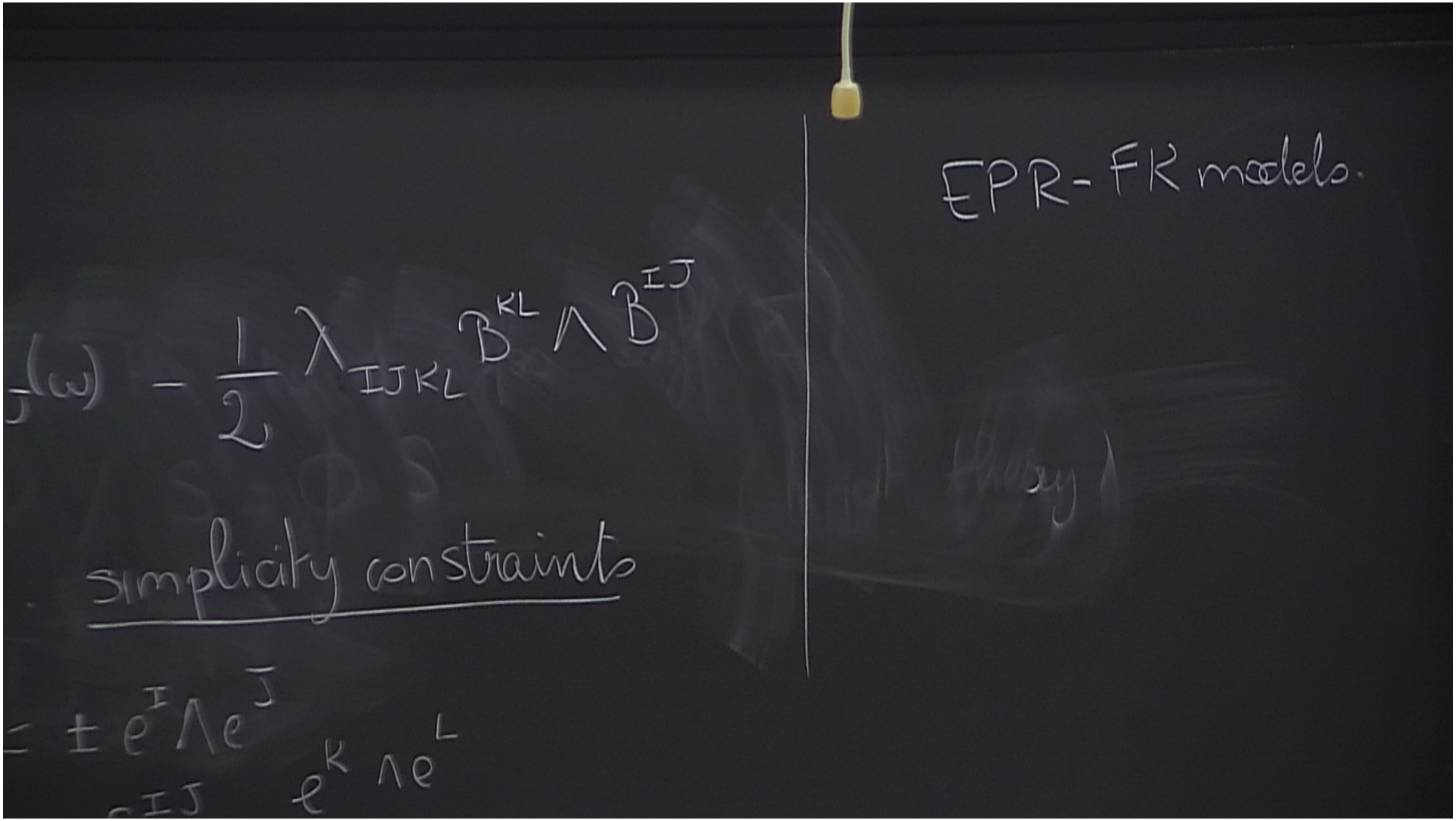
Title: PSI 2015/2016 Quantum Information - Lecture 14

Date: Mar 10, 2016 11:30 AM

URL: http://pirsa.org/16030061

Abstract:


EPR- FK models.
$(\omega) - \frac{1}{2}\lambda_{IJKL} B^{KL} \wedge B^{IJ}$
simplicity constraints
$= \pm e^I \wedge e^J$
$e^K \wedge e^L$

Q E Correction.

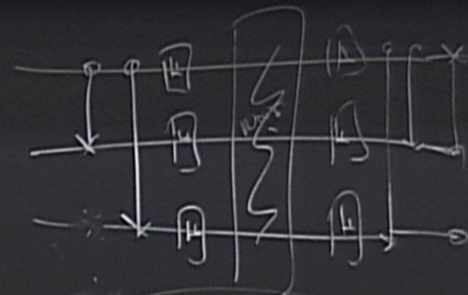
"Z" error

Noise: e.g. Kraus op. depolarising channel

Encoding.
Error Correction

$$R\left(\mathcal{E}(|\psi_e\rangle)\right) = |\psi_e\rangle$$

$$\langle i_L | E_\alpha^\dagger E_\beta | j_L\rangle = \delta_{ij} C_{\alpha\beta}$$



$$\alpha|+++\rangle + \beta|---\rangle$$

$$\mathcal{E} = \{ 1\!\!1, Z_1, Z_2, Z_3 \}$$

$\sqrt{1-3p}$ $\sqrt{p}$ $\sqrt{p}$ $\sqrt{p}$

measure

X_1X_2

X_1X_3

00

11

01

10

(syndrome

$p \to 3p^2$

$X|+\rangle = X(|0\rangle+|1\rangle) = |+\rangle$

$X|-\rangle = -|-\rangle$

X_1X_2

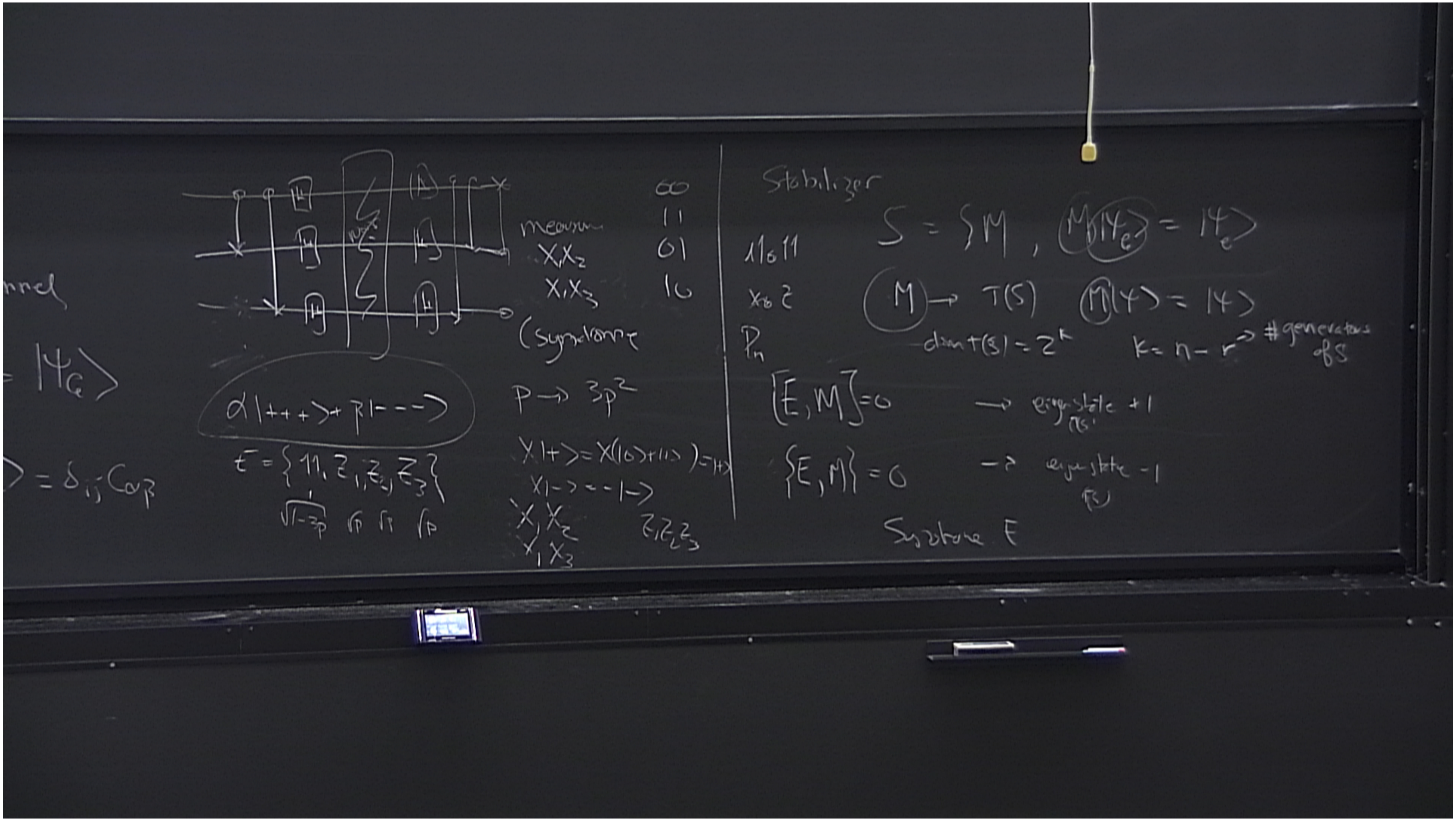
X_1X_3

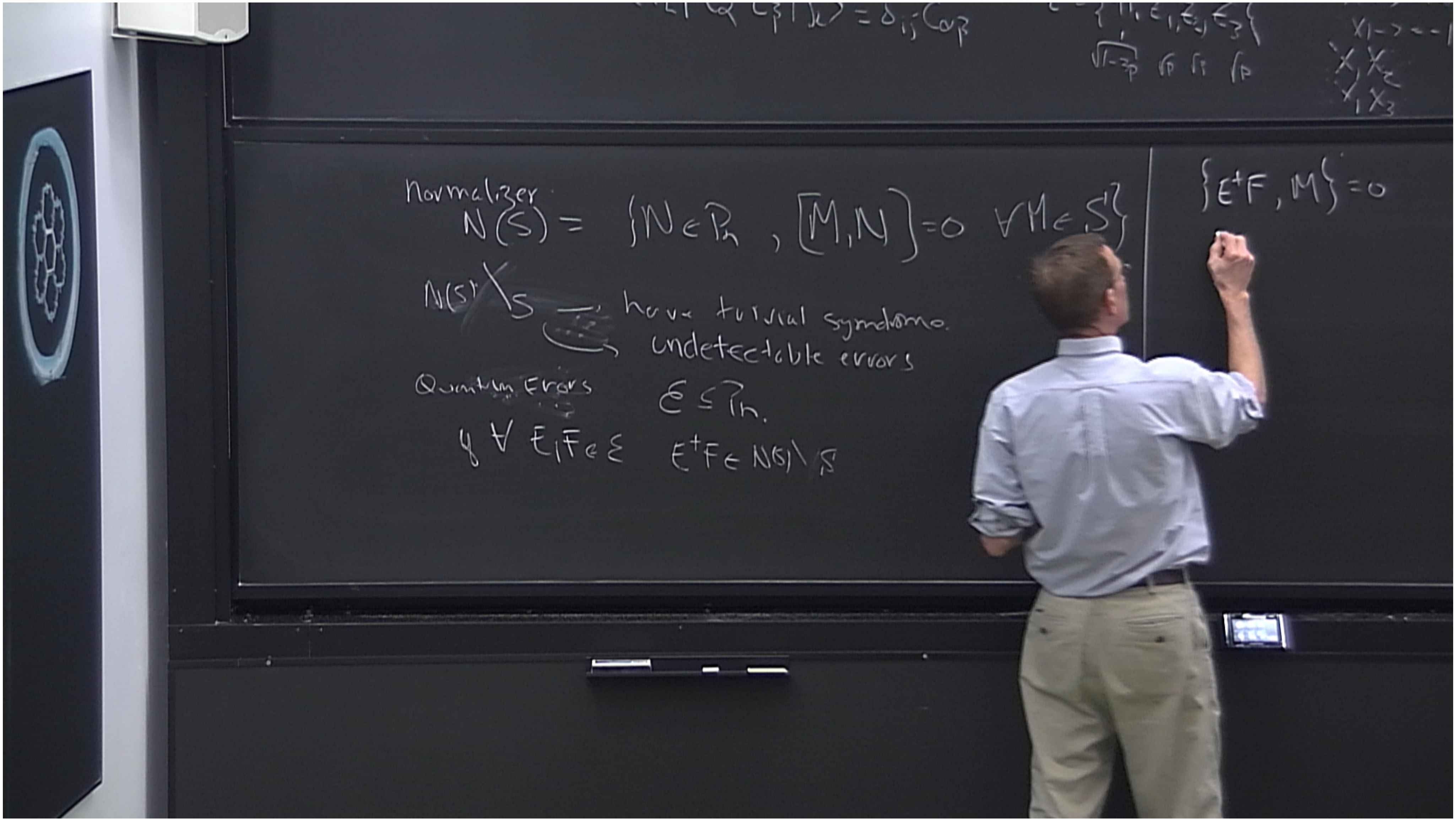
Stabilizer

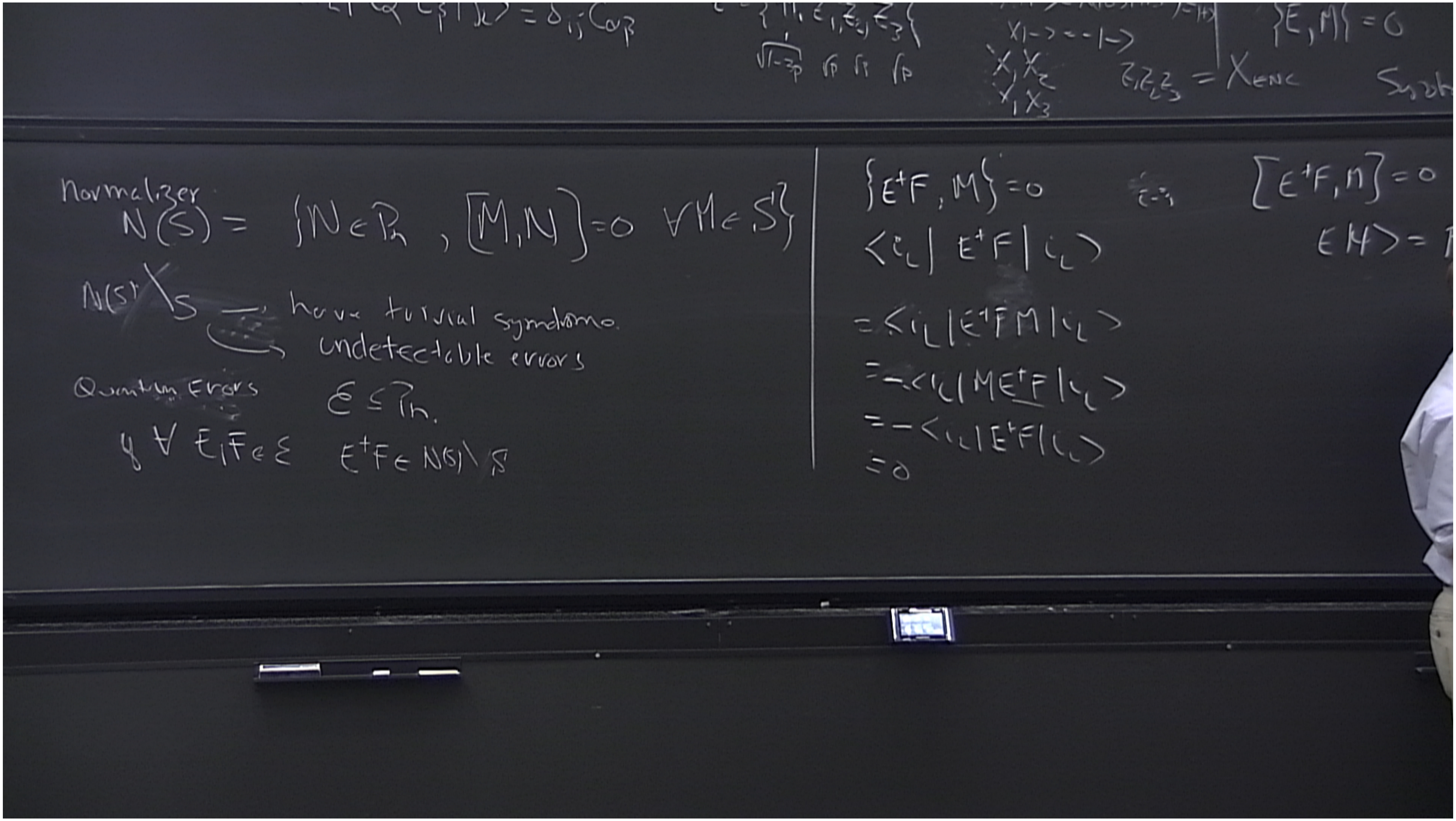
$1\!\!1 \otimes 1\!\!1$

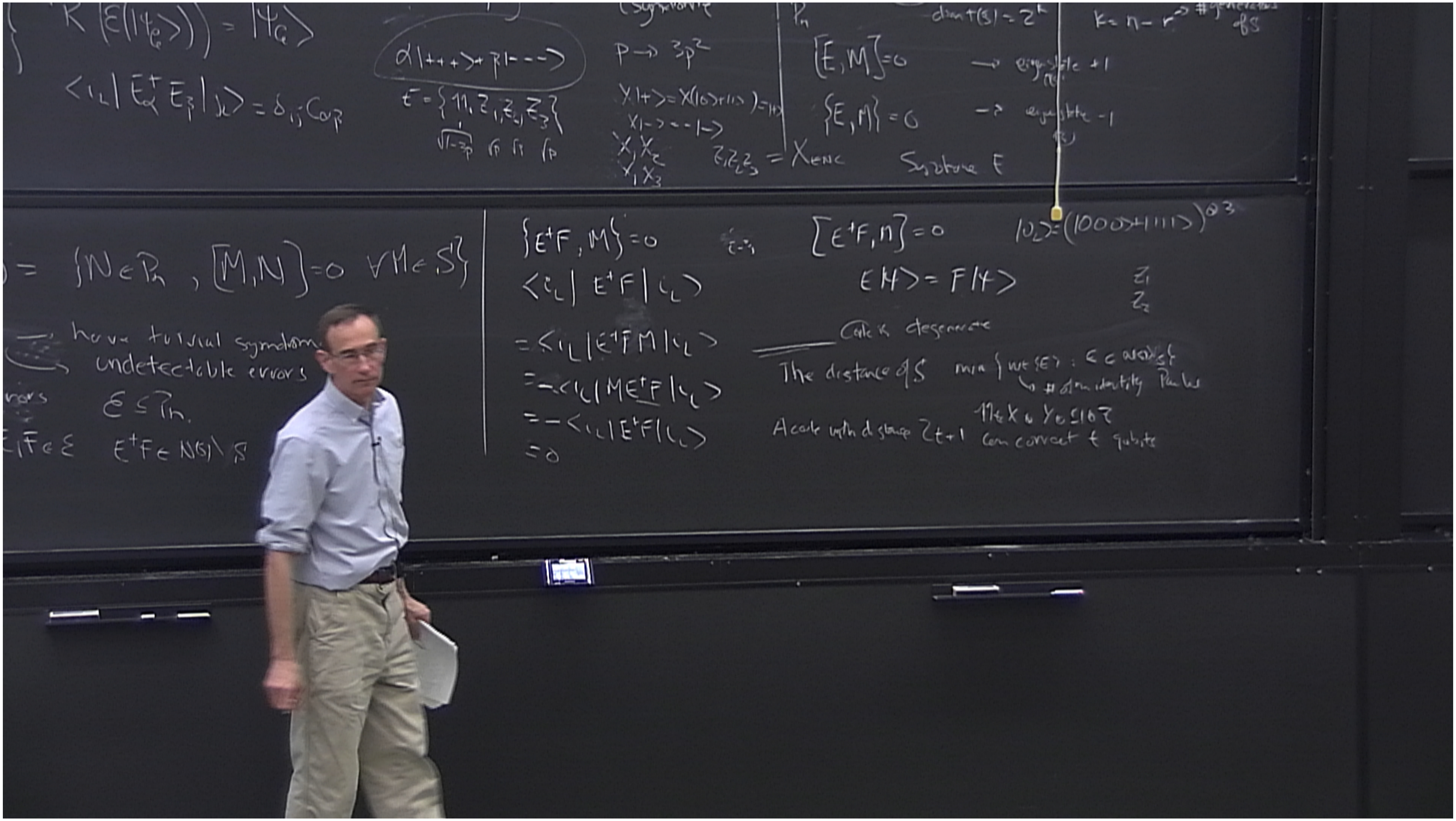
$X \otimes Z$

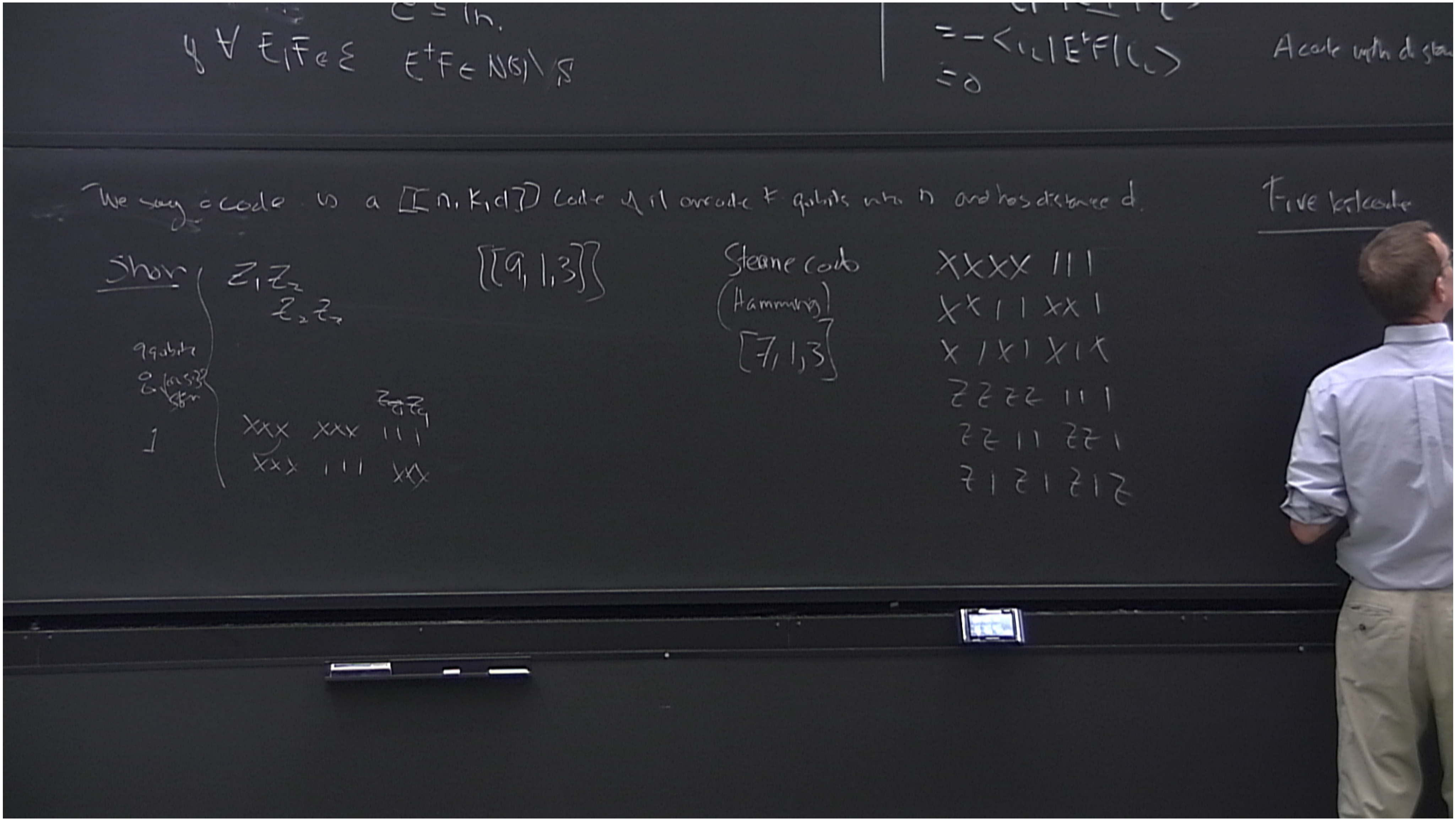
P_n


Stabilizer
measure
X_1X_2
X_1X_3
(syndrome
$P \to 3p^2$
$\alpha|{+}{+}{+}\rangle + \beta|{-}{-}{-}\rangle$
$S = \{M, \; M|\psi_e\rangle = |\psi_e\rangle$
$M \to T(S)$
$M|\psi\rangle = |\psi\rangle$
$\dim T(S) = 2^k$
$k = n - r$
generators of S
$[E, M] = 0$
eigenstate +1
$\{E, M\} = 0$
eigenstate −1
Syndrome E
$Z_1Z_2Z_3$


normalizer
N(S) = {N ∈ P_n , [M,N] = 0 ∀ M ∈ S}
N(S)\S → have trivial syndrome.
→ undetectable errors
Quantum Errors ℰ ⊆ P_n.
if ∀ E,F ∈ ℰ E†F ∈ N(S)\S
{E†F, M} = 0


Normalizer
$N(S) = \{N \in P_n , [M,N] = 0 \ \forall M \in S\}$
$N(S) \setminus S$ → have trivial syndrome.
→ undetectable errors
Quantum Errors $\mathcal{E} \subseteq P_n$.
if $\forall E, F \in \mathcal{E}$ $E^\dagger F \in N(S) \setminus S$
$\{E^\dagger F, M\} = 0$
$\langle i_L | E^\dagger F | i_L \rangle$
$= \langle i_L | E^\dagger F M | i_L \rangle$
$= -\langle i_L | M E^\dagger F | i_L \rangle$
$= -\langle i_L | E^\dagger F | i_L \rangle$
$= 0$
$[E^\dagger F, M] = 0$
$\{E, M\} = 0$
$Z_1 Z_2 Z_3 = X_{ENC}$


have trivial syndrome
undetectable errors
Code is degenerate
The distance of S
A code with distance 2t+1 can correct t qubits
Syndrome E


if ∀ E, F ∈ ℰ E†F ∈ N(S) \ S
= −⟨i_L|E†F|i_L⟩
= 0
We say a code is a [[n, k, d]] code if it encodes k qubits into n and has distance d.
Five qubit code
Shor
Z1Z2
Z2Z3
9 qubits
[[9, 1, 3]]
XXX XXX III
XXX III XXX
Steane code
(Hamming)
[7, 1, 3]
XXXX III
XXII XXI
XIXI XIX
ZZZZ III
ZZII ZZI
ZIZI ZIZ

$$= -\langle i_L | E^\dagger F | i_L \rangle$$
$$= 0$$

A code with distance $2t+1$ can correct t qubits

$\mathbb{1} \otimes X \otimes Y \otimes \mathbb{1} \otimes Z$

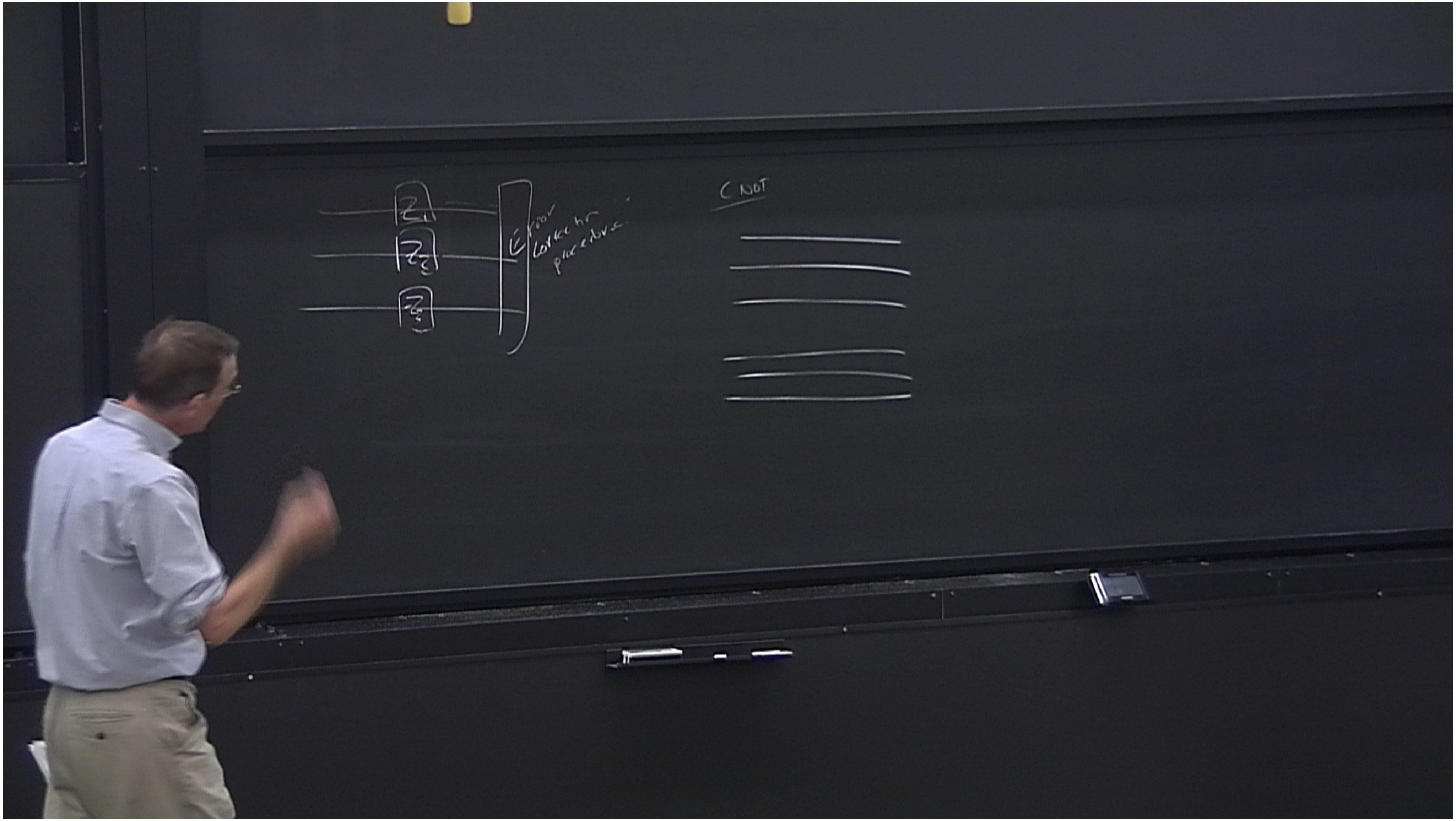
...code k qubits into n and has distance d.

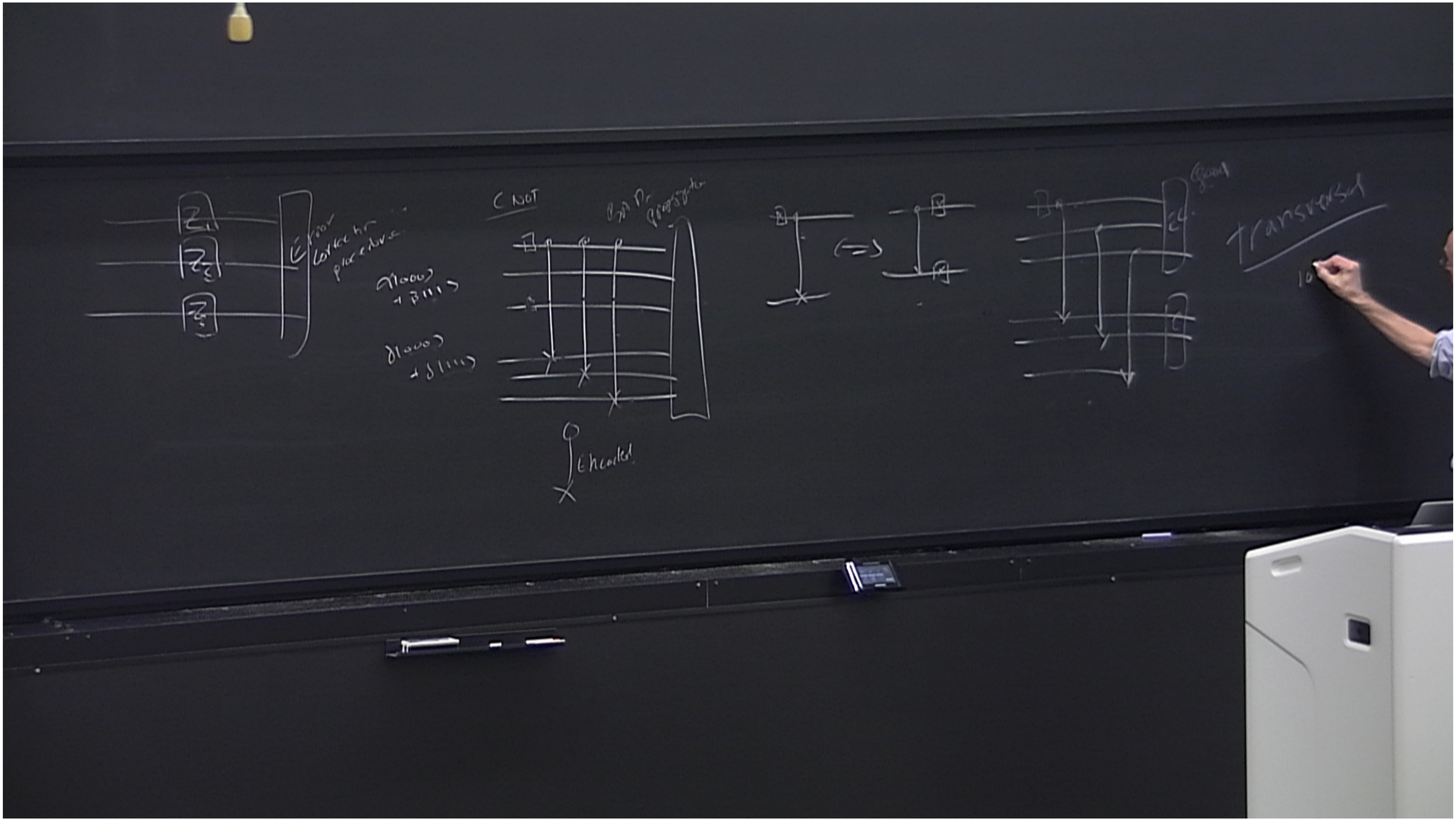
Steane code (Hamming) $[7,1,3]$

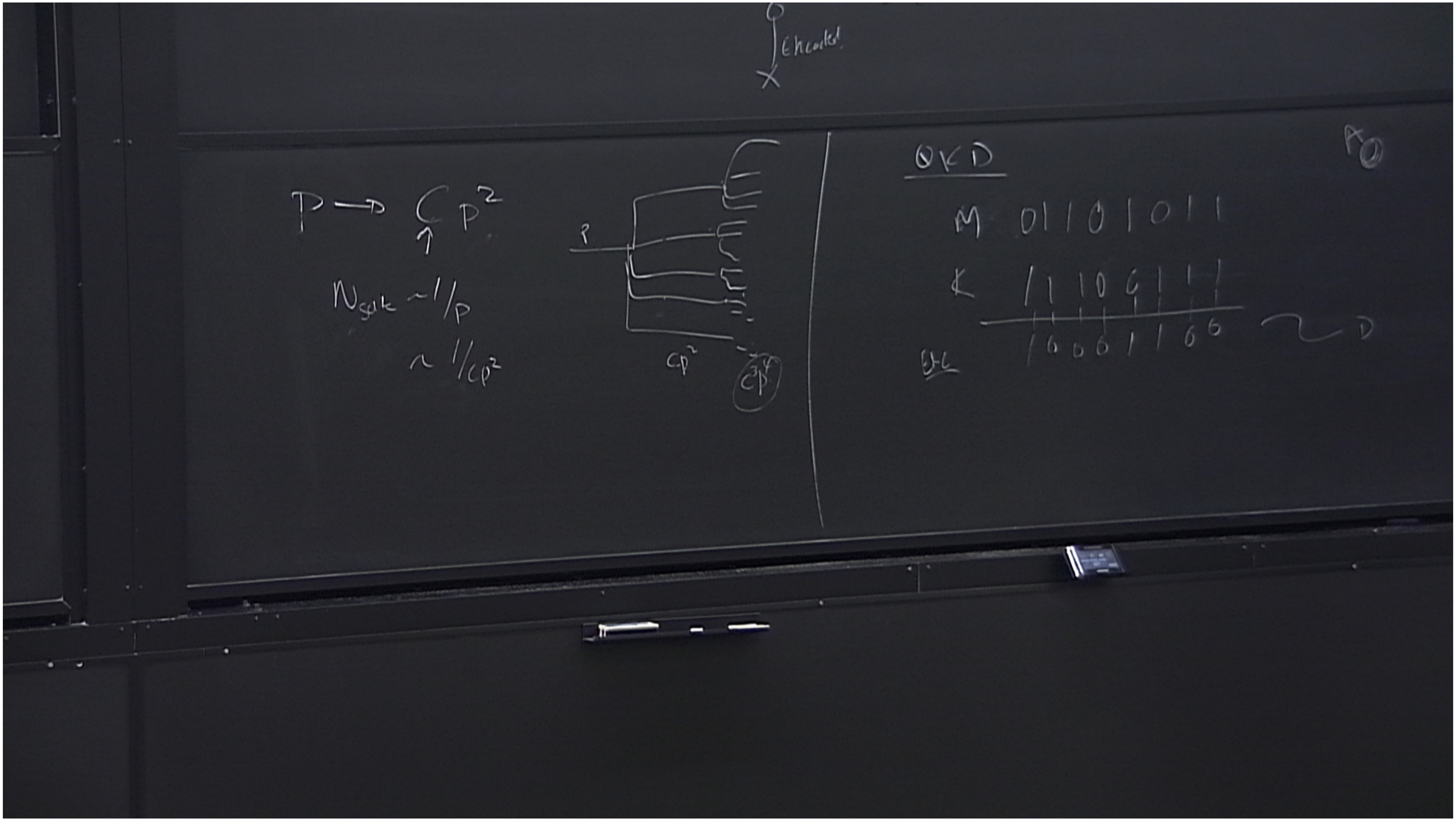
XXXX 111
XX11XX1
X1X1X1X
ZZZZ 111
ZZ11ZZ1
Z1Z1Z1Z

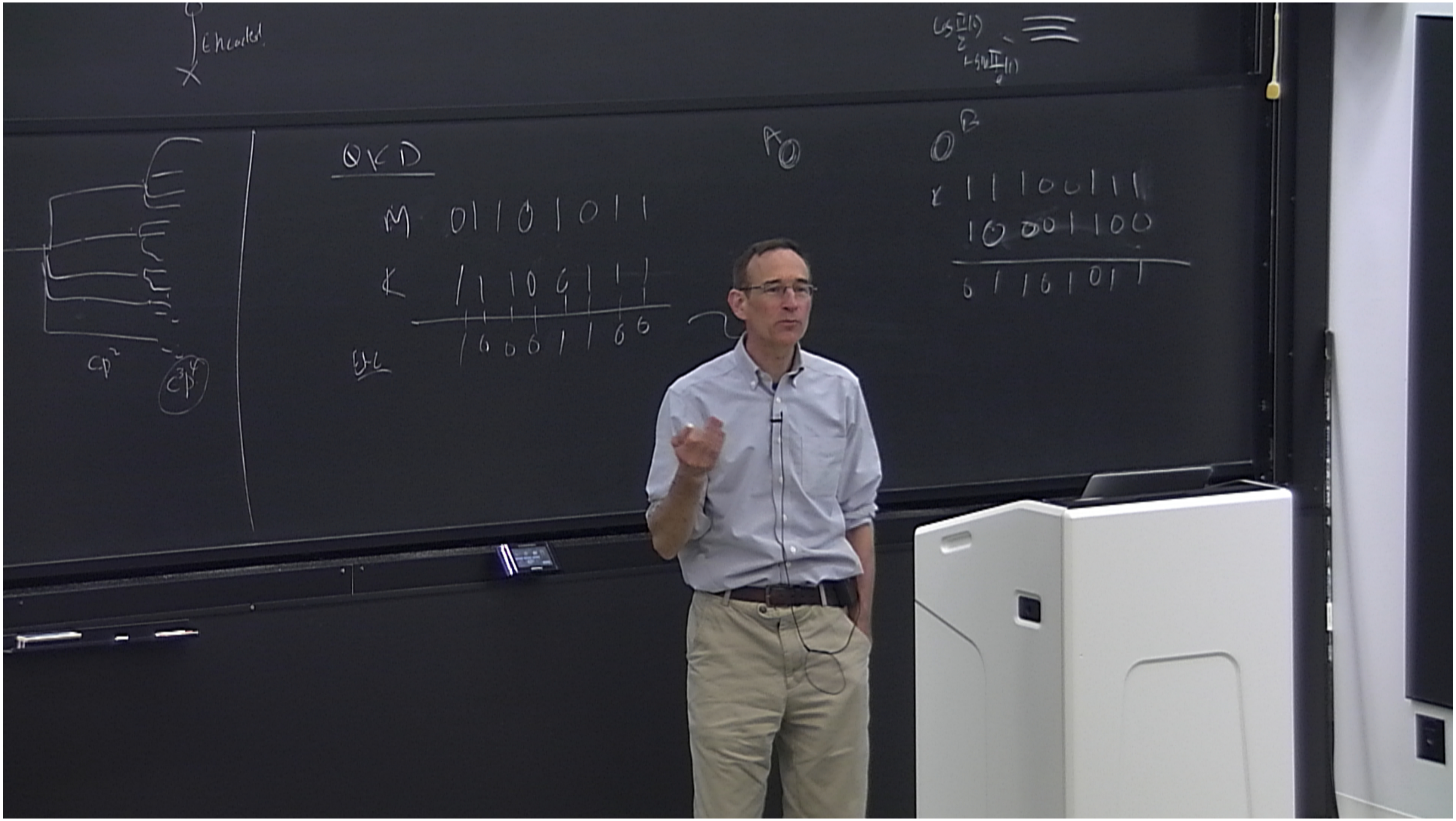
Five qubit code

XZZXI
IXZZX
XIXZZ
ZXIXZ

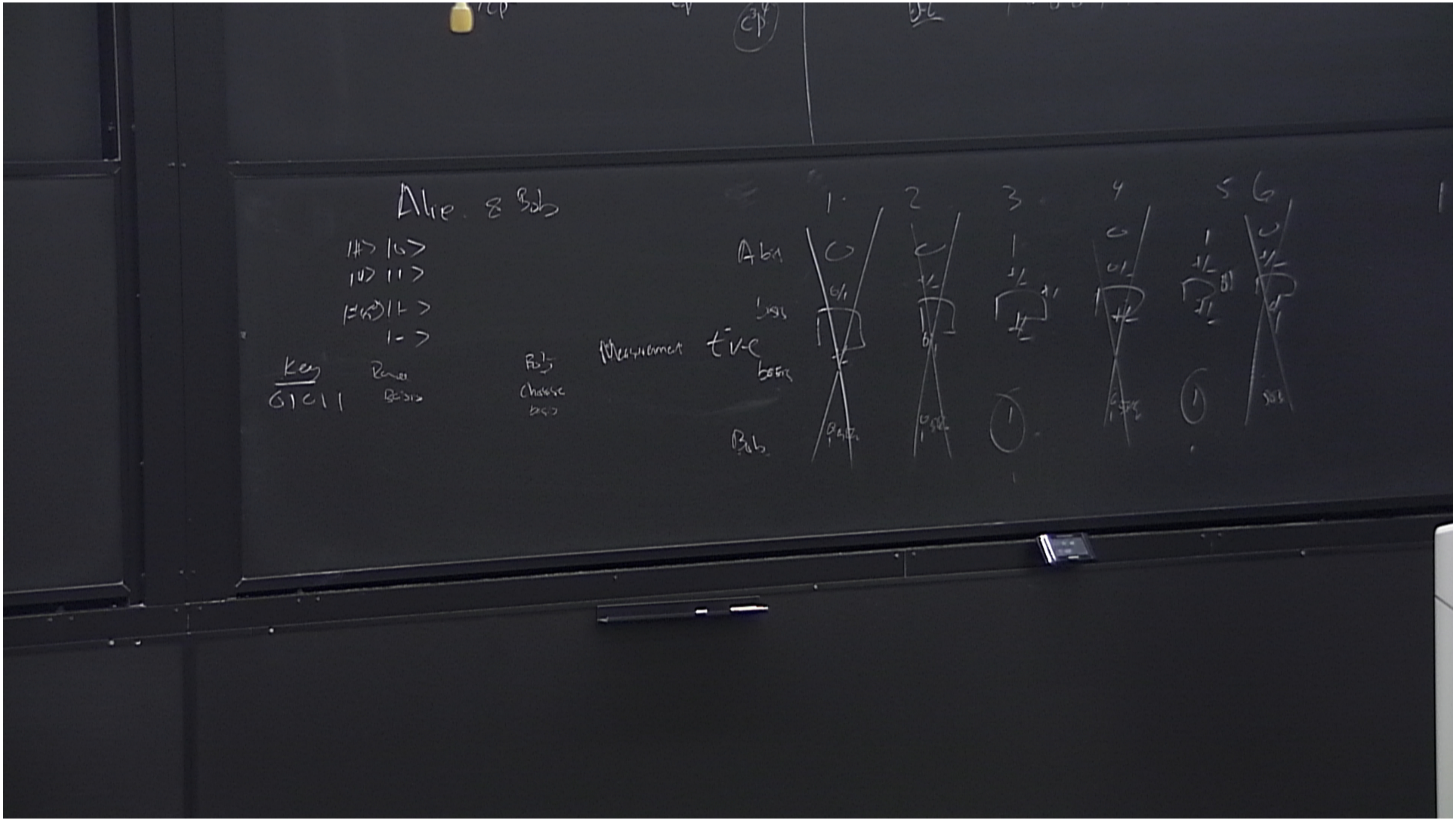

Error correction procedure
CNOT


Error correction procedure
CNOT
Encoded
EC
Transversal


Encoded
P → Cp²
N_gate ~ 1/p
~ 1/cp²
p
cp²
QKD
M 01101011
K 11100111
10001100
D



QKD
M 01101011
K 11106111
Enc


Key
Bob
1
2
3
4
5
6