Title: Exploring the Weak Gravity Conjecture

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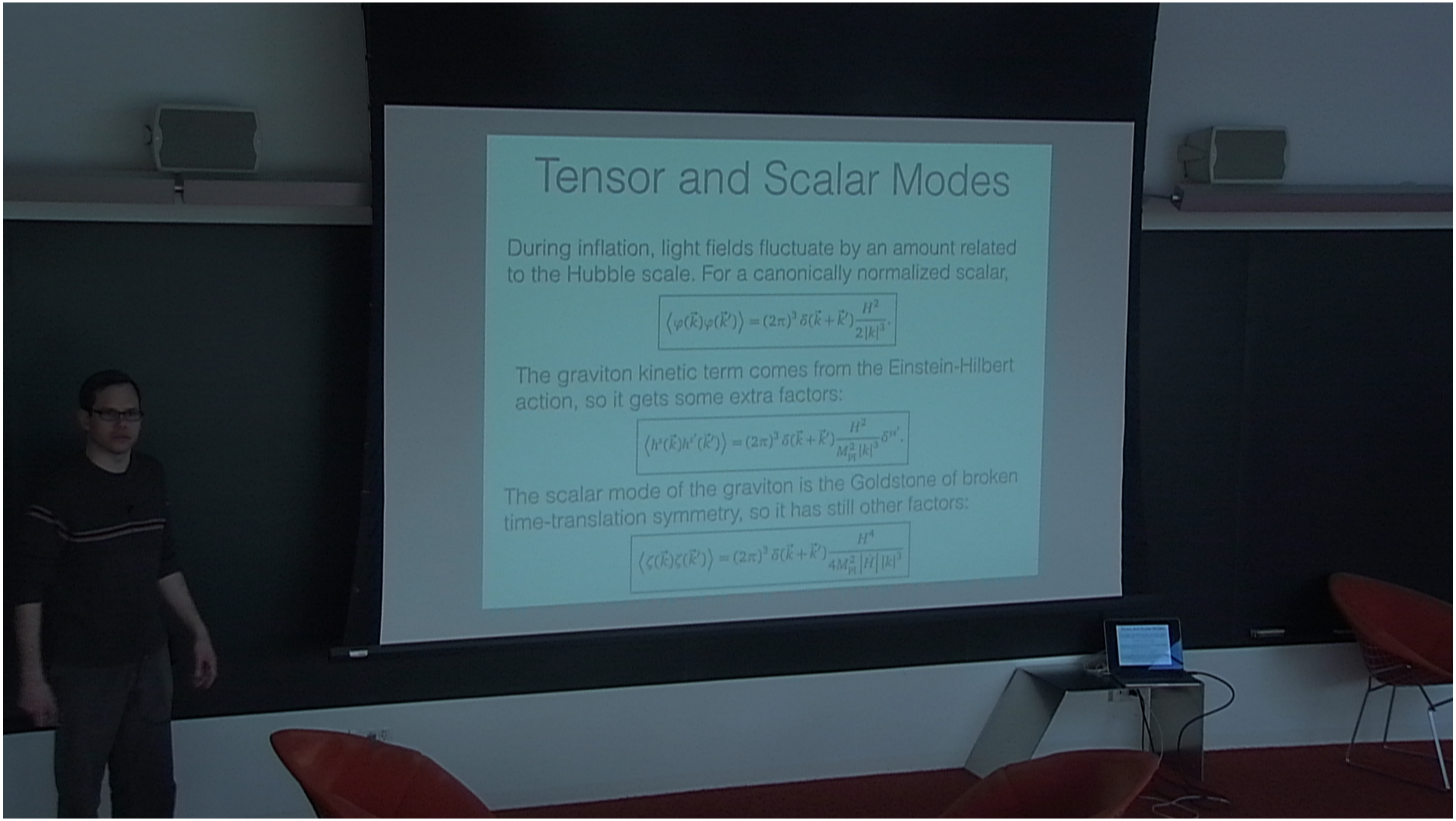
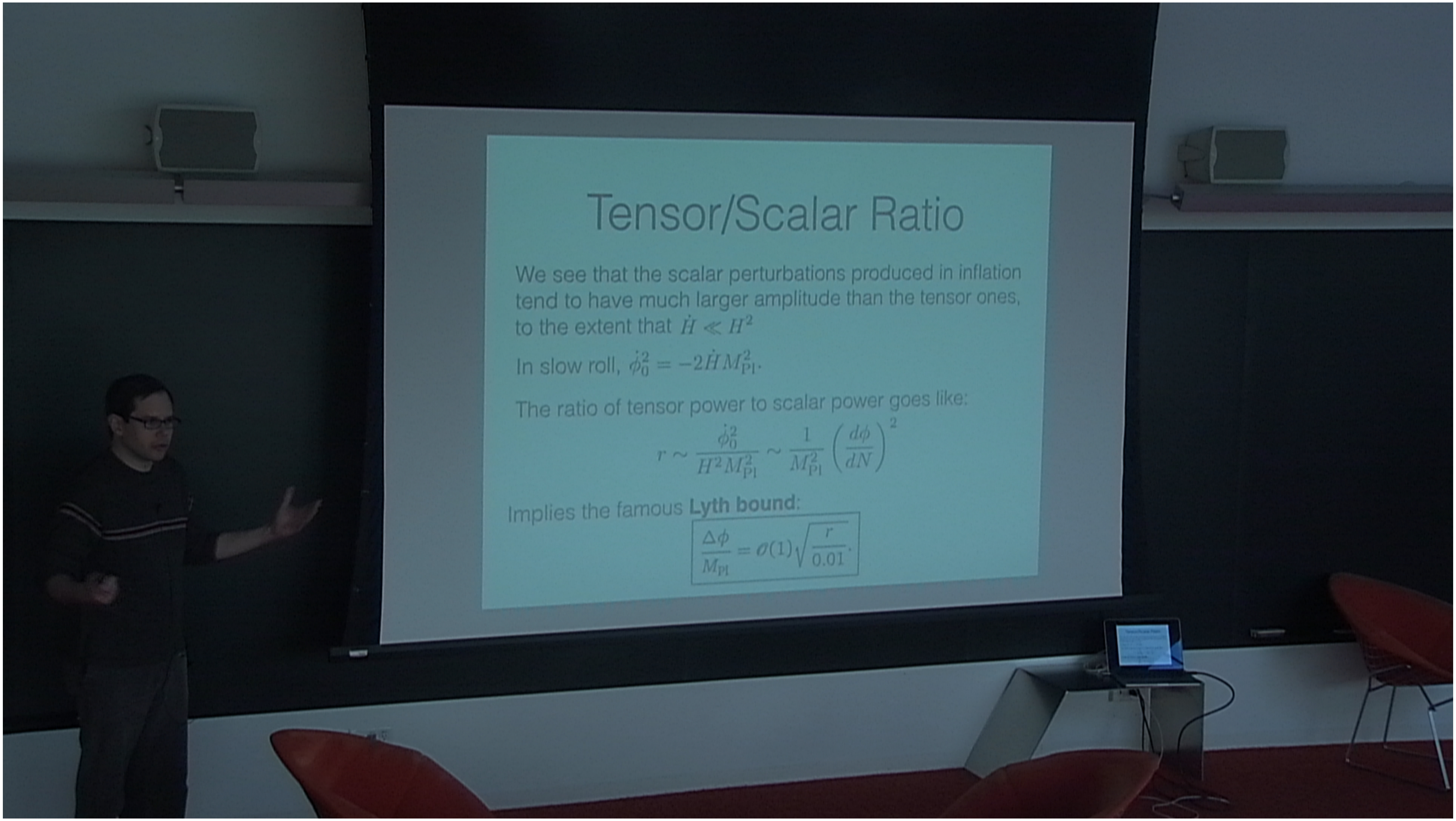
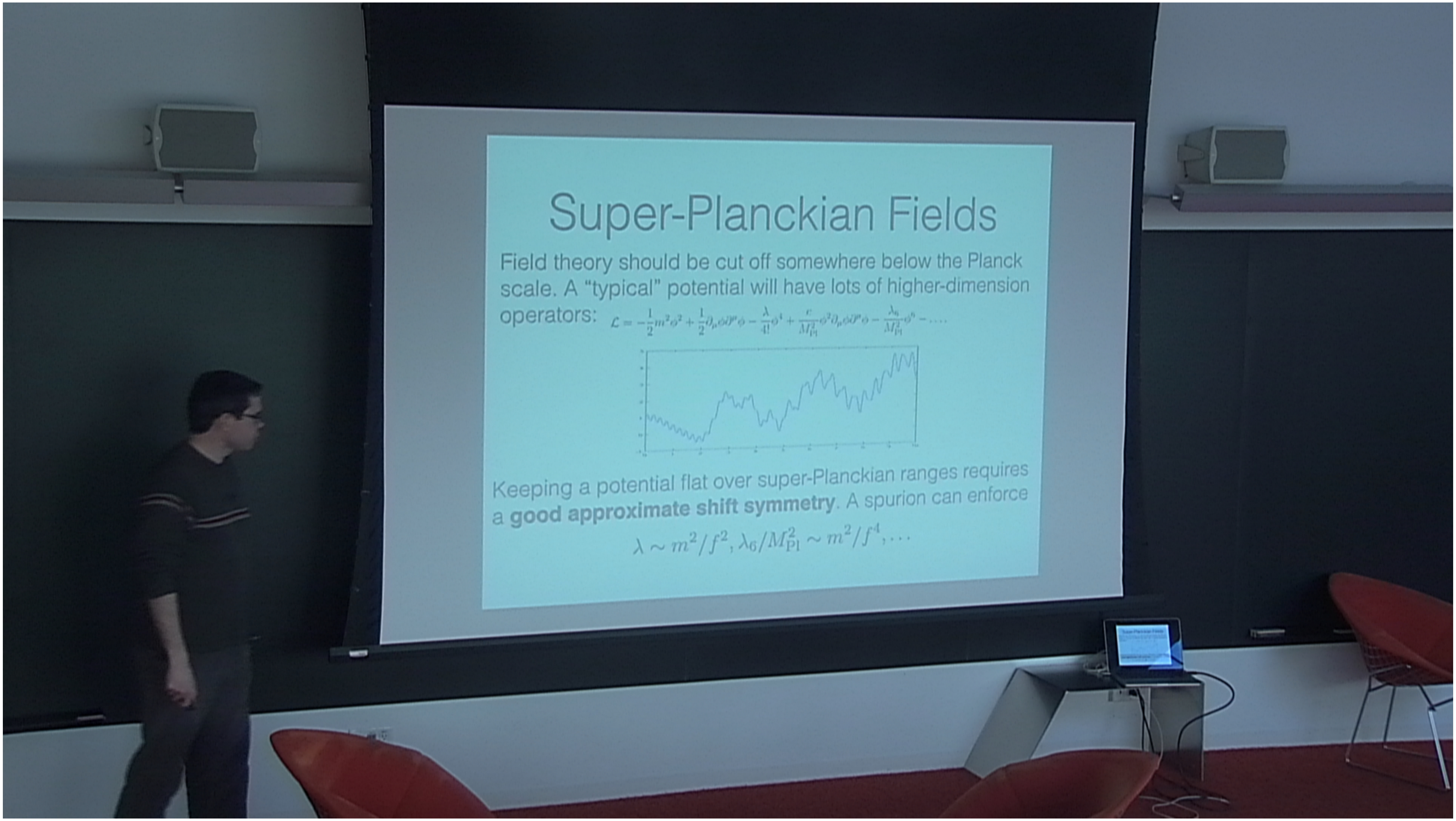
Abstract: <p>The Weak Gravity Conjecture (WGC), in its original form, says that given an abelian gauge theory there should be at least one charged particle whose charge is bigger than its mass in Planck units. This has surprisingly powerful implications for the possibility of large-field inflation. In this talk I will explore some of the arguments linking the WGC to inflation before taking a closer look at a different question: which version of the WGC should we be trying to prove? I suggest that the right version to focus on is much stronger than the original WGC, and requires a sufficiently light particle to exist in every representation of the gauge group. I will present some evidence for this statement from both weakly-coupled string theory and semiclassical GR. This talk is based on work with Ben Heidenreich and Tom Rudelius (arXiv:1506.03447, 1509.06374, and further work in progress).</p>


Tensor and Scalar Modes
During inflation, light fields fluctuate by an amount related to the Hubble scale. For a canonically normalized scalar,
$\langle \varphi(\vec{k})\varphi(\vec{k}')\rangle = (2\pi)^3\delta(\vec{k}+\vec{k}')\frac{H^2}{2|k|^3}$.
The graviton kinetic term comes from the Einstein-Hilbert action, so it gets some extra factors:
$\langle h^s(\vec{k})h^{s'}(\vec{k}')\rangle = (2\pi)^3\delta(\vec{k}+\vec{k}')\frac{H^2}{M_{\rm pl}^2|k|^3}\delta^{ss'}$.
The scalar mode of the graviton is the Goldstone of broken time-translation symmetry, so it has still other factors:
$\langle \zeta(\vec{k})\zeta(\vec{k}')\rangle = (2\pi)^3\delta(\vec{k}+\vec{k}')\frac{H^4}{4M_{\rm pl}^2|\dot{H}||k|^3}$


Tensor/Scalar Ratio
We see that the scalar perturbations produced in inflation tend to have much larger amplitude than the tensor ones, to the extent that $\dot{H} \ll H^2$
In slow roll, $\dot{\phi}_0^2 = -2\dot{H}M_{\rm Pl}^2$.
The ratio of tensor power to scalar power goes like:
$r \sim \frac{\dot{\phi}_0^2}{H^2 M_{\rm Pl}^2} \sim \frac{1}{M_{\rm Pl}^2}\left(\frac{d\phi}{dN}\right)^2$
Implies the famous **Lyth bound**:
$\frac{\Delta\phi}{M_{\rm Pl}} = \mathcal{O}(1)\sqrt{\frac{r}{0.01}}$.


Super-Planckian Fields
Field theory should be cut off somewhere below the Planck scale. A "typical" potential will have lots of higher-dimension operators: $\mathcal{L} = -\frac{1}{2}m^2\phi^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4!}\phi^4 + \frac{c}{M_{Pl}^2}\phi^2\partial_\mu\phi\partial^\mu\phi - \frac{\lambda_6}{M_{Pl}^2}\phi^6 - \ldots$
Keeping a potential flat over super-Planckian ranges requires a good approximate shift symmetry. A spurion can enforce
$\lambda \sim m^2/f^2, \lambda_6/M_{Pl}^2 \sim m^2/f^4, \ldots$

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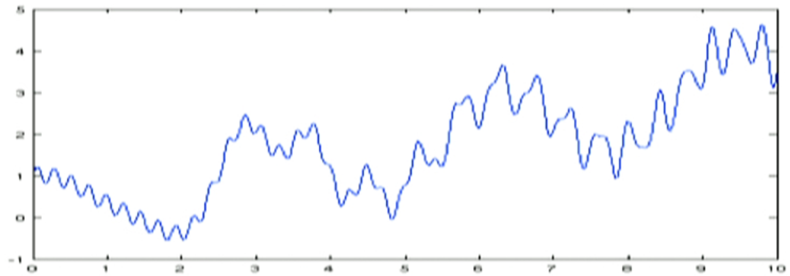

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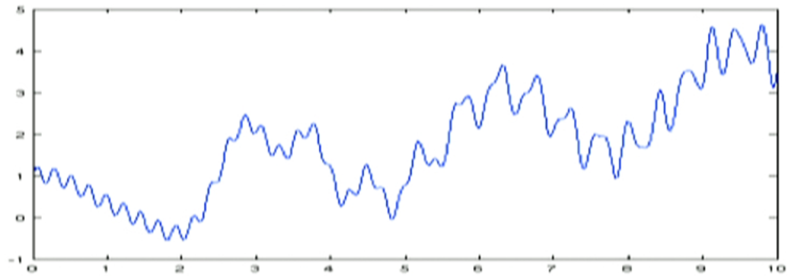

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Approximate Symmetries

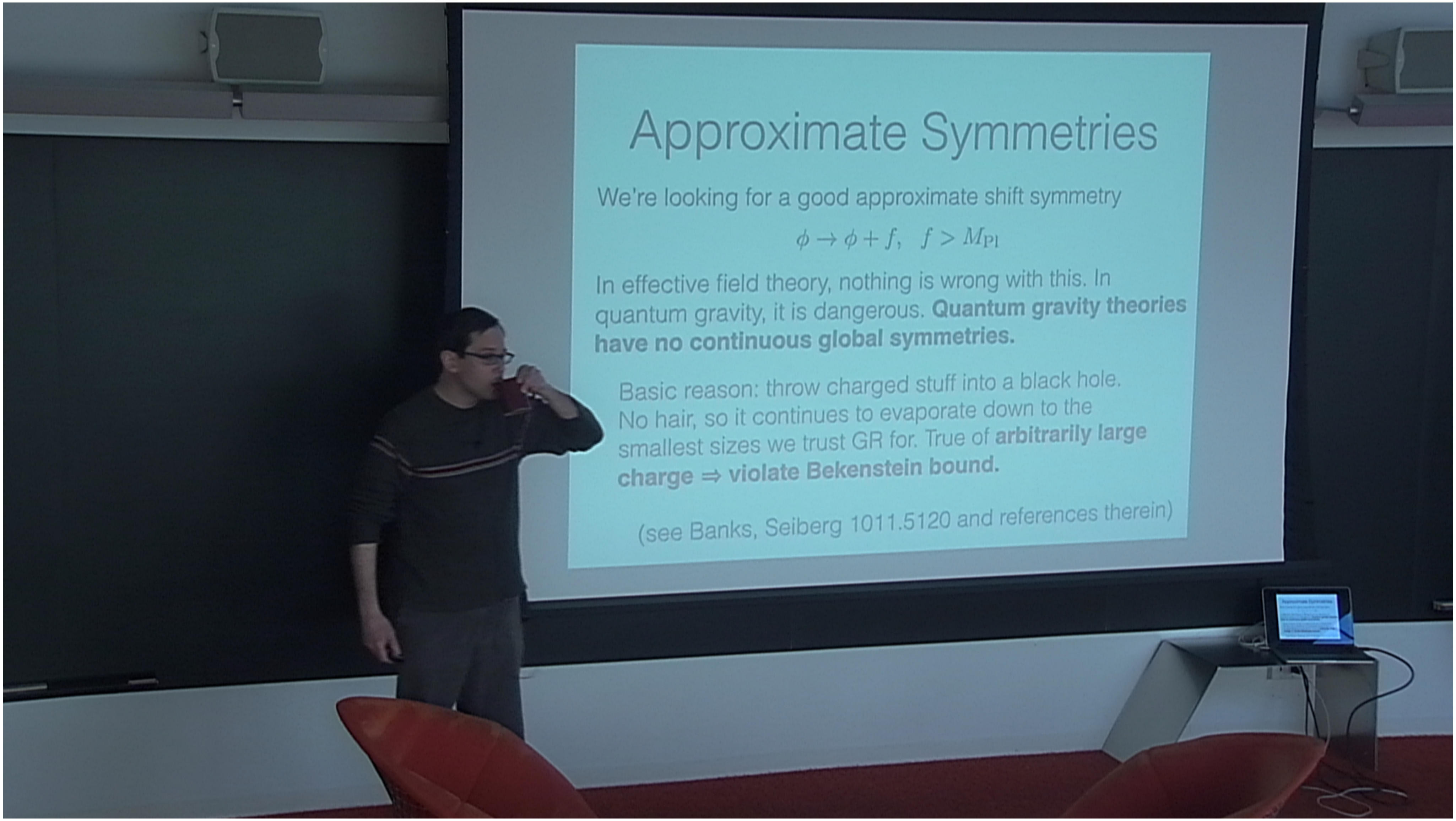
We're looking for a good approximate shift symmetry

$$\phi \to \phi + f, \quad f > M_{\rm Pl}$$

In effective field theory, nothing is wrong with this. In quantum gravity, it is dangerous. **Quantum gravity theories have no continuous global symmetries.**

Basic reason: throw charged stuff into a black hole. No hair, so it continues to evaporate down to the smallest sizes we trust GR for. True of **arbitrarily large charge ⇒ violate Bekenstein bound.**

(see Banks, Seiberg 1011.5120 and references therein)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Axions and Gauge Fields

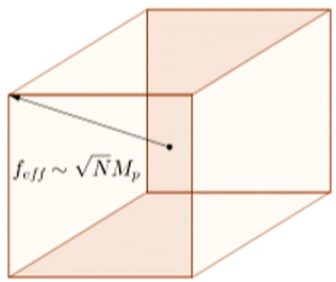
Good symmetries in quantum gravity are gauged. Either write down a continuous gauge theory, or a gauged discrete symmetry that can forbid dangerous operators to some level.

$$a = \int_{\Sigma_p} A_p$$

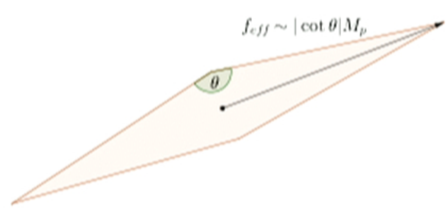
String axions come from dimensional reduction of *p*-form **gauge fields.** $a \to a + 2\pi$ gauge symmetry from large gauge transformations.

Super-Planckian Axions?

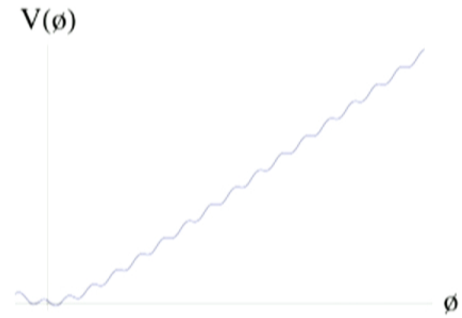
Axion fields in string theory are generally found to have ***f* < *M*$_{Pl}$**. (Banks, Dine, Fox, Gorbatov hep-th/0303252)
So can't immediately use one for large-field inflation.
People have tried **clever model-building** to improve:



N-flation
(Dimopoulos, Kachru, McGreevy, Wacker '05)



Alignment
(Kim, Nilles, Peloso '04)



Monodromy
(Silverstein, Westphal '08; McAllister, Flauger, ….)

Axions to Gauge Fields to the Swampland

Large field inflation has led us to seek good approximate symmetries, leading us to axions, leading us to higher-dimensional gauge theories.

From an EFT viewpoint these theories are very well-controlled. But in quantum gravity, EFT is not enough. Are these theories in the Swampland?

We'll take a detour to explore the Weak Gravity Conjecture and then return to inflation at the end.

Black Hole Extremality Bound

Reminder: semiclassical charged black holes exist in GR. For a given charge, there is a **minimum** mass the black hole can have. Being sloppy about order-one factors,

$$M_{\rm BH} > eQM_{\rm Pl}$$

e coupling constant

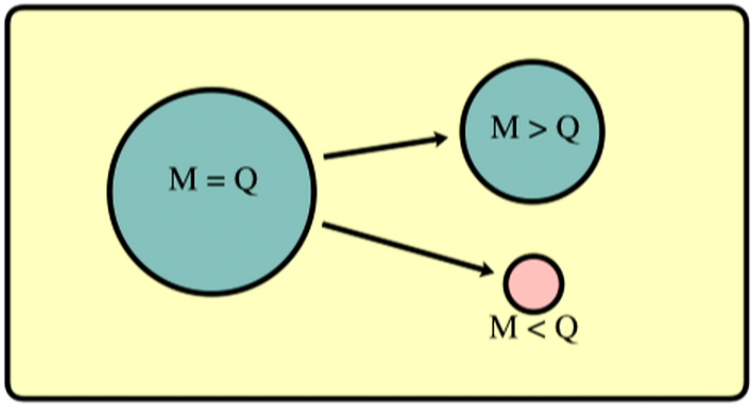
Q integer charge (quantized)

For black holes charged under multiple U(1) gauge groups,

$$M_{\rm BH} > \sqrt{(e_1Q_1)^2 + (e_2Q_2)^2 + \ldots + (e_NQ_N)^2}\,M_{\rm Pl}$$

(important later)

What is the Conjecture?

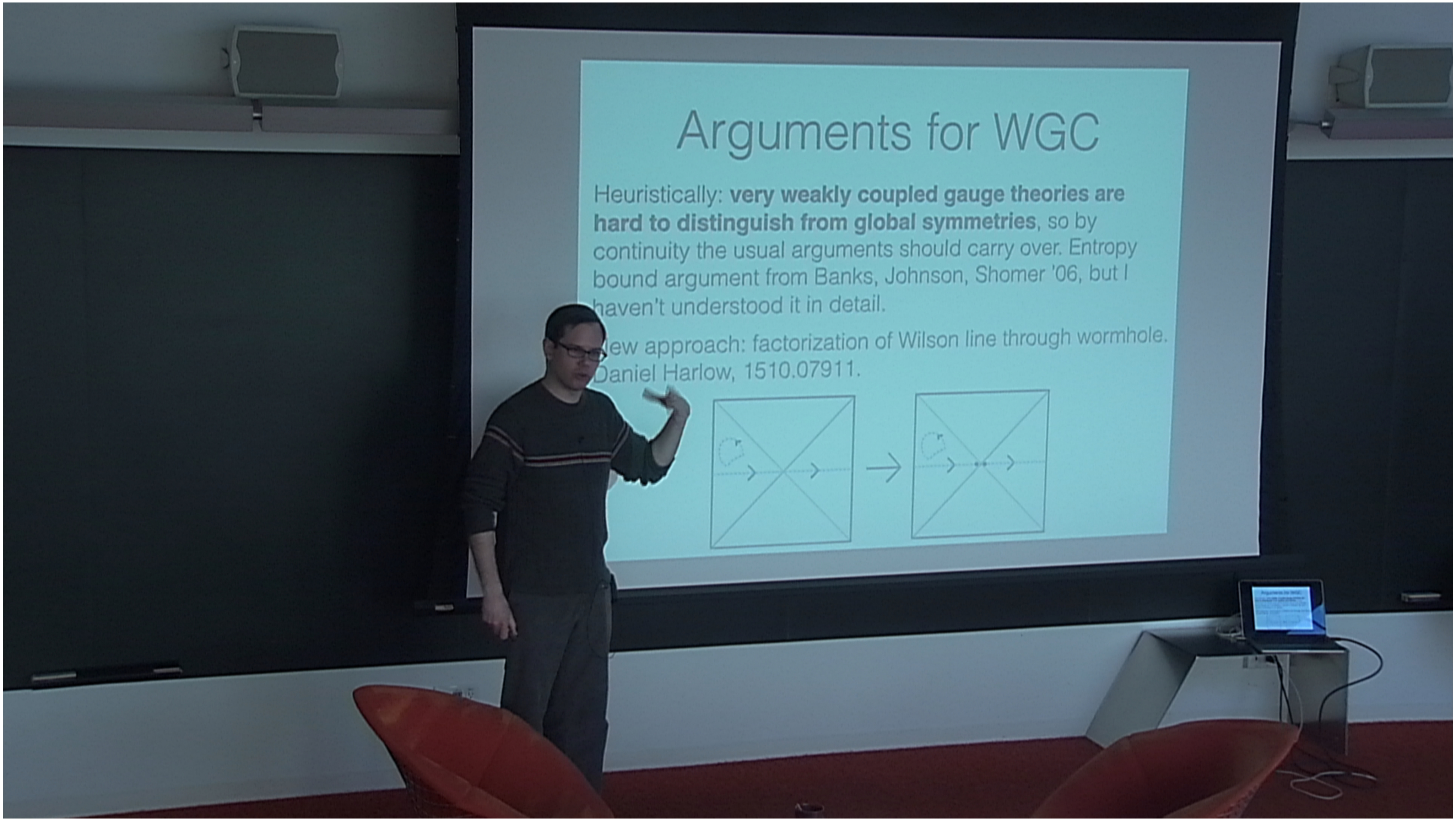


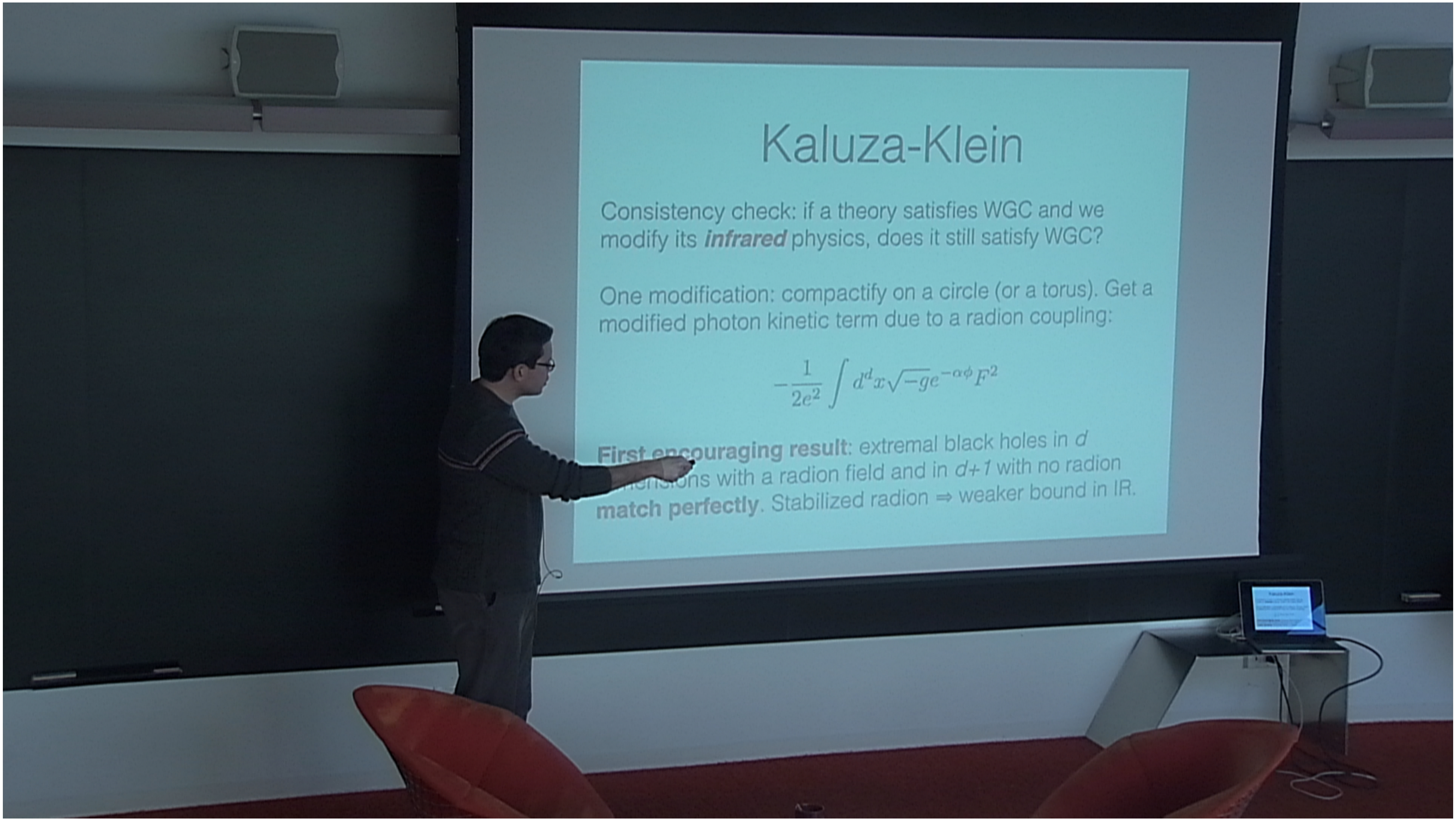
Arkani-Hamed, Motl, Nicolis, Vafa **("AMNV")** hep-th/0601001

Particle exists with M<Q (superextremal).

Why? Postulate that extremal black holes should decay.

Why? Else: nearly-extremal black holes spontaneously evolve toward extremality. Sequester information and entropy forever. Seems strange.


Arguments for WGC
Heuristically: very weakly coupled gauge theories are hard to distinguish from global symmetries, so by continuity the usual arguments should carry over. Entropy bound argument from Banks, Johnson, Shomer '06, but I haven't understood it in detail.
ew approach: factorization of Wilson line through wormhole.
Daniel Harlow, 1510.07911.


Kaluza-Klein
Consistency check: if a theory satisfies WGC and we modify its ***infrared*** physics, does it still satisfy WGC?
One modification: compactify on a circle (or a torus). Get a modified photon kinetic term due to a radion coupling:
$-\frac{1}{2e^2}\int d^dx\sqrt{-g}e^{-\alpha\phi}F^2$
First encouraging result: extremal black holes in *d*
ons with a radion field and in *d+1* with no radion
match perfectly. Stabilized radion ⇒ weaker bound in IR.

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Kaluza-Klein: $U(1)_{KK}$

A theory of pure gravity compactified on a circle produces a gauge theory: the Kaluza-Klein photon. Inverting the extremality bound, WGC asks:

$$m^2 \leq \frac{1}{2} e_{\mathrm{KK}}^2 q^2 M_{\mathrm{Pl}}^{d-2}$$

But the coupling is $e_{\mathrm{KK}}^2 = \dfrac{2}{R^2 M_{\mathrm{Pl}}^{d-2}}$

Result: every KK graviton *saturates* the bound, as

$$m = \frac{q}{R}$$

Kaluza-Klein and CHC

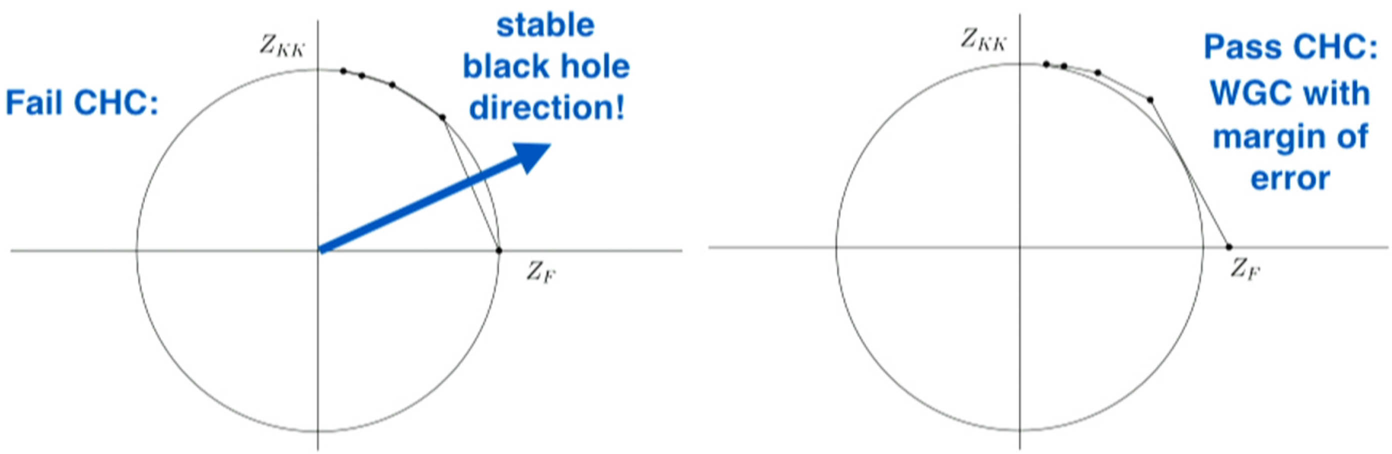
If we start with some U(1) that obeys WGC—call it $U(1)_{EM}$—and compactify, we have both $U(1)_{EM}$ in the reduced theory ***and*** $U(1)_{KK}$, individually satisfying WGC.

Can an extremal black hole charged under **both** $U(1)_{EM}$ and $U(1)_{KK}$ decay? Each KK mode is superextremal:

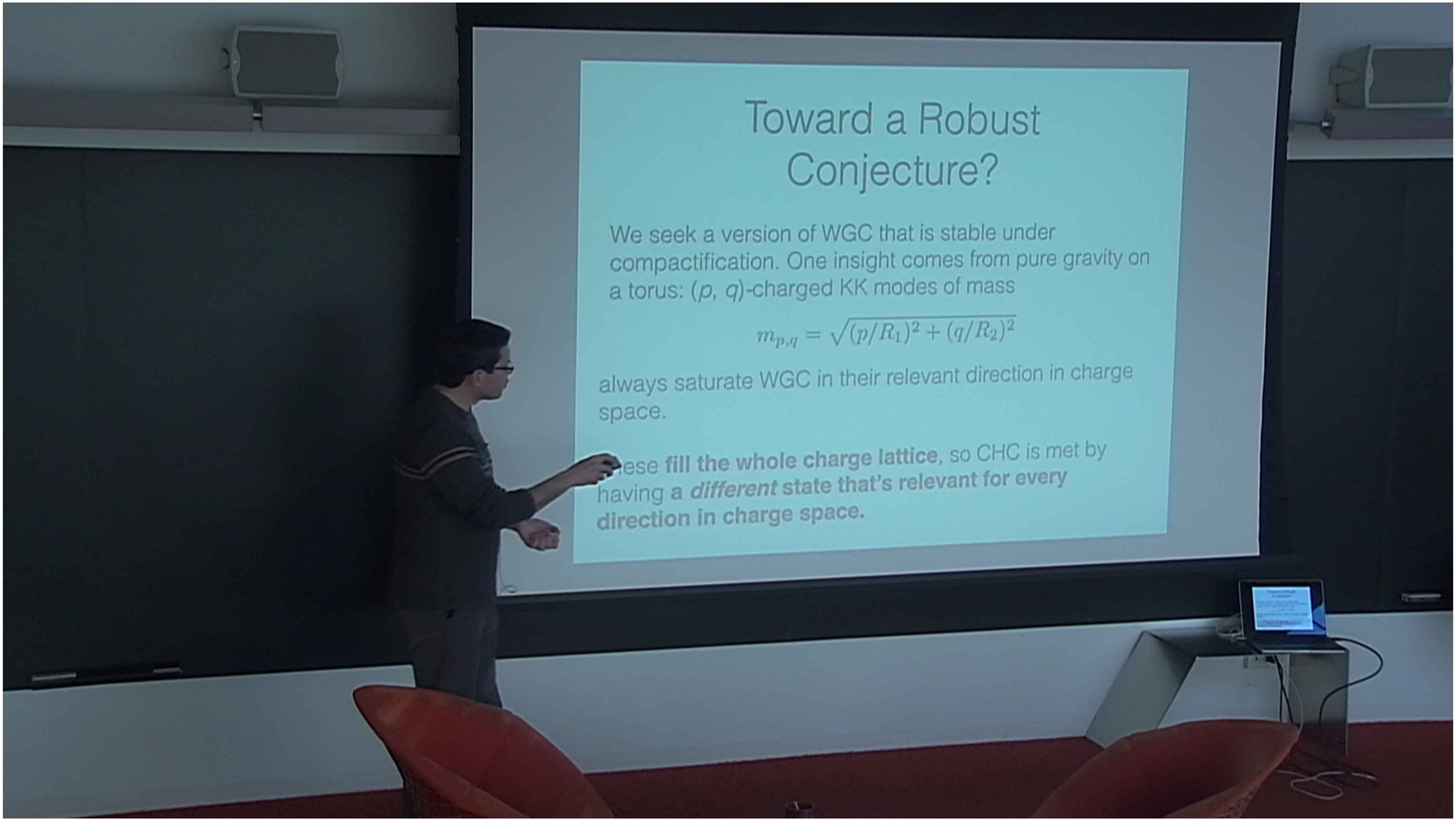
$$m_n = \sqrt{m_0^2 + \frac{n^2}{R^2}} < \sqrt{Q_{\rm EM}^2 M_{\rm Pl}^2 + n^2 e_{\rm KK}^2 M_{\rm Pl}^2}$$

U(1)EM and U(1)KK

Infinite tower of KK modes of the charged particle, each one (super)extremal. How “super”? Radius-dependent.



Not guaranteed to contain the unit ball! CHC can fail, at least for some compactification radii.

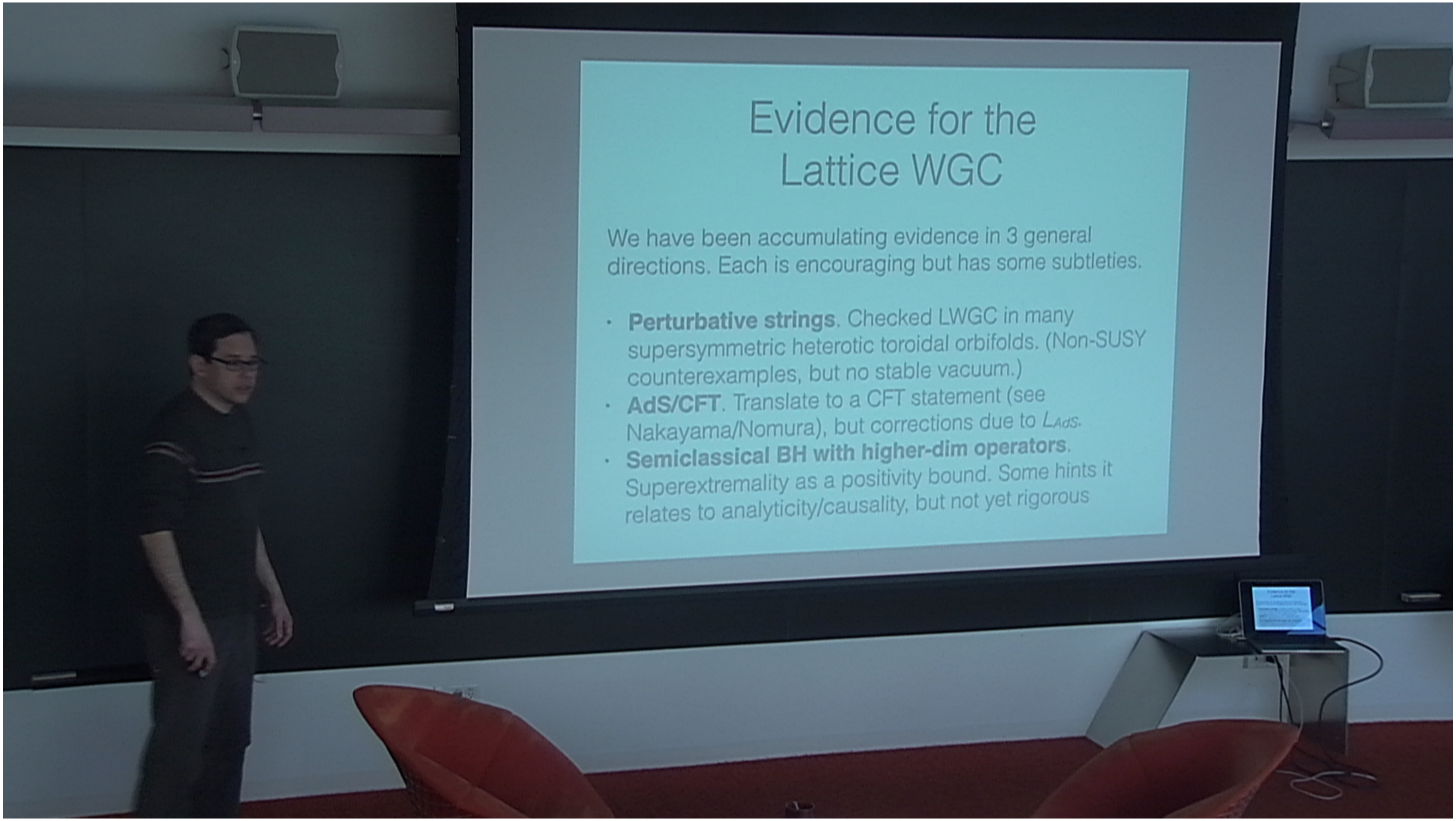

Toward a Robust Conjecture?
We seek a version of WGC that is stable under compactification. One insight comes from pure gravity on a torus: (p, q)-charged KK modes of mass
$m_{p,q} = \sqrt{(p/R_1)^2 + (q/R_2)^2}$
always saturate WGC in their relevant direction in charge space.
These fill the whole charge lattice, so CHC is met by having a different state that's relevant for every direction in charge space.

Lattice Weak Gravity Conjecture

This leads us to a generalization that is infinitely stronger than the original WGC, the Lattice Weak Gravity Conjecture:

For **any allowed charge** $\vec{Q}$ in the charge lattice, there is a particle with that charge and with a mass *m* that is smaller than the mass of a semiclassical extremal black hole with charge proportional to $\vec{Q}$.

For Reissner-Nordstrom this means there is always a state with $m < \left|\vec{Q}\right| M_{\mathrm{Pl}}$


Evidence for the
Lattice WGC
We have been accumulating evidence in 3 general directions. Each is encouraging but has some subtleties.
Perturbative strings. Checked LWGC in many supersymmetric heterotic toroidal orbifolds. (Non-SUSY counterexamples, but no stable vacuum.)
AdS/CFT. Translate to a CFT statement (see Nakayama/Nomura), but corrections due to LAdS.
Semiclassical BH with higher-dim operators. Superextremality as a positivity bound. Some hints it relates to analyticity/causality, but not yet rigorous

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Lattice WGC: Directions

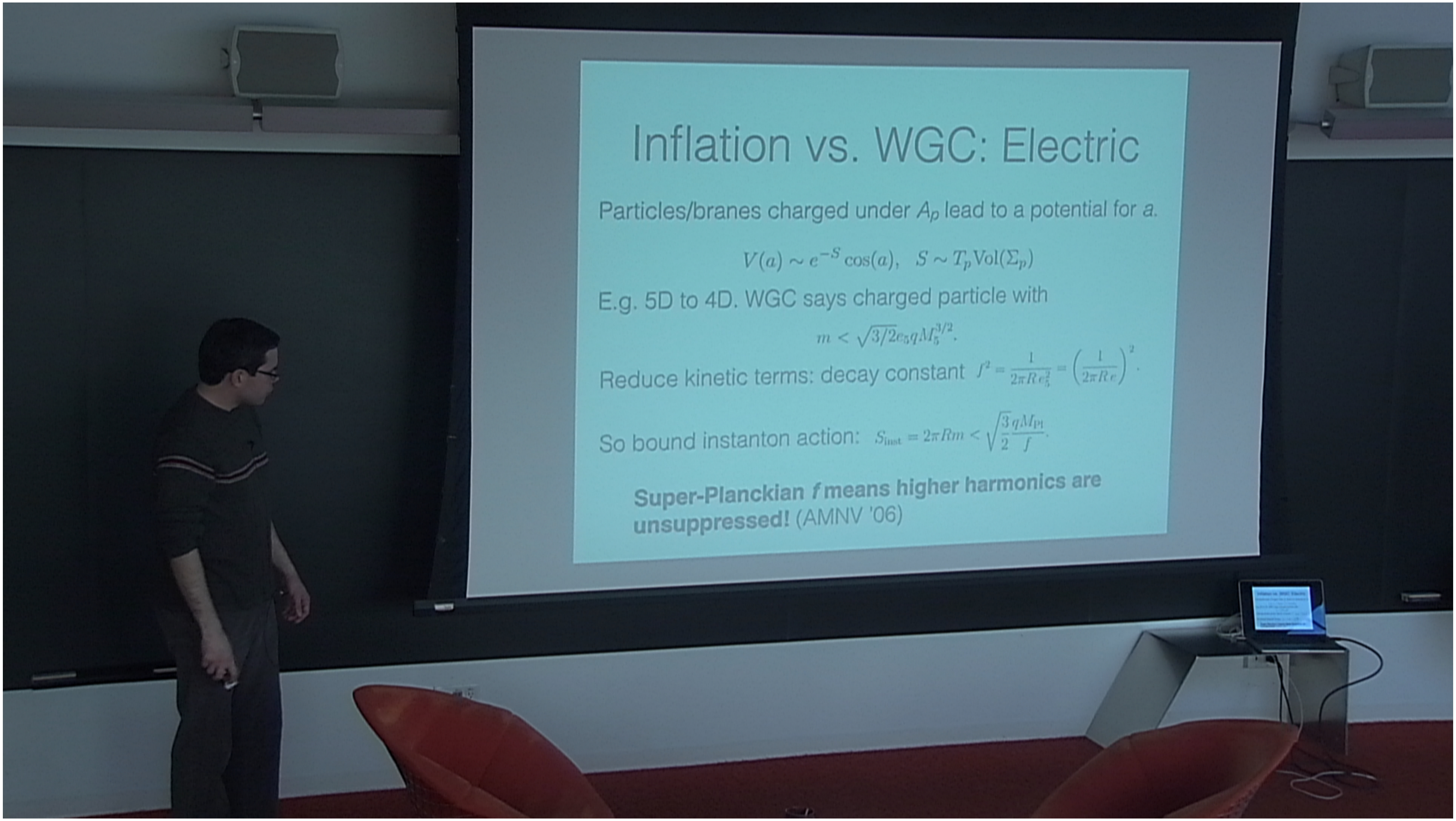
The Lattice WGC extends in a natural way to nonabelian groups (as we saw with SO(32) by dimensionally reducing). It suggests that **particles exist in all representations of the gauge group**, with masses obeying a WGC-like bound.

If we have gauge fields localized to defects—like the U(1) gauge field on a D3 brane in string theory, say—we must dimensionally reduce to apply the logic of the lattice WGC. In the parent theory it requires **strings ending on the brane.**

The Lattice WGC calls for infinite towers of particles. **Very suggestive of stringy physics** emerging from general principles.

Inflation and the WGC

(mostly *not* relying on the Lattice WGC because this work came first—still interesting work to do revisiting inflation questions with Lattice WGC)


Inflation vs. WGC: Electric
Particles/branes charged under A_p lead to a potential for a.
$V(a) \sim e^{-S}\cos(a), \quad S \sim T_p \mathrm{Vol}(\Sigma_p)$
E.g. 5D to 4D. WGC says charged particle with
$m < \sqrt{3/2}\, e_5 q M_5^{3/2}$.
Reduce kinetic terms: decay constant $f^2 = \frac{1}{2\pi R e_5^2} = \left(\frac{1}{2\pi R e}\right)^2$.
So bound instanton action: $S_{\rm inst} = 2\pi R m < \sqrt{\frac{3}{2}}\frac{q M_{\rm Pl}}{f}$.
Super-Planckian *f* means higher harmonics are unsuppressed! (AMNV '06)

Further Arguments

The preceding argument had a big assumption: **a simple kinetic term in the basis where magnetic charges satisfying magnetic WGC are simple.**

We can relax this assumption, considering $-\frac{1}{4}K_{ij}F^{i}_{\mu\nu}F^{j\mu\nu}$.

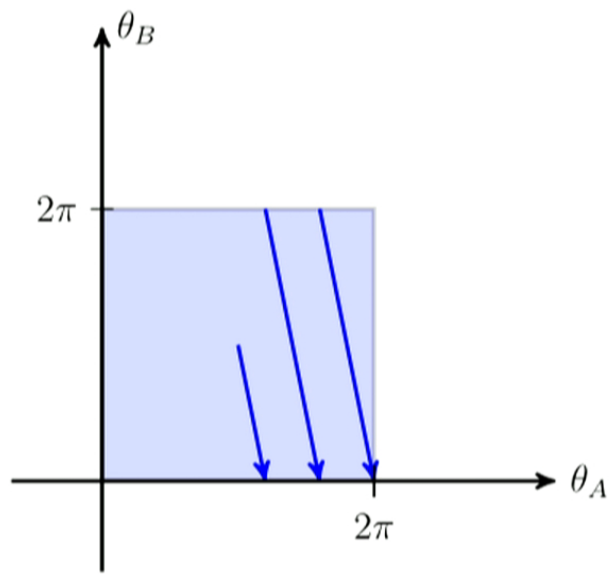
We find that we can still exclude super-Planckian effective decay constants, **provided we assume the electric charges contributing the dominant terms in the potential are simple in the same basis the magnetic charges are simple in.**

The Best Case

If magnetic and electric charges are **not simultaneously simple**, can evade WGC.

de la Fuente, Saraswat, Sundrum:

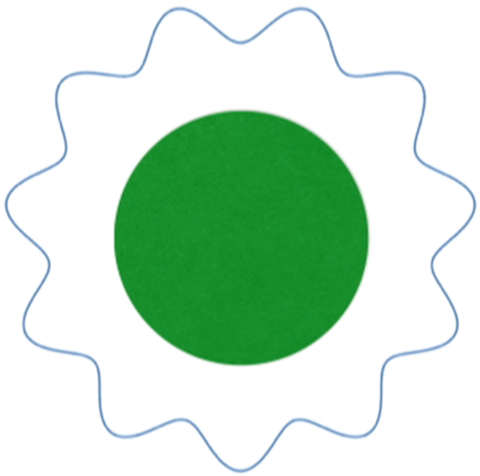
- two axions, *A* and *B,* from 5D gauge fields
- no kinetic mixing
- electrically charged particles (1,0) and (N,1)
- assume $f_A < M_{Pl}$, $f_B < M_{Pl}$



i.e. *implicitly* asumes magnetic WGC with (1,0) and (0,1)

Monopoles and Cutoffs

This example satisfies minimal versions of WGC. But it has strange features. The electrically charged particle of charge (N,1) and the magnetic monopole of charge (1,0) have a **nonminimal Dirac quantization**—especially at large N.



Is the classical monopole radius really the right cutoff?

$$\vec{L} = \int d^3\vec{r}\,\vec{r} \times \vec{E} \times \vec{B} \sim N$$

Charged particle wavefunction probes length scales shorter by *N*.

Lesson

The case that is hardest to exclude with the Weak Gravity Conjecture is one that involves **venturing far enough out in moduli space** that modes can descend from the cutoff to become light.

Is this a problem? To me, it suggests we might not trust the effective field theory. But many theorists strongly disagree with this—so we need better arguments that are conclusive one way or the other.

I think that charge/monopole scattering has the potential to provide such an argument.

Conclusions

Large-field inflation is a setting where a potential near-future experimental measurement—a large tensor-to-scalar ratio—could raise deep questions about quantum gravity.

The Weak Gravity Conjecture is a compelling criterion, which may be satisfied by all quantum gravity theories, that leads to at least *some* tension with large-field inflation.

We're still trying to find the sharpest, most convincing arguments for the WGC and how it relates to inflation. Much more to do.