

Title: PSI 2015/2016 Quantum Information - Lecture 13

Date: Mar 09, 2016 11:30 AM

URL: <http://pirsa.org/16030060>

Abstract:

Classical Error Correction

Noise: bit flips
(indep.)

Encoding

$0 \rightarrow 0$

000
001
010
100

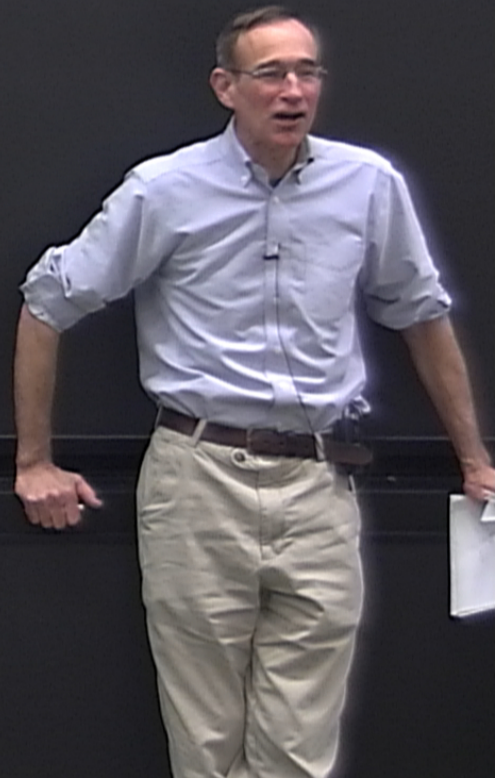
$(1-p)^3$

$p(1-p)^2$

Error Correction

↳ take majority

110
101
110
111



Classical Error Correction

Noise: bit flips
(indep.)

Encoding:

$0 \rightarrow 0$

000
001
010
100

$(1-p)^3$

$p(1-p)^2$

Error Correction

→ take majority

110
101
110
111

Noise

$$|0\rangle|E\rangle = |0\rangle|E^0\rangle + |1\rangle|E^1\rangle$$

$$|1\rangle|E\rangle = |1\rangle|E^1\rangle + |0\rangle|E^0\rangle$$

$\rightarrow$ take majority
 $\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \quad p(1-p)$
 $\begin{matrix} \geq 1/4 \\ \leq 1/4 \end{matrix}$
 $\frac{1}{2} (d|0\rangle - p|1\rangle)(|1\rangle - |0\rangle)$
 $\frac{1}{2} (d|1\rangle - p|0\rangle)(|1\rangle - |0\rangle)$

Encoding: Subspace $\mathcal{C}$ of $\mathcal{H}$

$\begin{matrix} |0\rangle \\ |1\rangle \end{matrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Code triplet $\mathcal{C}$: subspace of $\mathcal{H}$

E : noise
 R : recovery

$$R(E(|\psi_c\rangle)) = |\psi_c\rangle$$

$$E \rightarrow E_\alpha \quad \rho_c = \sum_\alpha E_\alpha \rho_c E_\alpha^\dagger$$

$$|\psi_c\rangle \rightarrow |\psi_c\rangle \begin{matrix} |0\rangle \\ -|1\rangle \end{matrix}$$

$$\langle \psi_c | E_\alpha^\dagger E_\beta | \psi_c \rangle = \delta_{\alpha\beta} C_{\alpha\beta}$$

adjointy $\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$ $P(1-P)$

subspace $\mathcal{C}$ of $\mathcal{H}$

$\rightarrow (\square)$

$\mathcal{C}$: subspace of $\mathcal{H}$

$\mathcal{E}$: noise CPTP map

$\mathcal{R}$: recovery operation

$\mathcal{R} \mathcal{E}(|\psi_e\rangle) = |\psi_e\rangle$

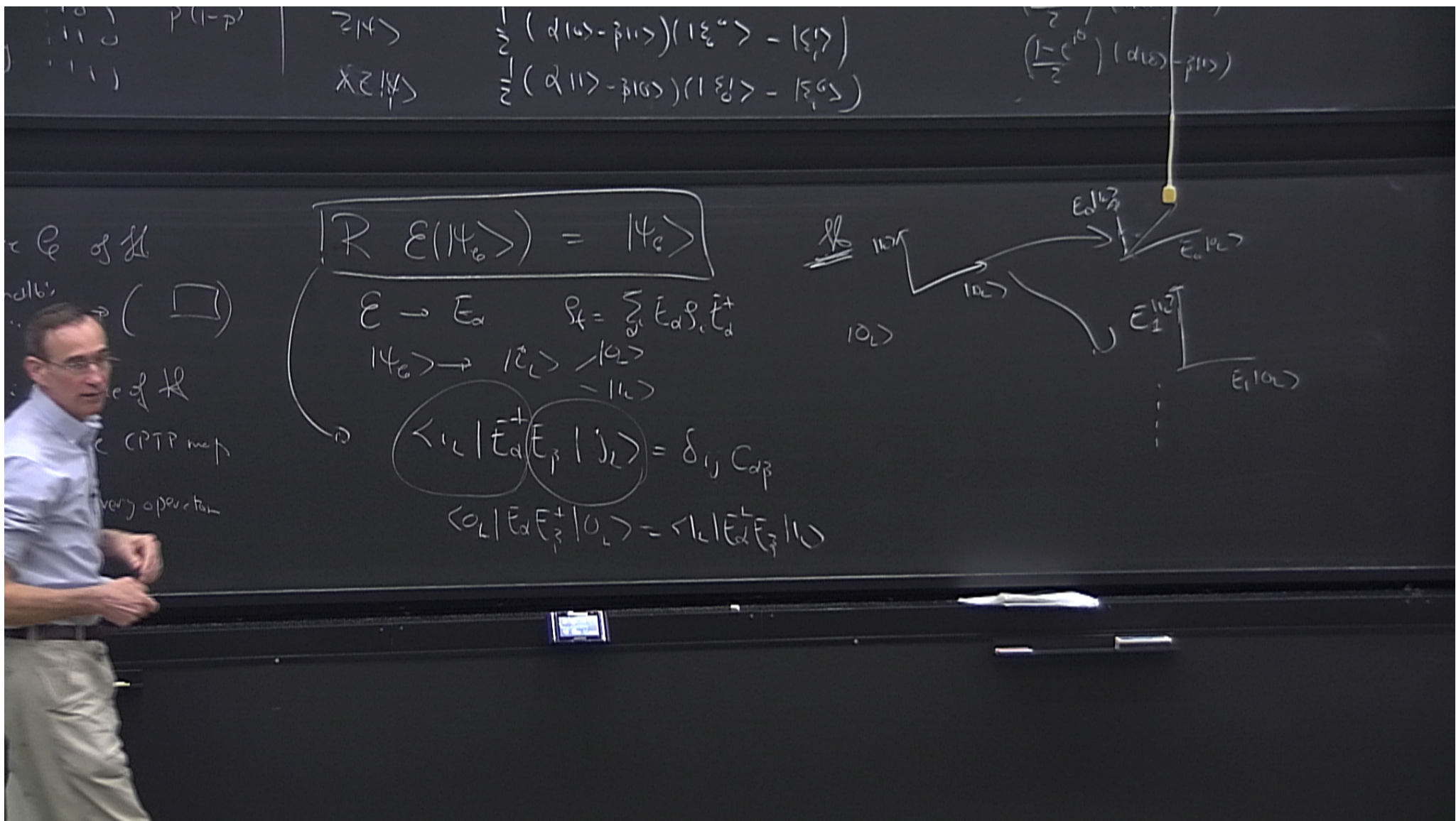
$\mathcal{E} \rightarrow \mathcal{E}_\alpha \quad \mathcal{P}_\mathcal{H} = \sum_\alpha \mathcal{E}_\alpha \mathcal{P}_\mathcal{H} \mathcal{E}_\alpha^\dagger$
 $|\psi_e\rangle \rightarrow |\psi_L\rangle, |\psi_R\rangle, |\psi_C\rangle$

$\langle \psi_L | \mathcal{E}_\alpha^\dagger \mathcal{E}_\beta | \psi_L \rangle = \delta_{\alpha\beta} C_{\alpha\beta}$
 $\langle \psi_L | \mathcal{E}_\alpha \mathcal{E}_\beta^\dagger | \psi_L \rangle = \langle \psi_L | \mathcal{E}_\alpha^\dagger \mathcal{E}_\beta | \psi_L \rangle$

$\frac{1}{2} (\alpha|0\rangle - \beta|1\rangle)(|1\rangle - |0\rangle)$
 $\frac{1}{2} (\alpha|1\rangle - \beta|0\rangle)(|1\rangle - |0\rangle)$

$\frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} (\alpha|0\rangle - \beta|1\rangle)$

Diagram illustrating a quantum channel $\mathcal{E}$ and its recovery operation $\mathcal{R}$. The input state $|\psi\rangle$ is mapped to a state in a 2D space (spanned by $|\psi_L\rangle$ and $|\psi_R\rangle$) via $\mathcal{E}$. The recovery operation $\mathcal{R}$ then maps this state back to the original state $|\psi\rangle$.



$$\begin{aligned} \sum (d_{10} - \beta_{11}) (|10\rangle - |\xi_1\rangle) \\ \sum (d_{11} - \beta_{10}) (|10\rangle - |\xi_1\rangle) \end{aligned}$$

$$\left(\frac{1-\epsilon^{10}}{2}\right) (d_{10} - \beta_{11})$$

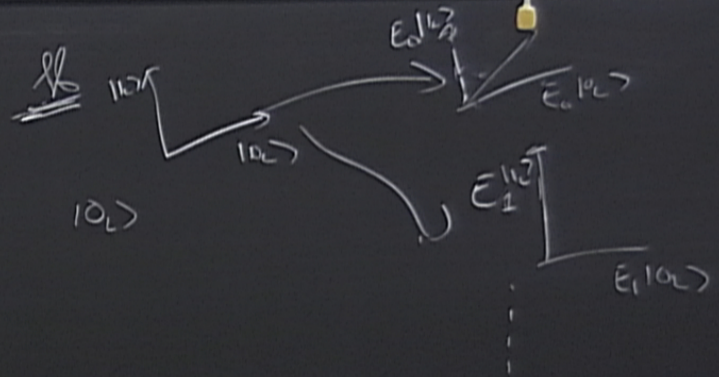
$$\boxed{R E(|14_e\rangle) = |14_e\rangle}$$

$$E \rightarrow E_\alpha \quad \rho_k = \sum_\alpha E_\alpha \rho_k E_\alpha^\dagger$$

$$|14_e\rangle \rightarrow |10_L\rangle, |10_L\rangle, |10_L\rangle$$

$$\langle 1_L | \bar{E}_\alpha^\dagger (E_\beta | 1_L \rangle) = \delta_{\alpha\beta} C_{\alpha\beta}$$

$$\langle 0_L | E_\alpha E_\beta^\dagger | 0_L \rangle = \langle 1_L | \bar{E}_\alpha^\dagger E_\beta | 1_L \rangle$$

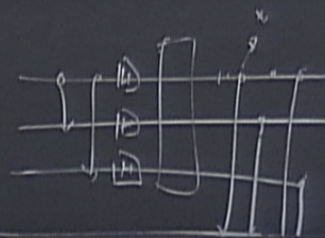


$$\begin{array}{ll}
 \alpha|000\rangle + \beta|111\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle)|00\rangle & \\
 \alpha|100\rangle + \beta|011\rangle \rightarrow (\alpha|1\rangle + \beta|0\rangle)|11\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) & \\
 \alpha|010\rangle + \beta|101\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle)|01\rangle & \vdots \\
 \alpha|101\rangle + \beta|010\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle)|10\rangle & \vdots
 \end{array}$$

$$\begin{array}{ll}
 \alpha 1000 > \beta 1111 > \rightarrow (\alpha 100 + \beta 111) 100 > \\
 \alpha 1100 > \beta 1011 > \rightarrow (\alpha 110 + \beta 111) 111 > \rightarrow (\alpha 10 + \beta 11) > \\
 \alpha 1010 > \beta 1101 > \rightarrow (\alpha 101 + \beta 111) 101 > & \vdots \\
 & (\alpha 101 + \beta 111) 110 > & \vdots
 \end{array}$$

$(\alpha|000\rangle + \beta|111\rangle)|0\rangle$
 $(\alpha|000\rangle + \beta|111\rangle) \rightarrow \alpha|111\rangle + \beta|000\rangle$
 $+ (\alpha|100\rangle + \beta|010\rangle)|1\rangle$
 $Z_1 Z_2 = Z_3$
 $Z_1 Z_3 = Z_2$
 $Z_2 Z_3 = Z_1$

$\alpha|111\rangle + \beta|000\rangle$
 L_1
 L_2



$|0_L\rangle = (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$
 $|1_L\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$

Show

$Z_1 Z_2$
 $Z_2 Z_3$
 $Z_1 Z_3$

$Z_1 Z_2$
 $Z_2 Z_3$

$+ (x_{11} x_{21} x_{31})^3$
 $E(I) = E(10111)$
 10
 1111
 $x_1 x_2$

$$|0_L\rangle = (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)$$

$$|1_L\rangle = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)$$

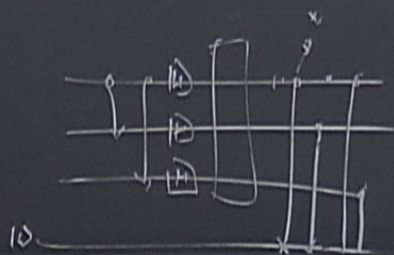
Shor
code

$z_1 z_2$
 $z_2 z_3$
 $z_3 z_4$

$x x x x x x$
 $x x x 1 1 1$

$z_1 z_2$

$$\alpha|11\rangle + \beta|11\rangle$$



$$(\alpha|10\rangle + \beta|11\rangle)|10\rangle$$

$$X_1 X_2$$

$$X_1 X_3$$

$$\{M \in P_n : M|1\rangle = |1\rangle \text{ for all codewords } |1\rangle\}$$

Eigenvalues of S identify errors. Consider $E \in P_n$, $M \in S$

$$\rightarrow [E] = 0 \text{ then } M(E|1\rangle) = E(M|1\rangle) = E|1\rangle \text{ so } E|1\rangle \text{ is a } +1 \text{ eigenvector.}$$

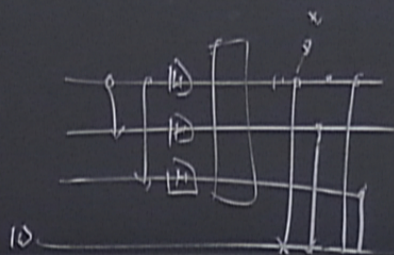
$$\rightarrow \text{if } [E] = -1 \text{ then } M(E|1\rangle) = -E(M|1\rangle) = -E|1\rangle \text{ so } E|1\rangle \text{ is a } -1 \text{ eigenvector.}$$

$$M \in S$$

eigenvalues of S

$$\alpha|+-\rangle + \beta|--\rangle$$

$$\begin{matrix} L \\ \downarrow \\ L \end{matrix}$$



$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle$$

$$X_1 X_2$$

$$X_1 X_3$$

$$\{M \in P_n : M| \psi \rangle = | \psi \rangle \text{ for all codewords } | \psi \rangle\}$$

Eigenvalues of S identify errors. Consider $E \in P_n$, $M \in S$

$$\rightarrow [E, M] = 0 \text{ then } E M | \psi \rangle = E | \psi \rangle \text{ so } E | \psi \rangle \text{ is a } +1 \text{ eigenvector.}$$

$$\rightarrow \{E, M\} = 0 \text{ then } E M | \psi \rangle = -E | \psi \rangle \text{ so } E | \psi \rangle \text{ is a } -1 \text{ eigenvector.}$$

$$EM + ME = 0$$

$M \in S$
eigenvectors of S