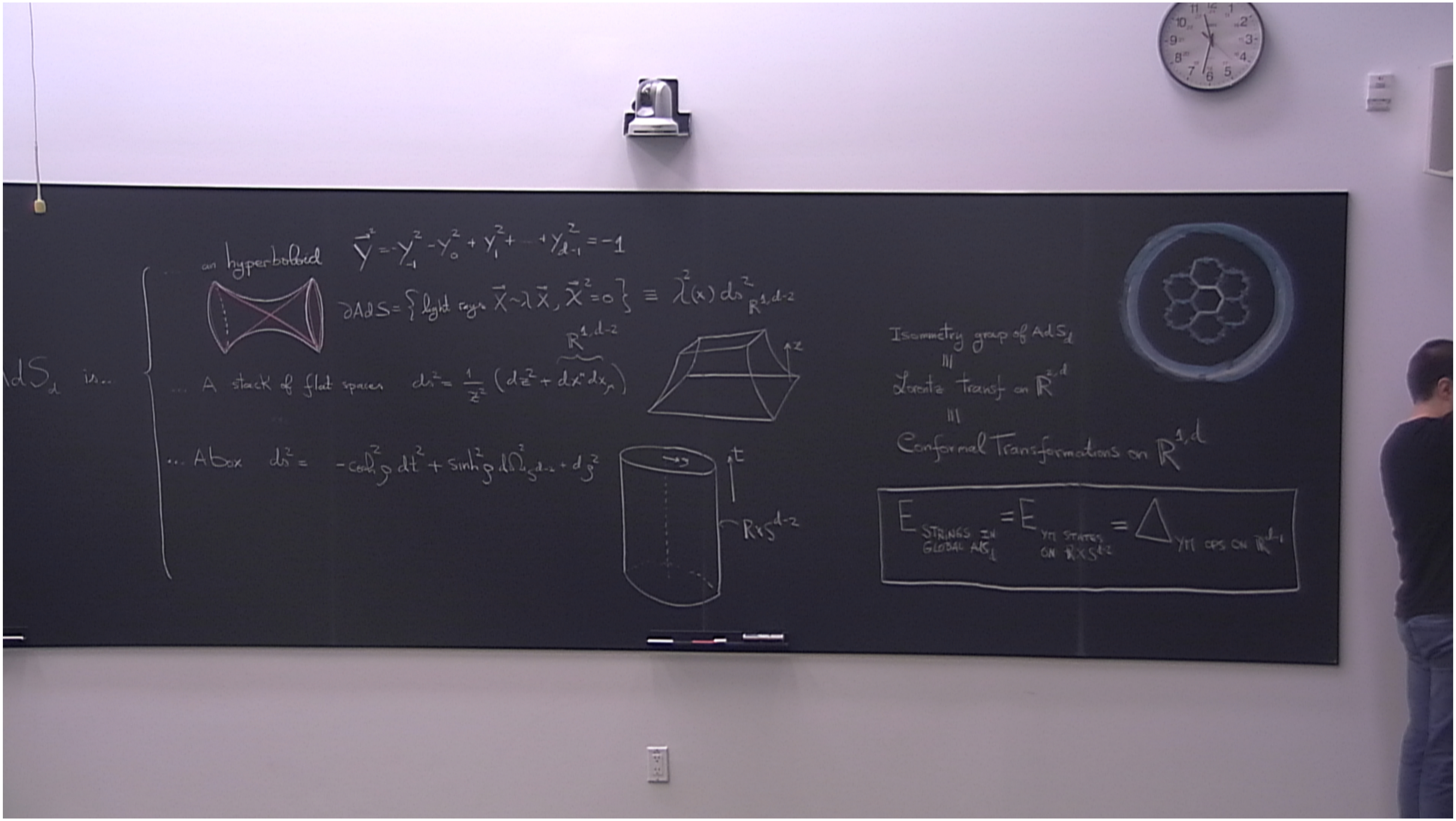
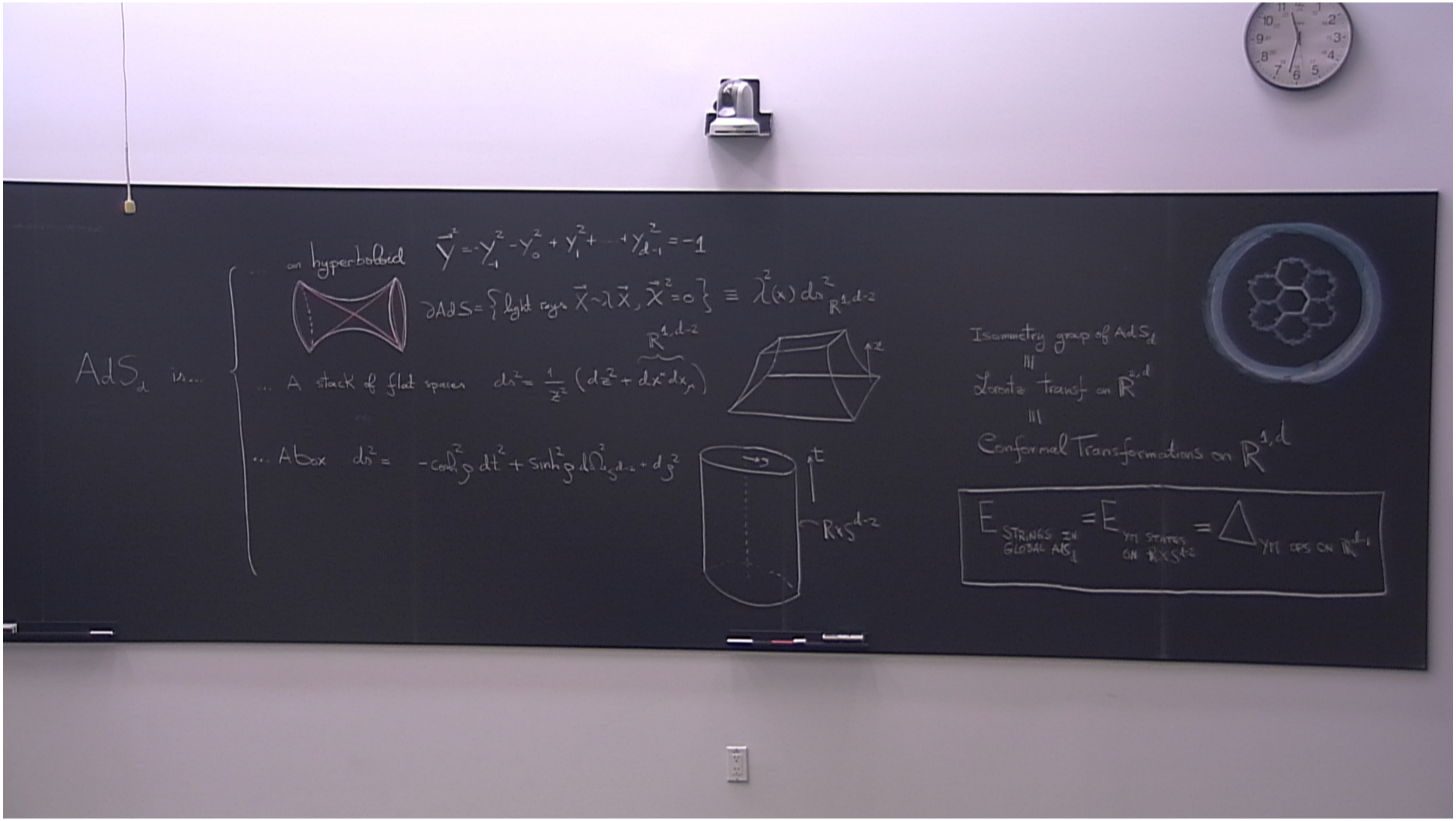
Title: PSI 2015/2016 Explorations in String Theory - Pedro Vieira - 6

Date: Mar 29, 2016 11:30 AM

URL: http://pirsa.org/16030069

Abstract:


AdS_d is...
... an hyperboloid $\vec{Y}^2 = -Y_{-1}^2 - Y_0^2 + Y_1^2 + \dots + Y_{d-1}^2 = -1$
$\partial AdS = \{$ light rays $\vec{X} \sim \lambda \vec{X}, \vec{X}^2 = 0 \} \equiv \lambda^2(x)\, ds^2_{\mathbb{R}^{1,d-2}}$
... A stack of flat spaces $ds^2 = \frac{1}{z^2}(dz^2 + \underbrace{dx^\mu dx_\mu}_{\mathbb{R}^{1,d-2}})$
z
... A box $ds^2 = -\cosh^2\rho\, dt^2 + \sinh^2\rho\, d\Omega^2_{S^{d-2}} + d\rho^2$
t
$\mathbb{R}\times S^{d-2}$
Isometry group of AdS_d
Lorentz transf on $\mathbb{R}^{2,d}$
Conformal Transformations on $\mathbb{R}^{1,d}$
$E_{\text{STRINGS IN GLOBAL AdS}_d} = E_{\text{YM STATES ON } \mathbb{R}\times S^{d-2}} = \Delta_{\text{YM OPS ON } \mathbb{R}^{d-1}}$


AdS$_d$ is...
... on hyperboloid $\vec{Y}^2 = -Y_{-1}^2 - Y_0^2 + Y_1^2 + \dots + Y_{d-1}^2 = -1$
$\partial AdS = \{$light rays $\vec{X} \sim \lambda \vec{X}, \vec{X}^2 = 0\} \equiv \lambda^2(x)\, ds^2_{R^{1,d-2}}$
... A stack of flat spaces $ds^2 = \frac{1}{z^2}(dz^2 + \underbrace{dx^\mu dx_\mu}_{R^{1,d-2}})$
z
... A box $ds^2 = -\cosh^2\rho\, dt^2 + \sinh^2\rho\, d\Omega^2_{S^{d-2}} + d\rho^2$
t
$R \times S^{d-2}$
Isometry group of AdS$_d$
$\parallel$
Lorentz transf on $R^{2,d}$
$\parallel$
Conformal Transformations on $R^{1,d}$
$E_{\text{STRINGS IN GLOBAL AdS}_d} = E_{\text{YM STATES ON } R\times S^{d-2}} = \Delta_{\text{YM OPS ON } R^{1,d-1}}$

$\mathbb{CFT}$

$\langle \mathcal{O}_1(x_1)$

scalar primary op of dim Δ_1

$[D, \mathcal{O}(0)] = i\Delta\mathcal{O}$

Lorentz $\mathbb{R}^{2,1}$ $\longrightarrow$ $\begin{cases} [K, D] = iK \\ [P, D] = -iP \end{cases}$

if $\mathcal{O}$ has dim Δ

$\partial\mathcal{O} \equiv P\mathcal{O}$ has dim $\Delta + 1$

$K\mathcal{O}$ has dim $\Delta - 1$

Δ bounded $\Rightarrow$ $\mathcal{O}$ $\begin{cases} \text{Primary } K\mathcal{O} = \end{cases}$

CFT

$$\langle O_1(x_1)\, O_2(x_2) \ldots \rangle_{\Omega^2(x)\, g} = \langle O_1(x_1) \ldots \rangle_g$$

O_1: scalar primary op of dim Δ_1

$[D, O(0)] = i\Delta O$

$[K, D] = iK$

$[P, D] = -iP$

if O has dim Δ

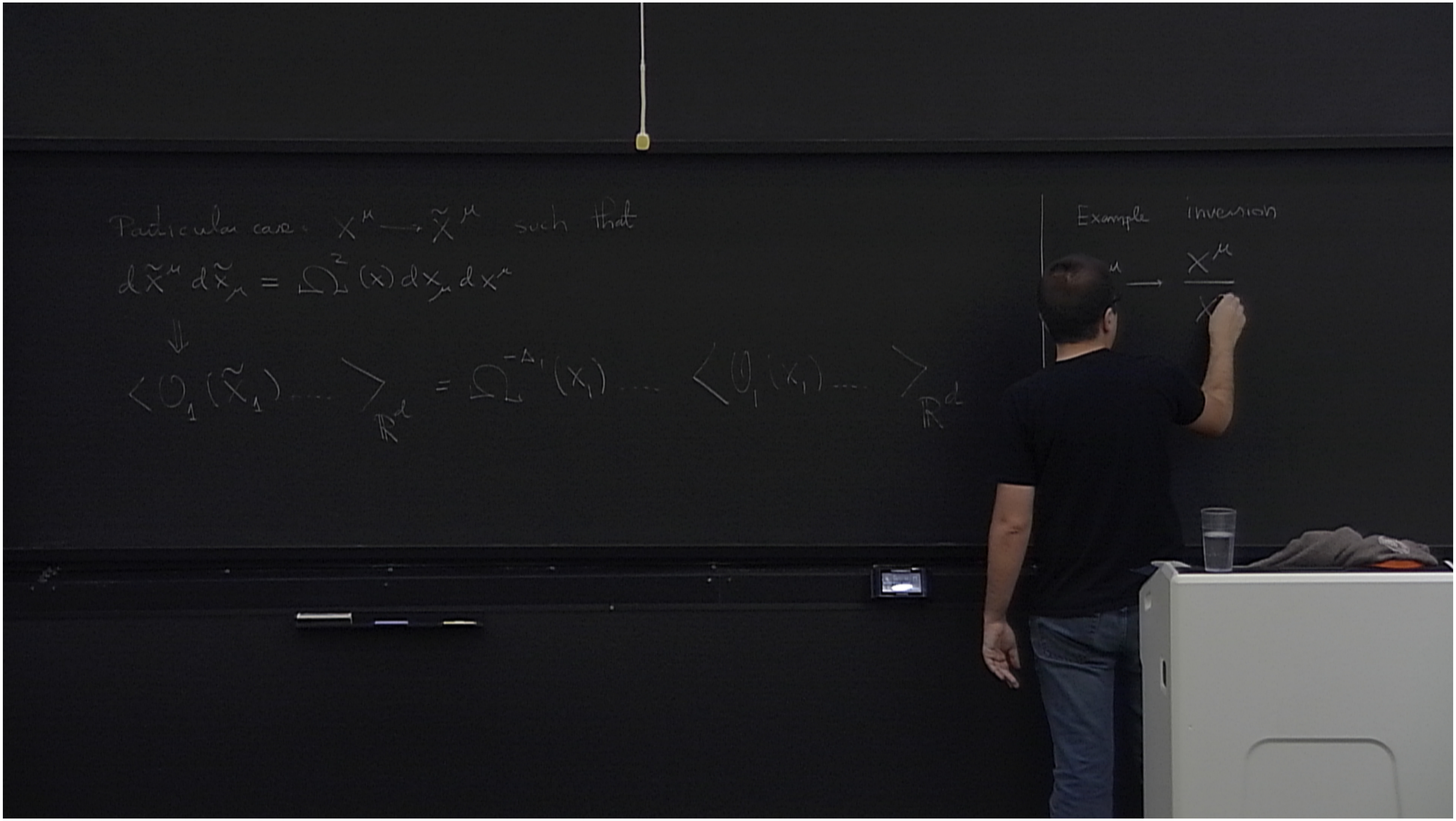
$\partial O = P\,O$ has dim $\Delta + 1$

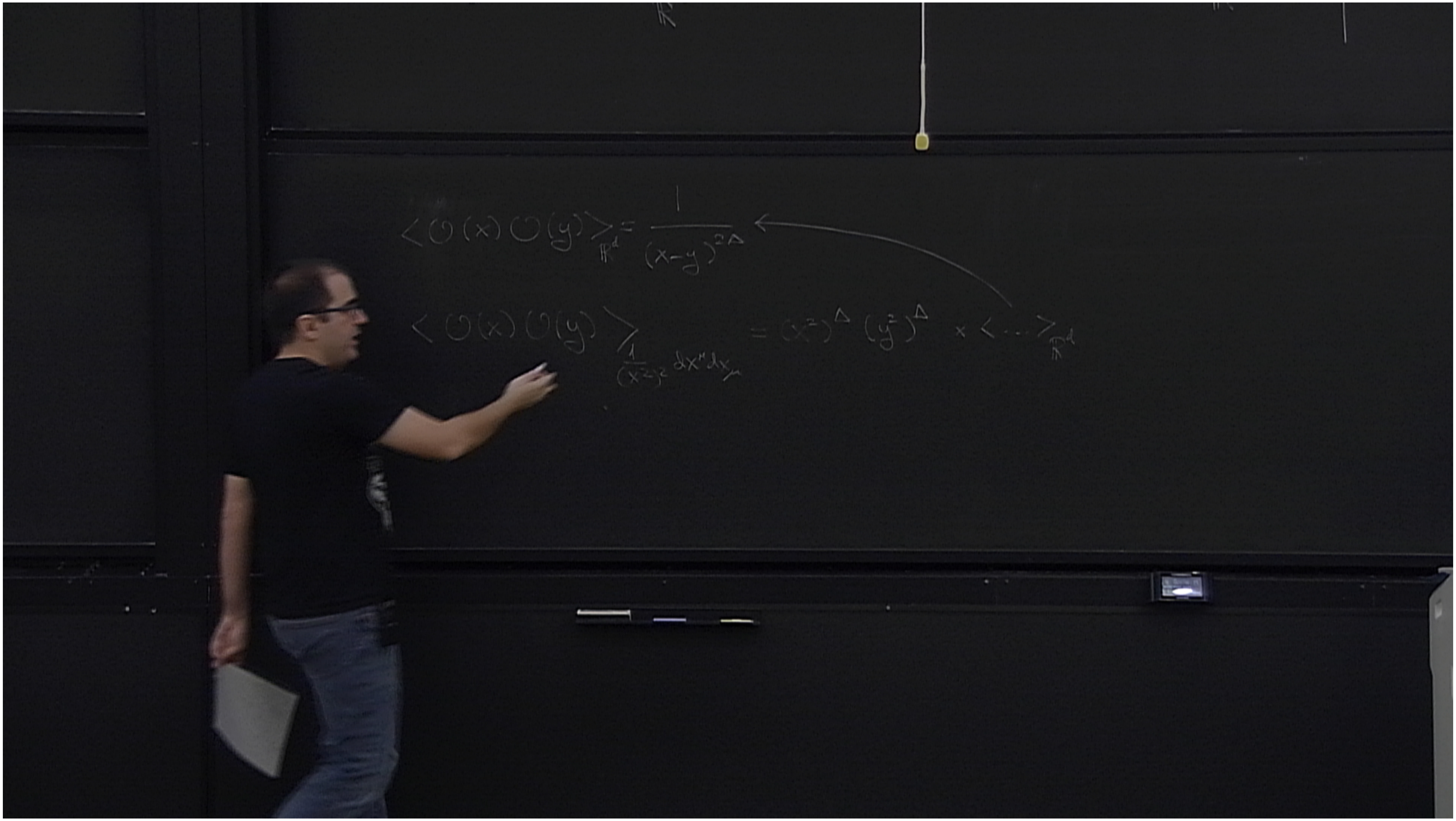
$K O$ has dim $\Delta - 1$

Δ bounded $\Rightarrow$ $O \begin{cases} \text{Primary} & K O = 0 \\ \text{Descendants} & O = P_{\mu_1} \ldots \end{cases}$

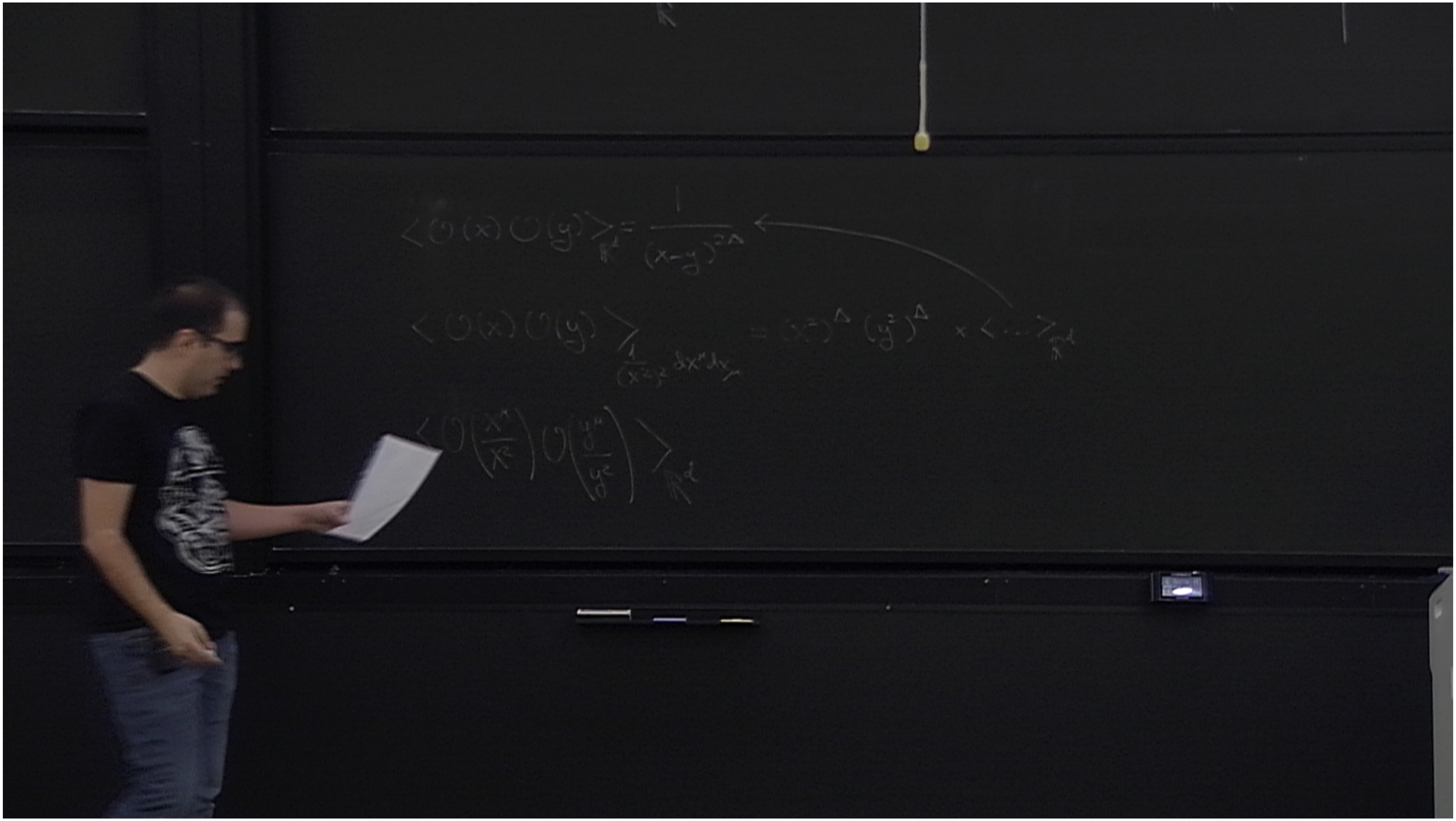
Particular case: $X^\mu \longrightarrow \tilde{X}^\mu$ such that

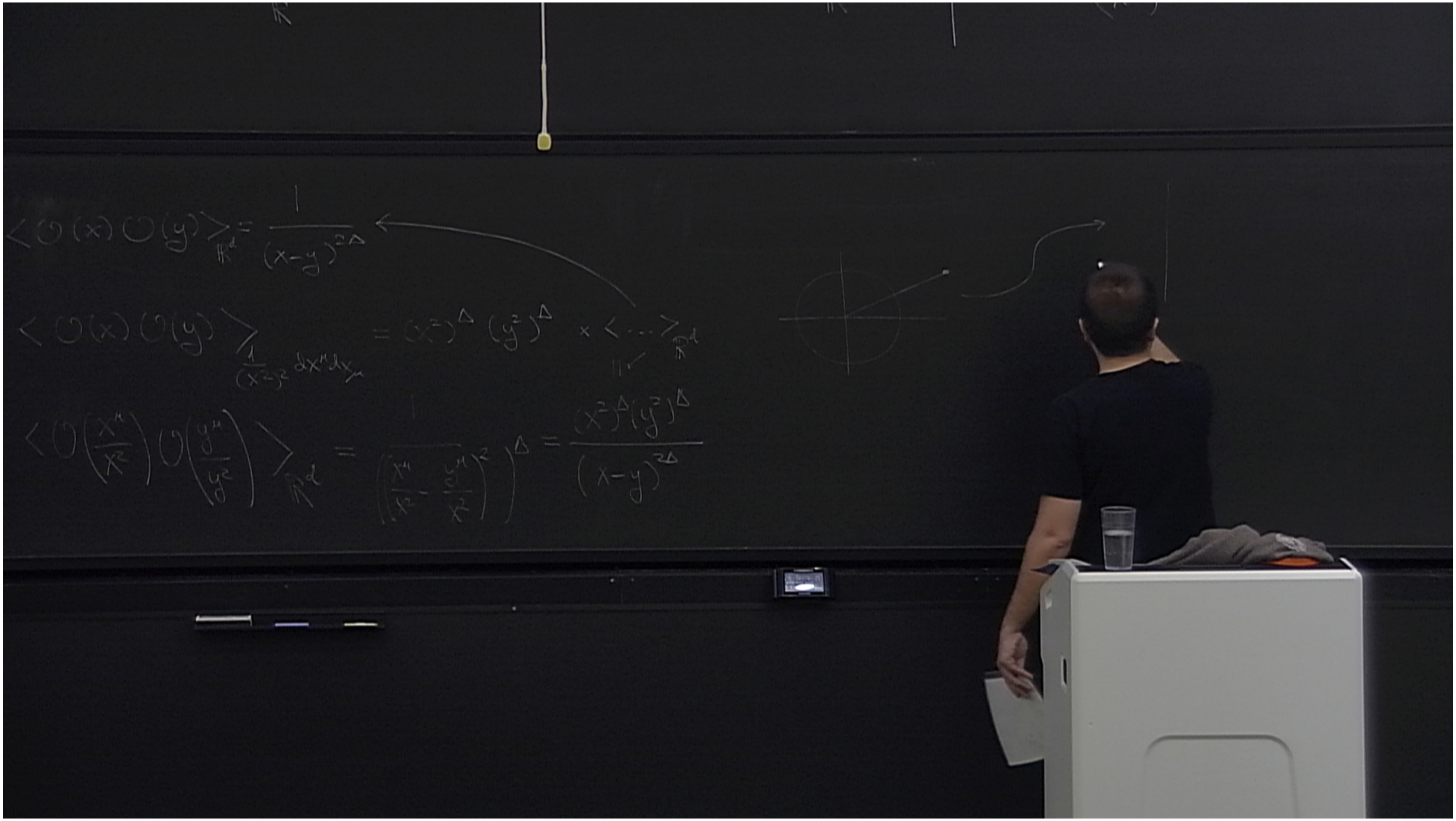
$$d\tilde{X}^\mu d\tilde{X}_\mu = \Omega^2(x)\, dx_\mu dx^\mu$$

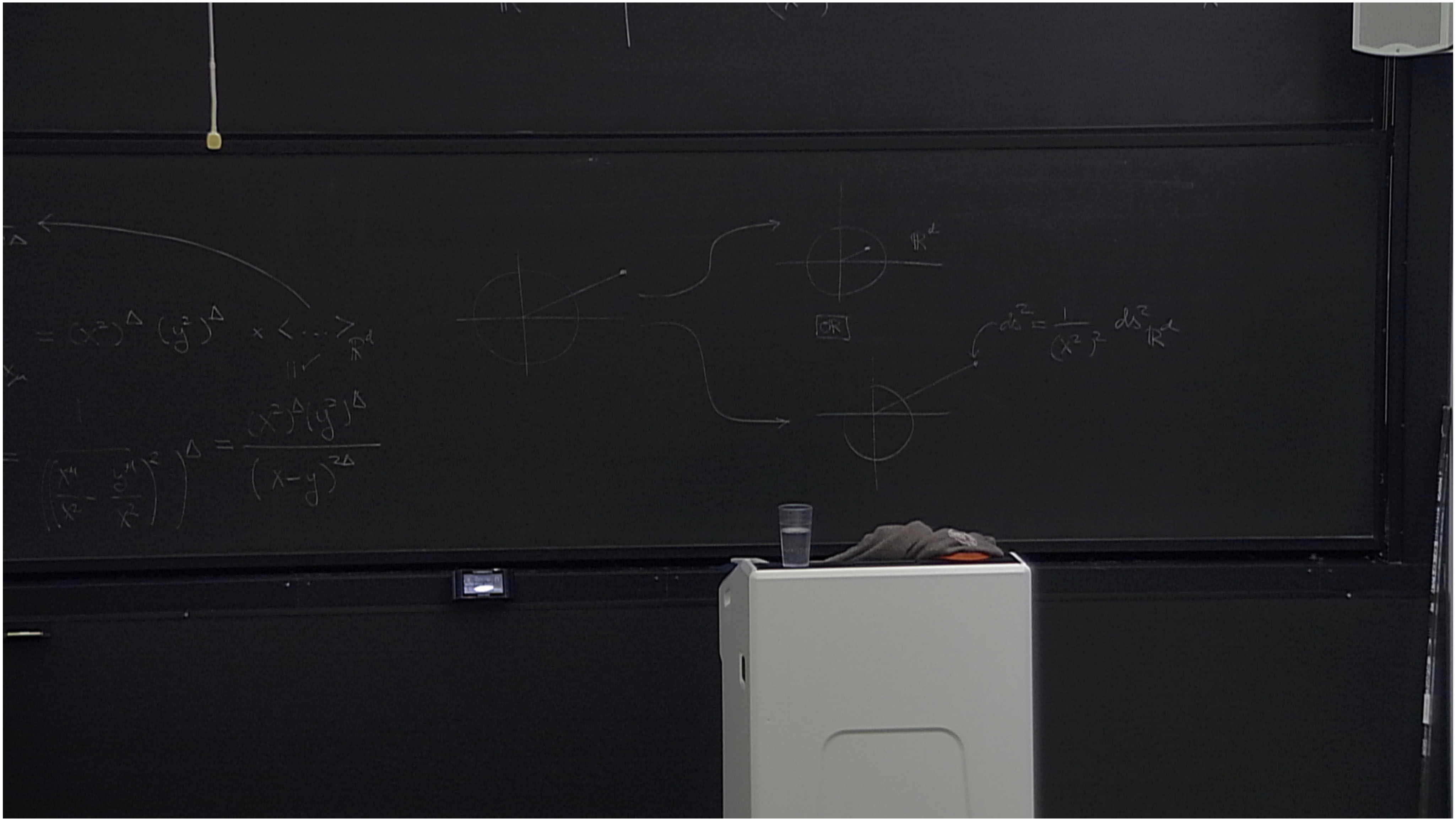

Particular case: $x^\mu \longrightarrow \tilde{x}^\mu$ such that
$d\tilde{x}^\mu d\tilde{x}_\mu = \Omega^2(x)\, dx_\mu dx^\mu$
$\Downarrow$
$\langle \mathcal{O}_1(\tilde{x}_1) \ldots \rangle_{\mathbb{R}^d} = \Omega^{-\Delta_1}(x_1) \ldots \langle \mathcal{O}_1(x_1) \ldots \rangle_{\mathbb{R}^d}$
Example inversion
$x^\mu \longrightarrow \frac{x^\mu}{x^2}$

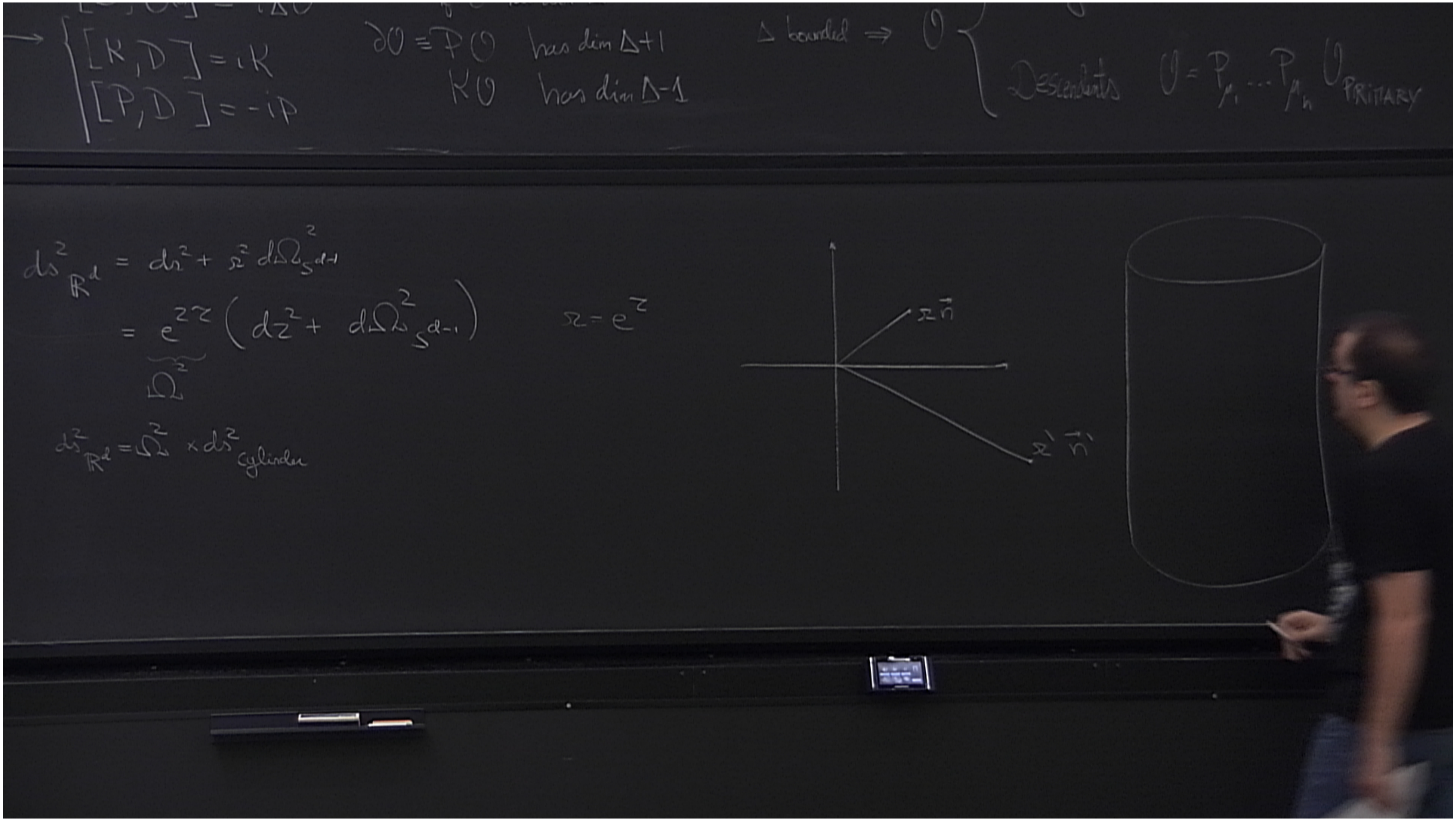


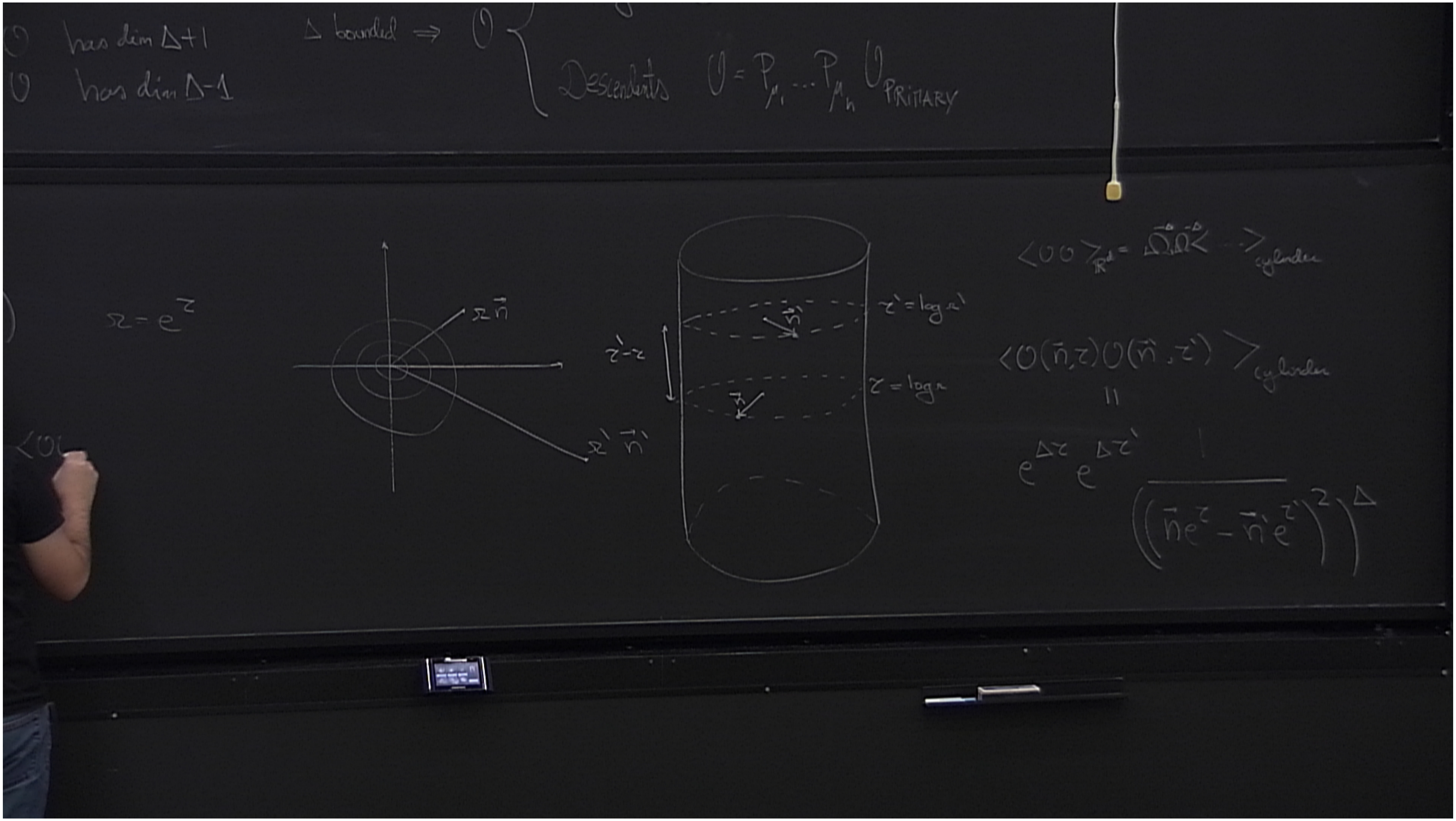
$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle_{\mathbb{R}^d} = \frac{1}{(x-y)^{2\Delta}}$
$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle_{\frac{1}{(x^2)^2}dx^\mu dx_\mu} = (x^2)^\Delta (y^2)^\Delta \times \langle \ldots \rangle_{\mathbb{R}^d}$


$\langle O(x) O(y) \rangle_{\mathbb{R}^d} = \frac{1}{(x-y)^{2\Delta}}$

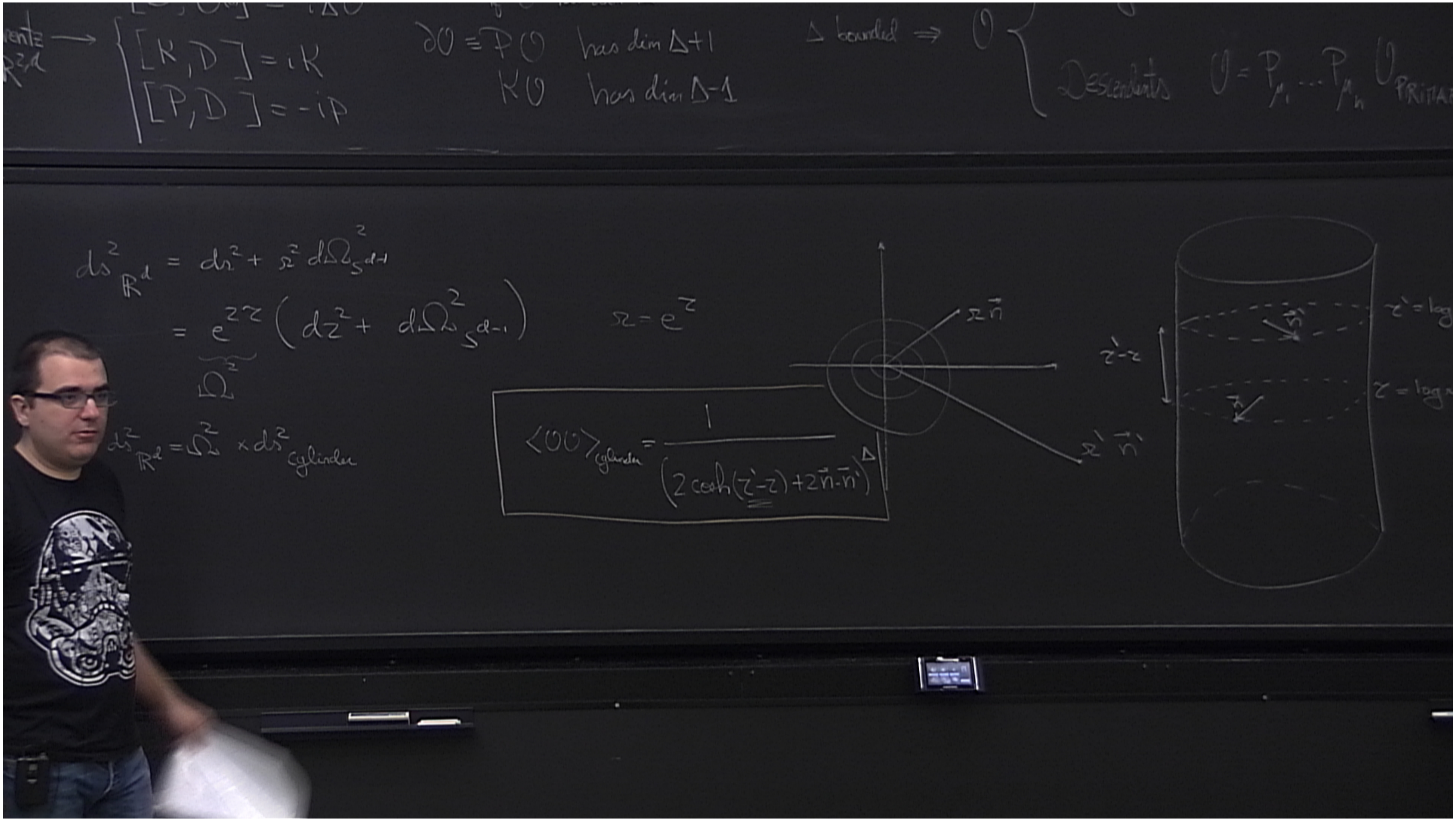

$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle_{\mathbb{R}^d} = \frac{1}{(x-y)^{2\Delta}}$
$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle_{\frac{1}{(x^2)^2}dx^\mu dx_\mu} = (x^2)^\Delta (y^2)^\Delta \times \langle \cdots \rangle_{\mathbb{R}^d}$
$\langle \mathcal{O}\left(\frac{x^\mu}{x^2}\right)\mathcal{O}\left(\frac{y^\mu}{y^2}\right)\rangle_{\mathbb{R}^d} = \frac{1}{\left(\left(\frac{x^\mu}{x^2}-\frac{y^\mu}{y^2}\right)^2\right)^\Delta} = \frac{(x^2)^\Delta (y^2)^\Delta}{(x-y)^{2\Delta}}$


OR


[K, D] = iK
[P, D] = -iP
∂O ≡ PO has dim Δ+1
KO has dim Δ-1
Δ bounded ⇒ O
Descendants O = P ... P O PRIMARY
ds² = dr² + r² dΩ²
= e^{2τ} (dτ² + dΩ²)
r = e^τ
ds² = Ω² × ds² cylinder



has dim Δ+1
has dim Δ-1
Δ bounded ⇒ O
Descendants O = P_μ1 ... P_μn O_PRIMARY
r = e^τ
r n⃗
r' n⃗'
τ' - τ
τ' = log r'
τ = log r
⟨O O⟩_R^d = Ω^Δ Ω'^Δ ⟨ ... ⟩_cylinder
⟨O(n⃗,τ) O(n⃗',τ')⟩_cylinder
e^Δτ e^Δτ' 1 / ((n⃗ e^τ - n⃗' e^τ')^2)^Δ


[K, D] = iK
[P, D] = -iP
∂O = PO has dim Δ+1
KO has dim Δ-1
Δ bounded ⇒
Descendants
r = e^τ
⟨OO⟩_cylinder = 1 / (2cosh(τ'-τ) + 2n·n')^Δ

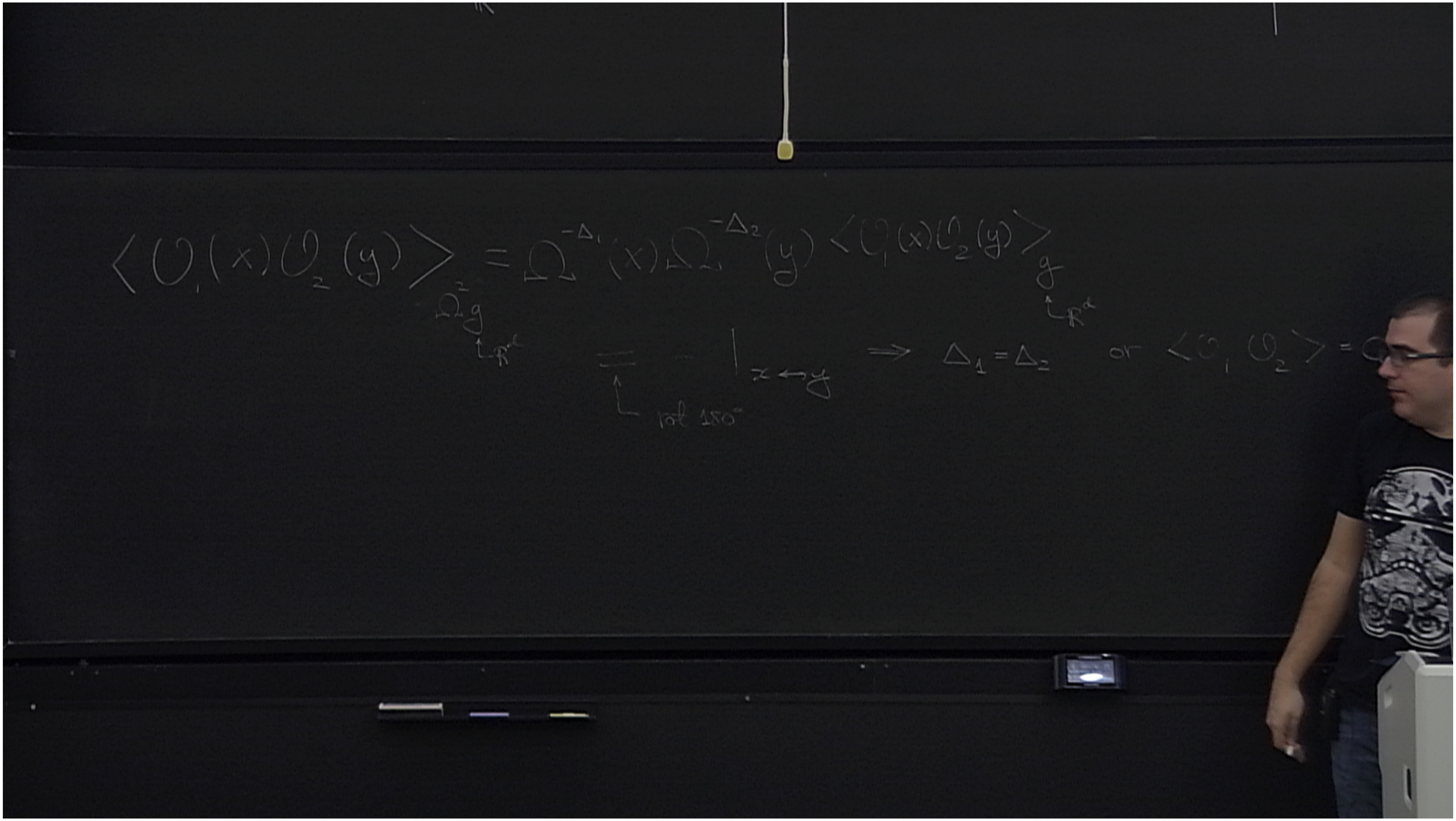
$$\langle \mathcal{O}_1(x)\,\mathcal{O}_2(y)\rangle_{\Omega^2 g} = \Omega^{-\Delta_1}(x)\,\Omega^{-\Delta_2}(y)\,\langle \mathcal{O}_1(x)\,\mathcal{O}_2(y)\rangle_g$$

$$\langle \mathcal{O}_1(x)\,\mathcal{O}_2(y)\rangle_{\Omega^2 g} = \Omega^{-\Delta_1}(x)\,\Omega^{-\Delta_2}(y)\,\langle \mathcal{O}_1(x)\,\mathcal{O}_2(y)\rangle_{g}$$

$\Omega^2 g \leftarrow \mathbb{R}^d$ $g \leftarrow \mathbb{R}^d$

$$= \quad \ldots \quad \Big|_{x \leftrightarrow y}$$

$\uparrow$ rot 180°


$\langle O_1(x) O_2(y) \rangle_{\Omega^2 g} = \Omega^{-\Delta_1}(x)\, \Omega^{-\Delta_2}(y) \langle O_1(x) O_2(y) \rangle_g$
$\mathbb{R}^d$
$\mathbb{R}^d$
$= \; \big|_{x \leftrightarrow y} \Rightarrow \Delta_1 = \Delta_2$ or $\langle O_1 O_2 \rangle = 0$
rot 180°

$$\langle \mathcal{O}_1(x)\,\mathcal{O}_2(y)\rangle_{\Omega^2 g} = \Omega^{-\Delta_1}(x)\,\Omega^{-\Delta_2}(y)\,\langle \mathcal{O}_1(x)\,\mathcal{O}_2(y)\rangle_g$$

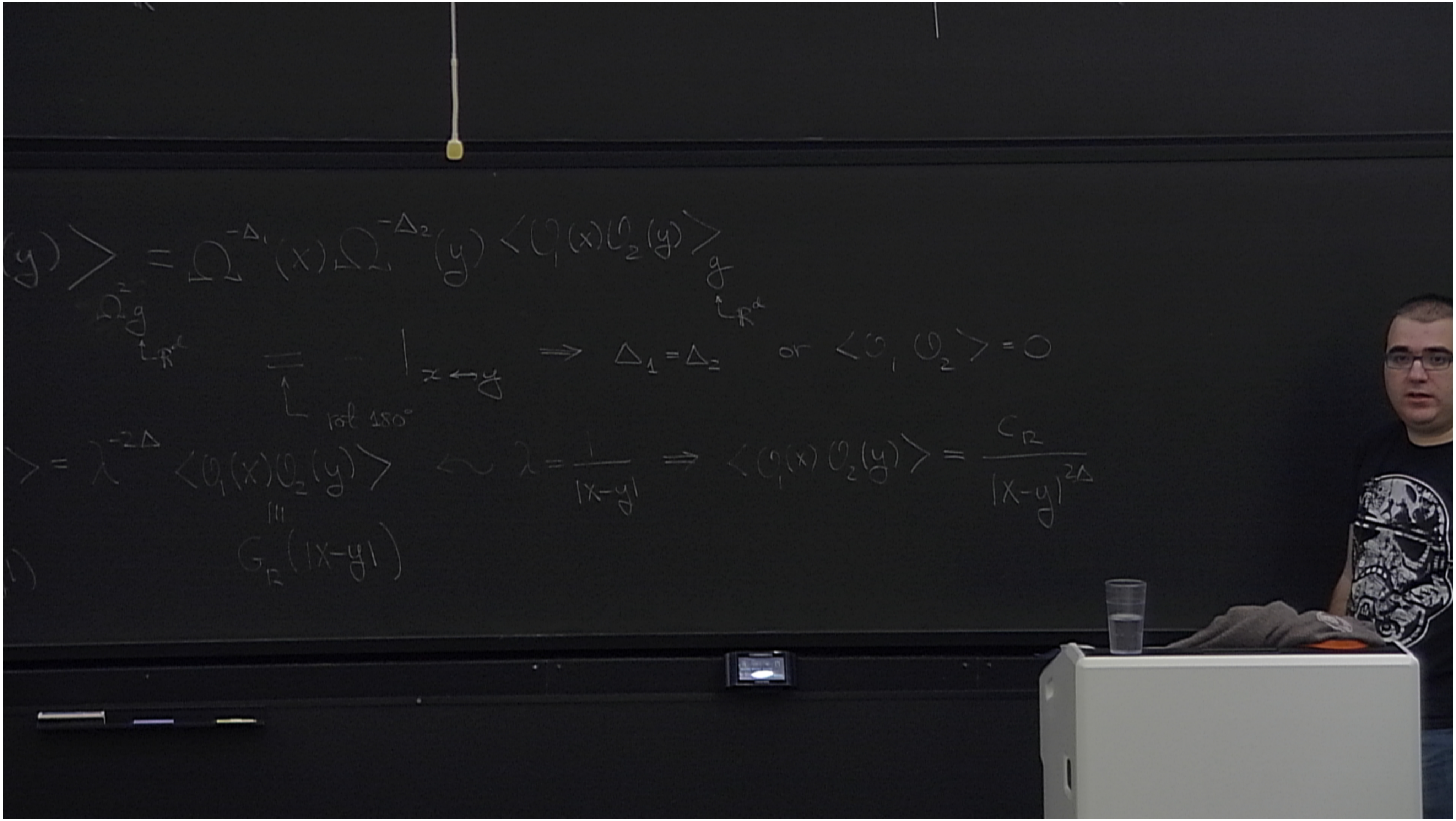
$\mathbb{R}^d$ $\qquad$ $\mathbb{R}^d$

$$= \ldots \Big|_{x \leftrightarrow y} \Rightarrow \Delta_1 = \Delta_2 \quad \text{or} \quad \langle \mathcal{O}_1\,\mathcal{O}_2\rangle = 0$$

rot 180°

$\Delta_1 = \Delta_2$

$\langle$


(y)⟩ = Ω^{-Δ₁}(x) Ω^{-Δ₂}(y) ⟨O₁(x) O₂(y)⟩_g
⇒ Δ₁ = Δ₂ or ⟨O₁ O₂⟩ = 0
rot 180°
⟩ = λ^{-2Δ} ⟨O₁(x) O₂(y)⟩ ~ λ = 1/|x-y| ⇒ ⟨O₁(x) O₂(y)⟩ = C₁₂/|x-y|^{2Δ}
G₁₂(|x-y|)

Lorentz $\mathbb{R}^{2,1}$ $\longrightarrow$ $\begin{cases} [D, \mathcal{O}(0)] = i\Delta\mathcal{O} \\ [K, D] = iK \\ [P, D] = -iP \end{cases}$

if $\mathcal{O}$ has dim Δ

$\partial\mathcal{O} \equiv P\mathcal{O}$ has dim $\Delta+1$

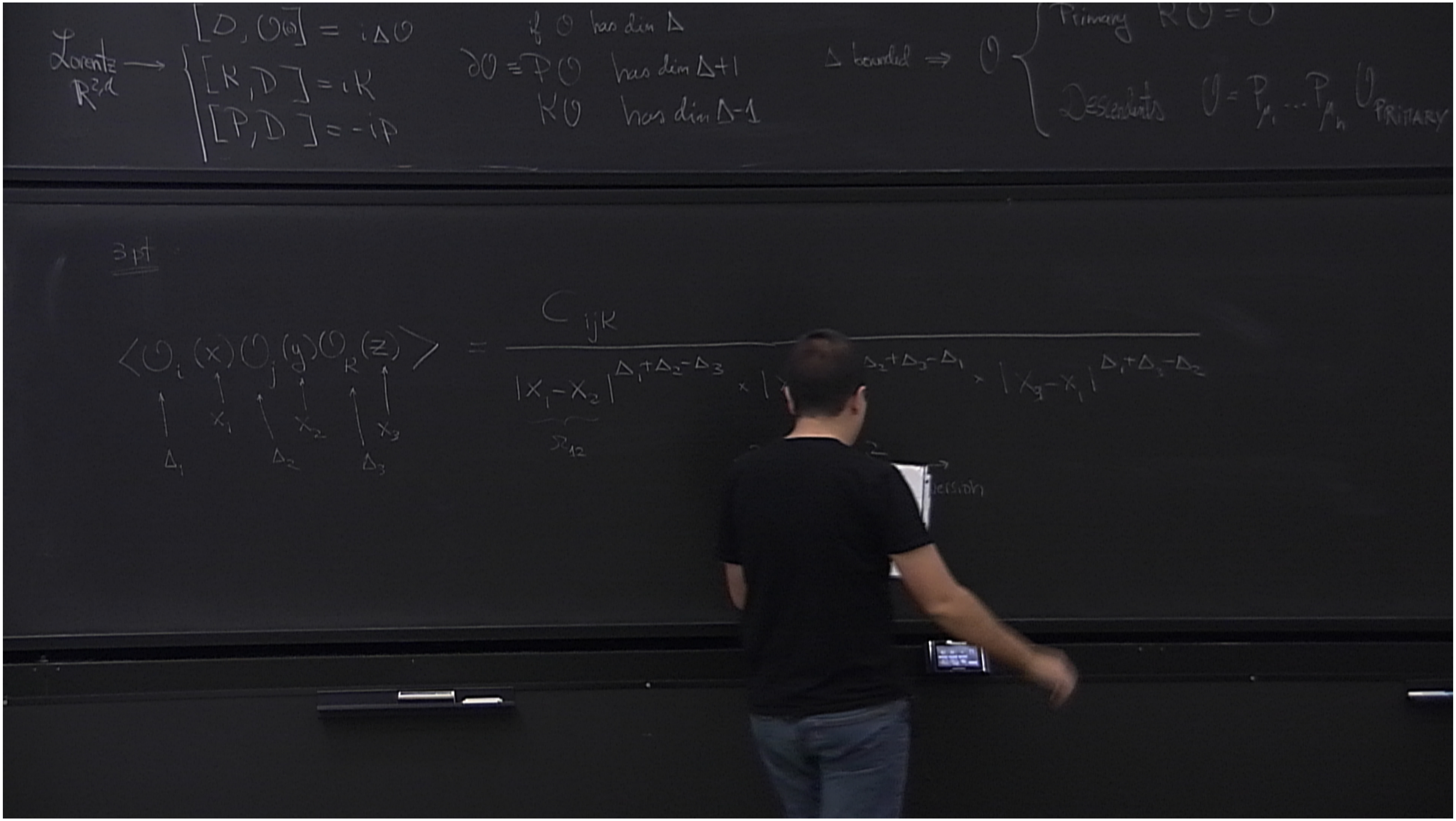
$K\mathcal{O}$ has dim $\Delta-1$

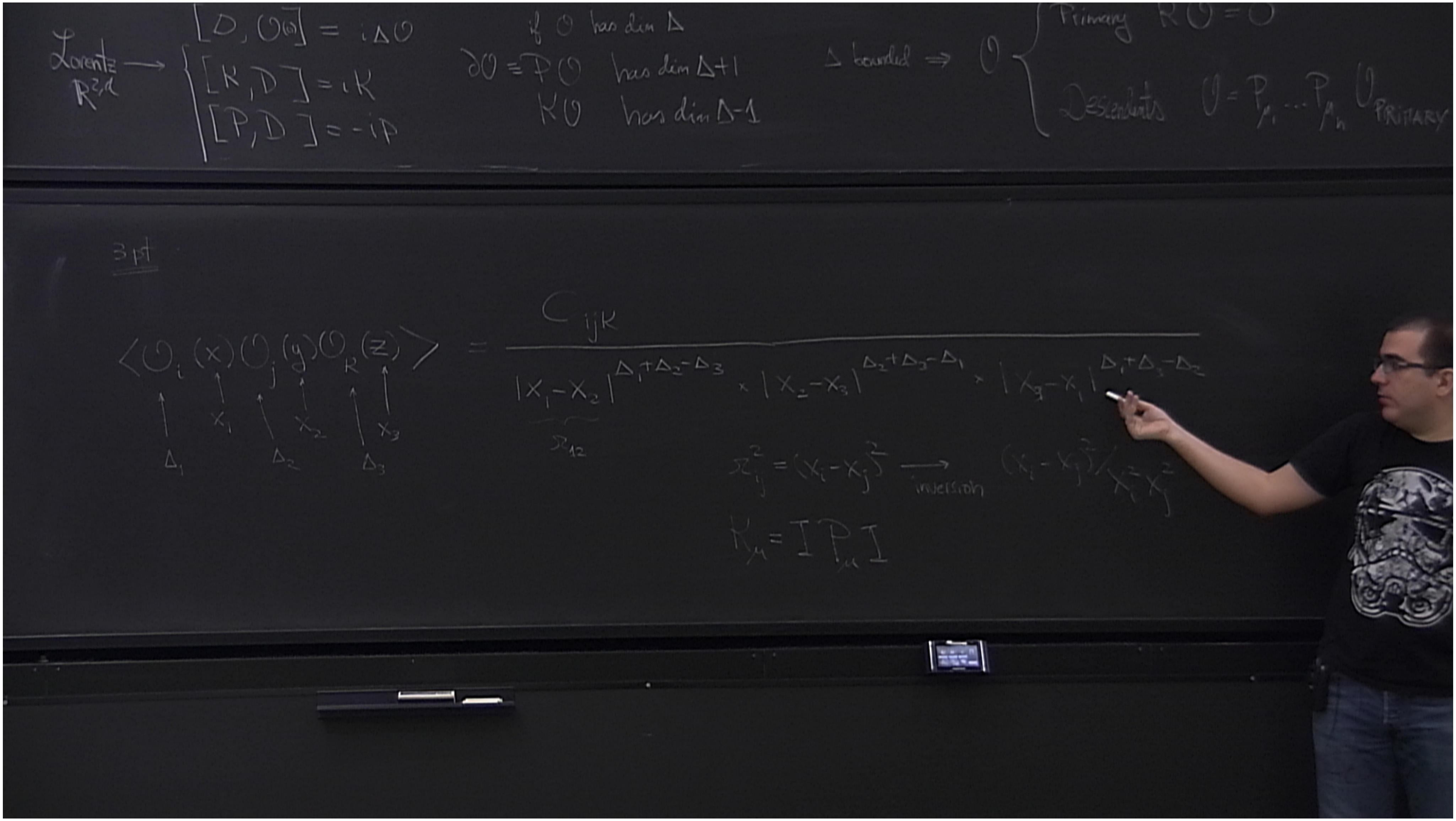
Δ bounded $\Rightarrow$ $\mathcal{O}$ $\begin{cases} \text{Primary } K\mathcal{O} = 0 \\ \text{Descendants } \mathcal{O} = P_{\mu_1} \cdots P \end{cases}$

3pt

$$\langle \mathcal{O}_i(x)\, \mathcal{O}_j(y)\, \mathcal{O}_k(z) \rangle = \frac{C_{ijk}}{\underbrace{|x_1 - x_2|}_{x_{12}}^{\Delta_1+\Delta_2-\Delta_3} \times |x_2 - x_3|^{\Delta_2+\Delta_3-\Delta_1} \times |x_3 - x_1|^{\Delta_3+\Delta_1-\Delta_2}}$$

Δ_1 x_1 Δ_2 x_2 Δ_3 x_3


Lorentz
$\mathbb{R}^{2,d}$
$[D, \mathcal{O}(0)] = i\Delta\mathcal{O}$
$[K, D] = iK$
$[P, D] = -iP$
if $\mathcal{O}$ has dim Δ
$\partial\mathcal{O} \equiv P\mathcal{O}$ has dim $\Delta+1$
$K\mathcal{O}$ has dim $\Delta-1$
Δ bounded $\Rightarrow$ $\mathcal{O}$
Primary $K\mathcal{O} = 0$
Descendants $\mathcal{O} = P_{\mu_1} \dots P_{\mu_n} \mathcal{O}_{PRIMARY}$
3 pt
$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\mathcal{O}_k(z)\rangle = \frac{C_{ijk}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3} \times \dots \times |x_3-x_1|^{\Delta_1+\Delta_3-\Delta_2}}$
x_1 x_2 x_3
Δ_1 Δ_2 Δ_3
x_{12}


Lorentz $\mathbb{R}^{2,1}$
$[D, \mathcal{O}(0)] = i\Delta\mathcal{O}$
$[K, D] = iK$
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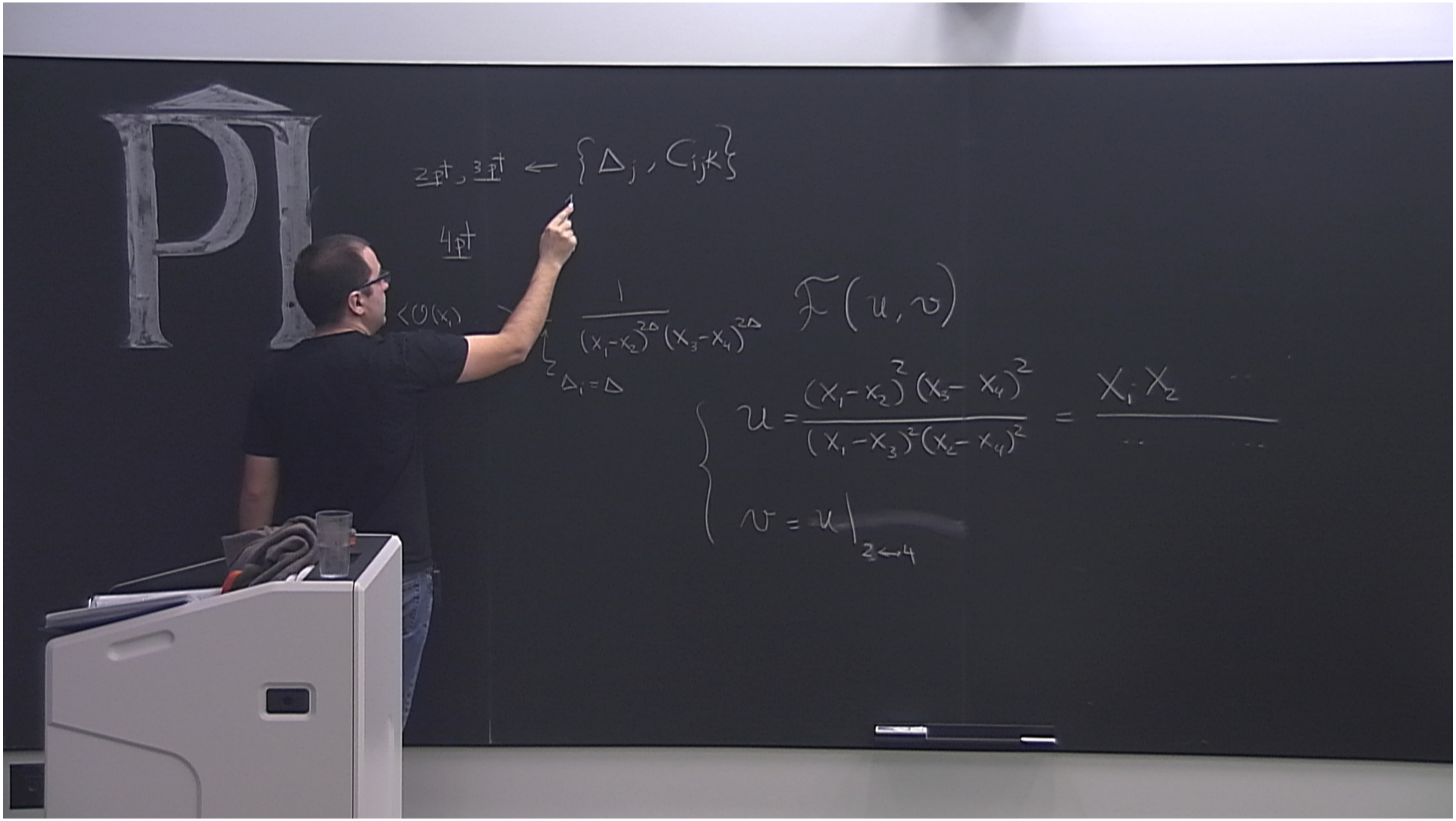
if $\mathcal{O}$ has dim Δ
$\partial\mathcal{O} \equiv P\mathcal{O}$ has dim $\Delta+1$
$K\mathcal{O}$ has dim $\Delta-1$
Δ bounded $\Rightarrow$ $\mathcal{O}$
Primary $K\mathcal{O}=0$
Descendants $\mathcal{O} = P_{\mu_1} \dots P_{\mu_n} \mathcal{O}_{PRIMARY}$
3pt
$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\mathcal{O}_k(z)\rangle = \dfrac{C_{ijk}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3} \times |x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1} \times |x_3-x_1|^{\Delta_1+\Delta_3-\Delta_2}}$
x_{12}
$x_{ij}^2 = (x_i-x_j)^2 \xrightarrow[\text{inversion}]{} (x_i-x_j)^2 / x_i^2 x_j^2$
$K_\mu = I P_\mu I$


2pt, 3pt ← {Δj, Cijk}
4pt
F(u, v)
Δi = Δ
v = u|2↔4