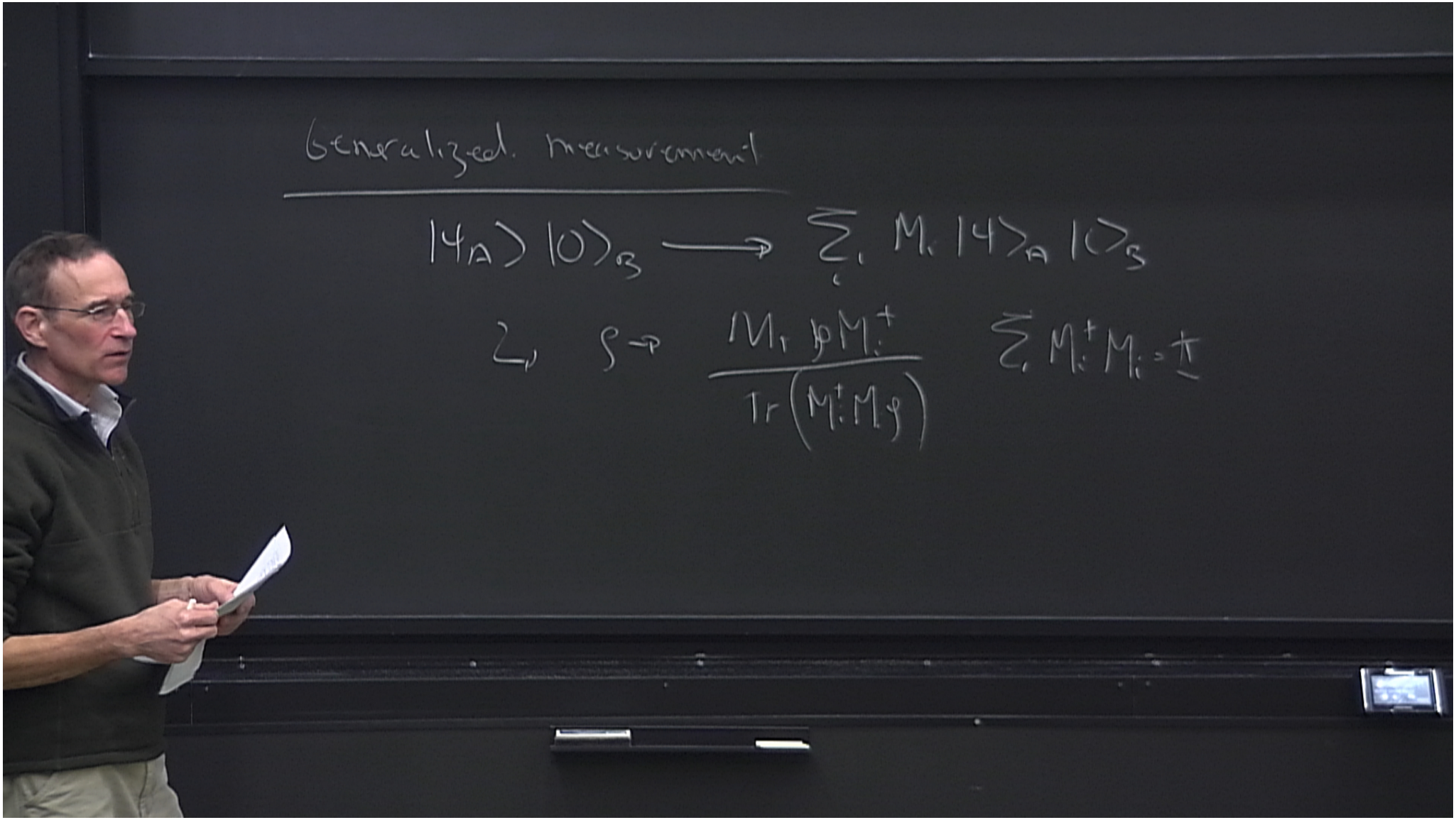
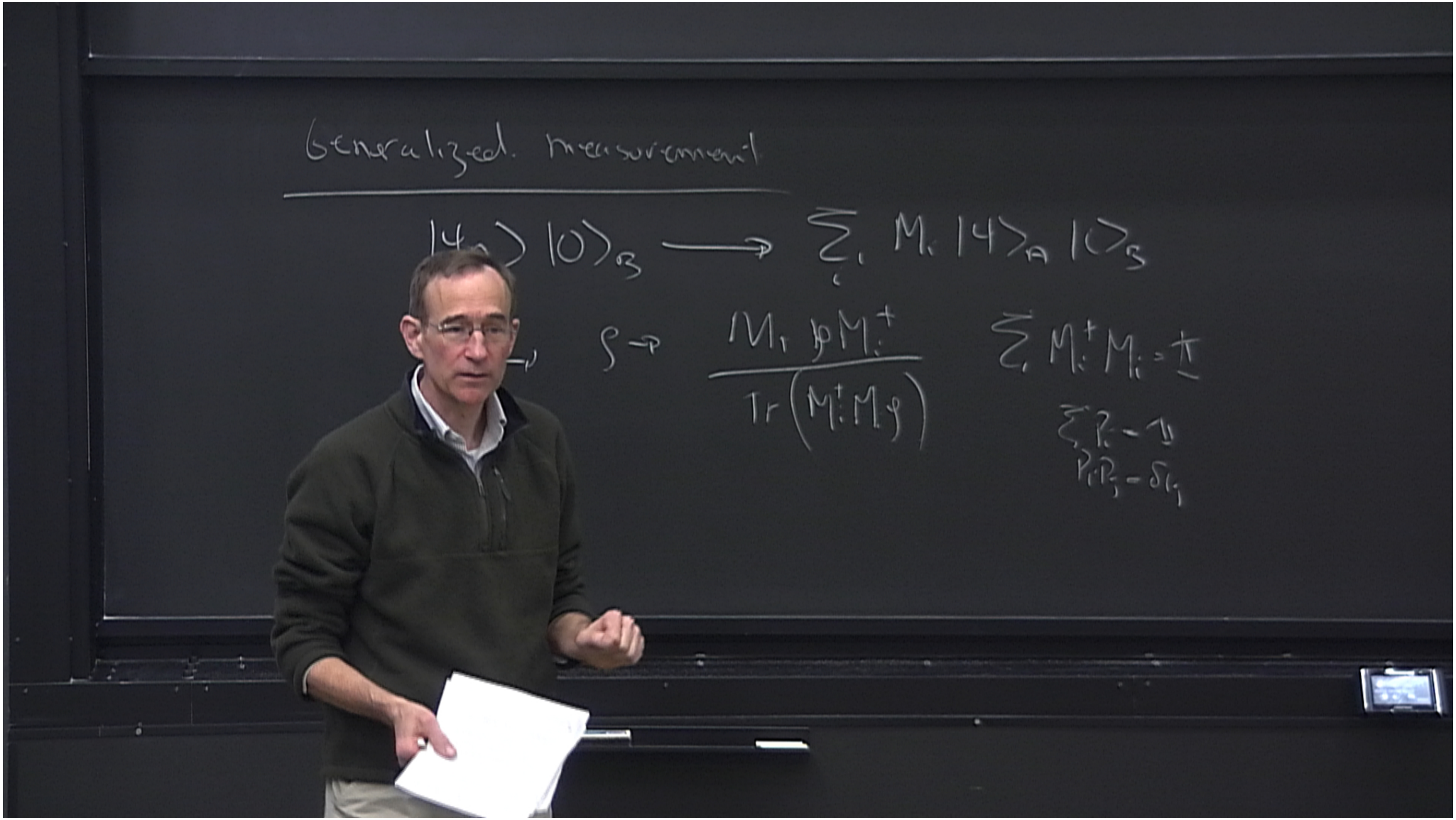
Title: PSI 2015/2016 Quantum Information - Lecture 12

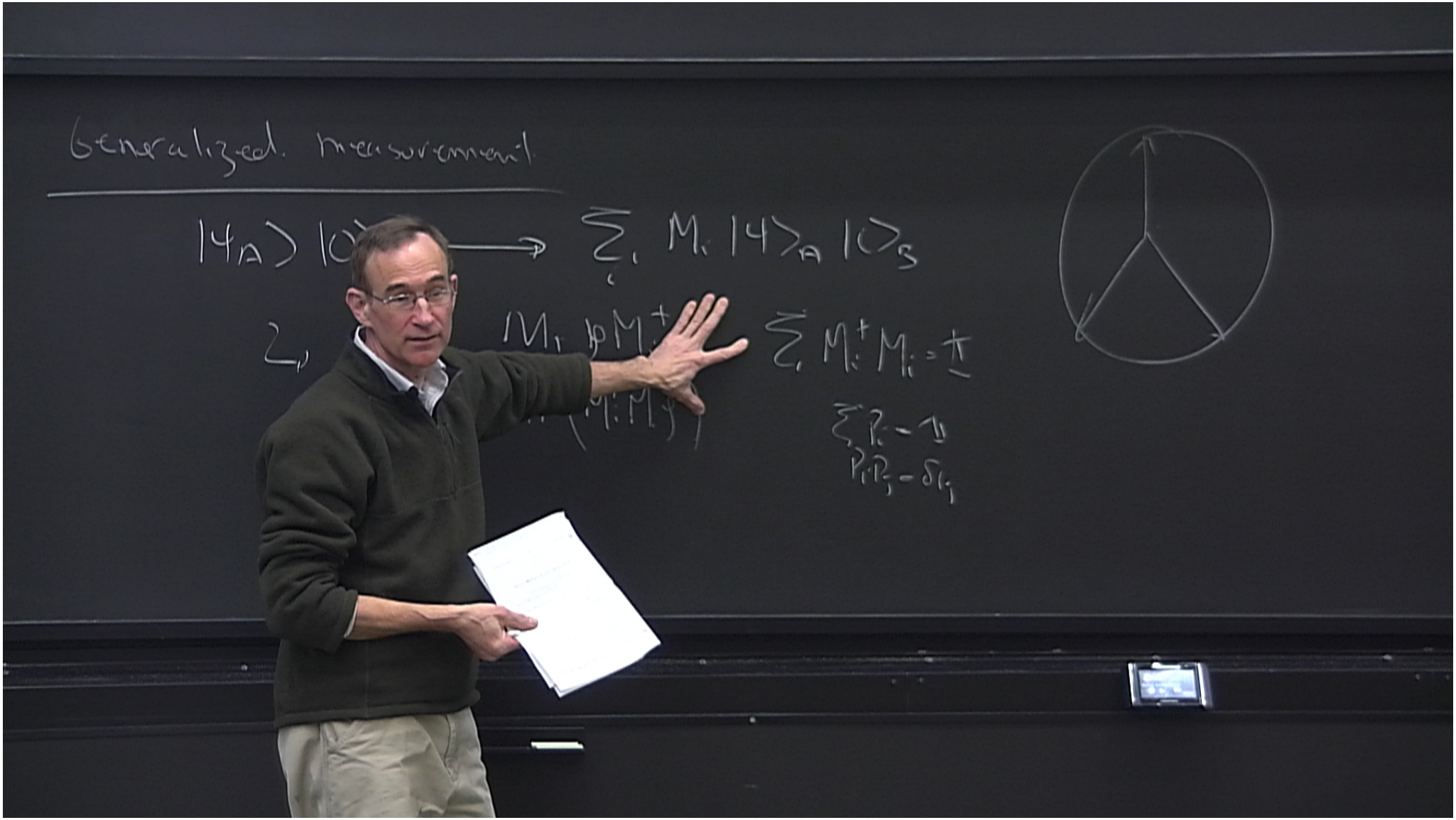
Date: Mar 08, 2016 11:30 AM

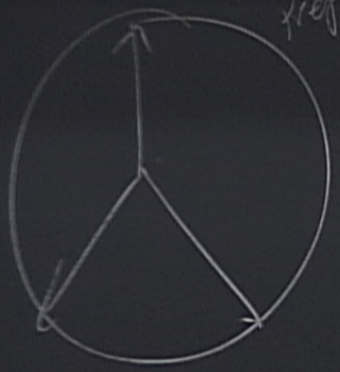
URL: http://pirsa.org/16030059

Abstract:


Generalized measurement
$|\psi_A\rangle|0\rangle_B \longrightarrow \sum_i M_i|\psi\rangle_A|i\rangle_B$
$\rho \to \frac{M_i \rho M_i^\dagger}{\mathrm{Tr}\left(M_i^\dagger M_i \rho\right)}$
$\sum_i M_i^\dagger M_i = \mathbb{1}$

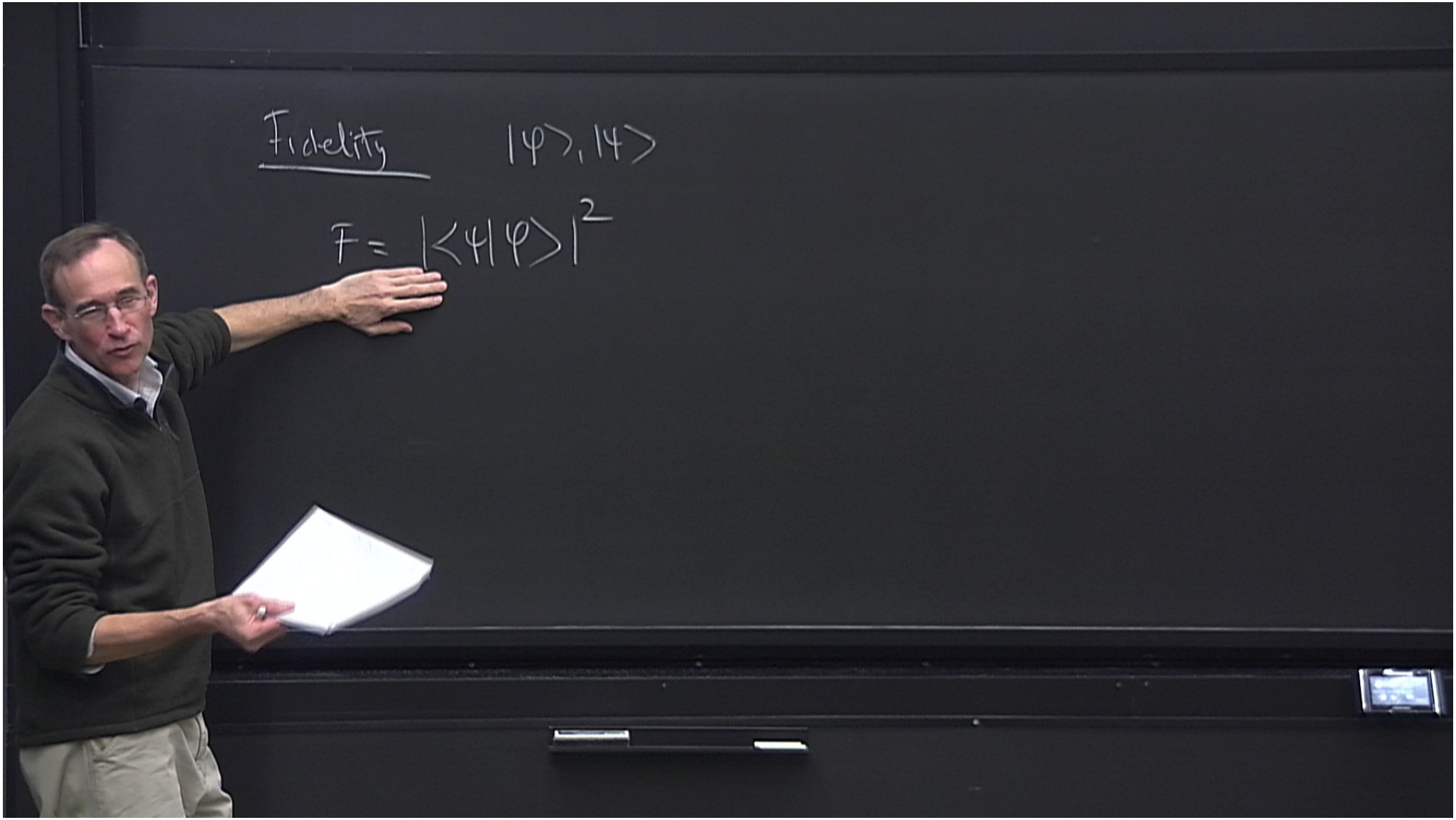

Generalized measurement
$\sum_i M_i |4\rangle_A |0\rangle_B$
$\rho \to \frac{M_i \rho M_i^\dagger}{\mathrm{Tr}(M_i^\dagger M_i \rho)}$
$\sum_i M_i^\dagger M_i = \mathbb{1}$
$\sum_i P_i = \mathbb{1}$
$P_i P_j = \delta_{ij}$

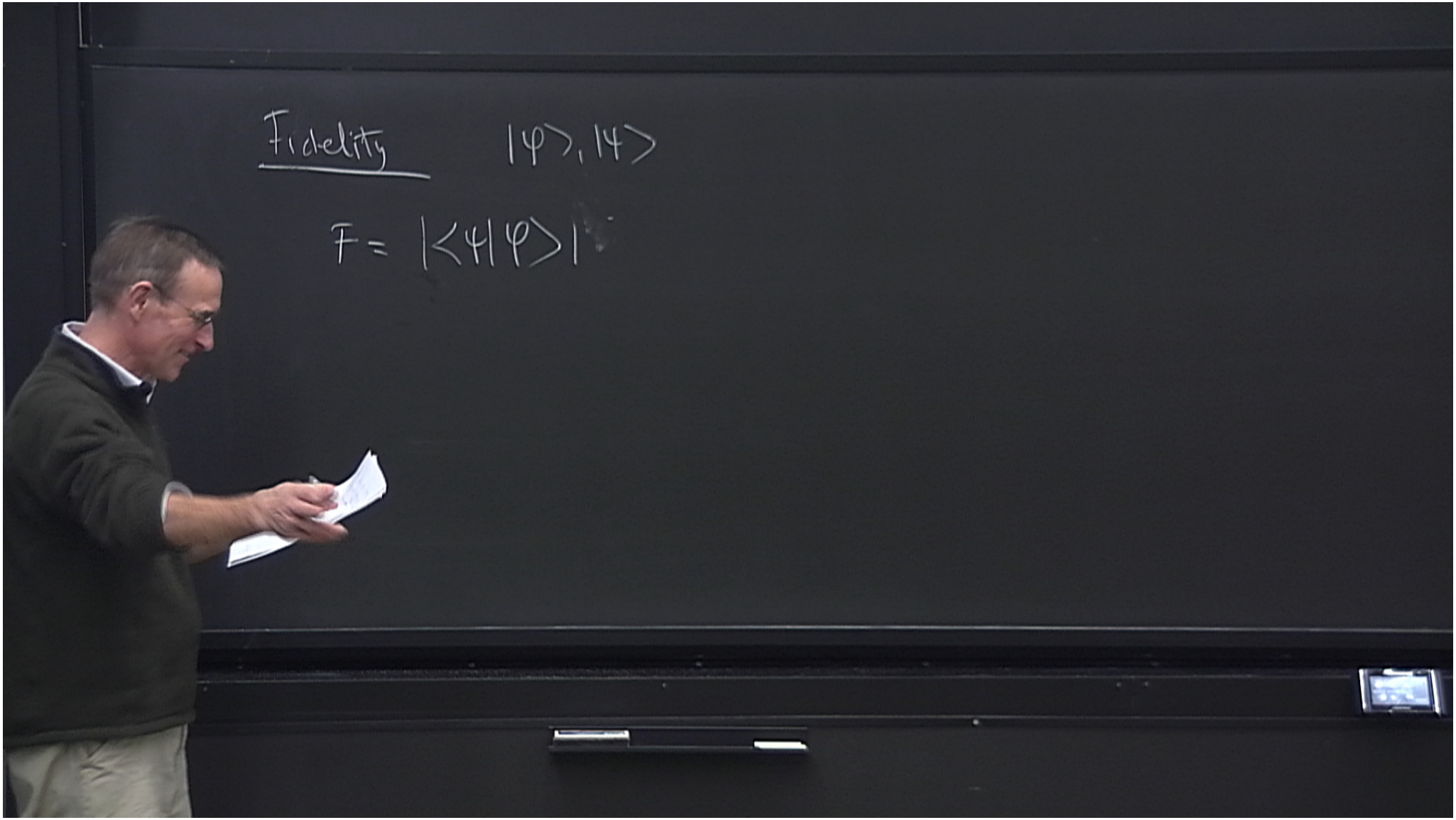

Generalized measurement
$\sum_i M_i |\psi\rangle_A |0\rangle_S$
$\sum_i M_i^\dagger M_i = \mathbb{1}$
$\sum_i P_i = \mathbb{1}$
$P_i P_j = \delta_{ij}$

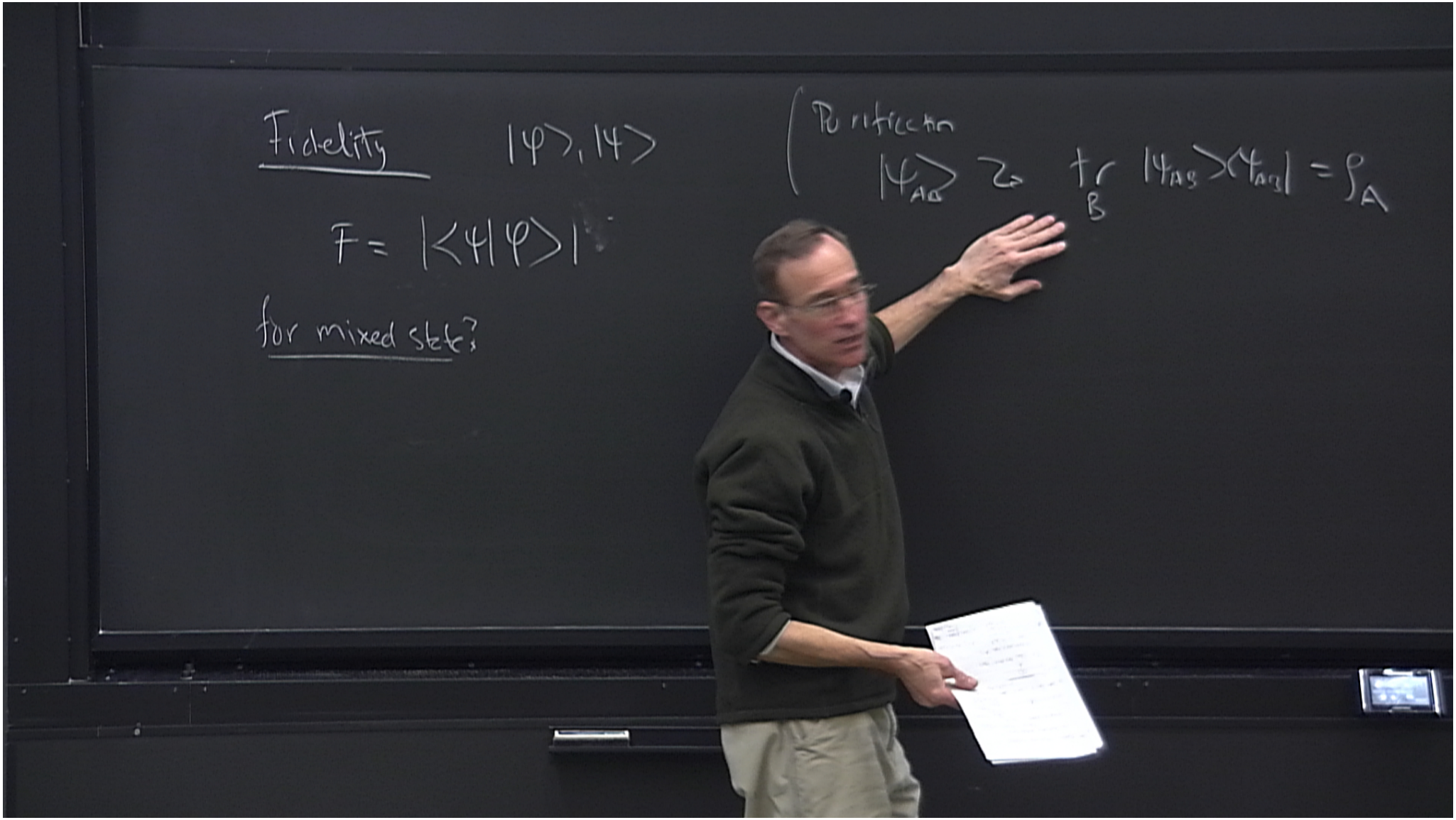


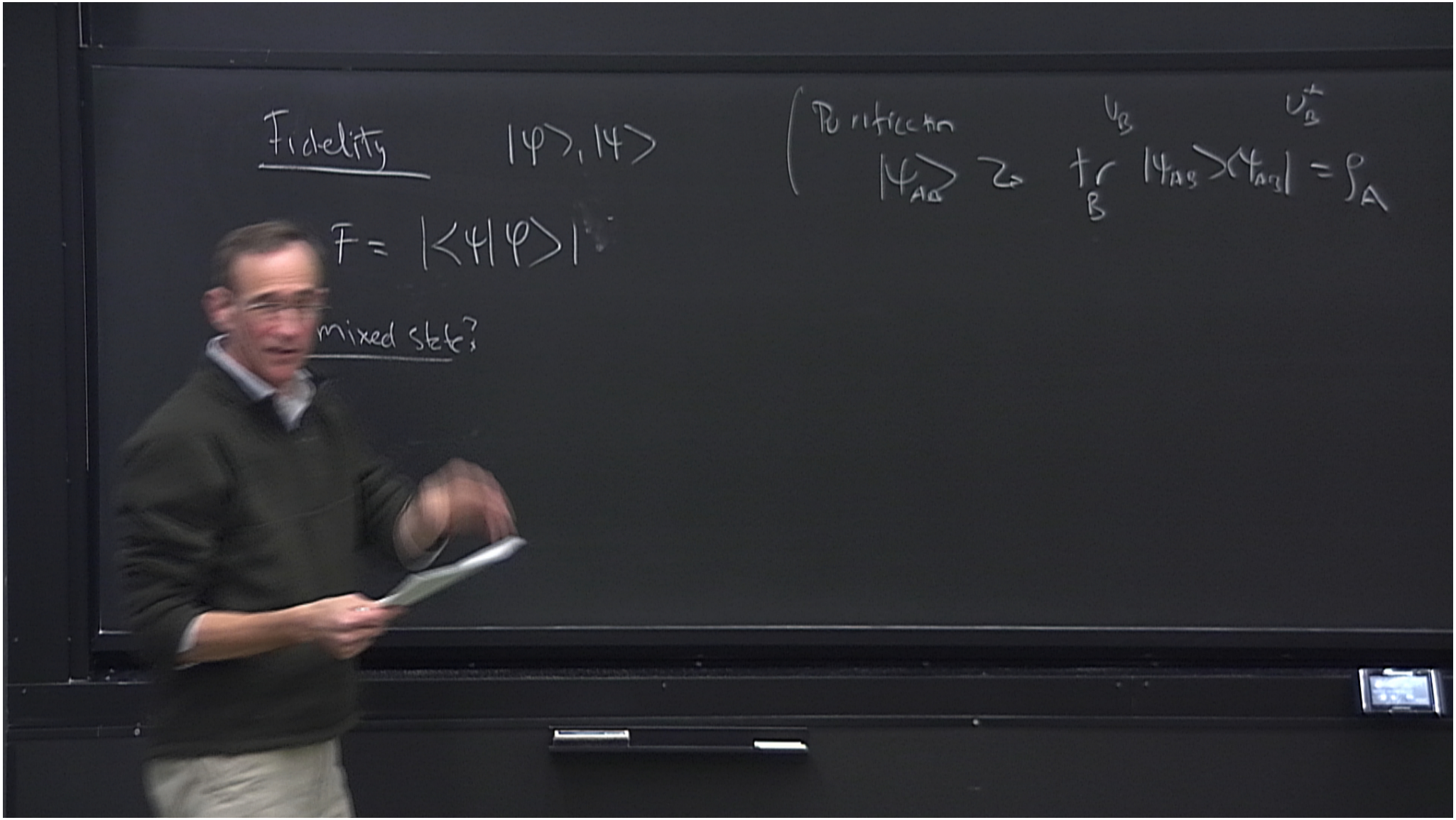
Prepare 3 states in the XZ plane of the Bloch sphere. (P. 1/3)

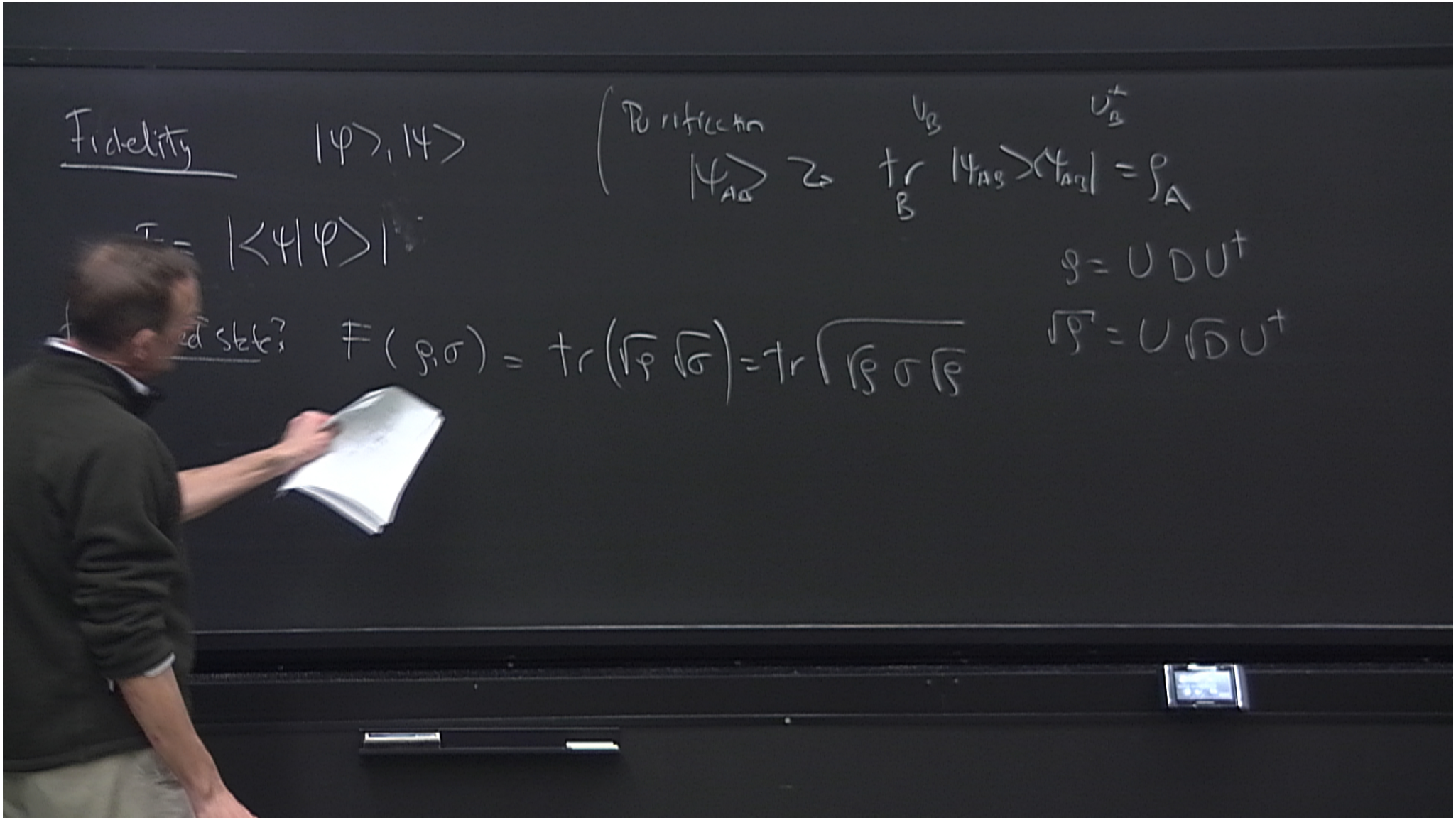
The measurement that guesses the state with smallest error probability is a 3-outcome generalized measurement (not a projective measurement).

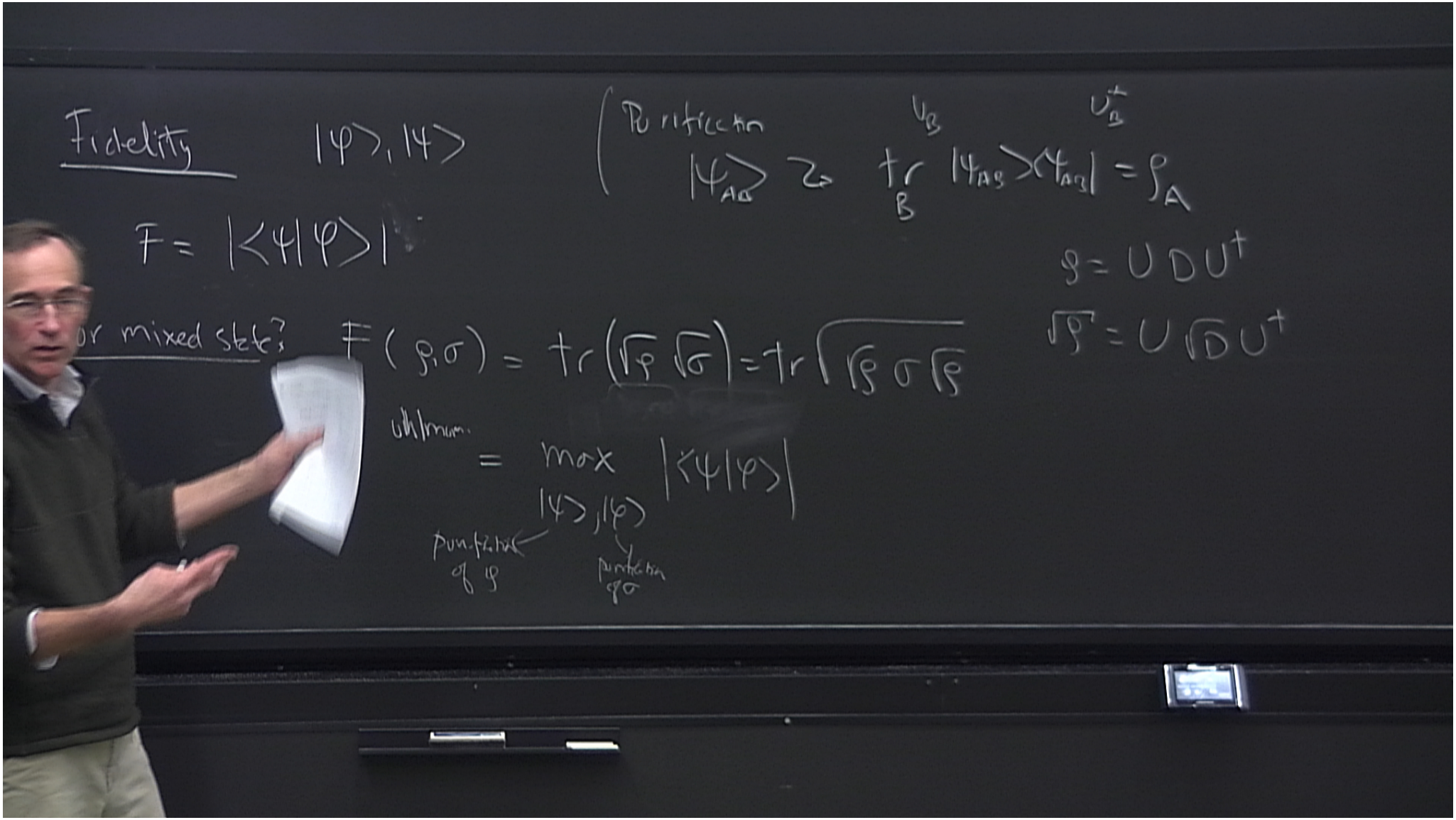

Fidelity
$|\psi\rangle, |\chi\rangle$
$F = |\langle\chi|\psi\rangle|^2$

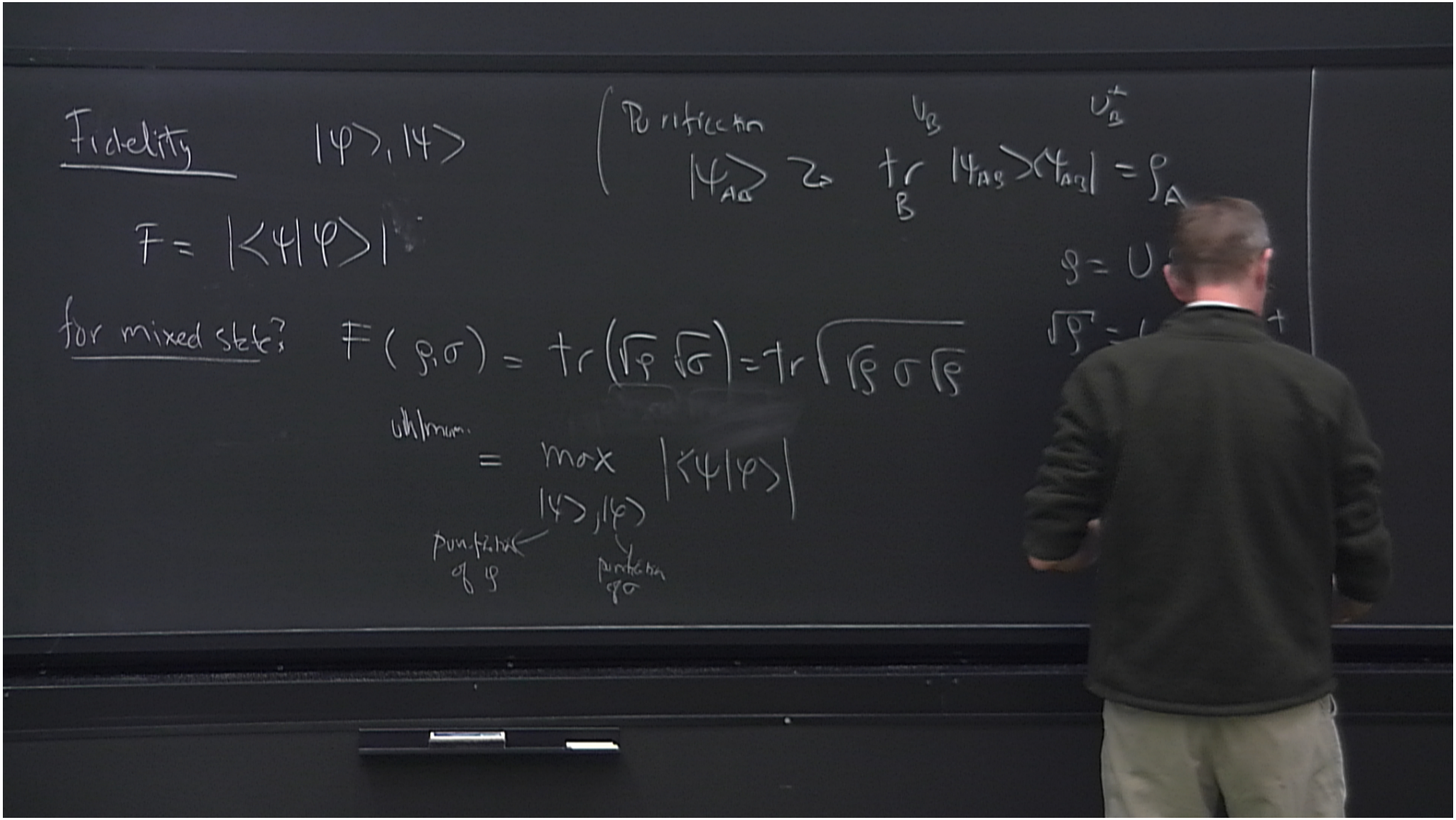

Fidelity
|ψ>, |ϕ>
F = |<ϕ|ψ>|²

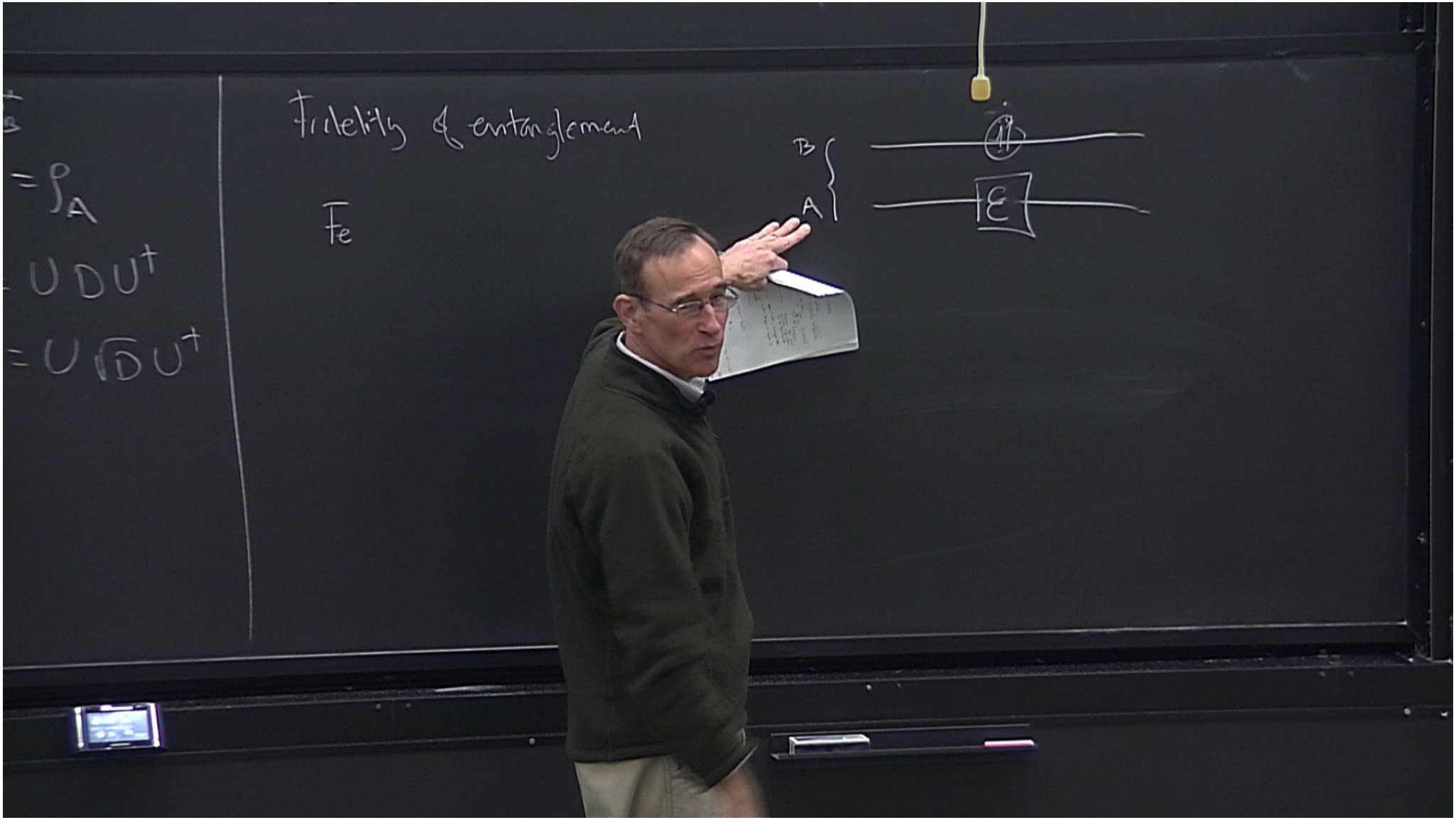

Fidelity
$|\psi\rangle, |\psi\rangle$
$F = |\langle\psi|\psi\rangle|^2$
for mixed state?
Purification
$|\psi_{AB}\rangle \rightarrow \mathrm{tr}_B |\psi_{AB}\rangle\langle\psi_{AB}| = \rho_A$

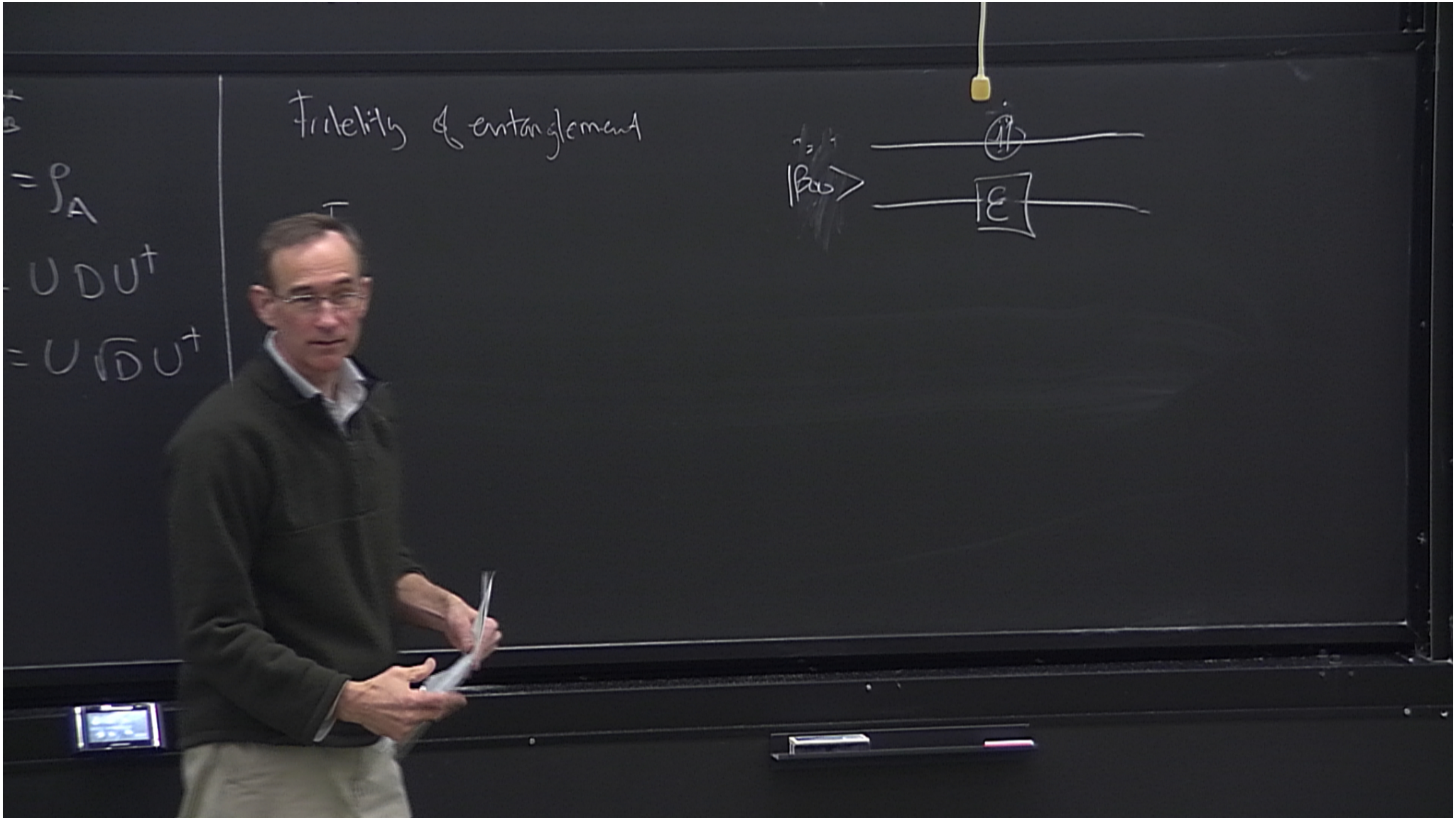

Fidelity
$|\psi\rangle, |\psi\rangle$
$F = |\langle\psi|\psi\rangle|^2$
mixed state?
Purification
$|\psi_{AB}\rangle \rightarrow \mathrm{tr}_B |\psi_{AB}\rangle\langle\psi_{AB}| = \rho_A$
U_B
$U_B^\dagger$


Fidelity
|φ⟩, |ψ⟩
F = |⟨ψ|φ⟩|²
d state?
F(ρ,σ) = tr(√ρ √σ) = tr √(√ρ σ √ρ)
Purification
|ψ_AB⟩ → tr_B |ψ_AB⟩⟨ψ_AB| = ρ_A
V_B
U_B†
ρ = U D U†
√ρ = U √D U†


Fidelity $|\psi\rangle, |\phi\rangle$
$F = |\langle\psi|\phi\rangle|$
or mixed state?
Purification
$|\psi_{AB}\rangle$
$\rho_A = \mathrm{tr}_B |\psi_{AB}\rangle\langle\psi_{AB}|$
U_B $U_B^\dagger$
$\rho = U D U^\dagger$
$\sqrt{\rho} = U \sqrt{D} U^\dagger$
$F(\rho,\sigma) = \mathrm{tr}(\sqrt{\rho}\sqrt{\sigma}) = \mathrm{tr}\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}$
ultimum.
$= \max_{|\psi\rangle,|\phi\rangle} |\langle\psi|\phi\rangle|$
purification of ρ
purification of σ


Fidelity $|\psi\rangle, |\phi\rangle$
$F = |\langle\psi|\phi\rangle|^2$
for mixed state? $F(\rho,\sigma) = tr\left(\sqrt{\sqrt{\rho}\,\sigma\sqrt{\rho}}\right)$
$= \max |\langle\psi|\phi\rangle|$
$|\psi\rangle, |\phi\rangle$
Purification of ρ
Purification of σ
Purification
$|\psi_{AB}\rangle$
$tr_B |\psi_{AB}\rangle\langle\psi_{AB}| = \rho_A$
U_B
$U_B^\dagger$
$\rho = U$
$\sqrt{\rho} =$


fidelity & entanglement
Fe
B
A
$\mathcal{E}$
$= \rho_A$
$= U D U^\dagger$
$= U \sqrt{D} U^\dagger$


fidelity & entanglement
$= \rho_A$
$= U D U^\dagger$
$= U \sqrt{D} U^\dagger$
$|\beta_{00}\rangle$
$\mathcal{E}$

$= \rho_A$

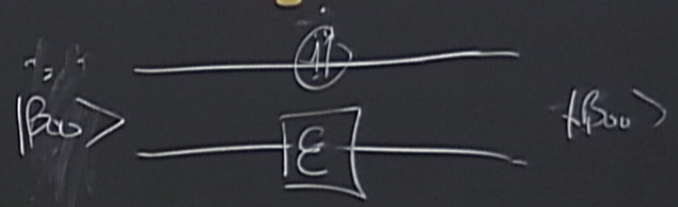
$= U D U^\dagger$

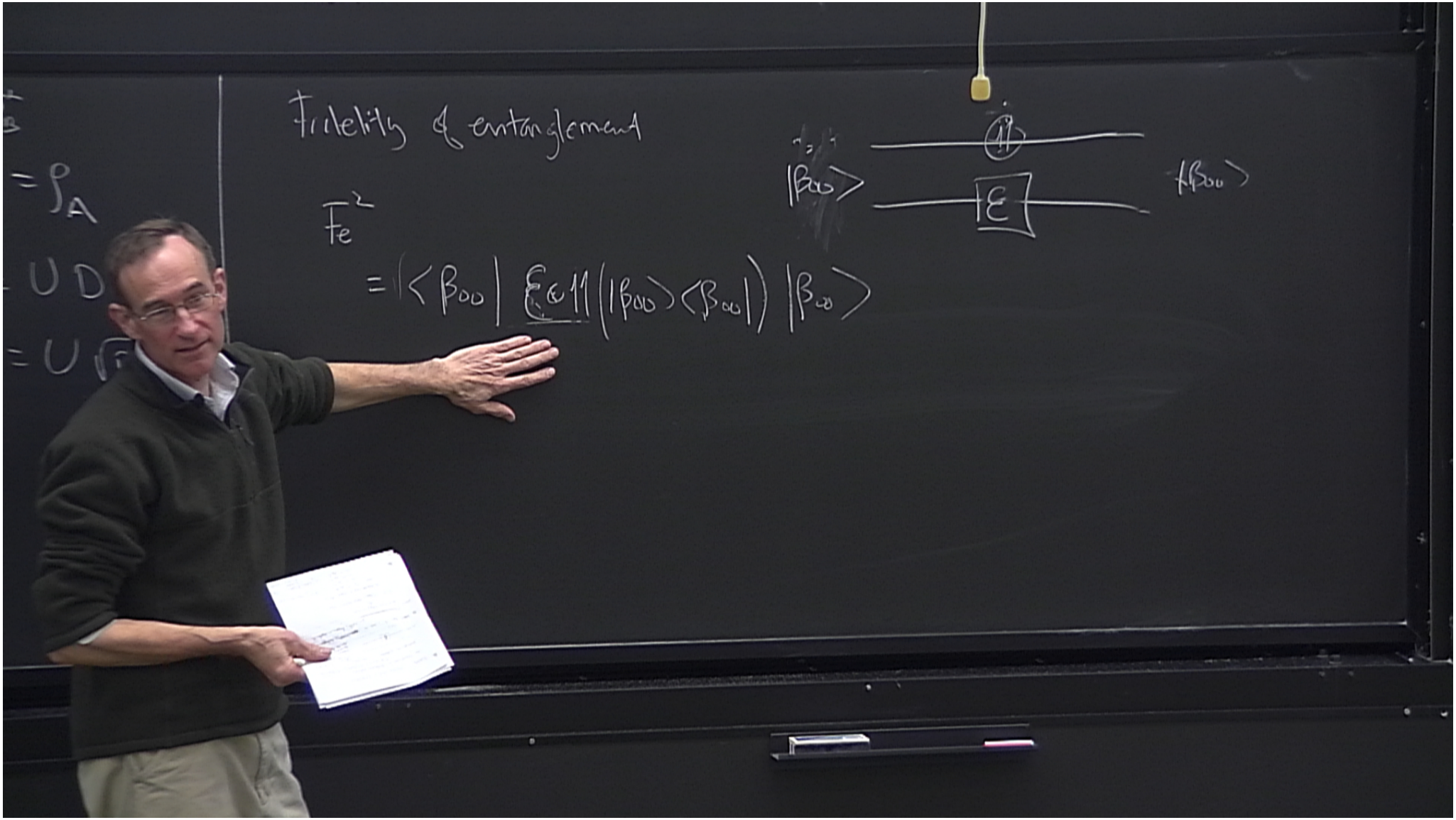
$= U \sqrt{D} U^\dagger$

Fidelity & entanglement

F_e

$= \langle \beta_{00} | \mathcal{E} \otimes \mathbb{1} (|\beta_{00}\rangle\langle\beta_{00}|) | \beta_{00} \rangle$




fidelity & entanglement
F_e^2
$= \langle \beta_{00} | \mathcal{E} \otimes 11 (|\beta_{00}\rangle\langle\beta_{00}|) | \beta_{00} \rangle$
$|\beta_{00}\rangle$
$\mathcal{E}$
$|\beta_{00}\rangle$

$= \rho_A$

$= U D U^\dagger$

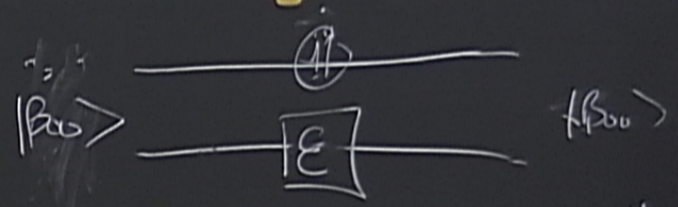
$= U \sqrt{D} U^\dagger$

Fidelity & entanglement

F_e^2

$= | \langle \beta_{00} | \mathcal{E} \otimes \mathbb{1} (|\beta_{00}\rangle\langle\beta_{00}|) | \beta_{00} \rangle$

$=$

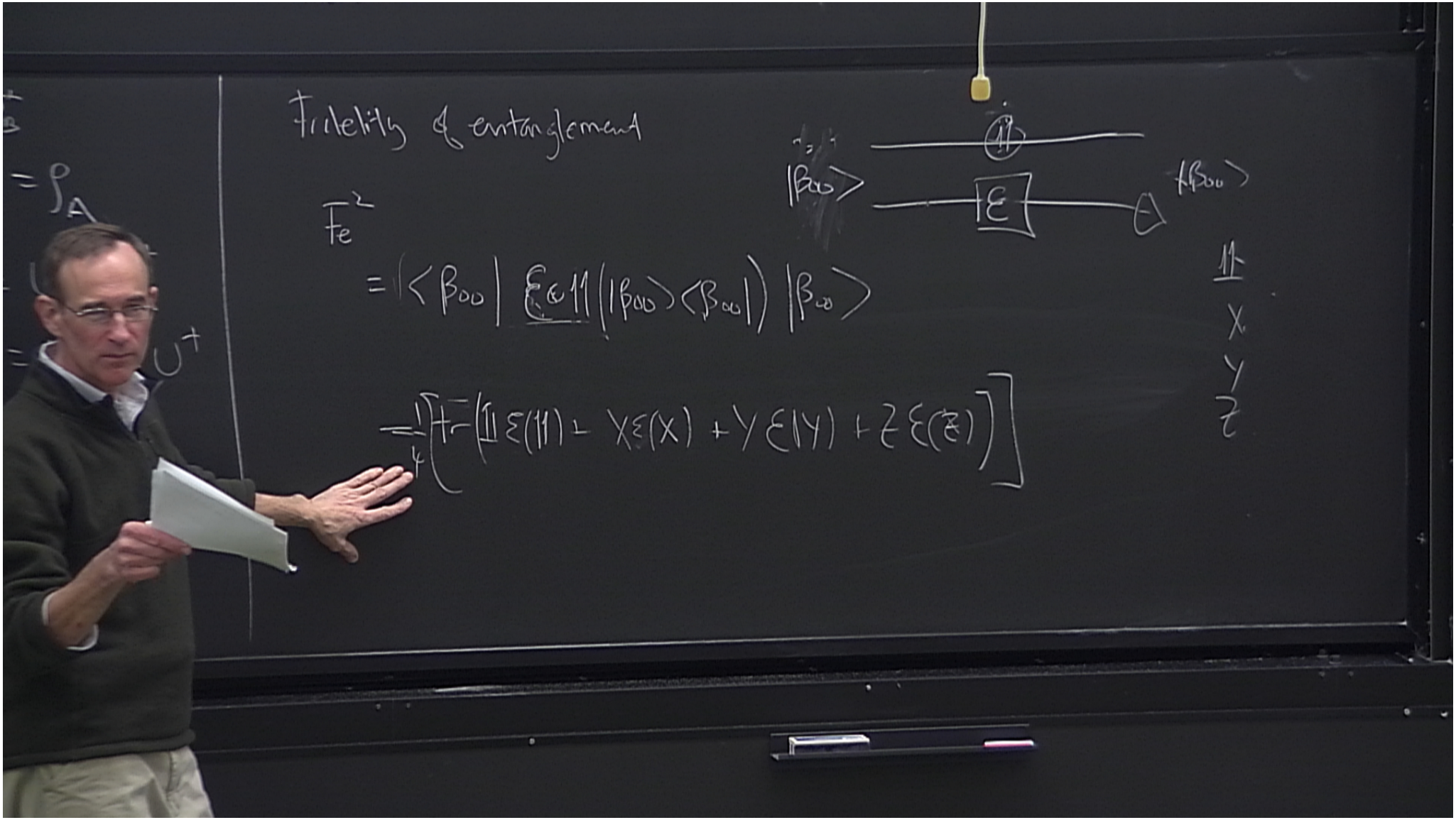


$\mathbb{1}$

X

Y

Z


fidelity & entanglement
F_e^2
$= \langle \beta_{00} | \mathcal{E} \otimes \mathbb{1} (|\beta_{00}\rangle\langle\beta_{00}|) | \beta_{00} \rangle$
$= \frac{1}{4} \left[\mathrm{Tr}(\mathbb{1}\mathcal{E}(\mathbb{1}) + X\mathcal{E}(X) + Y\mathcal{E}(Y) + Z\mathcal{E}(Z)) \right]$
$|\beta_{00}\rangle$
$\mathcal{E}$
$|\beta_{00}\rangle$
$\mathbb{1}$
X
Y
Z

$= \rho_A$

$= U D U^\dagger$

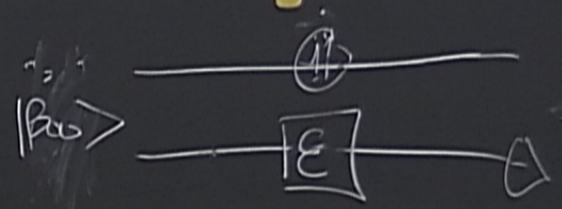
$= U \sqrt{D} U^\dagger$

Fidelity & entanglement

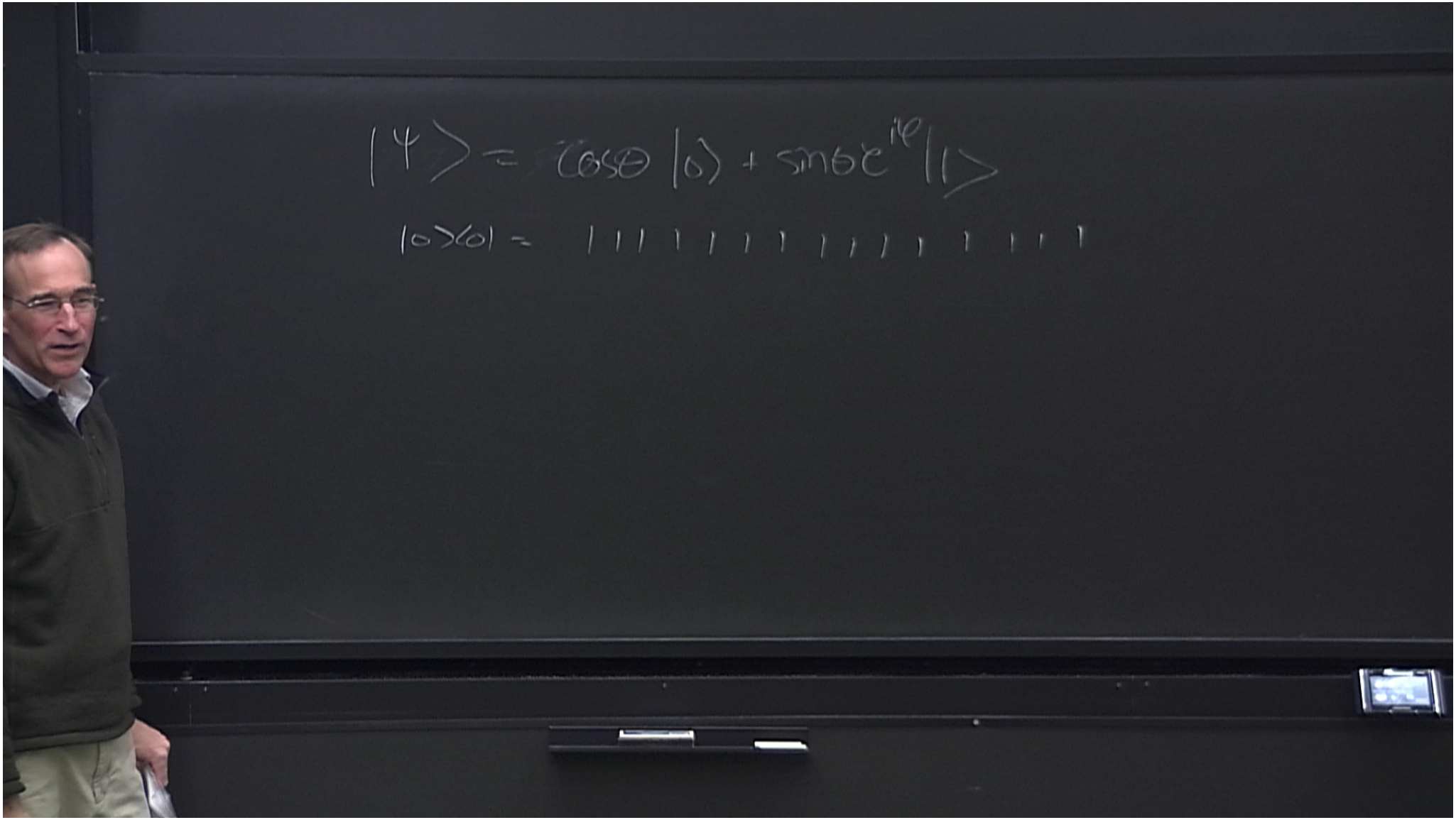
F_e^2

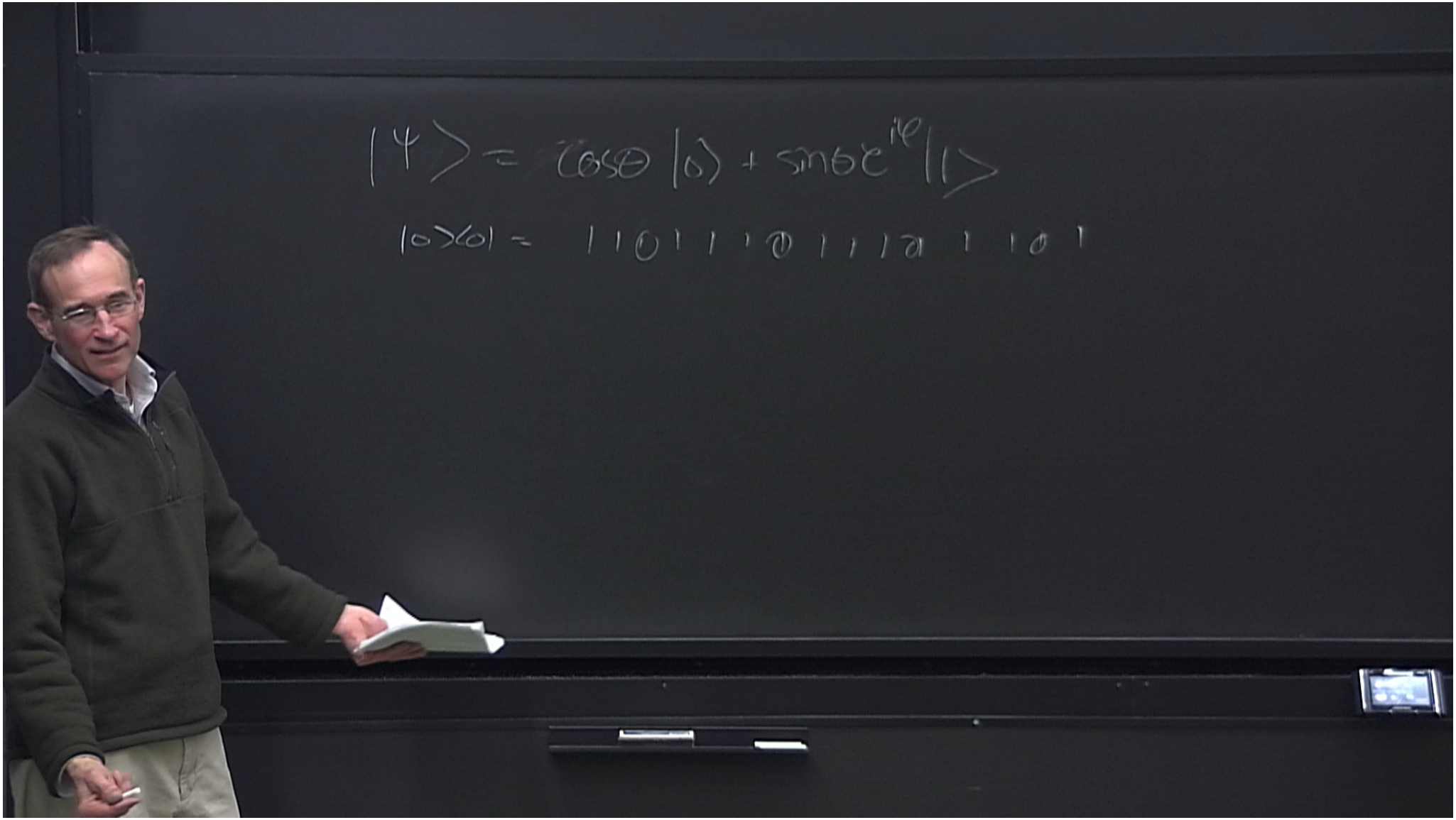
$= \langle \beta_{00} | \mathcal{E} \otimes \mathbb{1} (|\beta_{00}\rangle\langle\beta_{00}|) | \beta_{00} \rangle$

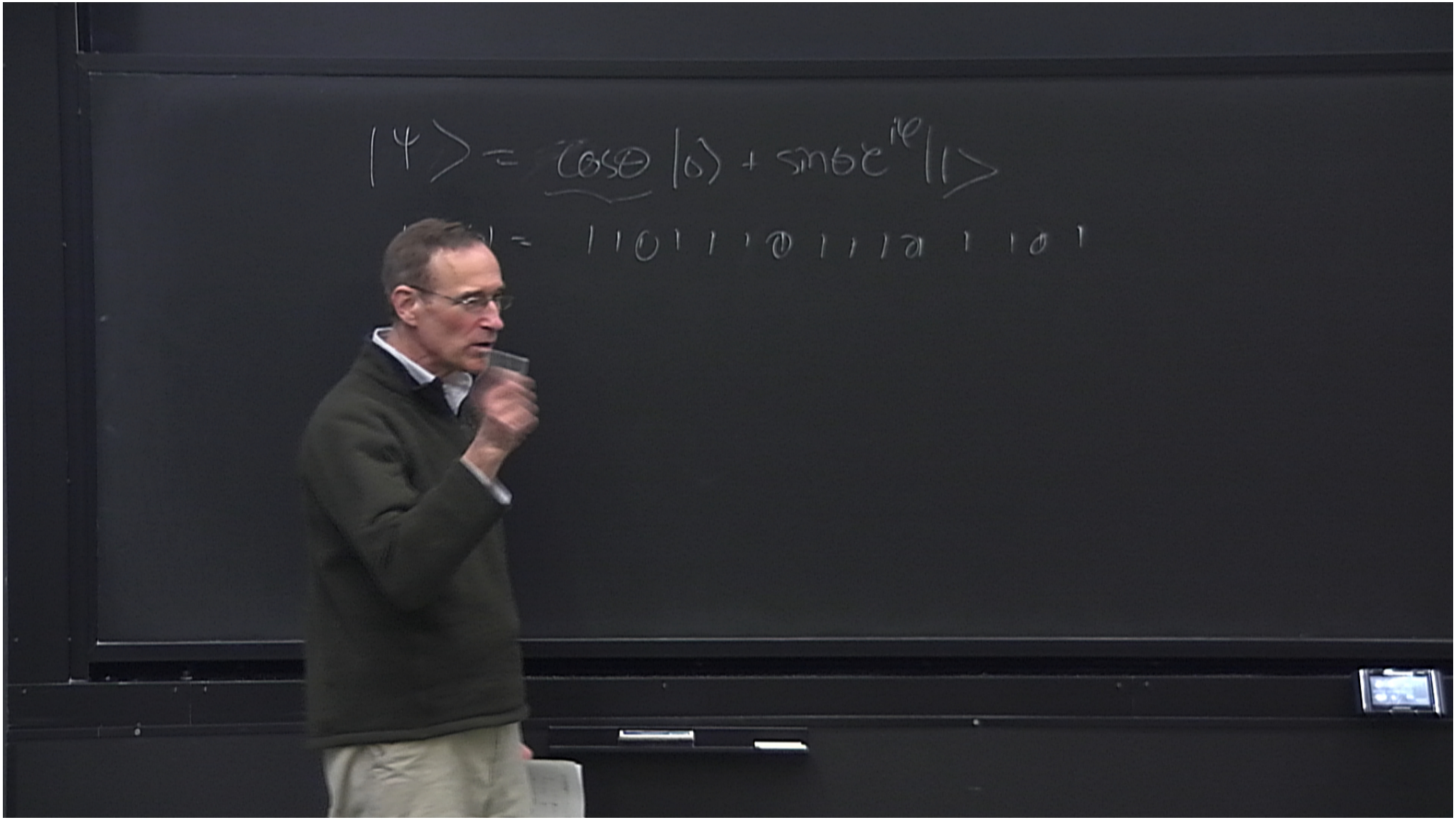
$= \frac{1}{4} \left[\mathrm{Tr}(\mathbb{1}\mathcal{E}(\mathbb{1}) + X\mathcal{E}(X) + Y\mathcal{E}(Y) + Z\mathcal{E}(Z)) \right]$

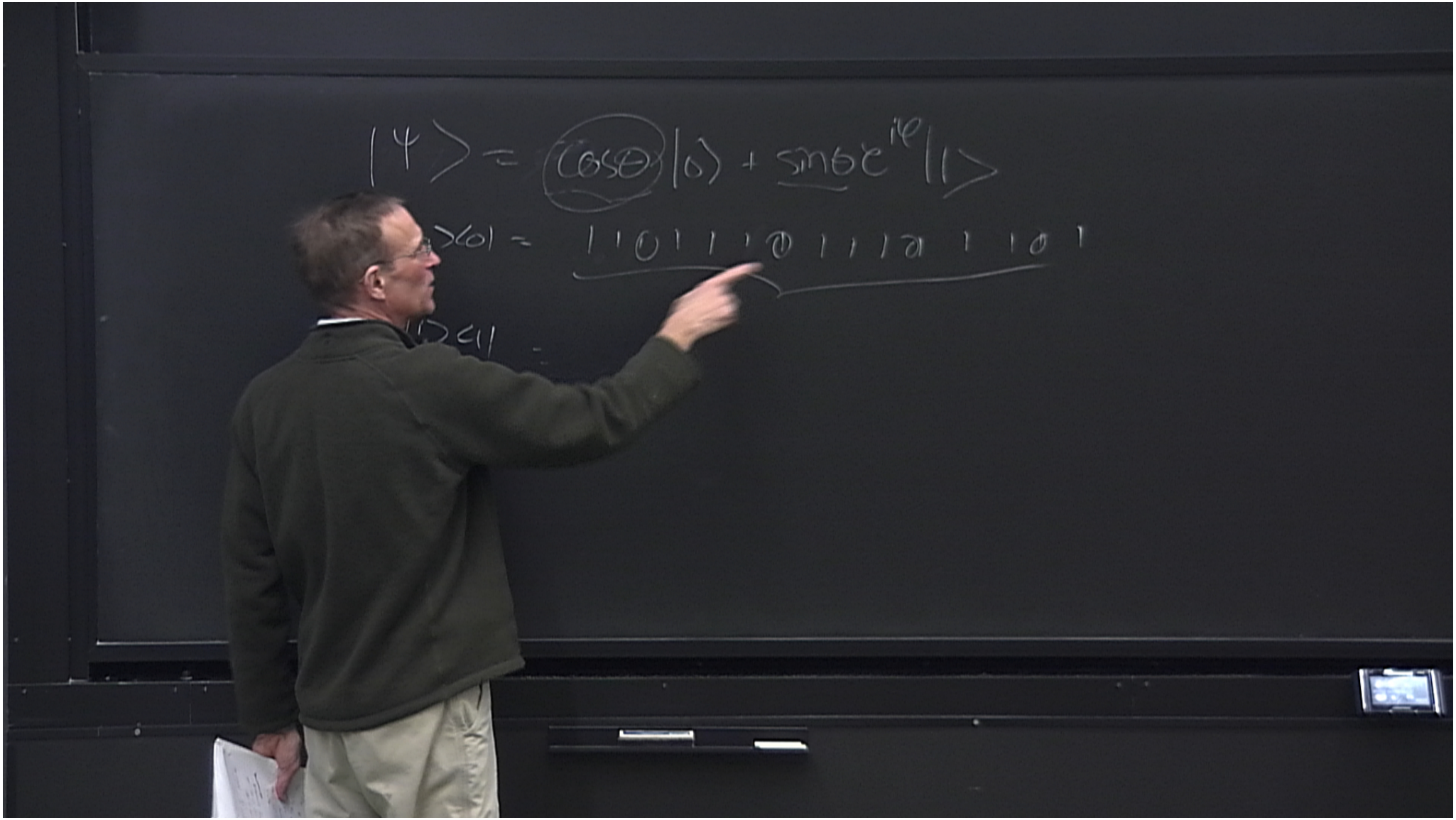


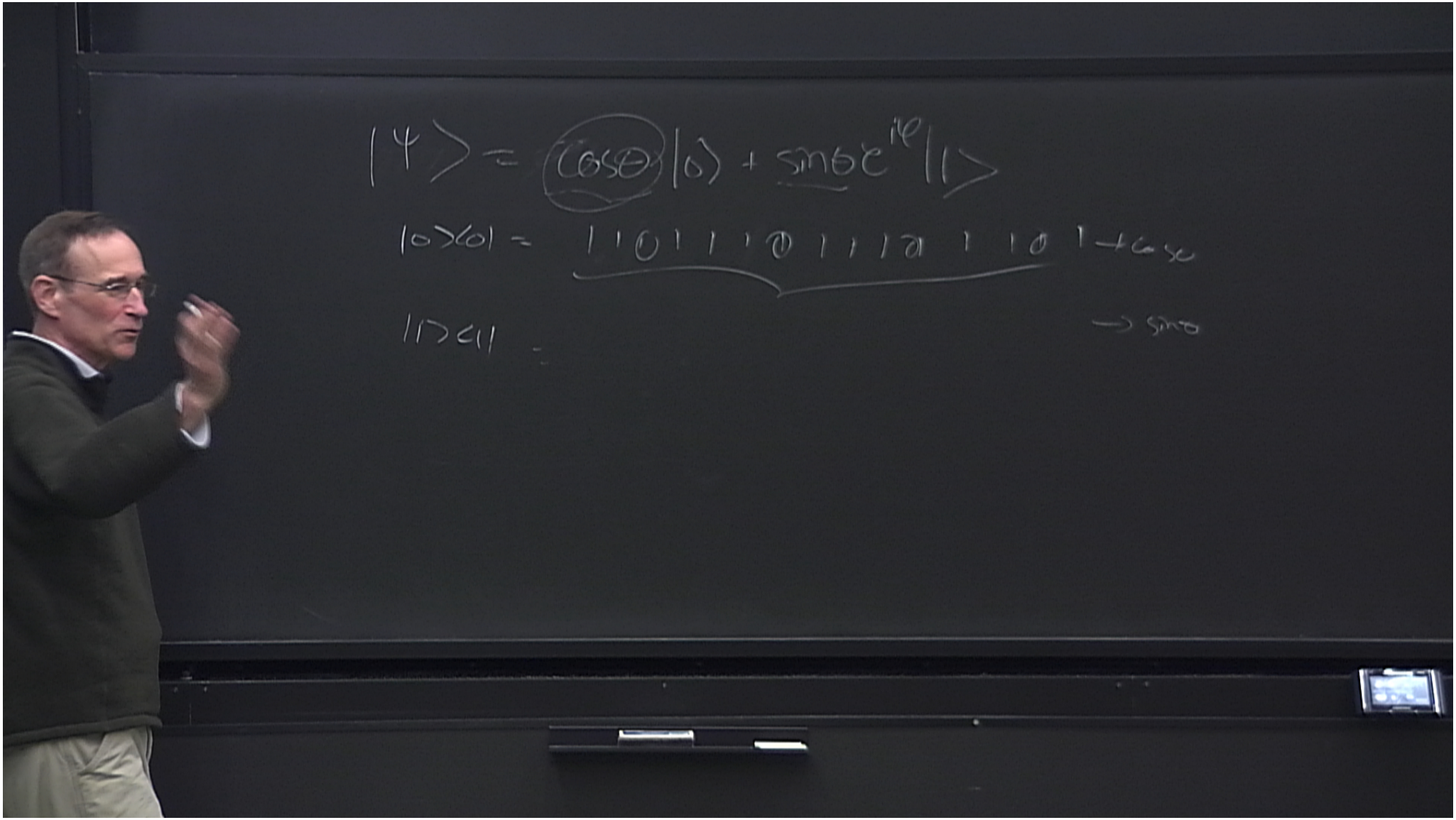
$\mathbb{1}$
X
Y
Z

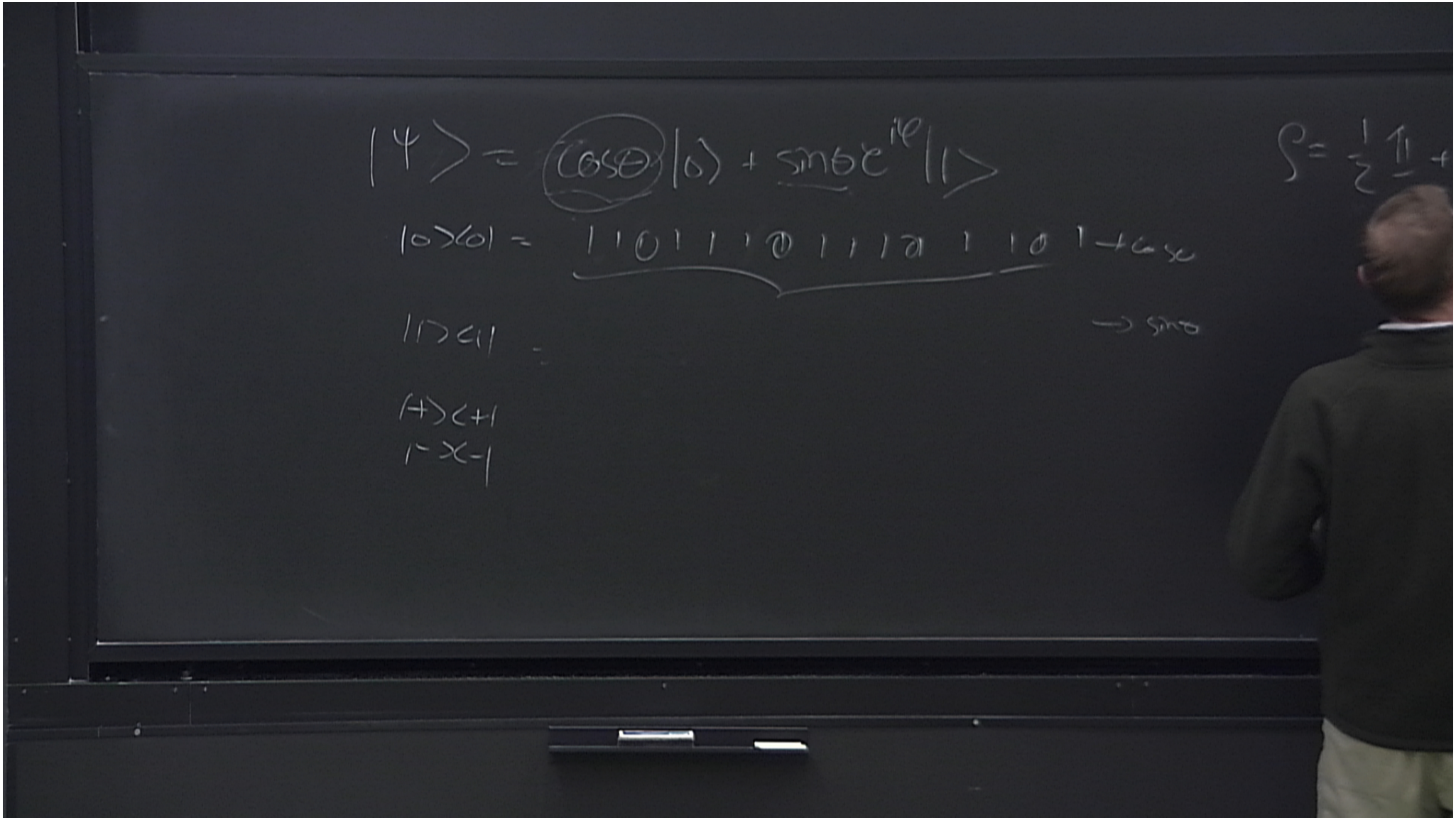

$|\psi\rangle = \cos\theta\,|0\rangle + \sin\theta\, e^{i\varphi}|1\rangle$
$|0\rangle\langle 0| =$

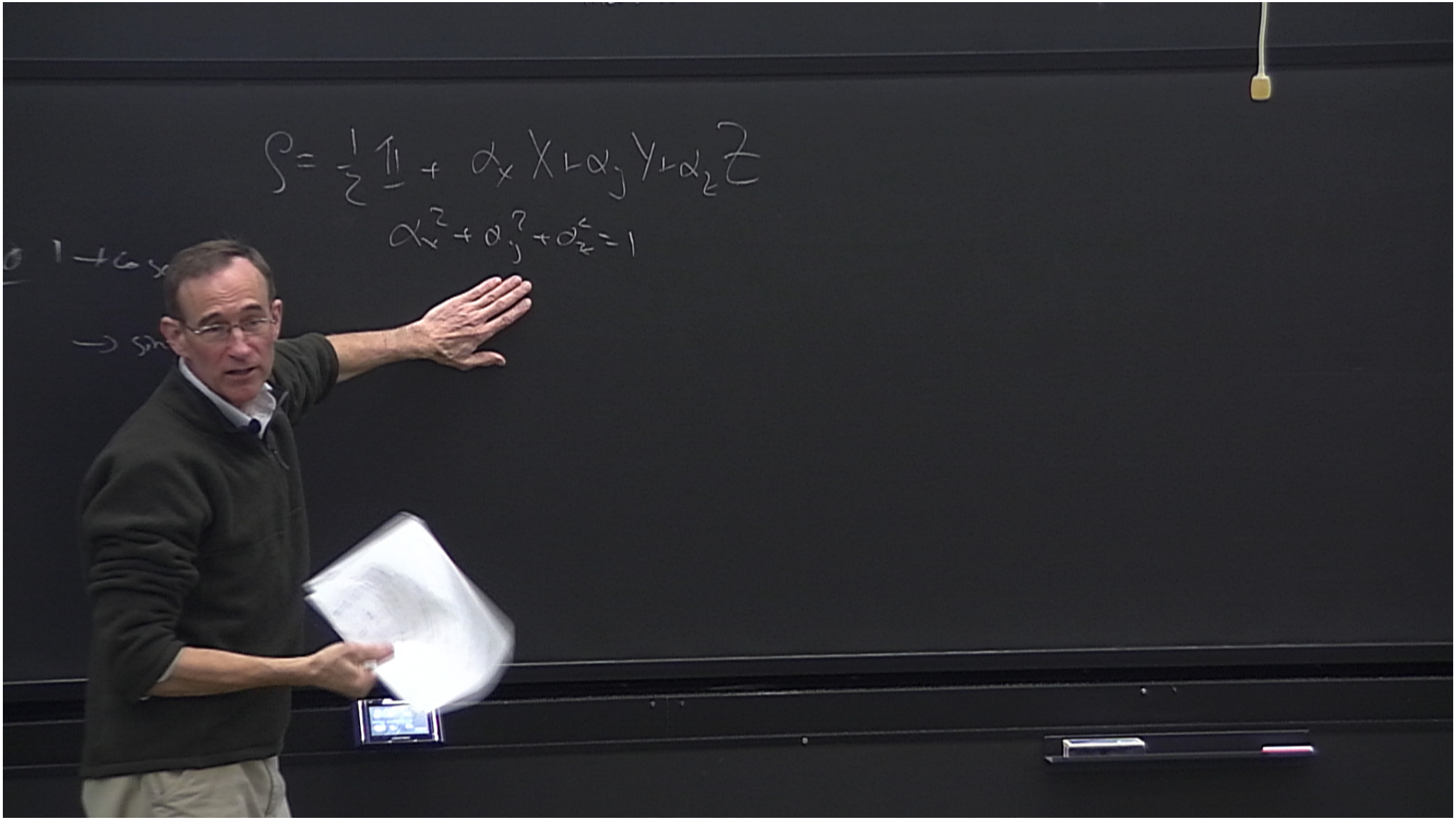

|ψ> = cosθ |0> + sinθ e^{iφ} |1>
|0><0| = |1><1| + |0><1| + |1><0| + |0><0|

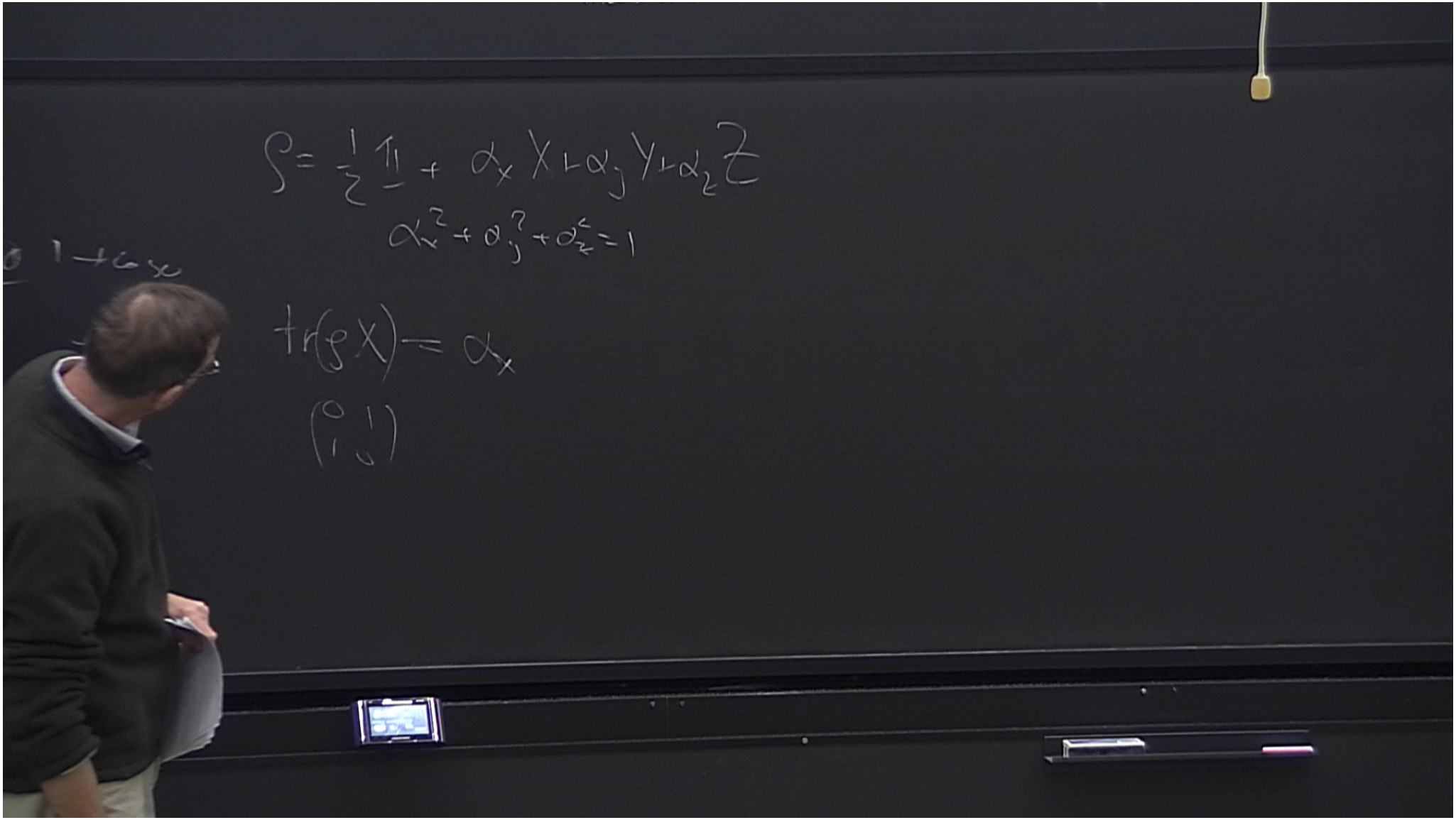

|ψ> = cosθ |0> + sinθ e^{iφ} |1>

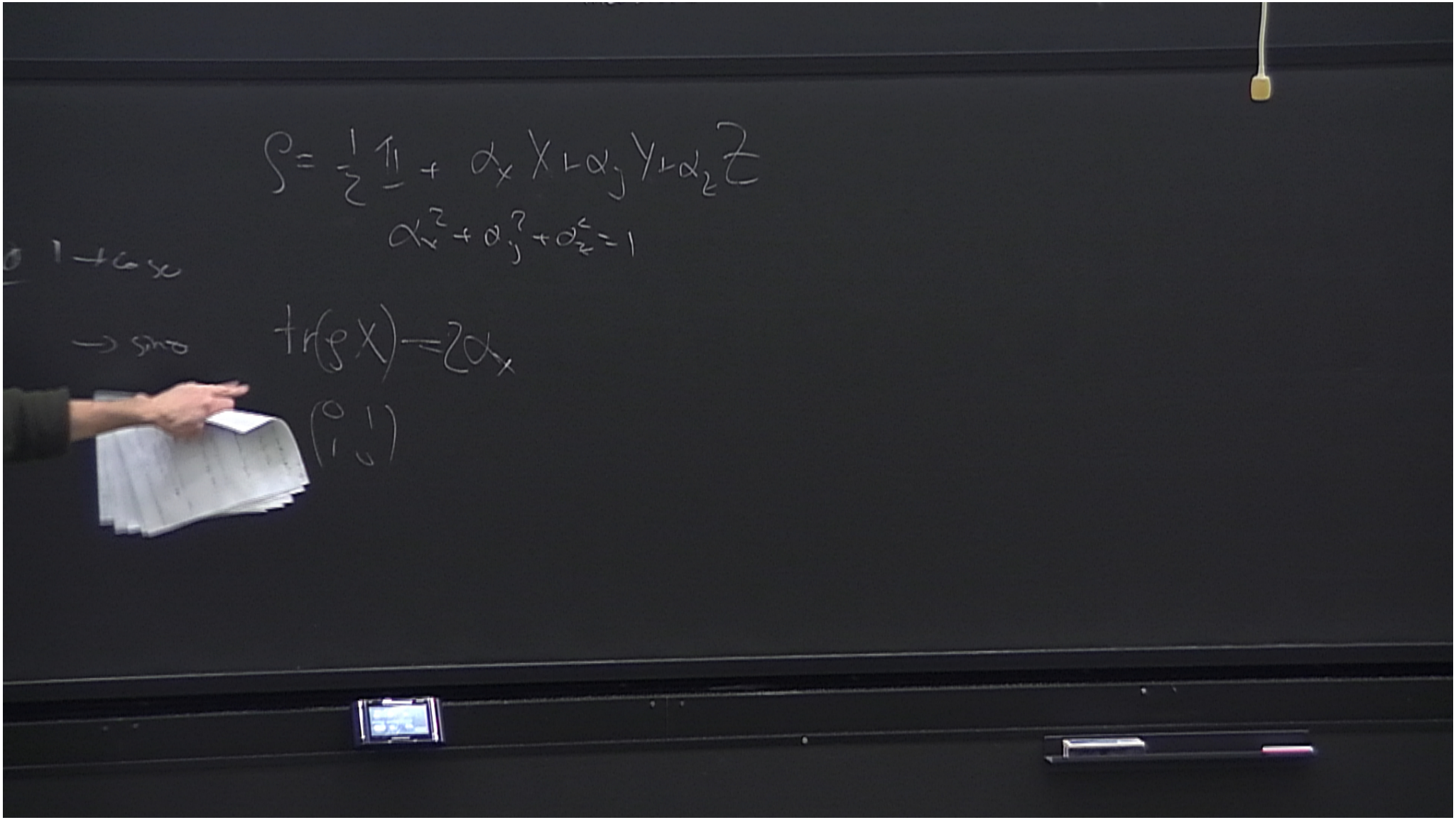

|ψ> = cosθ|0> + sinθe^{iφ}|1>


|ψ> = cosθ|0> + sinθe^{iφ}|1>
|0><0| = |1|0|1|1|0|1|1|0|1|1|0|1|+ conj
|1><1| .
→ sinθ


$|\psi\rangle = \cos\theta\,|0\rangle + \sin\theta\, e^{i\varphi}|1\rangle$
$|0\rangle\langle 0| =$
$|1\rangle\langle 1|$
$|+\rangle\langle +|$
$|-\rangle\langle -|$


$\rho = \frac{1}{2}\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z$
$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$


$\rho = \frac{1}{2}\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z$
$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$
$\mathrm{tr}(\rho X) = \alpha_x$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$


$\rho = \frac{1}{2}\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z$
$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$
$\rightarrow \sin\theta$
$\mathrm{tr}(\rho X) = 2\alpha_x$

$$\rho = \frac{1}{2}\left(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z\right)$$

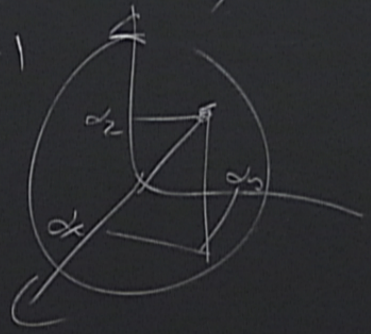
$$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$$

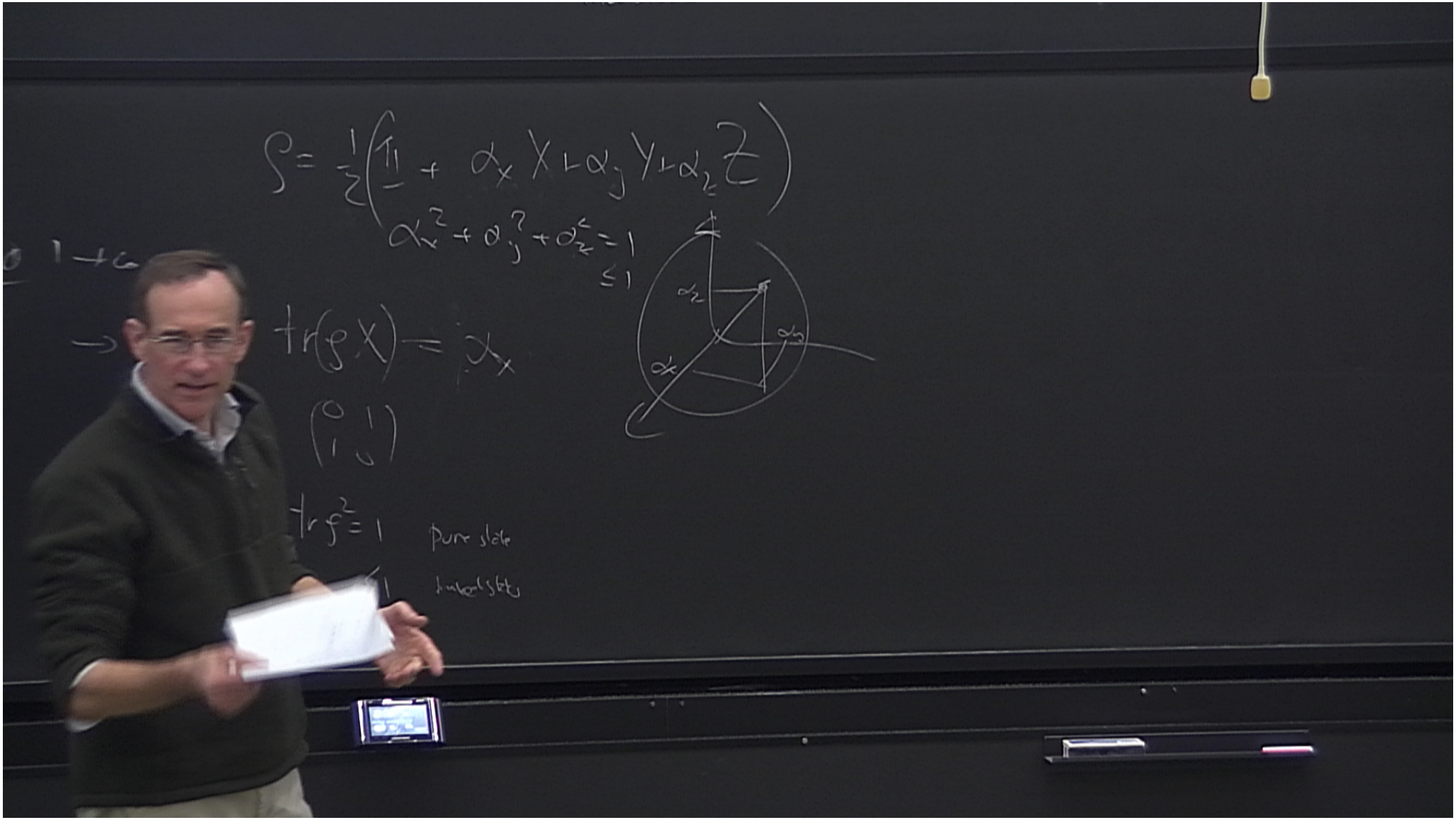
θ 1 + cos x

→ sin θ

$$\mathrm{tr}(\rho X) = \alpha_x$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$




$\rho = \frac{1}{2}(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z)$
$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$
≤ 1
α_z
α_y
α_x
$\mathrm{tr}(\rho X) = \alpha_x$
$\mathrm{tr}\,\rho^2 = 1$ pure state

$$\rho = \frac{1}{2}\left(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z\right)$$

$$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1 \quad \leq 1$$

$$|0\rangle\langle 0| = \frac{1}{2}(\mathbb{1} + Z)$$

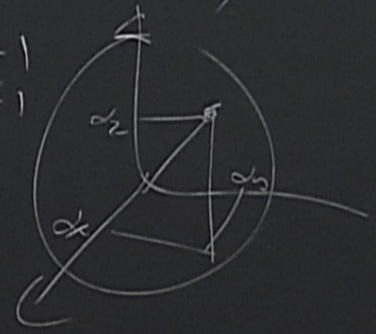
$$|1\rangle\langle 1| = \frac{1}{2}(\mathbb{1} - Z)$$

$1 + \cos x$

$\rightarrow \sin$

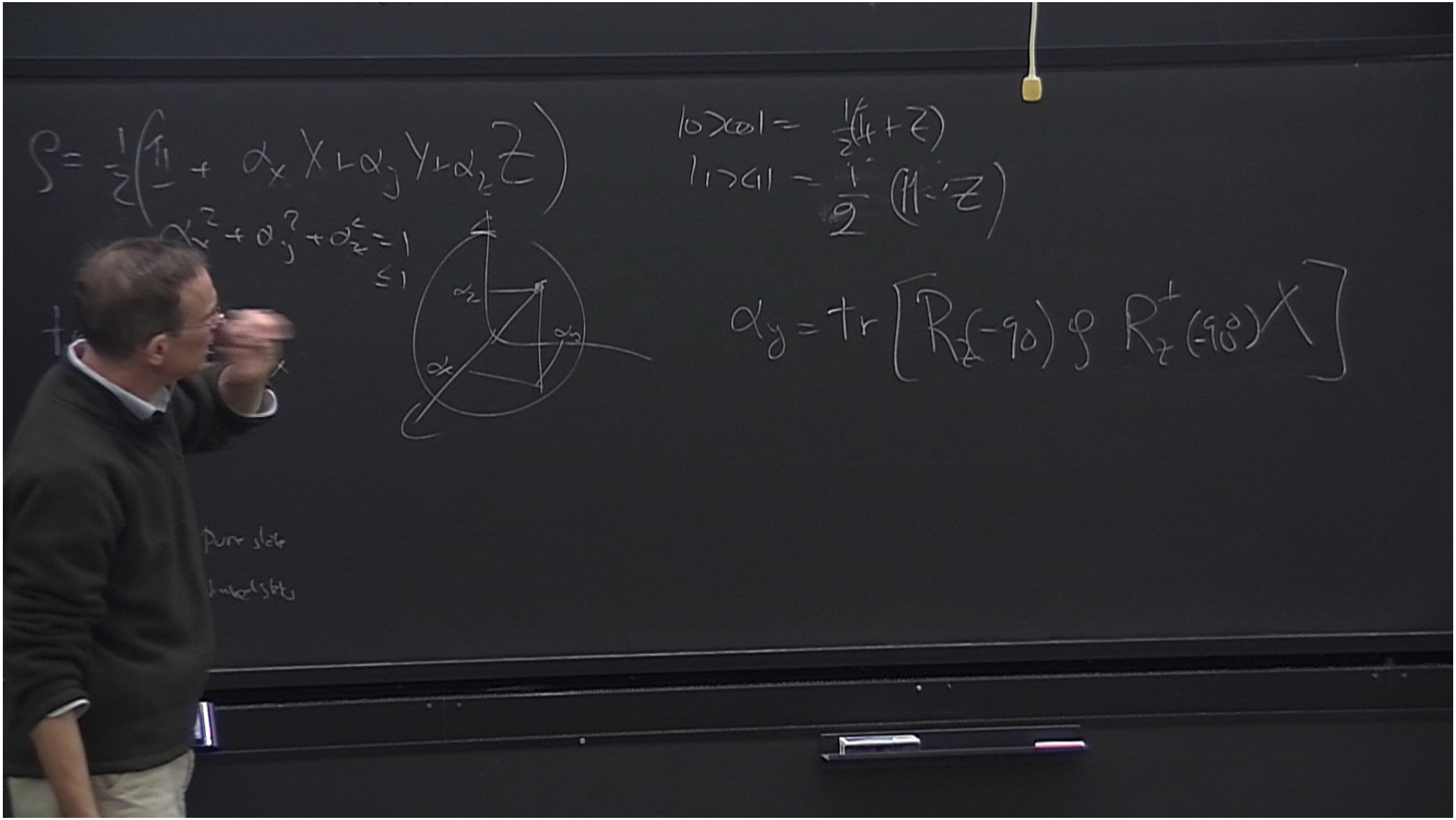
$$\mathrm{tr}(\rho X) = \alpha_x$$

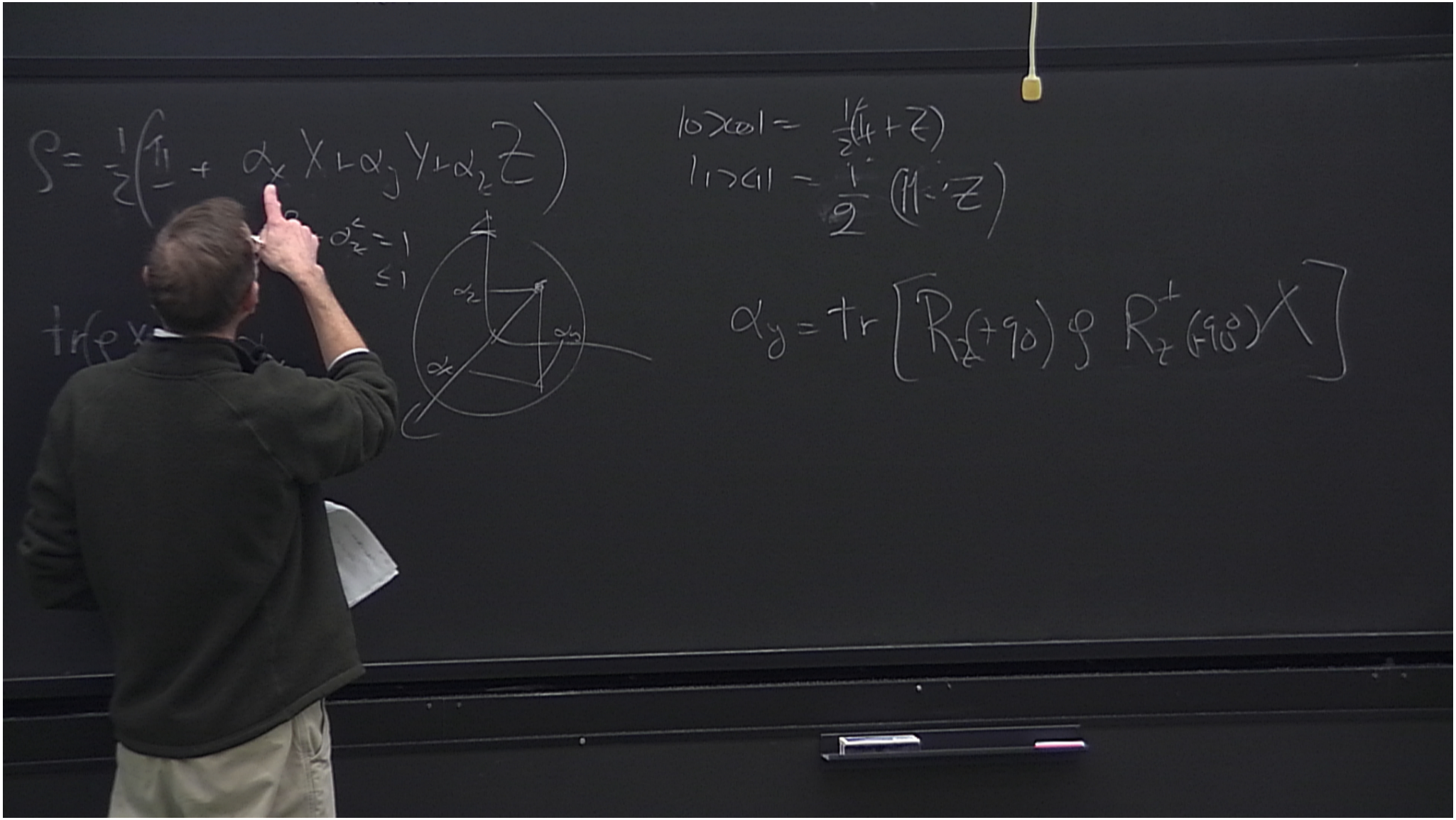
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

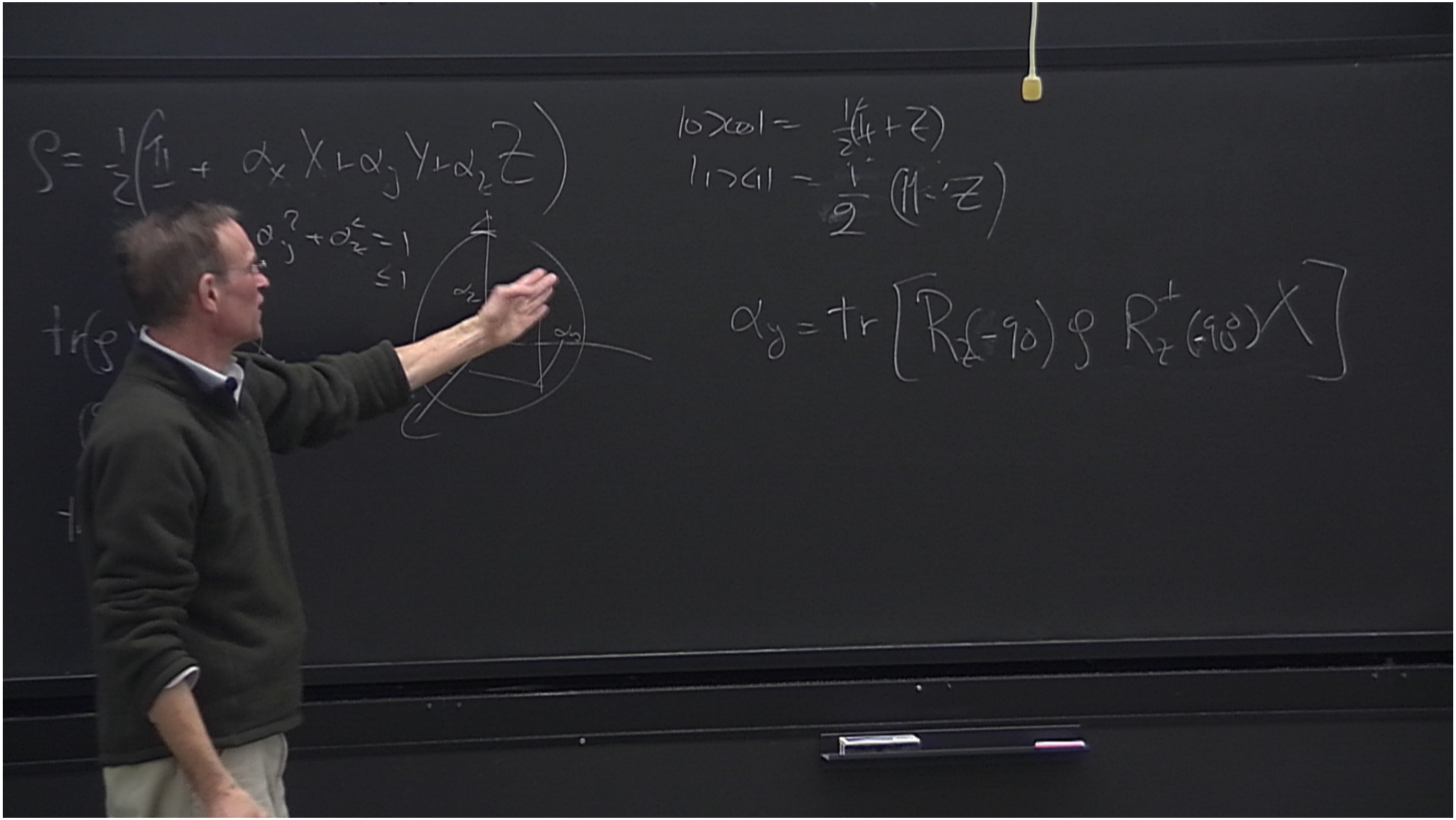


$\mathrm{tr}\,\rho^2 = 1$ pure state

< 1 mixed state


$\rho = \frac{1}{2}(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z)$
$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$
≤ 1
$|0\rangle\langle 0| = \frac{1}{2}(\mathbb{1} + Z)$
$|1\rangle\langle 1| = \frac{1}{2}(\mathbb{1} - Z)$
$\alpha_y = \mathrm{Tr}\left[R_z(-90)\,\rho\,R_z^\dagger(-90)\,X\right]$
pure state


$\rho = \frac{1}{2}(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z)$
$|0\rangle\langle 0| = \frac{1}{2}(\mathbb{1} + Z)$
$|1\rangle\langle 1| = \frac{1}{2}(\mathbb{1} - Z)$
$\alpha_y = \mathrm{tr}\left[R_z(+90)\,\rho\,R_z^\dagger(+90)\,X\right]$

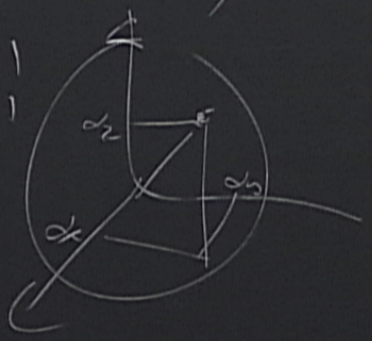

$\rho = \frac{1}{2}(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z)$
$\alpha_y^2 + \alpha_z^2 = 1$
≤ 1
$|0\rangle\langle 0| = \frac{1}{2}(\mathbb{1} + Z)$
$|1\rangle\langle 1| = \frac{1}{2}(\mathbb{1} - Z)$
$\alpha_y = \mathrm{tr}\left[R_z(-90)\,\rho\,R_z^\dagger(-90)\,X\right]$

$$\rho = \frac{1}{2}\left(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z\right)$$

$$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1 \leq 1$$

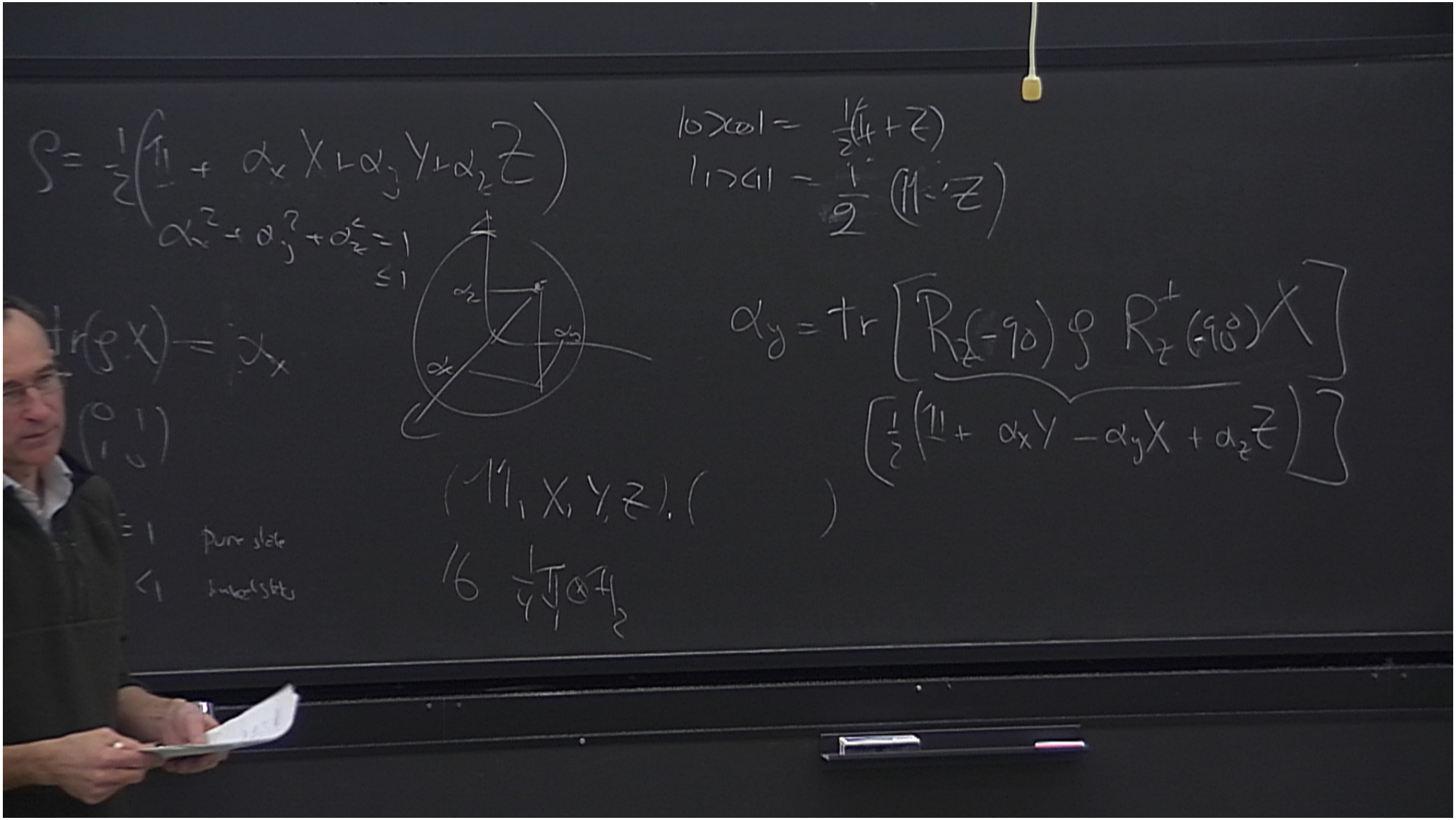
$$|0\rangle\langle 0| = \frac{1}{2}(\mathbb{1} + Z)$$

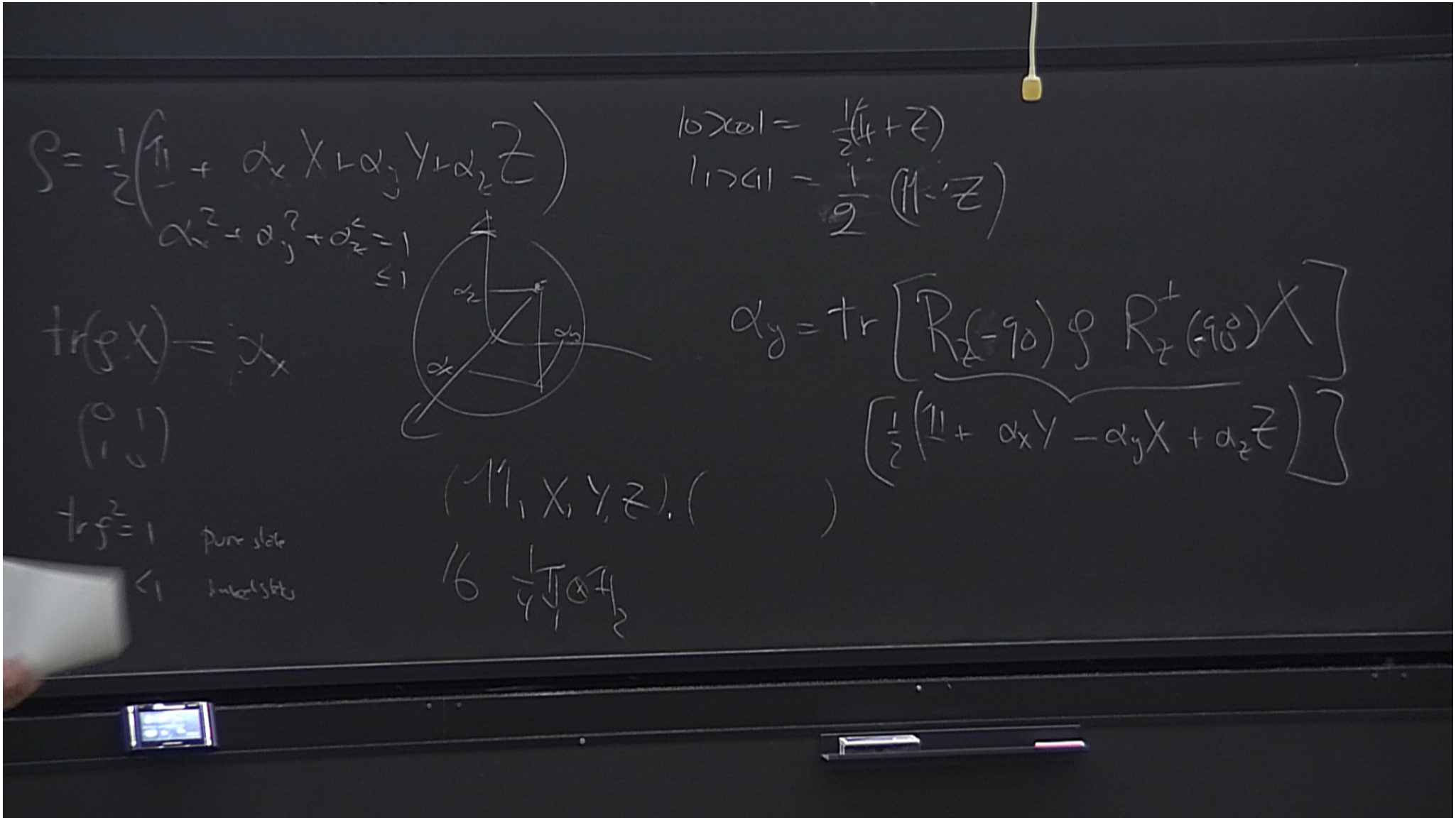
$$|1\rangle\langle 1| = \frac{1}{2}(\mathbb{1} - Z)$$



$$\alpha_y = \mathrm{tr}\left[R_z(-90)\,\rho\,R_z^\dagger(-90)\,X\right]$$

$$\left[\frac{1}{2}\left(\mathbb{1} + \alpha_x Y - \alpha_y X + \alpha_z Z\right)\right]$$


$\rho = \frac{1}{2}(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z)$
$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$
≤ 1
$|0\rangle\langle 0| = \frac{1}{2}(\mathbb{1} + Z)$
$|1\rangle\langle 1| = \frac{1}{2}(\mathbb{1} - Z)$
$\alpha_y = \mathrm{tr}\left[R_z(-90)\,\rho\,R_z^\dagger(-90)\,X\right]$
$\left[\frac{1}{2}(\mathbb{1} + \alpha_x Y - \alpha_y X + \alpha_z Z)\right]$
$\mathrm{tr}(\rho X) = \alpha_x$
$(\mathbb{1}, X, Y, Z) \cdot (\quad)$
$16\ \frac{1}{4}\mathbb{1} \otimes \mathbb{1}$
$= 1$ pure state
< 1 mixed state

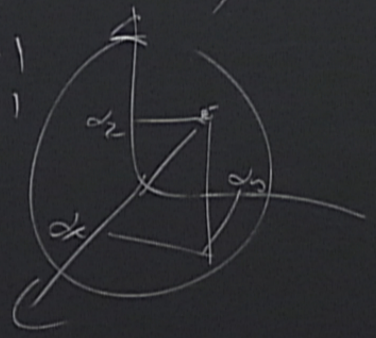

$\rho = \frac{1}{2}(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z)$
$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$
≤ 1
$|0\rangle\langle 0| = \frac{1}{2}(\mathbb{1} + Z)$
$|1\rangle\langle 1| = \frac{1}{2}(\mathbb{1} - Z)$
$\alpha_y = \mathrm{tr}\left[R_z(-90)\,\rho\,R_z^\dagger(-90)\,X\right]$
$\left[\frac{1}{2}(\mathbb{1} + \alpha_x Y - \alpha_y X + \alpha_z Z)\right]$
$\mathrm{tr}(\rho X) = \alpha_x$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\mathrm{tr}\rho^2 = 1$ pure state
< 1 mixed state
$(\mathbb{1}, X, Y, Z) \cdot (\quad)$
$16\ \frac{1}{4}\mathbb{1} \otimes \mathbb{1}_2$

$$\rho = \frac{1}{2}\left(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z\right)$$

$$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1 \quad \leq 1$$

$$|0\rangle\langle 0| = \frac{1}{2}(\mathbb{1} + Z)$$

$$|1\rangle\langle 1| = \frac{1}{2}(\mathbb{1} - Z)$$



$$\alpha_y = \mathrm{tr}\left[R_z(-90)\,\rho\,R_z^\dagger(-90)\,X\right]$$

$$\left[\frac{1}{2}\left(\mathbb{1} + \alpha_x Y - \alpha_y X + \alpha_z Z\right)\right]$$

$$\mathrm{tr}(\rho X) = \alpha_x$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$(\mathbb{1}, X, Y, Z)\cdot\begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$$

$$16\ \frac{1}{4}\frac{1}{4}\otimes\frac{1}{4}_2$$

$$\mathrm{tr}\rho^2 = 1 \quad \text{pure state}$$

$$< 1 \quad \text{mixed state}$$

$$\mathbb{1}: -1$$

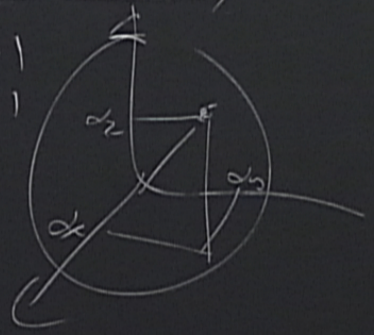
measurement

$$\rho = \frac{1}{2}\left(\mathbb{1} + \alpha_x X + \alpha_y Y + \alpha_z Z\right)$$

$$\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1 \quad \leq 1$$

$$|0\rangle\langle 0| = \frac{1}{2}(\mathbb{1} + Z)$$

$$|1\rangle\langle 1| = \frac{1}{2}(\mathbb{1} - Z)$$



$$\alpha_y = \mathrm{tr}\left[R_z(-90)\,\rho\,R_z^\dagger(-90)\,X\right]$$

$$\left[\frac{1}{2}\left(\mathbb{1} + \alpha_x Y - \alpha_y X + \alpha_z Z\right)\right]$$

$$\mathrm{tr}(\rho X) = \alpha_x$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

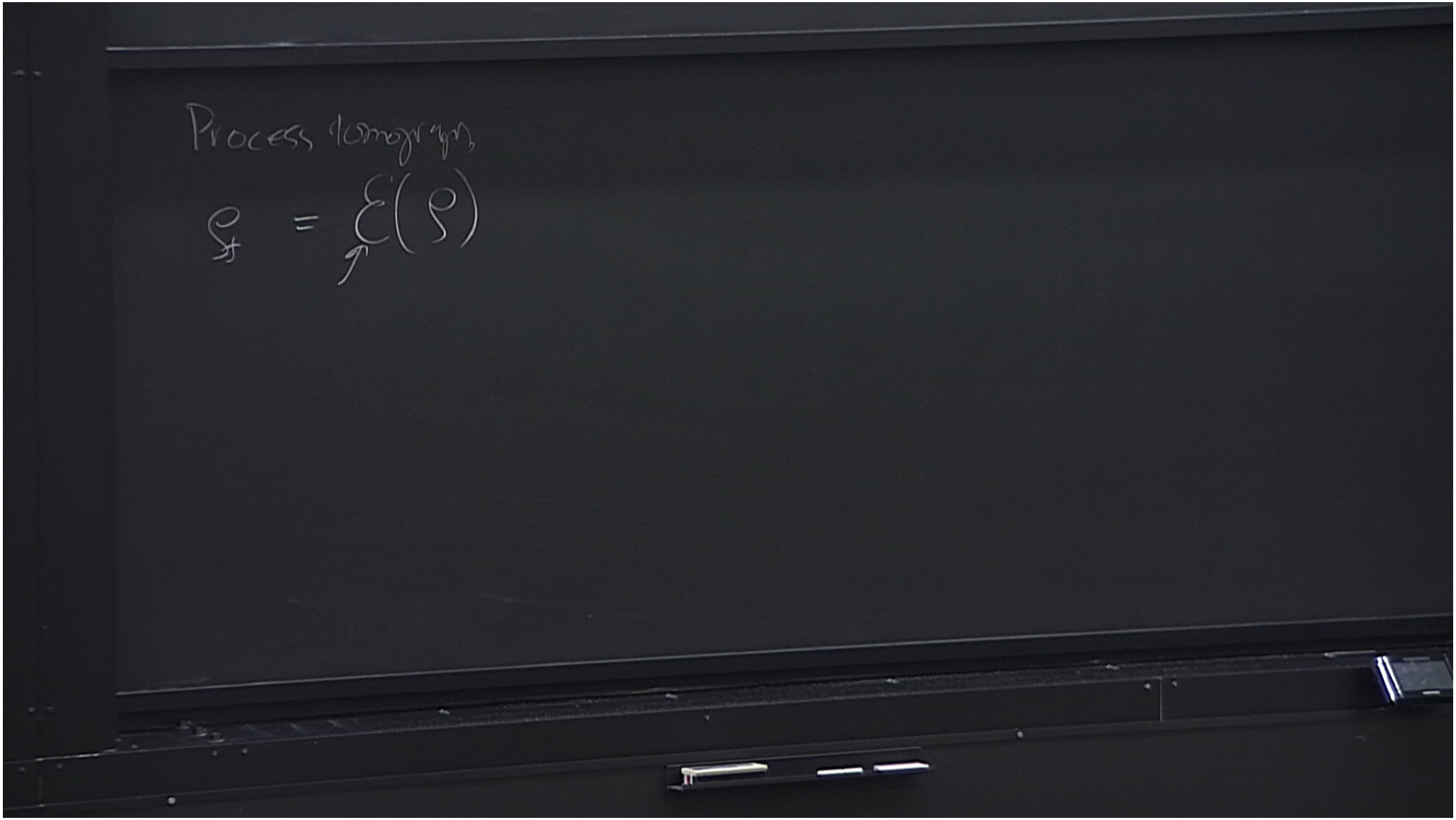
$$(\mathbb{1}, X, Y, Z)\cdot(\quad)$$

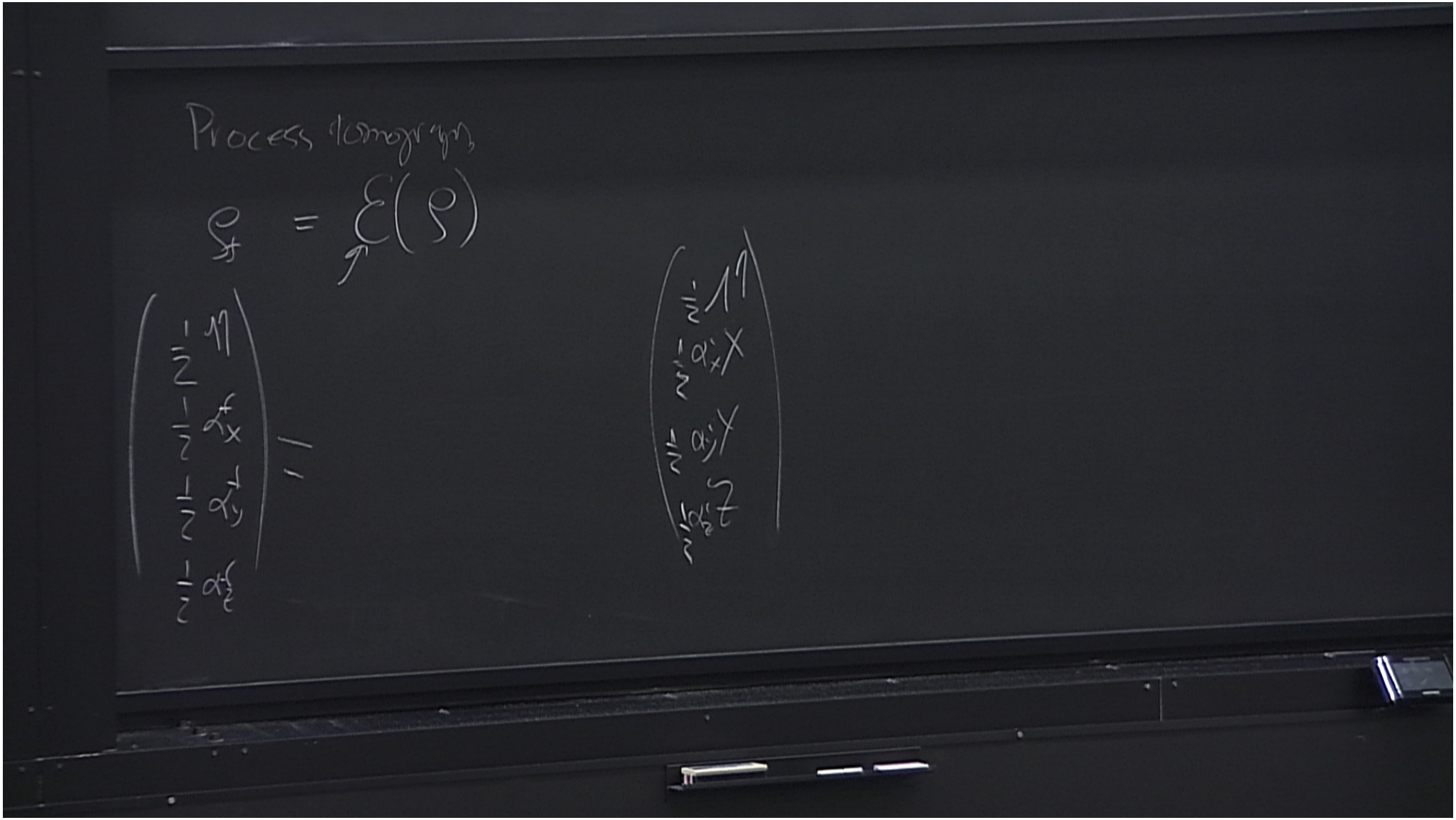
$$16\ \tfrac{1}{4}\,T \otimes T$$

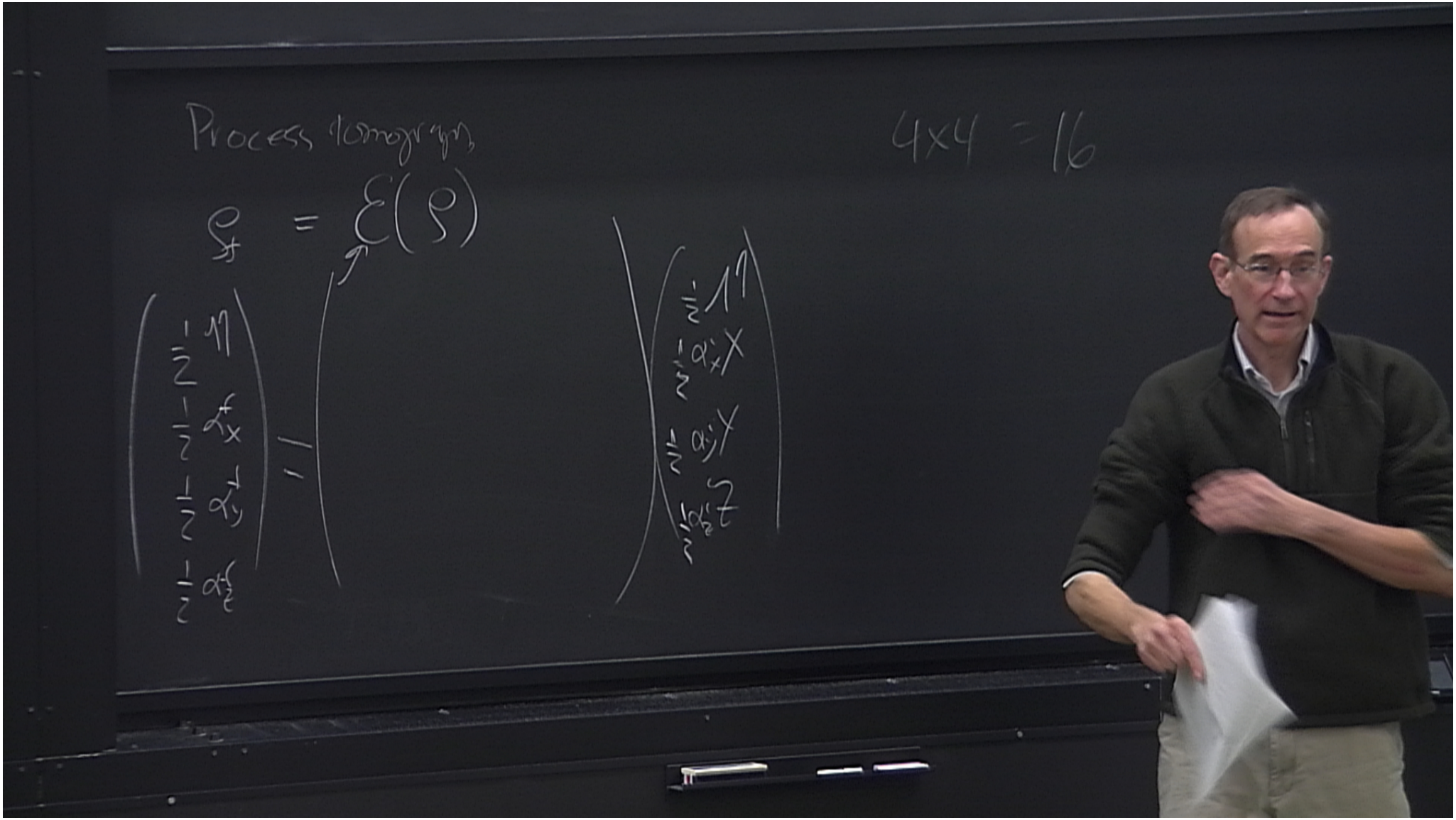
$$\mathrm{tr}\rho^2 = 1 \quad \text{pure state}$$

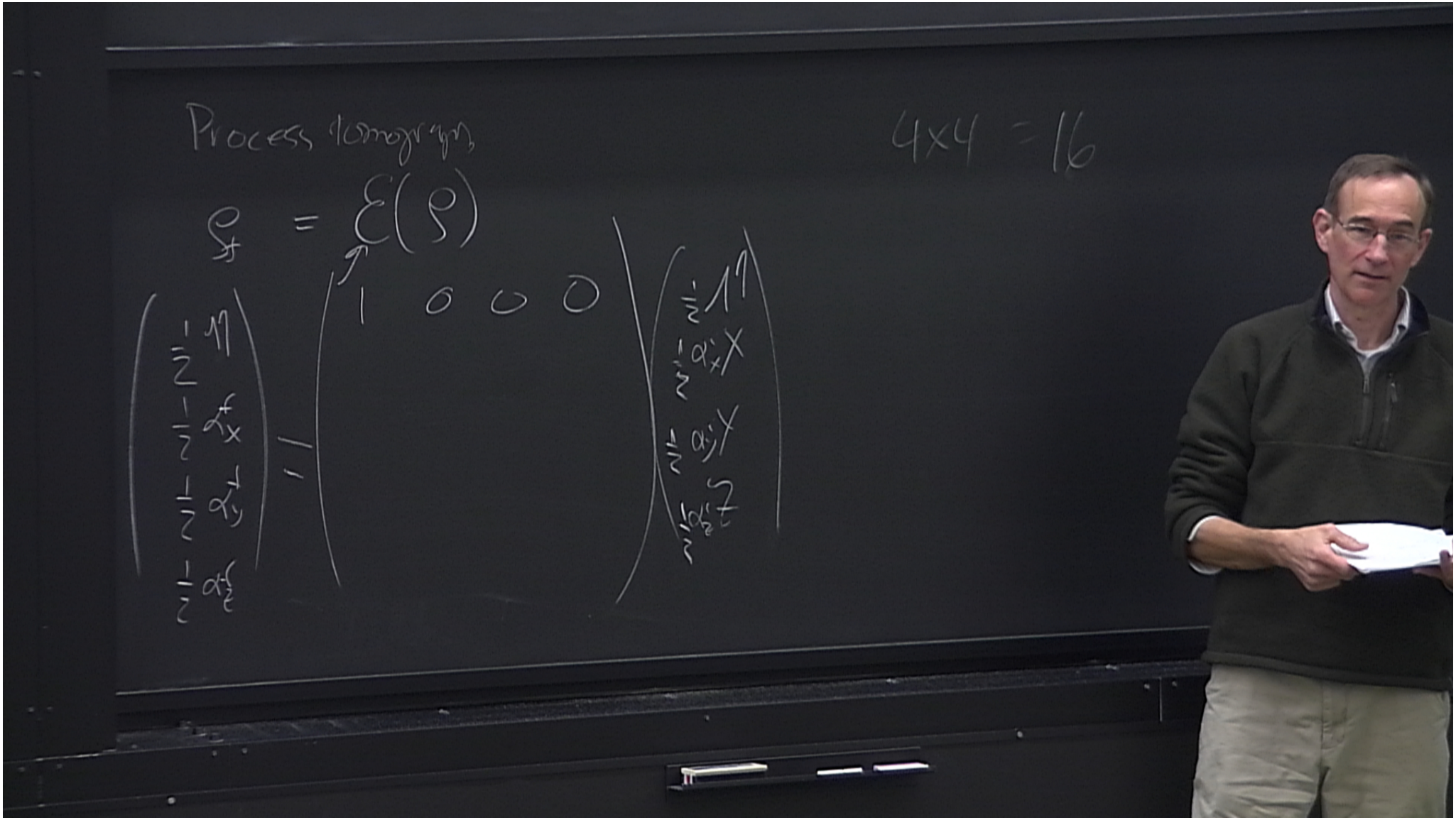
$$< 1 \quad \text{mixed state}$$

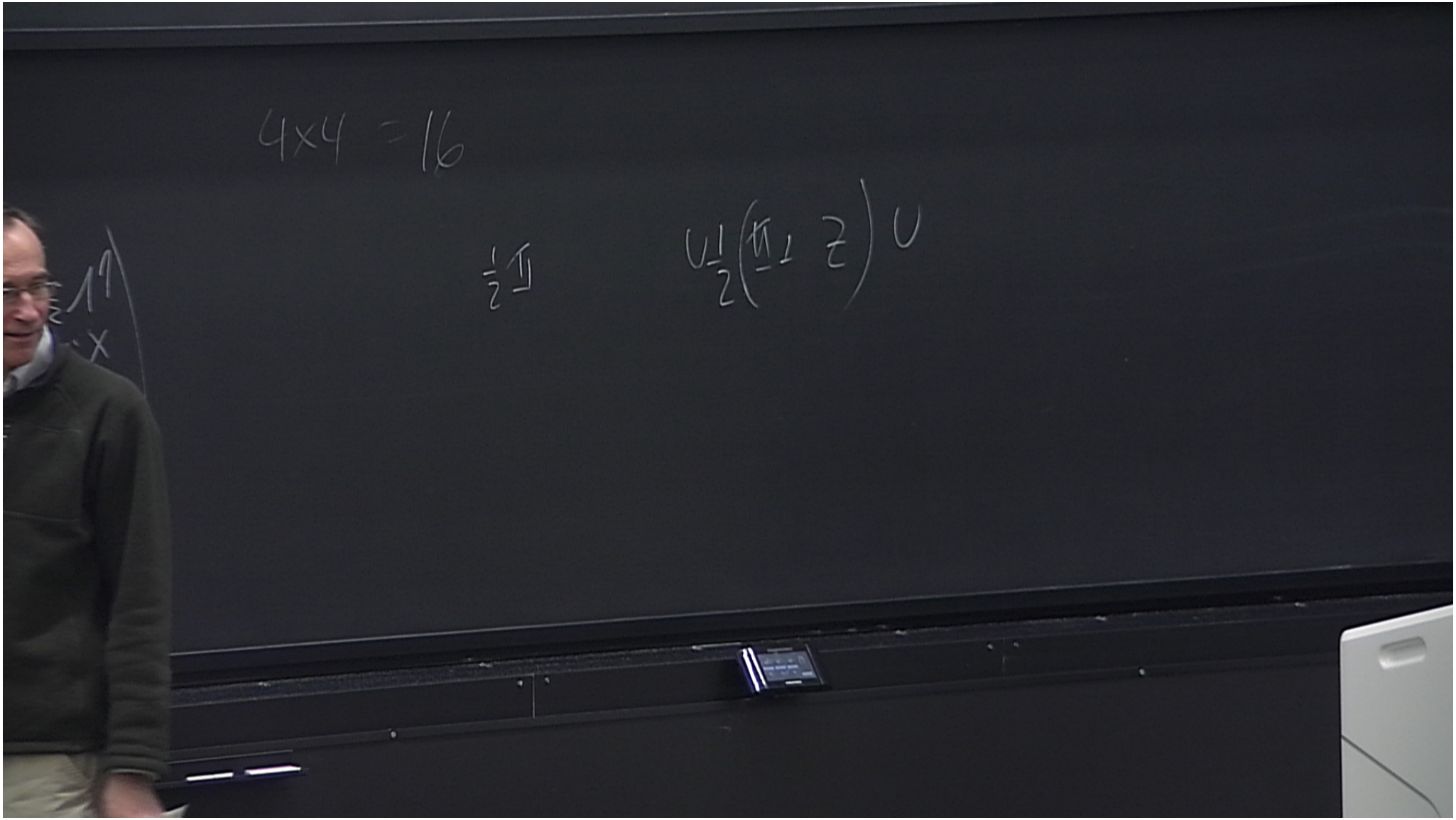
$$\mathbb{1}: -1$$

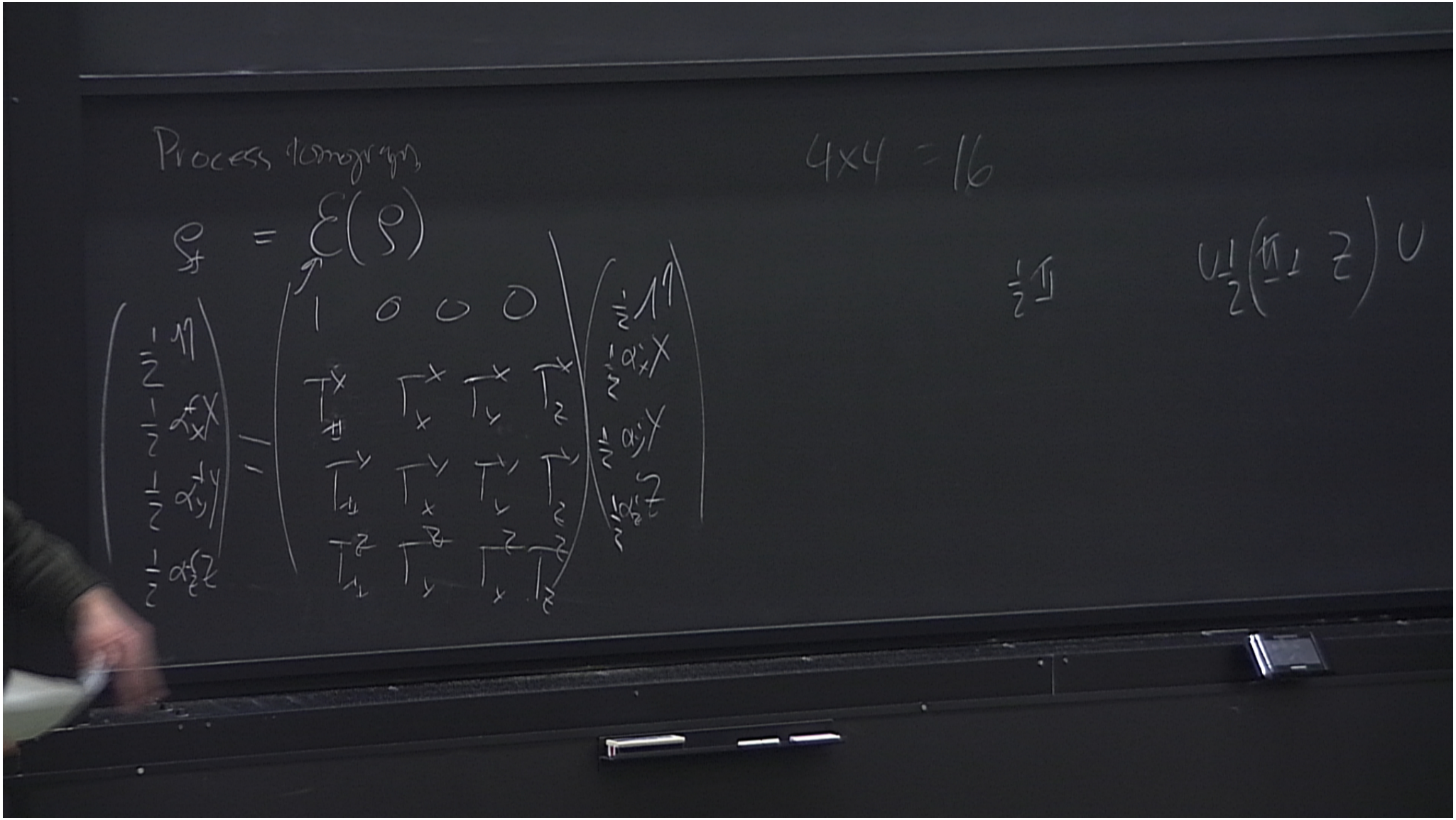

Process tomography
$\rho_{out} = \mathcal{E}(\rho)$

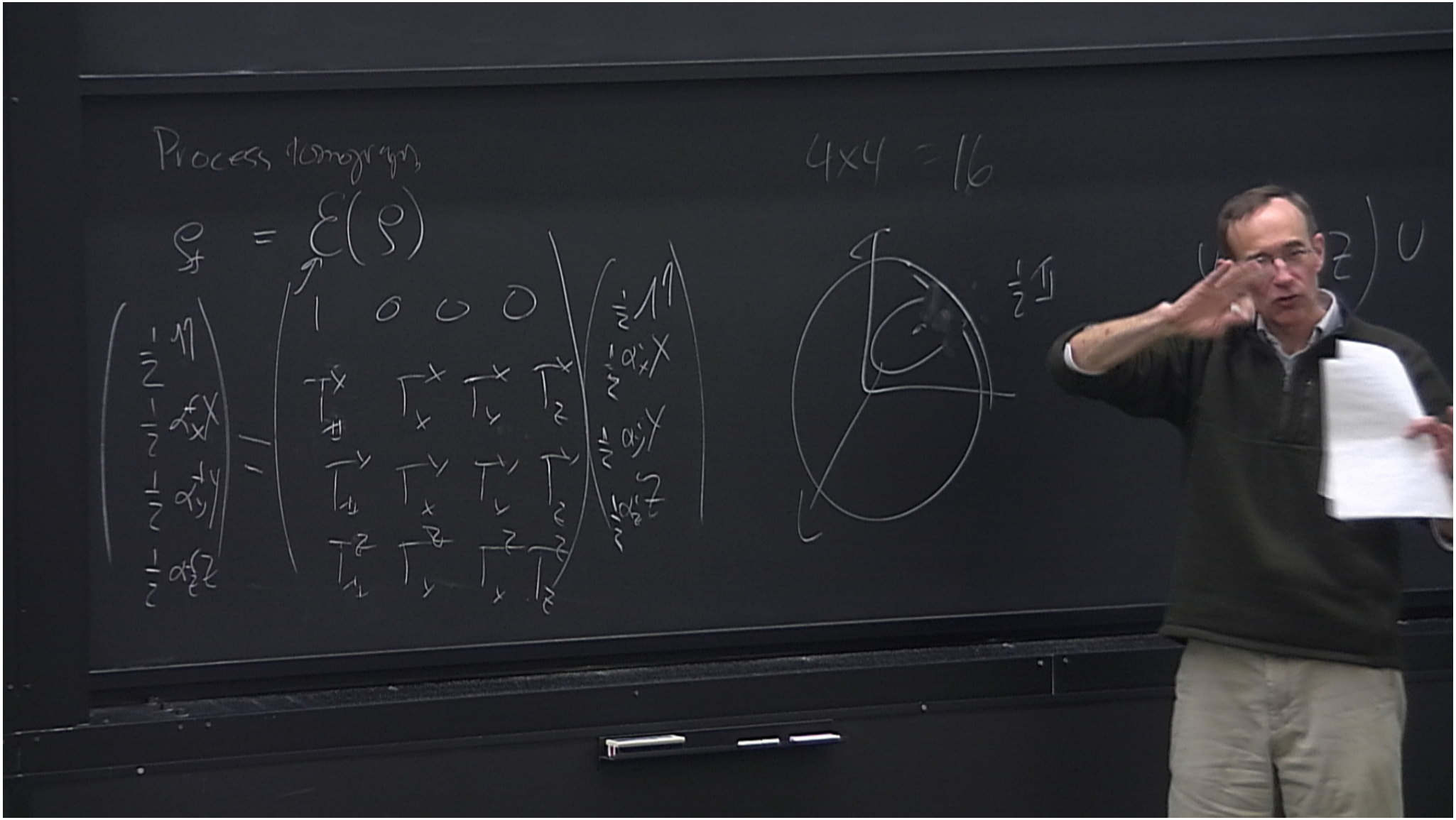

Process tomography
$\mathcal{E}(\rho) = \rho_{ft}$

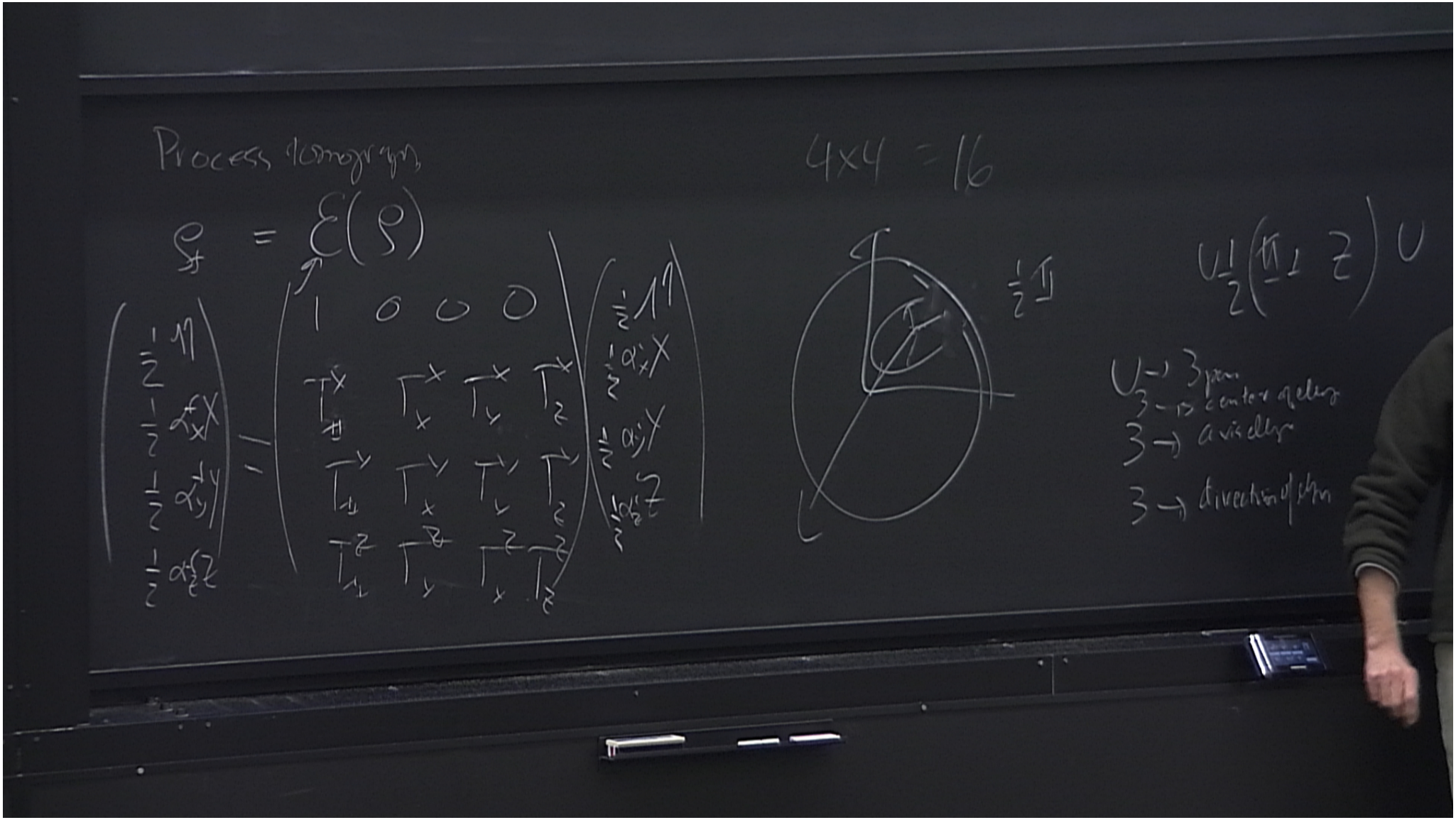

Process tomography
$\mathcal{E}(\rho) = \rho_{ft}$
$4 \times 4 = 16$

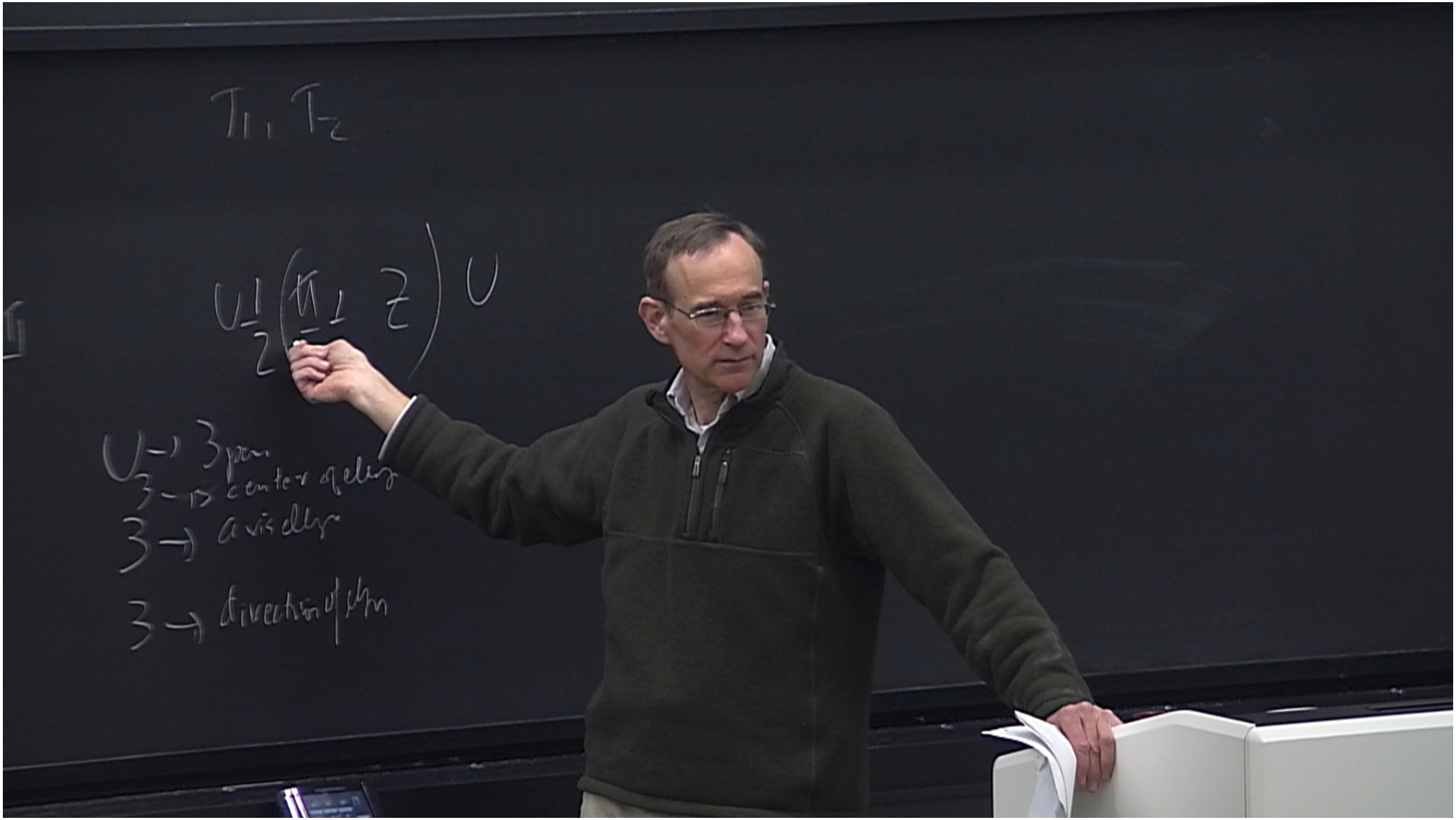

Process tomography
$\mathcal{E}(\rho)$
4x4 = 16

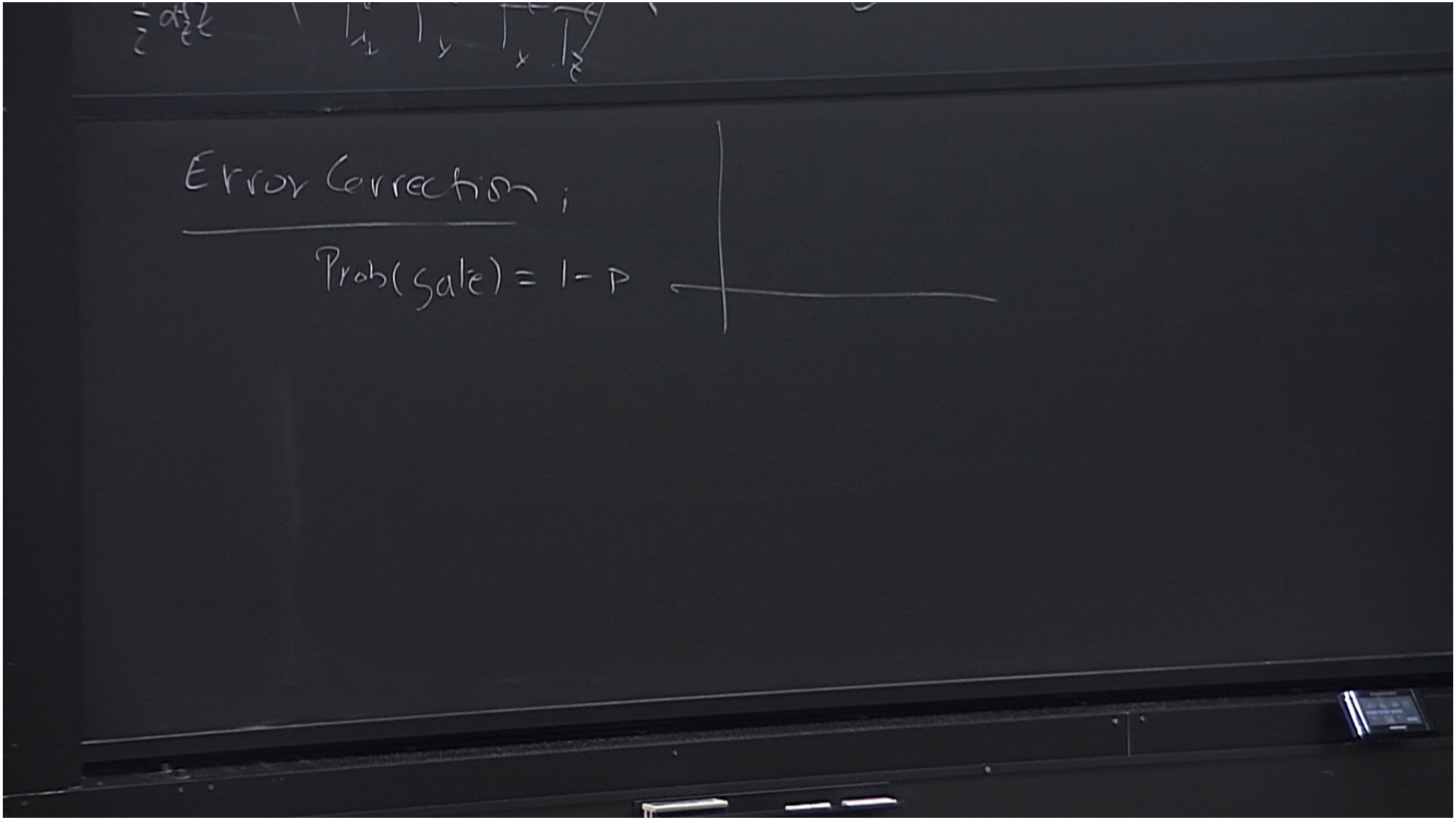

4x4 = 16

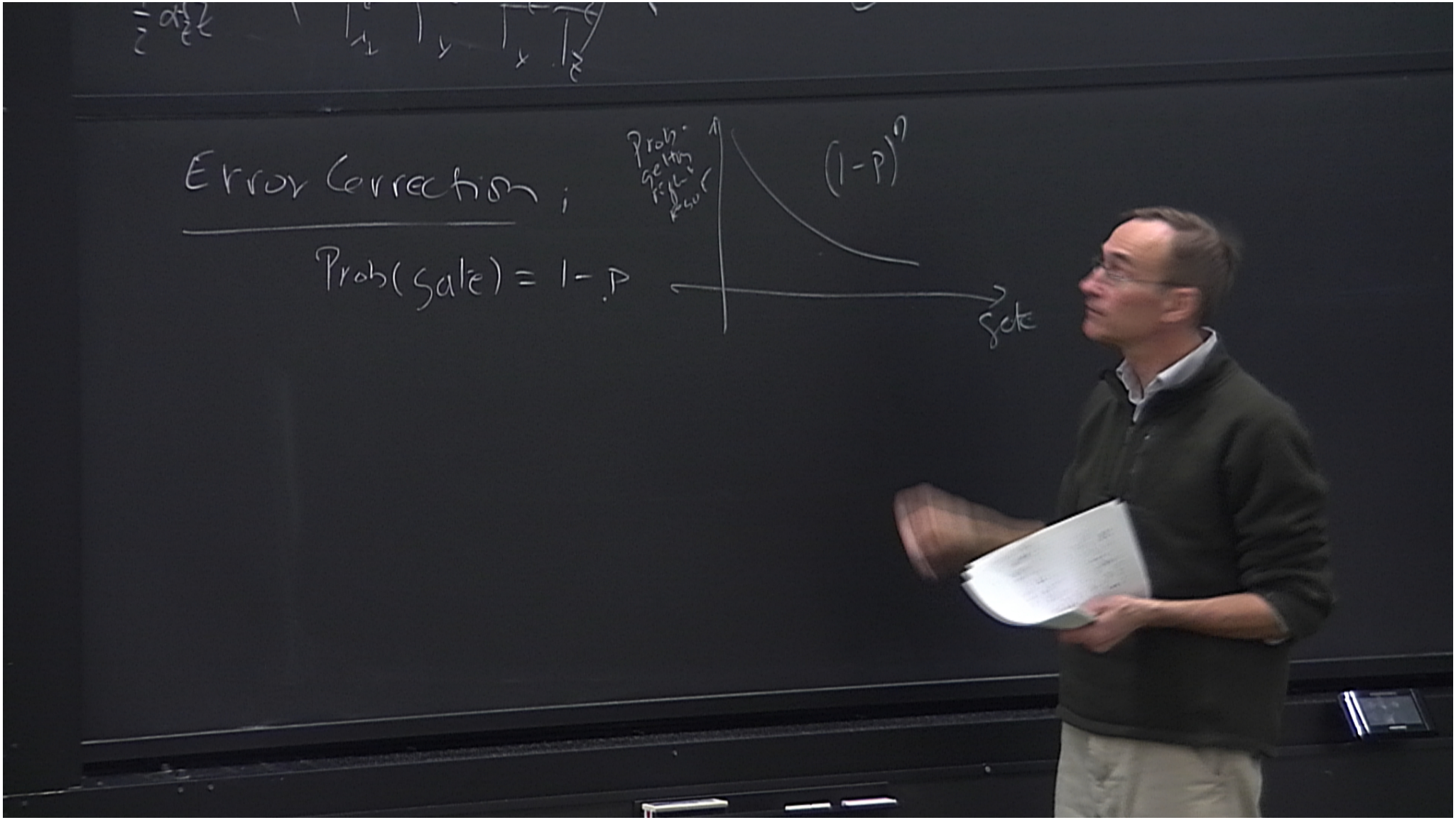

Process tomography
$\mathcal{E}(\rho)$
4×4 = 16

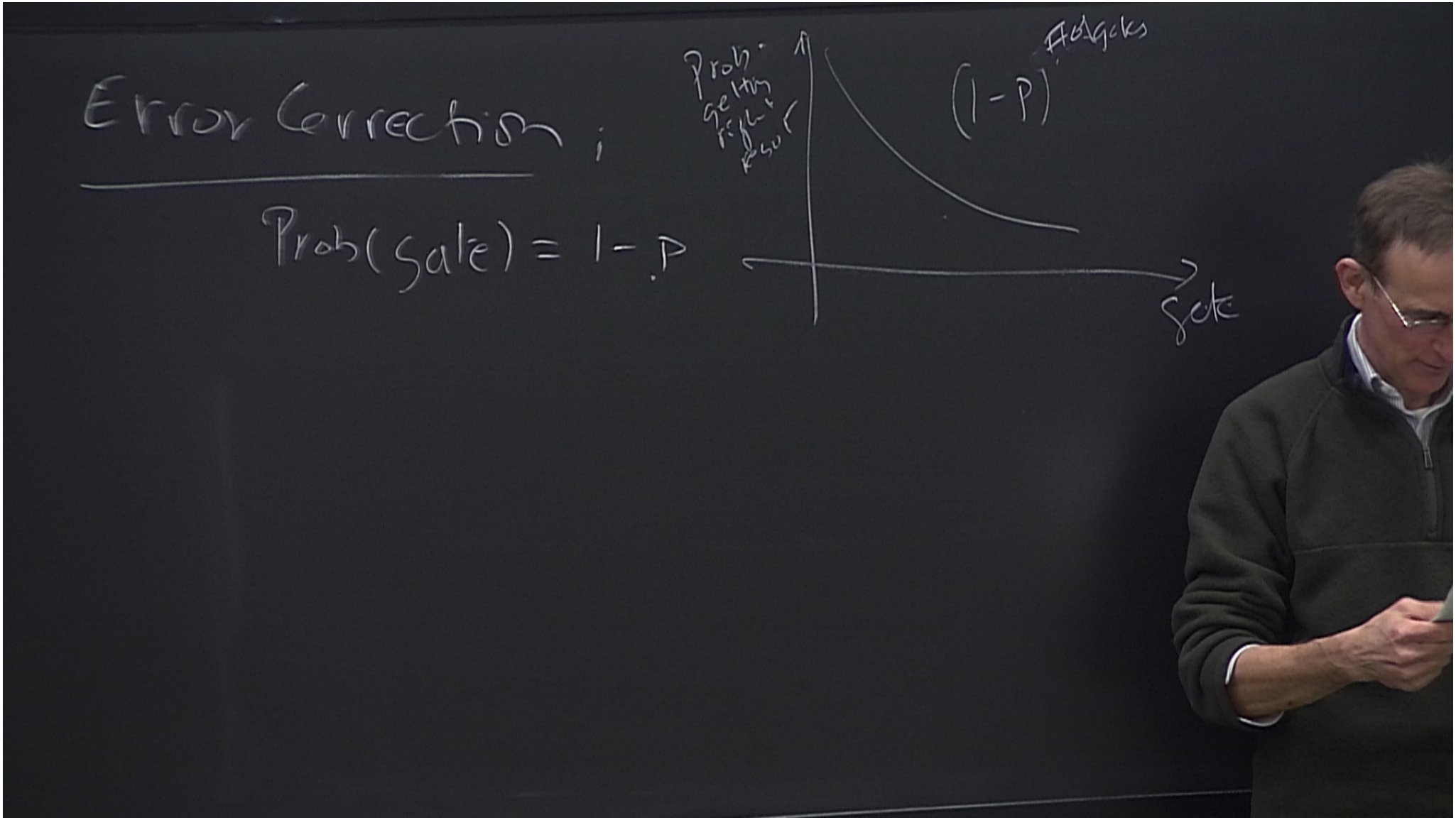

Process tomography
$\mathcal{E}(\rho)$
4x4 = 16

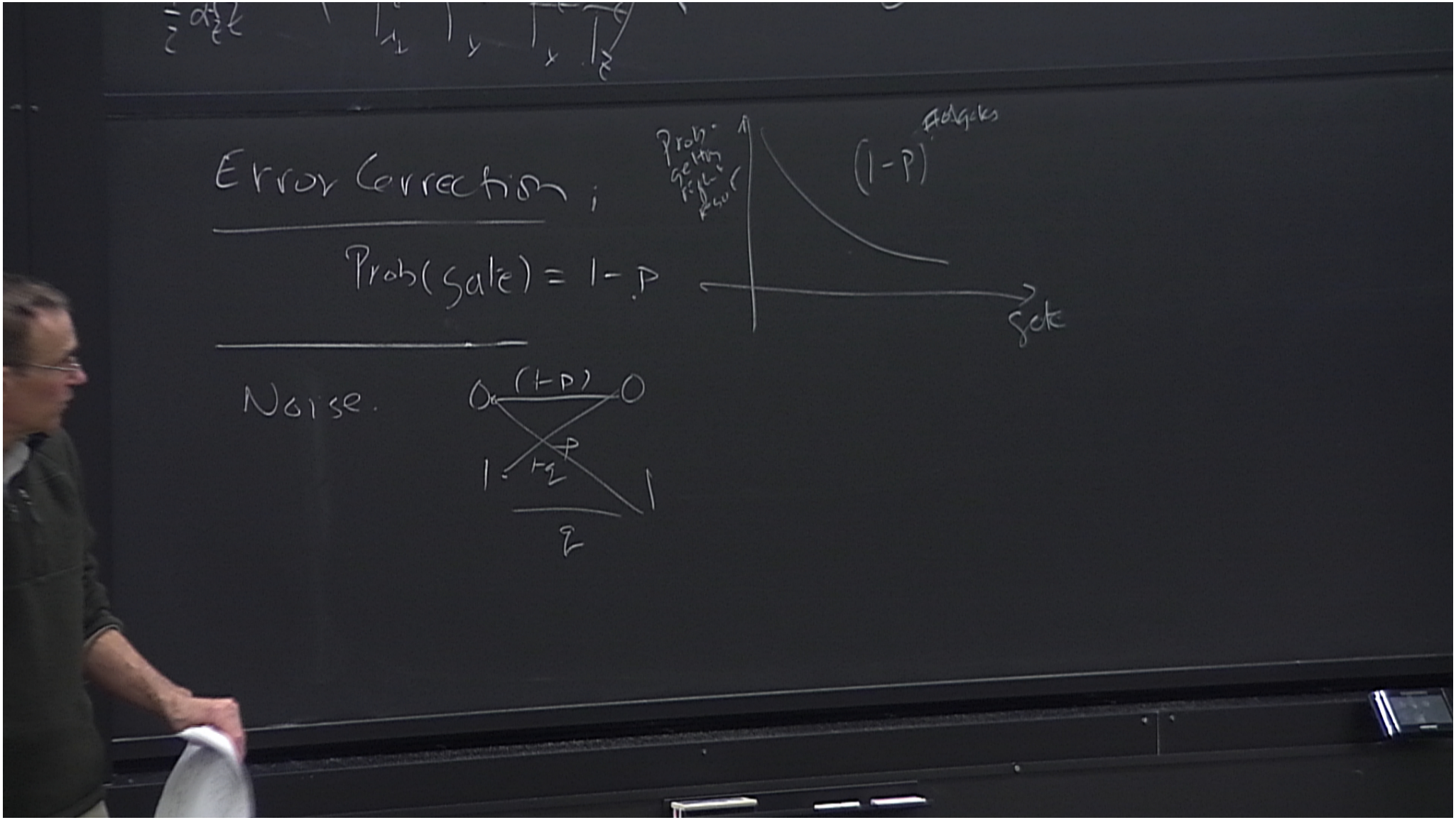

Process tomography
$\mathcal{E}(\rho)$
4x4 = 16
$\frac{1}{2}$ I
$U_{\frac{1}{2}}(\hat{n}, z)$ U
U → 3 pts
3 → center of ellipse
3 → axis of ellipse
3 → direction of spin

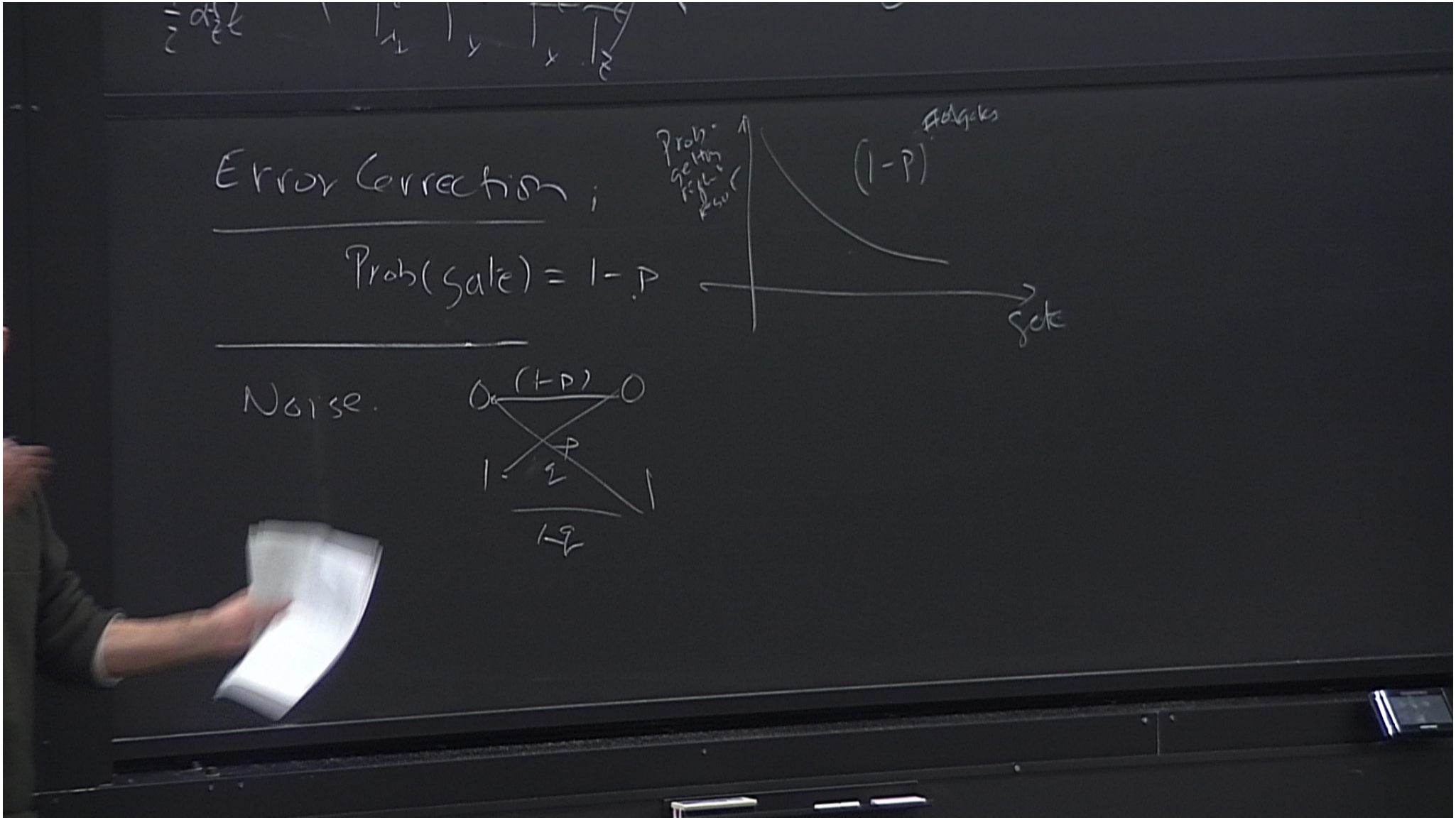

T_1, T_2
$U \frac{1}{2}(\vec{\pi}_1 \; \vec{z}) U$
$U \to$ 3 pos.
3 → center of ellip
3 → a vis ellip
3 → direction of sun

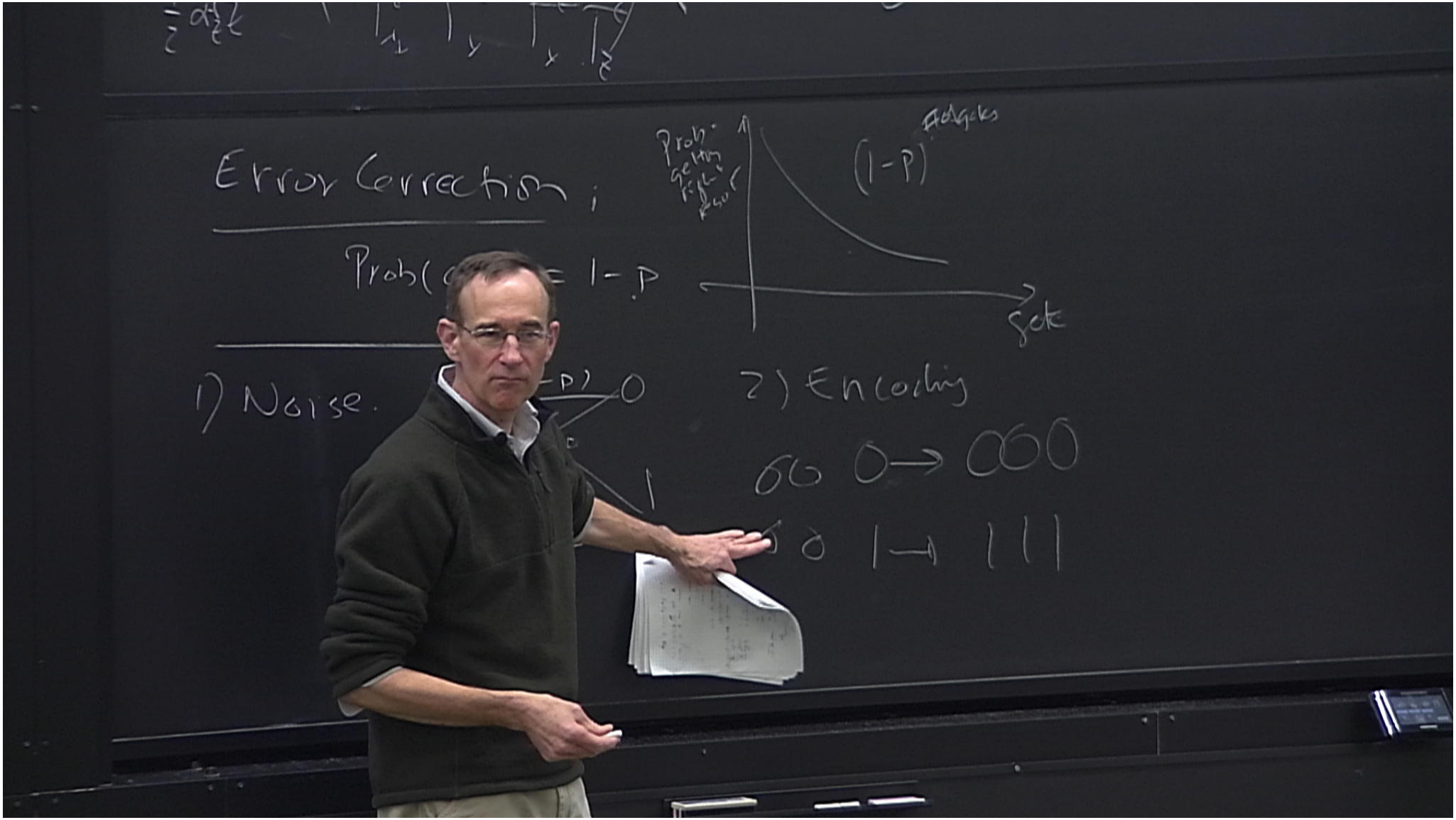

Error Correction;
Prob(gate) = 1 - p

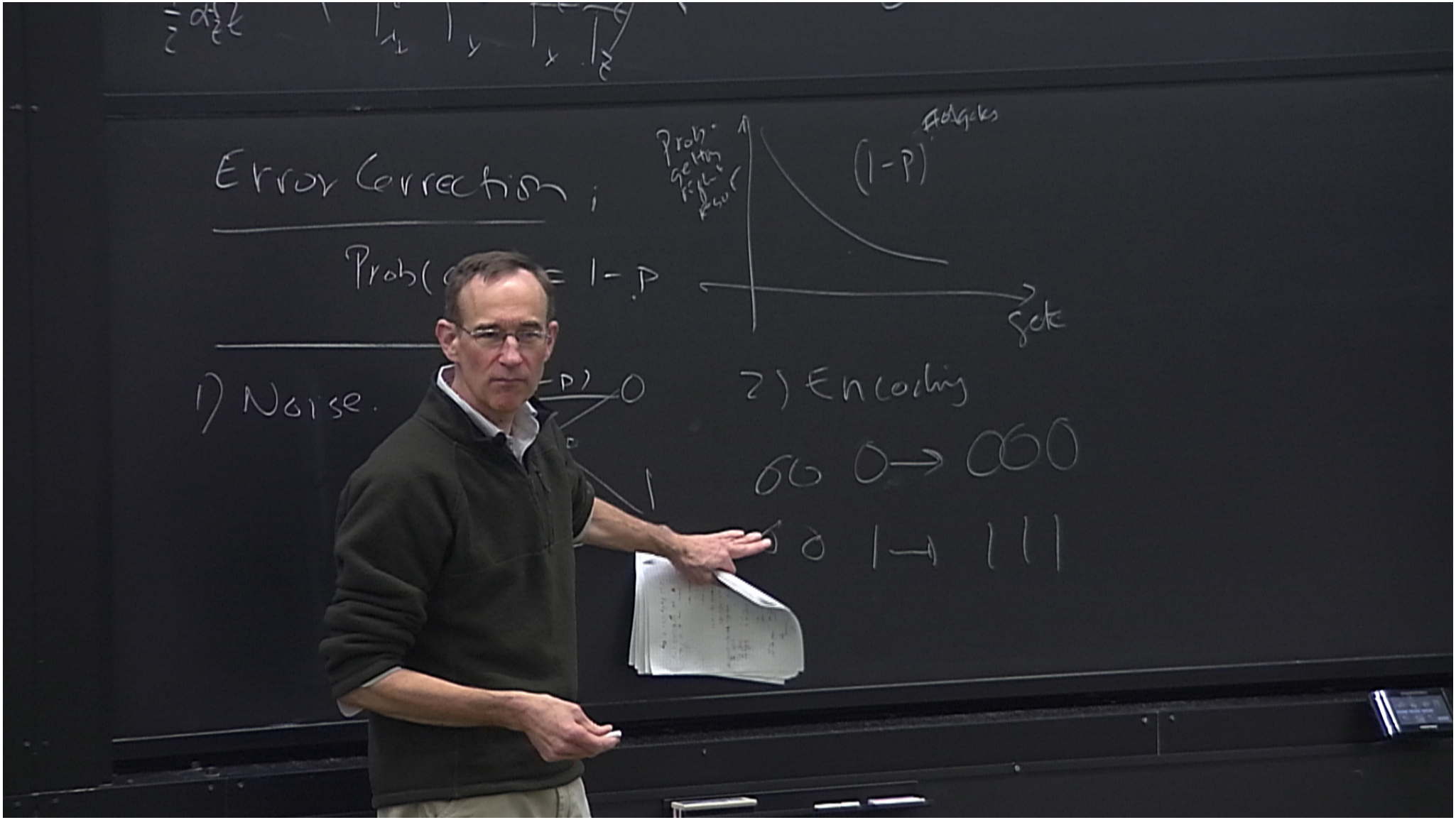

Error Correction;
Prob(gate) = 1 - p
Prob: getting right result
$(1-p)^n$
gate

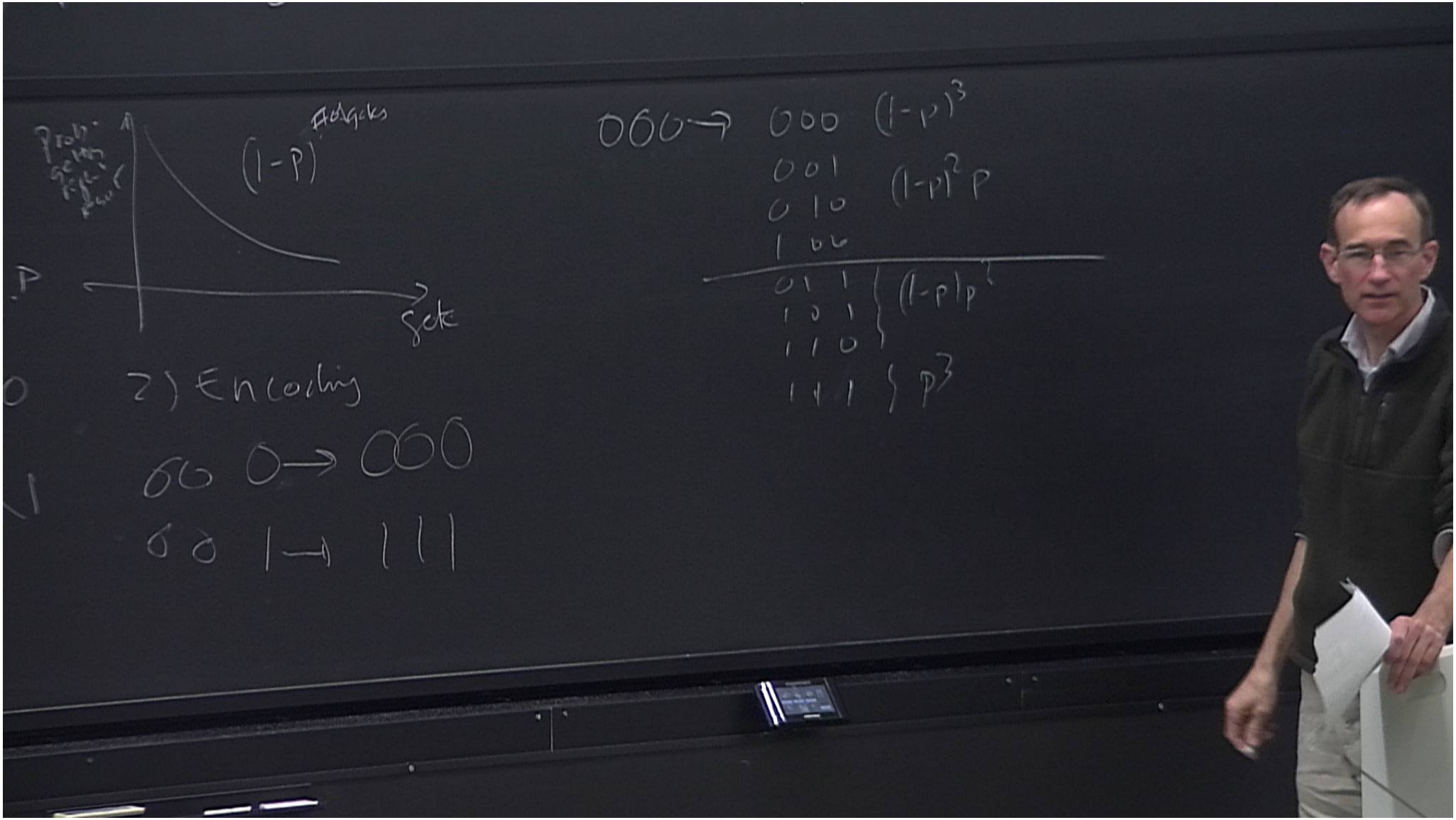

Error Correction:
Prob(gate) = 1 - p
Prob. getting final error
(1-p)^# of gates
gate

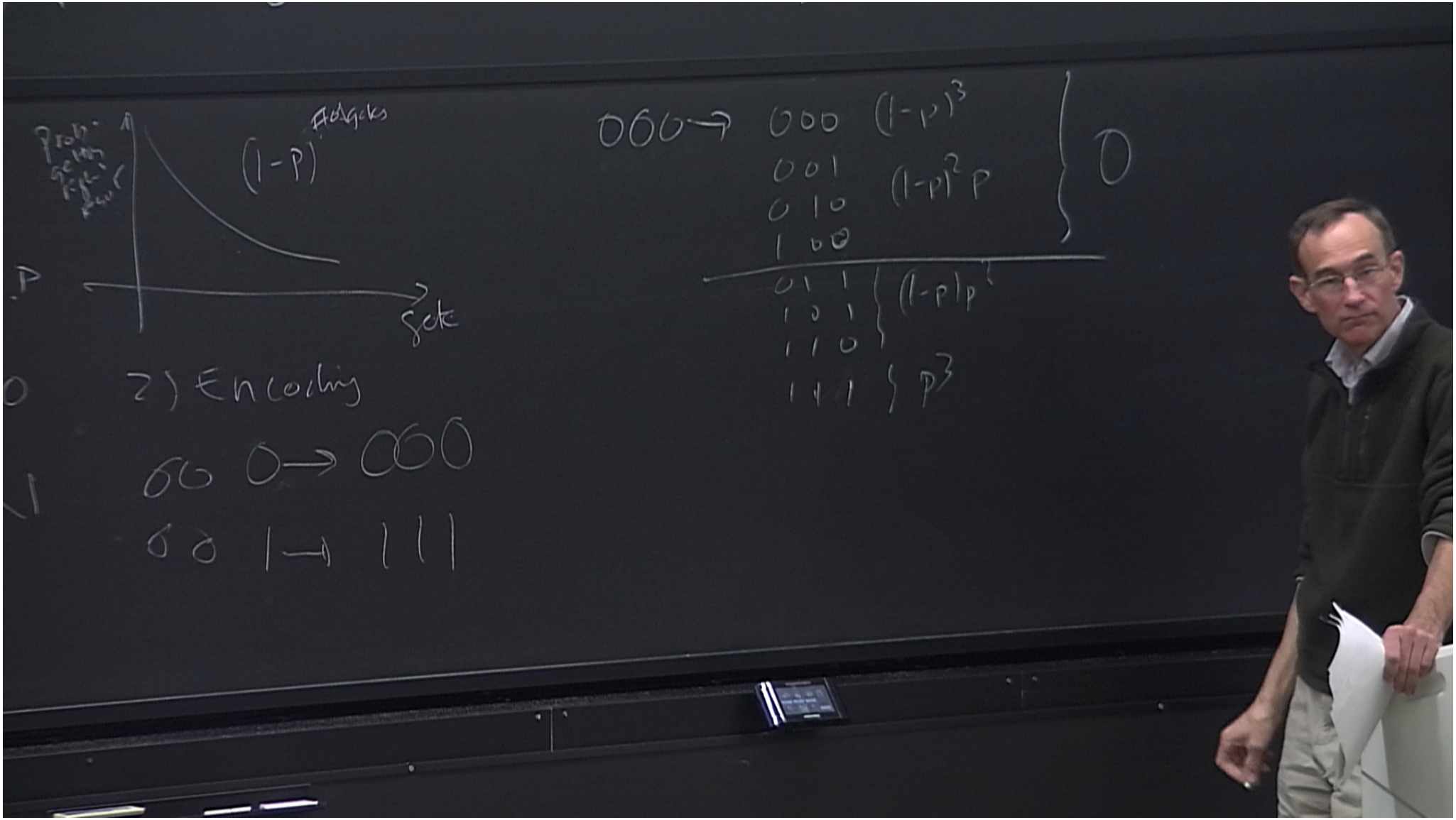

Error Correction;
Prob(gate) = 1 - p
Prob. getting right result
(1-p)
# of gates
gate
Noise.
0
(1-p)
0
p
1-p
1
2

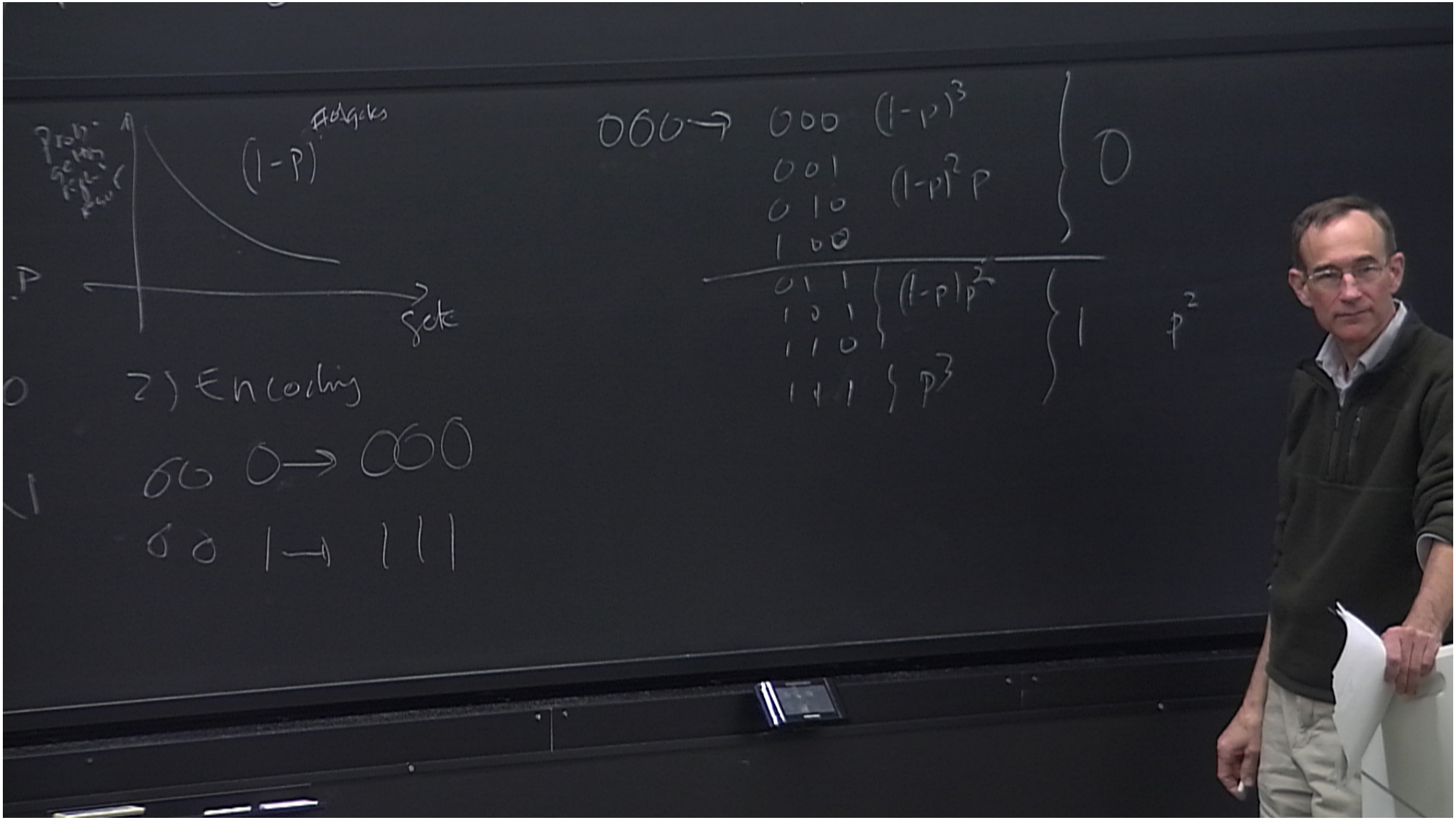

Error Correction;
Prob(gate) = 1 - p
Prob: getting right result
(1-p)
# of gates
gate
Noise.
0
(1-p)
0
p
1
1
1-p

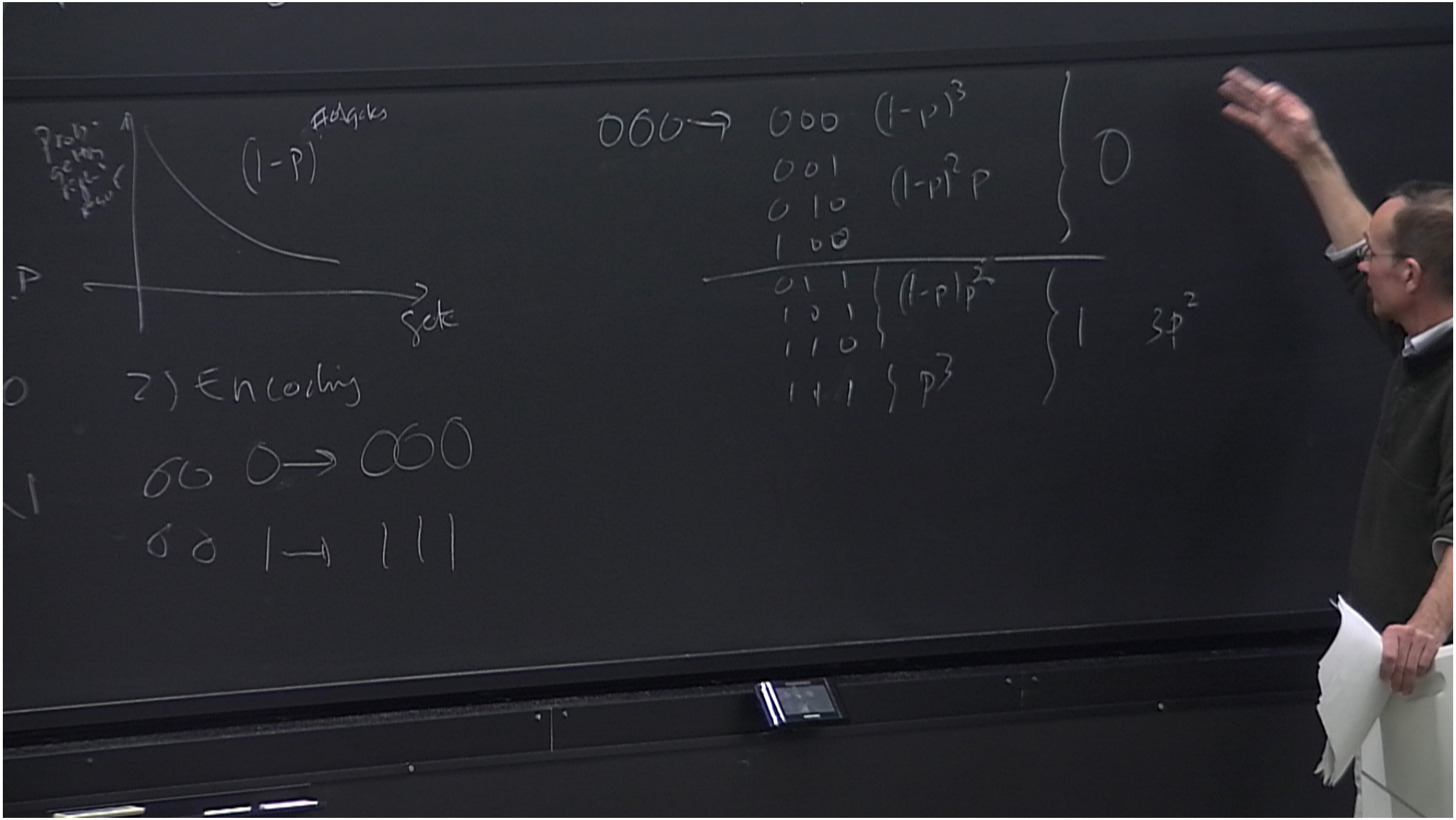

Error Correction;
Prob(
= 1-p
Prob: getting full-
gate
(1-p)
1) Noise.
p²
0
2) Encoding
000
111

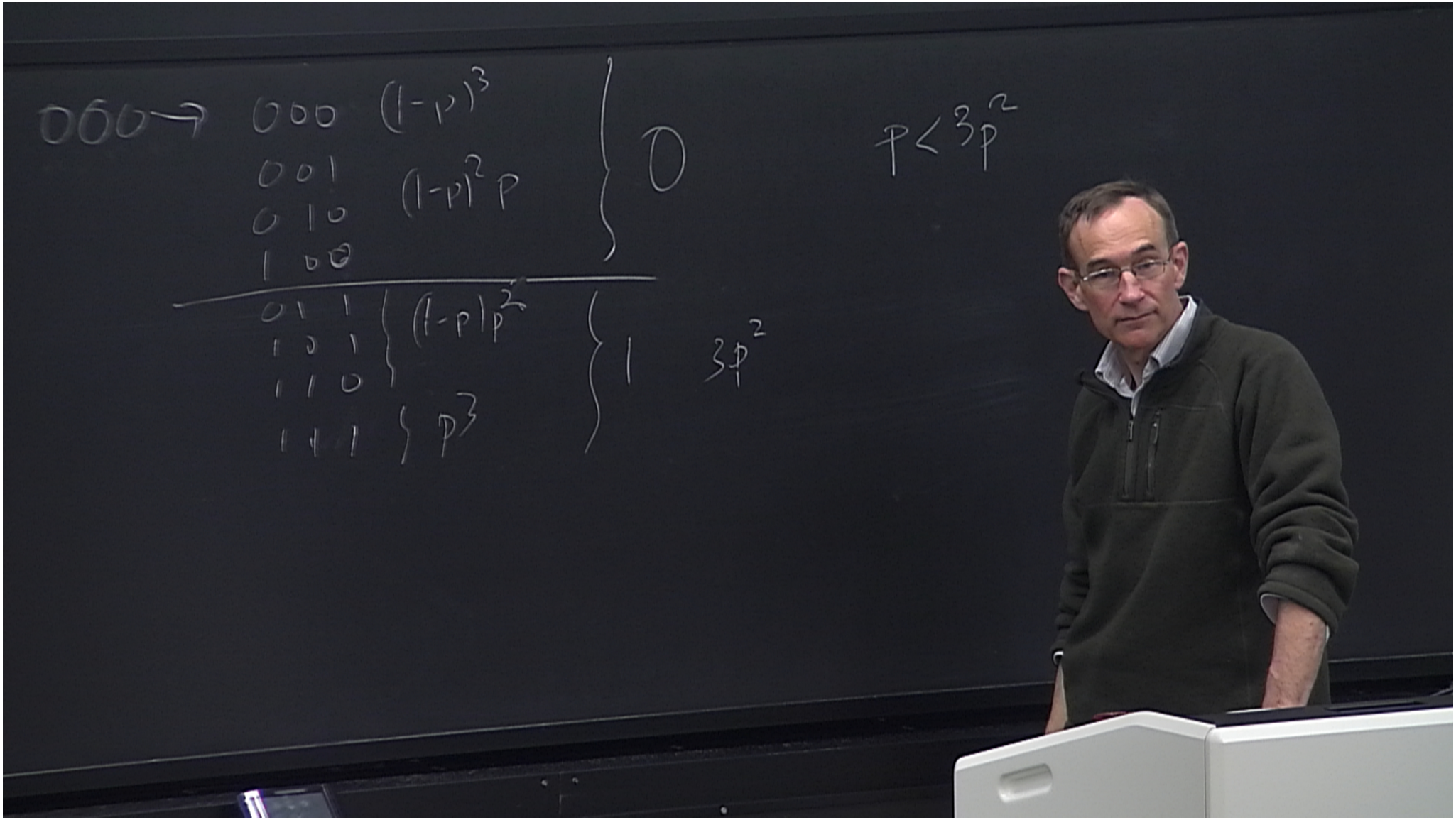

Error Correction;
Prob(
= 1-p
Prob: getting right result
(1-p)^# of gates
gate
1) Noise.
-p?
0
2) Encoding
000
111

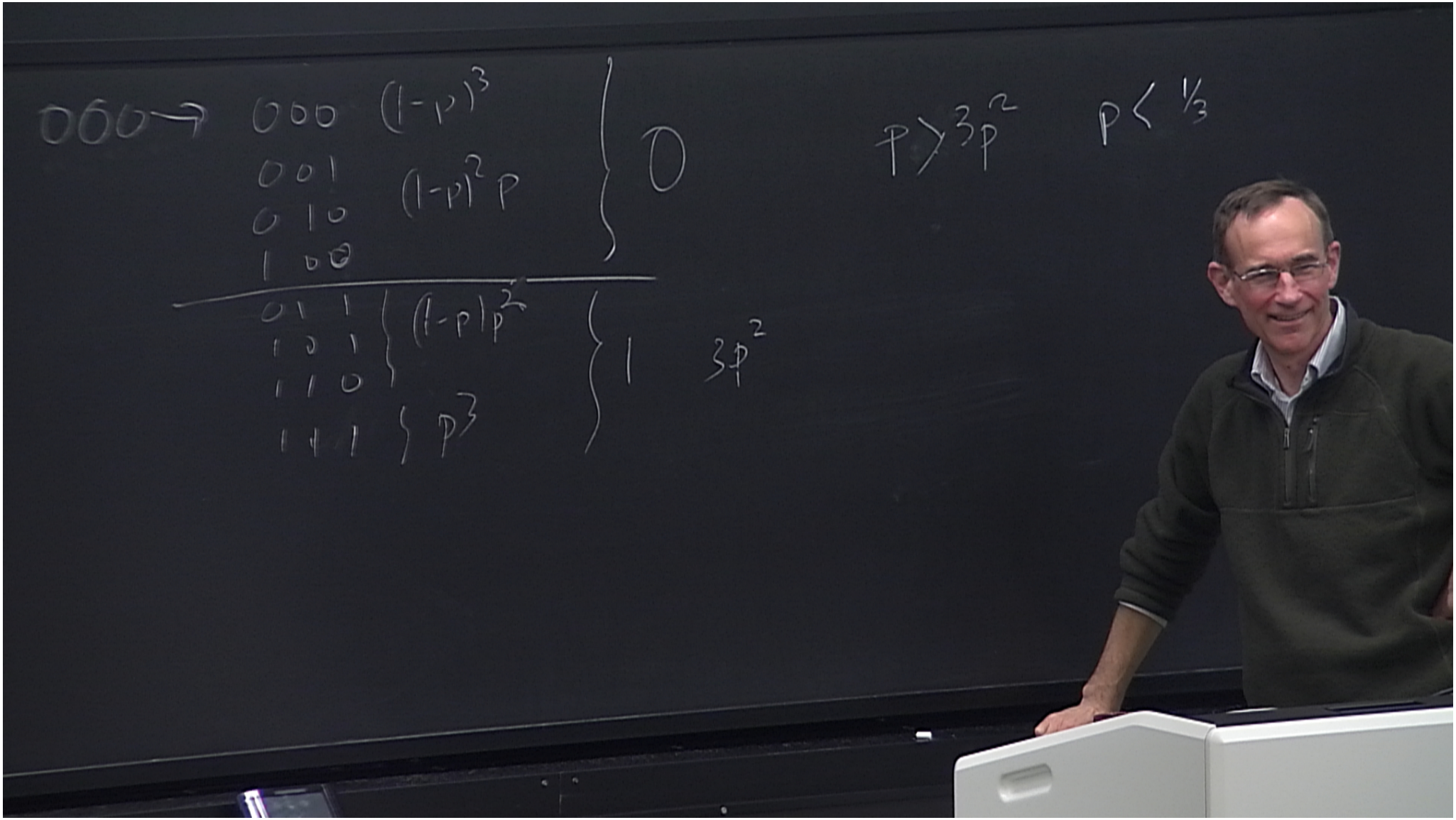

Prob. of flipping a fair coin
(1-p)
Hedgehogs
P
Sets
2) Encoding
000 → 000
001 → 111
000 → 000 (1-p)^3
001
010
100
011
101
110
(1-p)^2 p
(1-p) p^2
111
p^3

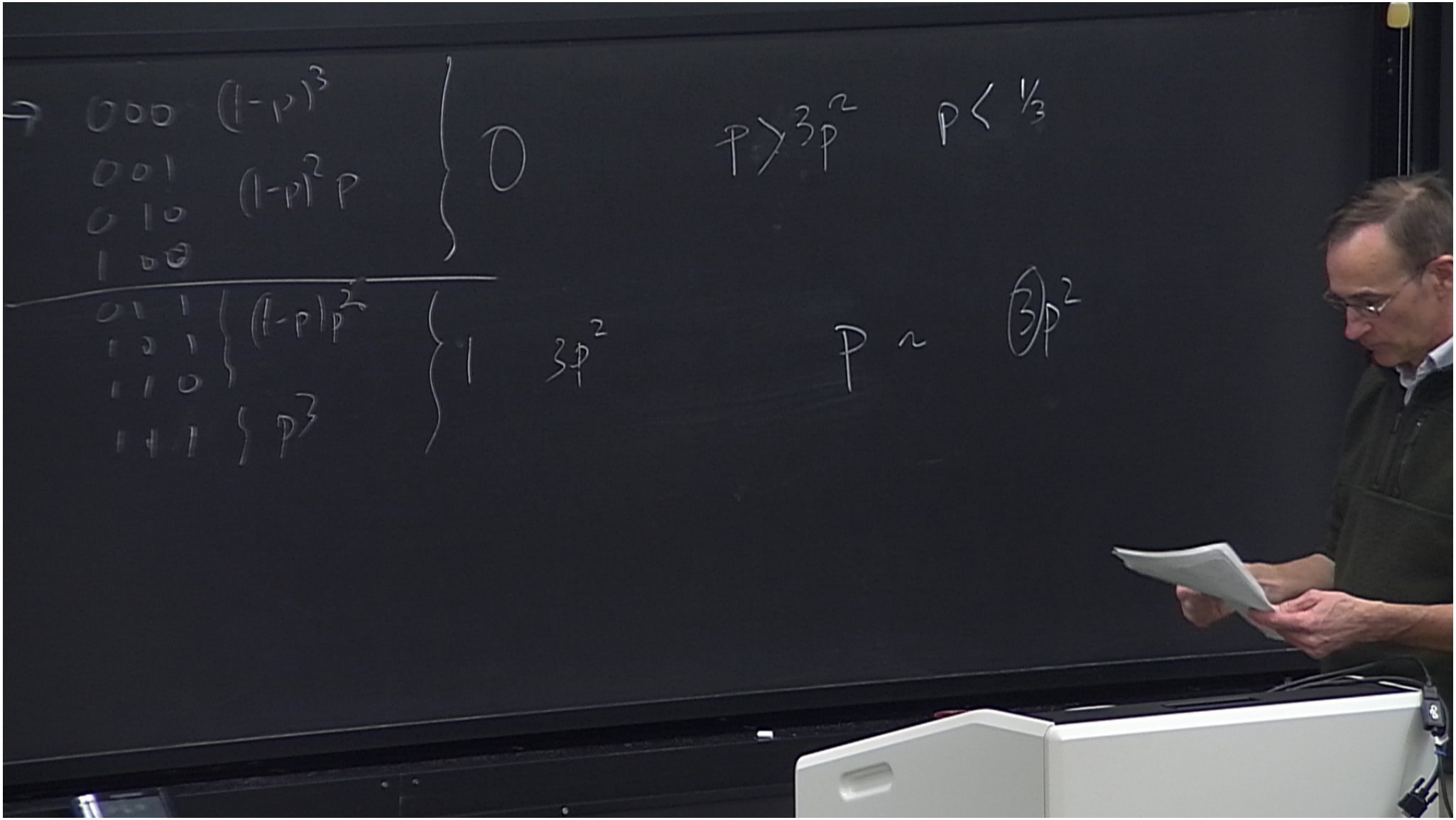

Prob.
(1-p)
Hedgehogs
p
Sets
2) Encoding
000 → 000
001 → 111
000 →
000 (1-p)^3
001
010
100
(1-p)^2 p
0
011
101
110
(1-p)p^2
111
p^3

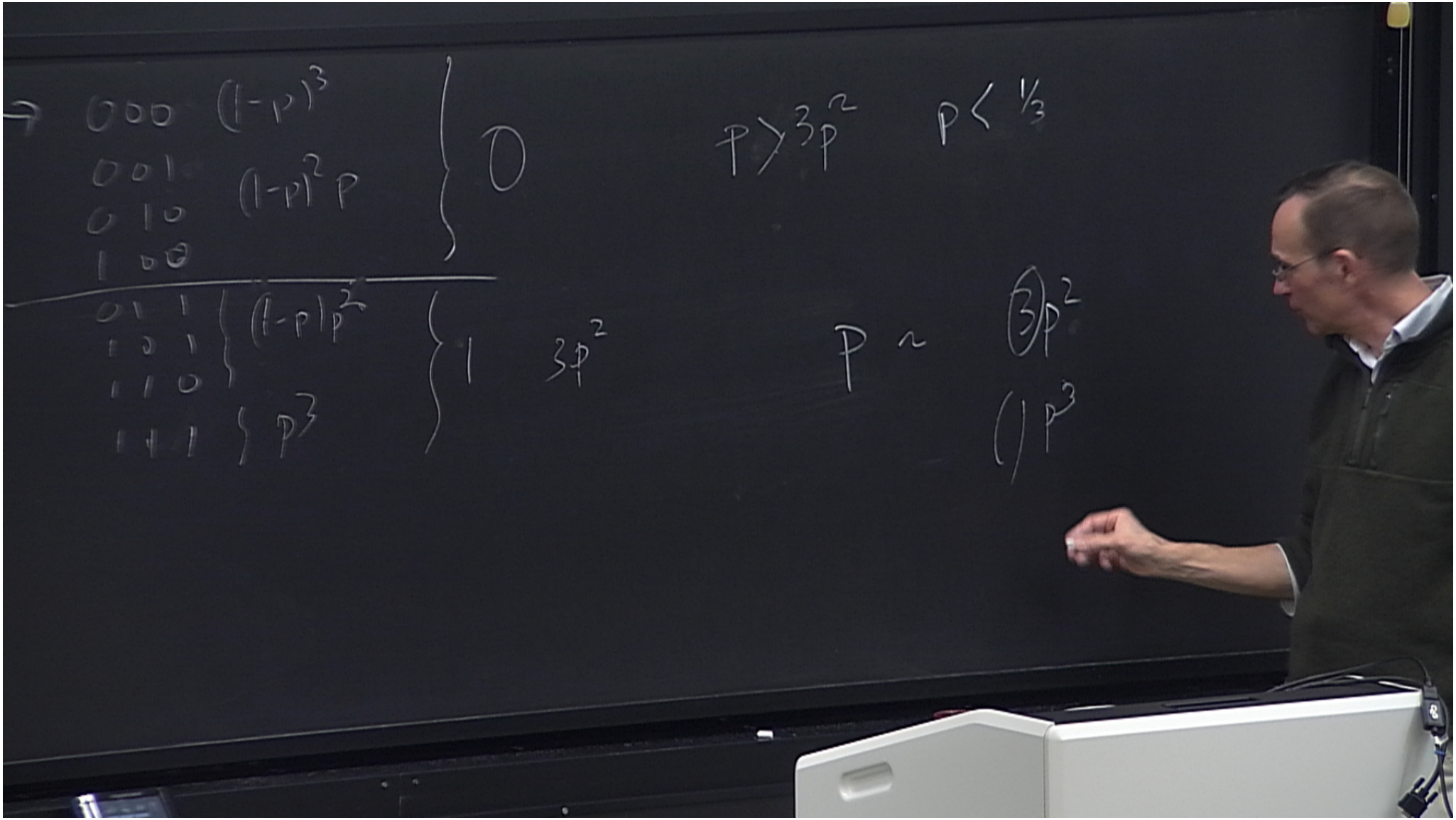

Prob.
(1-p)
flips
p
Seq
2) Encoding
000 → 000
001 → 111
000 → 000
001 (1-p)^3
010 (1-p)^2 p
100
011
101 (1-p) p^2
110
111 p^3
0
1
p^2

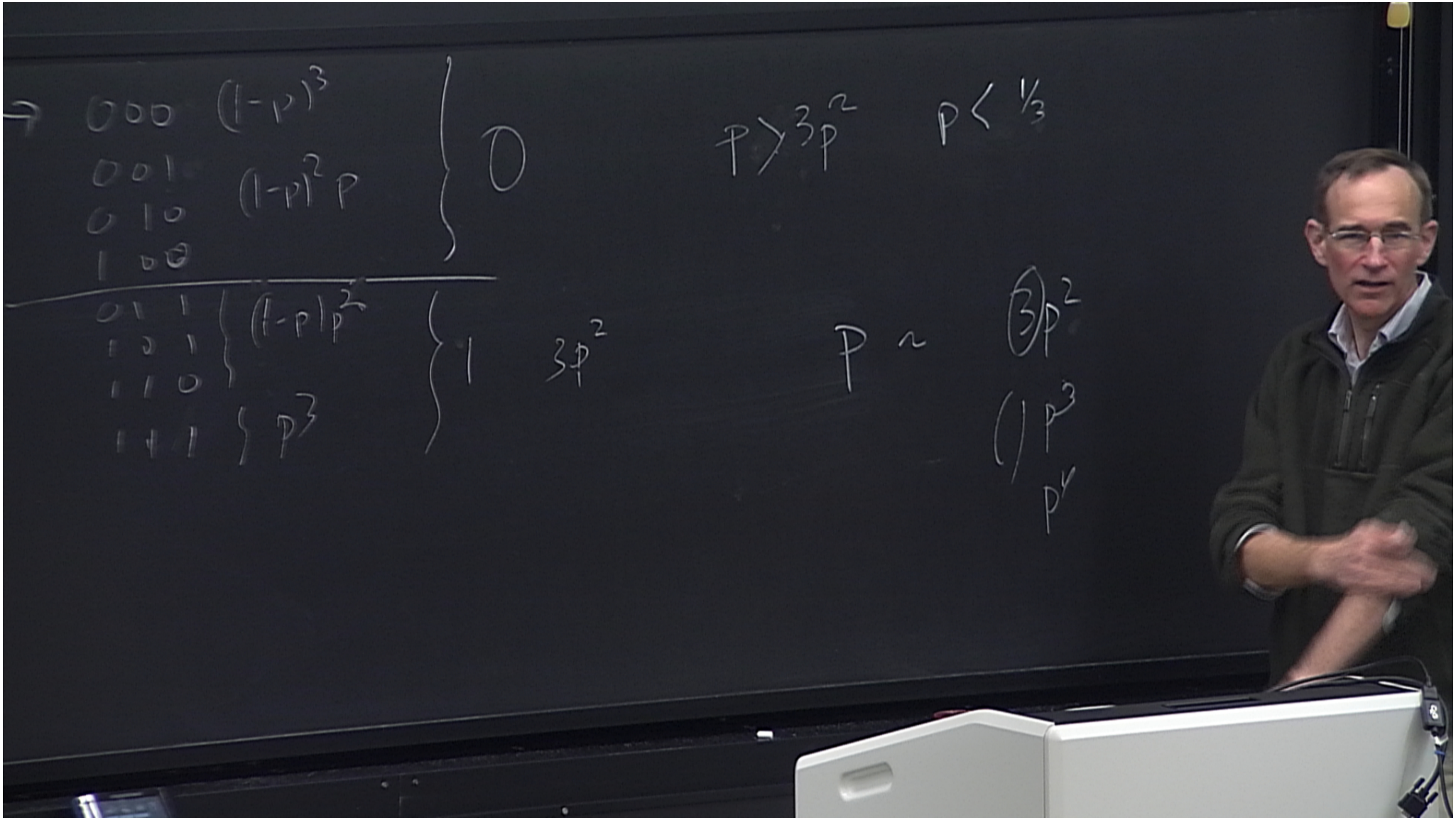

Prob.
p
Seuil
$(1-p)^{\text{Hodges}}$
$000 \to$ 000 $(1-p)^3$
001 $(1-p)^2 p$
010
100
0
011
101
110
$(1-p)p^2$
111 p^3
1 $3p^2$
2) Encoding
$0 \to 000$
$1 \to 111$

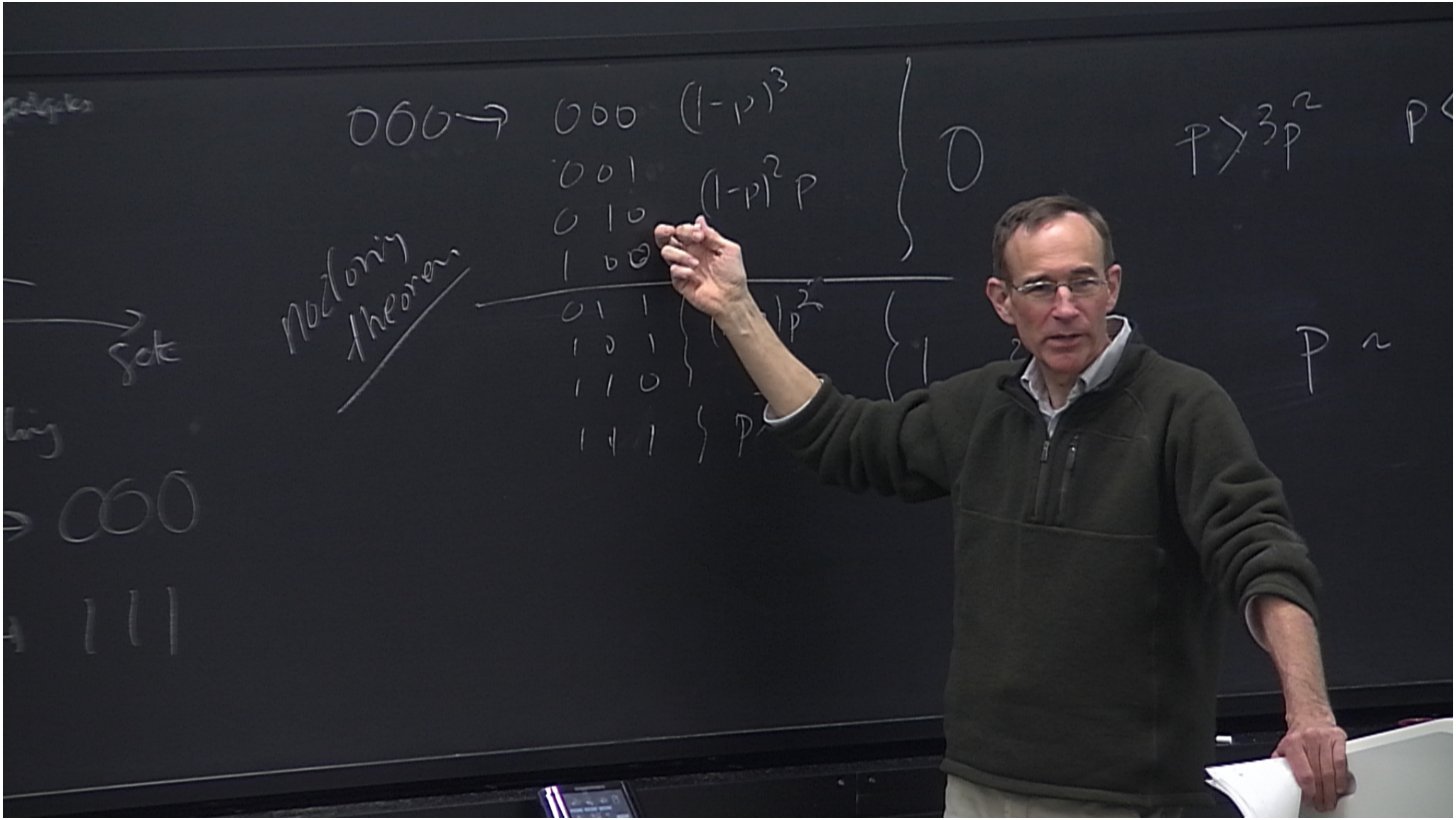

000 → 000 (1-p)^3
001
010
100
0
p < 3p^2
011
101
110
111
(1-p)p^2
p^3
1
3p^2

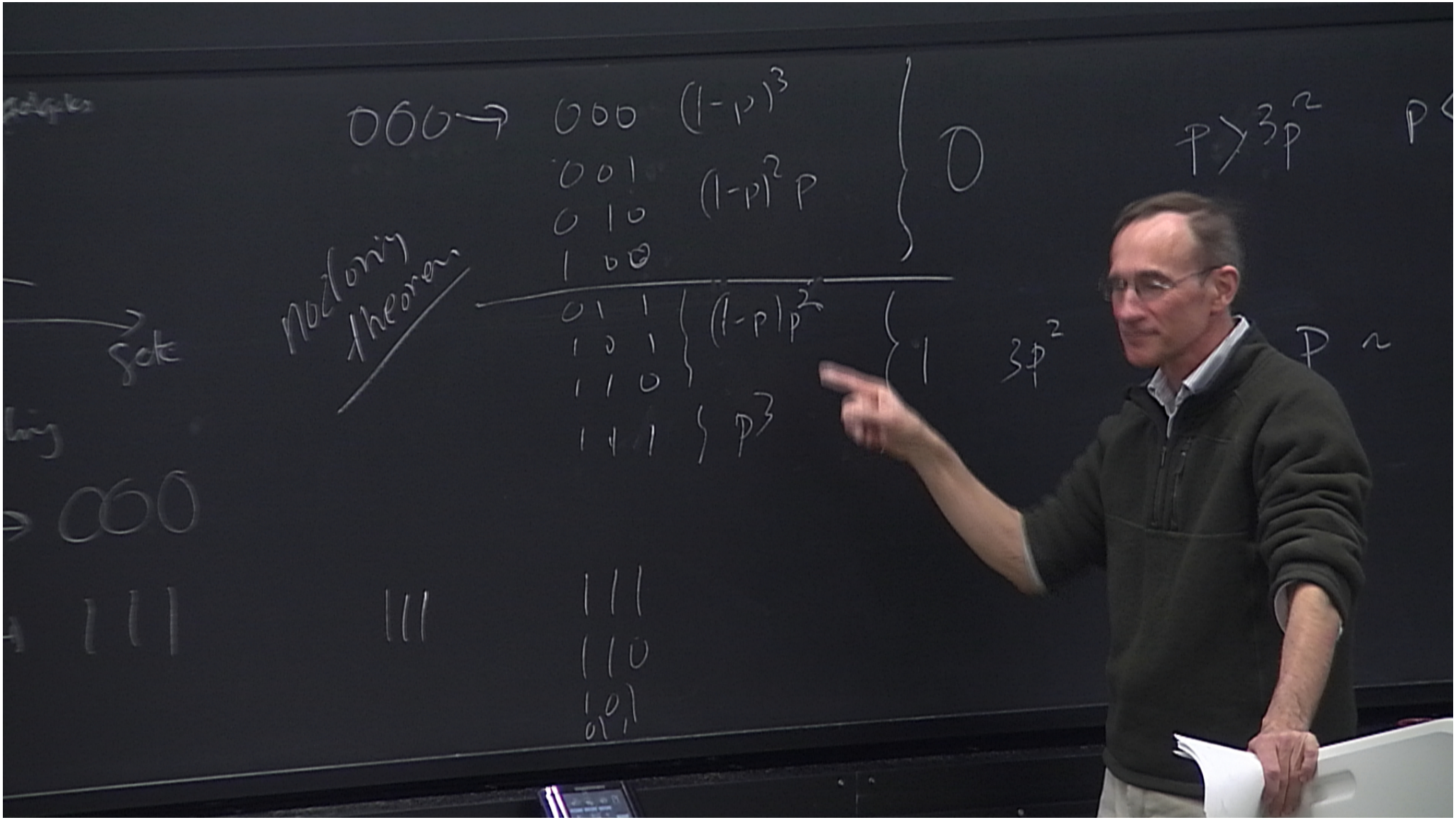

000 → 000 (1-p)^3
001
010
100
} 0
p > 3p^2
p < 1/3
011
101
110
} (1-p)p^2
111 } p^3
} 1
3p^2

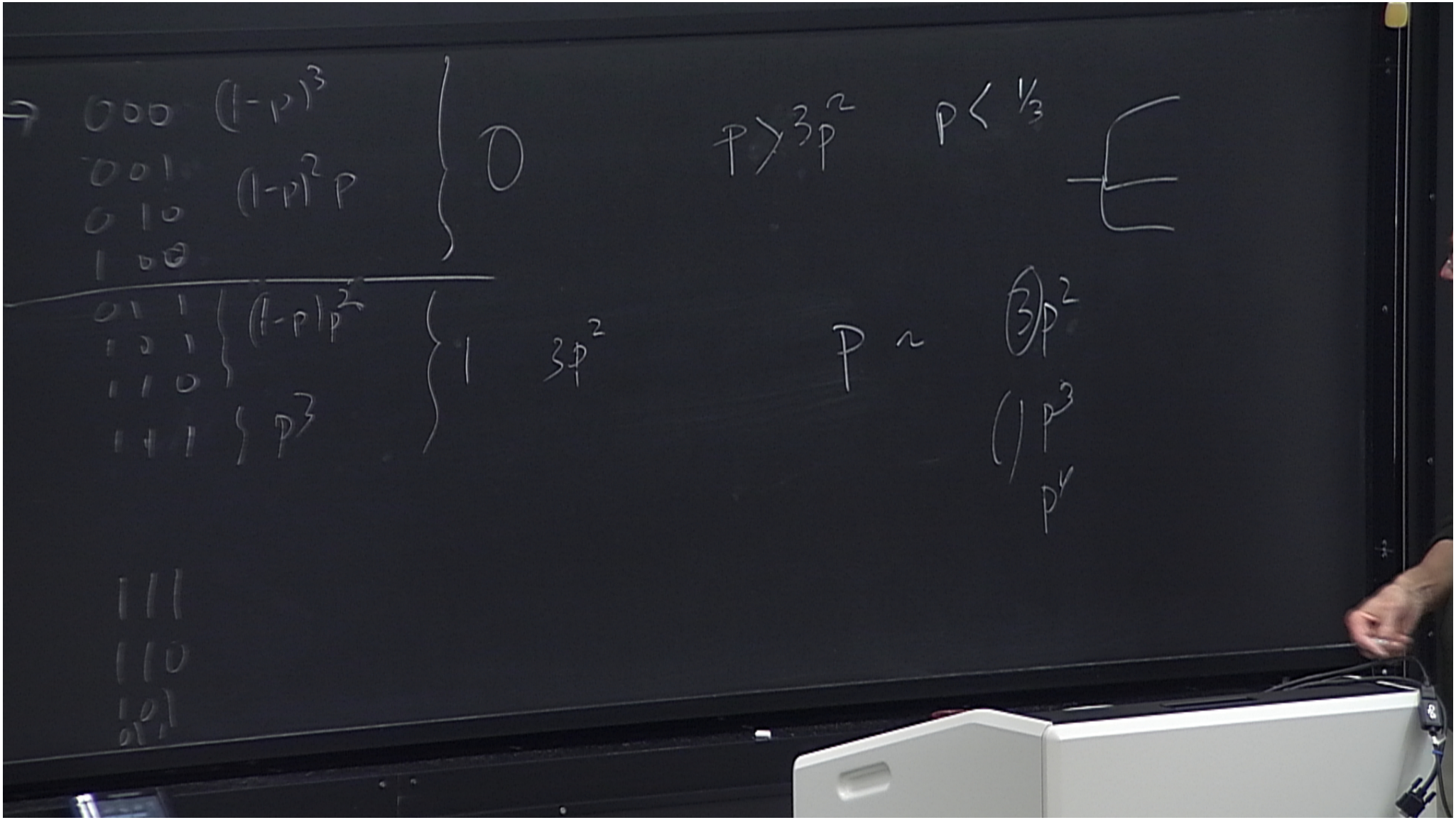

000 $(1-p)^3$
001 $(1-p)^2 p$
010
100
0
$p > 3p^2$
$p < 1/3$
011
101
110
$(1-p)p^2$
111 p^3
1
$3p^2$
$p \sim 3p^2$

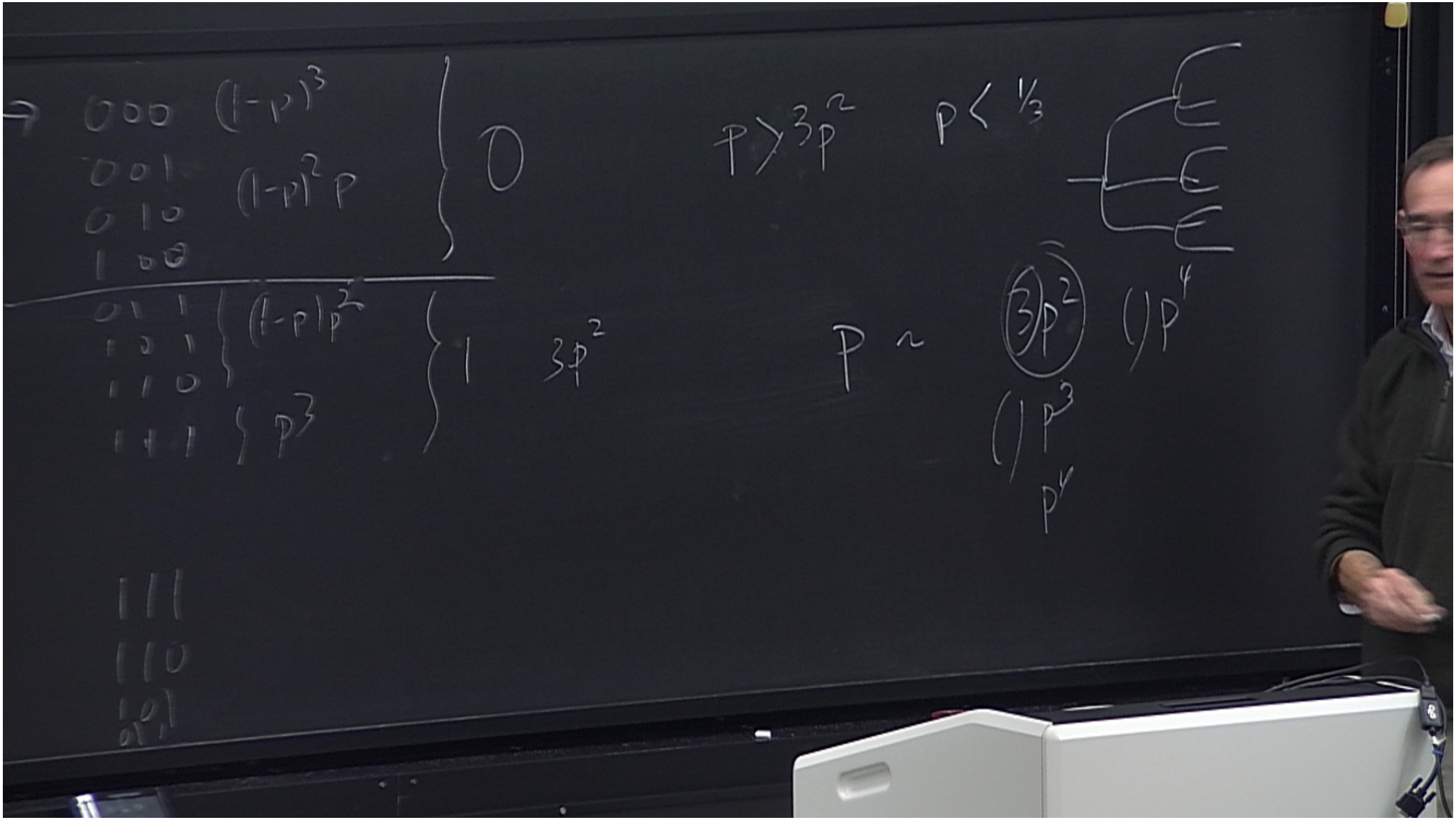

000 $(1-p)^3$
001 $(1-p)^2 p$
010
100
0
$p > 3p^2$
$p < 1/3$
011
101
110
$(1-p)p^2$
111 p^3
1
$3p^2$
$p \sim$
$3p^2$
p^3

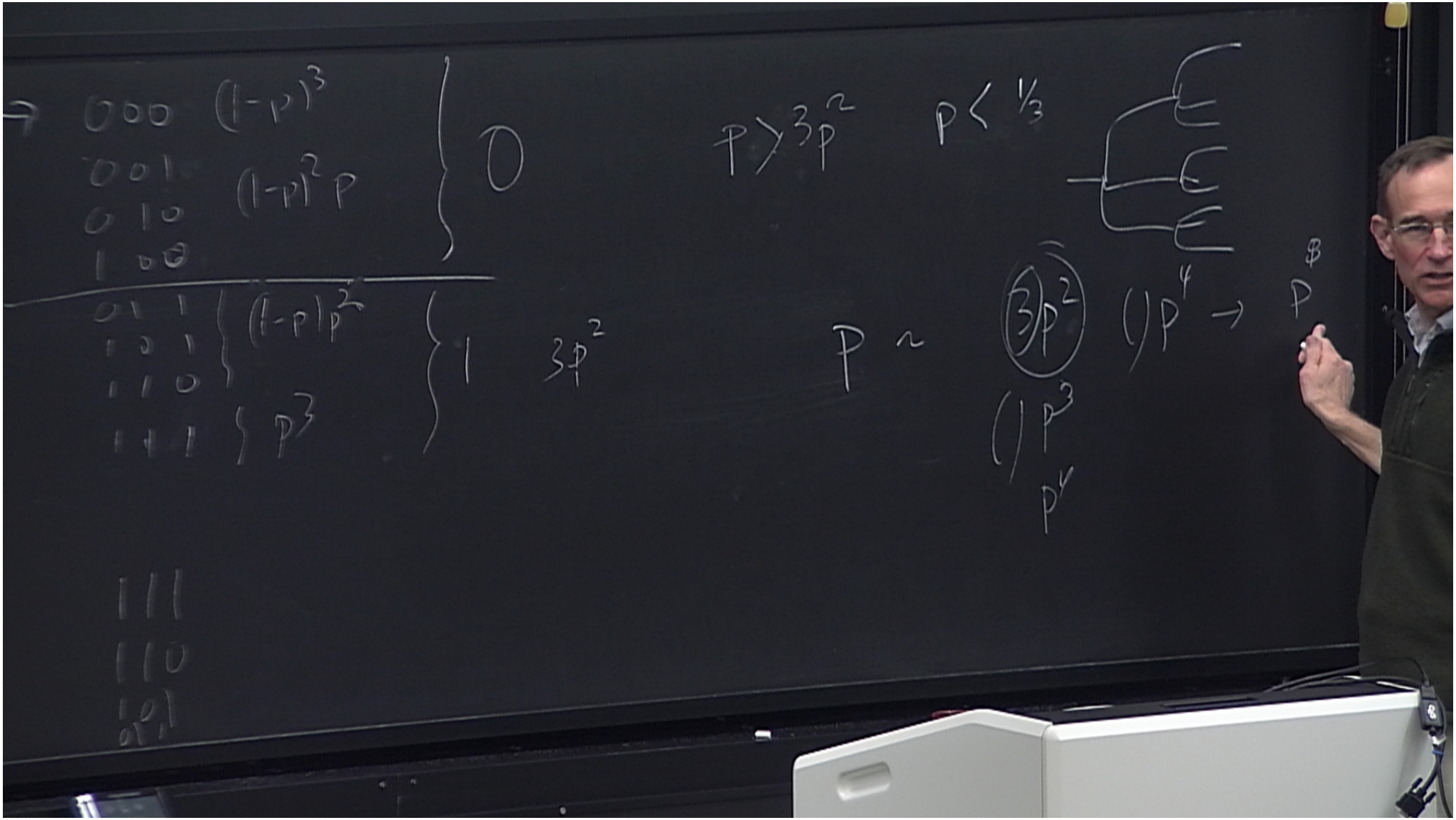

000 $(1-p)^3$
001 $(1-p)^2 p$
010
100
0
$p > 3p^2$
$p < 1/3$
011
101
110
$(1-p)p^2$
111 p^3
1
$3p^2$
$p \sim 3p^2$
p^3
p^4

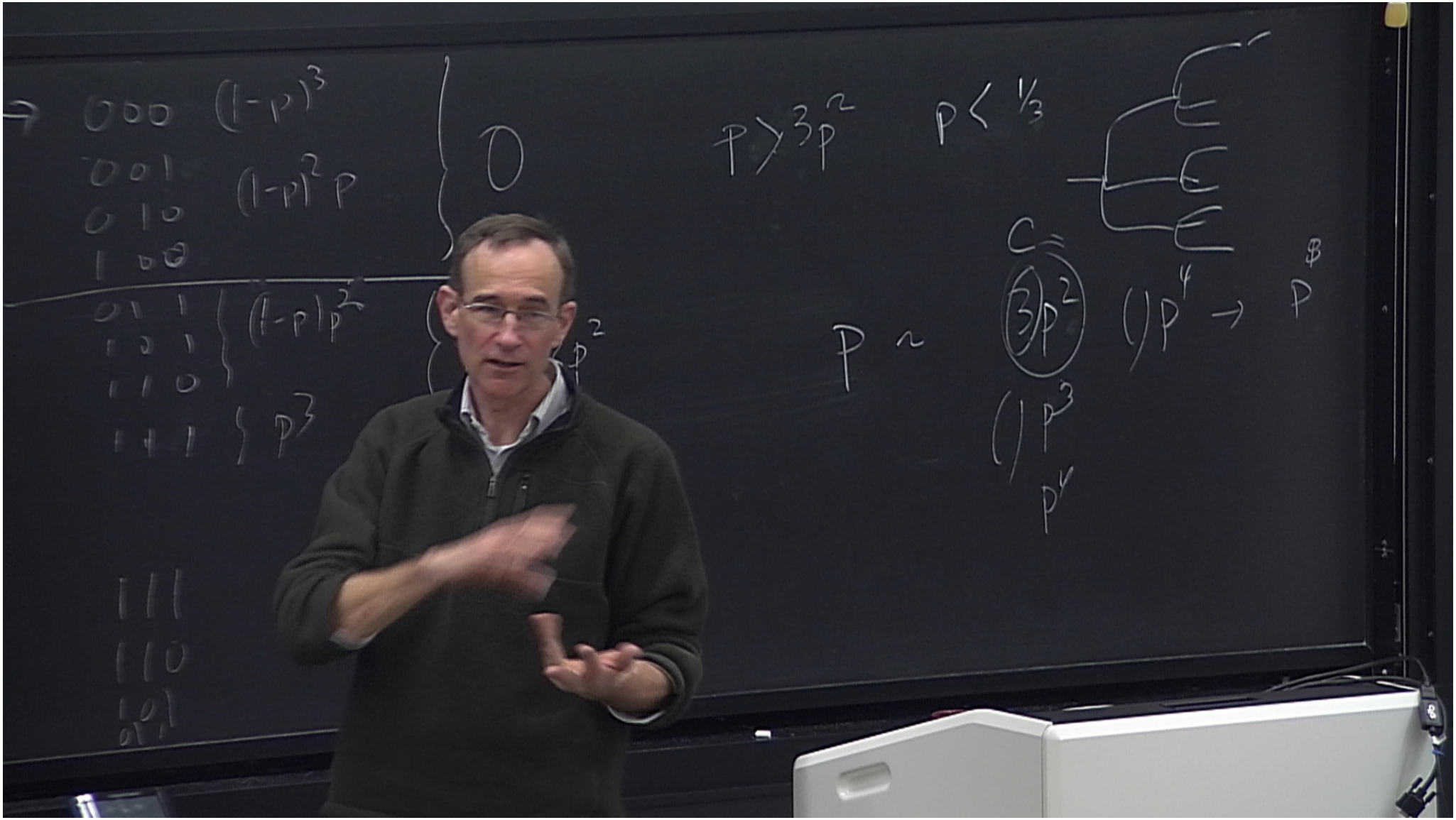

000 → 000 $(1-p)^3$
001
010
100
011
101
110
111
p
0
$p > 3p^2$
Gate
000
111


000 → 000 (1-p)^3
001
010
100
0
p > 3p^2
Nozick's theorem
011
101
110
(1-p)p^2
111
p^3
1
3p^2
111
111
110
101
011


000 $(1-p)^3$
001 $(1-p)^2 p$
010
100
0
$p > 3p^2$
$p < 1/3$
011 $(1-p)p^2$
101
110
1
$3p^2$
111 p^3
$p \sim 3p^2$
p^3
p^4
111
110
101
011


000 $(1-p)^3$
001 $(1-p)^2 p$
010
100
0
$p > 3p^2$
$p < 1/3$
011
101
110
$(1-p)p^2$
111 p^3
1
$3p^2$
$p \sim$
$3p^2$
$1)p^4$
$1)p^3$
p^4
111
110
101
011


000 $(1-p)^3$
001 $(1-p)^2 p$
010
100
0
$p > 3p^2$
$p < 1/3$
011
101
110
$(1-p)p^2$
111 p^3
1
$3p^2$
$p \sim$
$3p^2$
p^3
p^4
$p^4 \to$
p^3


000 $(1-p)^3$
001 $(1-p)^2 p$
010
100
011
101
110
p^2
111
110
101
011
$p > 3p^2$
$p < 1/3$
$p \sim$
$3p^2$
p^3
p^4
$p^4 \to p^3$