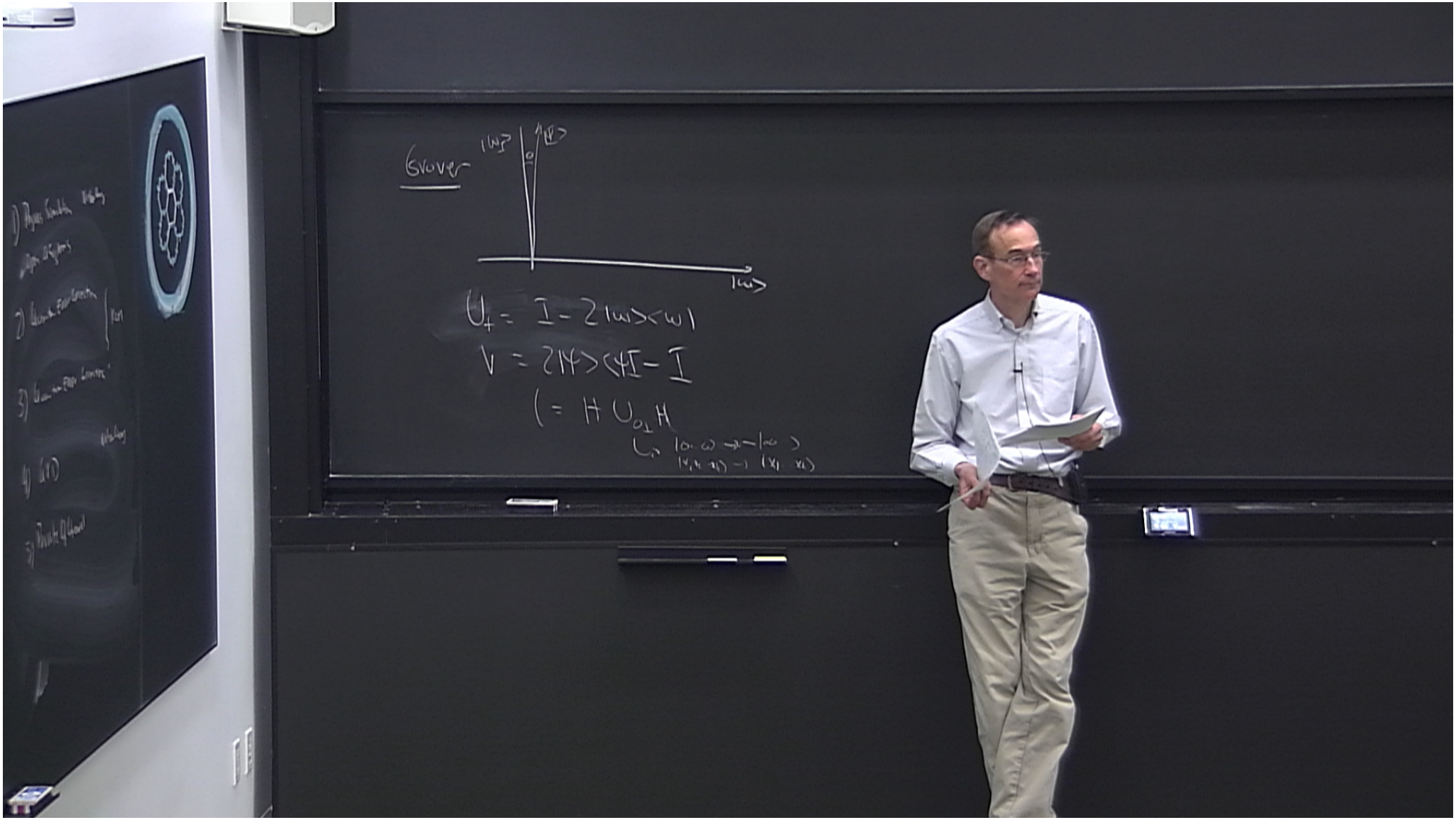
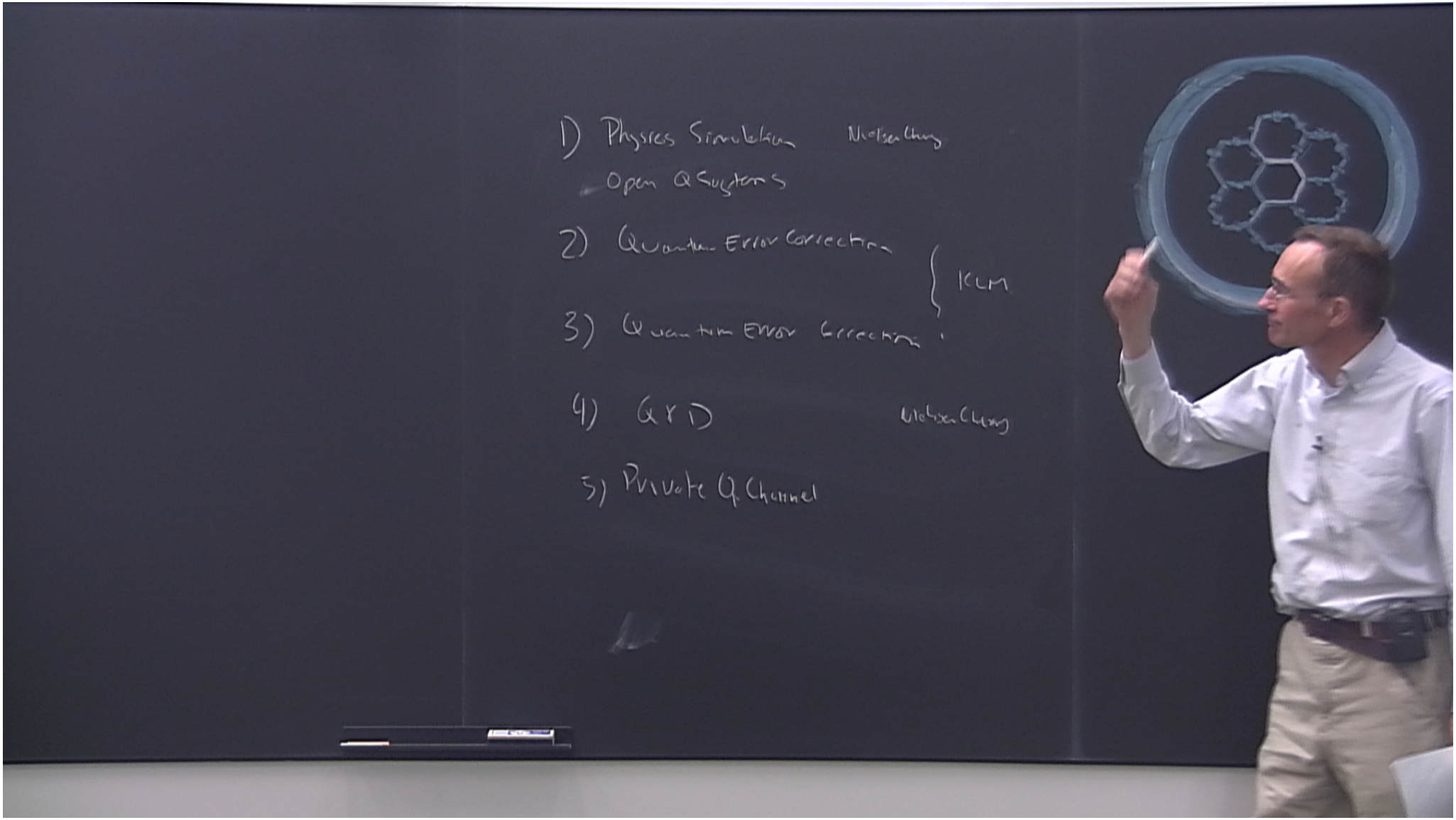
Title: PSI 2015/2016 Quantum Information - Lecture 11

Date: Mar 07, 2016 11:30 AM

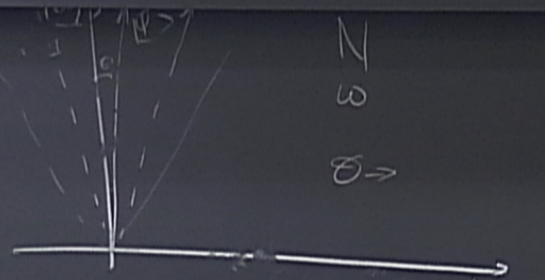
URL: http://pirsa.org/16030058

Abstract:


Gover
$U_f = I - 2|w\rangle\langle w|$
$V = 2|\psi\rangle\langle\psi| - I$
$(= H U_{0_1} H$


1) Physics Simulation Nielsen Chuang
Open Q Systems
2) Quantum Error Correction
3) Quantum Error Correction
KLM
4) QRD Nielsen Chuang
5) Private Q. Channel

Grover



$U_f = I - 2|\omega\rangle\langle\omega|$

$V = 2|\psi\rangle\langle\psi| - I$

$(= H U_{0\perp} H$

↳ $|0\ldots0\rangle \to -|0\ldots0\rangle$
$|x_1 \ldots x_n\rangle \to |x_1 \ldots x_n\rangle$

$\omega = 7$

$|\psi\rangle = \frac{1}{2\sqrt{2}}\left(|1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle - |6\rangle - |7\rangle - |8\rangle\right)$

$U_f|\psi\rangle =$

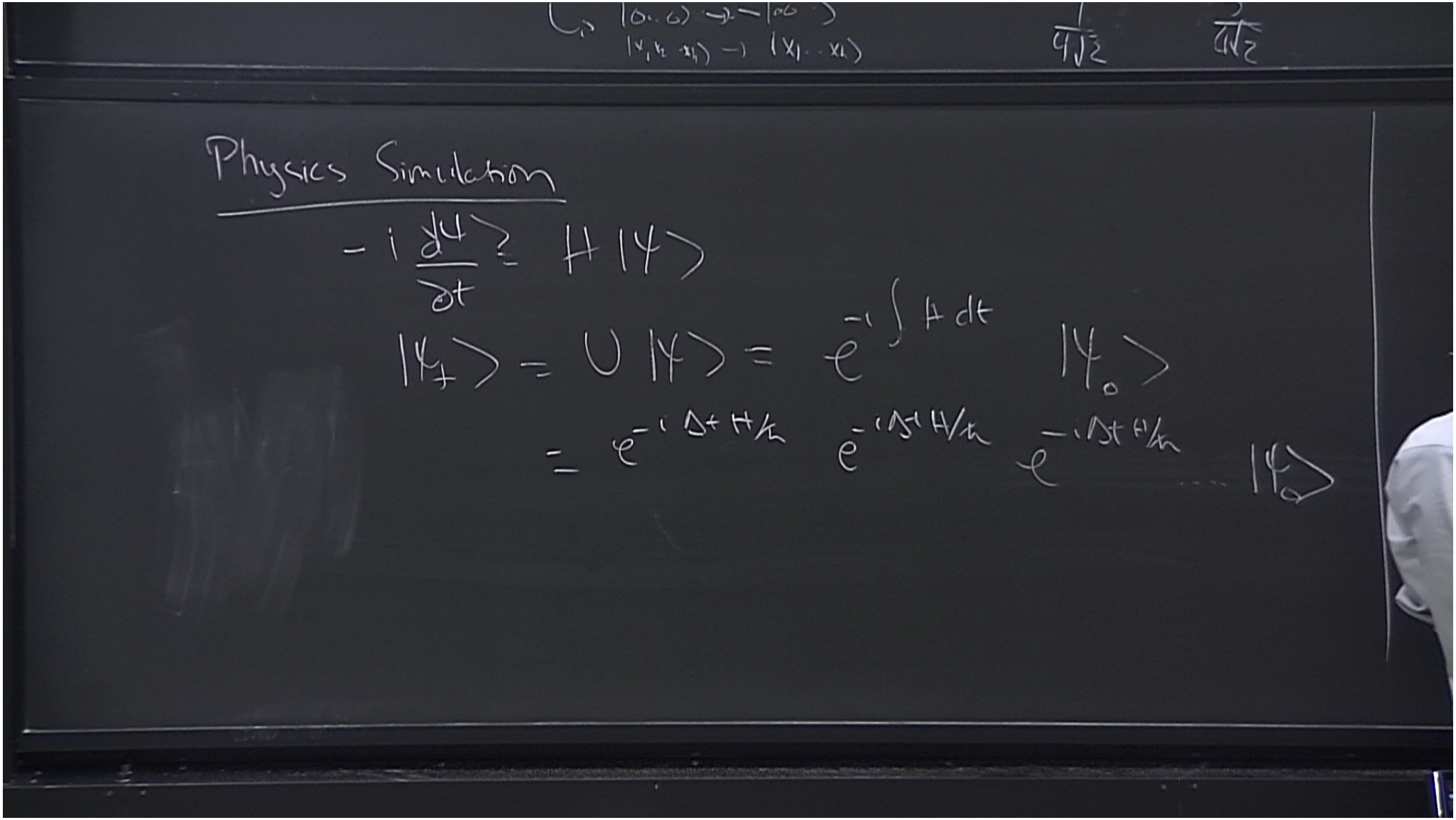
$V U_f|\psi\rangle = (2|\psi\rangle\langle\psi| - I)\, U_f|\psi\rangle$

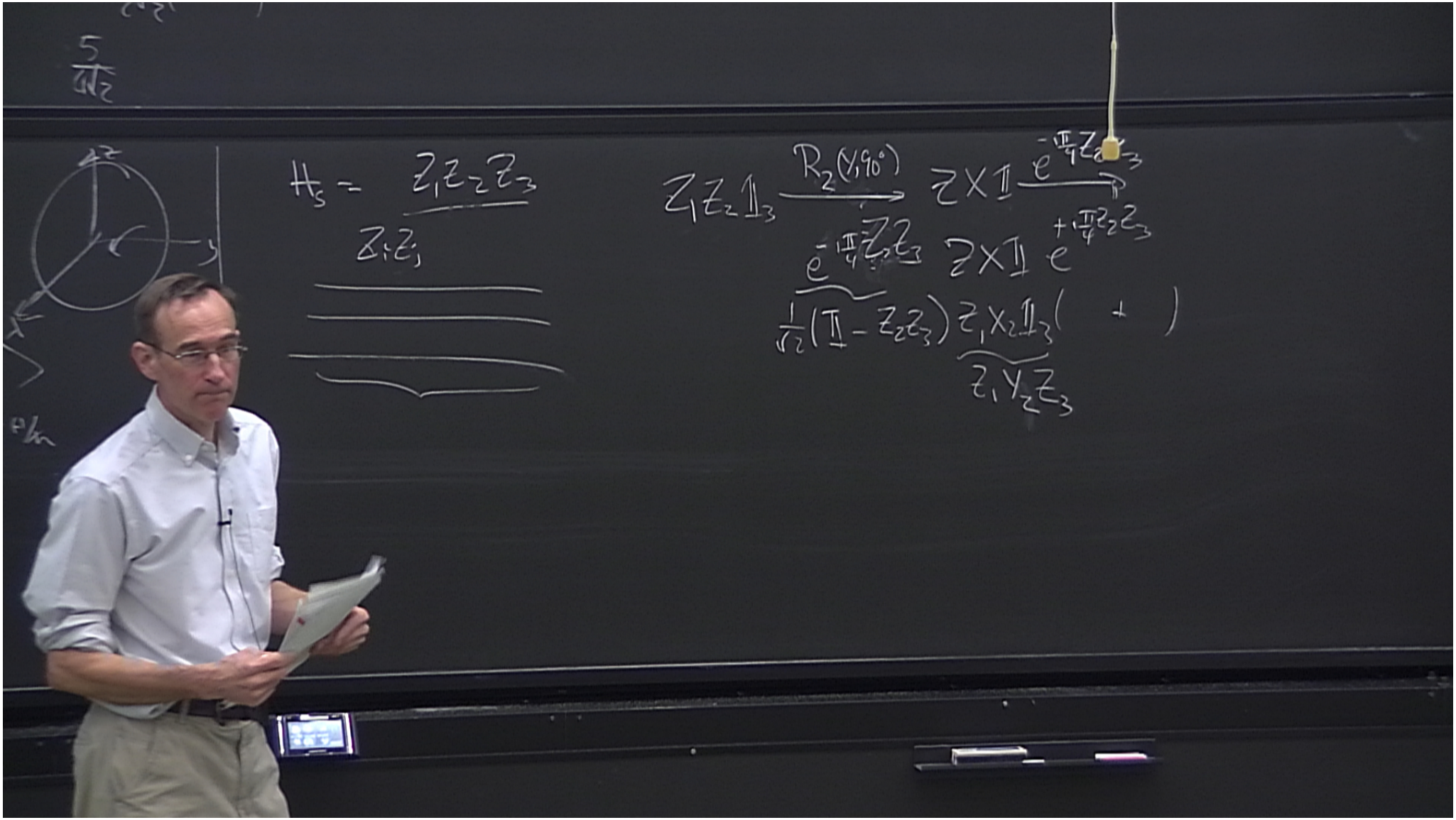
$= 2\left(\frac{1}{2\sqrt{2}}\right)^2 \frac{6}{8}|\psi\rangle - \frac{1}{2}(\quad)$

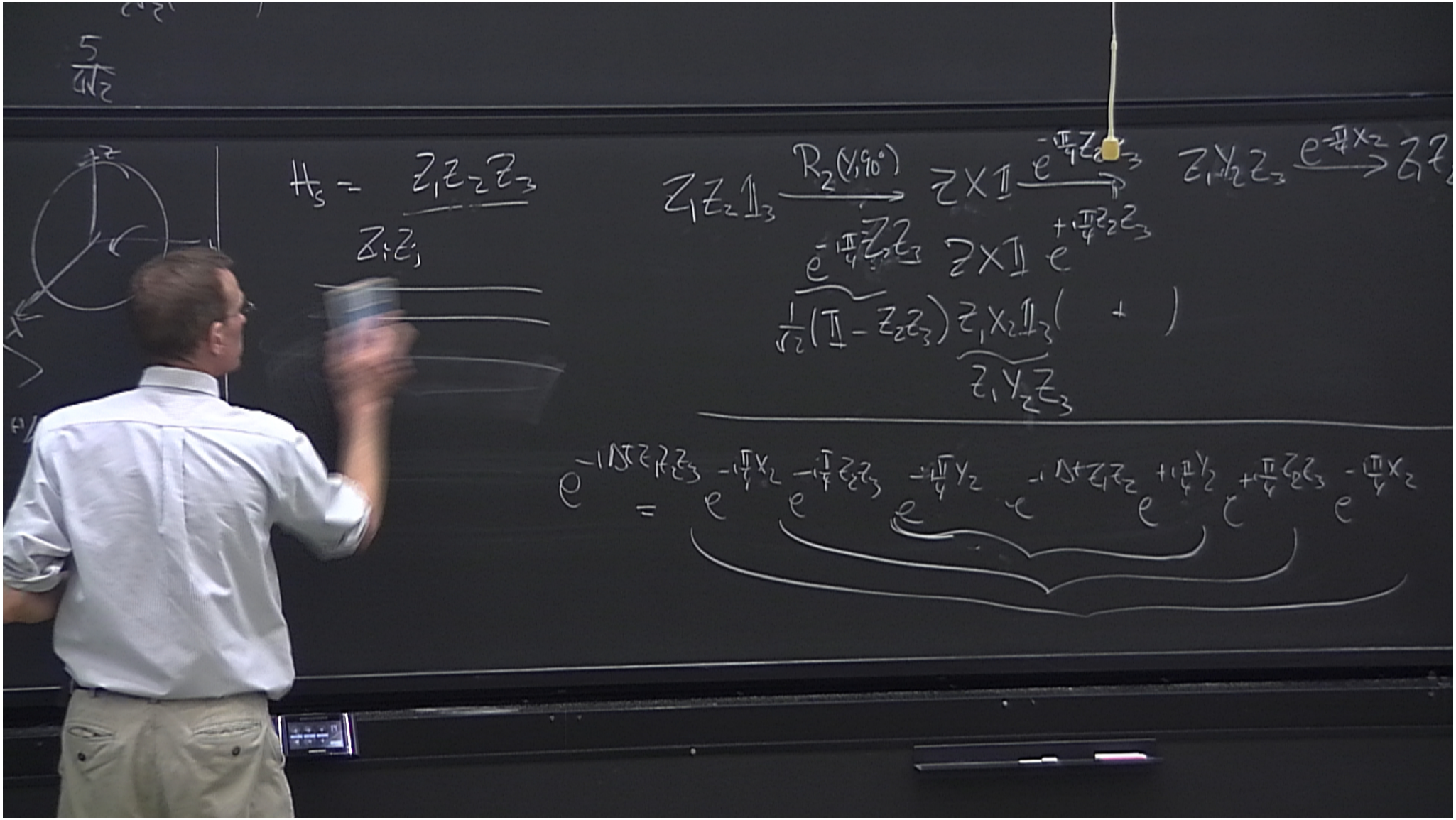
$= \frac{3}{4\sqrt{2}}|\psi\rangle - \frac{1}{2\sqrt{2}}(\quad)$

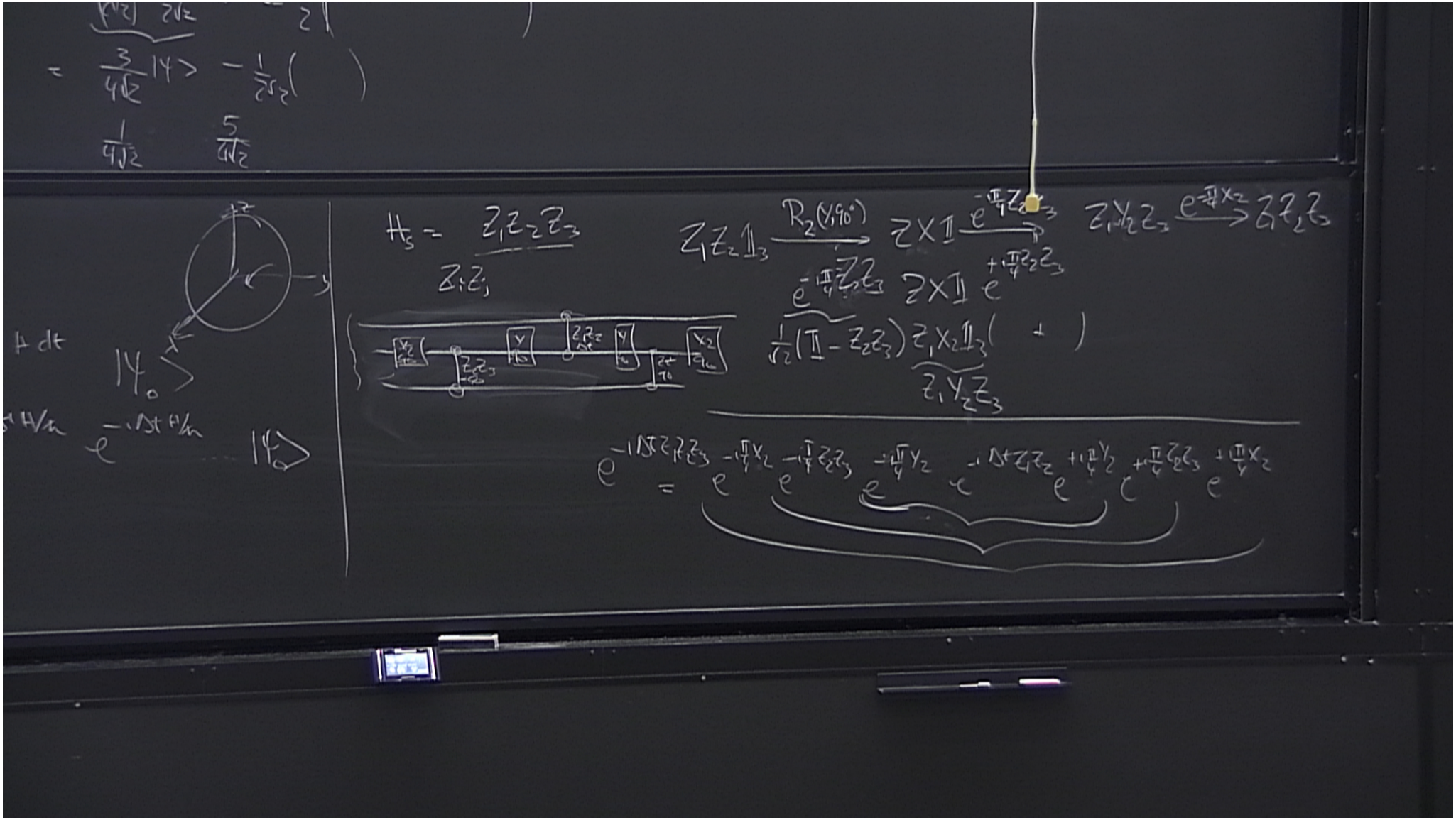
$\frac{1}{4\sqrt{2}} \qquad \frac{5}{4\sqrt{2}}$

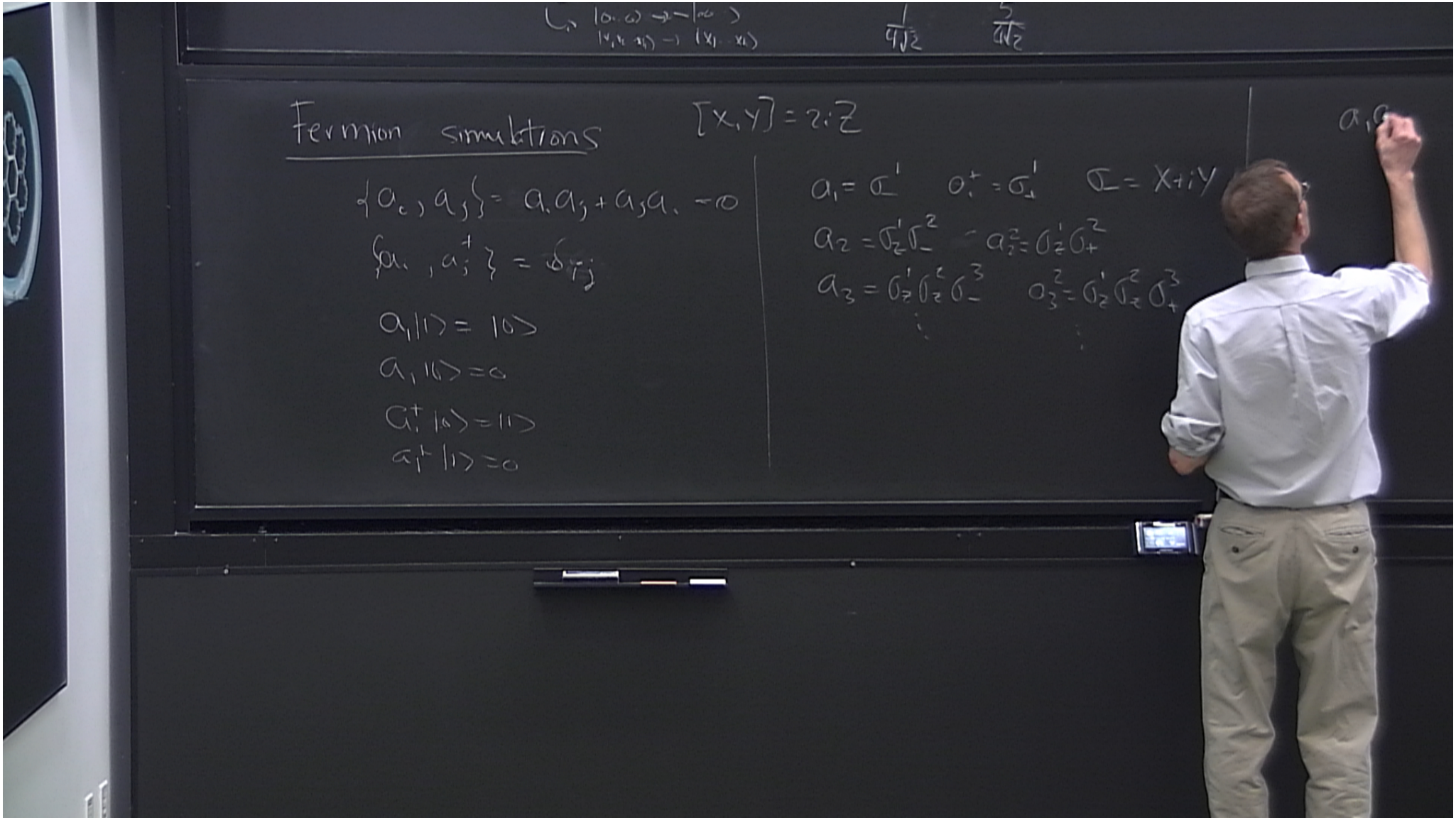
Physics Simulat

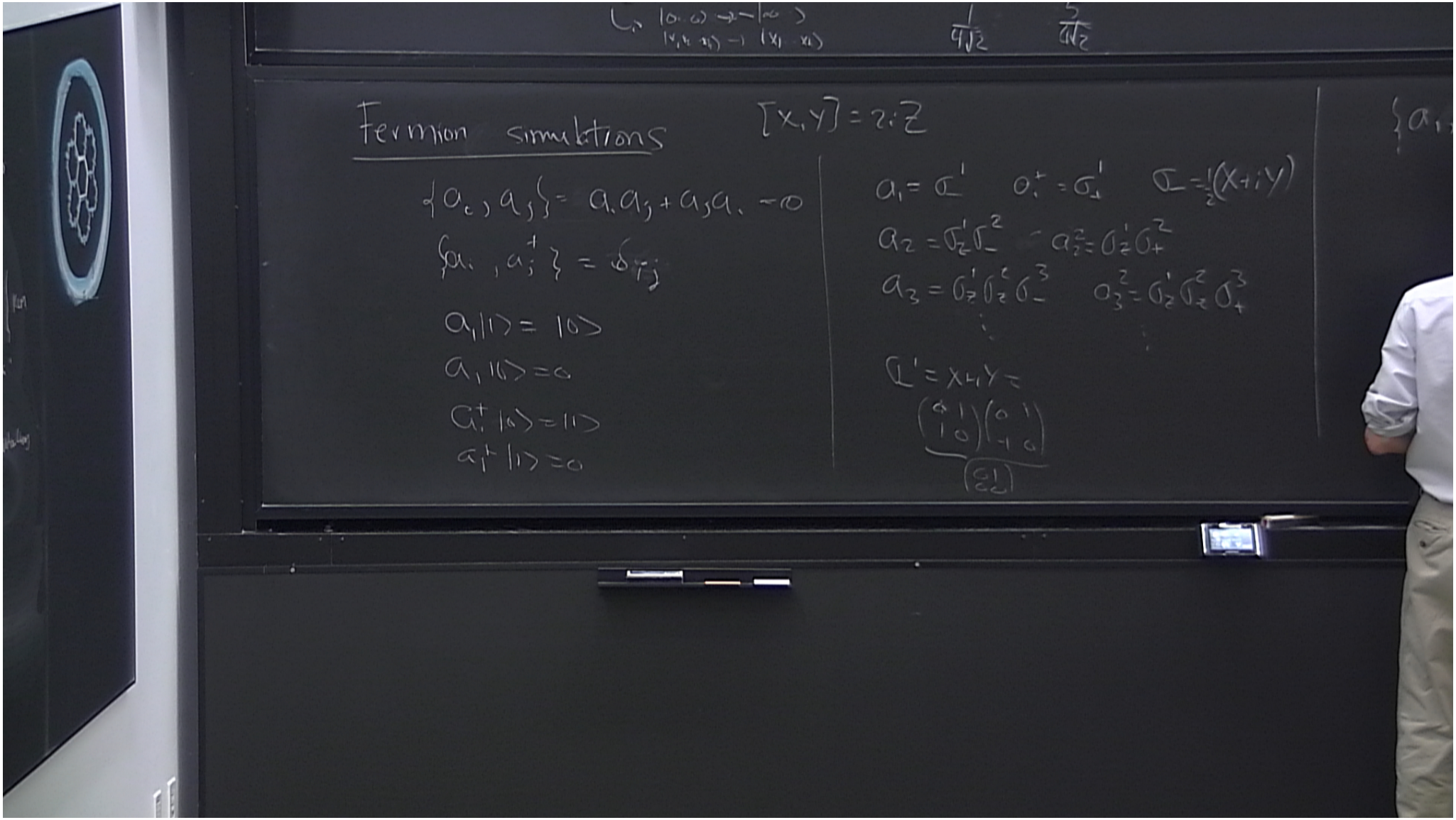

Physics Simulation
$-i\frac{d|\psi\rangle}{dt} = H|\psi\rangle$
$|\psi_t\rangle = U|\psi\rangle = e^{-i\int H\,dt}|\psi_0\rangle$
$= e^{-i\Delta t H/\hbar}\, e^{-i\Delta t H/\hbar}\, e^{-i\Delta t H/\hbar} \cdots |\psi\rangle$

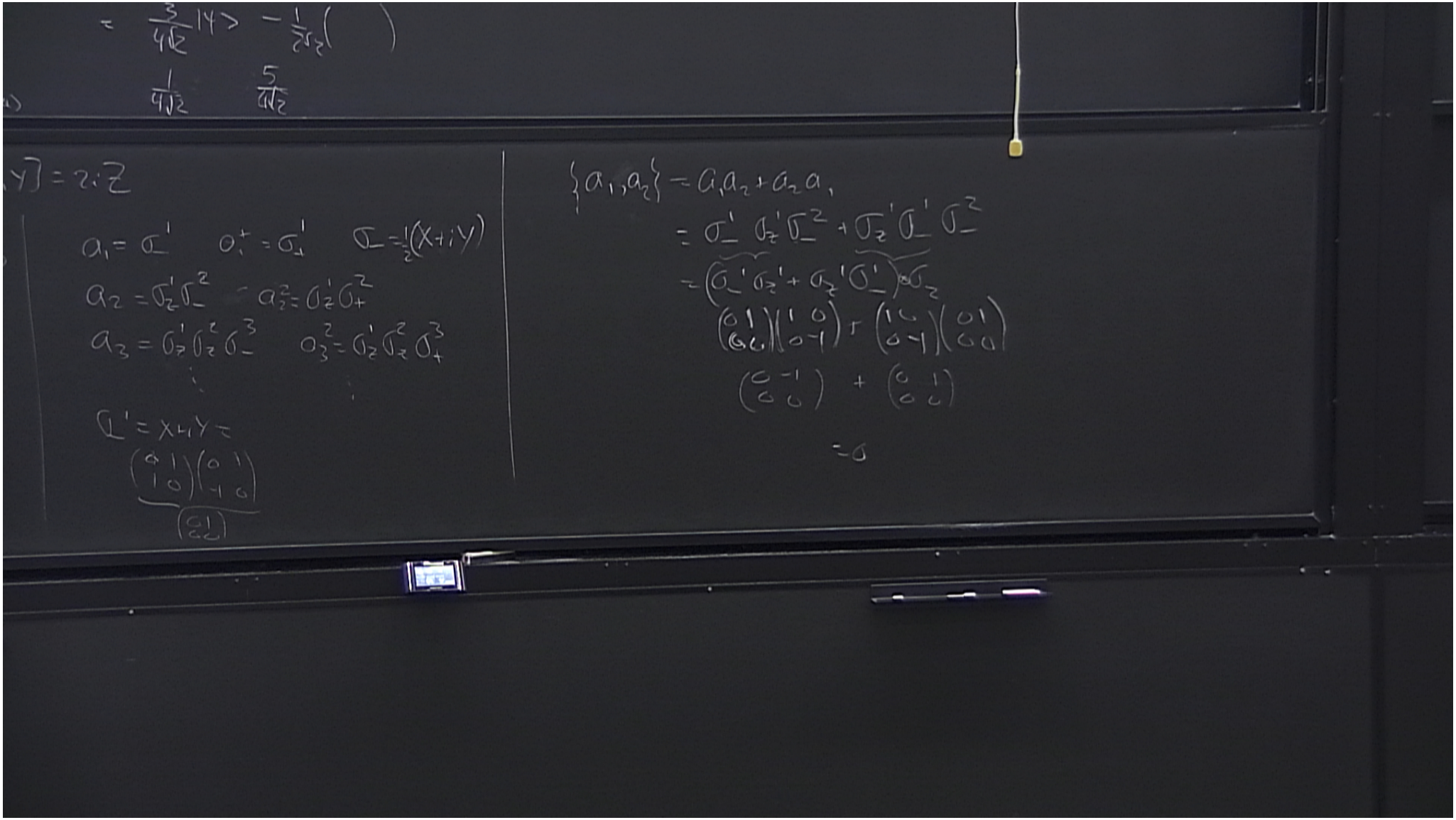

$H_5 = \frac{Z_1 Z_2 Z_3}{Z_1 Z_3}$
$Z_1 Z_2 \mathbb{1}_3 \xrightarrow{R_2(Y,90°)} Z \times \mathbb{1} \xrightarrow{e^{-i\frac{\pi}{4} Z_2 Z_3}}$
$e^{-i\frac{\pi}{4} Z_1 Z_3}$ $Z \times \mathbb{1}$ $e^{+i\frac{\pi}{4} Z_2 Z_3}$
$\frac{1}{\sqrt{2}}(\mathbb{1} - Z_2 Z_3)\, Z_1 X_2 \mathbb{1}_3 (\quad + \quad)$
$Z_1 Y_2 Z_3$


$H_5 = \frac{Z_1 Z_2 Z_3}{Z_1 Z_3}$
$Z_1 Z_2 \mathbb{1}_3 \xrightarrow{R_2(\gamma, 90°)} Z \times \mathbb{1}$
$Z_1 Y_2 Z_3$
$\frac{1}{\sqrt{2}}(\mathbb{1} - Z_2 Z_3) Z_1 X_2 \mathbb{1}_3 (+)$
$Z_1 Y_2 Z_3$




Fermion simulations
[X,Y] = 2iZ
$\{a_i, a_j\} = a_i a_j + a_j a_i = 0$
$\{a_i, a_j^\dagger\} = \delta_{ij}$
$a_1|1\rangle = |0\rangle$
$a_1|0\rangle = 0$
$a_1^\dagger|0\rangle = |1\rangle$
$a_1^\dagger|1\rangle = 0$
$a_1 = \sigma^1 \quad a_1^\dagger = \sigma_+^1 \quad \sigma = X + iY$
$a_2 = \sigma_z^1 \sigma_-^2 \quad a_2^\dagger = \sigma_z^1 \sigma_+^2$
$a_3 = \sigma_z^1 \sigma_z^2 \sigma_-^3 \quad a_3^\dagger = \sigma_z^1 \sigma_z^2 \sigma_+^3$


Fermion simulations
[X, Y] = 2iZ
$\{a_i, a_j\} = a_i a_j + a_j a_i = 0$
$\{a_i, a_j^\dagger\} = \delta_{ij}$
$a_1|1\rangle = |0\rangle$
$a_1|0\rangle = 0$
$a_1^\dagger|0\rangle = |1\rangle$
$a_1^\dagger|1\rangle = 0$
$a_1 = \sigma^1 \quad a_1^\dagger = \sigma_+^1 \quad \sigma_- = \frac{1}{2}(X + iY)$
$a_2 = \sigma_z^1 \sigma_-^2 \quad a_2^\dagger = \sigma_z^1 \sigma_+^2$
$a_3 = \sigma_z^1 \sigma_z^2 \sigma_-^3 \quad a_3^\dagger = \sigma_z^1 \sigma_z^2 \sigma_+^3$
$Q' = X + iY =$


$= \frac{3}{\sqrt{2}}|4\rangle - \frac{1}{\sqrt{2}}($
$\frac{1}{\sqrt{2}} \quad \frac{5}{\sqrt{2}}$
$Y] = 2iZ$
$a_1 = \sigma_-^1 \quad a_1^+ = \sigma_+^1 \quad \sigma_- = \frac{1}{2}(X+iY)$
$a_2 = \sigma_z^1 \sigma_-^2 \quad a_2^2 = \sigma_z^1 \sigma_+^2$
$a_3 = \sigma_z^1 \sigma_z^2 \sigma_-^3 \quad a_3^2 = \sigma_z^1 \sigma_z^2 \sigma_+^3$
$\sigma^1 = X+iY =$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$
$\{a_1, a_2\} = a_1 a_2 + a_2 a_1$
$= \sigma_-^1 \sigma_z^1 \sigma_-^2 + \sigma_z^1 \sigma_-^1 \sigma_-^2$
$= (\sigma_-^1 \sigma_z^1 + \sigma_z^1 \sigma_-^1)\sigma_-^2$
$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$
$= 0$

$U_+ = I - 2|w><w|$

$V = 2|+><+| - I$

$(= H U_{0,1} H$

↳ $|0\cdots0\rangle \to |-\infty\rangle$, $|x_1 x_2 x_3\rangle \to |x_1 x_2\rangle$

$= 2\left(\frac{1}{\sqrt{2}}\right)^2 \frac{6-1}{2\sqrt{2}}|+\rangle - \frac{1}{2}($ $)$

$= \frac{3}{4\sqrt{2}}|+\rangle - \frac{1}{2\sqrt{2}}($ $)$

$\frac{1}{4\sqrt{2}}$ $\frac{5}{4\sqrt{2}}$

Fermion simulations

$[X,Y] = 2iZ$

$\{a_0, a_3\} = a_0 a_3 + a_3 a_0 = 0$

$\{a_i, a_j^\dagger\} = \delta_{ij}$

$a_1|1\rangle = |0\rangle$

$a_1|0\rangle = 0$

$a_1^\dagger|0\rangle = |1\rangle$

$a_1^\dagger|1\rangle = 0$

$a_1 = \sigma_-^1 \quad a_1^\dagger = \sigma_+^1 \quad \mathbb{C} = \frac{1}{2}(X+iY)$

$a_2 = \sigma_z^1 \sigma_-^2 \quad a_2^\dagger = \sigma_z^1 \sigma_+^2$

$a_3 = \sigma_z^1 \sigma_z^2 \sigma_-^3 \quad a_3^\dagger = \sigma_z^1 \sigma_z^2 \sigma_+^3$

$\mathbb{C}^1 = X+iY =$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

$\begin{pmatrix} 2i & \\ & 2i \end{pmatrix}$

$\{a_1, a_2\} = a_1 a_2 + a_2 a_1$

$= \sigma_-^1 \sigma_z^1 \sigma_-^2 + \sigma_z^1 \sigma_-^1 \sigma_-^2$

$= (\sigma_-^1 \sigma_z^1 + \sigma_z^1 \sigma_-^1)\sigma_-^2$

$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

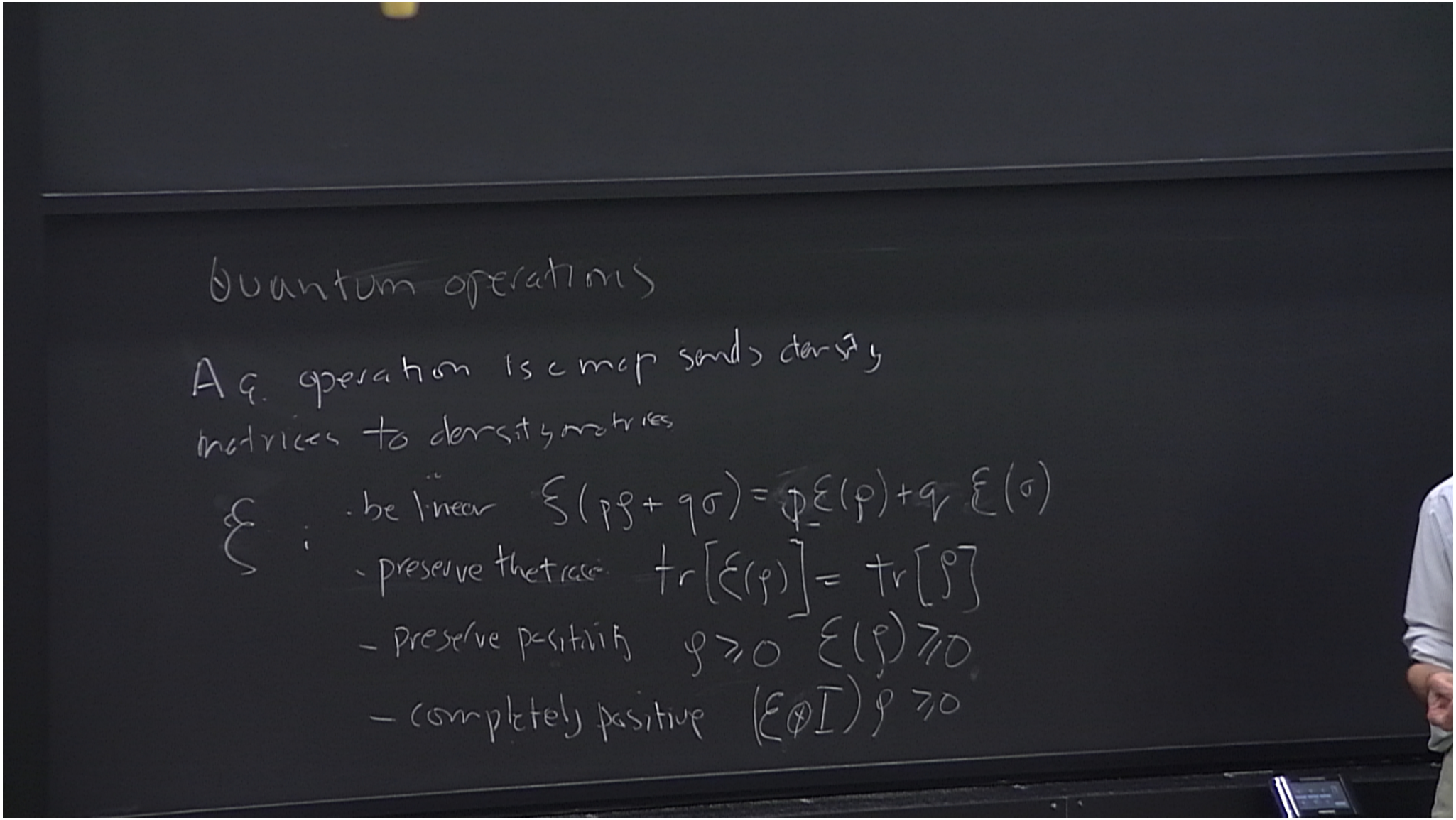
$\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$= 0$

ZZ_3

$X_2 \to \frac{1}{-i}\frac{Z}{2}Y_3$

$Z_1 Z_2 X_3$


Quantum operations
A q. operation is a map sends density matrices to density matrices
$\mathcal{E}$: - be linear $\mathcal{E}(p\rho + q\sigma) = p\mathcal{E}(\rho) + q\mathcal{E}(\sigma)$
- preserve the trace $tr[\mathcal{E}(\rho)] = tr[\rho]$
- preserve positivity $\rho \geq 0$ $\mathcal{E}(\rho) \geq 0$
- completely positive $(\mathcal{E} \otimes I)\rho \geq 0$

Quantum operations

A q. operation is a map sends density matrices to density matrices

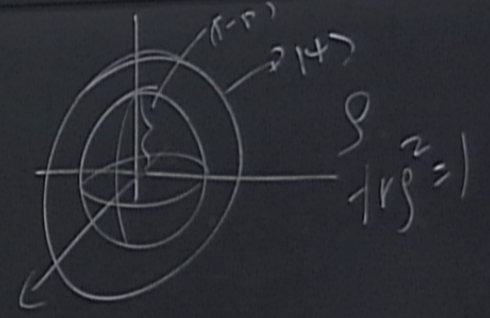
$\mathcal{E}$:
- be linear $\mathcal{E}(pf + q\sigma) = p\mathcal{E}(f) + q\mathcal{E}(\sigma)$
- preserve the trace $tr[\mathcal{E}(f)] = tr[f]$
- preserve positivity $f \geq 0$ $\mathcal{E}(f) \geq 0$
- completely positive $(\mathcal{E} \otimes I) f \geq 0$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Preserve positivity $\rho \geq 0 \quad \mathcal{E}(\rho) \geq 0$

i) Depolarizing channel

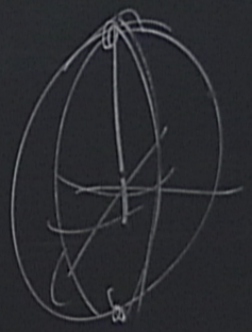
$$\rho = (1-p)\rho + \frac{p}{3}\left[X\rho X + Y\rho Y + Z\rho Z\right]$$

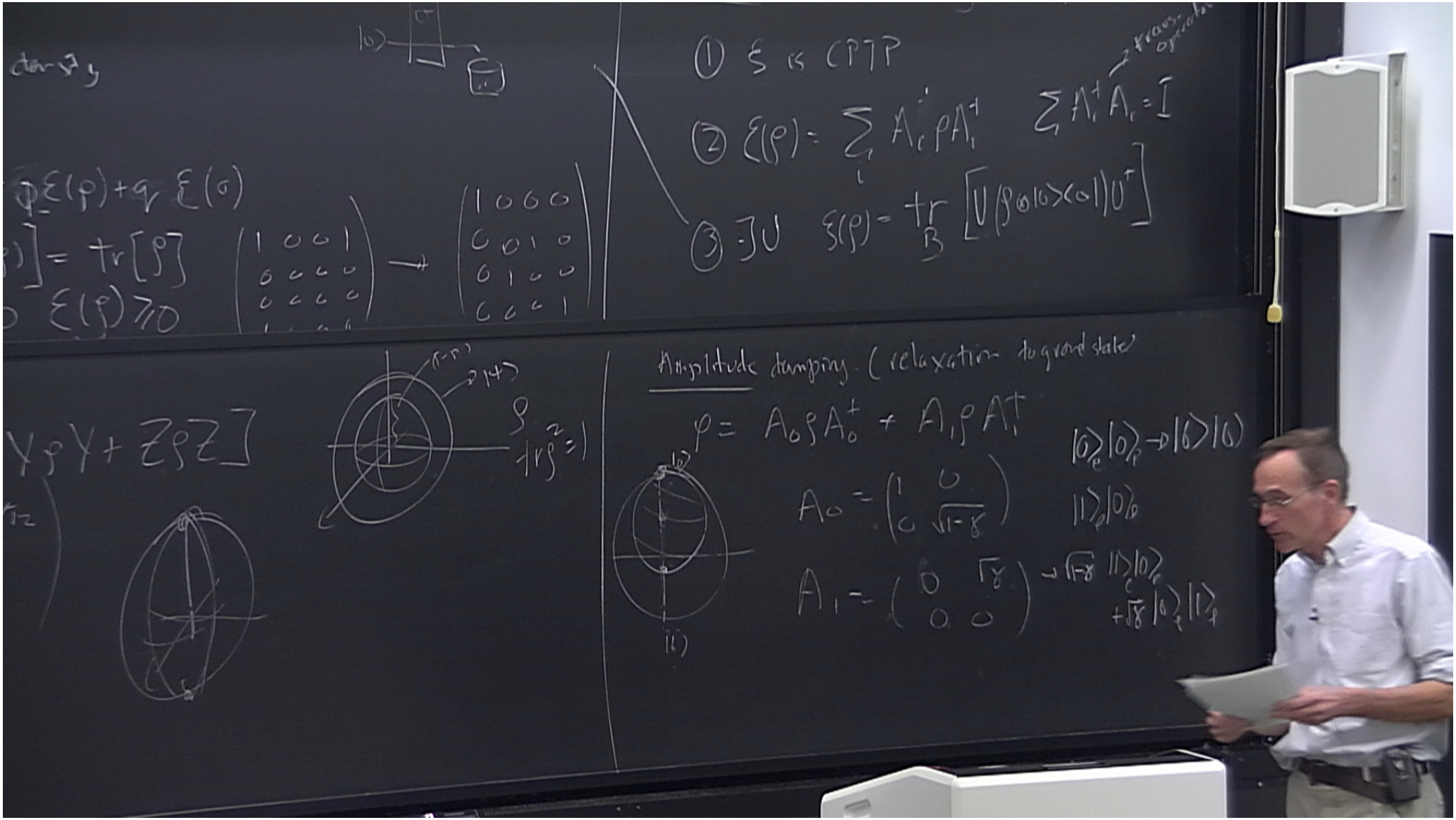


ii) Dephasing

$$\rho = (1-p)\rho + Z\rho Z$$

$$\begin{pmatrix} \alpha\alpha^* & \alpha\beta^* e^{-t/T_2} \\ \alpha^*\beta e^{-t/T_2} & \beta\beta^* \end{pmatrix}$$




① $\mathcal{E}$ is CPTP
② $\mathcal{E}(\rho) = \sum_i A_i \rho A_i^\dagger$ $\sum A_i^\dagger A_i = I$
③ $\mathcal{E}(\rho) = \mathrm{tr}_B [U(\rho \otimes |0\rangle\langle 0|)U^\dagger]$
Amplitude damping (relaxation to ground state)
$\rho = A_0 \rho A_0^\dagger + A_1 \rho A_1^\dagger$
$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}$
$A_1 = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix}$
$|0\rangle_e|0\rangle_g \to |0\rangle|0\rangle$
$|1\rangle_e|0\rangle_g$
$\to \sqrt{1-\gamma}\,|1\rangle_e|0\rangle_g + \sqrt{\gamma}\,|0\rangle_e|1\rangle_g$