

Title: PSI 2015/2016 Explorations in String Theory - Pedro Vieira - 6

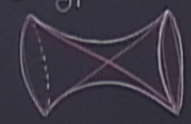
Date: Mar 29, 2016 11:30 AM

URL: <http://pirsa.org/16030069>

Abstract:



an hyperboloid



$$\vec{Y}^2 = -Y_{-1}^2 - Y_0^2 + Y_1^2 + \dots + Y_{d-1}^2 = -1$$

$$\partial \text{AdS} = \{ \text{light rays } \vec{X} \sim \lambda \vec{X}, \vec{X}^2 = 0 \} \equiv \int_{\mathbb{R}^{1,d-2}} \lambda^2(x) ds^2_{\mathbb{R}^{1,d-2}}$$

AdS_d is...

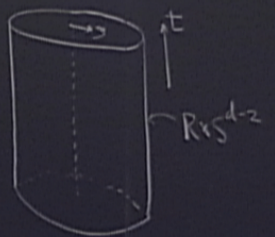
... A stack of flat spaces

$$ds^2 = \frac{1}{z^2} (dz^2 + dx^i dx_i)$$



... A box

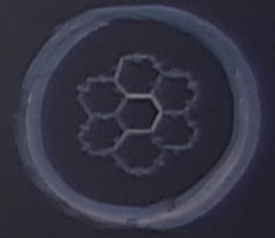
$$ds^2 = -\cos^2 \rho dt^2 + \sinh^2 \rho d\Omega_{d-2}^2 + d\rho^2$$



Isometry group of AdS_d

||| Lorentz transf on $\mathbb{R}^{2,d}$

||| Conformal Transformations on $\mathbb{R}^{1,d}$

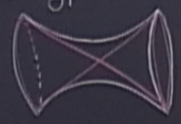


$$E_{\text{STRINGS IN GLOBAL AdS}_d} = E_{\text{YIT STATES ON } \mathbb{R}^{1,d-2}} = \Delta_{\text{YIT OPS ON } \mathbb{R}^{d-1}}$$



AdS_d is...

... an hyperboloid $\vec{Y}^2 = -Y_{-1}^2 - Y_0^2 + Y_1^2 + \dots + Y_{d-1}^2 = -1$

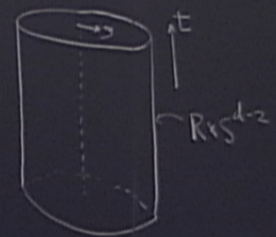


$\partial\text{AdS} = \{ \text{light rays } \vec{X} \sim \lambda \vec{X}, \vec{X}^2 = 0 \} \equiv \lambda^2(x) ds^2_{\mathbb{R}^{1,d-2}}$

... A stack of flat spaces $ds^2 = \frac{1}{z^2} (dz^2 + dx^i dx_i)$



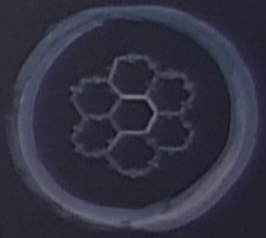
... A box $ds^2 = -\cosh^2 g dt^2 + \sinh^2 g d\Omega_{d-2}^2 + dg^2$



Isometry group of AdS_d

||| Lorentz transf on $\mathbb{R}^{1,d}$

||| Conformal Transformations on $\mathbb{R}^{1,d}$



$$E_{\text{STRINGS IN GLOBAL } \text{AdS}_d} = E_{\text{YIE STATES ON } \mathbb{R}^{1, d-2}} = \triangleleft \text{YIE OPS ON } \mathbb{R}^{1, d-1}$$

CFT

$$\langle \mathcal{O}(x_1) \rangle$$

↑ scalar primary of dim Δ

Lorentz $\mathbb{R}^{2,d}$ →

$$\begin{cases} [D, \mathcal{O}] = i\Delta \mathcal{O} \\ [K, D] = iK \\ [P, D] = -iP \end{cases}$$

if \mathcal{O} has dim Δ

$$\begin{aligned} \partial \mathcal{O} &\equiv \mathcal{P} \mathcal{O} \quad \text{has dim } \Delta + 1 \\ K \mathcal{O} &\quad \text{has dim } \Delta - 1 \end{aligned}$$

Δ bounded $\Rightarrow \mathcal{O}$ { Primary $K \mathcal{O} =$

FTT

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \dots \rangle_{\Omega_2(x) \mathcal{g}} = \langle \mathcal{O}_1(x_1) \dots \rangle_{\mathcal{g}}$$

↑ scalar primary of dim Δ_1

$$\begin{cases} [D, \mathcal{O}] = i\Delta \mathcal{O} \\ [K, \mathcal{O}] = iK \\ [P, \mathcal{O}] = -iP \end{cases}$$

if \mathcal{O} has dim Δ
 $\partial \mathcal{O} \equiv \mathcal{P} \mathcal{O}$ has dim $\Delta+1$
 $K \mathcal{O}$ has dim $\Delta-1$

Δ bounded $\Rightarrow \mathcal{O} \begin{cases} \text{Primary } K \mathcal{O} = 0 \\ \text{Descendants } \mathcal{O} = \mathcal{P}_\mu \dots \end{cases}$

Particular case: $x^\mu \rightarrow \tilde{x}^\mu$ such that

$$d\tilde{x}^\mu d\tilde{x}_\mu = \Omega^2(x) dx_\mu dx^\mu$$

Particular case: $x^\mu \rightarrow \tilde{x}^\mu$ such that

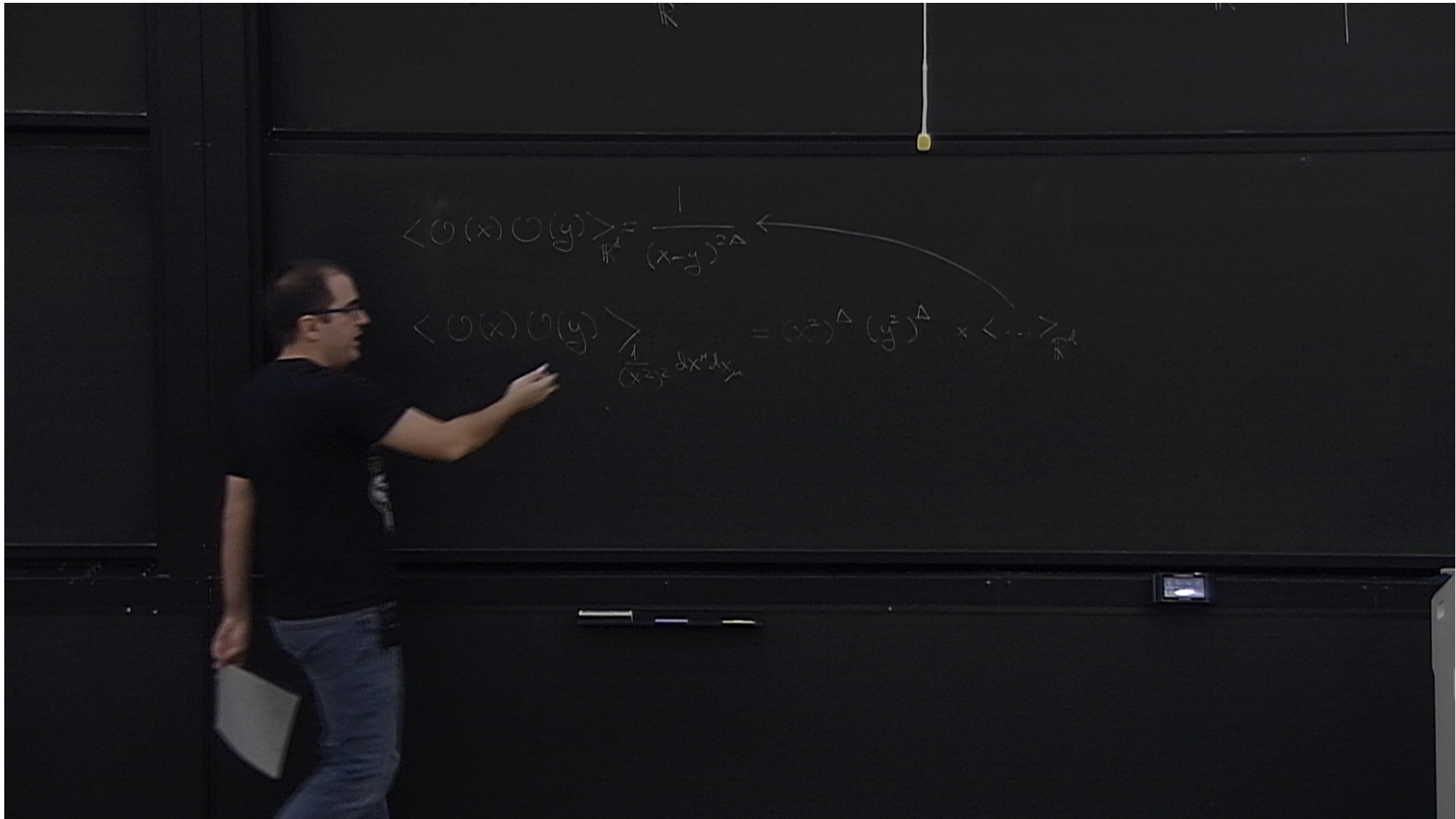
$$d\tilde{x}^\mu d\tilde{x}_\mu = \Omega^2(x) dx^\mu dx_\mu$$

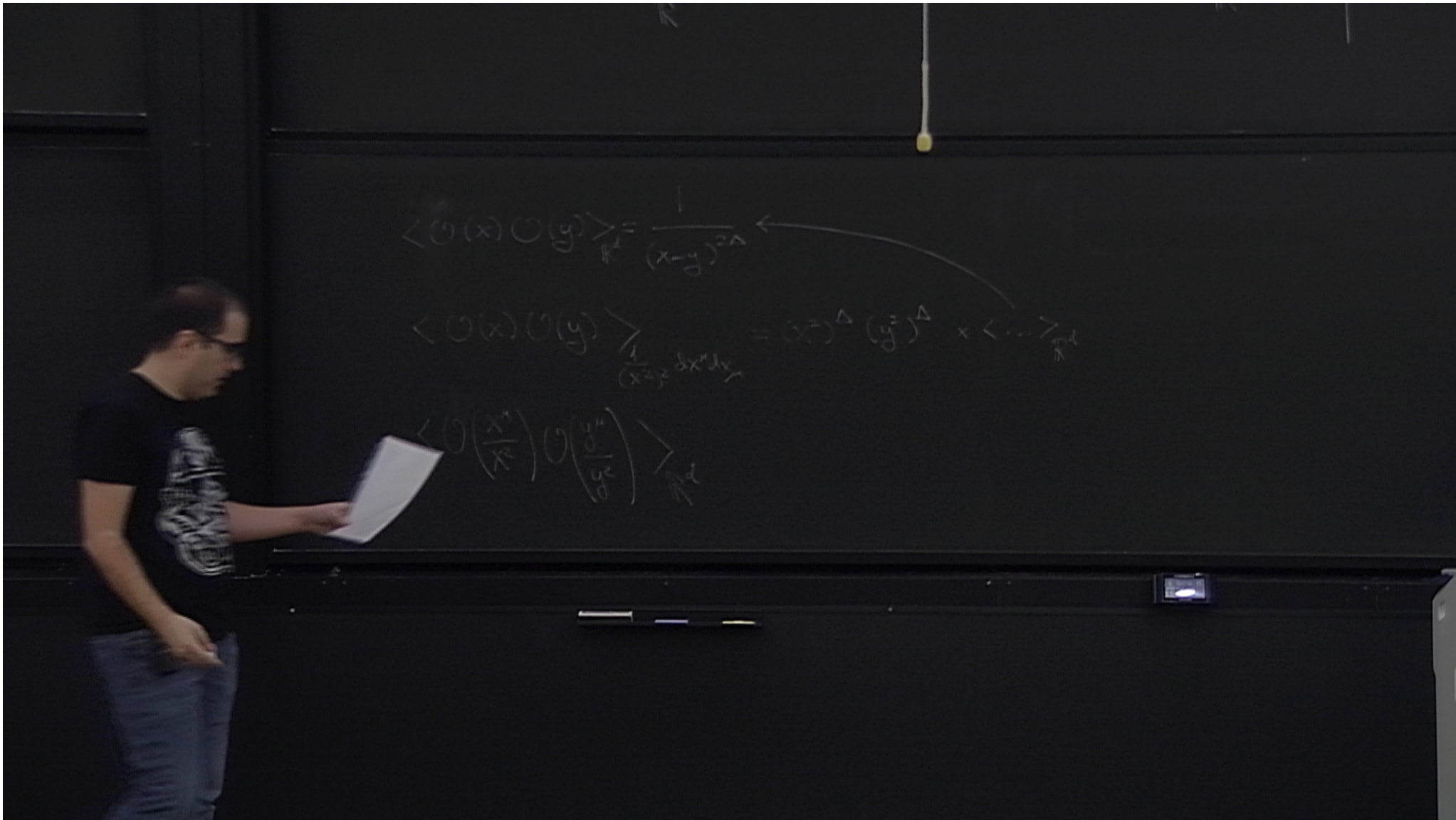
$$\downarrow$$

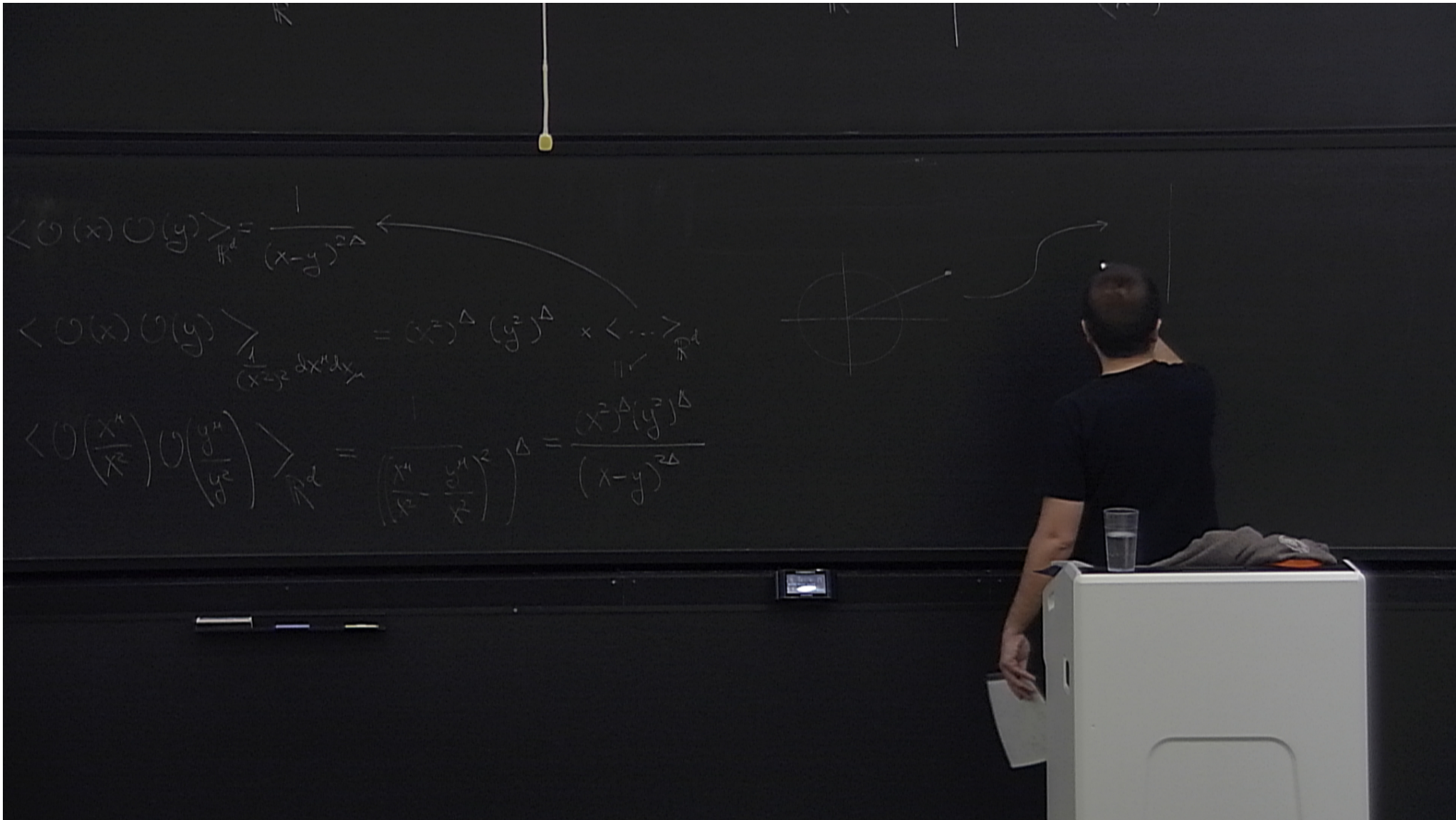
$$\langle \Omega_1(\tilde{x}_1) \dots \rangle_{\mathbb{R}^d} = \Omega^{-\Delta_1}(x) \dots \langle \Omega_1(x) \dots \rangle_{\mathbb{R}^d}$$

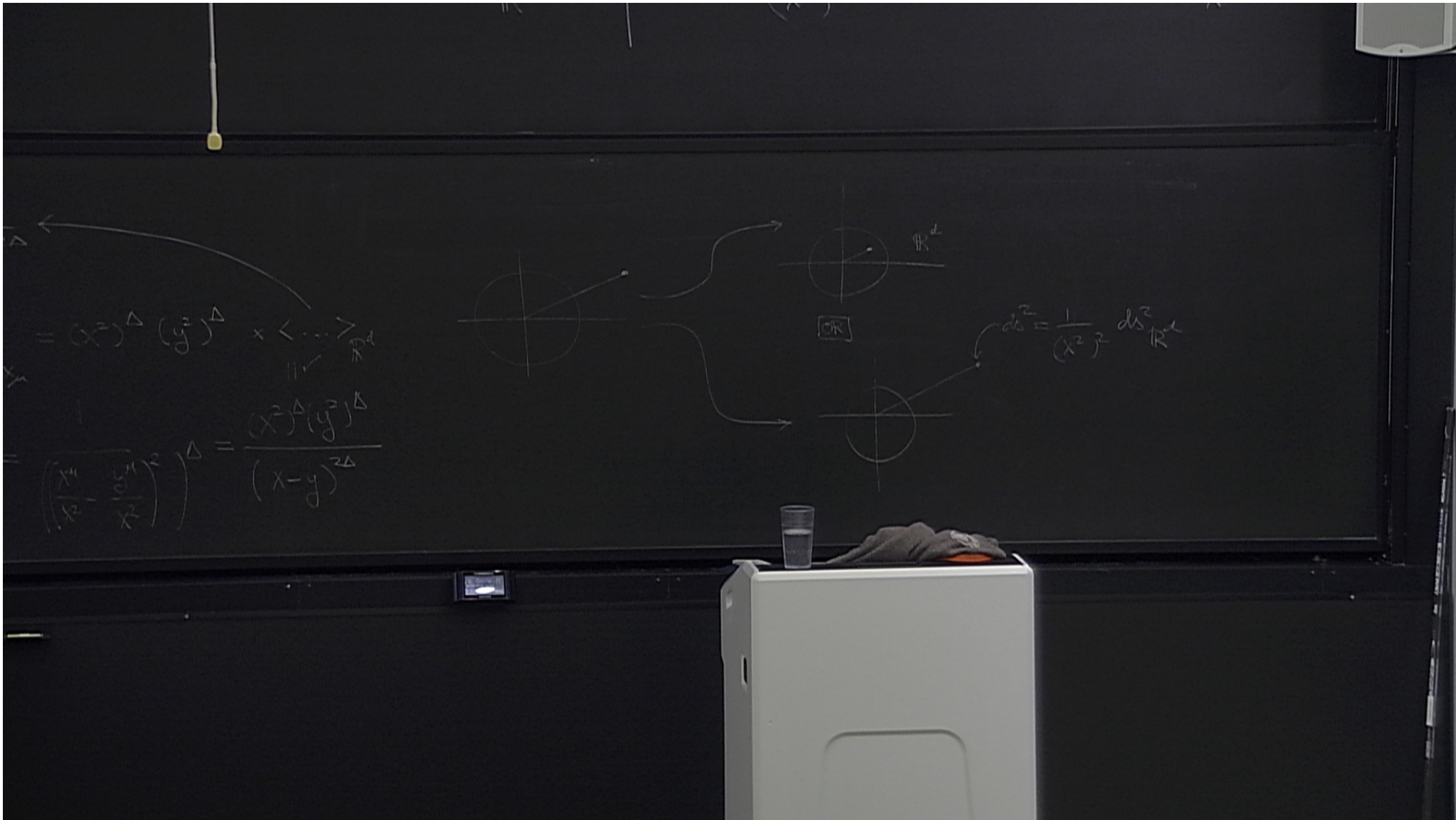
Example inversion

$$\mu \rightarrow \frac{x^\mu}{x}$$









$\rightarrow \begin{cases} [K, D] = iK \\ [P, D] = -iP \end{cases}$

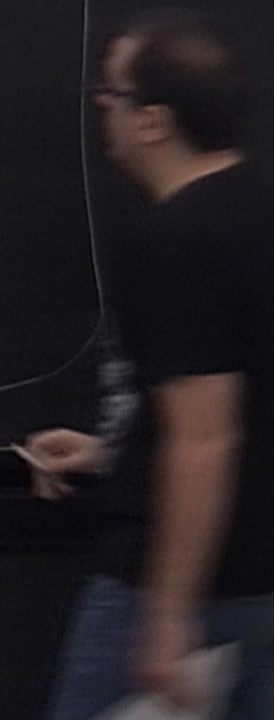
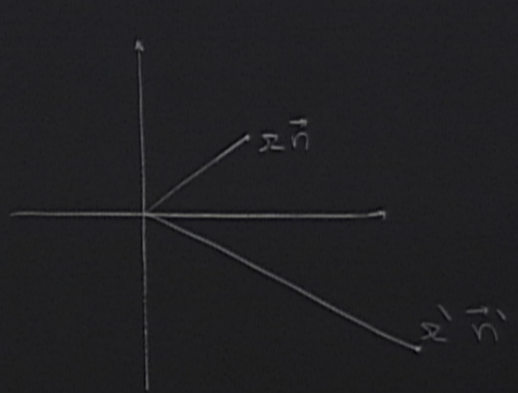
$\partial\mathcal{V} \equiv \mathcal{P}(\mathcal{V})$ has $\dim \Delta + 1$ Δ bounded $\Rightarrow \mathcal{V}$

$K\mathcal{V}$ has $\dim \Delta - 1$

Descendants $\mathcal{V} = \mathcal{P}_{A_1} \dots \mathcal{P}_{A_n} \mathcal{V}_{\text{PRIMARY}}$

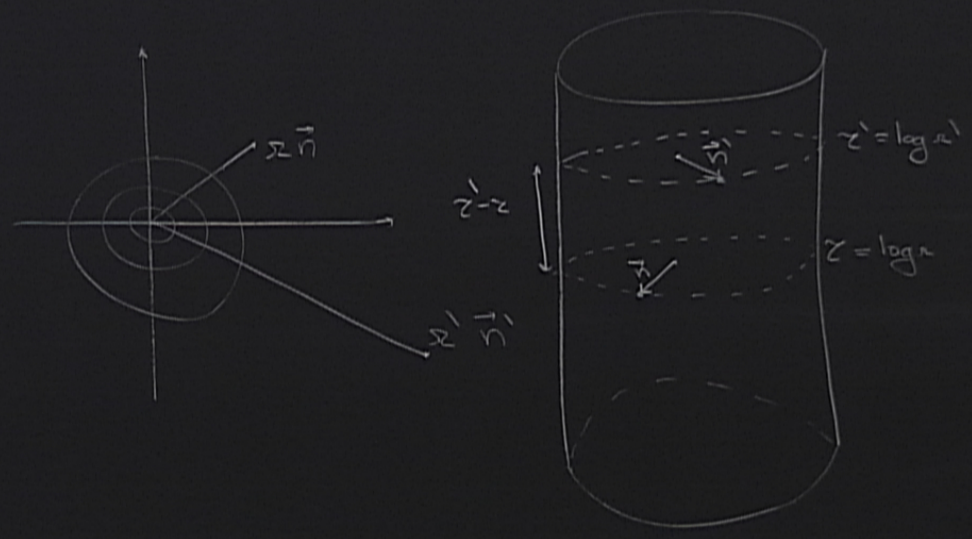
$$\begin{aligned}
 ds^2_{\mathbb{R}^d} &= dr^2 + r^2 d\Omega_{d-1}^2 \\
 &= e^{2z} \left(dz^2 + d\Omega_{d-1}^2 \right) \quad r = e^z \\
 &\quad \underbrace{\hspace{10em}}_{\Omega^2}
 \end{aligned}$$

$$ds^2_{\mathbb{R}^d} = \Omega^2 \times ds^2_{\text{cylinder}}$$



Δ bounded \Rightarrow \mathcal{O} $\left\{ \begin{array}{l} \text{has dim } \Delta+1 \\ \text{has dim } \Delta-1 \end{array} \right.$ Descendants $\mathcal{O} = P_{\mu_1} \dots P_{\mu_n} \mathcal{O}_{\text{PRIMARY}}$

$z = e^z$



$\langle \mathcal{O} \mathcal{O} \rangle_{\mathbb{R}^d} = \langle \mathcal{O} \mathcal{O} \rangle_{\text{cylinder}}$

$\langle \mathcal{O}(\vec{n}, z) \mathcal{O}(\vec{n}, z') \rangle_{\text{cylinder}}$

\parallel
 $e^{\Delta z} e^{\Delta z'}$
 $\left(\vec{n} e^z - \vec{n}' e^{z'} \right)^2 \Delta$

$\partial\Omega \equiv \mathbb{P}\Omega$ has $\dim \Delta + 1$ Δ bounded $\Rightarrow \emptyset$
 $K\Omega$ has $\dim \Delta - 1$

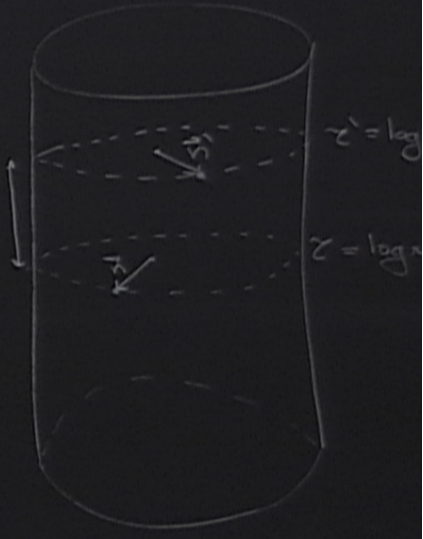
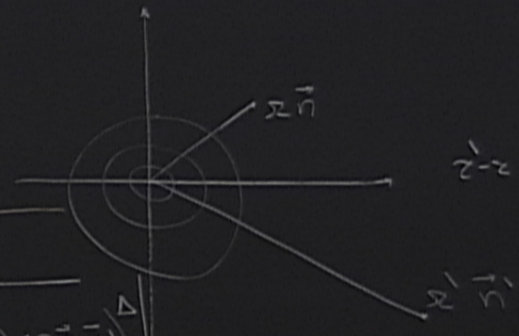
$\left\{ \begin{array}{l} [K, D] = iK \\ [P, D] = -iP \end{array} \right.$ Descendants $\mathcal{O} = P_{\mu_1} \dots P_{\mu_n} \mathcal{O}_{PRIM}$

$$\begin{aligned}
 ds^2_{\mathbb{R}^d} &= dr^2 + r^2 d\Omega_{S^{d-1}}^2 \\
 &= e^{2z} (dz^2 + d\Omega_{S^{d-1}}^2)
 \end{aligned}$$

$$r = e^z$$

$$ds^2_{\mathbb{R}^d} = \Omega^2 \times ds^2_{\text{cylinder}}$$

$$\langle \mathcal{O} \mathcal{O}' \rangle_{\text{cylinder}} = \frac{1}{(2 \cosh(z' - z) + 2\vec{n} \cdot \vec{n}')^\Delta}$$



$$\langle U_1(x) U_2(y) \rangle_{\Omega_2 g} = \Omega_1^{-\Delta_1}(x) \Omega_2^{-\Delta_2}(y) \langle U_1(x) U_2(y) \rangle_g$$

$$\langle \psi_1(x) \psi_2(y) \rangle = \Omega_1^{-\Delta_1}(x) \Omega_2^{-\Delta_2}(y) \langle \psi_1(x) \psi_2(y) \rangle$$

$\Omega_1^{-\Delta_1}(x)$ and $\Omega_2^{-\Delta_2}(y)$ are labeled with \mathbb{R}^d and \mathbb{R}^d respectively.

$\Rightarrow \Delta_1 = \Delta_2$ or $\langle \psi_1 \psi_2 \rangle = 0$

A diagram shows a horizontal line with a vertical line intersecting it at a point labeled $x \leftrightarrow y$. A right-angle symbol is drawn at the intersection, with the text "rot 180°" written below it.



$$\langle \psi_1(x) \psi_2(y) \rangle = \Omega_1^{-\Delta_1}(x) \Omega_2^{-\Delta_2}(y) \langle \psi_1(x) \psi_2(y) \rangle$$

$\Omega_2^{-\Delta_2}(y) \xrightarrow{R^2} \Omega_1^{-\Delta_1}(x)$ $\Omega_1^{-\Delta_1}(x) \xrightarrow{R^2} \Omega_2^{-\Delta_2}(y)$

$\xrightarrow{180^\circ} \quad | \quad x \leftrightarrow y \quad \Rightarrow \quad \Delta_1 = \Delta_2 \quad \text{or} \quad \langle \psi_1 \psi_2 \rangle = 0$

$$\Delta_1 = \Delta_2$$

<

$$\begin{aligned}
 \langle \psi_1(x) \psi_2(y) \rangle &= \Omega_1^{-\Delta_1}(x) \Omega_2^{-\Delta_2}(y) \langle \psi_1(x) \psi_2(y) \rangle \\
 &\stackrel{\text{rot } 180^\circ}{=} \Omega_2^{-\Delta_2}(y) \Omega_1^{-\Delta_1}(x) \langle \psi_2(y) \psi_1(x) \rangle \\
 &\stackrel{x \leftrightarrow y}{=} \Omega_1^{-\Delta_1}(y) \Omega_2^{-\Delta_2}(x) \langle \psi_1(y) \psi_2(x) \rangle \\
 &\Rightarrow \Delta_1 = \Delta_2 \quad \text{or} \quad \langle \psi_1, \psi_2 \rangle = 0
 \end{aligned}$$

$$\begin{aligned}
 \langle \psi_1(x) \psi_2(y) \rangle &\sim \lambda^{-2\Delta} \langle \psi_1(x) \psi_2(y) \rangle \\
 &\stackrel{\lambda = \frac{1}{|x-y|}}{\Rightarrow} \langle \psi_1(x) \psi_2(y) \rangle = \frac{c_R}{|x-y|^{2\Delta}} \\
 &\stackrel{=}{=} G_R(|x-y|)
 \end{aligned}$$



$$\text{Lorentz } \mathbb{R}^{2,d} \rightarrow \begin{cases} [D, \mathcal{O}] = i\Delta \mathcal{O} \\ [K, D] = iK \\ [P, D] = -iP \end{cases}$$

if \mathcal{O} has $\dim \Delta$
 $\partial \mathcal{O} \equiv \mathcal{P} \mathcal{O}$ has $\dim \Delta + 1$
 $K \mathcal{O}$ has $\dim \Delta - 1$

Δ bounded $\Rightarrow \mathcal{O}$

Primary $K \mathcal{O} = 0$

Descendants $\mathcal{O} = \mathcal{P}_1 \dots \mathcal{P}_n \mathcal{O}$

3 pt

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(y) \mathcal{O}_k(z) \rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1 + \Delta_2 - \Delta_3} \times |x_2 - x_3|^{\Delta_2 + \Delta_3 - \Delta_1} \times |x_3 - x_1|^{\Delta_1 + \Delta_3 - \Delta_2}}$$

$\underbrace{\hspace{10em}}_{\mathcal{R}_{12}}$

Lorentz $R^{3,1}$

$$\begin{cases} [D, \mathcal{O}] = i\Delta \mathcal{O} \\ [K, D] = iK \\ [P, D] = -iP \end{cases}$$

if \mathcal{O} has dim Δ
 $\partial \mathcal{O} \equiv \mathcal{P} \mathcal{O}$ has dim $\Delta+1$
 $K \mathcal{O}$ has dim $\Delta-1$

Δ bounded $\Rightarrow \mathcal{O} \begin{cases} \text{Primary } K \mathcal{O} = 0 \\ \text{Descendants } \mathcal{O} = P_1 \dots P_n \mathcal{O}_{\text{PRIMARY}} \end{cases}$

3pt

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(y) \mathcal{O}_k(z) \rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1 + \Delta_2 - \Delta_3} \times |x_2 - x_3|^{\Delta_2 + \Delta_3 - \Delta_1} \times |x_3 - x_1|^{\Delta_1 + \Delta_3 - \Delta_2}}$$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ \Delta_1 & \Delta_2 & \Delta_3 \end{matrix}$

Lorentz $R_{z,t}$

$$\begin{cases} [D, \mathcal{O}] = i\Delta \mathcal{O} \\ [K, D] = iK \\ [P, D] = -iP \end{cases}$$

if \mathcal{O} has dim Δ

$$\partial \mathcal{O} \equiv \mathcal{P} \mathcal{O} \text{ has dim } \Delta + 1$$

$$K \mathcal{O} \text{ has dim } \Delta - 1$$

Δ bounded $\Rightarrow \mathcal{O}$

Primary $K \mathcal{O} = 0$

Descendants $\mathcal{O} = P_{\mu_1} \dots P_{\mu_n} \mathcal{O}_{\text{PRIMARY}}$

3 pt

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(y) \mathcal{O}_k(z) \rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1 + \Delta_2 - \Delta_3} \times |x_2 - x_3|^{\Delta_2 + \Delta_3 - \Delta_1} \times |x_3 - x_1|^{\Delta_1 + \Delta_3 - \Delta_2}}$$

$\mathcal{R}_{12}^2 = (x_1 - x_2)^2 \xrightarrow{\text{inversion}} \frac{(x_1 - x_2)^2}{x_1^2 x_2^2}$

$K_{\mu\nu} = I P_{\mu} I$

PI

$$\underline{z_{pt}}, \underline{z_{pt}} \leftarrow \{\Delta_j, C_{ijk}\}$$

4pt

$\langle U(x_i) \rangle$

$$\frac{1}{(x_1 - x_2)^{2\Delta} (x_3 - x_4)^{2\Delta}} \mathcal{F}(u, v)$$

$\Delta_i = \Delta$

$$\left\{ \begin{array}{l} u = \frac{(x_1 - x_2)^2 (x_3 - x_4)^2}{(x_1 - x_3)^2 (x_2 - x_4)^2} = \frac{x_1 x_2}{x_3 x_4} \\ v = u \Big|_{2 \leftrightarrow 4} \end{array} \right.$$