

Title: Plasma effects on extra galactic ultra high energy cosmic ray hadron beams in cosmic voids

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Abstract:

# Plasma effects on extragalactic ultrahighenergy cosmic ray hadron beams in cosmic voids II. Kinetic instability of parallel electrostatic waves

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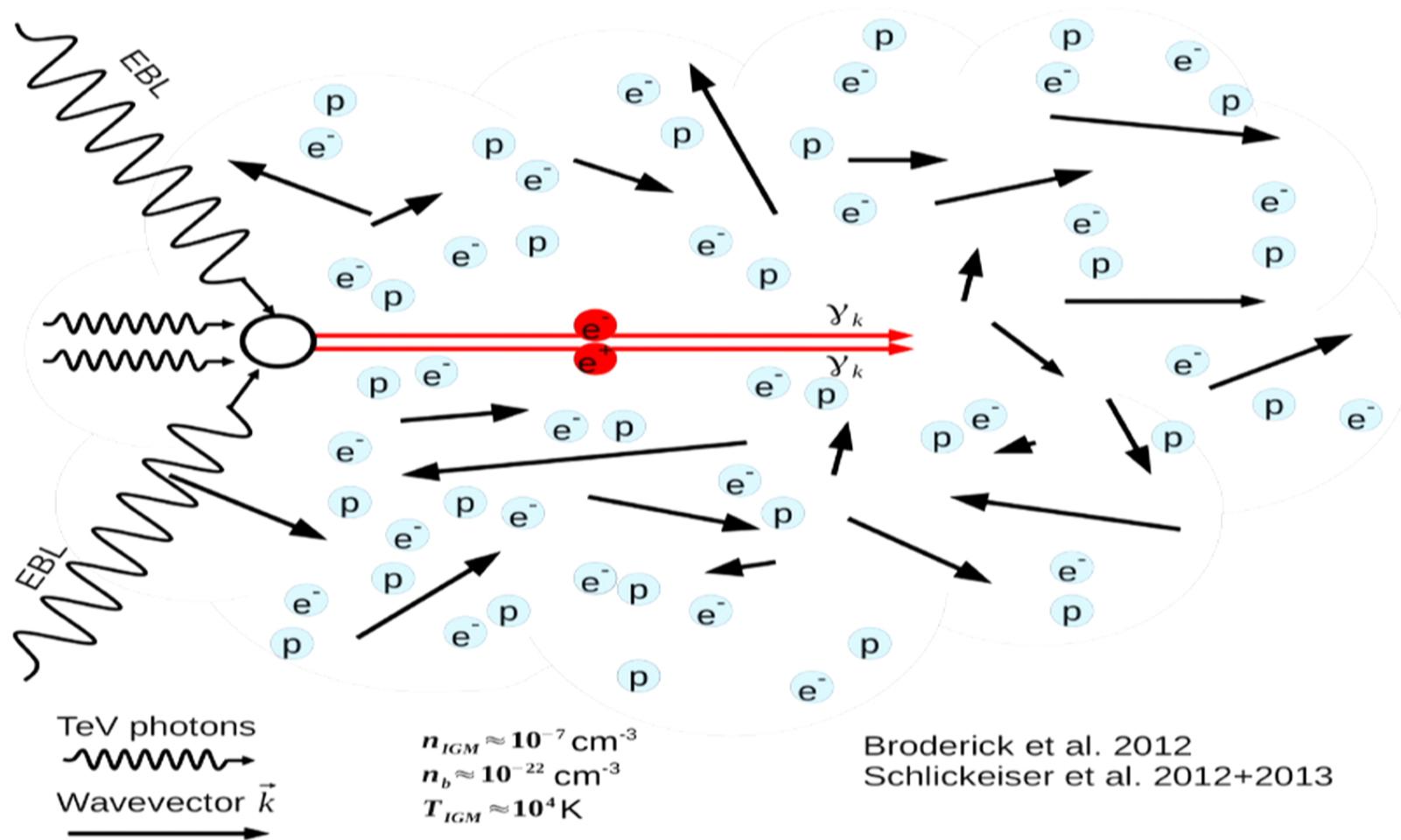
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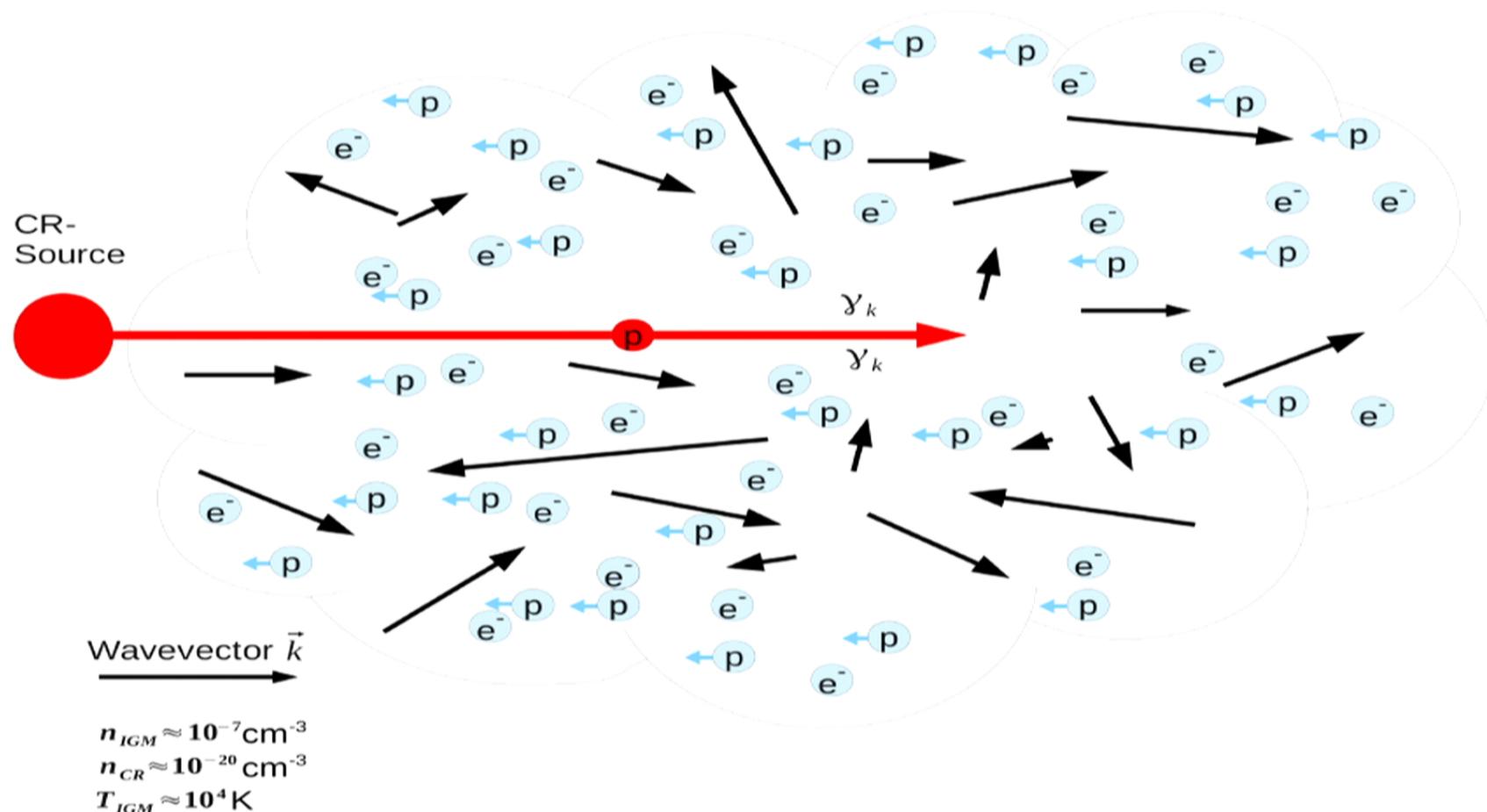
March 14, 2016

- ① Motivation and previous calculation
- ② Kinetic instability of parallel electrostatic waves
- ③ Conclusions

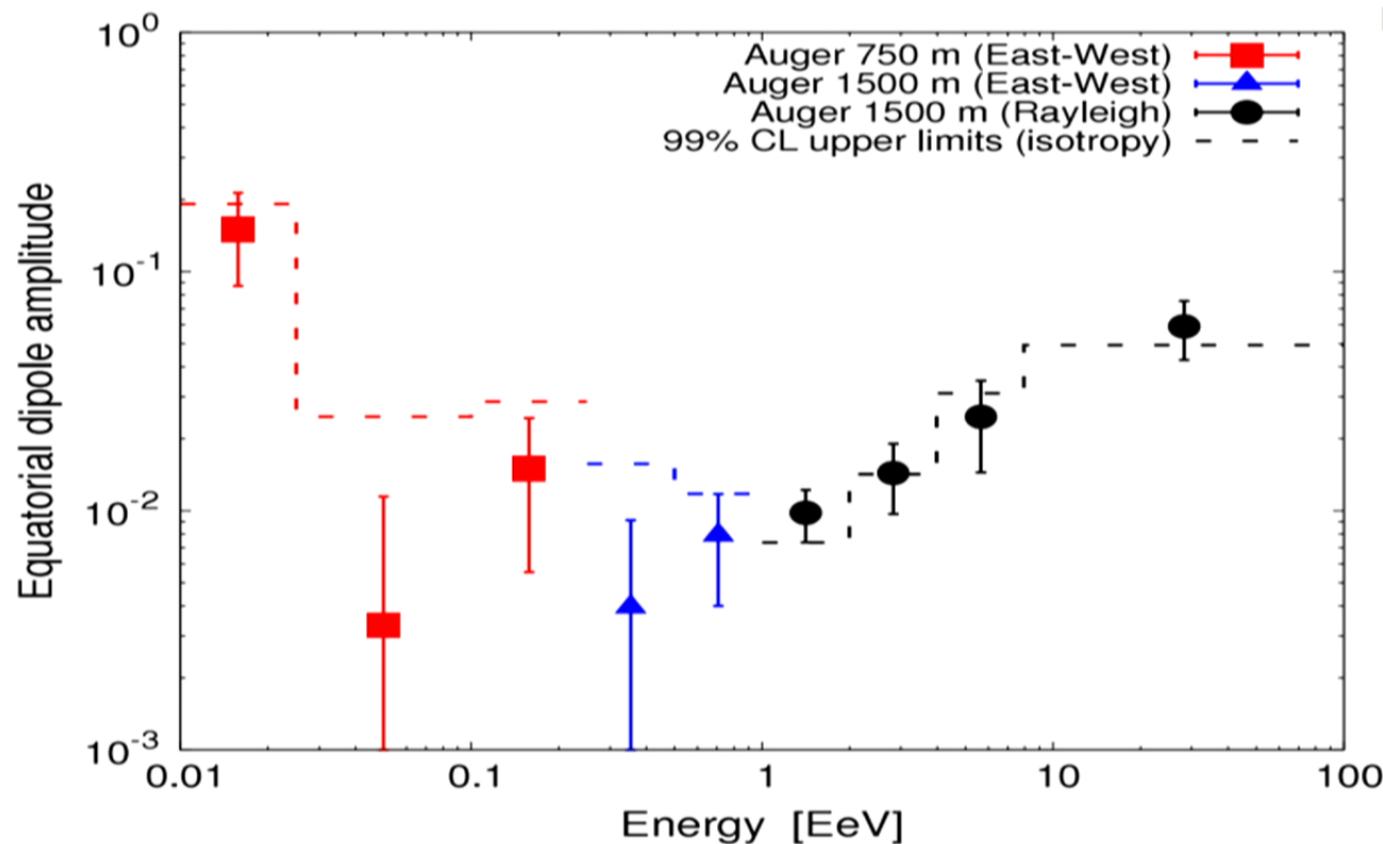
# Pair beam instability



# Cosmic ray beam instability



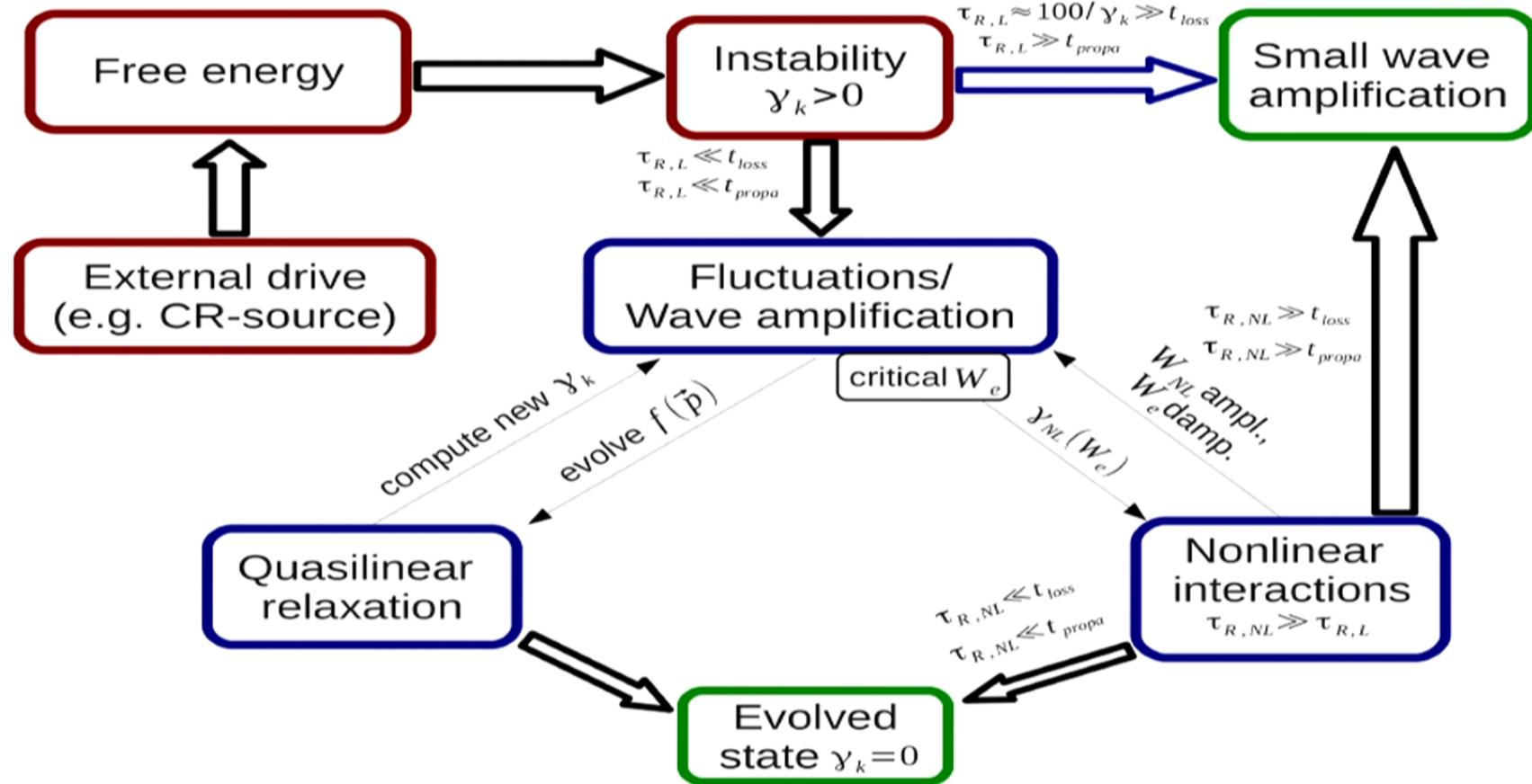
- ultrahighenergy cosmic rays are believed to be extragalactic
- ECR have to propagate through a highly charged plasma, the intergalactic medium (IGM) for roughly  
 $t_F = L/c = 1.6 \cdot 10^7 L_{50}$  yrs
- the excitation of electrostatic or electromagnetic fluctuations can hugely affect the propagation of cosmic rays and lead to an energy loss or even change the initial gyrotropic function to an isotropic one
- extragalactic cosmic rays (ECR) are isotropically distributed on large scales



Harari 2014 ([10.1016/j.dark.2014.04.003](https://doi.org/10.1016/j.dark.2014.04.003))

- monoenergetic beam traversing the *cold* IGM was investigated in 2014 (SK+RS ApJ 789, 84, 2014)
- electrostatic and electromagnetic growth time is much smaller than propagation time
- strong nonlinear effect (modulation instability) stabilizes the monoenergetic beam and thus the beam can propagate stable to the earth without loosing a significant amount of energy to fluctuations
- recently Schlickeiser (ApJ 809, 124, 2015) showed that cosmic rays accelerated at relativistic (like in active galactic nuclei or gamma-ray bursts) and nonrelativistic shocks (e.g. supernova explosions/remnants) are Lorentzian distributed

# Relaxation of an instability



- ① Motivation and previous calculation
- ② Kinetic instability of parallel electrostatic waves
- ③ Conclusions

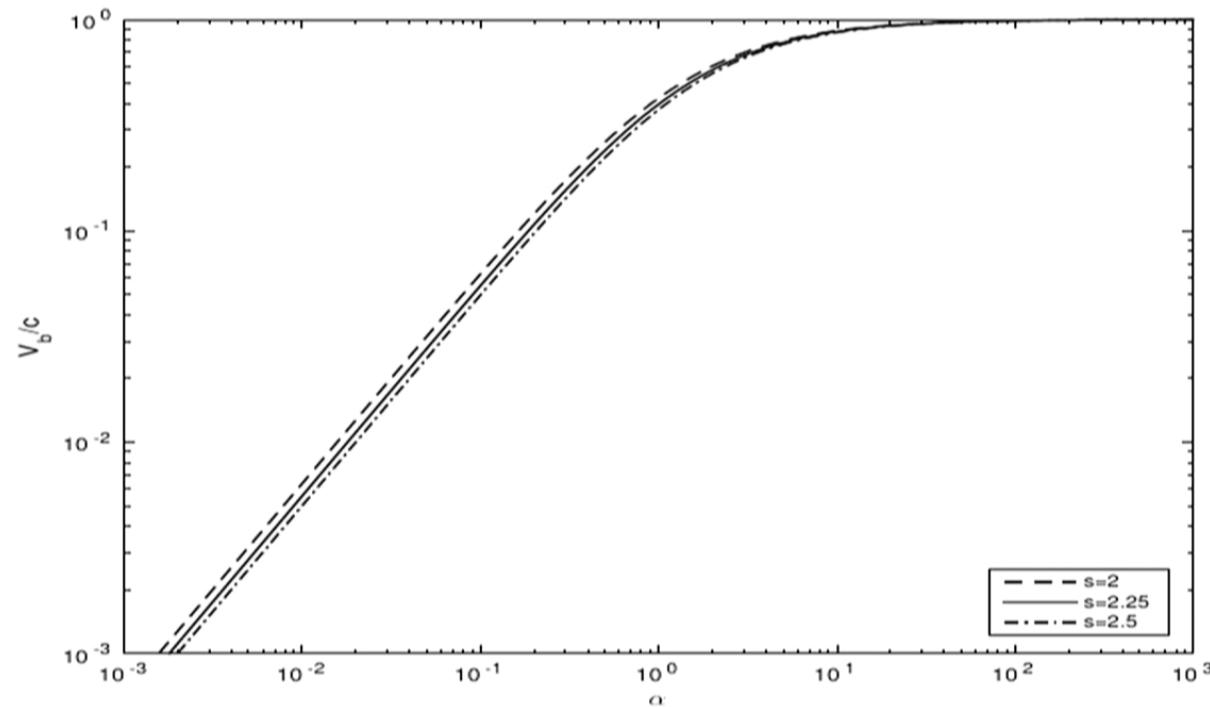
## Restframe of IGM electrons

$$\begin{aligned}
 f(p_{\parallel}, p_{\perp}) &= f_{IGM}(p_{\parallel}, p_{\perp}) + f_b(p_{\parallel}, p_{\perp}) \\
 &= \frac{n_p}{(2\pi m_p k_B T)^{3/2}} \cdot \exp\left(-\frac{(\mathbf{p} - \mathbf{p}_d)^2}{2m_p k_B T}\right) \\
 &+ \frac{n_e}{(2\pi m_e k_B T)^{3/2}} \cdot \exp\left(-\frac{\mathbf{p}^2}{2m_e k_B T}\right) \\
 &+ \frac{N_b C(a, s)}{2\pi p_{\perp}} \delta(p_{\perp}) (p_{\parallel}^2 + a^2)^{-s} H[p_{\parallel}]
 \end{aligned} \tag{1}$$

$$n_p = n_e - N_b, \quad \chi = \frac{N_b}{n_e} \approx 10^{-13} \quad (?) \tag{2}$$

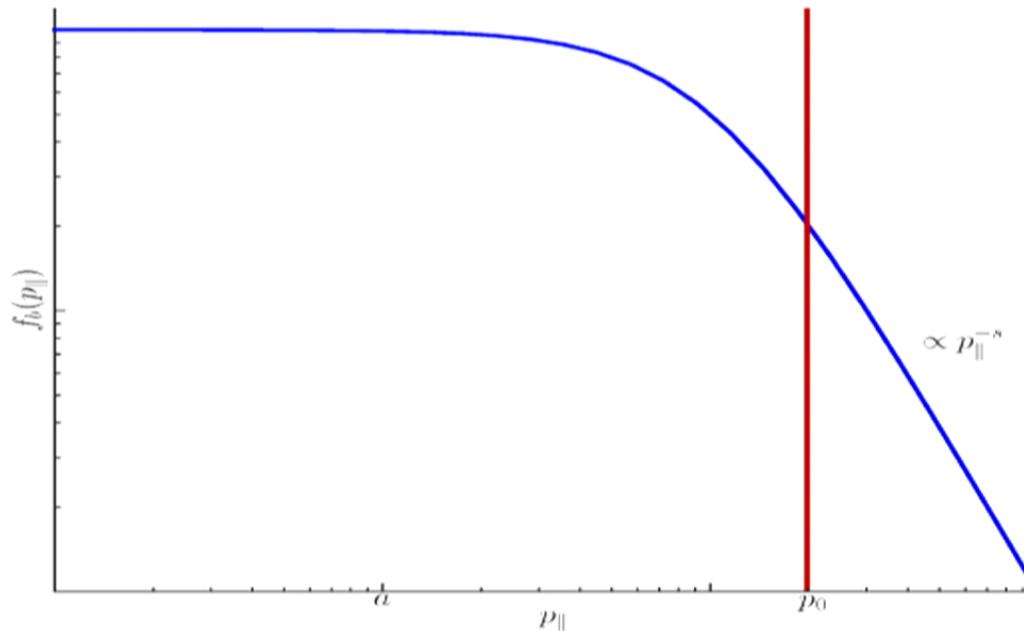
$$\mathbf{v}_d = -\frac{\chi}{1-\chi} \mathbf{v}_b, \quad \mathbf{p}_d = -\frac{\chi}{1-\chi} \frac{\Gamma_p}{\Gamma_b} \mathbf{p}_b \tag{3}$$

$$V_b = \frac{\int d^3 p v f_b(\mathbf{p})}{N_b} = A(s) \alpha c {}_2F_1 \left( 1, \frac{1}{2}; s + \frac{1}{2}; 1 - \alpha^2 \right), \quad \alpha = \frac{a}{mc}$$



## Beam distribution functions

$$f_b \propto (p_{\parallel}^2 + a^2)^{-s}$$
$$f_b \propto \delta(p_{\parallel} - p_0)$$



$$\Lambda_T = 1 - \frac{k^2 c^2}{\omega^2} + \pi \sum_a \frac{\omega_{p,a}^2}{\omega^2 n_a} \int_{-\infty}^{\infty} dp_{||} \int_0^{\infty} dp_{\perp} \frac{p_{\perp}^2}{\Gamma_a} \times \left[ \frac{\partial f_a}{\partial p_{\perp}} + \frac{kv_{\perp}}{\omega - kv_{||}} \frac{\partial f_a}{\partial p_{||}} \right] = 0 \quad (4)$$

$$\omega = \pm \sqrt{c^2 k^2 + \sum_a \frac{2\pi \omega_{p,a}^2}{n_a} \int_{-\infty}^{\infty} dp_{||} \frac{\tilde{f}(p_{||})}{\Gamma_a(p_{||})}} \in \Re$$

⇒ no electromagnetic instability for parallel wavevector and momentum orientation

## Complete electrostatic dispersion function

$$\Lambda_e(z, k) = 1 - \frac{\omega_{p,e}^2}{2k^2 v_e^2} Z' \left( \frac{zc}{\sqrt{2}v_e} \right) - \frac{\omega_{p,i}^2}{2k^2 v_i^2} Z' \left( \frac{zc - v_d}{\sqrt{2}v_i} \right) - \frac{2sC(\alpha, s)\omega_{p,b}^2}{zk^2 c^2} \int_0^1 dy \frac{f(y, \alpha)}{z - y} = 0 \quad (5)$$

$$f(y, \alpha) = \frac{y^2(1 - y^2)^{s-1}}{(\alpha^2 + y^2(1 - \alpha^2))^{s+1}}, \quad y = \frac{p_{||}}{\sqrt{p_{||}^2 + m^2 c^2}}$$

$$Z'(x) = \pi^{-1/2} \int_{-\infty}^{\infty} dt \frac{\exp(-t^2)}{(t - x)^2}$$

$$z = \frac{\omega}{kc} = R + \imath l$$

## Complete electrostatic dispersion function

$$\Lambda_e(z, k) = 1 - \frac{\omega_{p,e}^2}{2k^2 v_e^2} Z' \left( \frac{zc}{\sqrt{2}v_e} \right) - \frac{\omega_{p,i}^2}{2k^2 v_i^2} Z' \left( \frac{zc - v_d}{\sqrt{2}v_i} \right) - \frac{2sC(\alpha, s)\omega_{p,b}^2}{zk^2 c^2} \int_0^1 dy \frac{f(y, \alpha)}{z - y} = 0 \quad (5)$$

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$$z = \frac{\omega}{kc} = R + \imath l$$

## Weakly amplified electrostatic fluctuations $|R| \gg |I|$

1.

$$|z| \gg \frac{\sqrt{2}v_e}{c} \approx 3 \cdot 10^{-3}$$

2.

$$|z| \ll \frac{\sqrt{2}v_i}{c} \approx 6 \cdot 10^{-5}$$

## Weakly propagating electrostatic fluctuations $|R| \ll |I|$

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$$|z| \ll \frac{\sqrt{2}v_i}{c} \approx 6 \cdot 10^{-5}$$

$$|R| \gg |I| \quad (6)$$

Taylor-expansion around  $I = 0$

$$\Re \Lambda_e(R, I = 0, k) = 0 \quad (7)$$

$$I = -\frac{\Im(\Lambda_e(R, I = 0, k))}{[\partial \Re \Lambda_e(R, I = 0, k) / \partial R]} \quad (8)$$

Plemelj's formula

$$\lim_{\epsilon \rightarrow 0} \frac{1}{x - (x_0 \pm i\epsilon)} = \mathcal{P} \frac{1}{x - x_0} \pm i\pi \delta(x - x_0) \quad (9)$$

$$\int_0^1 dy \frac{f(y, \alpha)}{z - y} \approx - \left( \mathcal{P} \int_0^1 dy \frac{f(y, \alpha)}{y - R} + i\pi \frac{\alpha R^2 (1 - R^2)^{s-1}}{(\alpha^2 + R^2(1 - \alpha^2))^{s+1}} \right) \quad (10)$$

## Large phase velocity

$$\frac{\omega_{p,e/i}^2}{2k^2 v_{e/i}^2} Z' \left( \frac{zc}{\sqrt{2}v_{e/i}} \right) \approx \frac{\omega_{p,e/i}^2}{z^2 k^2 c^2} \quad (11)$$

$$|z| \gg \frac{\sqrt{2}v_e}{c} \approx 3 \cdot 10^{-3}$$

$$c \gg v_e \gg v_i \gg v_d$$

## Total plasma dispersion function

$$\Lambda_e(R, I = 0, k) = 1 - \frac{\omega_{p,e}^2 + \omega_{p,i}^2}{R^2 k^2 c^2} + \frac{2sC(\alpha, s)\omega_{p,b}^2}{Rk^2 c^2}$$

$$\times \left[ \mathcal{P} \int_0^1 dy \frac{f(y, \alpha)}{y - R} + i\pi \frac{\alpha R^2 (1 - R^2)^{s-1}}{(\alpha^2 + R^2(1 - \alpha^2))^{s+1}} \right] \quad (12)$$

- growth rates for power law indices between 1.5 and 2.5 are about  $(2 - 7) \cdot 10^{-16}$  Hz
  - relaxation time must be at least  $\tau = 100 \cdot (\gamma^{max})^{-1} \approx 4.3 \cdot 10^9$  yrs
- ⇒ orders of magnitude longer than the propagation time of  $t_F = L/c \approx 1.6 \cdot 10^7$  yrs

## Weakly amplified electrostatic fluctuations $|R| \gg |I|$

1. large phase velocity

$$\gamma_{max} \simeq (2 - 7) \cdot 10^{-16} \text{ Hz} \quad (14)$$

2. small phase velocity

$$I \ll 10^{-20} \quad (15)$$

## Weakly propagating electrostatic fluctuations $|R| \ll |I|$

1. large phase velocity

$\Rightarrow$  no instability

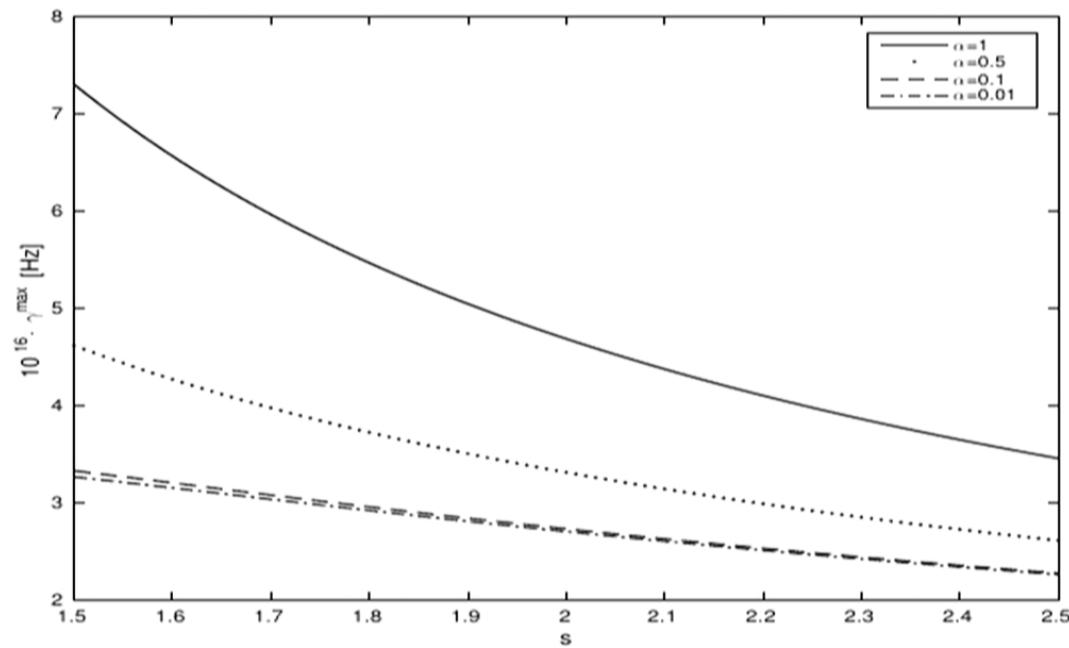
2. small phase velocity

$$I \approx 10^{-26} \frac{s}{\alpha^2} \quad (16)$$

- instabilities are proportional to the beam to background ratio
- in the calculation we normalized the CR-density to the measured one at Earth
- closer to the CR-source the density can increase and thus the instabilities can get potentially more important

## Maximum growth rate

$$\gamma^{max}(s) = \omega_{p,e} \frac{2\sqrt{2(2\alpha^2 + s - 1)}s\pi A(s)\chi\xi(s-1)^{s-1}}{\sqrt{1+\xi}(1+s)^{s+1}} \quad (13)$$



- electrostatic and electromagnetic interaction of a cosmic ray beam with the unmagnetized intergalactic medium was investigated
  - no electromagnetic instabilities are possible, when the wavevector is parallel to the beam direction
  - in contrast to a monoenergetic beam, a Lorentzian distributed beam can only excite negligibly small parallel electrostatic instabilities for very small density ratio
- ⇒ very dilute Lorentzian distributed cosmic rays do not lose a significant energy amount to parallel electrostatic and electromagnetic fluctuations, while propagating through the IGM
- in future we will work on the density issue and oblique propagation of the cosmic ray beam to the wavevector, which potentially could lead to a greater electrostatic and electromagnetic instability