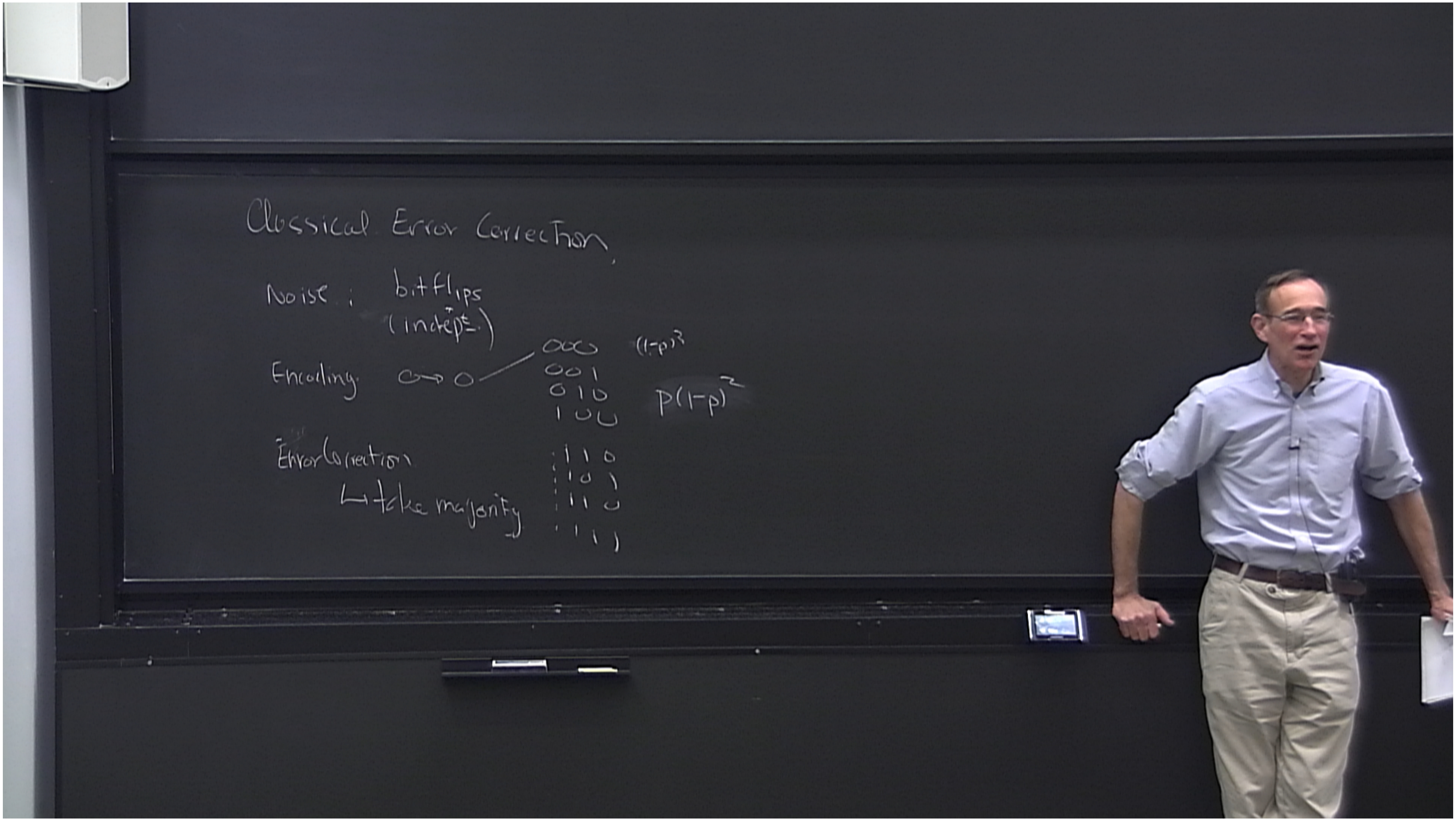
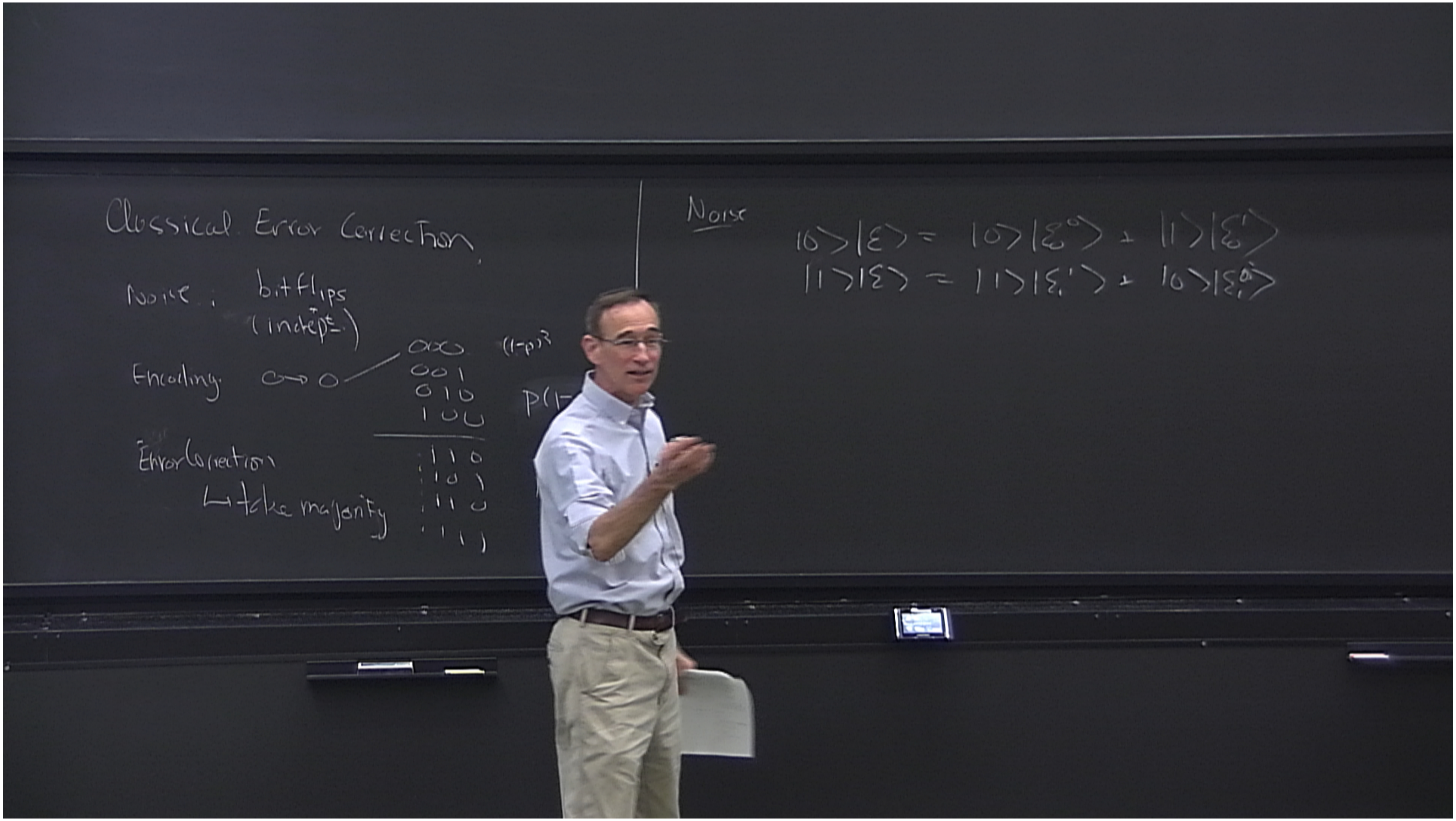
Title: PSI 2015/2016 Quantum Information - Lecture 13

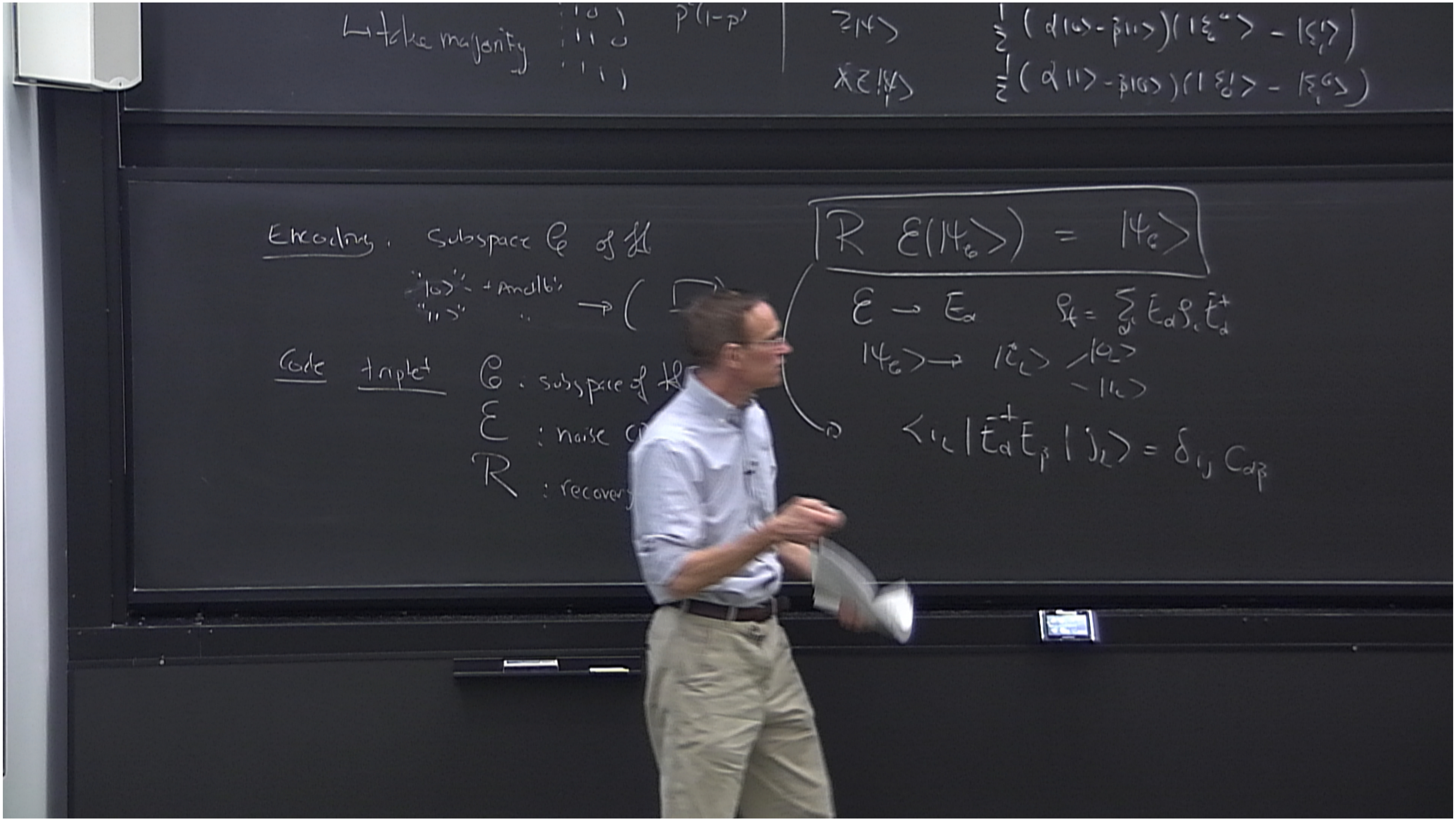
Date: Mar 09, 2016 11:30 AM

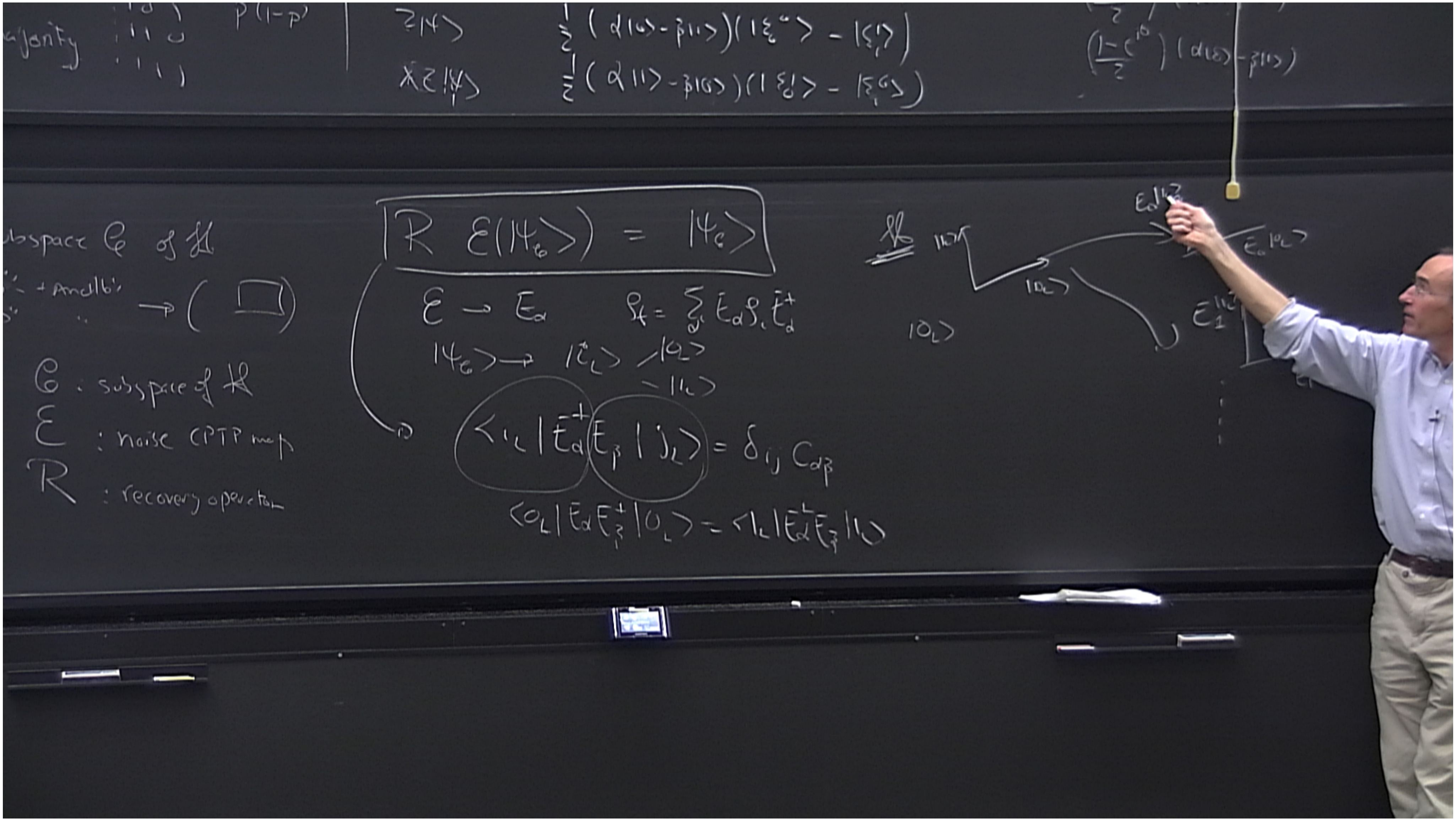
URL: http://pirsa.org/16030060

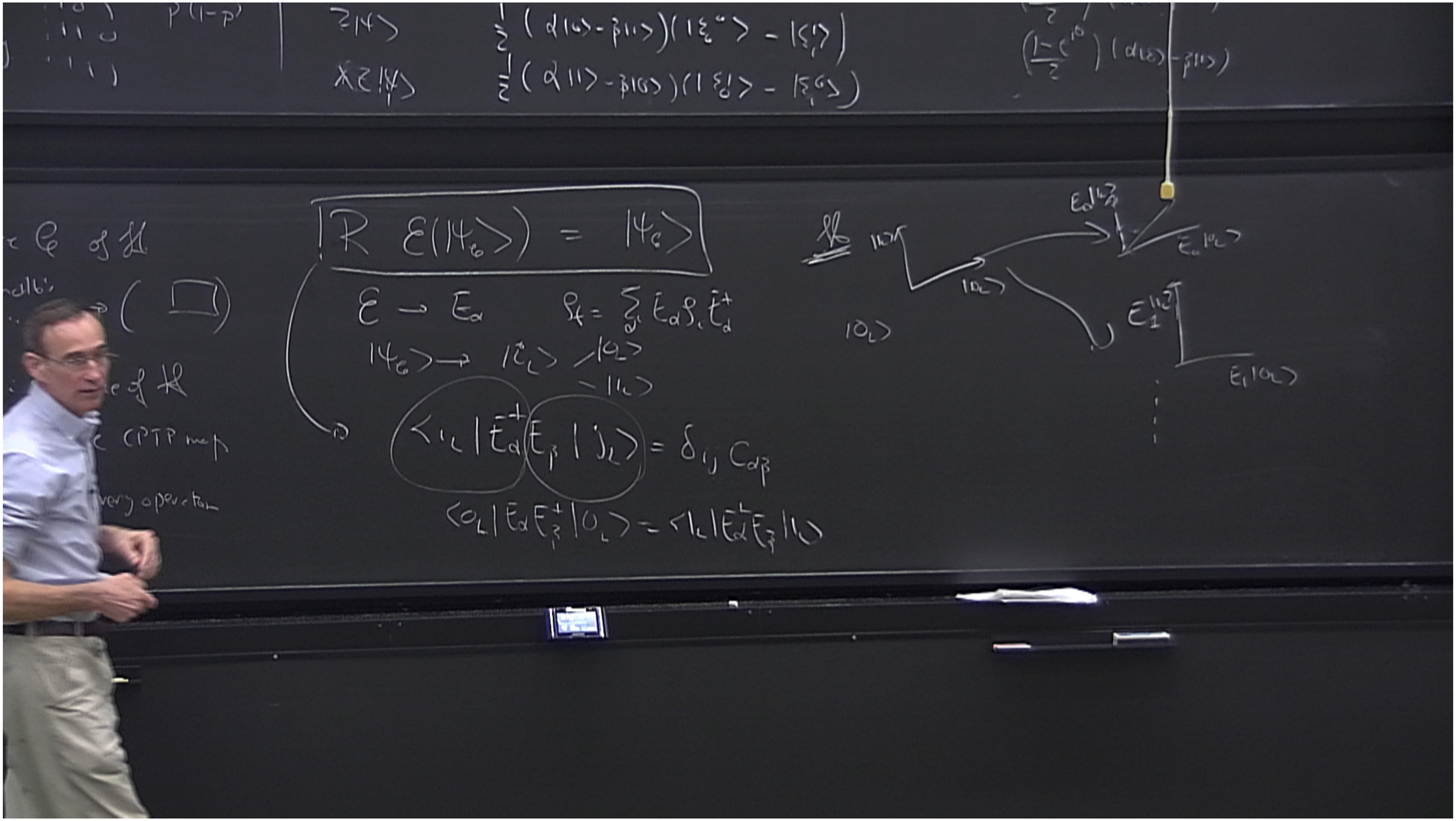
Abstract:

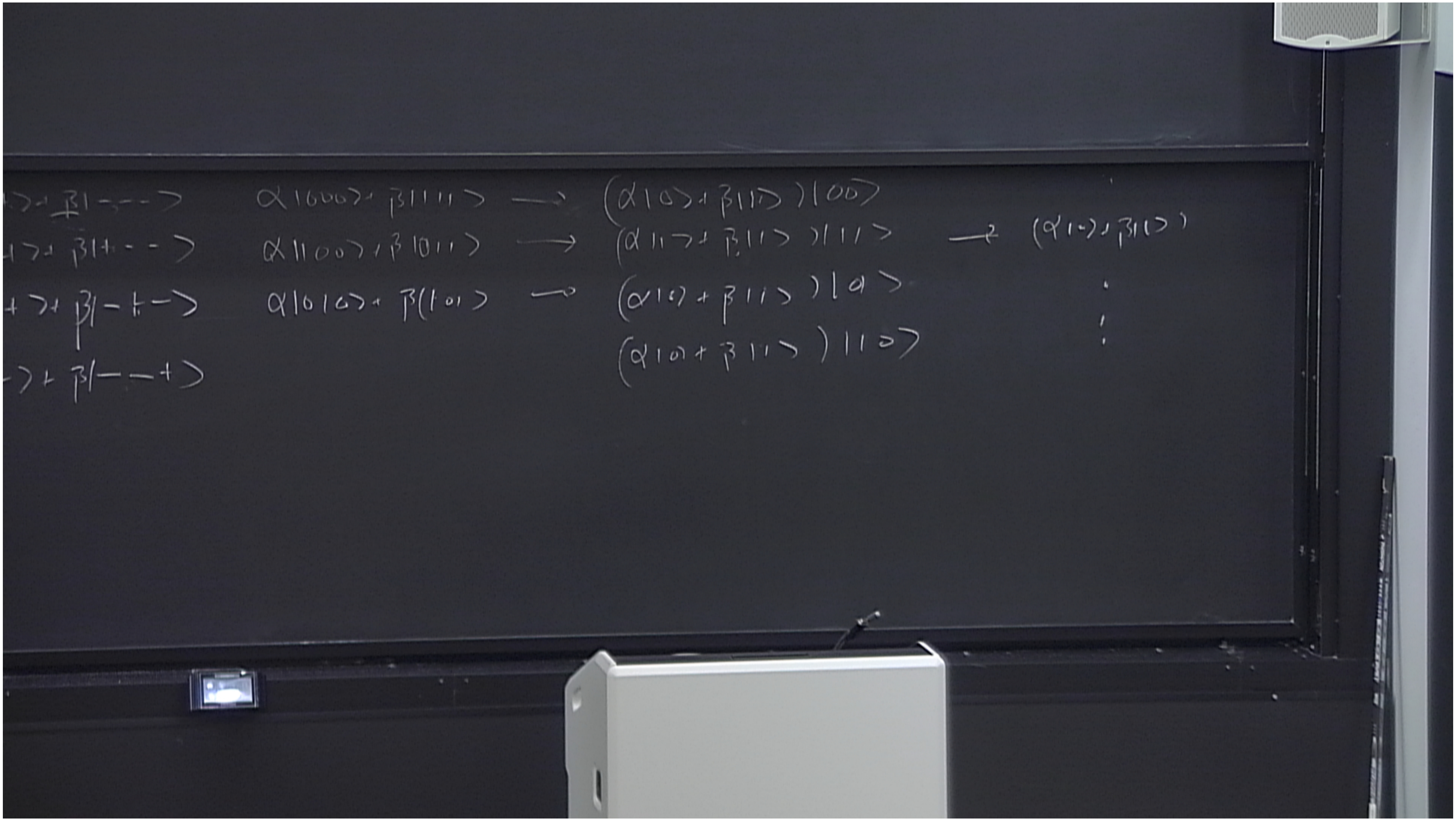

Classical Error Correction
Noise: bit flips
(indep't)
Encoding 0 → 000
000
001
010
100
(1-p)^3
p(1-p)^2
Error Correction
↳ take majority
110
101
011
111

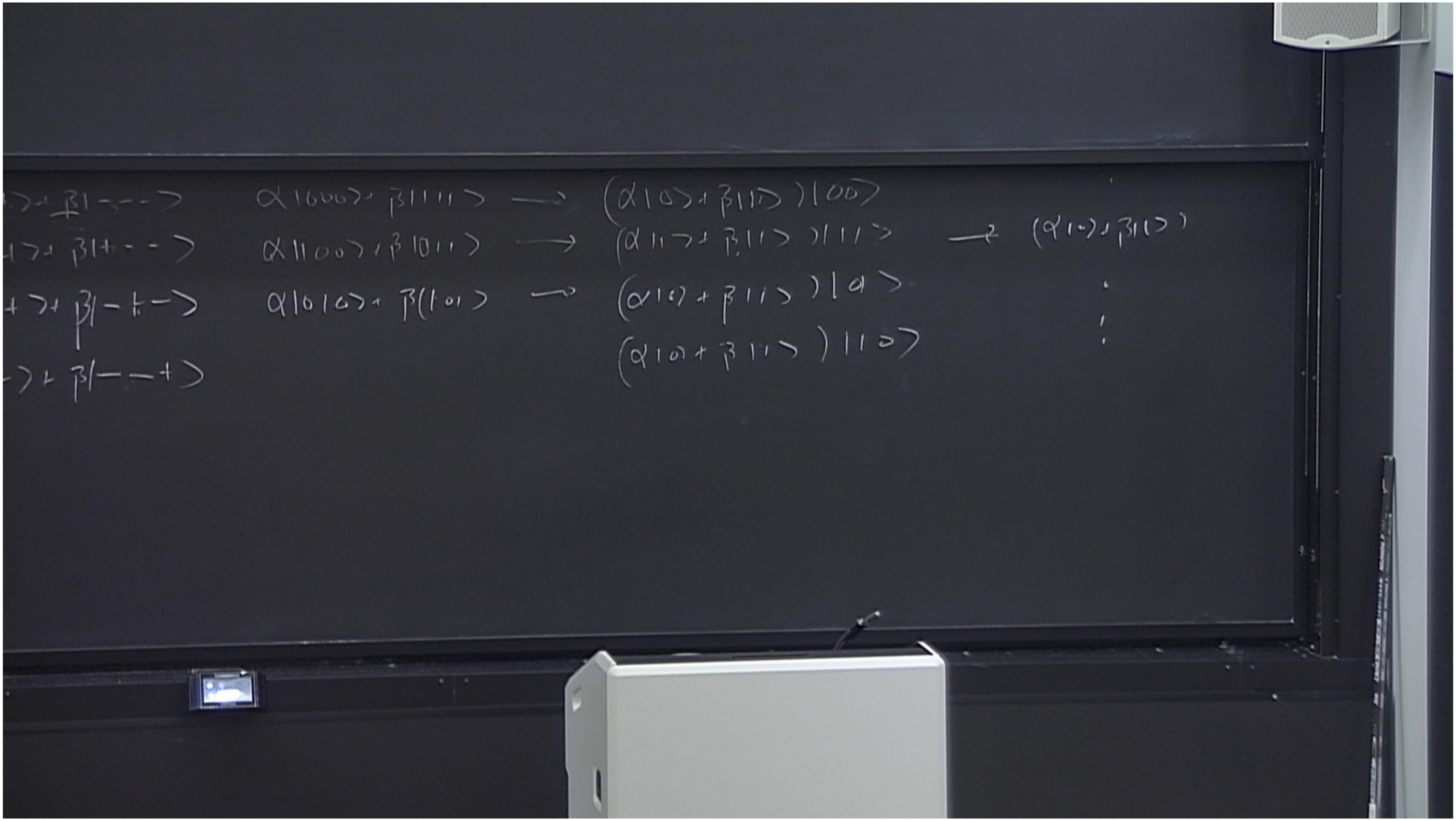

Classical Error Correction
Noise: bitflips
Encoding
Error Correction
take majority
Noise


take majority
Encoding. Subspace C of H
Code triplet
C . subspace of H
E : noise
R : recovery
R E(|ψ_c>) = |ψ_c>
E → E_α
ρ_f = Σ_α E_α ρ_i E_α^†
<i_L| E_α^† E_β |j_L> = δ_ij C_αβ


R E(|ψ_C>) = |ψ_C>
E → E_α
ρ_f = Σ_α E_α ρ_i E_α^†
|ψ_C> → |i_L>
|0_L>
|1_L>
<i_L| E_α^† E_β |j_L> = δ_ij C_αβ
<0_L| E_α E_β^† |0_L> = <1_L| E_α^† E_β |1_L>
subspace C of H
C : subspace of H
E : noise CPTP map
R : recovery operator
majority


R ε(|ψ_c>) = |ψ_c>
ε → E_α
ρ_f = Σ_α E_α ρ_i E_α^†
<i_L | E_α^† E_β | j_L> = δ_ij C_αβ
<0_L | E_α E_β^† | 0_L> = <1_L | E_α^† E_β | 1_L>
CPTP map
every operator


>+ β|−−−−>
α|0000> + β|1111> → (α|0> + β|1>)|00>
>+ β|+−−−>
α|1100> + β|0011> → (α|1> + β|1>)|11> → (α|0> + β|1>)
+>+ β|−−+−>
α|0101> + β|1010> → (α|0> + β|1>)|01>
(α|0> + β|1>)|10>
−>+ β|−−−+>



|-> + β|---> α|000> + β|111> ⟶ (α|0> + β|1>)|00>
|+> + β|+--> α|100> + β|011> ⟶ (α|1> + β|1>)|11> ⟶ (α|0> + β|1>)
|+> + β|-+-> α|010> + β|101> ⟶ (α|0> + β|1>)|0>
(α|0> + β|1>)|10>
|-> + β|--+>

$(\alpha|00\rangle+\beta|11\rangle)|0\rangle$

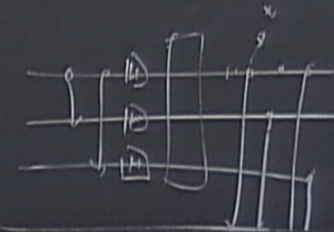
$\xi_2 = Z_2$

$\xi_3 = Z_3$

$(\alpha|000\rangle+\beta|111\rangle) \to \alpha|+++\rangle+\beta|---\rangle$

$Z|\pm\rangle = Z(|0\rangle \pm |1\rangle)$

$\alpha|++-\rangle + \beta|--+\rangle$



$$|0_L\rangle = (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$

$$|1_L\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

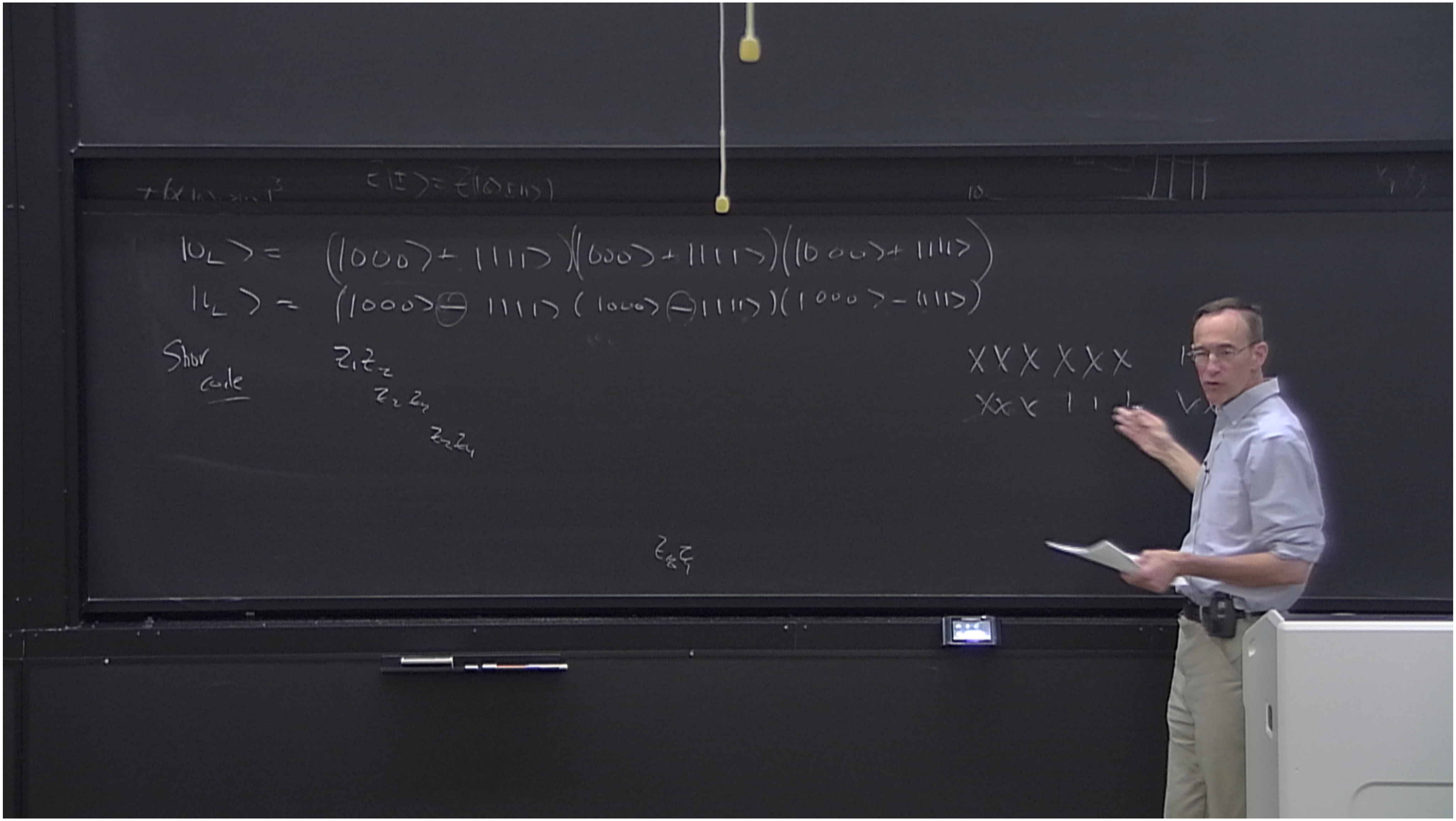
Shor

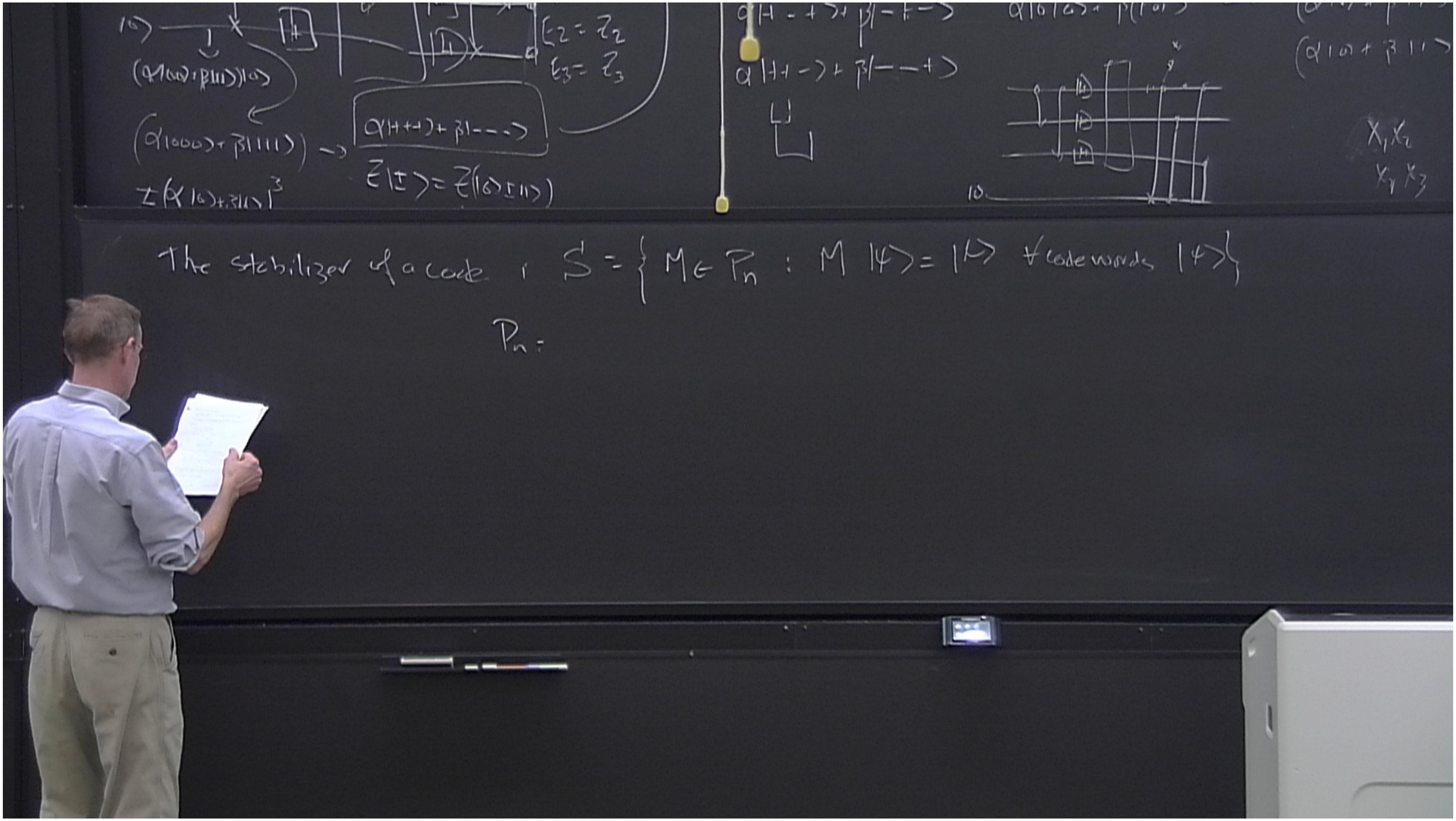
$Z_1 Z_2$

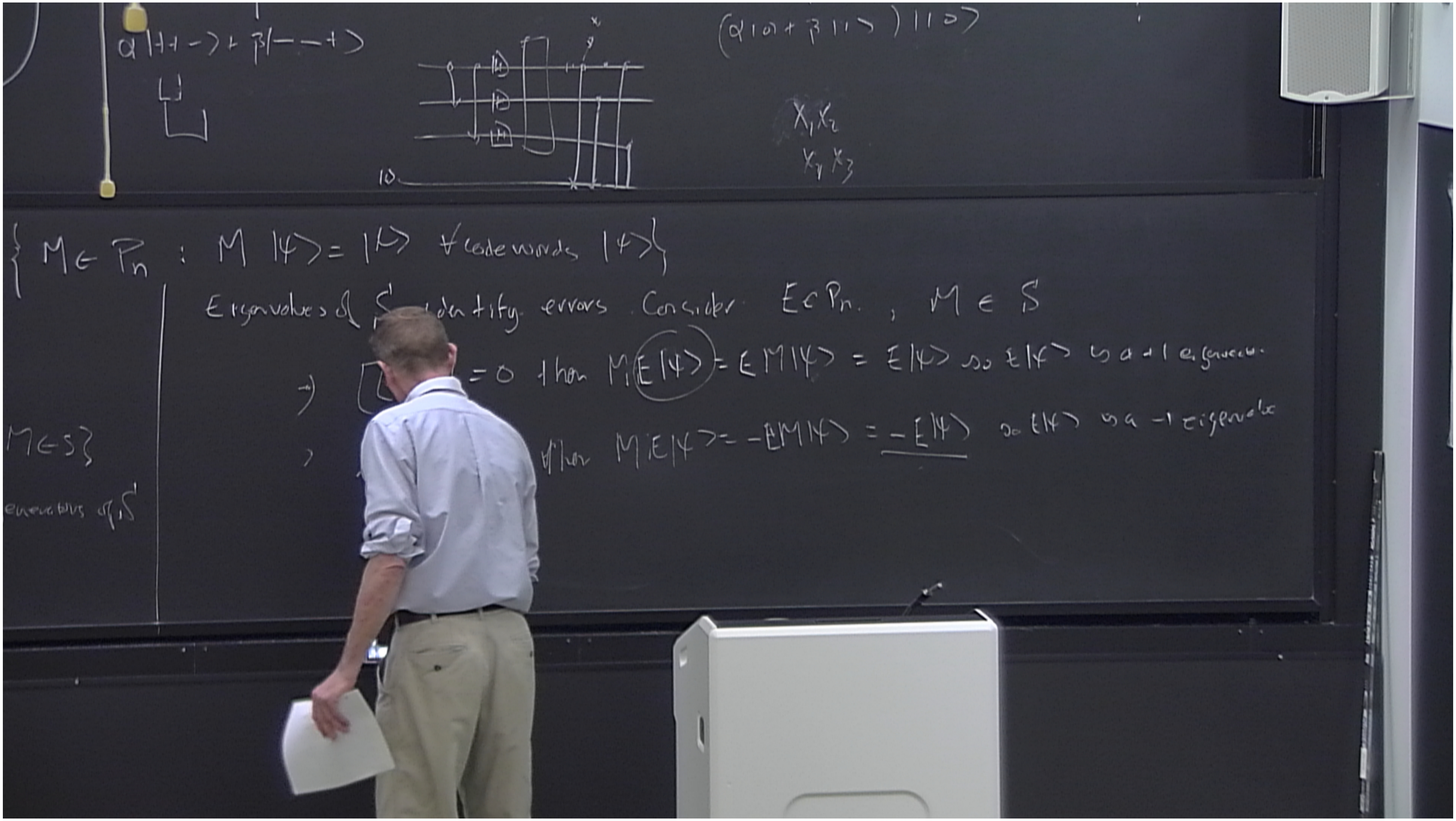
$Z_2 Z_3$

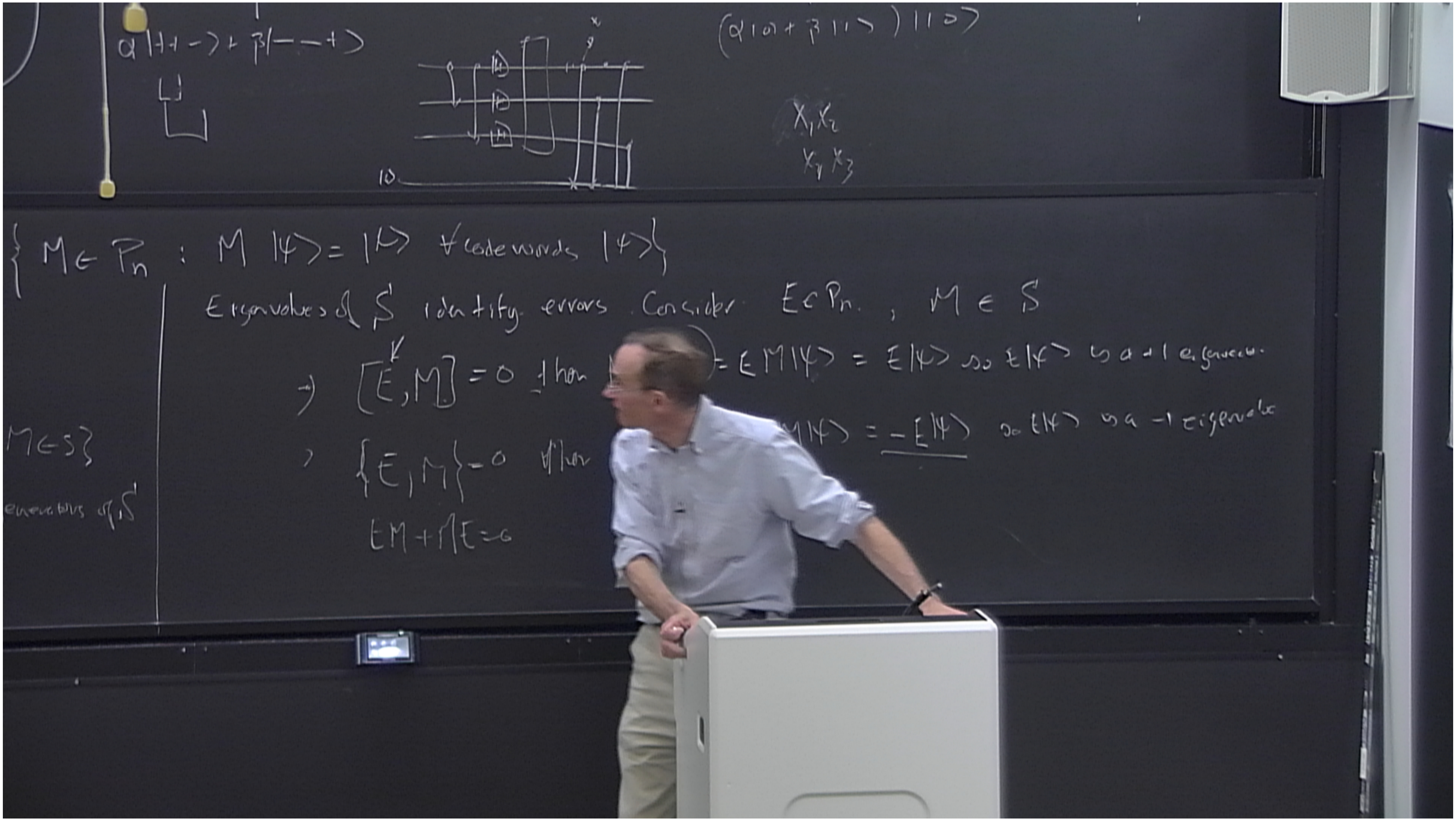
$Z_2 Z_4$

$Z_8 Z_9$


|0_L> = (|000> + |111>)(|000> + |111>)(|000> + |111>)
|1_L> = (|000> − |111>)(|000> − |111>)(|000> − |111>)
Shor code
XXXXXX


The stabilizer of a code: S = { M ∈ P_n : M |ψ⟩ = |ψ⟩ ∀ codewords |ψ⟩ }
P_n :


α|++-> + β|--+>
(α|0> + β|1>)|10>
X1X2
X2X3
{M ∈ Pn : M|ψ> = |ψ> ∀ codewords |ψ>}
Eigenvalues of S identify errors. Consider E ∈ Pn, M ∈ S
= 0 then M(E|ψ>) = EM|ψ> = E|ψ> so E|ψ> is a +1 eigenvector.
then ME|ψ> = -EM|ψ> = -E|ψ> so E|ψ> is a -1 eigenvalue
generators of S


α|++−> + β|−−+>
(α|0> + β|1>)|10>
X1X2
X2X3
{M ∈ Pn : M |ψ> = |ψ> ∀ codewords |ψ>}
Eigenvalues of S identify errors. Consider E ∈ Pn, M ∈ S
[E,M] = 0 then
= EM|ψ> = E|ψ> so E|ψ> is a +1 eigenvector
{E,M} = 0 then
M|ψ> = −E|ψ> so E|ψ> is a −1 eigenvalue
EM + ME = 0
M ∈ S}
generators of S