


Title: PSI 2015/2016 Explorations in String Theory - Pedro Vieira - 4

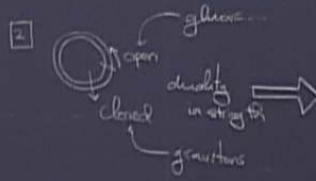
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URL: <http://pirsa.org/16030066>

Abstract:

1  Large N \leadsto 't Hooft limit

4 Holography $S_{BH} \propto Area$



4d $\mathcal{N}=4$ Super Yang-Mills
 \equiv λ, N
 $AdS_5 \times S^5$ superstring theory


3 Brane int vs Background Decoupling argument



 free SUGRA

\mathbb{Z} far from branes at ∞
 Δ

$AdS_5 \times S^5$ physics due to brane
 $=$ $\left\{ \begin{array}{l} \oplus \\ \otimes \end{array} \right\}$ free SUGRA
 $\mathcal{R} \ll L$ $\mathcal{R} \gg L$

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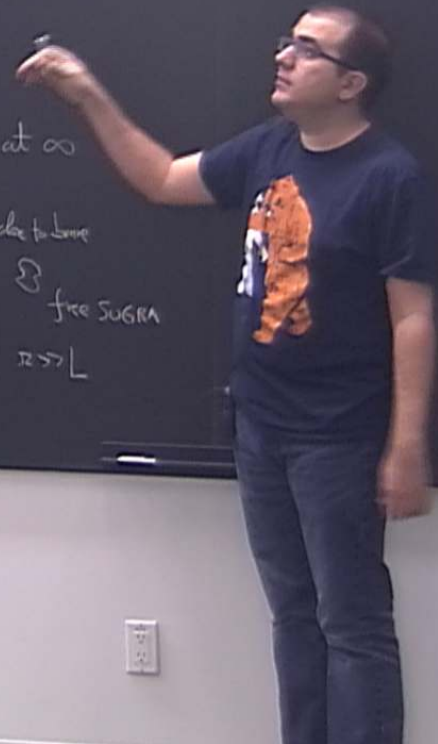
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\rightarrow
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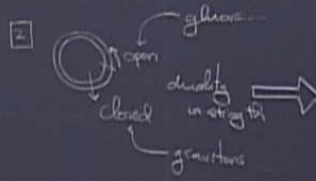
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I  =  Large N \approx 't Hooft limit

II Holography $S_{BH} \propto Area$




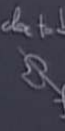
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~~free SUGRA~~

* $\mathcal{N}=4$ SYM is a CFT (no scale)

$X^M \rightarrow \lambda X^M$ is a sym.

$$ds^2 = \underbrace{dx^M dx^N}_{\mathbb{R}^{13}} \eta_{MN} \quad \text{not invariant}$$

simplest modification would be:

$$ds^2 = \frac{dx^M dx_M + dz^2}{z^2} \quad \leftarrow \text{this is AdS}_5$$

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In fact $\text{Isometries of AdS} \equiv \text{conformal group of } \mathbb{R}^{1,3}$



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In fact $\text{Isom}(\text{AdS}) \equiv \text{conformal group of } \mathbb{R}^{1,3}$
Killing $\leftrightarrow K_{\mu\nu}, P_\mu, D, M_{\mu\nu}$

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In fact (isometries of AdS) \equiv conformal group of $\mathbb{R}^{1,3}$

→ $K_{\mu\nu}, P_\mu, D, M_{\mu\nu}$



G_n transform like a G_n in a Conformal Th. in 4d
Well, then $AdS_5 \times S^5$ string \equiv Some CFT in ∂AdS
Which one? it has $\mathcal{N}=4$ supersym

$z=0$
($x=\infty$)
 $U = \mathbb{R}^{1,3}$
)



G_n transform like a G_n in a Conformal Th. in 4d

Well, then $AdS_5 \times S^5$ string \equiv Some CFT in $\mathcal{D}AB$

Which one? it has $\mathcal{N}=4$ supersym on susy of super string. It has an $SO(6)$ global sym.

So it is $\mathcal{N}=4$ SYM.

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ormal Th. in 4d

≡ Some CFT in $\mathcal{D}AB$

Supersym on susy of superstring. It has an $SO(6)$ global sym.

Consider heavy pt
P_{optimal}
geodesics



$\sim \ell$ - m length of geodesic



Formal Th. in 4d

≡ Some CFT in $\mathcal{D}AB$

Supersym on susy of superstring. It has an $SO(6)$ sym

Consider heavy pt
p=optimal
geodesics



$$\sim e^{-m \times \text{length of geodesics}}$$

$$\sim \frac{1}{(x_1 - x_2)(x_2 - x_3) \dots}$$



can $dx dx y_{\mu\nu}$ not invariant

simplest modification would be

$$ds^2 = \frac{dx^\mu dx_\mu + dz^2}{z^2} \quad \leftarrow \text{this is AdS}_5$$

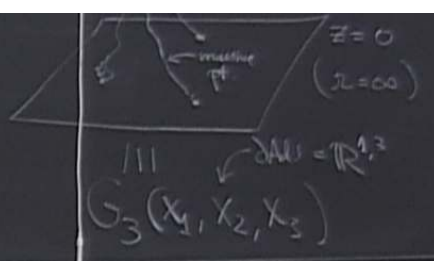


$$\mathcal{L}_{\mathcal{N}=4\text{SYM}} = \text{Reduction to 4d} \left[\text{tr} \left(F_{MN} F^{MN} + \bar{\psi} \not{D} \psi \right) \right]$$

$$\text{tr} \bar{\psi} \not{D} \psi = (\bar{\Psi}) \Gamma \left(\partial \Psi + i [A, \Psi] \right)$$

first modification would be

$$ds^2 = \frac{dx^i dx_i + dz^2}{z^2} \leftarrow \text{this is AdS}_5$$



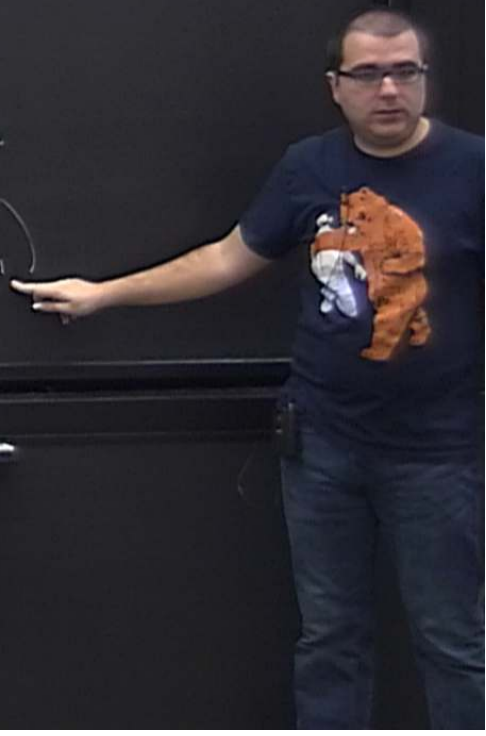
= Reduction to 4d $\left[\text{tr} \left(F_{MN} F^{MN} + \bar{\psi} \not{D} \psi \right) \right] \rightarrow F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi_m D_\mu \Phi_m +$

$$\sum_{A, a, b, M, B} (\bar{\Psi})^A \Gamma^{ab AB M} (\partial_M \psi_B + i [A_M, \psi_B])_{bc}$$

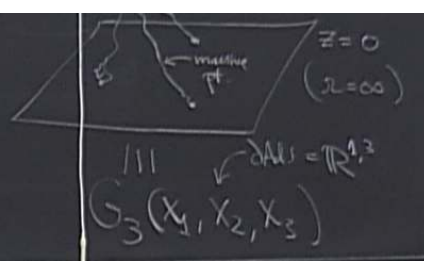
Index labels: $A \xrightarrow{1-16} ab \xrightarrow{1-16} AB \xrightarrow{1-16} M$; $M \xrightarrow{0-9} \psi_B$

$$M = \underbrace{0 \dots 3}_\mu \underbrace{4 \dots 6}_m$$

$$A_M = (A_\mu, \Phi_m)$$



AdS₅



Which one? If ha
So if $\mathcal{N} = 4$ SYM.

$$\left[\dots + \psi \not{D} \psi \right] \rightarrow F_{\mu\nu} F^{\mu\nu} + D_{\mu} \Phi_m D_{\mu} \Phi_m + [\Phi_m, \Phi_n]^2$$

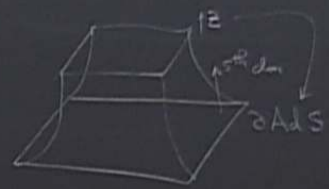
$$\gamma^M \left(\partial_M \psi_B + i [A_M, \psi_B] \right)_{bc}$$

$\begin{matrix} AB \\ \dots \\ LN \end{matrix}$
 $\begin{matrix} M \\ \dots \\ 6 \end{matrix}$
 $\begin{matrix} B \\ \dots \\ 6 \end{matrix}$

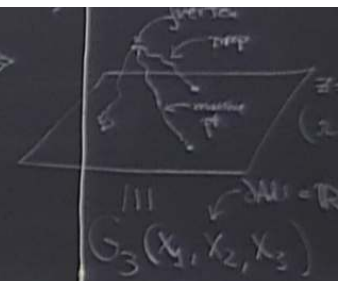
$$M = \underbrace{0 \dots 3}_{\mu} \underbrace{4 \dots 6}_m$$

$$A_M = (A_{\mu}, \Phi_m)$$

$dx^2 = dx^M dx^N \gamma_{MN}$ not invariant
 simplest modification would be:
 $ds^2 = \frac{dx^\mu dx_\mu + dz^2}{z^2}$ ← this is AdS₅



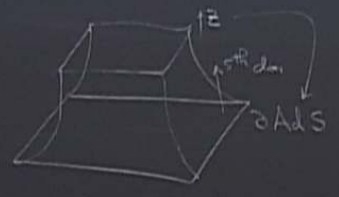
AdS isometries ⇒



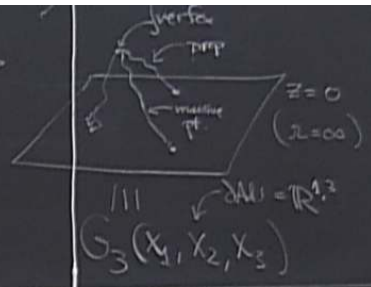
$\mathcal{L}_{\mathcal{N}=4S/M} = \text{Reduction to 4d} \left[\int (F_{MN} F^{MN} + \bar{\psi} \not{D} \psi) \right] \rightarrow \int F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi_m D_\mu \Phi_m + [\Phi_m, \Phi_n]^2$
 $+ \bar{\psi} \Gamma^M (\partial_M \psi + i [A_M, \psi]) + \bar{\psi} \Gamma^m [\Phi_m, \psi]$
 $\int \bar{\psi} \not{D} \psi = \int \bar{\psi}_A \Gamma_{ab}^M (\partial_M \psi_B + i [A_M, \psi_B])$
 $M = 0, 1, 2, \dots, 5$
 $\mu = 0, 1, 2, \dots, 3$
 $m = 1, 2, 3, \dots, 5$
 $\Gamma^M \Gamma^N + \Gamma^N \Gamma^M = 2g^{MN}$
 $\int_M = (A_\mu, \Phi_m)$



not invariant
 could be
 dz^2
 ← this is AdS_5



AdS isometric \rightarrow



Well, then AdS_5
 Which one?
 So the $d=4$ part

$$\left[\text{tr} \left(F_{MN} F^{MN} + \psi \not{D} \psi \right) \right] \rightarrow F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi_m D_\mu \Phi_m + [\Phi_m, \Phi_n]^2$$

$$+ \psi \Gamma^\mu (\partial_\mu \psi + i [A_\mu, \psi]) + \psi \Gamma^m [\Phi_m, \psi] = \mathcal{L}_{N=4}$$

$$\left(\bar{\psi}_A \Gamma_{ab}^{AB} \Gamma^M (\partial_M \psi_B + i [A_M, \psi_B]) \right)_{bc}$$

$M = \underbrace{0, 1, 2, 3}_{\mu} \dots \underbrace{4, \dots, 6}_m$
 $A_M = (A_\mu, \Phi_m)$
 $\underbrace{4 \text{ group}}_{\mu} \quad \underbrace{6 \text{ Scalars}}_m$

$$\Gamma^M \Gamma^N + \Gamma^N \Gamma^M = 2\eta^{MN} \mathbb{1}$$

$$\partial_m \equiv 0$$

So it is $\mathcal{N} = 4$ SYM.

* Weinberg-Witten Theorem

A theory with a Lorentz covariant energy-momentum tensor $T^{\mu\nu}$ for which

$$P^0 \equiv \int T^{00} d^3x$$



can not contain massless particles of spin $j > 1$.

A, a, b, M, B

$A \xrightarrow{ab} B$
 \downarrow
 $1-16$

\downarrow
 $0-9$

Consider

$$\underbrace{\langle P' |}_{\text{single pt}} T^{\mu\nu} | P \rangle \xrightarrow{P' \rightarrow P}$$

$$P^\mu P^\nu \frac{f}{(2\pi)^3 E} \quad (f=1)$$

\uparrow energy

$$\Psi \Psi \Psi = \dots$$

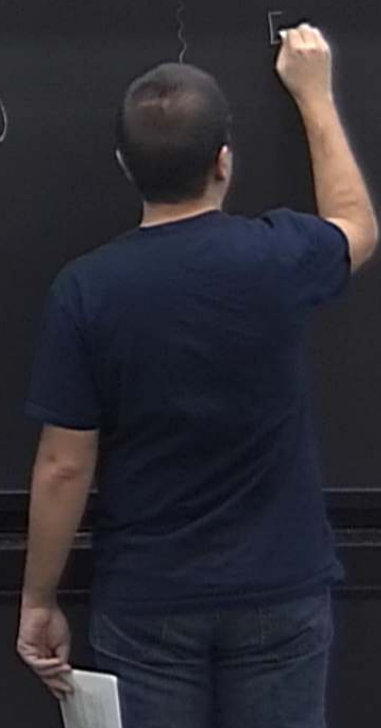
A, a, b, M, B
 $\begin{matrix} A & ab & AB \\ \uparrow & \leftarrow & \leftarrow \\ 1 & 16 & 1-N \end{matrix}$
 $\begin{matrix} \Psi & \Psi & \Psi \\ \uparrow & \uparrow & \uparrow \\ M & B & \end{matrix}$
 $[A, M, B]$
 $\begin{matrix} \mu & \mu \\ \underbrace{\quad} & \underbrace{\quad} \\ 4\text{group} & 6\text{S} \end{matrix}$

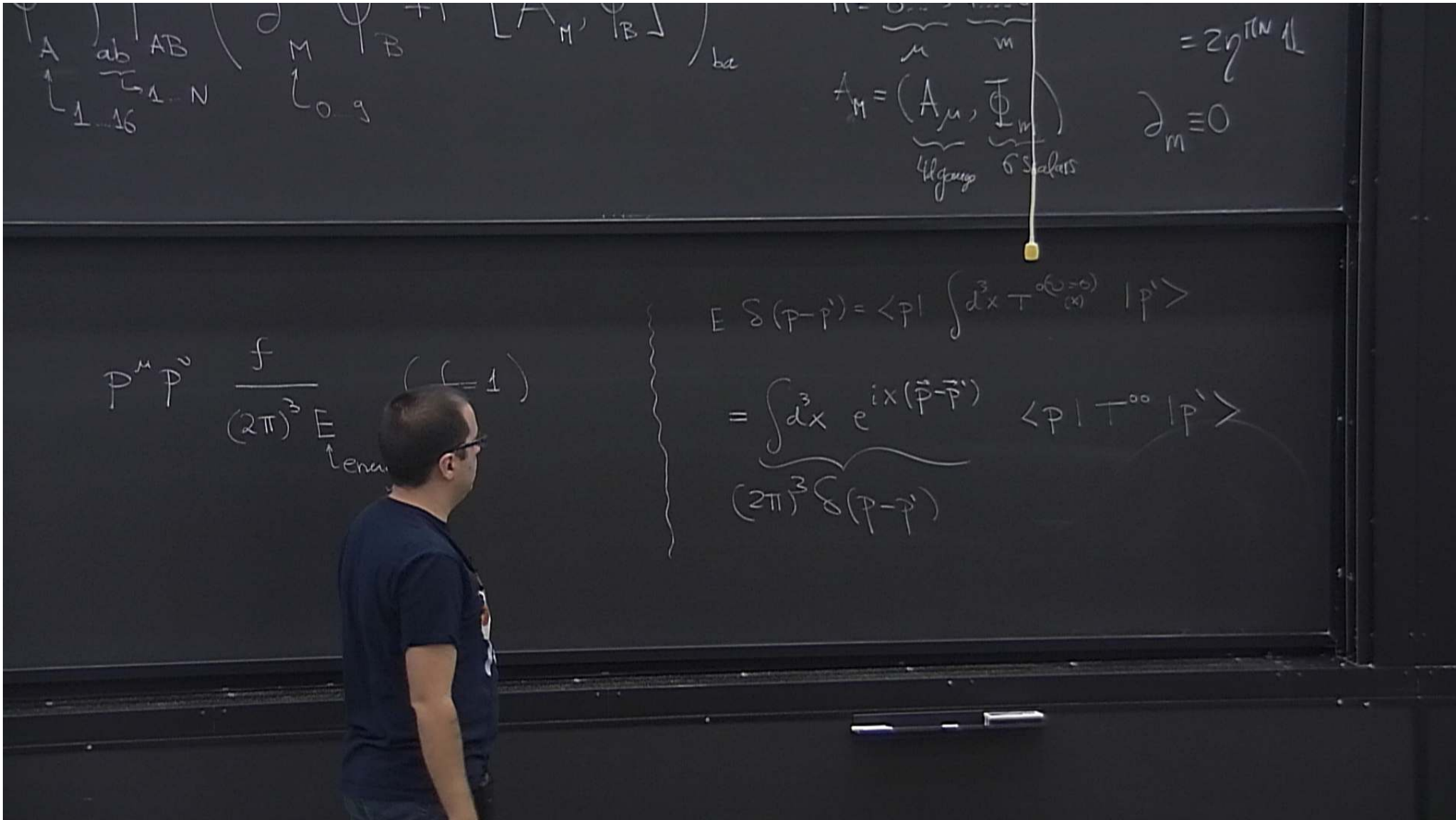
Consider

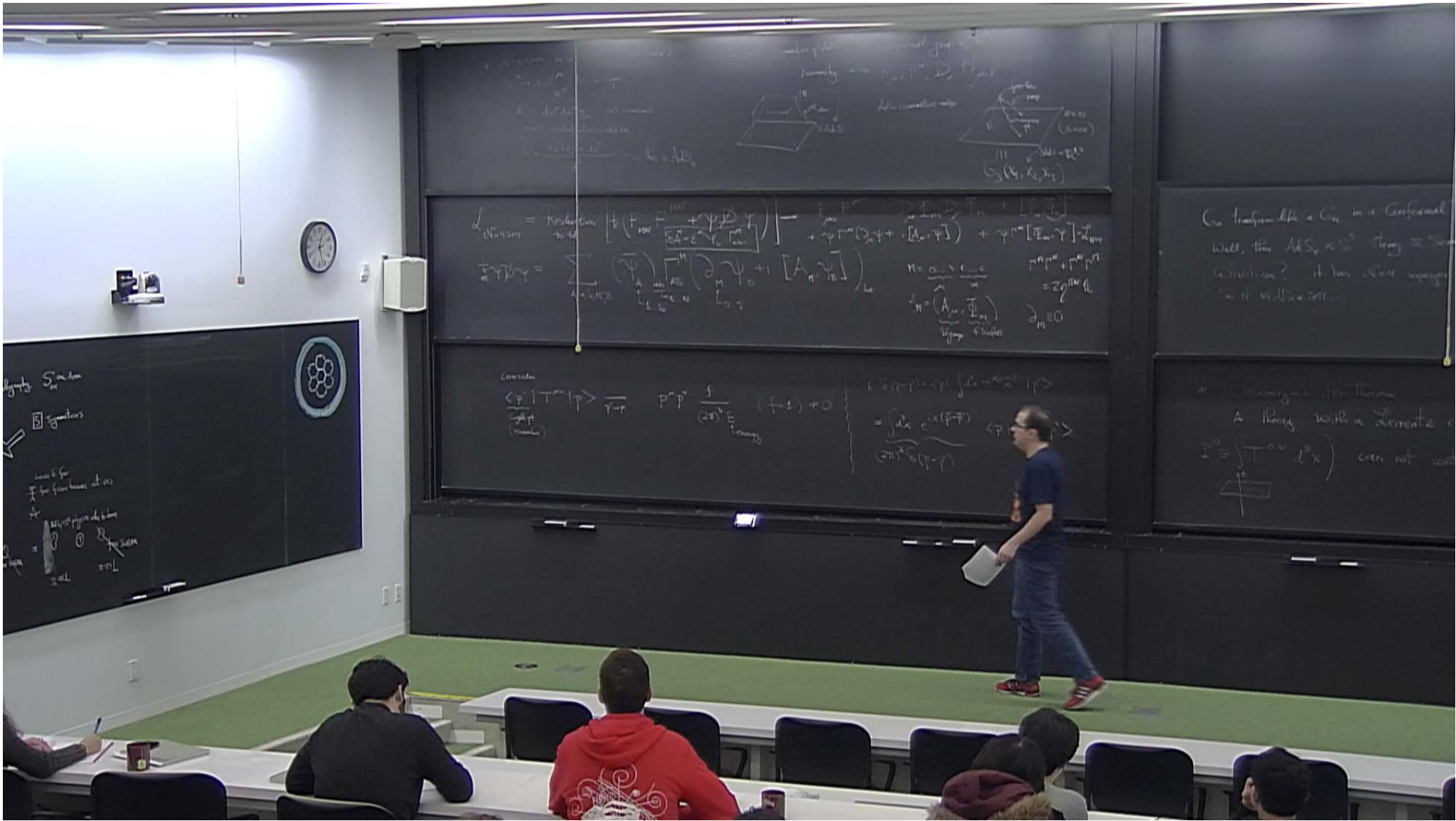
$$\langle \underbrace{P'}_{\text{Single pt (manus)}} | T^{\mu\nu} | P \rangle \xrightarrow{P' \rightarrow P}$$

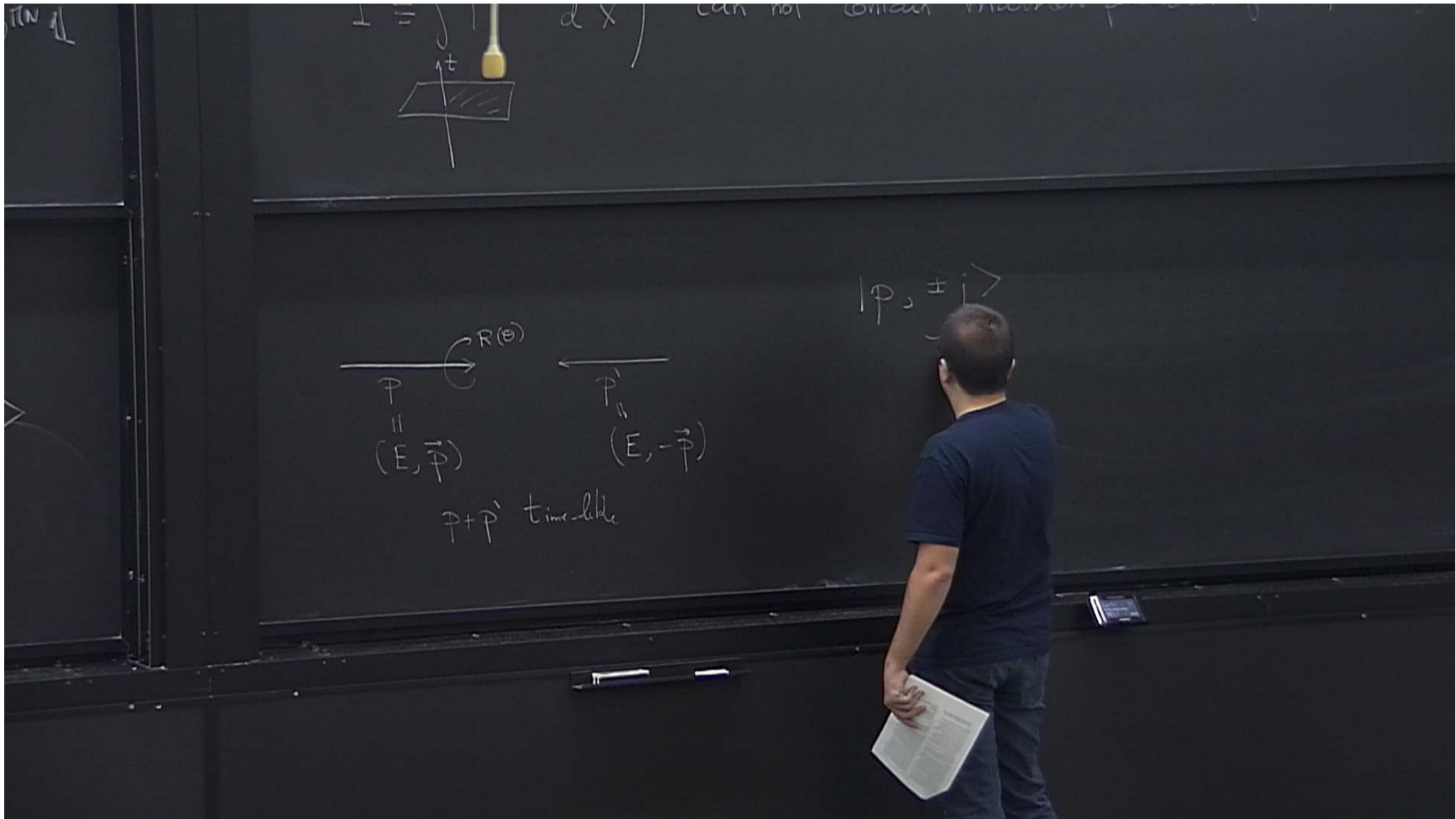
$$P^\mu P^\nu \frac{f}{(2\pi)^3 E} \quad (f=1)$$

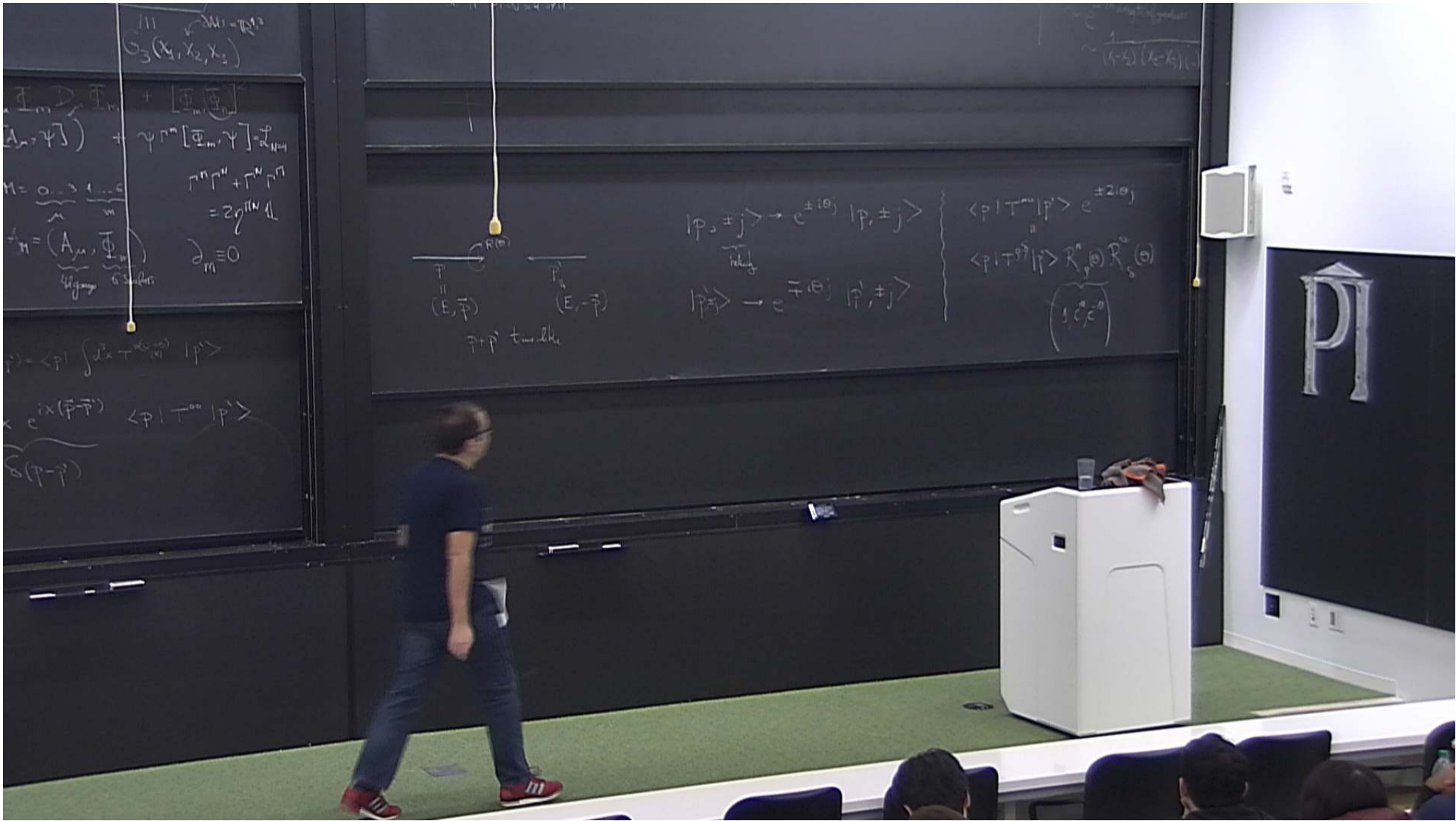
↑ energy







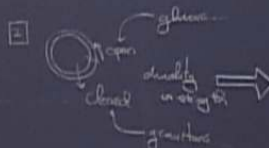






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IV Holography $S_{BH} \propto Area$



4d $\mathcal{N}=4$ Super Yang-Mills
III λ, N
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V symmetries

VI Weinberg-witten

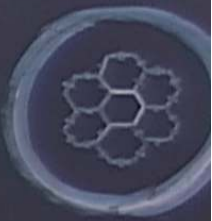
Low E for \mathbb{F} for fixed masses at ∞
A

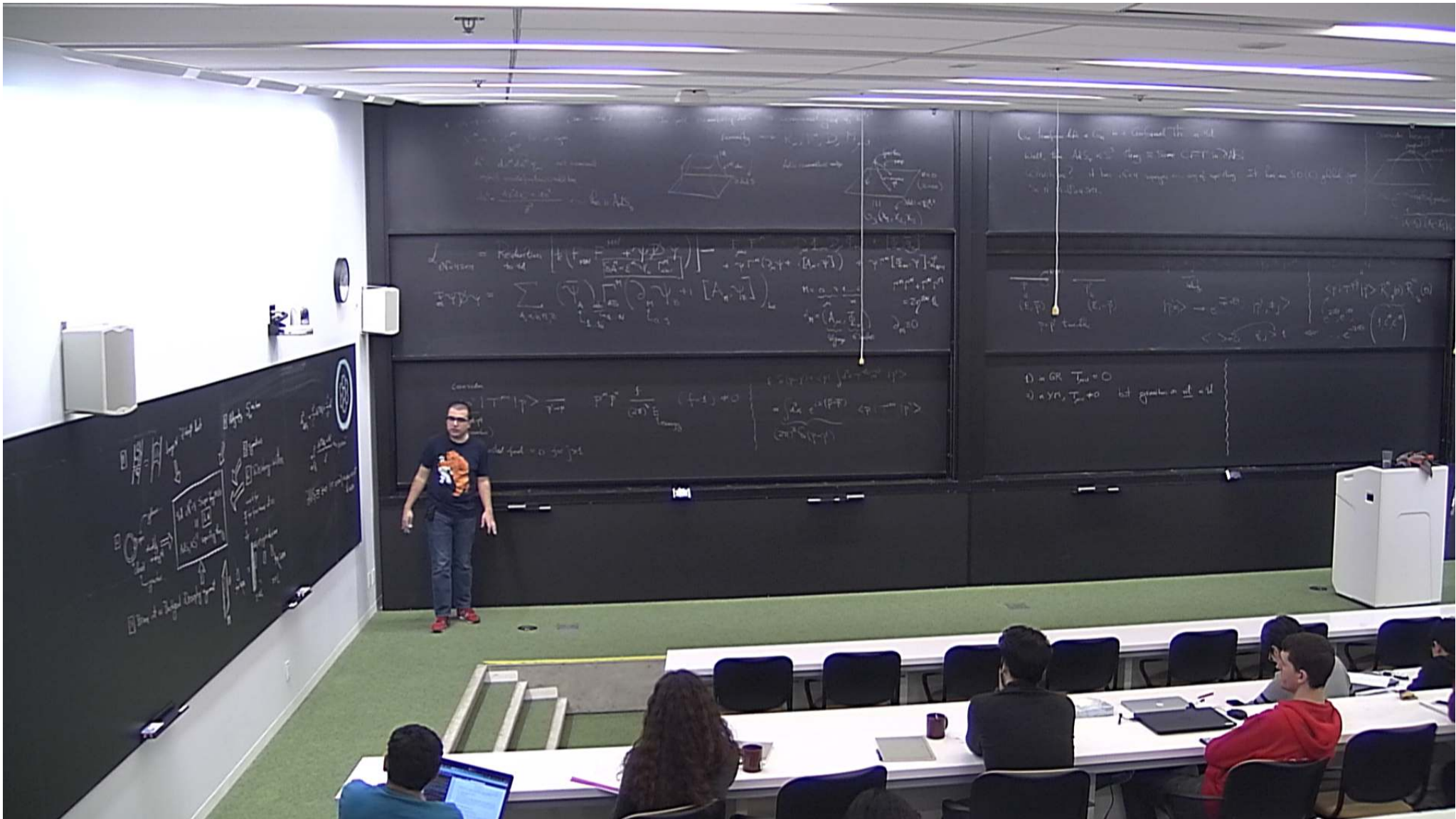
III Brane int vs Background Decoupling argument



~~free SUSYM~~
= $AdS_5 \times S^5$ physics close to brane
 \mathbb{F} ~~free SUSYM~~
 $L \ll L$ $L \gg L$

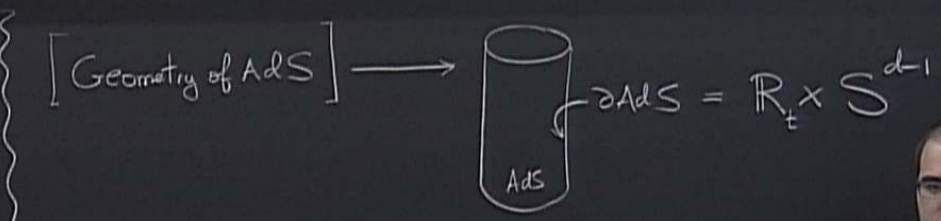
$$ds^2_{AdS_5} = \frac{r^2}{L^2} dx^\mu dx_\mu + \frac{L^2}{r^2} dr^2$$





$\vec{p}' = (E, -\vec{p})$
like
 $|\vec{p}'_{\neq j}\rangle \rightarrow e^{i\theta_j} |\vec{p}'_{\neq j}\rangle$ *helicity*
 $\langle \dots \rangle = 0$ $|\vec{p}'_{\neq j}\rangle = 1$
 $\langle p | T^{\mu\nu} | p' \rangle \sim R^{\mu}_{\nu}(\theta) R^{\nu}_{\sigma}(\theta)$
 $e^{2i\theta}, e^{i\theta}, \dots, e^{-2i\theta}$
 $\begin{pmatrix} 1 & e^{i\theta} & e^{-i\theta} \end{pmatrix}$

$T_{\mu\nu} = 0$
 $T_{\mu\nu} \neq 0$ but graviton in rot in 4d



P_{μ}^{ν}
 $(E, -\vec{p})$
 helicity
 $|p, \pm j\rangle \rightarrow e^{\mp i\theta j} |p', \pm j\rangle$
 $\langle \dots \rangle = 0$ for $j > 1$
 $\langle p | T^{\mu\nu} | p' \rangle R_p^{\mu}(\theta) R_p^{\nu}(\theta)$
 $e^{2i\theta}, e^{i\theta}, \dots, e^{-2i\theta}$
 $\begin{pmatrix} 1 & e^{i\theta} & e^{-i\theta} \end{pmatrix}$

$T_{\mu\nu} = 0$
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