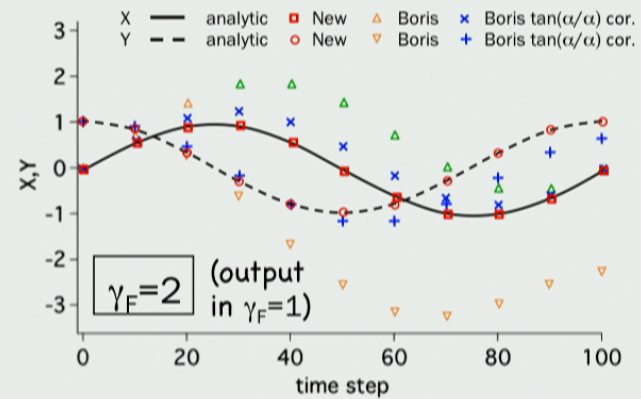
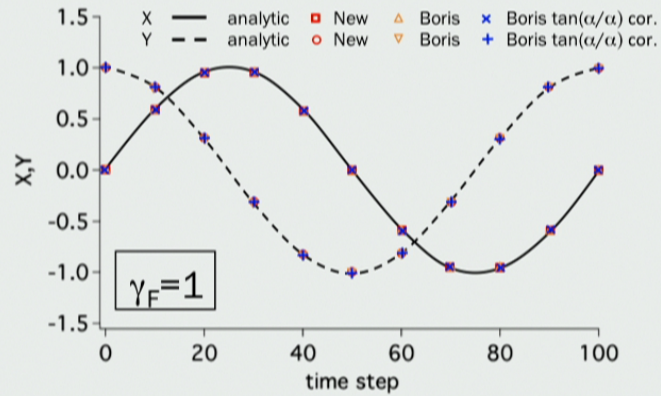
Lab frame

particle cycling in constant B field



Boosted frame $\gamma=2$

ExB drift adds to gyration



Vay - IPAM 2012



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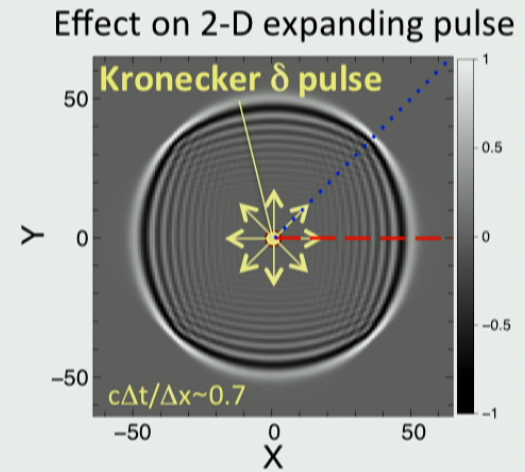
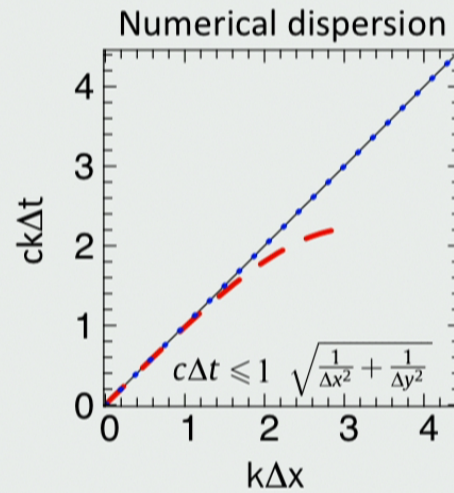
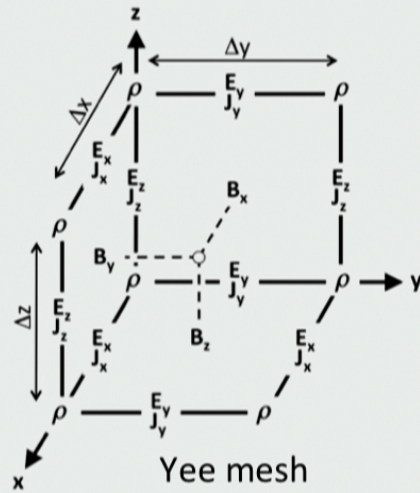
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Field solver most commonly 2nd order “Yee” solver*



*K. S. Yee, IEEE Trans. Antennas Prop. 14 (1966).



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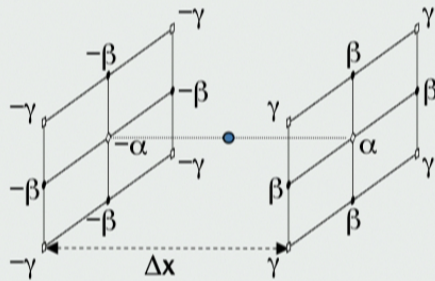
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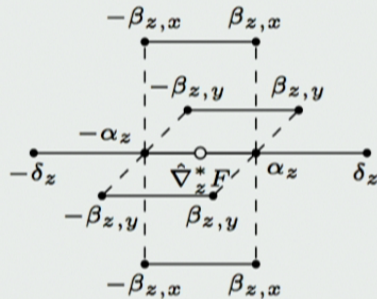
Non-Standard FD solvers offer some tunability

NSFD^{1,2}: weighted average of quantities transverse to FD

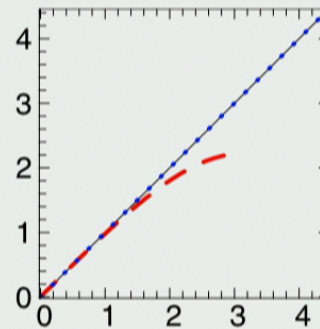
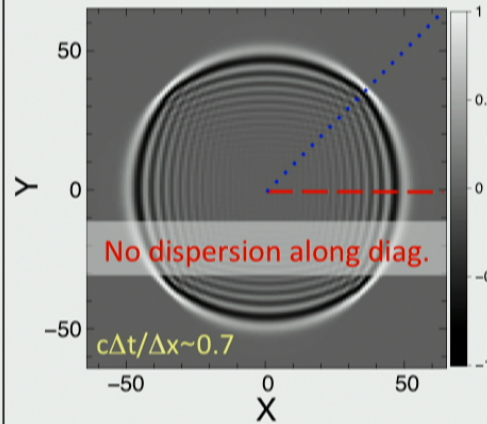


- Adaptations to PIC^{3,4,5}

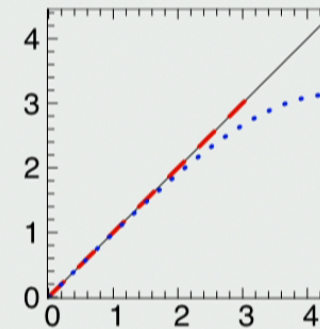
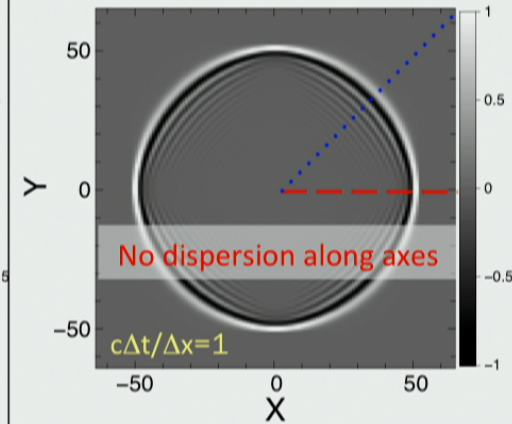
Lehe⁶ algorithm:



FDTD (Yee)



NS-FDTD (Karkkainen/Lehe)



¹J. B. Cole, IEEE Trans. Microw. Theory Tech. **45** (1997).

²M. Karkkainen et al., Proc. ICAP, Chamonix, France (2006).

³A. Pukhov, J. Plasma Physics **61** (1999) 425.

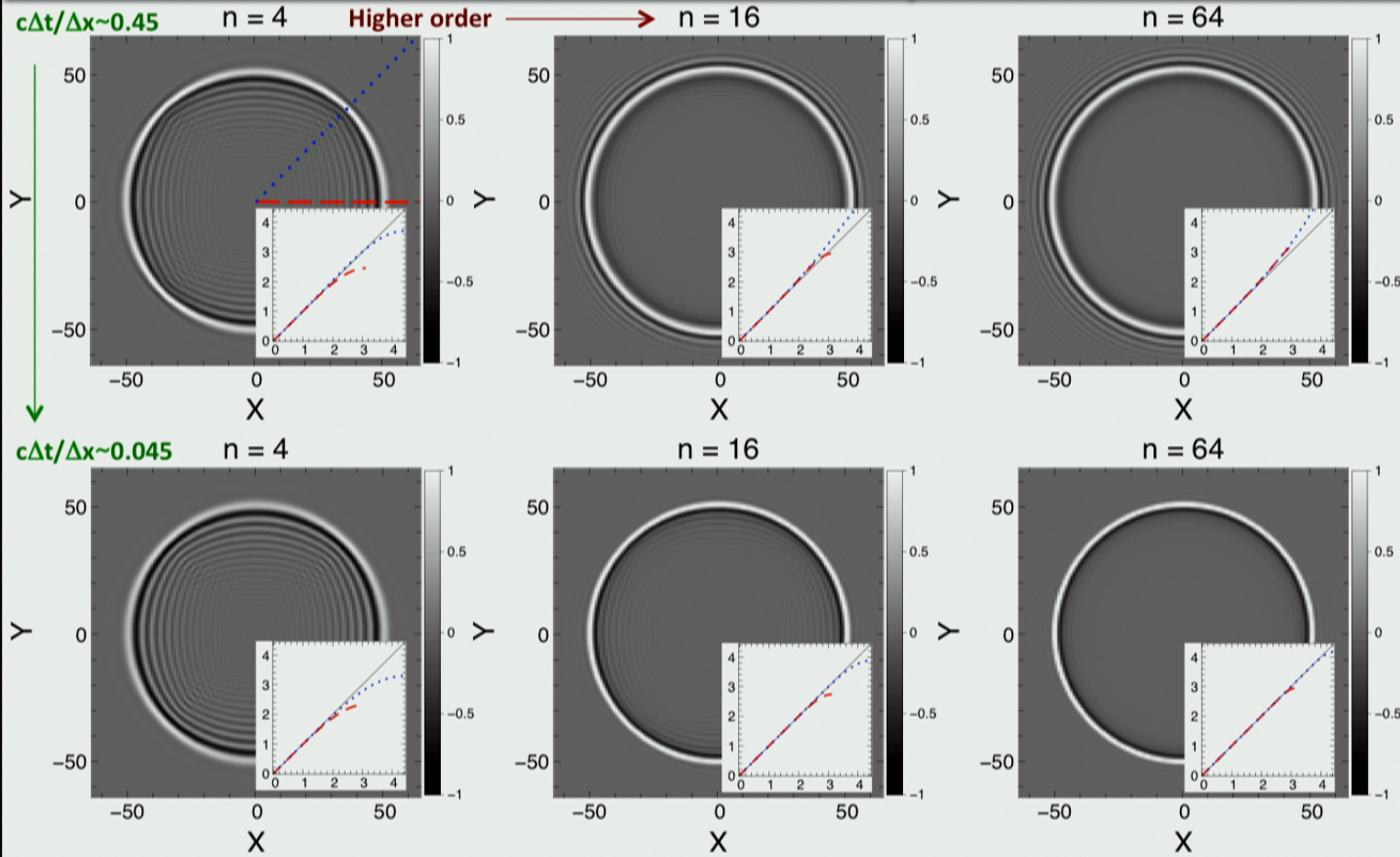
⁴J.-L. Vay et al, J. Comput. Phys. **230** (2011) 5908.

⁵B. Cowan et al, PRST-AB **16** (2013) 041303.

⁶R. Lehe et al, PRST-AB **16** (2013) 021301.

High-order stencils + small time steps

→ exact solution but expensive



Spectral solver offers “infinite order” but still needs small time steps

Finite-Difference Time-Domain
(FDTD)

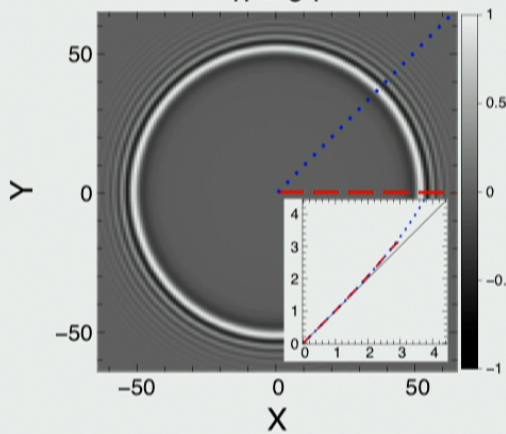
Pseudo-Spectral Time-Domain
(PSTD)

$\mathcal{F}=\text{FFT}$

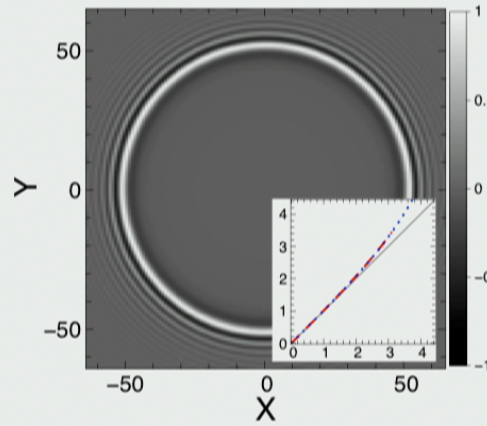
$$B_z^{n+1} = B_z^n + \Delta t \left(\frac{\Delta E_x}{\Delta y} - \frac{\Delta E_y}{\Delta x} \right)$$

$$B_z^{n+1} = B_z^n + \Delta t \left[\mathcal{F}^{-1} \left(ik_y \mathcal{F}(E_x) \right) - \mathcal{F}^{-1} \left(ik_x \mathcal{F}(E_y) \right) \right]$$

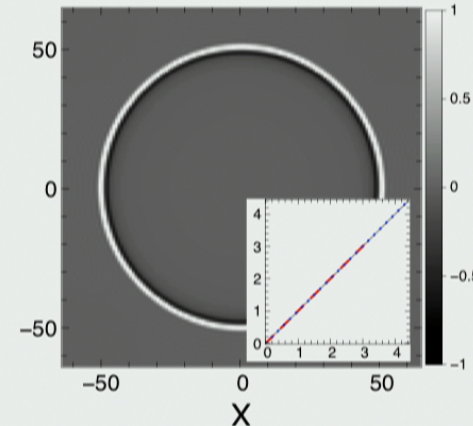
FDTD $c\Delta t/\Delta x \sim 0.45$
 $n = 64$



PSTD $c\Delta t/\Delta x \sim 0.45$



PSTD $c\Delta t/\Delta x \sim 0.045$



PSTD is limit of high-order FDTD when $n \rightarrow \text{infinity}$.

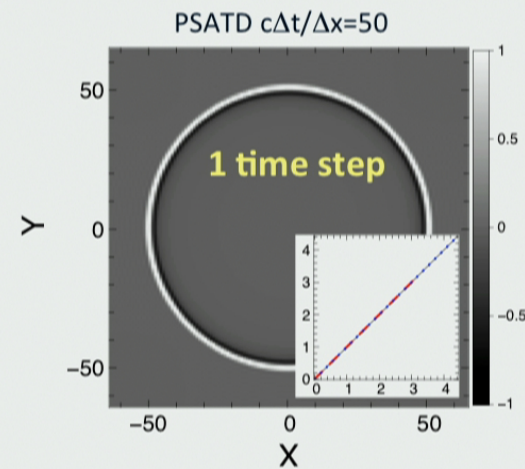
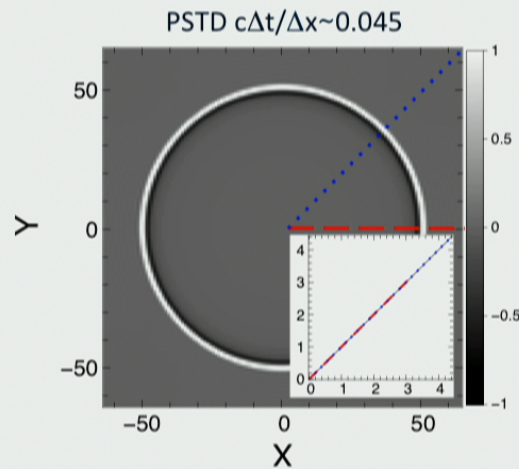
PSTD converges to exact solution (on grid) for $\Delta t \rightarrow 0$.

Analytical pseudo-spectral solver offers **exact solution** with **no Courant condition**

Pseudo-Spectral Analytical Time-Domain¹ (PSATD)

$$B_z^{n+1} = \mathcal{F}^{-1} \left(C \mathcal{F} (B_z^n) \right) + \mathcal{F}^{-1} \left(i S k_y \mathcal{F} (E_x) \right) - \mathcal{F}^{-1} \left(i S k_x \mathcal{F} (E_y) \right)$$

with $C = \cos(kc\Delta t)$; $S = \sin(kc\Delta t)$; $k = \sqrt{k_x^2 + k_y^2}$

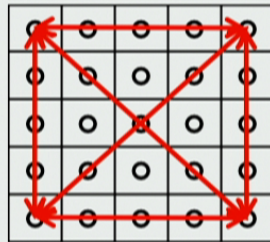


¹I. Haber, R. Lee, H. Klein & J. Boris, *Proc. Sixth Conf. on Num. Sim. Plasma*, Berkeley, CA, 46-48 (1973)

But **spectral solvers** involve **global operations**:
→ **harder to scale to large # of cores**

Spectral

**global “costly”
communications**

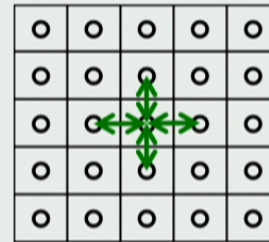


Harder to scale

VS

Finite Difference (FDTD)

**local “cheap”
communications**



Easier to scale

*J.-L. Vay, I. Haber, B. Godfrey, *J. Comput. Phys.* **243**, 260-268 (2013)



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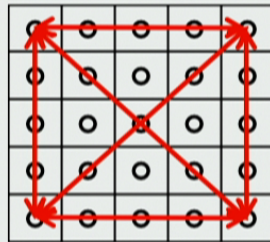
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But **spectral solvers** involve **global operations**:
→ **harder to scale to large # of cores**

Spectral

**global “costly”
communications**

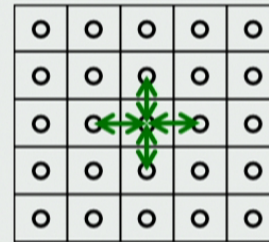


Harder to scale

VS

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Easier to scale

*J.-L. Vay, I. Haber, B. Godfrey, *J. Comput. Phys.* **243**, 260-268 (2013)



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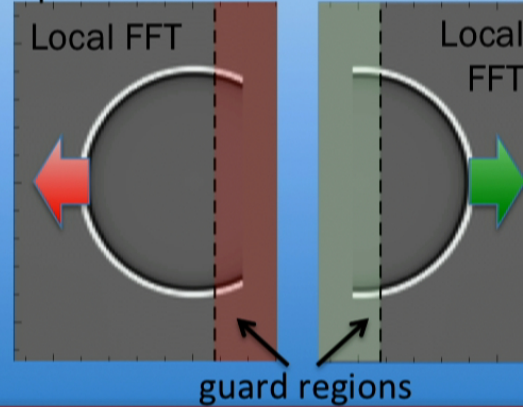


New concept on single pulse – part 1

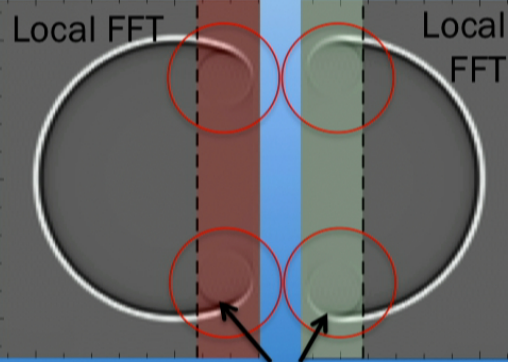
Example: unit pulse expansion at time T



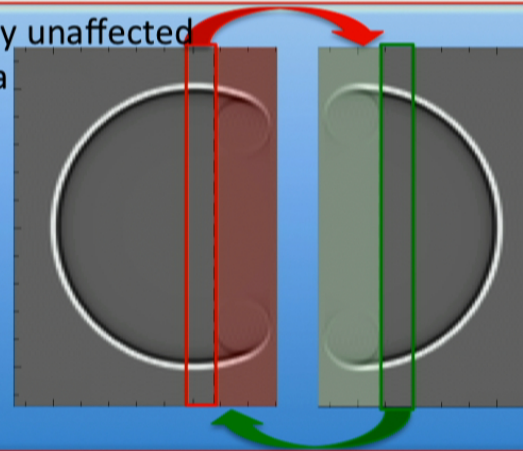
Separate calculation in two domains



Advance to time $T+DT$



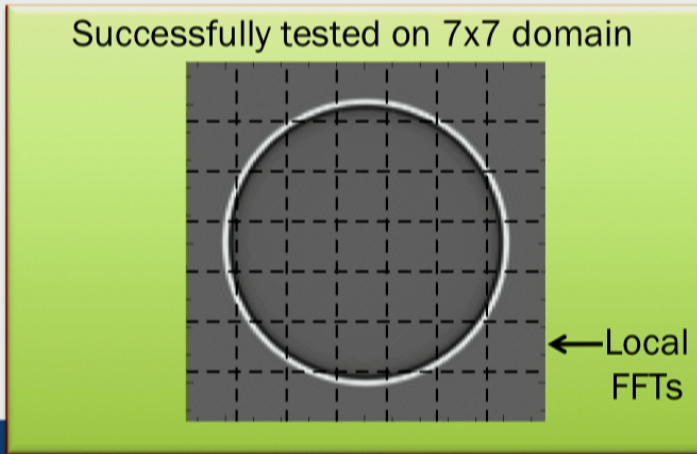
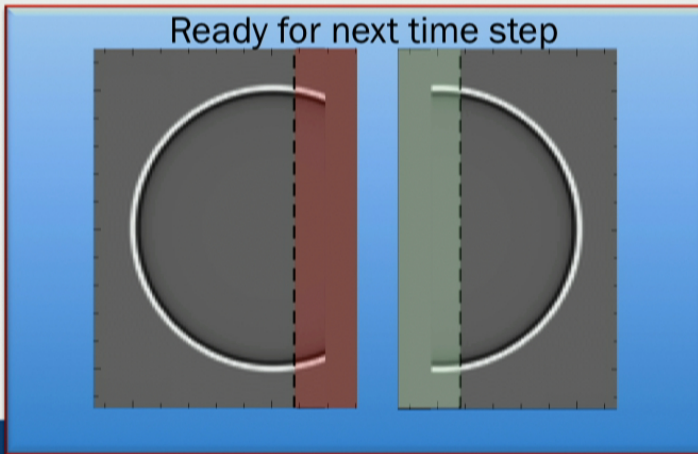
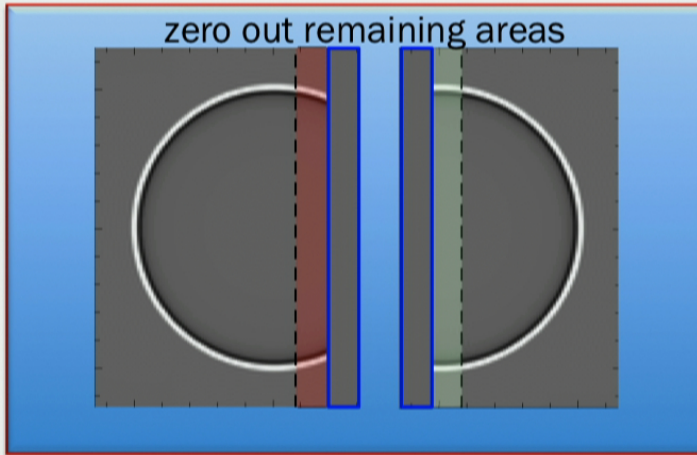
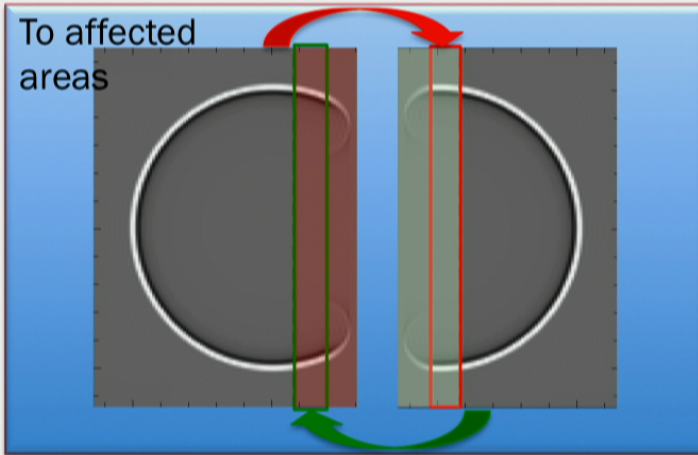
Copy unaffected data



*J.-L. Vay, I. Haber, B. Godfrey, *J. Comput. Phys.* **243**, 260-268 (2013)

29

New concept on single pulse – part 2



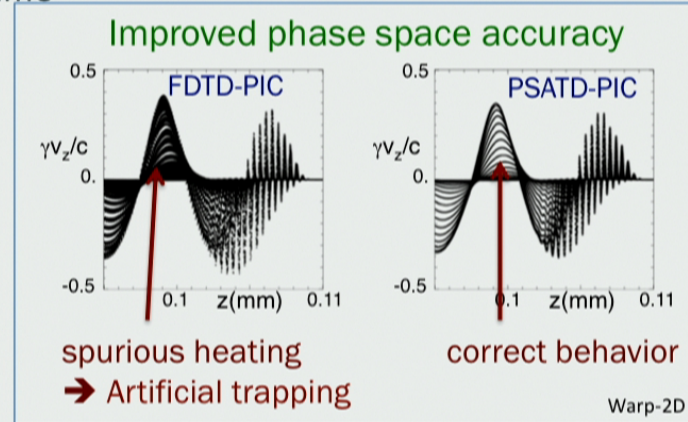
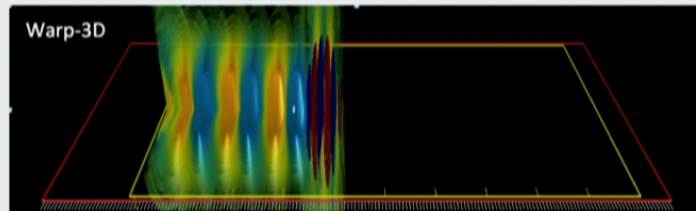
*J.-L. Vay, I. Haber, B. Godfrey, *J. Comput. Phys.* **243**, 260-268 (2013)

30

Successfully tested on 2-D modeling of short LPA stages

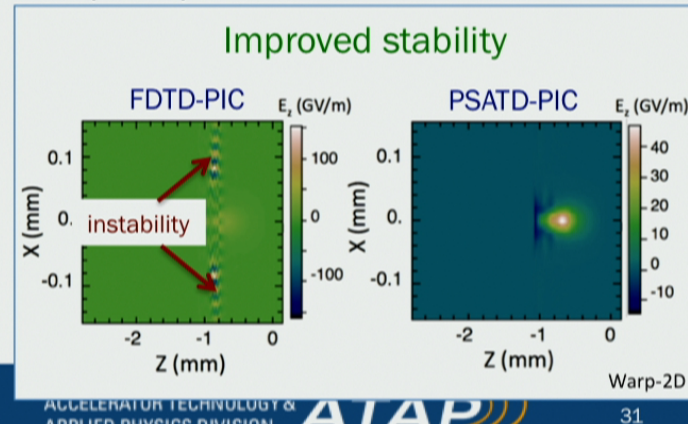
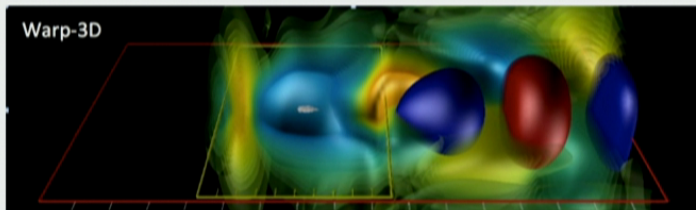
Lab frame

Short laser propagates into long plasma channel, electron beam accelerated in wake.



Lorentz boosted frame (wake)

Modeling in a boosted frame reduces # time steps. Plasma drifting near C leads to Num. Cherenkov.



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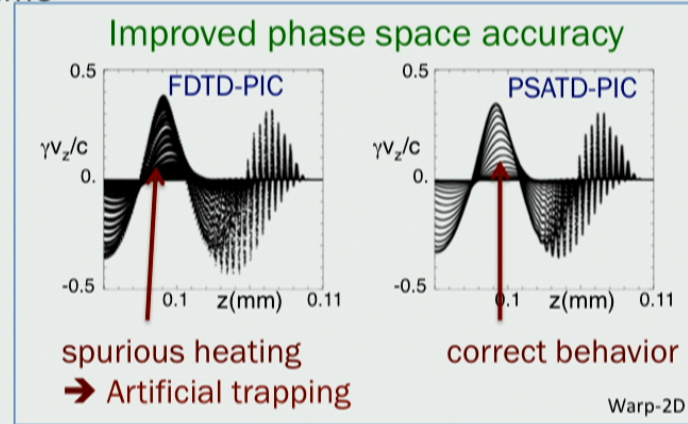
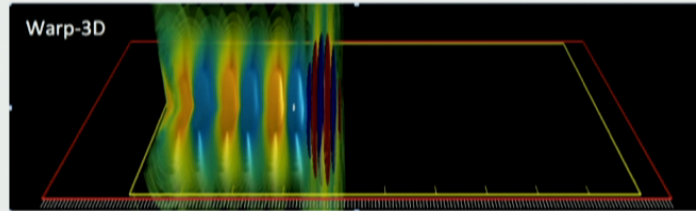


31

Successfully tested on 2-D modeling of short LPA stages

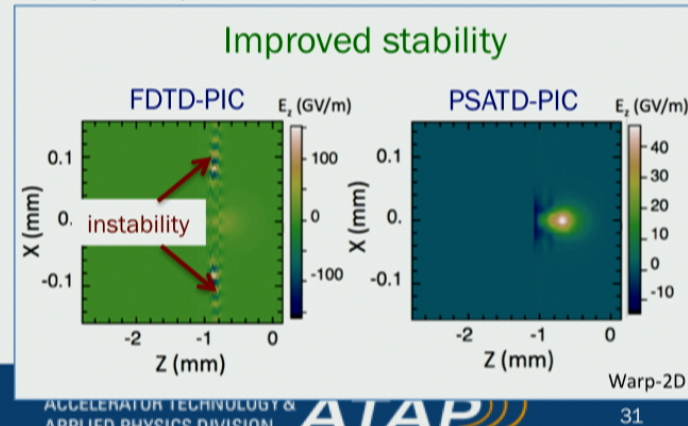
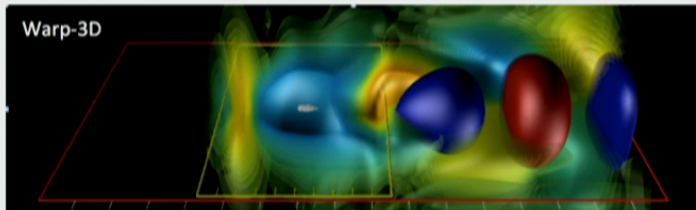
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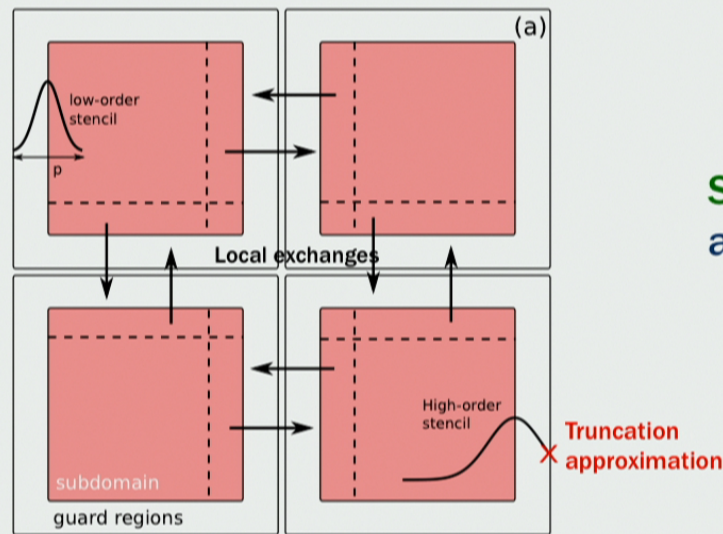
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31

Domain decomposition with high/infinite-order stencil implies stencil truncation



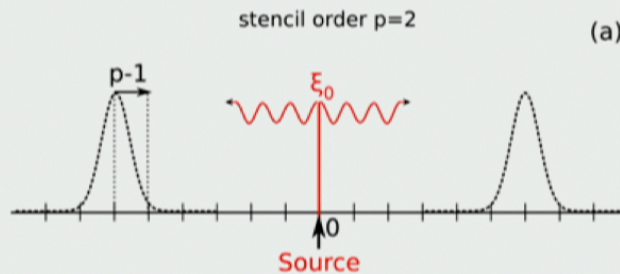
Scales but approximation
at very high/infinite orders!

Need to characterize/mitigate truncation errors

to enable new method to scale to millions or billions of cores!

New error-predictive analytical model for stencil spatial variations/truncations

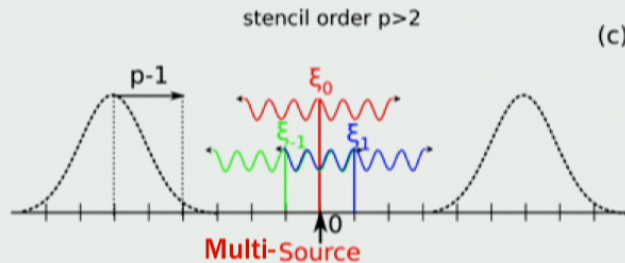
Order 2 stencil: stencil modification at one point accounted with a single source term*



- Yields inaccurate results for orders $p > 2$
- Total truncation error amplitude:

$$\zeta = \xi_0$$

Higher order: stencil modification at one point accounted by multi-source error terms**



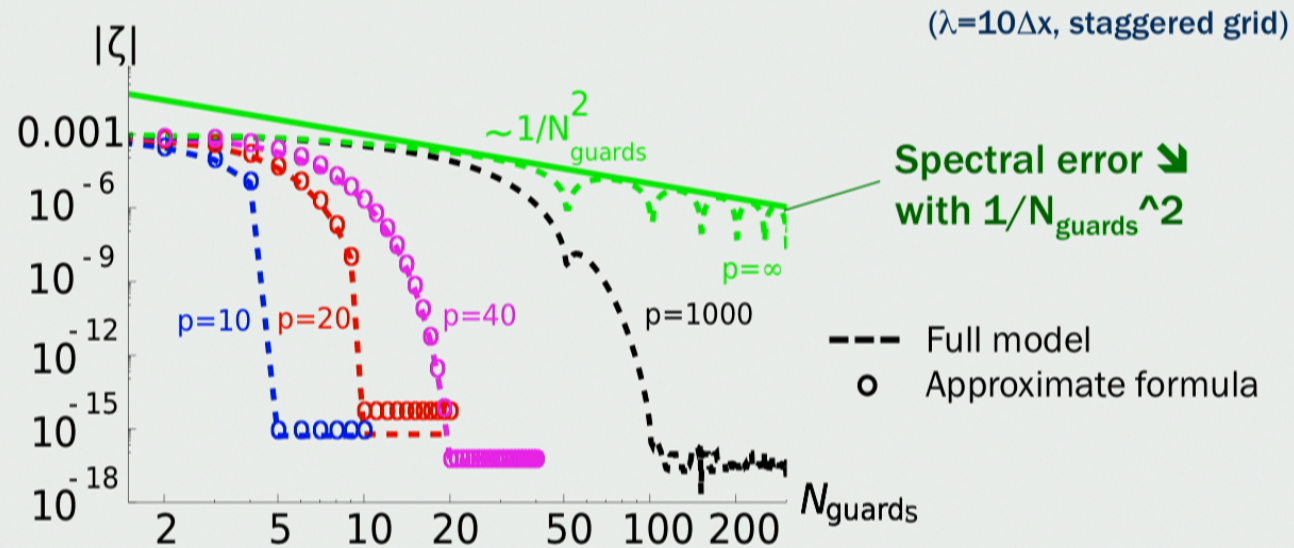
- Solves discrepancy at orders $p > 2$
- Total truncation error amplitude:

$$\zeta = \sum_l \xi_l e^{jkl\delta x}$$

*J.-L. Vay, *J. Comput. Phys.* **183**, 367 (2002).

H. Vincenti, J.-L. Vay, *Comp. Phys. Comm.* **196, 221 (2015).

Model predicts error from stencil truncation



Variation with wavelength (not shown) are also perfectly reproduced

Finite very-high order requires \ll # guard cells than spectral.

*H. Vincenti, J.-L. Vay, *Comp. Phys. Comm.* **196**, 221 (2015).

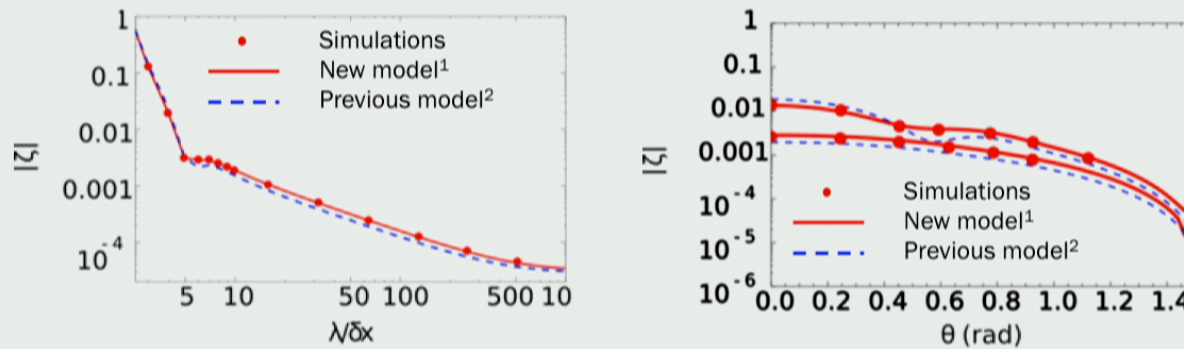


New analysis confirms efficacy of PML with spectral

Perfectly Matched Layer (PML) enables efficient open boundary conditions



New analysis predicts exact coefficients of reflections



¹H. Vincenti, J.-L. Vay, *Comp. Phys. Comm.* **196**, 221 (2015).

²P. Lee, J.-L. Vay, *Comp. Phys. Comm.* **194**, 1-9 (2015).



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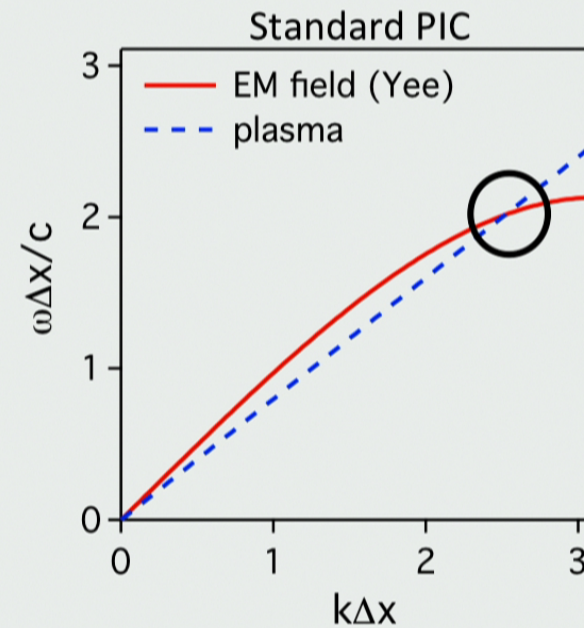
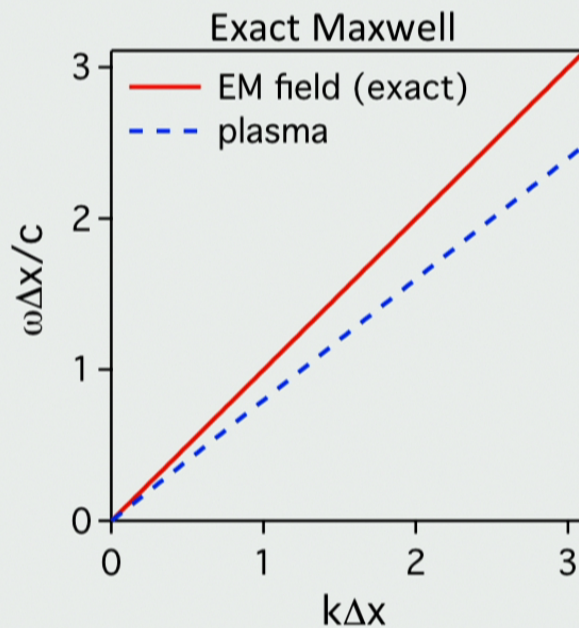
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Relativistic plasmas PIC subject to “numerical Cherenkov”

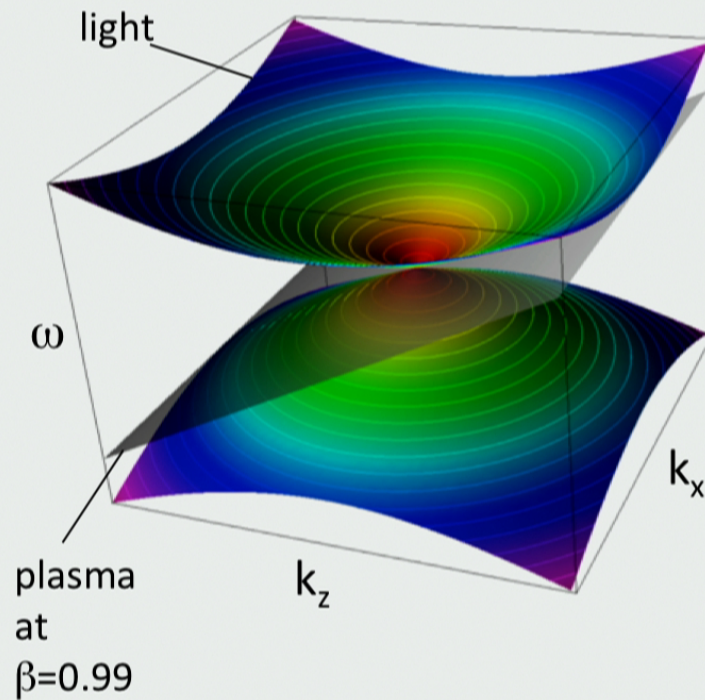
B. B. Godfrey, “Numerical Cherenkov instabilities in electromagnetic particle codes”,
J. Comput. Phys. **15** (1974)

Numerical dispersion leads to crossing of EM field and plasma modes \rightarrow instability.

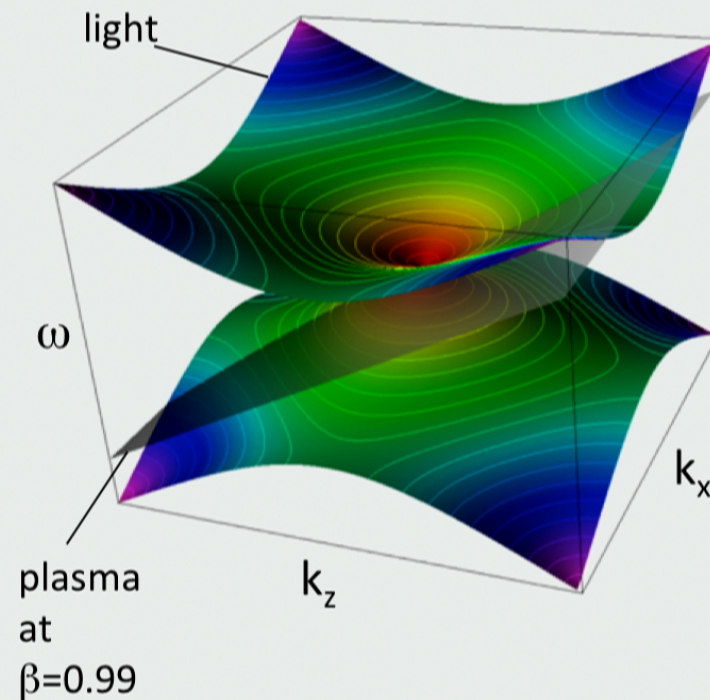


Space/time discretization **aliases** → more **crossings** in 2/3-D

Exact Maxwell



Standard PIC



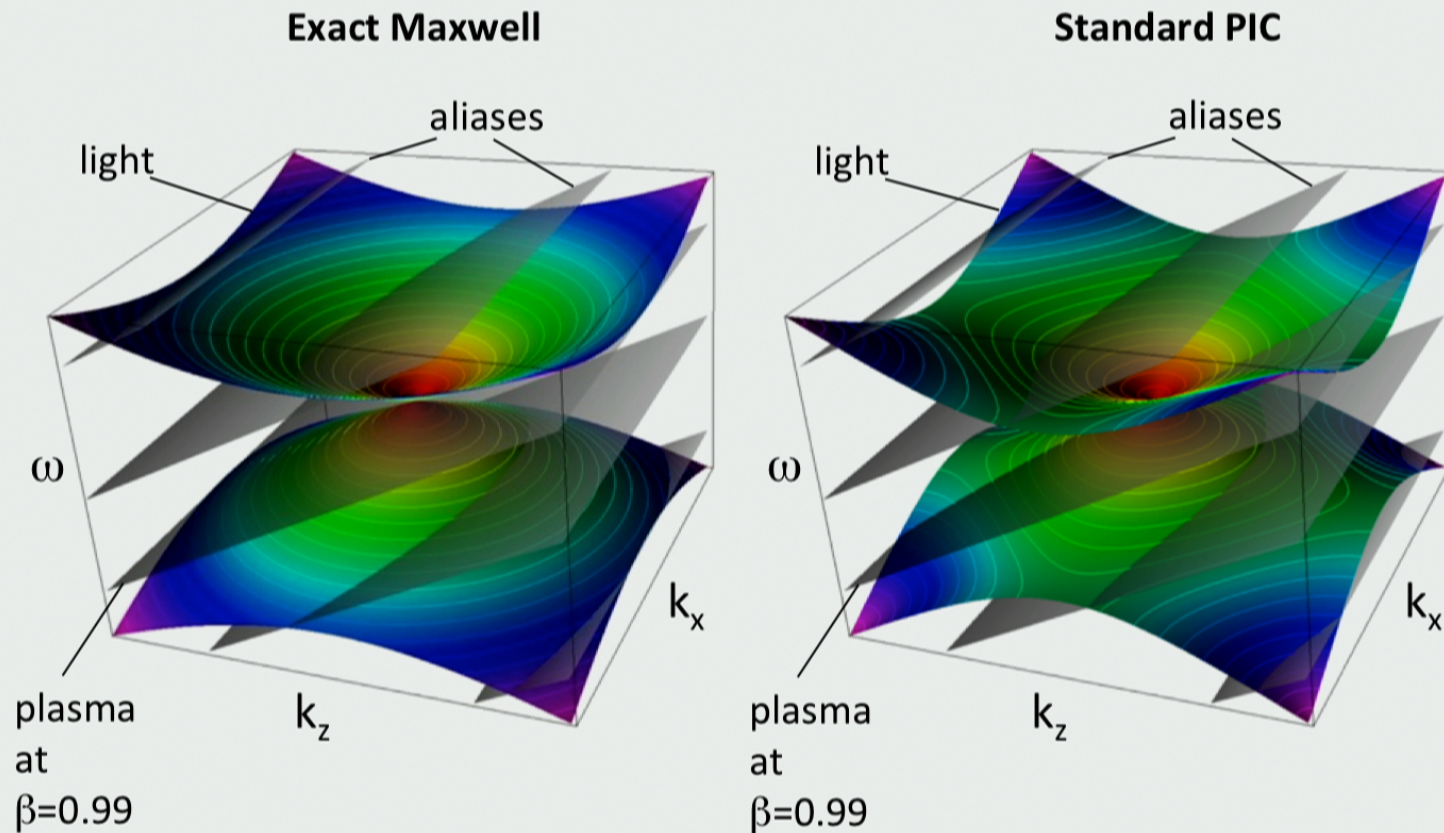
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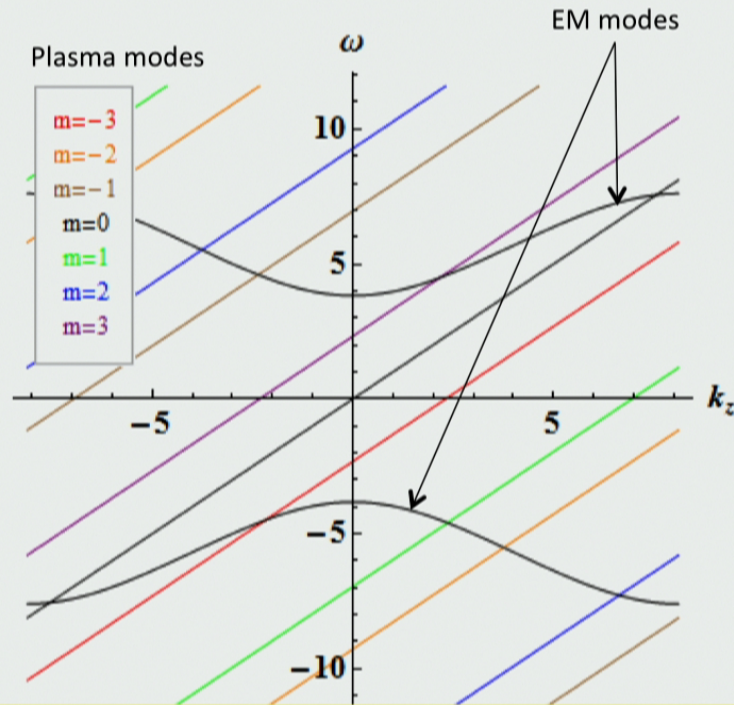
Space/time discretization **aliases** → more **crossings** in 2/3-D



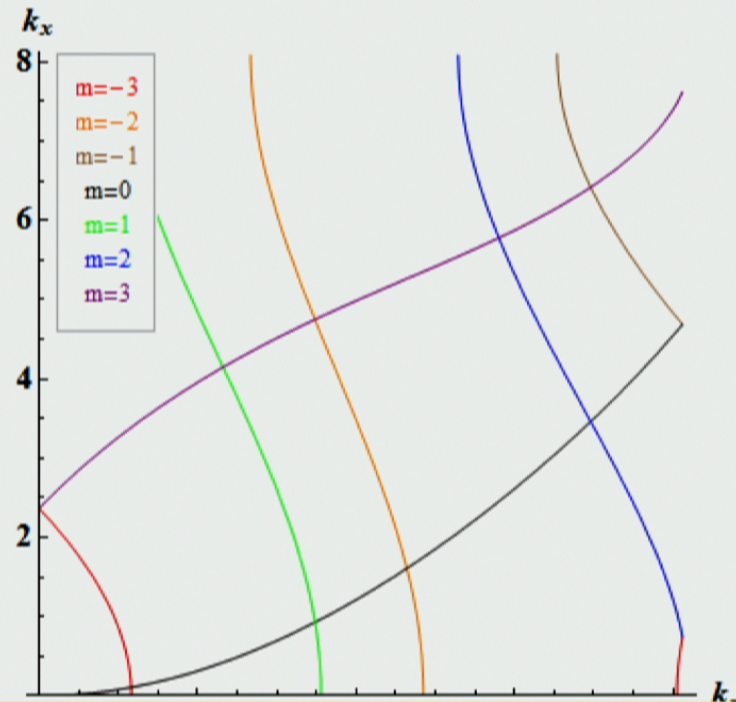
Need to consider at least first aliases $m_x = \{-3 \dots +3\}$ to study stability.

Maps of unstable modes

Normal modes
at $k_x=0.5\pi/\Delta x$ for $c\Delta t=0.7\Delta z$



Projection of normal
modes intersection



Analysis calls for full PIC numerical dispersion relation

Numerical dispersion relation of full-PIC algorithm*

$$\text{E.g.: 2-D relation (Fourier space): } \begin{pmatrix} \xi_{z,z} + [\omega] & \xi_{z,x} & \xi_{z,y} + [k_x] \\ \xi_{x,z} & \xi_{x,x} + [\omega] & \xi_{x,y} - [k_z] \\ [k_x] & -[k_z] & [\omega] \end{pmatrix} \begin{pmatrix} E_z \\ E_x \\ B_y \end{pmatrix} = 0.$$

$$[\omega] = \sin\left(\omega \frac{\Delta t}{2}\right) / \left(\frac{\Delta t}{2}\right) \quad [k_z] = k_z \sin\left(k \frac{\Delta t}{2}\right) / \left(k \frac{\Delta t}{2}\right) \quad [k_x] = k_x \sin\left(k \frac{\Delta t}{2}\right) / \left(k \frac{\Delta t}{2}\right)$$

$$S^J = \left[\sin\left(k'_z \frac{\Delta z}{2}\right) / \left(k'_z \frac{\Delta z}{2}\right) \right]^{\ell_z+1} \left[\sin\left(k'_x \frac{\Delta x}{2}\right) / \left(k'_x \frac{\Delta x}{2}\right) \right]^{\ell_x+1},$$

$$S^{E_z} = \left[\sin\left(k'_z \frac{\Delta z}{2}\right) / \left(k'_z \frac{\Delta z}{2}\right) \right]^{\ell_z} \left[\sin\left(k'_x \frac{\Delta x}{2}\right) / \left(k'_x \frac{\Delta x}{2}\right) \right]^{\ell_x+1} (-1)^{m_z},$$

$$S^{E_x} = \left[\sin\left(k'_z \frac{\Delta z}{2}\right) / \left(k'_z \frac{\Delta z}{2}\right) \right]^{\ell_z+1} \left[\sin\left(k'_x \frac{\Delta x}{2}\right) / \left(k'_x \frac{\Delta x}{2}\right) \right]^{\ell_x} (-1)^{m_x},$$

$$S^{B_y} = \cos\left(\omega \frac{\Delta t}{2}\right) \left[\sin\left(k'_z \frac{\Delta z}{2}\right) / \left(k'_z \frac{\Delta z}{2}\right) \right]^{\ell_z} \left[\sin\left(k'_x \frac{\Delta x}{2}\right) / \left(k'_x \frac{\Delta x}{2}\right) \right]^{\ell_x} (-1)^{m_z+m_x}.$$

*B. B. Godfrey, J. L. Vay, I. Haber, J. Comp. Phys. 248 (2013)



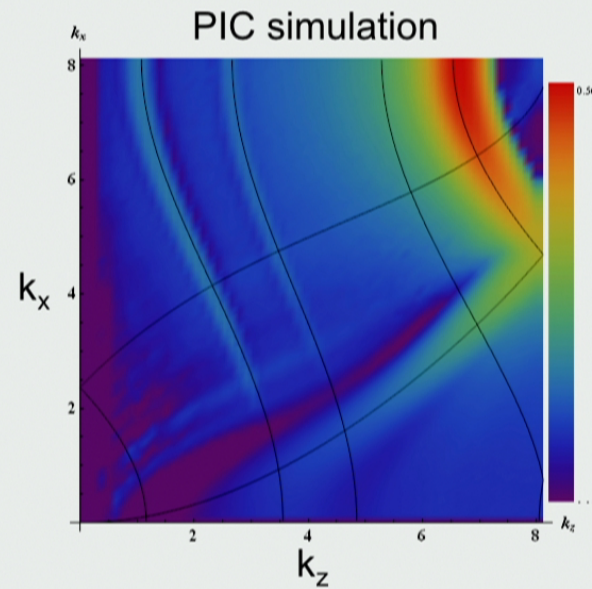
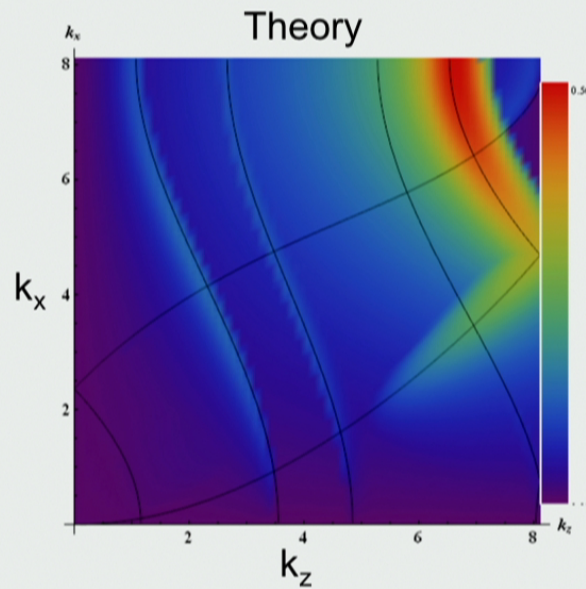
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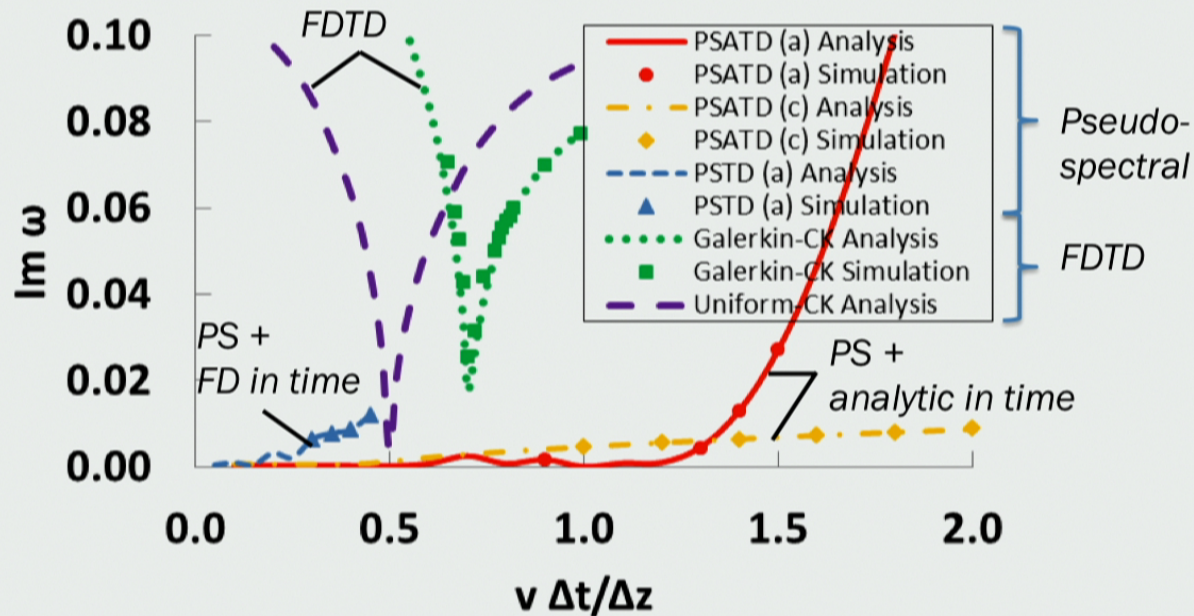
Growth rates from theory match PIC simulations



PIC: uniform drifting plasma
with periodic BC.

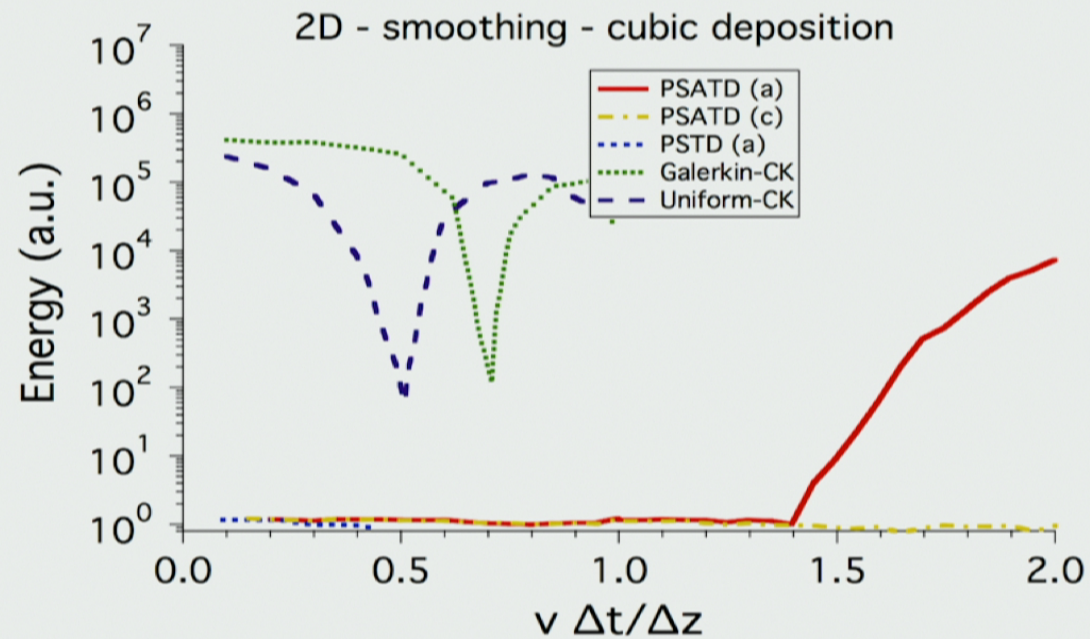
Yee finite difference, energy conserving gather ($c\Delta t/\Delta x=0.7$)

Stability analysis also shows that pseudo-spectral algorithms are more stable*



*B. B. Godfrey, J.-L. Vay, I. Haber, "Numerical stability analysis of the Pseudo-Spectral Analytical Time-Domain PIC algorithm", *J. Comp. Phys.* **258** (2014) 689.

Stability over wide range of time steps confirmed with Warp*



*B. B. Godfrey, J.-L. Vay, I. Haber, "Numerical stability analysis of the Pseudo-Spectral Analytical Time-Domain PIC algorithm", *J. Comp. Phys.* **258** (2014) 689.



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Rapid recent progress on analysis and mitigation of numerical Cherenkov instability

- **Analysis of Numerical Cherenkov has been generalized:**
 - **to finite-difference PIC codes (“Magical” time step explained):**
 - B. B. Godfrey and J.-L. Vay, J. Comp. Phys. 248 **(2013)** 33.
 - X. Xu, et. al., Comp. Phys. Comm., 184 **(2013)** 2503.
 - **to pseudo-spectral PIC codes:**
 - B. B. Godfrey, J. -L. Vay, I. Haber, J. Comp. Phys., 258 **(2014)** 689.
 - P. Yu et. al, J. Comp. Phys. 266 **(2014)** 124.
- **Efficient suppression techniques were recently developed:**
 - **for finite-difference PIC codes:**
 - B. B. Godfrey and J.-L. Vay, J. Comp. Phys. 267 **(2014)** 1.
 - B. B. Godfrey and J.-L. Vay, Comp. Phys. Comm. 196, **(2015)** 221.
 - **for pseudo-spectral PIC codes:**
 - B. B. Godfrey, J.-L. Vay, I. Haber, IEEE Trans. Plas. Sci. 42 **(2014)** 1339.
 - P. Yu, et. al., Comp. Phys. Comm., 192/197 **(2015)** 32/144.



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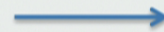
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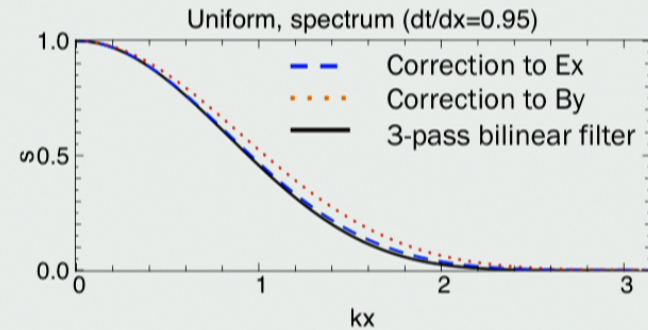
47

Local special filter efficiently suppresses instability

Filters fields seen by particles using 9-points linear smoothing in direction of plasma drift;



- key feature:
 - slightly different smoothing for E_x and B_y
- inexpensive and easy to implement
- local: amount of smoothing similar to standard 3-pass bilinear filter



*B. Godfrey & J.-L. Vay, *J. Comput. Phys.* **267** (2014)



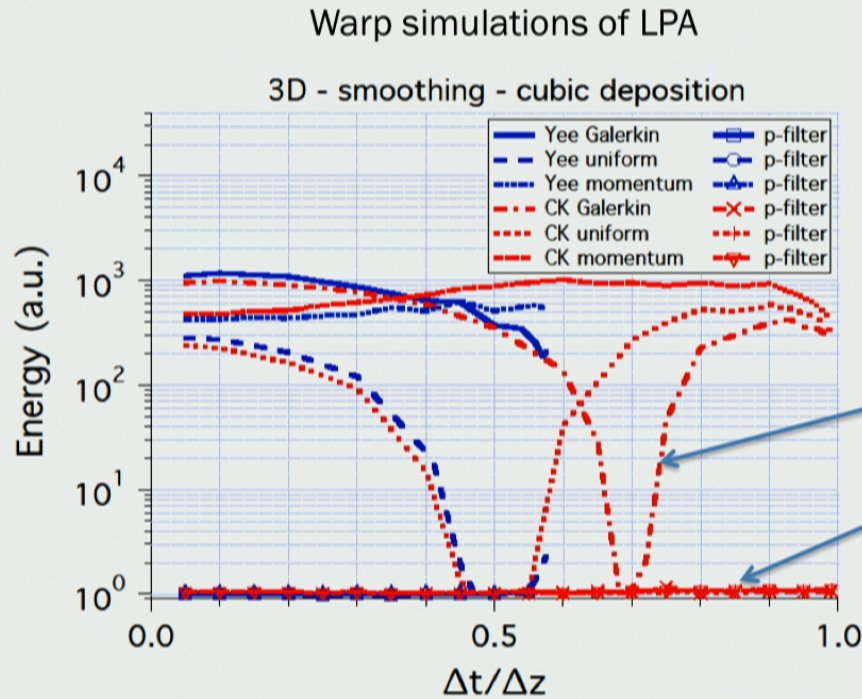
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Filtering scheme yields excellent stability



Without filtering, stability only around “magical time steps”*

New filter** enables stability at all time steps.

*J.-L. Vay, et al., *J. Comput. Phys.* **230**, 5908 (2011)

B. Godfrey & J.-L. Vay, *J. Comput. Phys.* **267 (2014)

Exascale Supercomputing



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Leadership and Production Computing Facilities



Titan:

- Peak performance of 27.1 PF
- 18,688 Hybrid Compute Nodes
- 8.9 MW peak power



Mira:

- Peak performance of 10 PF
- 49,152 Compute Nodes
- 4.8 MW peak power

Edison XC30:

- Peak performance 2.4 PF
- 124,608 processing cores
- 2.1 MW peak power



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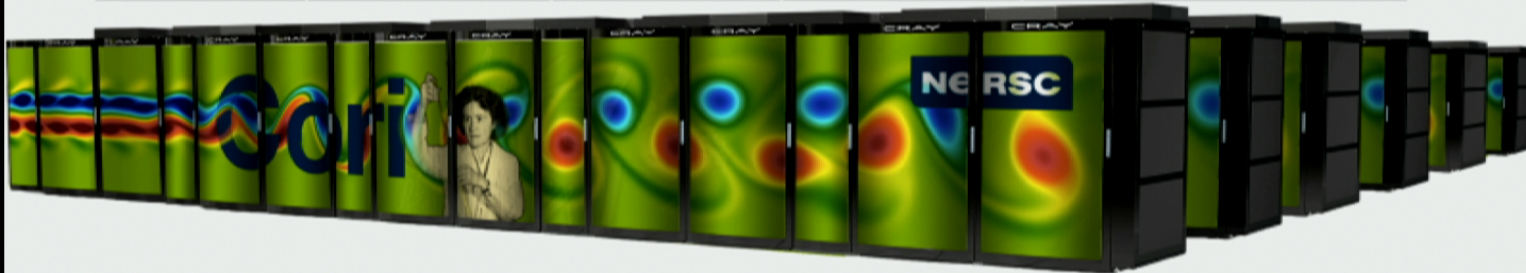


NERSC systems to reach 1 ExaFlops by 2024

NERSC Systems Timeline



2007/2009	NERSC-5	Franklin	Cray XT4	102/352 TF
2010	NERSC-6	Hopper	Cray XE6	1.28 PF
2014	NERSC-7	Edison	Cray XC30	2.57 PF
2016	NERSC-8	Cori	Cray XC	30 PF
2020	NERSC-9			100PF-300PF
2024	NERSC-10			1EF
2028	NERSC-11			5-10EF



- 13 -
Courtesy Sudip Dosanjh, IXPUG 2015

Computing Sciences Area

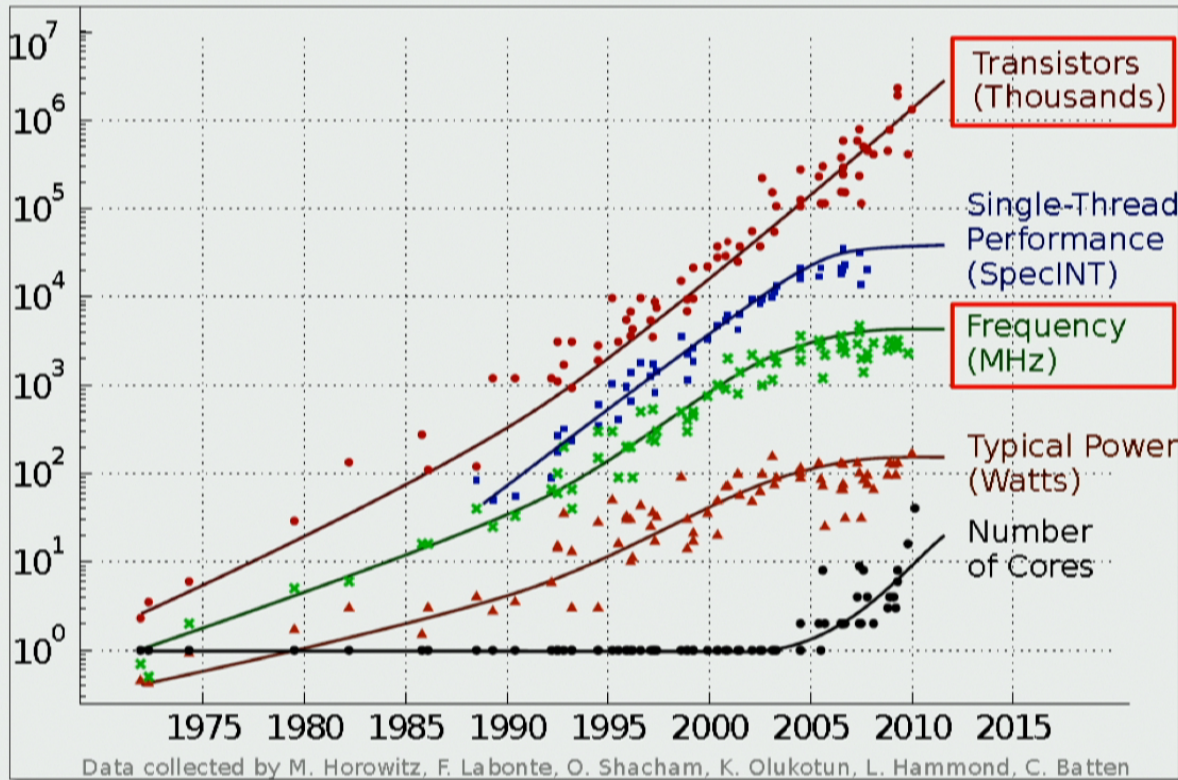


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Progress in CMOS CPU Technology



Moore's Law continues

- Transistor count still doubles every 24 months

Dennard scaling stalls

- Voltage
- Clock Speed
- Power
- Performance/clock



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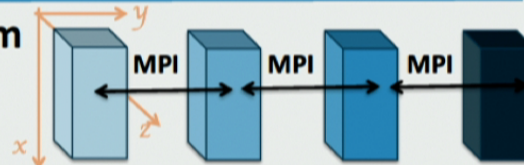
Novel exascale supercomputers require restructuration with “multi-level parallelism”

To run effectively on future systems



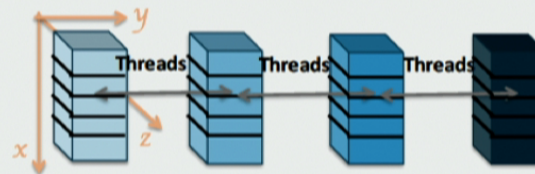
- **Manage Domain Parallelism**

- independent program units; explicit



- **Increase Thread Parallelism**

- independent execution units within the program; generally explicit



- **Exploit Data Parallelism**

- Same operation on multiple elements

- **Improve data locality**

- Cache blocking;
Use on-package memory

```

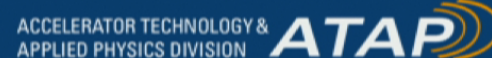
|--> DO I = 1, N
|      R(I) = B(I) + A(I)
|--> ENDDO
    
```



- 12 -



Courtesy Katherine Riley, FES Exascale Review, 2016

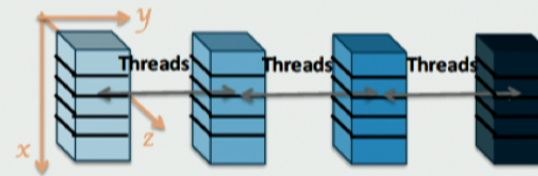
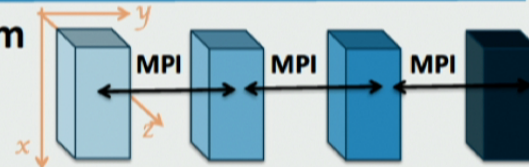


Novel exascale supercomputers require restructuration with “multi-level parallelism”

To run effectively on future systems



- **Manage Domain Parallelism**
 - independent program units; explicit
- **Increase Thread Parallelism**
 - independent execution units within the program; generally explicit
- **Exploit Data Parallelism**
 - Same operation on multiple elements
- **Improve data locality**
 - Cache blocking;
 - Use on-package memory

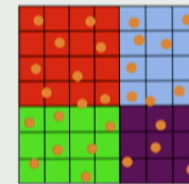


```

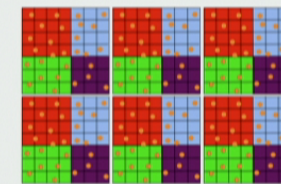
|--> DO I = 1, N
|      R(I) = B(I) + A(I)
|--> ENDDO
    
```

Particle-In-Cell

Domain decomposition



Domain decomposition + tiling



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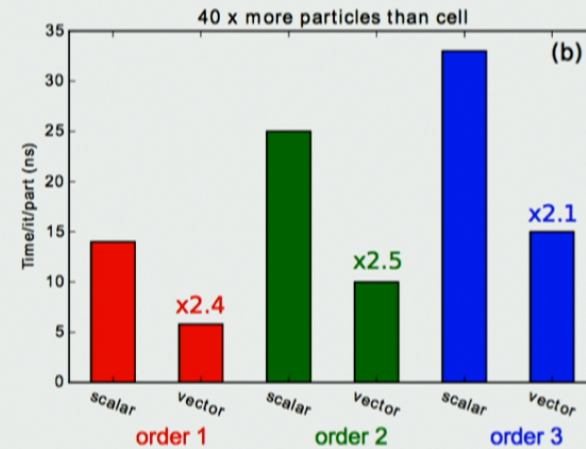
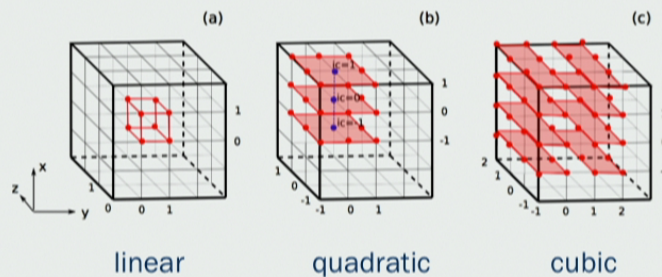


Novel vectorization algorithm leads to >2x speedup on charge/current deposition routines

- Previous algorithms developed on vector supercomputers in 70s-90s do not work
- Novel algorithm developed* (implemented in Warp/PICSAR)

➤ Benchmarks on Cori (Haswell CPU)

Tests demonstrate speedups >2x
(max. theoretical=4)



*H. Vincenti, R. Lehe, R. Sasanka, J-L. Vay, « An efficient and portable SIMD algorithm for charge/current deposition in Particle-In-Cell codes », arXiv:1601.02056, submitted to Comp. Phys. Comm.



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Developing **common** modules/data format is **beneficial**

- OpenPMD:
 - a common I/O format for simulations with particles and meshes
 - standardized layout of data in file (using hdf5, netcdf, ADIOS, ..)
 - for easy comparisons between codes, common visualization tools
 - OpenPMD Viewer based on IPython+Matplotlib available, Visit reader in dev.
 - implemented in Warp, PConGPU, FBPIC, ...
 - More at <https://github.com/openPMD> at <http://www.openpmd.org>
- PICSAR (Particle-In-Cell Scalable Application Resource)
 - Open Source collection of PIC kernel subroutines (current deposition, field gather, field solve, particle pusher, ...)
 - Collaborative development of multi-level parallel implementations (vectorization+OpenMP+MPI+GPU+...)
 - For testing, comparing, distributing production-level PIC functionalities
 - To be available to wider community soon
 - send email to jlvey@lbl.gov if interested



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Conclusion

Modeling of astrophysical jets/shocks and plasma-based accelerators share common Particle-In-Cell methods and codes

Recent advances enable more stable and accurate codes

- Lorentz-invariant particle pusher
- Pseudo-spectral solvers
- Numerical Cherenkov theory and mitigation

Better tools on novel (and future exascale) supercomputers will enable full 3-D parametric studies

Looking forward to opportunities in cross-fertilizing collaborations



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