Title: Nonlinear Plasma Instabilities

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Abstract:

Nonlinear Physics of Beam-Plasma Systems

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Feedback over 44 orders of magnitude

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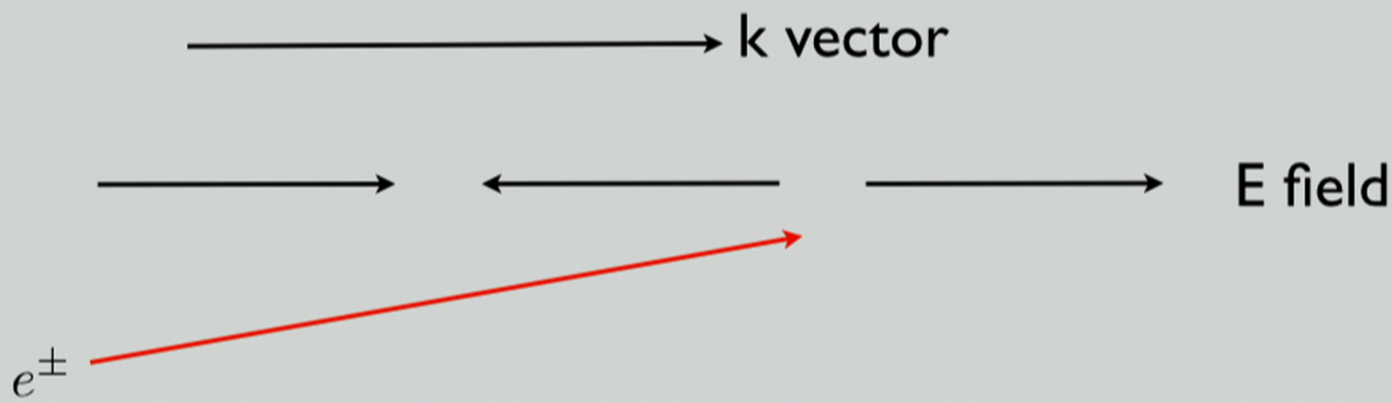
Outline

- Review of the linear instability: intuitive picture and impact
- Current numerical understanding of oblique instability
- Analytical approach to nonlinear plasma physics
  - Quasilinear Theory - Wave-background interaction
  - Wave-Particle Interaction
  - Wave-Wave Interaction
  - Strong Turbulence Effects
- Summary

Oblique Instability: Intuitive Picture

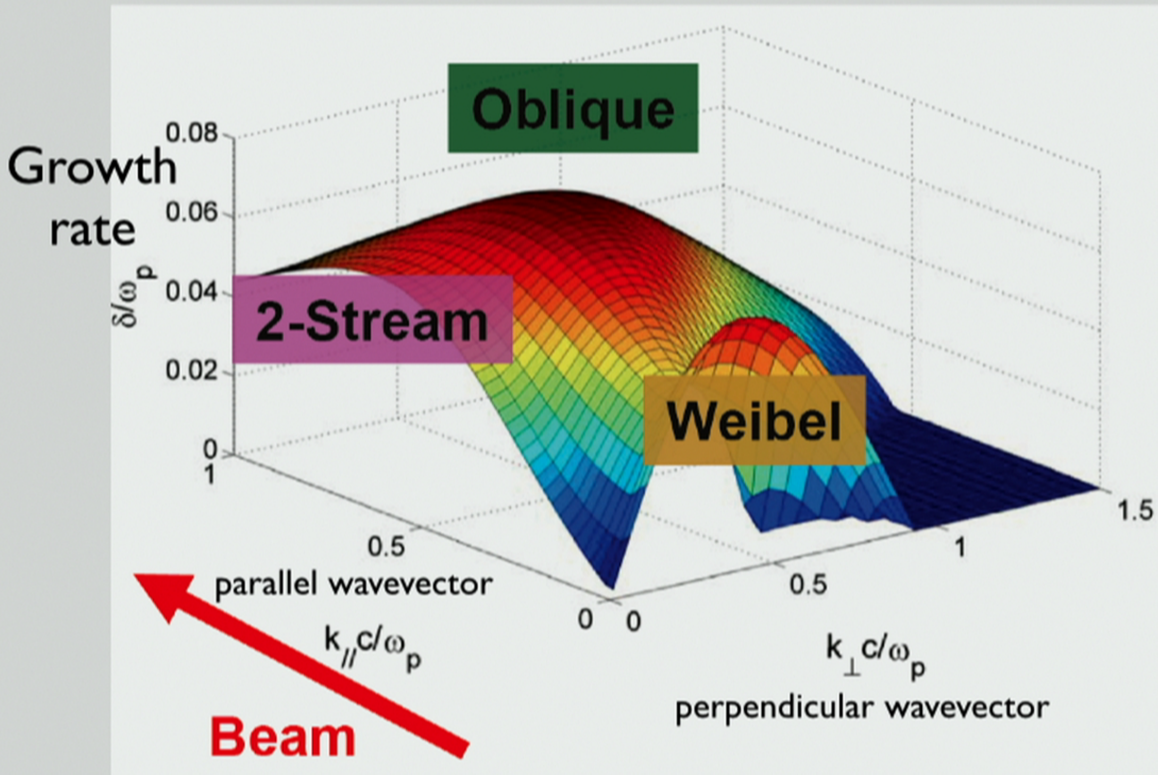
- Basically an overstable Langmuir wave (plasma oscillation)
- Move to the reference frame of the wave



- Resonant particles exchange energy with the wave.
- Deflections of particle trajectory instead of particle straight-line velocity
- Greater growth rate than two-stream because ultrarelativistic particles are easier to deflect than to change their parallel velocities (Nakar, Bret & Milosavljevic 2011).

Oblique Instability

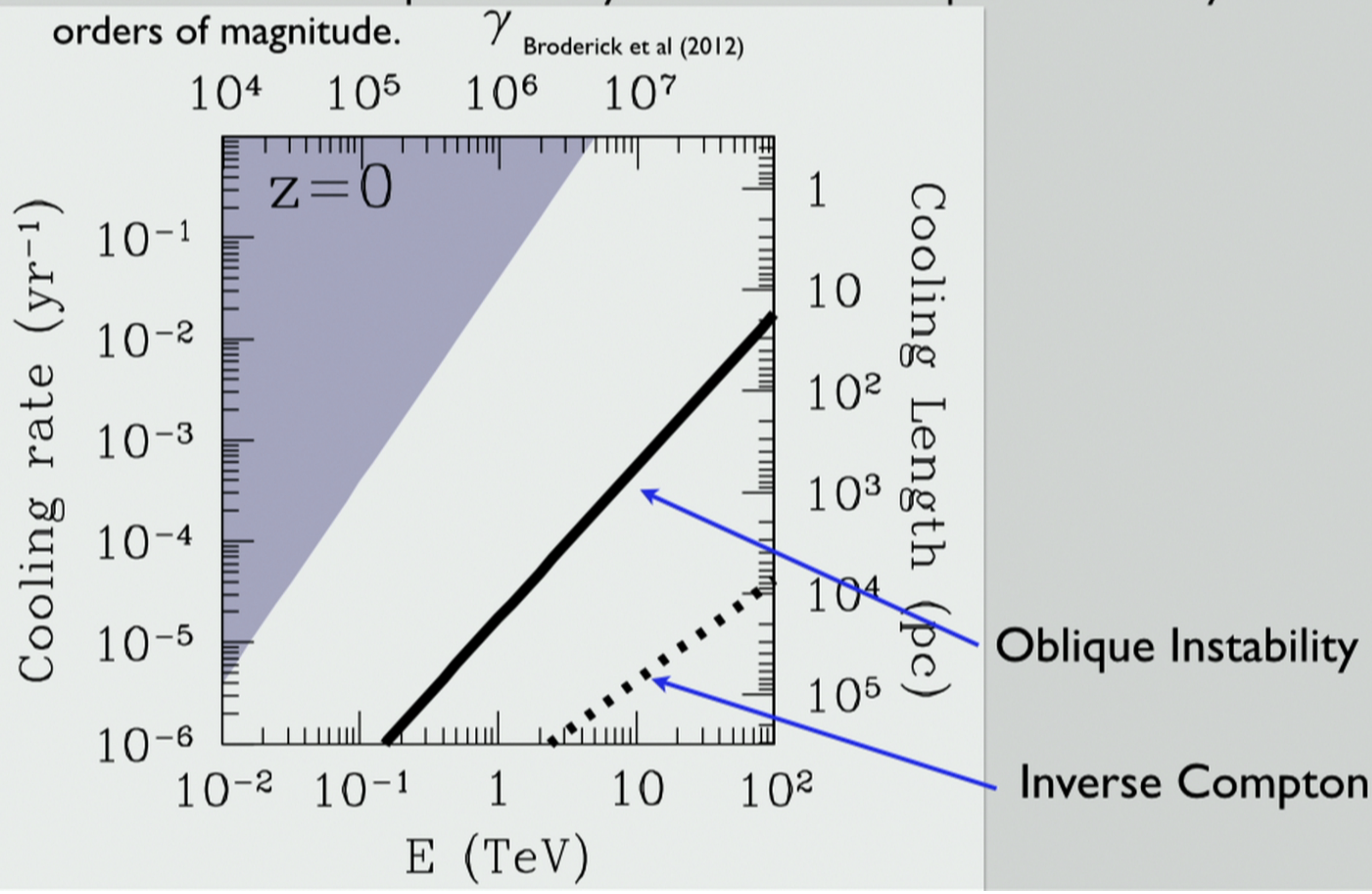
- Generalization of the classical beam plasma and Weibel instability
- Shows greatest growth for ultrarelativisitic beams $n_b/n_0 \ll 1, \; \gamma_b \gg 1$



Bret et al. (2010)

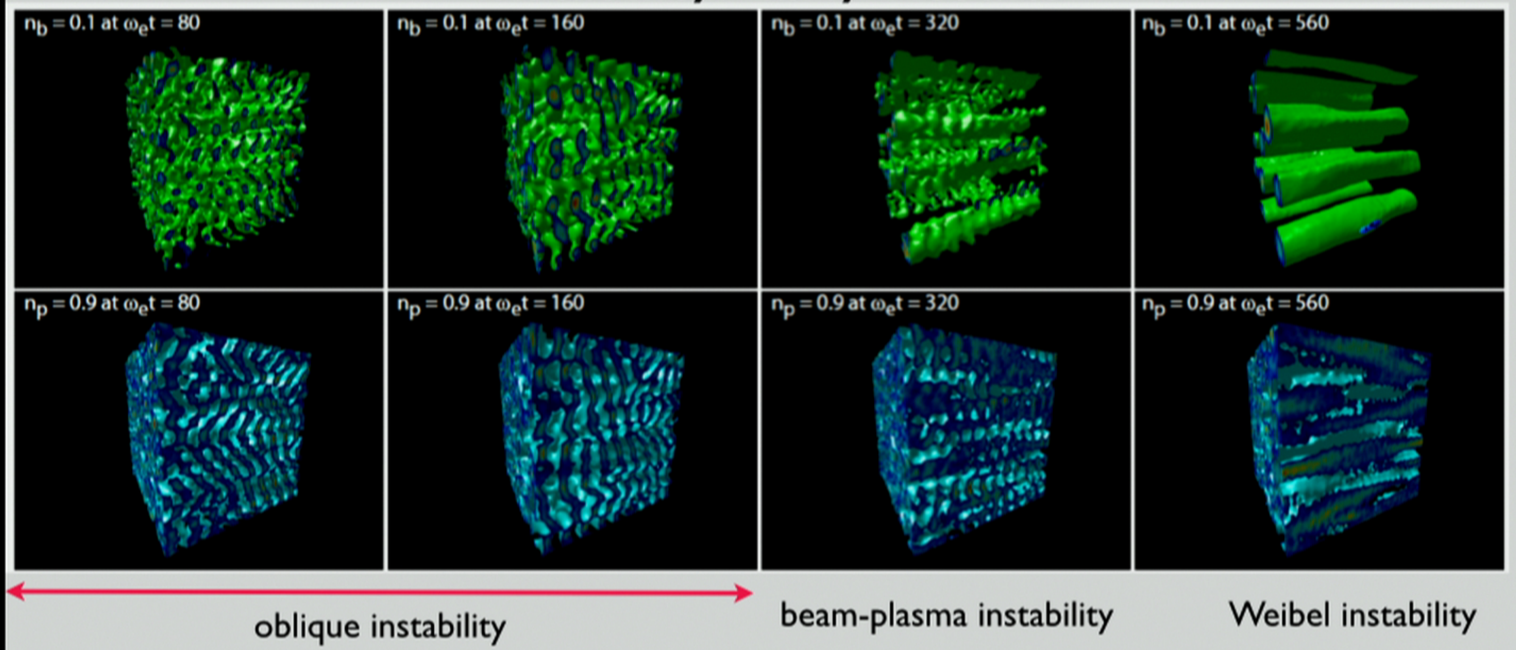
Effect of instabilities

Growth rate of Oblique instability beats inverse Compton off CMB by ~2 orders of magnitude.



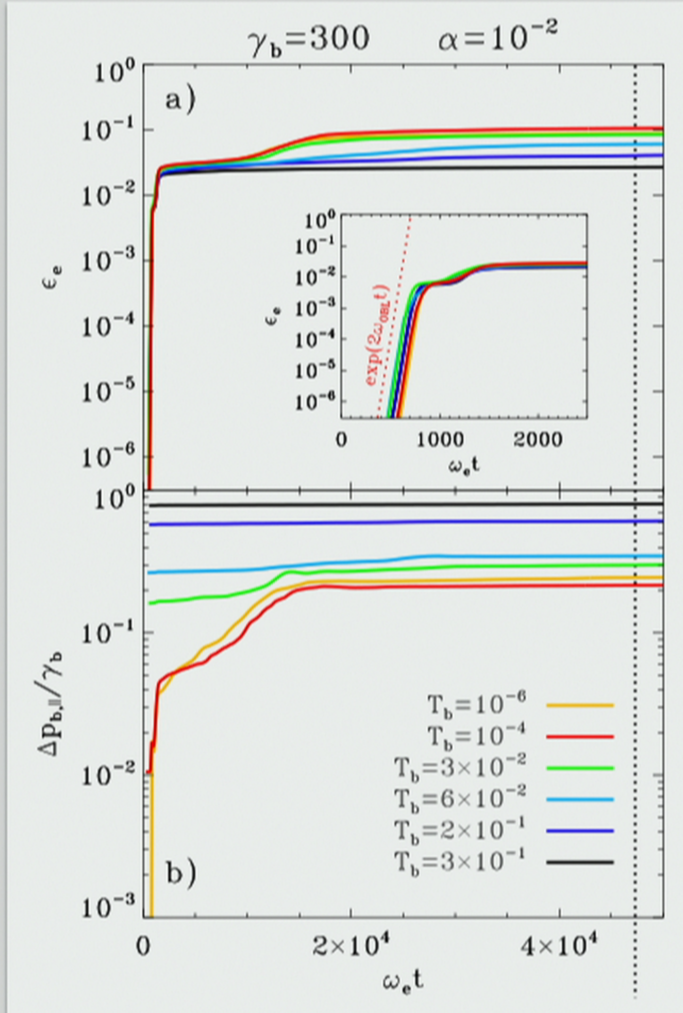
Nonlinear Saturation of the Oblique Instability: Early Results

Bret et al. (2010)



- 3D PIC simulation of a beam of hot electrons moving through an e-p plasma
  - 0.1 density ratio between beam and background and \gamma = 3

- ~30 % of energy dumped into the e-p plasma before end of simulation
- For the TeV case: we will assume O(1) of energy is dumped as heating, but back to this later.



Sironi & Giannos (2013)

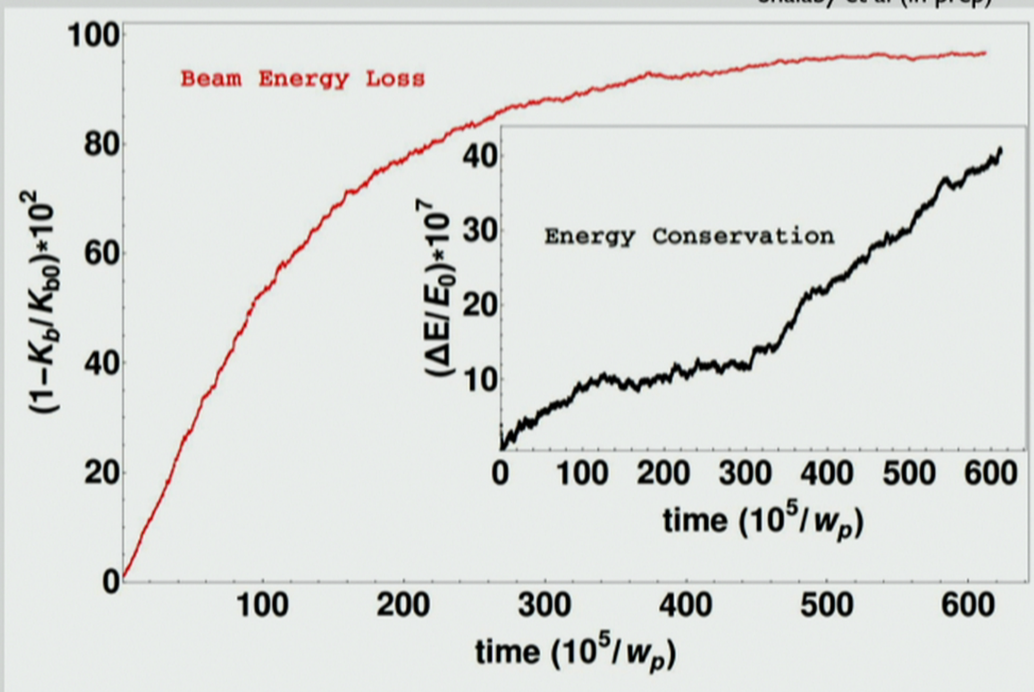
Numerical Simulations

- Sironi & Giannios (2014) used simulations to argued that the efficiency of conversion from beam energy to heat < 10%
- Conversion is expected to be lower as they showed that higher T beams has lower conversion efficiency
- T_b expect ~1 for TeV pair beams from TeV blazers
- Conversion is also expected to be lower for more tenuous and relativistic beams.

What is to be done?

- "Better" Numerical Simulations
- Constructed a higher order code **MAXWELL** with superior energy conservation
- Shows near 100% thermalization of beam energy

Shalaby et al (in prep)



Modes of Nonlinear Saturation

- Another method is via analytical methods via perturbation theory
  - $E_{beam} << E_{thermal}$ -> wave amplitudes is small
- Possibly allows for exploration of physics that may be difficult to resolve numerically
- Allows comparison with numerical results.
- Waves are saturated
  - quasilinear theory
  - weak wave-wave coupling
  - wave-particle coupling
  - strong turbulence effects.

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Quasilinear theory

- In quasilinear theory, the effect of instabilities is the change the background such that waves are no longer unstable.
- Start with the Vlasov equation:

$$\frac{\partial f_s}{\partial t} + \boldsymbol{v}_s \cdot \boldsymbol{\nabla} f_s + \frac{q_s}{m_e} \boldsymbol{E} \cdot \boldsymbol{\nabla}_p f_s = 0,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 4\pi\rho,$$

- Consider perturbations of the form: $f_s \rightarrow f_{s,0} + \delta f_s$
-
- Leads the linear equations and dispersion relation:

$$\frac{\partial \delta f_s}{\partial t} + \boldsymbol{v}_s \cdot \boldsymbol{\nabla} \delta f_s + \frac{q_s}{m_e} \delta \boldsymbol{E} \cdot \boldsymbol{\nabla}_p f_{s,0} = 0. \qquad \boldsymbol{\nabla} \cdot \delta \boldsymbol{E} = 4\pi \delta\rho,$$

$$1 - \frac{\omega_p^2}{k} \int \frac{\boldsymbol{k} \cdot \boldsymbol{\nabla}_p F}{\omega - \boldsymbol{k} \cdot \boldsymbol{v}} d^3p = 0.$$

Quasilinear theory

- Now consider $f_{s,0}(t)$ where it is slow compared to $\exp(i\omega t)$

$$\frac{\partial f_s}{\partial t} + \boldsymbol{v}_s \cdot \boldsymbol{\nabla} f_s + \frac{q_s}{m_e} \boldsymbol{E} \cdot \boldsymbol{\nabla}_p f_s = 0,$$

$$\frac{\partial f_{s,0}}{\partial t} = -\frac{q_s}{m_e} \int \delta \boldsymbol{E}_k \cdot \boldsymbol{\nabla}_p \delta f_k^* dk$$

- This gives a diffusion equation that depends on

$$\frac{\partial f_{s,0}}{\partial t} = \boldsymbol{\nabla}_p \cdot \boldsymbol{D} \cdot \boldsymbol{\nabla}_p f_{s,0}$$

- The diffusion coefficient depends on wave energy and the wave energy depends on the background through the dispersion relationship.

- Leads to the background changing on the same timescale as the wave energy, i.e., background changes in such a way that the growth is sapped.

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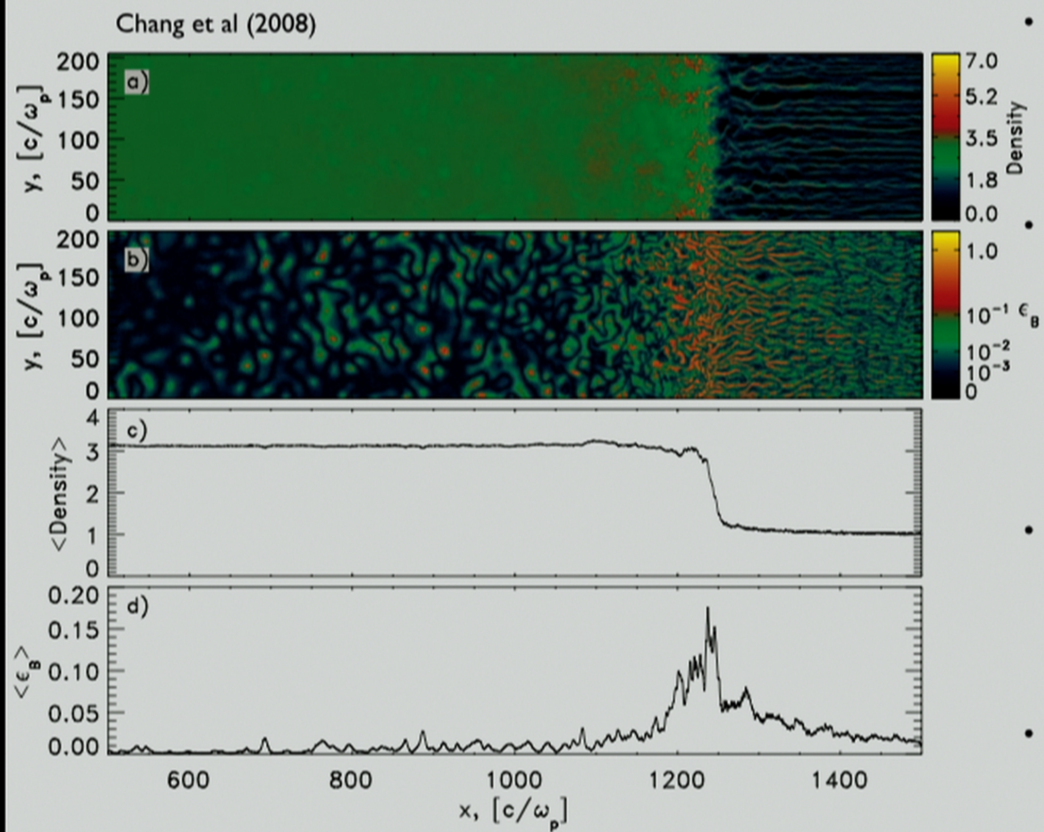

- Most astrophysical plasma instabilities are strong, i.e., shocks
- Leads to large wave amplitudes that nonlinearly drive the system toward suppressing instability
- Example is Weibel mediated internal shock in GRB's
- Growing magnetic fluctuations lead to thermalization of particles in O(1) growth time.

Wave-Particle Interaction

- Again an averaging effect of the Vlasov equation

$$\frac{\partial f_s}{\partial t} + \boldsymbol{v}_s \cdot \boldsymbol{\nabla} f_s + \frac{q_s}{m_e} \boldsymbol{E} \cdot \boldsymbol{\nabla}_p f_s = 0,$$

- Again an averaging/two time scaling of the Vlasov equation

$$\delta \to \delta(t) \exp(-i\omega_k t + i\mathbf{k} \cdot \mathbf{x})$$

$$\frac{\partial \delta f_{s,k'}}{\partial t} + \mathbf{v}_s \cdot \nabla \delta f_{s,k'} + \frac{e}{m_e} \sum \delta \mathbf{E}'_{\mathbf{k}} \cdot \nabla_p \delta f_{s,k'}$$

- Here the beat wave may have a really small phase velocity.

$$\frac{\omega' - \omega}{k' - k} \sim v_{\rm ph} \frac{v_e^2}{v_{\rm ph}^2}$$

- Is resonant with slow particles, i.e., ions
- Process is also known as nonlinear Landau damping

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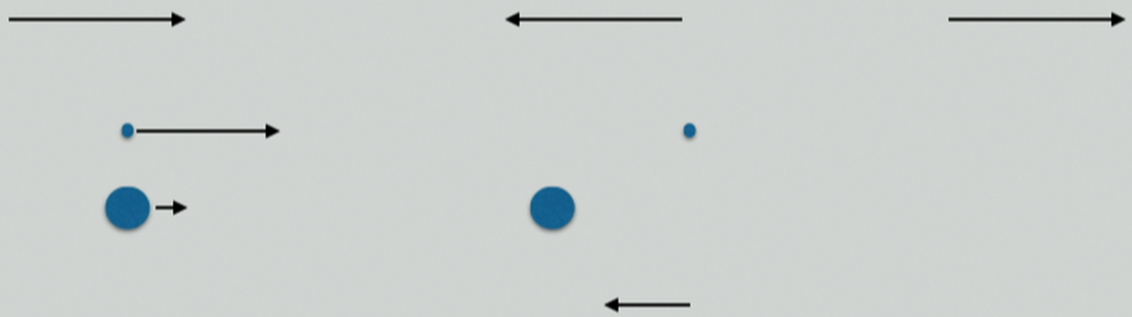
Nonlinear Landau Damping: Intuitive Picture



- Electric field of Langmuir wave accelerates electrons more strongly than protons
- Leads to charge separation and resulting in an induced E-field
- The oscillation of the induced E-field from the driving Langmuir wave, produces a scattered Langmuir wave.

Nonlinear Landau Damping: OOM Estimate

Miniati & Elyiv (2013) argued that particle-wave interaction (nonlinear Landau damping) limits the growth of the instability to exceedingly small values.

$$\frac{dW_{\mathbf{k}}}{dt} = 2\Gamma_{\mathbf{k}} W_{\mathbf{k}} - \frac{W_{\mathbf{k}}\omega_p}{8(2\pi)^{5/2} n_e m_e v_e^2} \int \frac{(\mathbf{k}\cdot\mathbf{k}')^2}{k^2 k'^2} \phi(\mathbf{k},\mathbf{k}') W_{\mathbf{k}'} d\mathbf{k}'$$

$$\phi(\mathbf{k},\mathbf{k}') = \frac{3v_e^2\left(k^2 - k'^2\right)}{4\omega_p |\mathbf{k}-\mathbf{k}'| v_i} \exp\left[-2\left(\frac{3v_e^2\left(k^2 - k'^2\right)}{4\omega_p |\mathbf{k}-\mathbf{k}'| v_i}\right)^2\right]$$

OOM estimate for the timescale is:

$$\frac{1}{\tau} \sim \omega_p \frac{W}{n_e m_e v_{\rm ph} v_i}$$

Balancing against the growth rate, we can show:

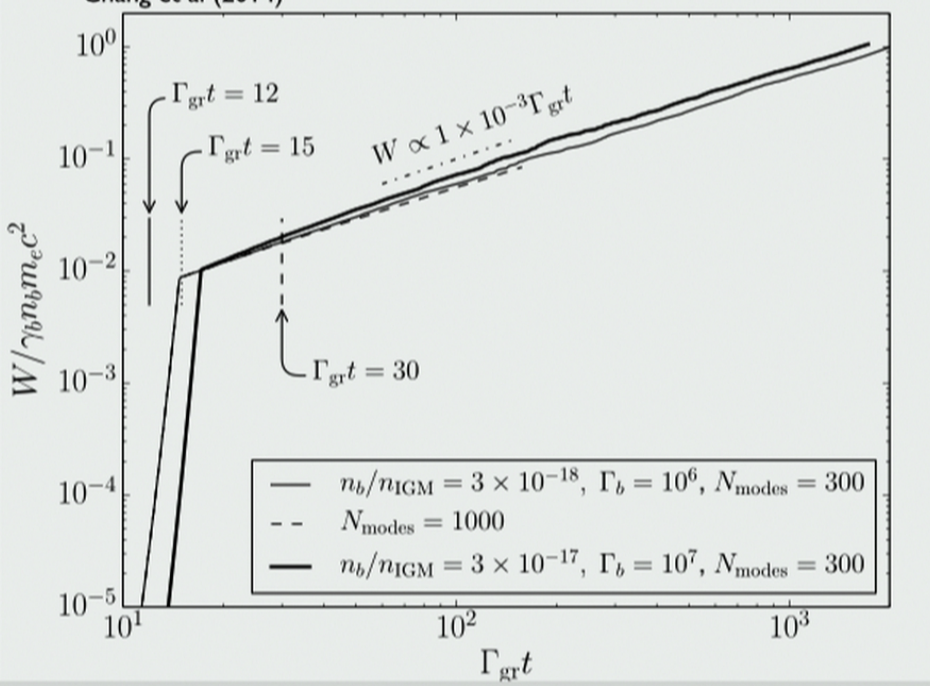
$$W \sim \sqrt{\frac{kT}{m_p c^2}} \gamma_b n_b m_e c^2 \sim 3\times 10^{-5} \gamma_b n_b m_e c^2 \left(\frac{T}{10^4\,\mathrm{K}}\right)^{1/2}$$

Saturation at a low level

Nonlinear Landau Damping: Detailed Calculation

To check this we calculate the mode evolution numerically for N=300 and 1000 modes - logarithmically spaced.

Chang et al (2014)



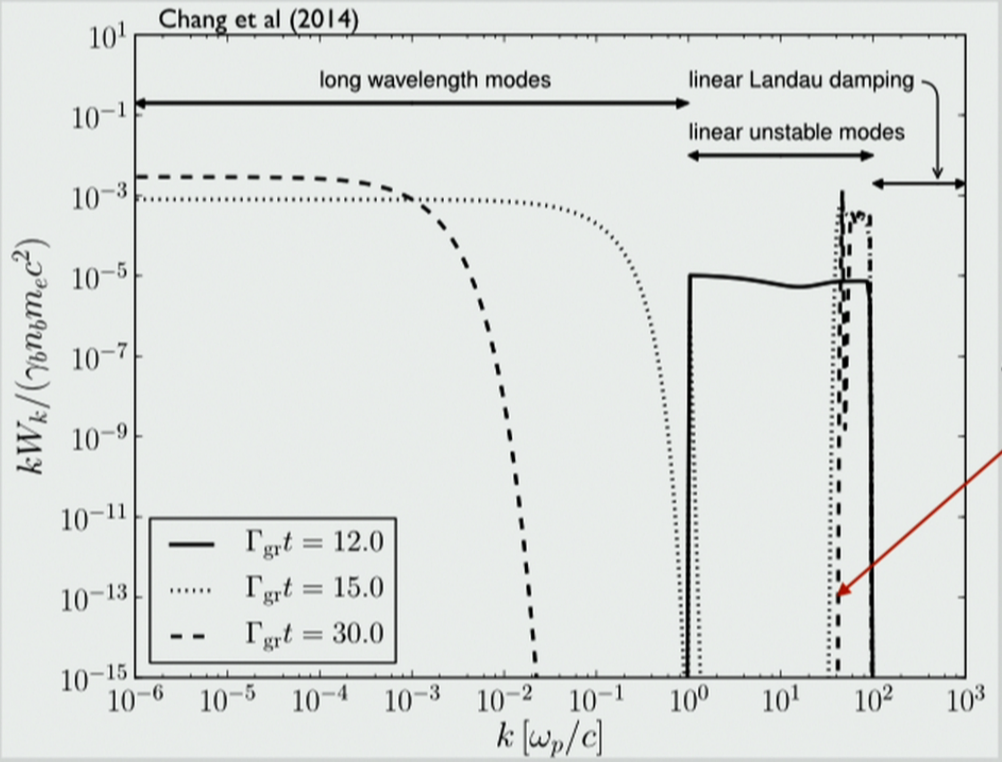
Saturation happens around 1% of beam energy

Continues growth of wave energy at a rate of 0.1% of initial linear growth rate.

Sufficient to allow plasma instability to overcome inverse Compton scattering

Nonlinear Landau Damping: Detailed Calculation

Plot of wave energy shows why this growth continues



Suppression is important at

$$\frac{v_e^2\left(k^2-k'^2\right)}{\omega_p|\mathbf{k}-\mathbf{k}'|v_i} \sim \frac{v_e^2 k}{\omega_p v_i} \gtrsim 1$$

Gives condition on k vector

$$k \gtrsim \sqrt{\frac{m_e}{m_i}}\frac{\omega_p}{c}\frac{c}{v_e} \approx 20\left(\frac{T}{10^4\,\mathrm{K}}\right)^{-1/2}\frac{\omega_p}{c}$$

High k modes can continue to dump energy to low k modes.

Nonlinear Landau Damping: Detailed Calculation

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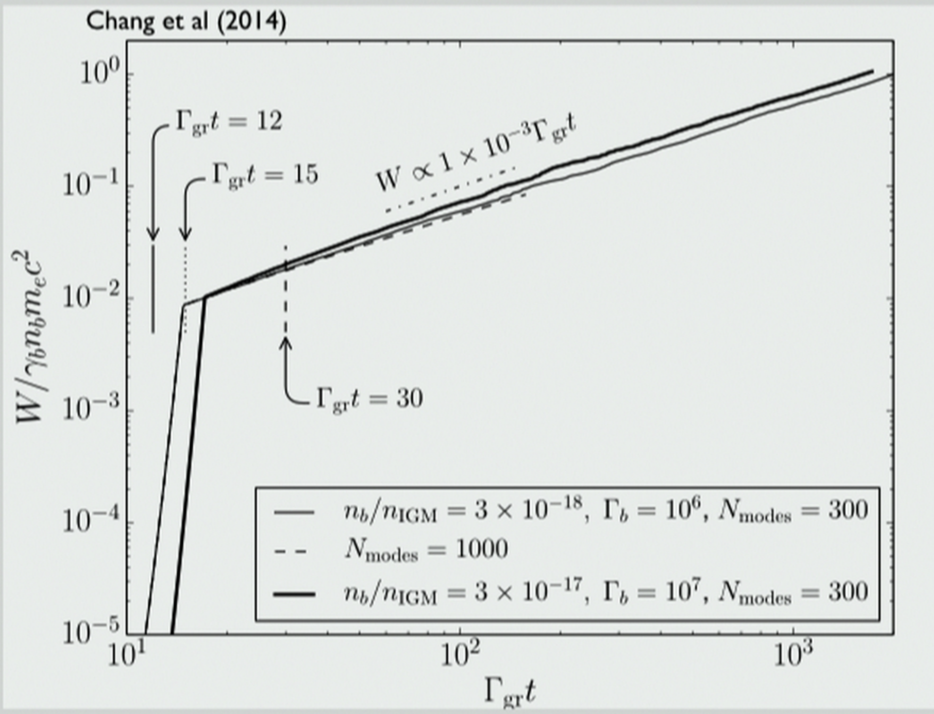

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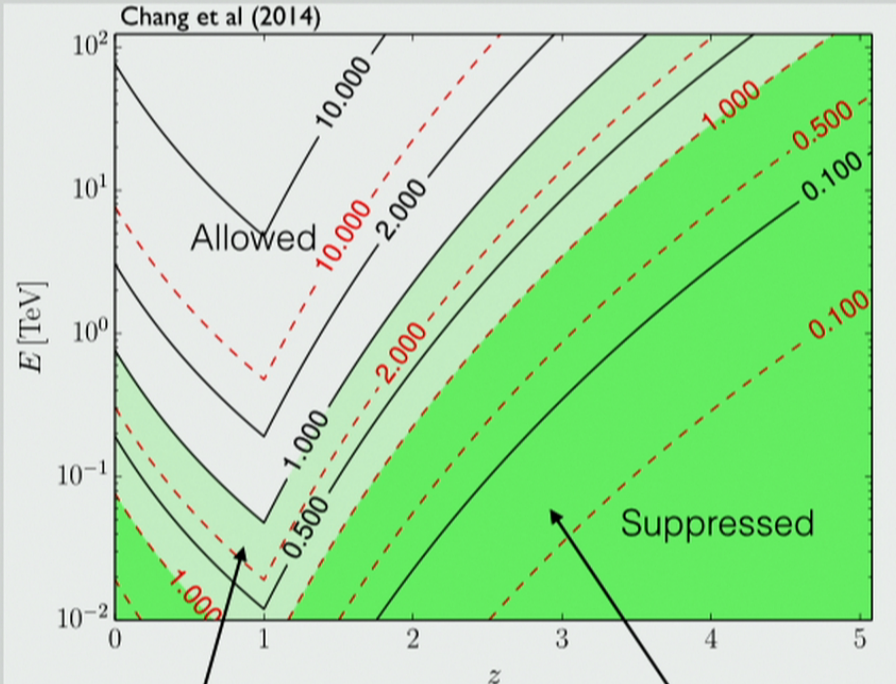
Nonlinear Landau Damping: Effects



$L_{\rm iso} = 10^{45}\,{\rm ergs\,s^{-1}}$

$L_{\rm iso} = 10^{46}\,{\rm ergs\,s^{-1}}$

Effects depend on luminosity of TeV blazer and photon energy.

Make a difference at higher redshift.

Weakest at z=1: peak stellar photon density (higher beam density) wins out over increased inverse Compton of CMB

Wave-Wave Interactions

- Unstable waves can also couple via wave-wave interactions
- Averaging of perturbed Vlasov equations gives:

$$\frac{\partial \delta f_{s,k''}}{\partial t} + \mathbf{v}_s \cdot \nabla \delta f_{s,k''} + \frac{e}{m_e} \sum \delta \mathbf{E}'_k \cdot \nabla_p \delta f_{s,k} = 0$$

- With the condition: $\omega'' = \omega' + \omega, \; k'' = k' + k$
- Beat wave is typically slow

$$\frac{\omega''}{k''} = \frac{\omega' - \omega}{k' - k} \sim v_{\rm ph} \frac{v_e^2}{v_{\rm ph}^2} \ll v_{\rm ph} \sim \frac{\omega_p}{k}$$

- But can couple to ion sound waves.

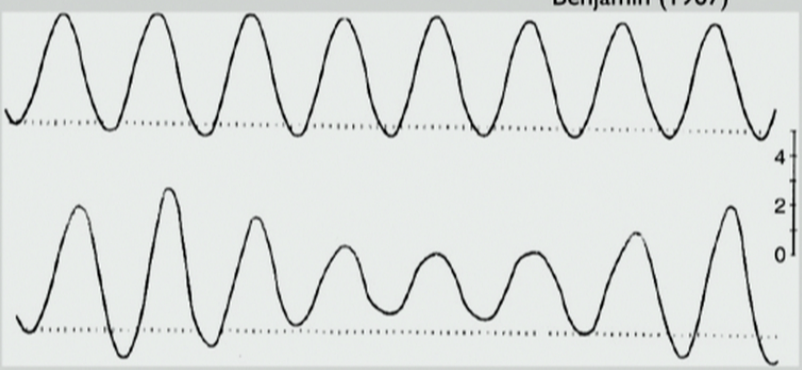
$$\omega^2 = \frac{k^2 v_i^2}{1 + k^2 \lambda_D^2}$$

- Can be numerically solved similar to wave-particle interactions

Strong Turbulence Effects - Modulation Instability

- The amplitude of the wave affects how it propagates.
- Best illustrated with surface gravity water waves: $\omega^2 \approx gk(1 + k^2a^2)$
- The group velocity increases toward increasing amplitude and decrease toward decreasing amplitude -> concentration in regions of high amplitude.
- Leads to regions are high wave amplitude and regions of low wave amplitude and eventual breakup of wave into high-k modes.

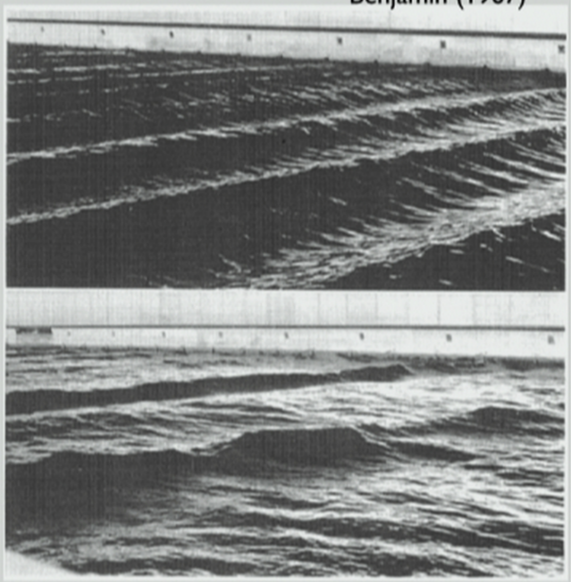
Benjamin (1967)



60 m

120 m

Benjamin (1967)



Strong Turbulence Effects - Modulation Instability

- In plasmas, regions of high wave intensity leads to lower charge density -> increasing amplitude
- Condition for instability is:

$$\frac{W}{n_e k_B T} \gg k^2 \lambda_D^2 = \frac{k^2 v_e^2}{\omega_p^2} \approx 2 \times 10^{-10} \left(\frac{kc/\omega_p}{10^{-2}} \right) \left(\frac{T}{10^4\,\mathrm{K}} \right)$$

- Long wavelength modes are especially suspectible
- At a saturation level of 0.1% of beam energy, this gives

$$\frac{W}{n_e k_B T} \approx 10^{-9} \left(\frac{\gamma_b}{10^6} \right)^2 \left(\frac{T}{10^4\,\mathrm{K}} \right)^{-1}$$

- Strong turbulence effects might be important at long wavelengths that are pumped by NLD
- Effect of strong turbulence leads to breakup of waves into high k-modes

Summary

- The linear oblique instability dominates over inverse Compton for cooling the pair beam.
- Ultimate fate relies on the nonlinear evolution/saturation of the instability
  - Some preliminary evidence for optimism
  - Some later evidence for pessimism
- Current state of the art suggest that nonlinear physics do not sufficiently suppress the instability
  - Numerical evidence - see Shalaby's talk.
  - Analytical evidence on NLD
- Suppression due to NLD may be significant for some systems at some z's
- Unknown effect of wave-wave coupling
- Strong turbulent effects of modulation instability may lead to thermalization of background electrons.