

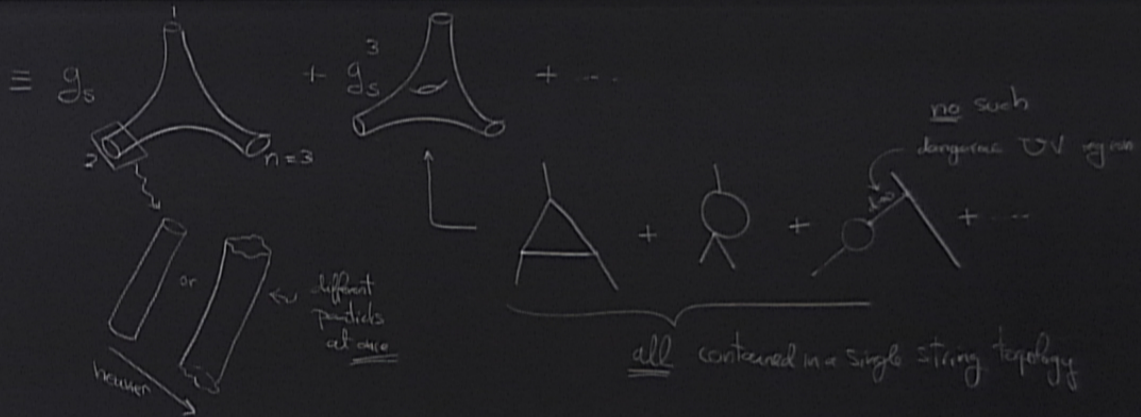
Title: PSI 2015/2016 Explorations in String Theory - Pedro Vieira - 1

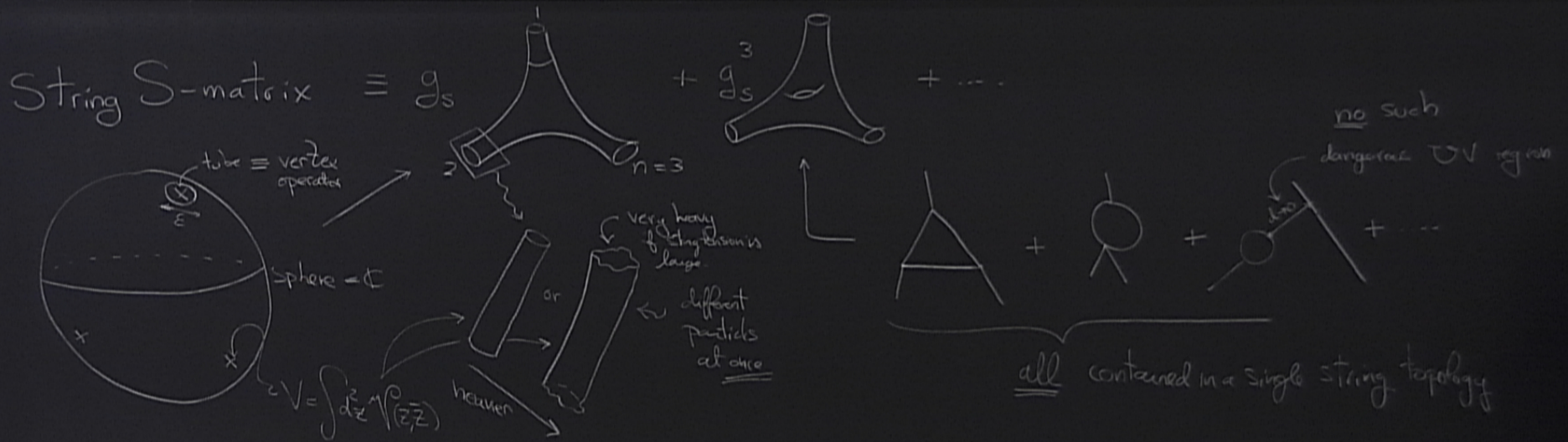
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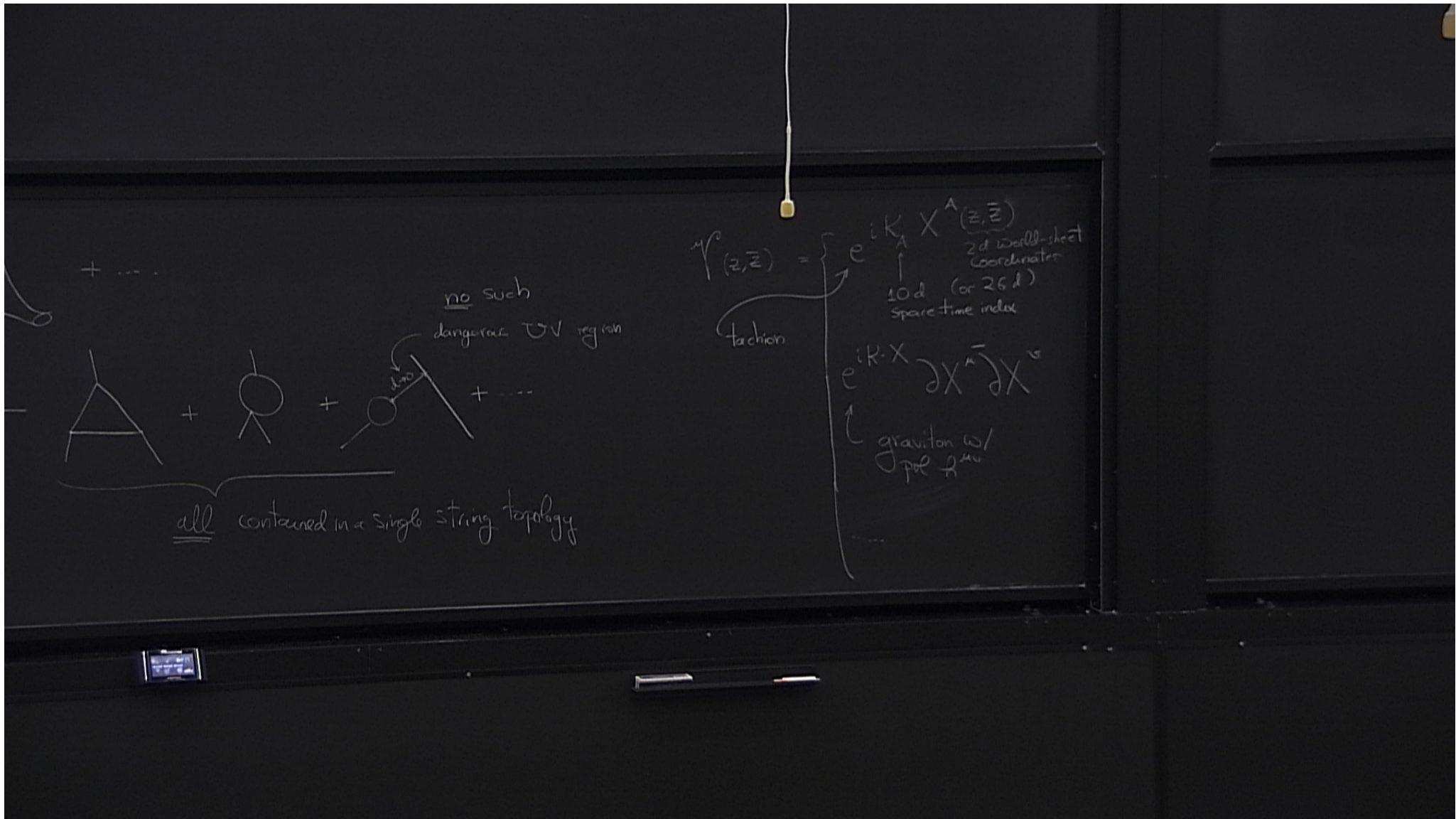
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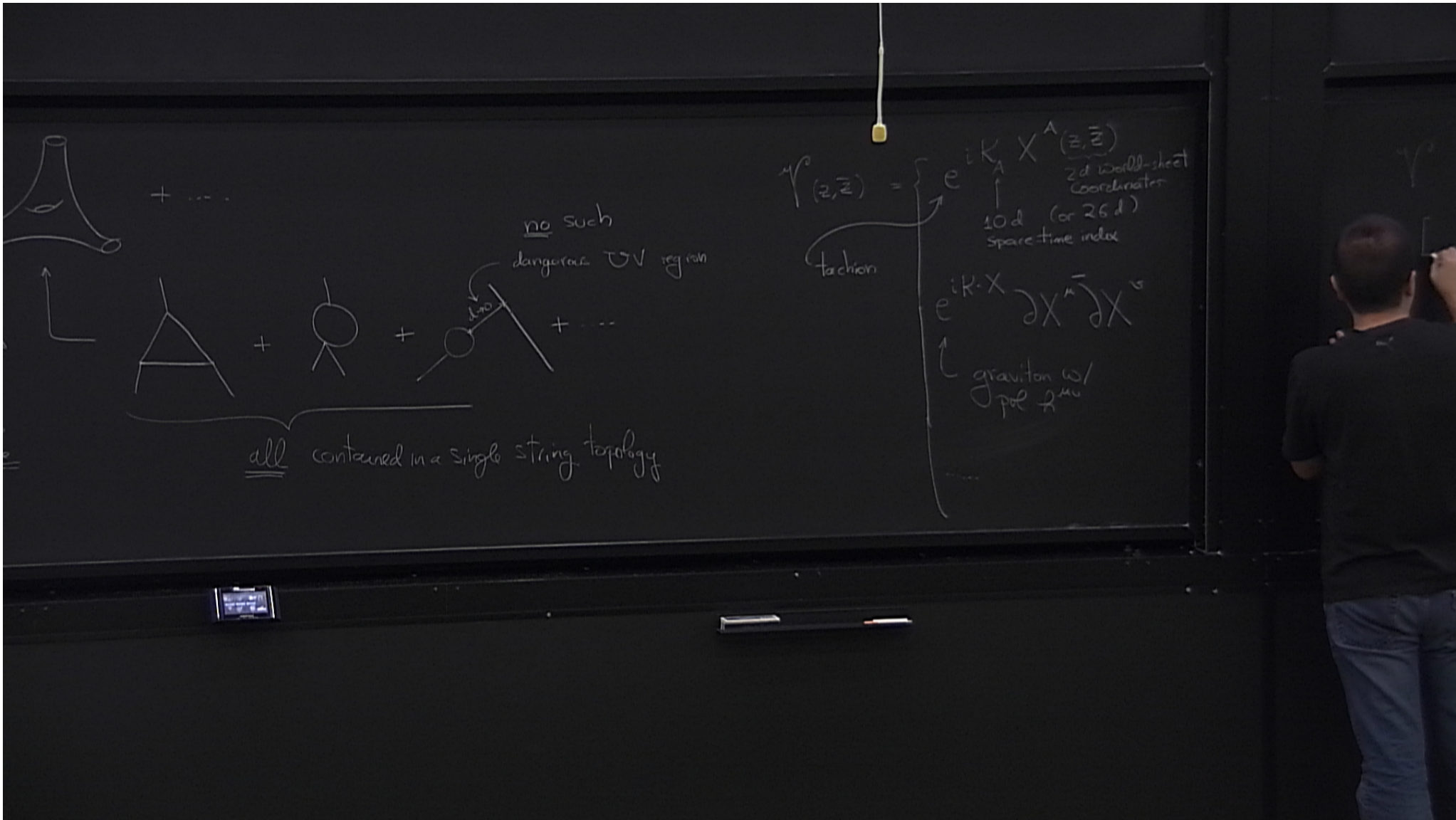
Abstract:

String S-matrix









(z, \bar{z})
 2d world-sheet
 coordinates
 or 26 d)
 the index

$$\partial X^\mu$$

ω/μ

\mathcal{V} dim 2 (1+1)

$$[X] = 0 \text{ classically}$$

$$\langle X_\mu X_\nu \rangle = \delta_{\mu\nu} \log|z-w|$$

$$\langle e^{ik \cdot X} e^{ik' \cdot X} \rangle = e^{k^2 \log|z-w|} = \frac{1}{|z-w|^{2k^2}} \quad \text{if } k^2 = -2 \leftarrow \text{tachyon}$$

(graviton)
 $[\alpha] + [\bar{\alpha}] = 2$
 $k^2 = 0 \leftarrow \Delta_{\text{grav}}$



γ dim 2 (1+1)

$[X] = 0$ classically

$$\langle X_\mu X_\nu \rangle = \delta_{\mu\nu} \# \log|z-w|$$

$$\langle e^{iK \cdot X} e^{iK \cdot X} \rangle = e^{K^2 \log|z-w|} = \frac{1}{|z-w|^2} \quad \text{if } K^2 = -2 \leftarrow \text{tachyon}$$

Graviton:

$$[\partial] + [\bar{\partial}] = 2 \checkmark$$

$$K^2 = 0 \leftarrow \Delta_{\text{rest}} = 0$$

$$\langle T_n X_\mu \rangle = \int_{\mu\nu} \log|z-w|$$

$$\langle T_n X_\mu X_\nu \rangle = e^{k^2 \log|z-w|} = \frac{1}{|z-w|^2} \quad (k^2 = -2 \text{ tension})$$

$$= \sum_{\text{genus } g} (g_s)^{2g-2+n} \int \mathcal{D}X \quad V_1 - V_n \exp(-\text{Tension } S[X])$$

\uparrow string coupling \uparrow 2d surfaces of genus g

$$\int_{\mu\nu} G_{\mu\nu} X^\mu \partial X^\nu + B_{\mu\nu} \epsilon^{\mu\nu} \partial X^\mu \partial X^\nu + \mathbb{I}(X) R^{(2)}$$

\uparrow background

$$= \sum_{\text{genus } g} (g_s)^{2g-2+n} \int \mathcal{D}X \quad V_1 \dots V_n \quad \exp(-\text{Tens})$$

\uparrow String coupling 2d Surfaces of genus g

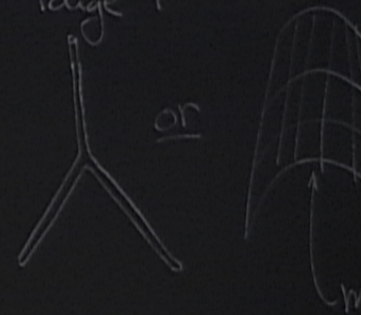
2d QFT is a CFT $\Rightarrow \beta_{G_{\mu\nu}} = 0$

$\Rightarrow G$ obeys (gen) Einstein Eqs.

small T

$$\int \mathcal{D}X!$$

large T





$V_1 \dots V_n$

$\exp\left(-\frac{\text{Tension}}{T} S[X]\right) \sim 2d \text{ CFT}$

$\int G_{\mu\nu}(X) \partial X^\mu \partial X^\nu + B_{\mu\nu}(X) \epsilon^{\alpha\beta} \partial X^\alpha \partial X^\beta + \Phi(X) R^{(2)}$

small T $\int \mathcal{D}X!$

large T or 

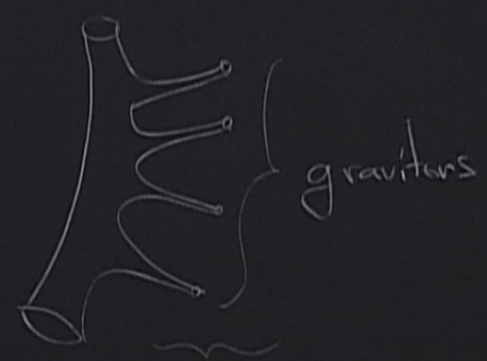

 minimal area surface

$\int G_{\mu\nu}(X) \partial X^\mu \partial X^\nu$ background

$$\int dz \sqrt{z \bar{z}}$$

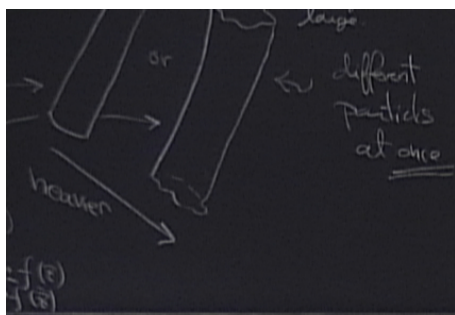
cutien

$$\begin{matrix} z \rightarrow f(z) \\ \bar{z} \rightarrow \bar{f}(\bar{z}) \end{matrix}$$



$$\Sigma_{MO} \rightarrow \eta + h_{MO}$$

$$\frac{M}{n!} \left(\Delta_{grav} \right)^n \rightarrow SS = \int h_{\mu\nu}(x) \partial^\mu \bar{\psi}^\nu \times \eta$$



all contained in a single string topology

graviton w/
pol $h_{\mu\nu}$

$f(z)$
 $f(\bar{z})$

$$\eta_{\mu\nu} \rightarrow \eta_{\mu\nu} + h_{\mu\nu}$$

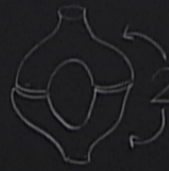
$$\int h_{\mu\nu}(X) \partial X^\mu \bar{\partial} X^\nu$$

$$(g_s)^{2g-2+n}$$



$g=1, n=2$

$$2-2+2=2$$



$2 \times \begin{pmatrix} g=0 \\ n=3 \end{pmatrix}$

$$2(0-2+3)=2$$

all contained in a single string topology

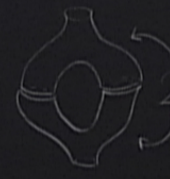
graviton $\omega_{\mu\nu}$

$$(g_s)^{2g-2+n}$$



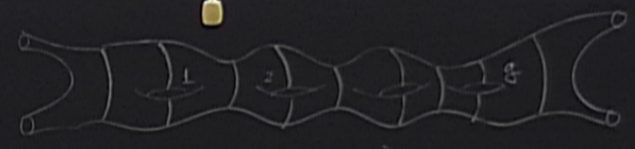
$$g=1, n=2$$

$$2-2+2=2$$



$$2 \times \begin{pmatrix} g=0 \\ n=3 \end{pmatrix}$$

$$2(0-2+3)=2$$



$$\left(\# \text{ of } \text{diagram} = 2g-2 \right) \times (1)$$

$$2g-2+4 \checkmark$$

$$+2$$

$$= \sum_{g \in \mathbb{N}} \dots$$

2d QFT

$$\Rightarrow G$$

all contained in a single string topology

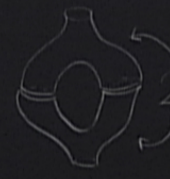
graviton w/
pol μ

$(g_s) \quad 2g - 2 + n$



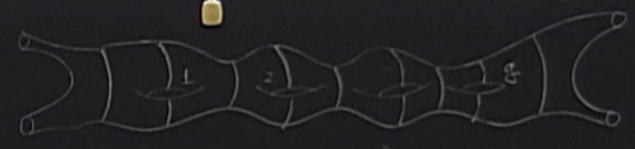
$g=1, n=2$

$2 - 2 + 2 = 2$



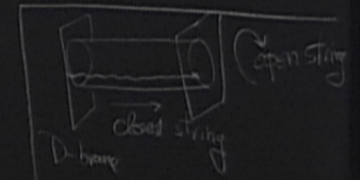
$2 \times \begin{pmatrix} g=0 \\ n=3 \end{pmatrix}$

$2(0 - 2 + 3) = 2$



$(\# \text{ of } \text{ } = 2g - 2) \times (1)$

$2g - 2 + 4$
 $+ 2$



$= \sum_{g \in \text{gen}}$

2d QFT

$\Rightarrow G$

Large N_c (in QCD, $N_c = 3$)

$$\mathcal{L}_{\text{pure gluon}} \propto \frac{1}{g^2} \text{Tr} (F_{\mu\nu} F^{\mu\nu} + \dots)$$

$+ \dots$)

$$\mu \frac{d}{d\mu} g_{\text{YM}} = \frac{g_{\text{YM}}^3 N_c}{16\pi^2} \left(-\frac{11}{3} + \dots \right)$$

$g_{\text{YM}} \rightarrow 0$
small
but N_c large
 $\rightarrow \infty$

$$\mu \frac{d}{d\mu} \lambda = \# \lambda^2$$

$$\lambda = \frac{g_{\text{YM}}^2 N_c}{\dots}$$

fixed

't Hooft coupling

• Another example: matrix valued scalar $U(N)$

$$\mathcal{L} = \frac{1}{g^2} \text{tr}_{N \times N} \left(\partial_\mu \Phi \partial_\mu \bar{\Phi} + c_3 \Phi^3 + c_4 \bar{\Phi}^4 \right) \stackrel{\text{rescale}}{=} \text{tr}(\partial \tilde{\Phi})^2 + c_3$$



$$\begin{aligned}
 & \text{or } U(N) \\
 & + C_4 \bar{\Phi}^4 \Big) \stackrel{\text{rescale}}{=} \text{tr} \left((\partial \tilde{\Phi})^2 + C_3 g \tilde{\Phi}^3 + C_4 g^2 \tilde{\Phi}^4 \right)
 \end{aligned}$$

$$\text{or } U(N) + C_4 \bar{\Phi}^4 \Big) \stackrel{\text{rescale}}{=} \text{tr} \left((\partial \tilde{\Phi})^2 + C_3 g \tilde{\Phi}^3 + C_4 g^2 \tilde{\Phi}^4 \right)$$

goal: λ fixed w/
 $N_c \rightarrow \infty$
 $g_{YM} \rightarrow 0$

- Another example. matrix valued scalar $U(N)$

$$\mathcal{L} = \frac{1}{g^2} \text{tr}_{N \times N} \left(\partial_\mu \Phi \partial_\mu \bar{\Phi} + c_3 \Phi^3 + c_4 \bar{\Phi}^4 \right) \stackrel{\text{rescale}}{=} \text{tr} (\partial \tilde{\Phi})^2 + c_3 g$$

$$\Rightarrow \langle \Phi^i_j \Phi^k_l \rangle$$

- Another example. matrix valued scalar $U(N)$

$$\mathcal{L} = \frac{1}{g^2} \frac{1}{N^2} \left(\partial_\mu \Phi^i \partial_\mu \bar{\Phi}^i + c_3 \Phi^3 + c_4 \bar{\Phi}^4 \right) \quad \text{rescale } \Rightarrow \dots^2 + c_3 g$$

$$\Rightarrow \langle \Phi^i_j \bar{\Phi}^k_l \rangle = \dots$$

$$\langle e^{ikx} e^{ikx} \rangle = e^{k^2 \log |z-w|} = \frac{1}{|z-w|^2} \quad \text{if } k^2 = -2 \leftarrow \text{tachyon}$$

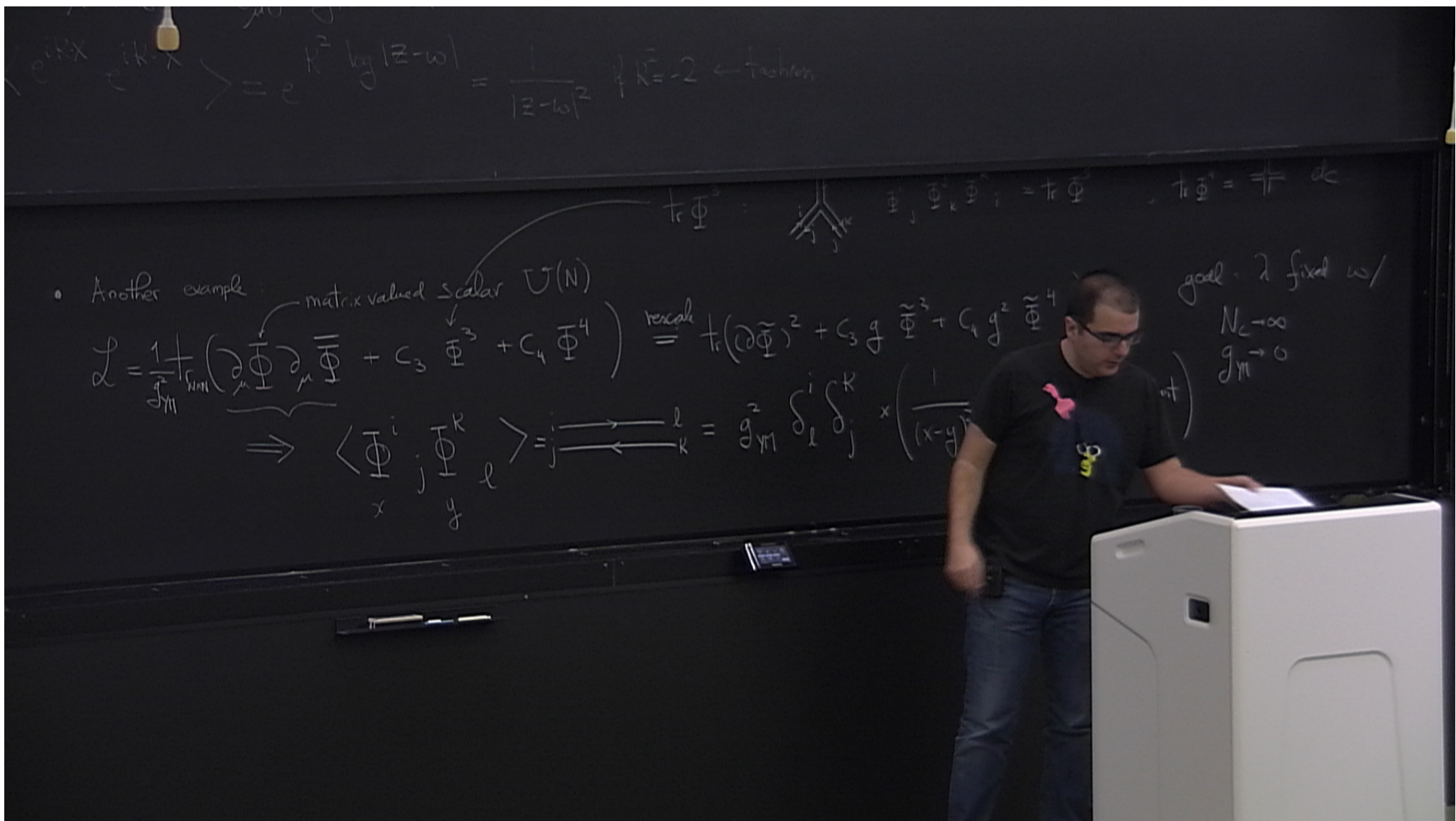
• Another example

matrix valued scalar $U(N)$

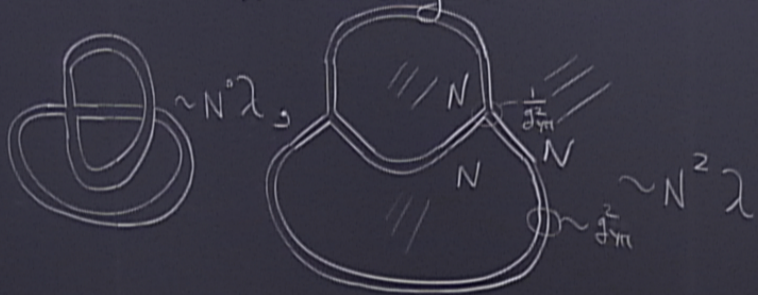
$$\mathcal{L} = \frac{1}{g^2} \text{tr}_{N \times N} \left(\partial_\mu \Phi^\dagger \partial_\mu \Phi + C_3 \Phi^3 + C_4 \Phi^4 \right) \stackrel{\text{rescale}}{=} \text{tr} \left((\partial \tilde{\Phi})^2 + C_3 g \tilde{\Phi}^3 + C_4 g^2 \tilde{\Phi}^4 \right)$$

$$\Rightarrow \langle \Phi^i_x \Phi^k_y \rangle = \begin{matrix} \xrightarrow{l} \\ \xleftarrow{k} \end{matrix} = g^2 \int_l^i \int_j^k \times \left(\frac{1}{(x-y)^2} \right) \leftarrow G(x-y) \text{ irrelevant}$$

goal: λ
 $N_c \rightarrow \infty$
 $g_m \rightarrow 0$



A vacuum diagram with V vertices, E propagators (edges), F closed loops of colour (faces)



$$V=2, E=3, F=3$$

scales as

$$\left(\frac{\lambda}{N}\right)^E \left(\frac{N}{\lambda}\right)^V N^F$$

all contained in a single string topology

graviton w/ pol $\lambda^{\mu\nu}$

$$\mu \frac{d}{d\mu} g_{\text{YM}} = \frac{g_{\text{YM}}^3 N_c}{16\pi^2} \left(-\frac{11}{3} + \dots \right)$$

$$\mu \frac{d}{d\mu} \lambda = + \lambda^2 \Rightarrow \lambda = \frac{g_{\text{YM}}^2 N_c}{\mu}$$

fixed

small
 N_c large
 graviton

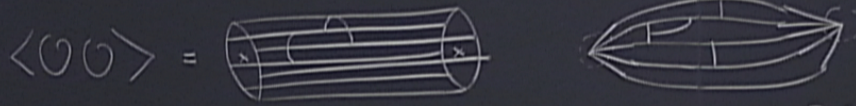
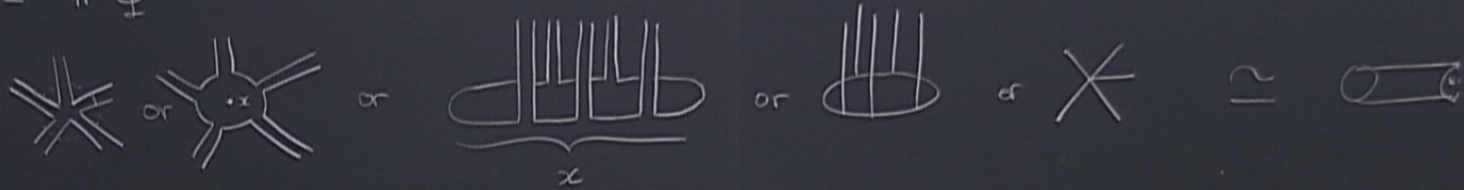
• Another example: matrix-valued scalars $\mathcal{U}(N)$

$$\mathcal{L} = \frac{1}{2} \text{tr} \left(\partial_\mu \Phi^i \partial_\mu \bar{\Phi}^i + c_3 \Phi^3 + c_4 \Phi^4 \right)$$

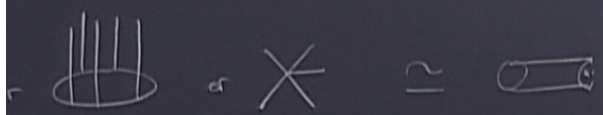
rescale $\Rightarrow \frac{1}{\mu}$

$$\Rightarrow \langle \Phi^i_j \Phi^k_l \rangle = \delta^i_l \delta^k_j$$

$$\langle \emptyset \rangle = \frac{1}{r} \Phi^{\sqrt{2}}$$



$$\langle \mathcal{O}_1 \mathcal{O}_n \rangle_{\text{connected}} = \sum_{\text{genus } g} \left(\frac{1}{N} \right)^{2g-2+n} \mathcal{Z}_g(\mathcal{O}) = g_s \text{ (diagram)} + g_s^3 \text{ (diagram)} + \dots$$



Conclusion

(Large N) gauge theories are string theories...
 $g_s = \frac{1}{N}$ mixed.

$\left(\frac{1}{N}\right)^{2g-2+n}$
 $= g_s$
 $+ \dots$

all contained in a single string topology

$$\mu \frac{d}{d\mu} g_{\text{YM}} = \frac{g_{\text{YM}}^3 N_c}{16\pi^2} \left(-\frac{11}{3} + \dots \right)$$

$g_{\text{YM}} \rightarrow 0$
 small
 but N_c large
 $\rightarrow \infty$

$$\mu \frac{d}{d\mu} \lambda = \# \lambda^2$$

$$\lambda = \frac{g_{\text{YM}}^2 N_c}{4\pi^2}$$

fixed

λ 't Hooft coupling \leftarrow relevant combination

• Another example \rightarrow matrix valued scalars $U(N)$

$$\mathcal{L} = \frac{1}{g_{\text{YM}}^2} \text{tr} \left(\partial_\mu \Phi^i \partial_\mu \bar{\Phi}^i + c_3 \text{tr} \Phi^3 + c_4 \text{tr} \Phi^4 \right)$$

$$\Rightarrow \langle \Phi^i_j \Phi^k_l \rangle = \delta^i_l \delta^k_j$$