Title: Cosmological singularities in holography

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Abstract: <p>I will discuss recent work on big crunch singularities produced in asymptotic AdS cosmologies using gauge/gravity duality. The dual description consists of a constant mass deformation of ABJM theory on de Sitter space and is well-defined and stable for small deformations.</p>

<p> There is a critical deformation where the theory becomes unstable at weak and at strong coupling. I will discuss a field theory diagnostic of this instability as well as boundary two-point correlators calculated via the geodesic approximation. Near the critical deformation a second saddle point contribution enters, in which the spacelike geodesics probe the high curvature region near the singularity. Its contribution strongly enhances the long-distance correlations. This has a natural interpretation in the weakly coupled boundary theory where the critical point corresponds to a massless limit.</p>

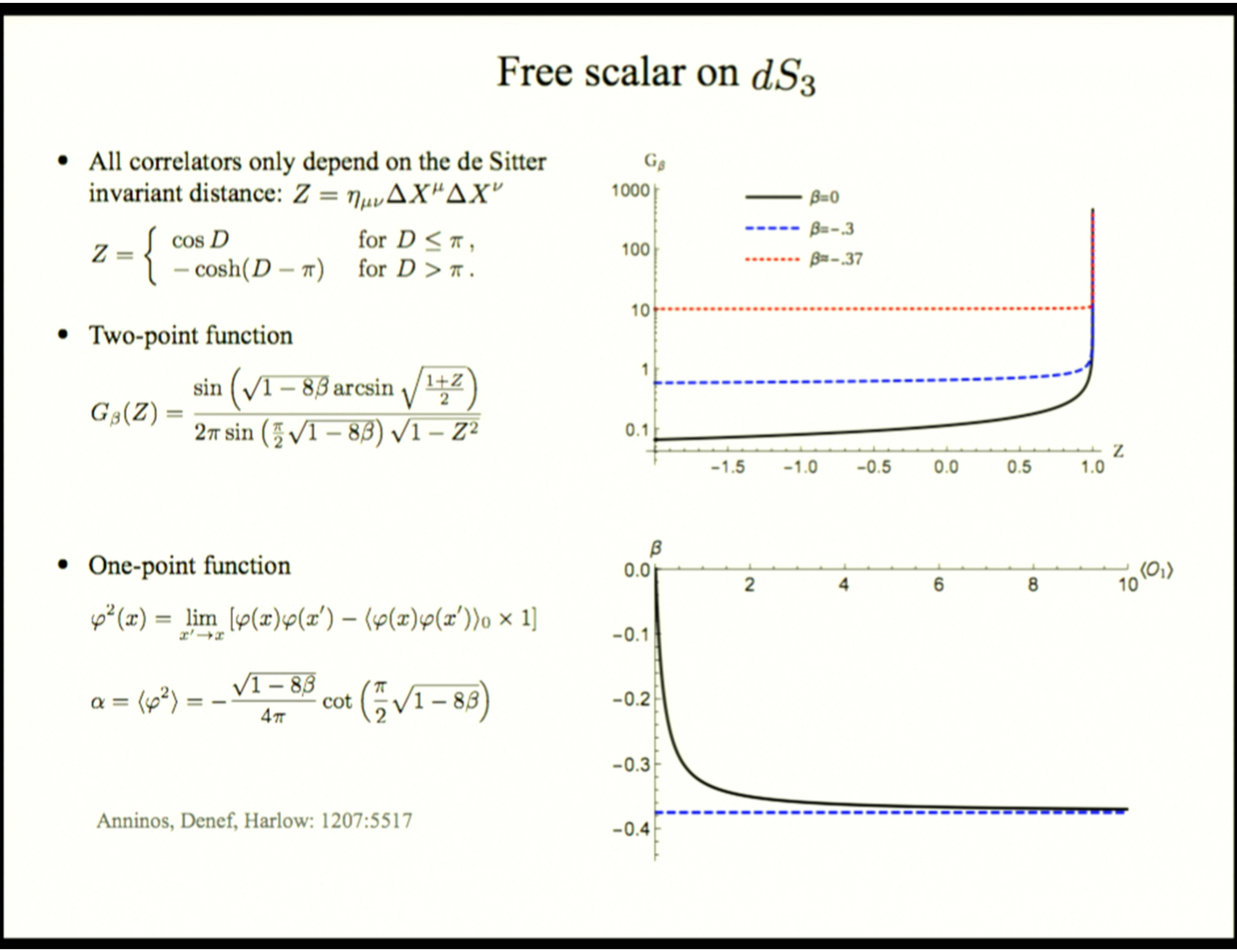
Plan:

- The stage is set: a regular instanton geometry evolves into a cosmological singularity in the bulk, and is dual to a deformation of ABJM - a strongly coupled massive scalar on a de Sitter space boundary.

- How do we extract information about the singularity?

  1. Compute an effective potential for the deformation operator VEV. Notice an instability in the theory. For intuition, we will compare these results with analogous calculations in the free theory.

  2. Compute two-point correlation function for heavy operators in the boundary theory using the geodesic method. Signature of the singularity corresponds to the massless limit in the free boundary theory.



Free scalar on dS_3

- All correlators only depend on the de Sitter invariant distance: $Z = \eta_{\mu\nu}\Delta X^\mu \Delta X^\nu$

$$Z = \begin{cases} \cos D & \text{for } D \leq \pi\,, \\ -\cosh(D-\pi) & \text{for } D > \pi\,. \end{cases}$$

- Two-point function

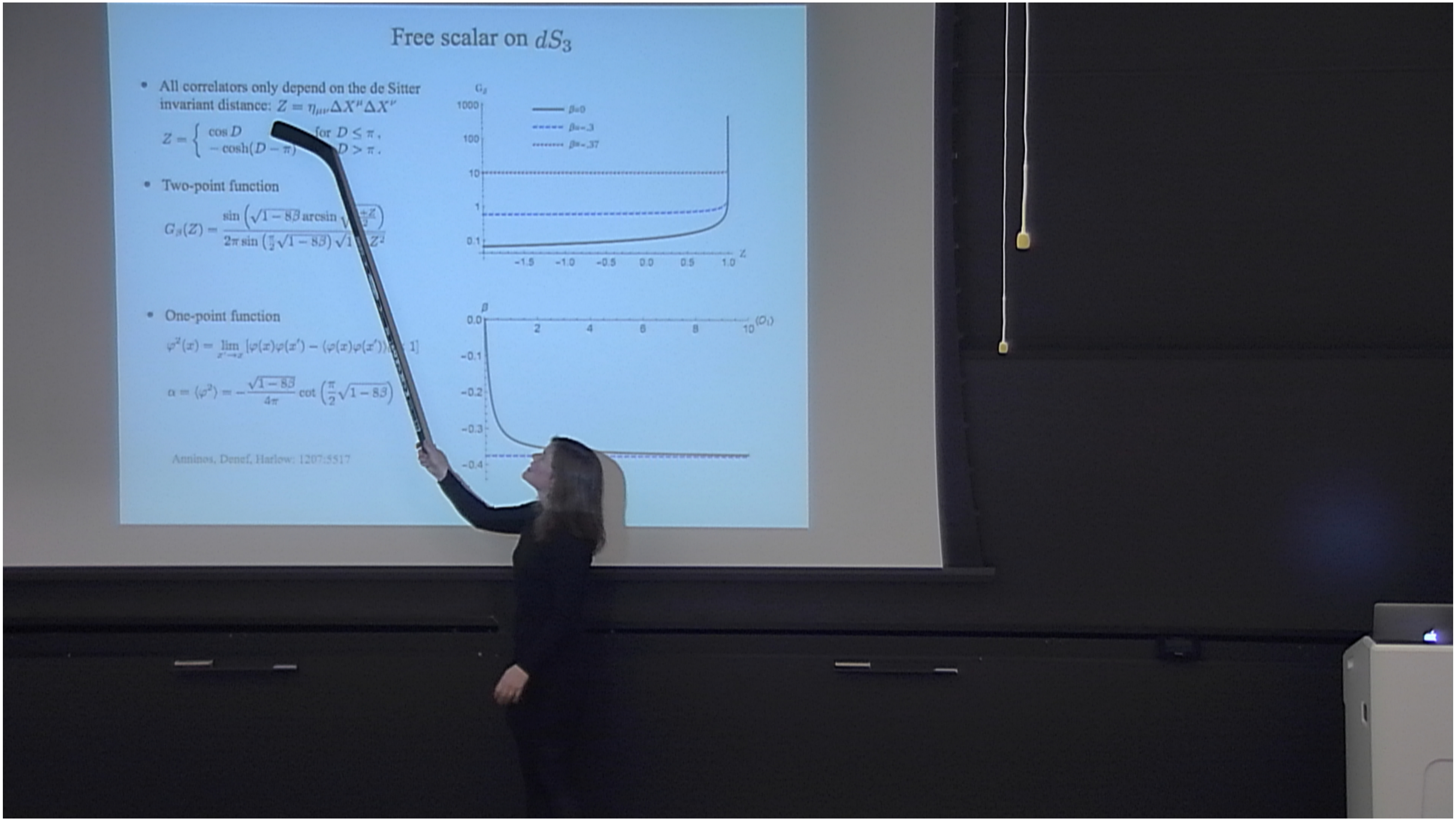
$$G_\beta(Z) = \frac{\sin\left(\sqrt{1-8\beta}\,\arcsin\sqrt{\frac{1+Z}{2}}\right)}{2\pi \sin\left(\frac{\pi}{2}\sqrt{1-8\beta}\right)\sqrt{1-Z^2}}$$

- One-point function

$$\varphi^2(x) = \lim_{x' \to x}\left[\varphi(x)\varphi(x') - \langle\varphi(x)\varphi(x')\rangle_0 \times 1\right]$$

$$\alpha = \langle\varphi^2\rangle = -\frac{\sqrt{1-8\beta}}{4\pi}\cot\left(\frac{\pi}{2}\sqrt{1-8\beta}\right)$$

Anninos, Denef, Harlow: 1207:5517



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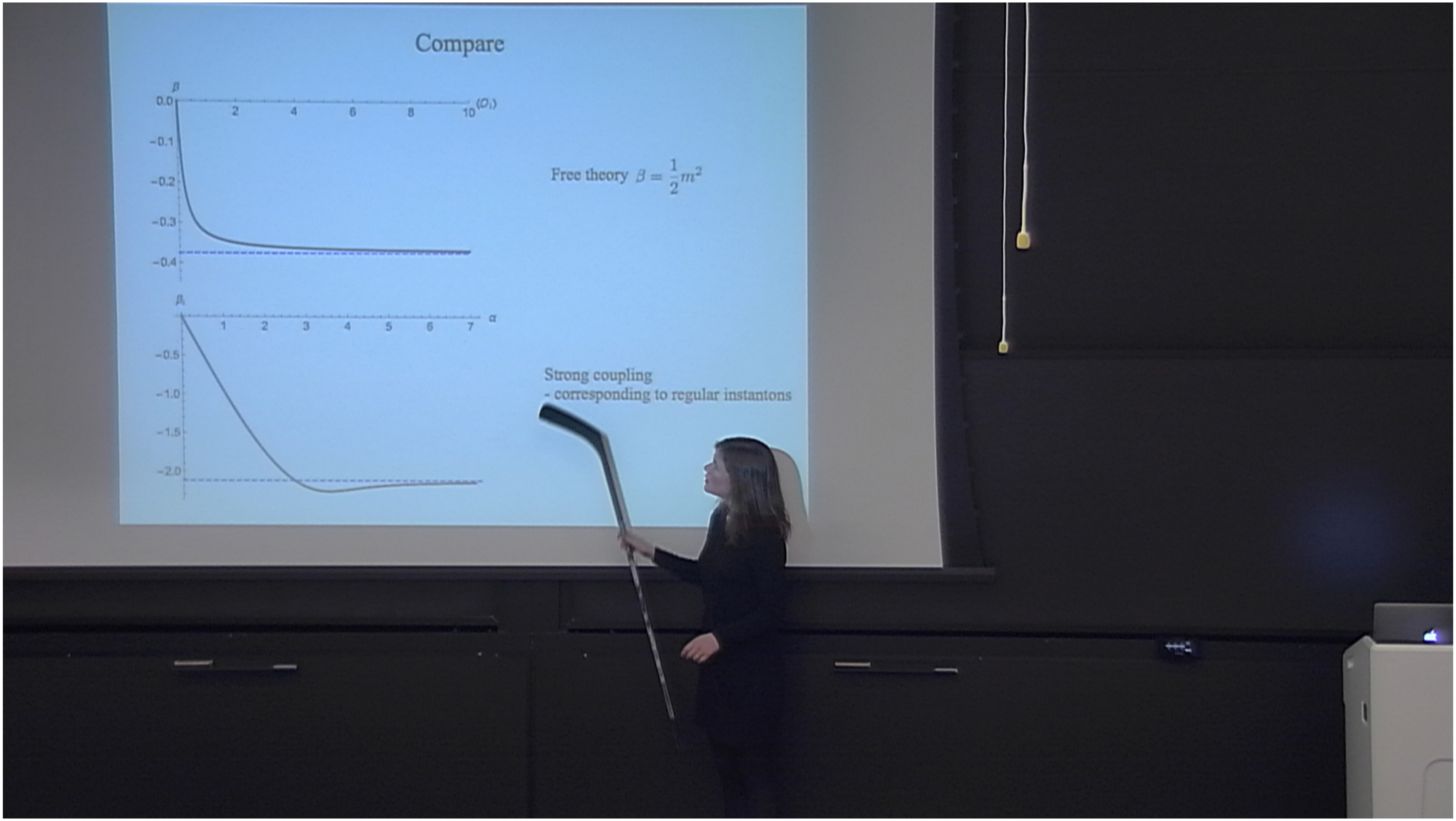
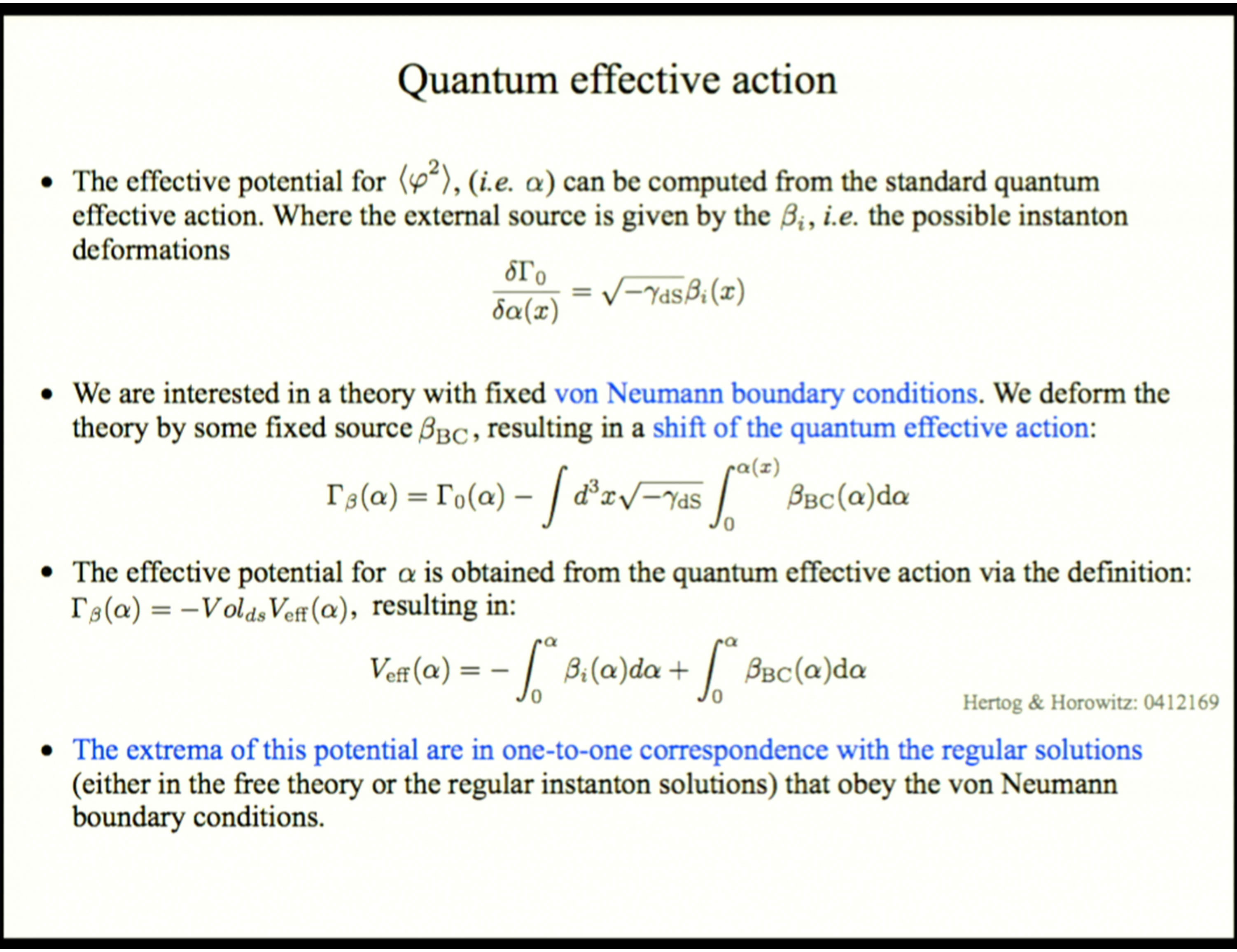


Compare

Free theory $\beta = \frac{1}{2}m^2$

Strong coupling
- corresponding to regular instantons



Quantum effective action

- The effective potential for $\langle\varphi^2\rangle$, (*i.e.* α) can be computed from the standard quantum effective action. Where the external source is given by the β_i, *i.e.* the possible instanton deformations

$$\frac{\delta\Gamma_0}{\delta\alpha(x)} = \sqrt{-\gamma_{\rm dS}}\beta_i(x)$$

- We are interested in a theory with fixed von Neumann boundary conditions. We deform the theory by some fixed source $\beta_{\rm BC}$, resulting in a shift of the quantum effective action:

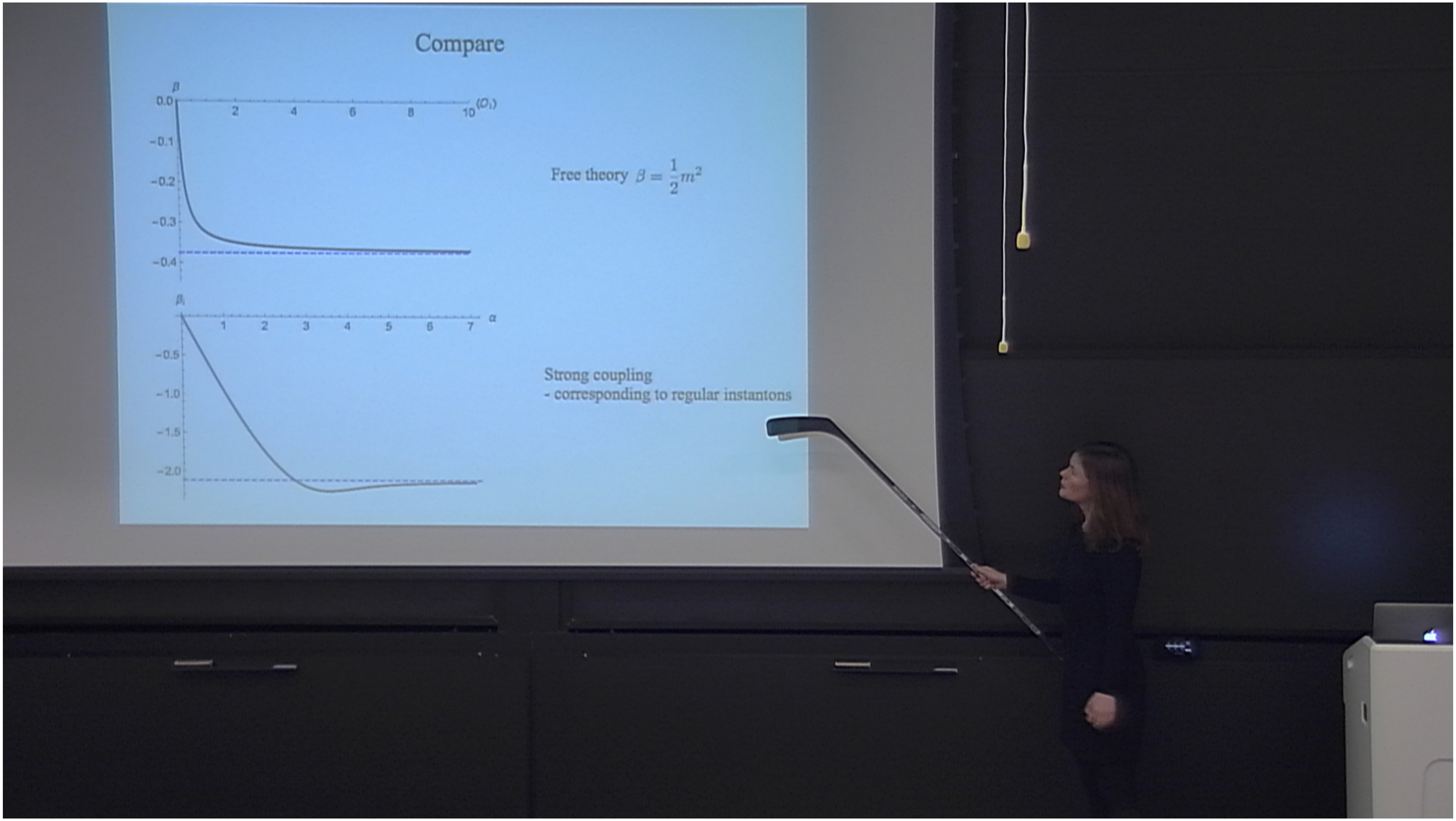
$$\Gamma_\beta(\alpha) = \Gamma_0(\alpha) - \int d^3x\sqrt{-\gamma_{\rm dS}}\int_0^{\alpha(x)}\beta_{\rm BC}(\alpha){\rm d}\alpha$$

- The effective potential for α is obtained from the quantum effective action via the definition: $\Gamma_\beta(\alpha) = -Vol_{ds}V_{\rm eff}(\alpha)$, resulting in:

$$V_{\rm eff}(\alpha) = -\int_0^\alpha \beta_i(\alpha)d\alpha + \int_0^\alpha \beta_{\rm BC}(\alpha){\rm d}\alpha$$

Hertog & Horowitz: 0412169

- The extrema of this potential are in one-to-one correspondence with the regular solutions (either in the free theory or the regular instanton solutions) that obey the von Neumann boundary conditions.



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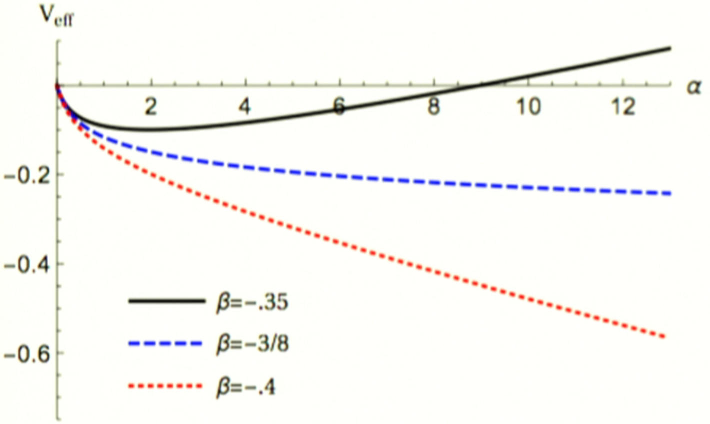
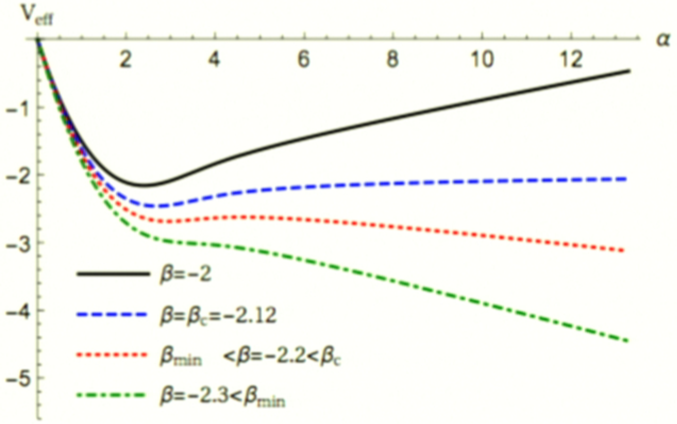
Effective Potential

Free theory

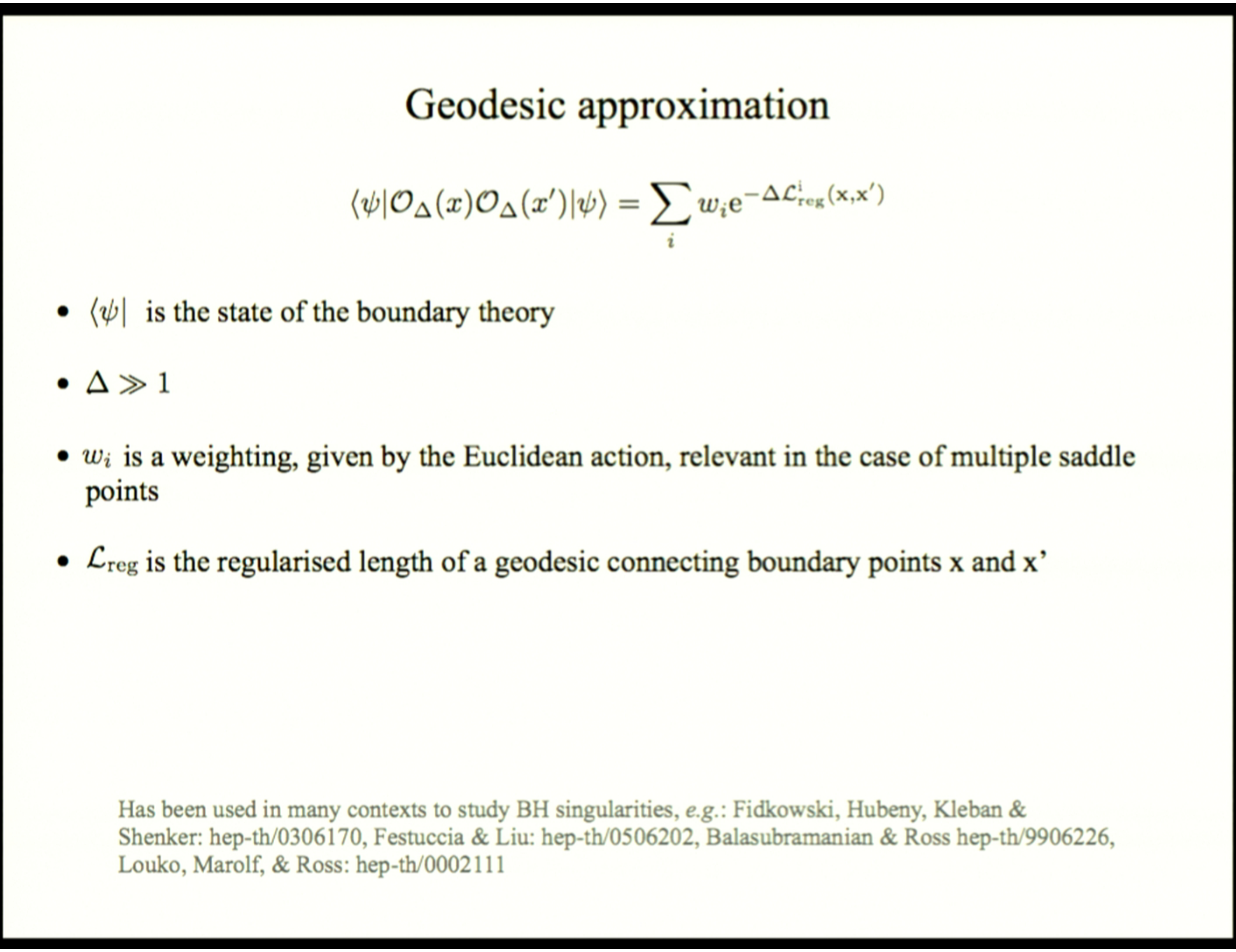


- Stable for $\beta > \beta_c^{fr} = -3/8$
- No stable vacua for tachyonic scalars

Strong coupling



- Stable for $\beta > \beta_c \approx -2.12$
- Regime of with a metastable minimum corresponding to the first saddle point, and an unstable maxima corresponding to the second saddle point and no overall stable vacuum.
- No (meta)stable vacua for large deformations

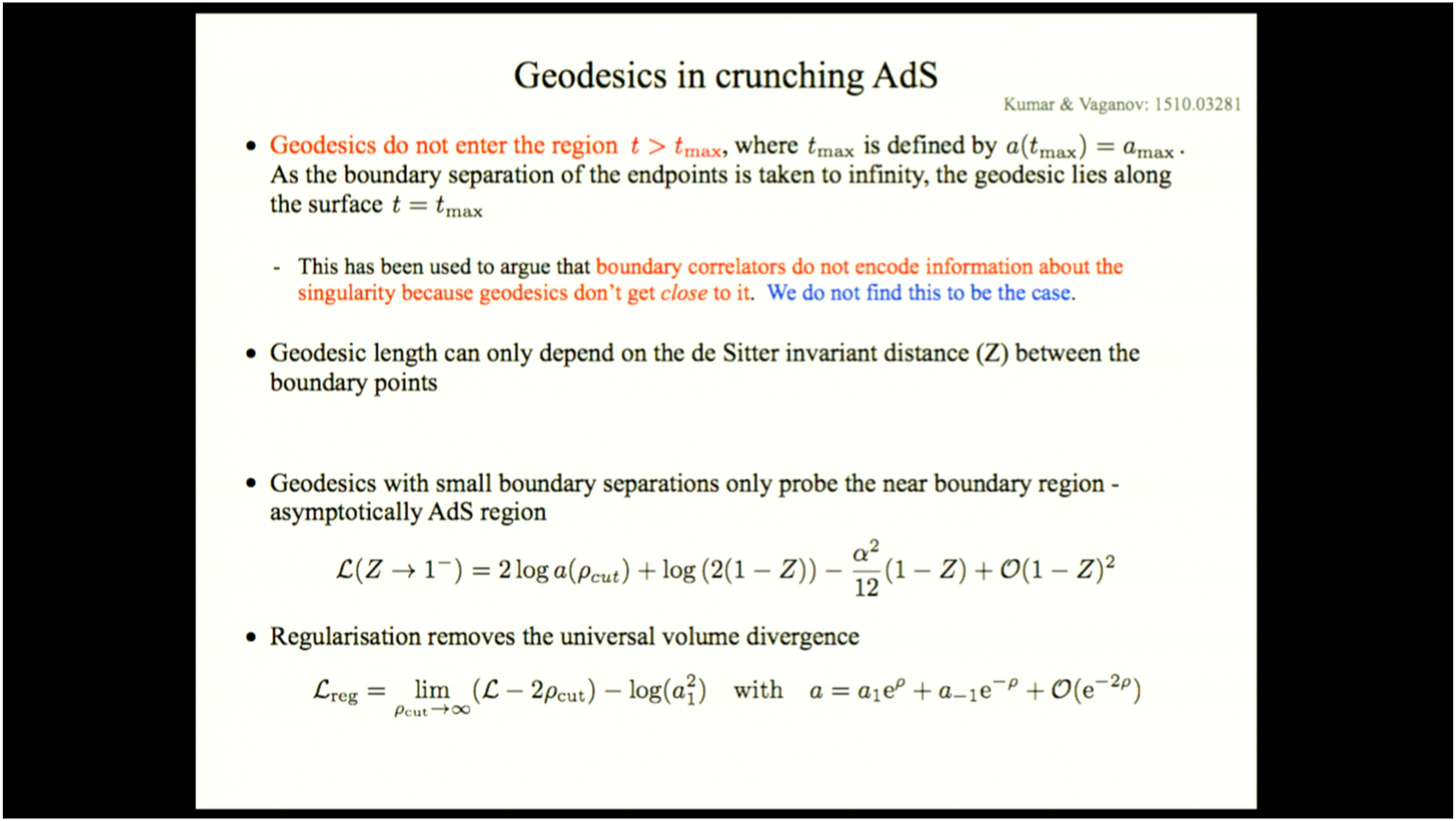


Geodesic approximation

$$\langle\psi|\mathcal{O}_\Delta(x)\mathcal{O}_\Delta(x')|\psi\rangle = \sum_i w_i \mathrm{e}^{-\Delta\mathcal{L}^{\mathrm{i}}_{\mathrm{reg}}(\mathrm{x},\mathrm{x}')}$$

- $\langle\psi|$ is the state of the boundary theory
- $\Delta \gg 1$
- w_i is a weighting, given by the Euclidean action, relevant in the case of multiple saddle points
- $\mathcal{L}_{\mathrm{reg}}$ is the regularised length of a geodesic connecting boundary points x and x'

Has been used in many contexts to study BH singularities, *e.g.*: Fidkowski, Hubeny, Kleban & Shenker: hep-th/0306170, Festuccia & Liu: hep-th/0506202, Balasubramanian & Ross hep-th/9906226, Louko, Marolf, & Ross: hep-th/0002111



Geodesics in crunching AdS

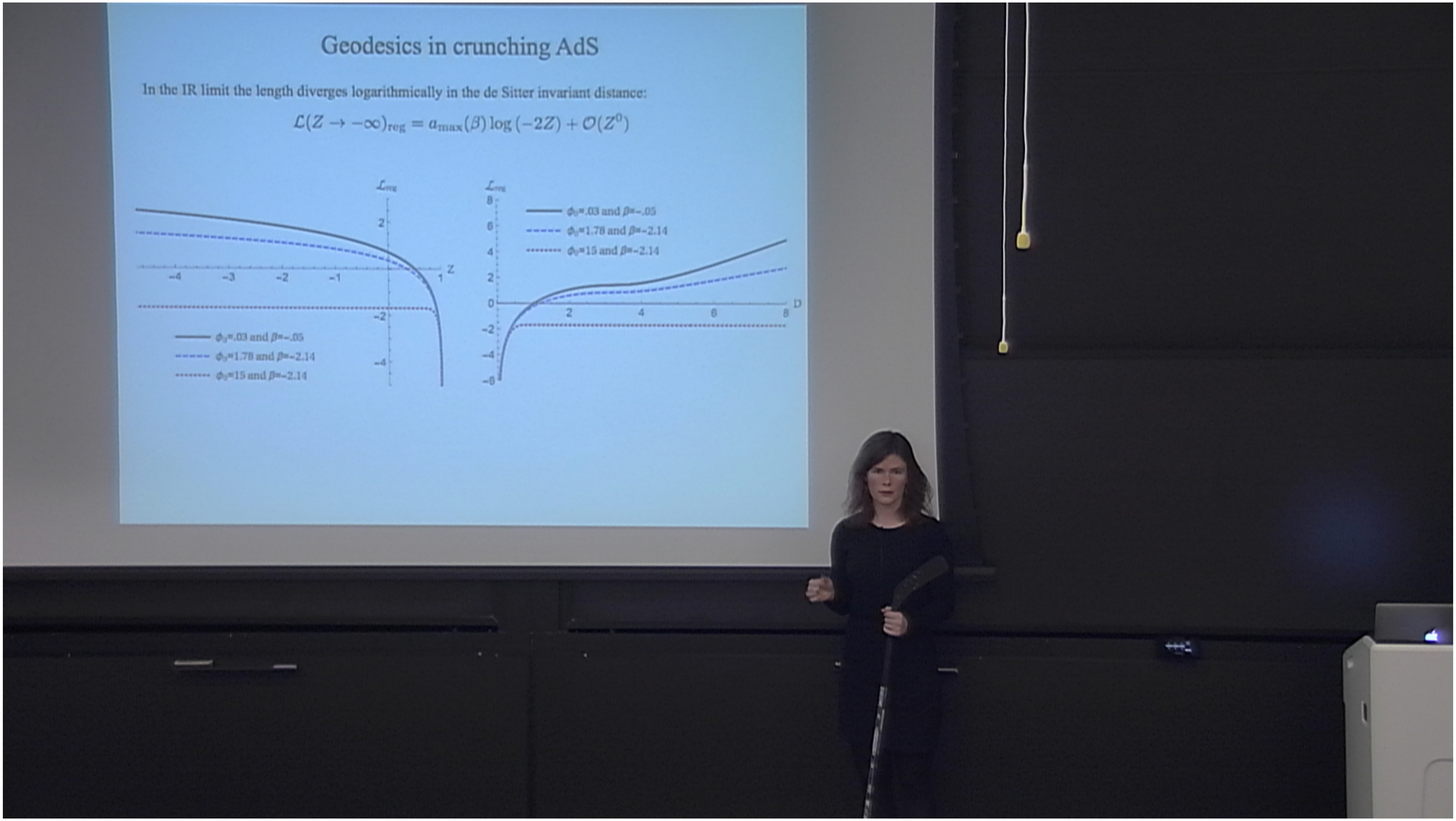
Kumar & Vaganov: 1510.03281

- Geodesics do not enter the region $t > t_{\text{max}}$, where t_{max} is defined by $a(t_{\text{max}}) = a_{\text{max}}$. As the boundary separation of the endpoints is taken to infinity, the geodesic lies along the surface $t = t_{\text{max}}$
  - This has been used to argue that boundary correlators do not encode information about the singularity because geodesics don't get *close* to it. We do not find this to be the case.
- Geodesic length can only depend on the de Sitter invariant distance (Z) between the boundary points
- Geodesics with small boundary separations only probe the near boundary region - asymptotically AdS region

$$\mathcal{L}(Z \to 1^-) = 2\log a(\rho_{cut}) + \log\left(2(1-Z)\right) - \frac{\alpha^2}{12}(1-Z) + \mathcal{O}(1-Z)^2$$

- Regularisation removes the universal volume divergence

$$\mathcal{L}_{\text{reg}} = \lim_{\rho_{\text{cut}}\to\infty}(\mathcal{L} - 2\rho_{\text{cut}}) - \log(a_1^2) \quad \text{with} \quad a = a_1 e^{\rho} + a_{-1}e^{-\rho} + \mathcal{O}(e^{-2\rho})$$



Geodesics in crunching AdS

In the IR limit the length diverges logarithmically in the de Sitter invariant distance:

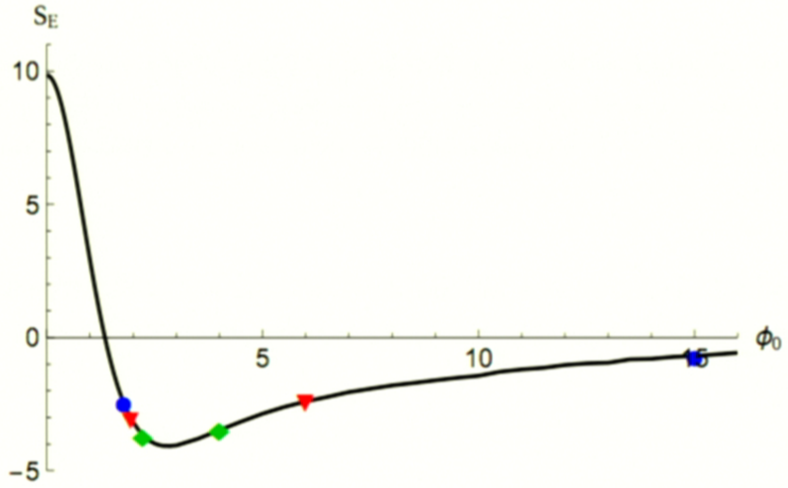
$$\mathcal{L}(Z \to -\infty)_{\rm reg} = a_{\rm max}(\beta)\log(-2Z) + \mathcal{O}(Z^0)$$

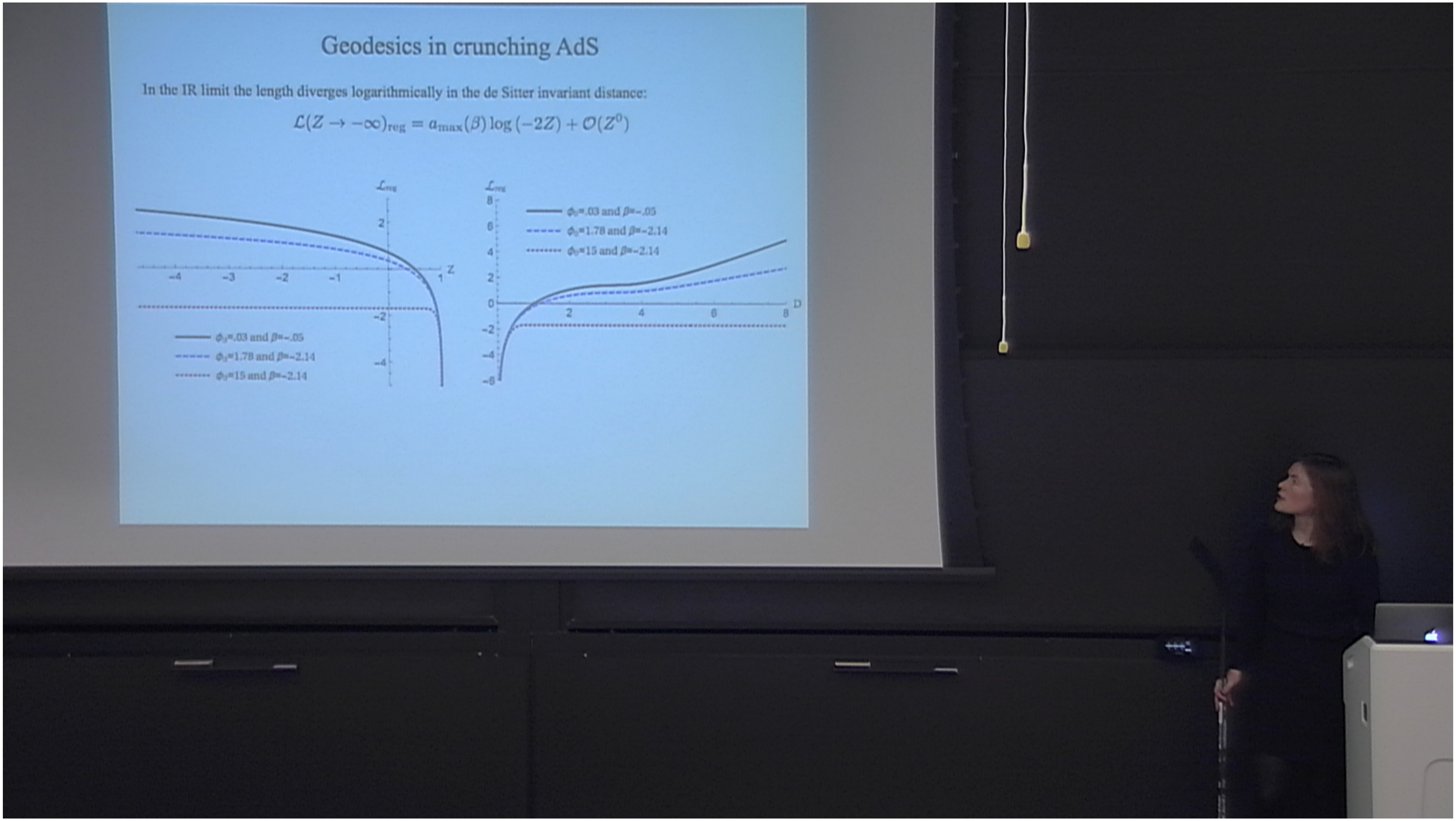
Weight saddle points via the Euclidean action

- Weights are given by: $w_i = \begin{cases} 1 & \text{for } \beta > \beta_c\,, \\ \frac{\mathrm{e}^{-S_{\mathrm{E}\,i}}}{\sum_j \mathrm{e}^{-S_{\mathrm{E}\,j}}} & \text{for } \beta_{\min} \le \beta \le \beta_c\,, \end{cases}$

- When the second saddle point enters, it has a less negative Euclidean action, and is therefore subdominant
  - reminiscent of Maldacena's eternal BH in AdS story

    hep-th/0106112





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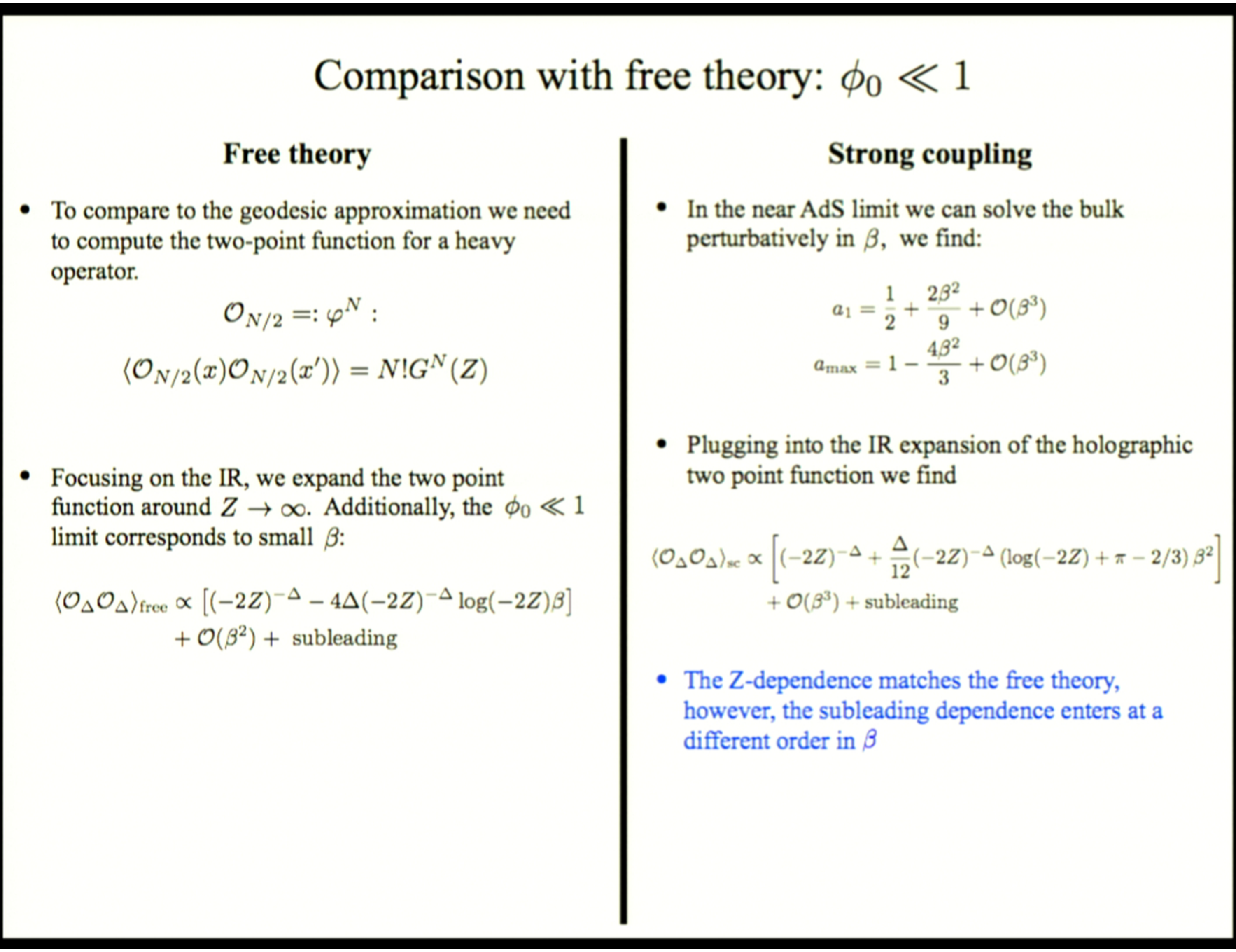


Comparison with free theory: $\phi_0 \ll 1$

Free theory

- To compare to the geodesic approximation we need to compute the two-point function for a heavy operator.

$$\mathcal{O}_{N/2} =: \varphi^N :$$

$$\langle \mathcal{O}_{N/2}(x) \mathcal{O}_{N/2}(x') \rangle = N! G^N(Z)$$

- Focusing on the IR, we expand the two point function around $Z \to \infty$. Additionally, the $\phi_0 \ll 1$ limit corresponds to small β:

$$\langle \mathcal{O}_\Delta \mathcal{O}_\Delta \rangle_{\text{free}} \propto \left[(-2Z)^{-\Delta} - 4\Delta (-2Z)^{-\Delta} \log(-2Z) \beta \right] + \mathcal{O}(\beta^2) + \text{ subleading}$$

Strong coupling

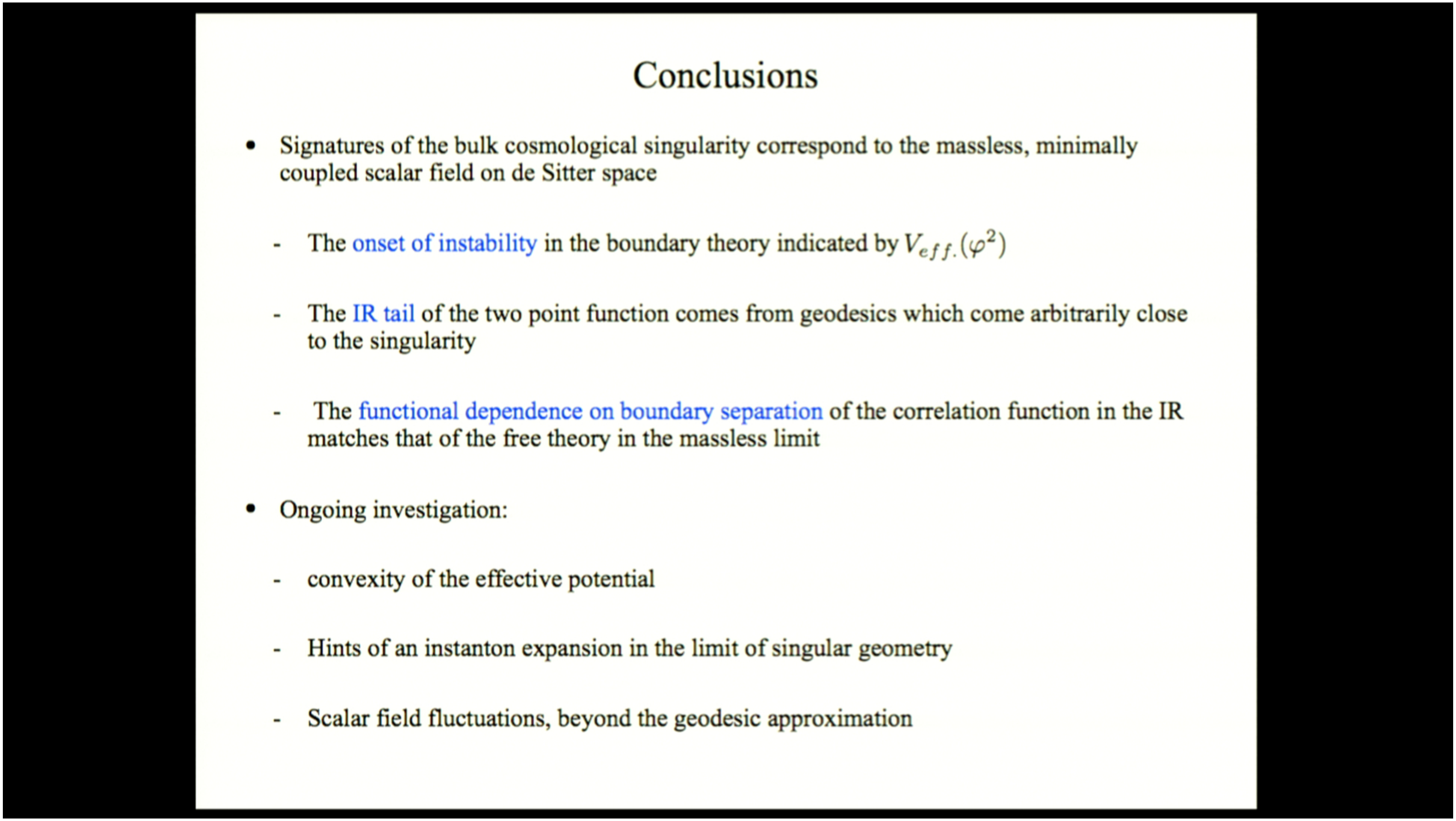
- In the near AdS limit we can solve the bulk perturbatively in β, we find:

$$a_1 = \frac{1}{2} + \frac{2\beta^2}{9} + \mathcal{O}(\beta^3)$$
$$a_{\max} = 1 - \frac{4\beta^2}{3} + \mathcal{O}(\beta^3)$$

- Plugging into the IR expansion of the holographic two point function we find

$$\langle \mathcal{O}_\Delta \mathcal{O}_\Delta \rangle_{\text{sc}} \propto \left[(-2Z)^{-\Delta} + \frac{\Delta}{12} (-2Z)^{-\Delta} \left(\log(-2Z) + \pi - 2/3 \right) \beta^2 \right] + \mathcal{O}(\beta^3) + \text{subleading}$$

- The Z-dependence matches the free theory, however, the subleading dependence enters at a different order in β



Conclusions

- Signatures of the bulk cosmological singularity correspond to the massless, minimally coupled scalar field on de Sitter space
  - The onset of instability in the boundary theory indicated by $V_{eff.}(\varphi^2)$
  - The IR tail of the two point function comes from geodesics which come arbitrarily close to the singularity
  - The functional dependence on boundary separation of the correlation function in the IR matches that of the free theory in the massless limit
- Ongoing investigation:
  - convexity of the effective potential
  - Hints of an instanton expansion in the limit of singular geometry
  - Scalar field fluctuations, beyond the geodesic approximation