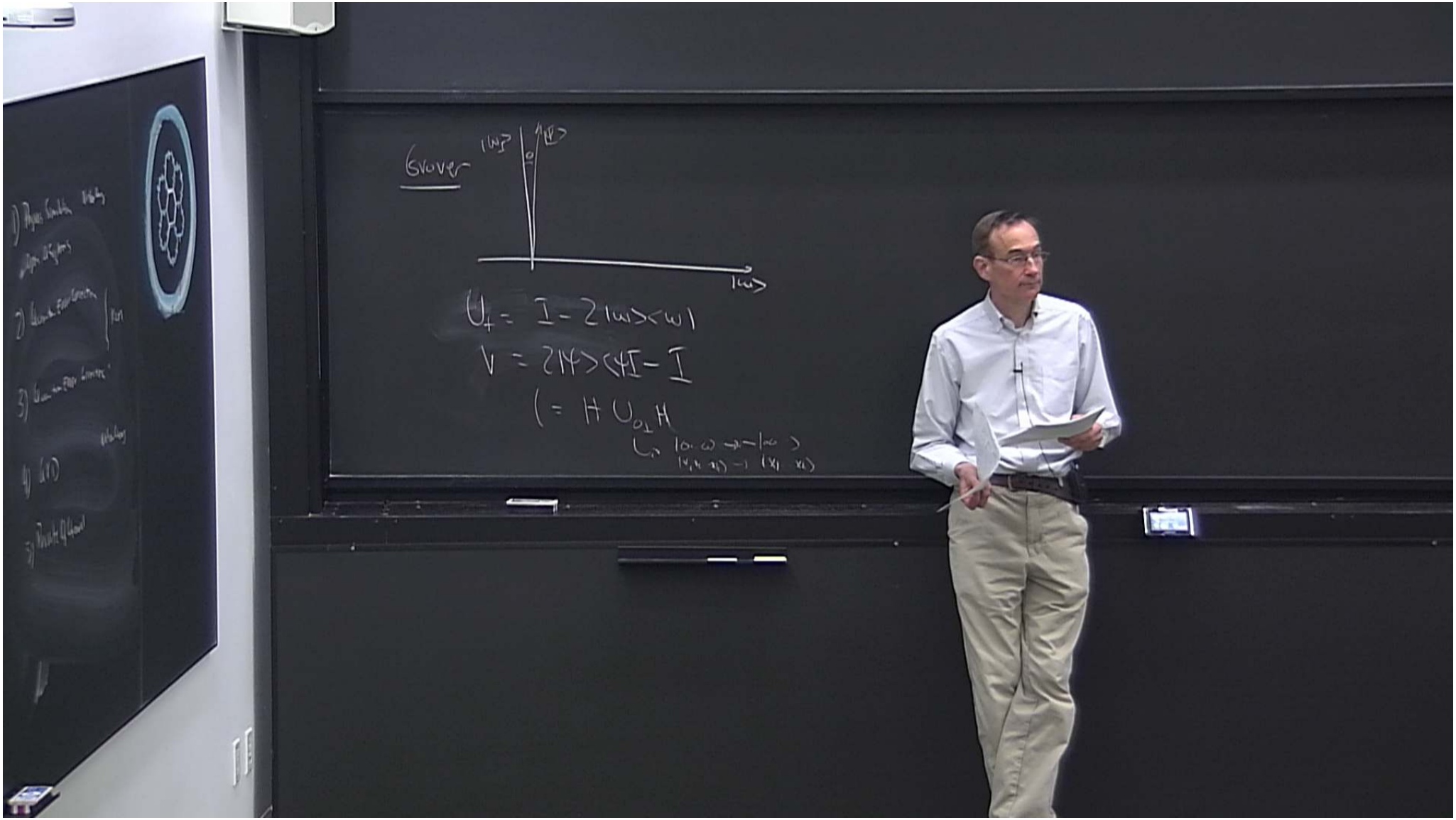
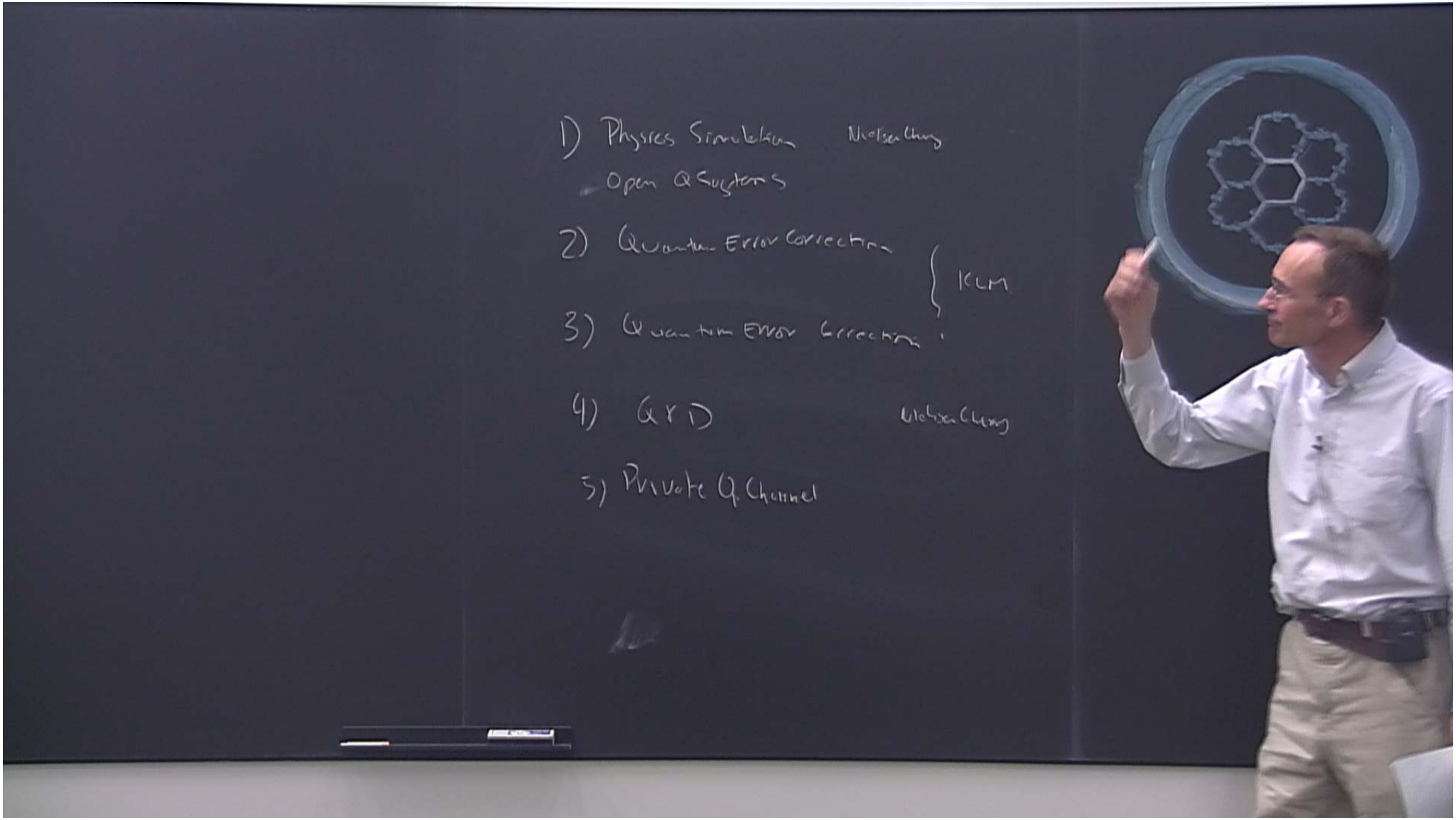
Title: PSI 2015/2016 Quantum Information - Lecture 11

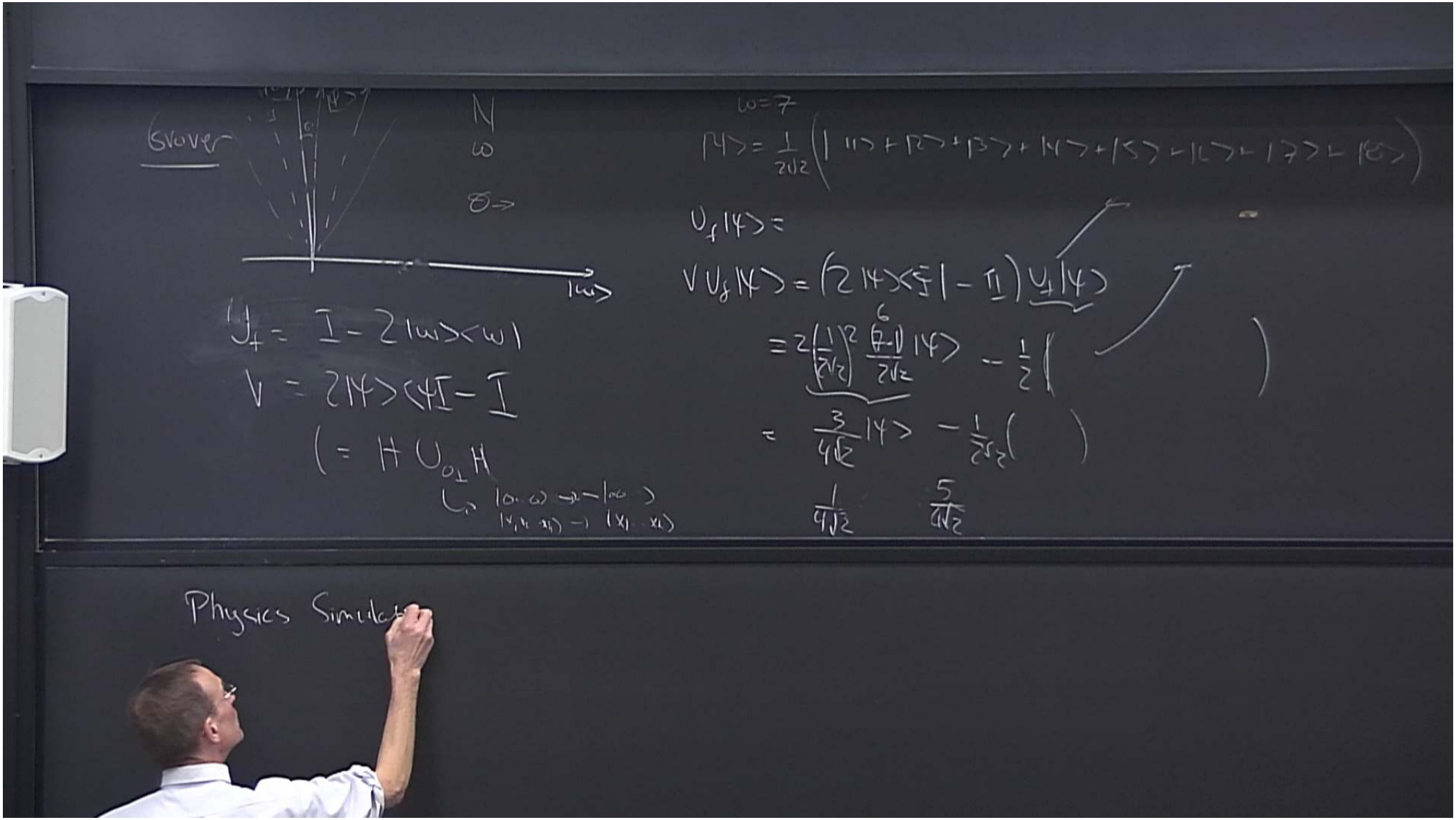
Date: Mar 07, 2016 11:30 AM

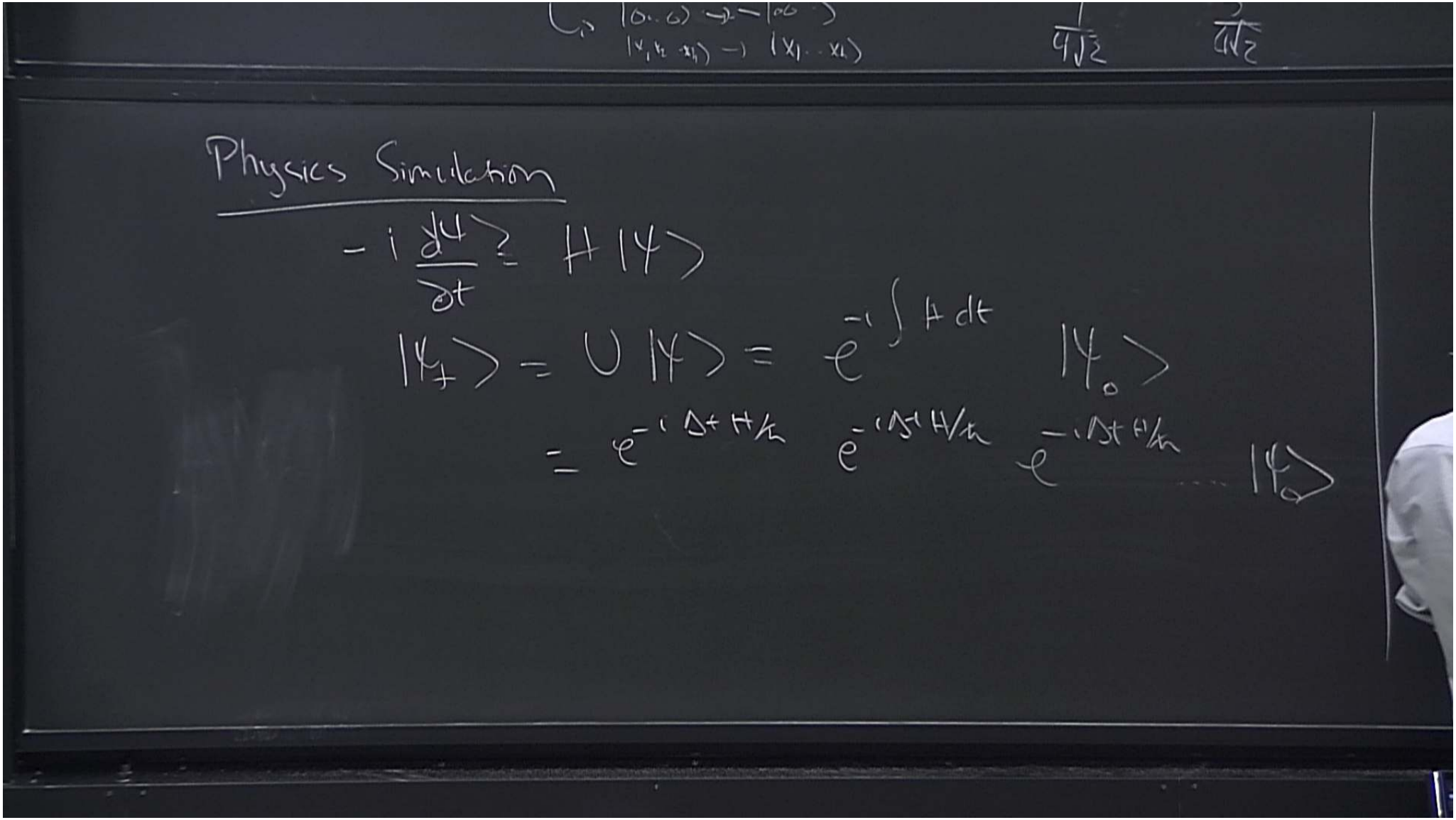
URL: http://pirsa.org/16030058

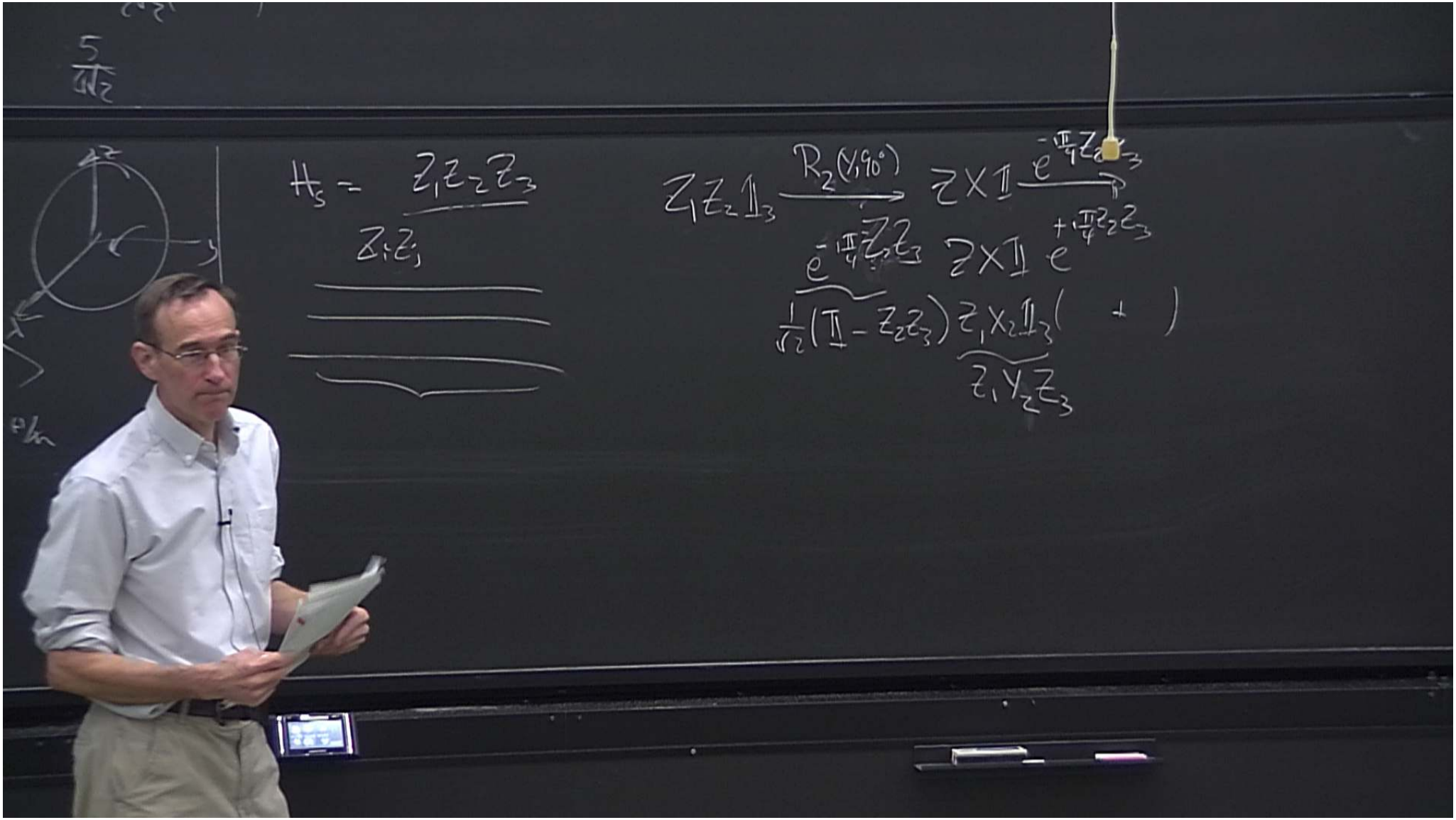
Abstract:

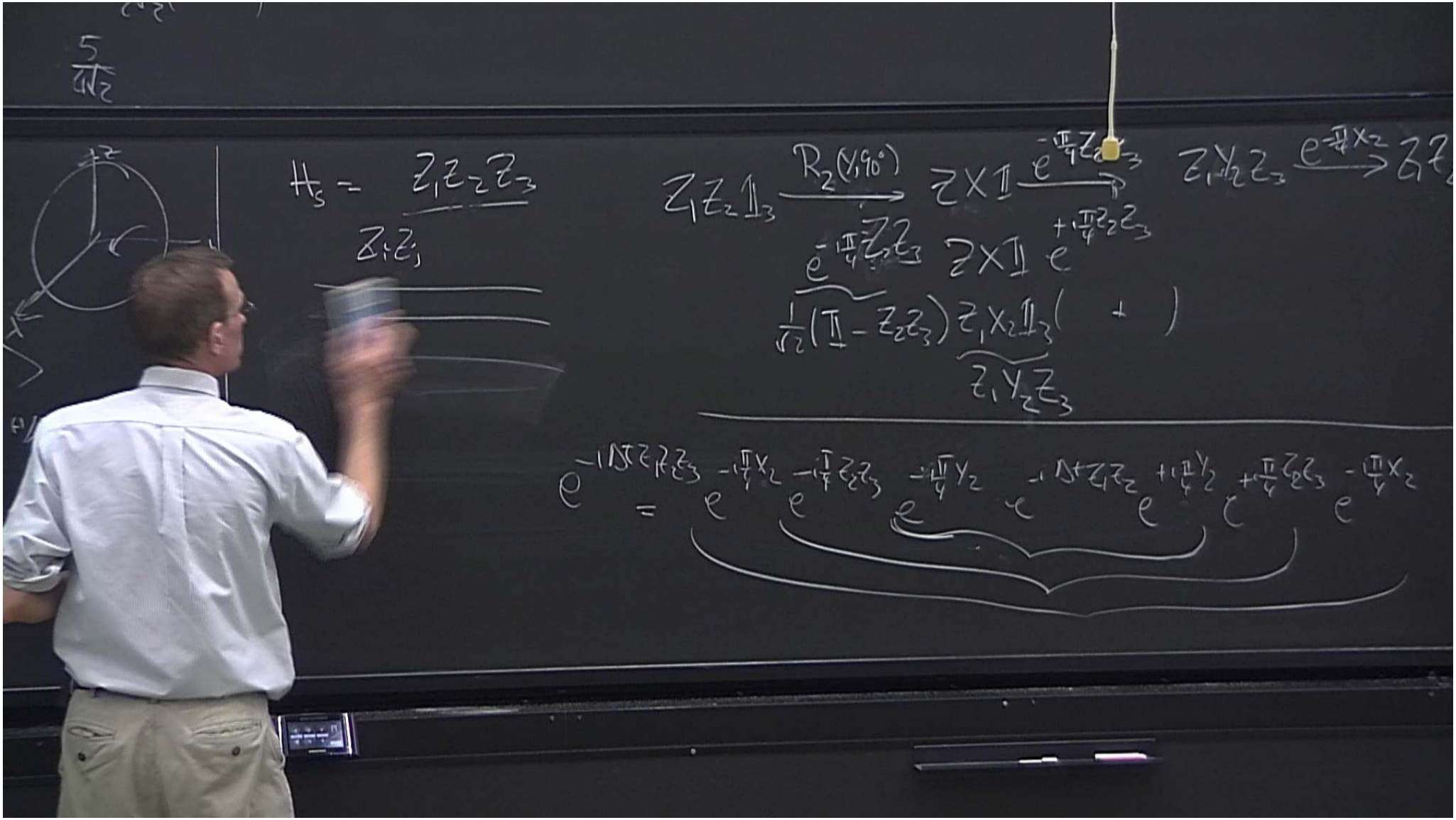

Grover
$|\omega\rangle$
$|\psi\rangle$
$U_f = I - 2|\omega\rangle\langle\omega|$
$V = 2|\psi\rangle\langle\psi| - I$
$(= H U_{0\perp} H$

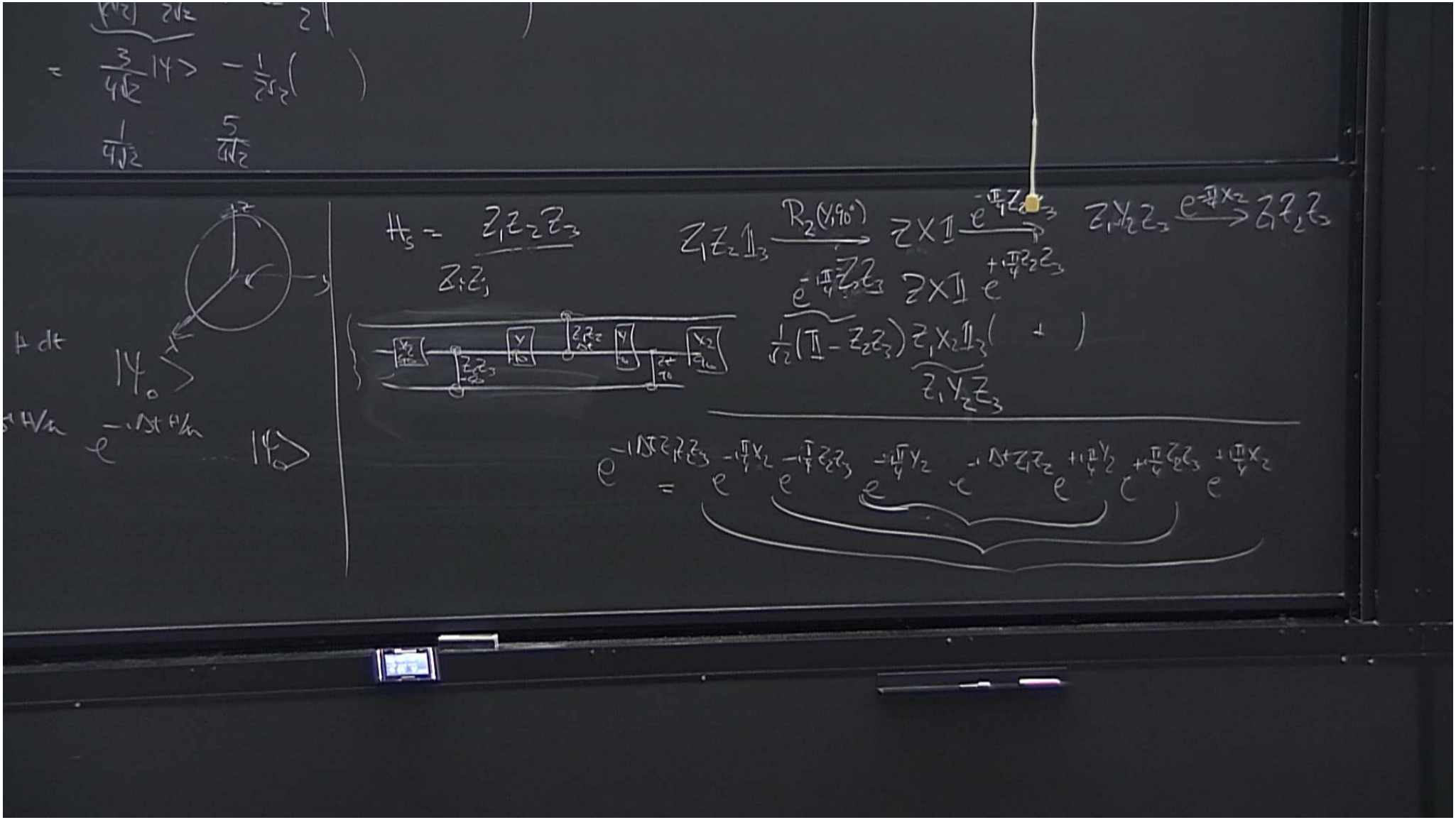

1) Physics Simulation Nielsen Chuang
Open Q Systems
2) Quantum Error Correction
ICLM
3) Quantum Error Correction
4) QKD Nielsen Chuang
5) Private Q. Channel


Grover
N
ω
θ →
|ω⟩
U_f = I − 2|ω⟩⟨ω|
V = 2|ψ⟩⟨ψ| − I
(= H U_{0⊥} H
ω = 7
|ψ⟩ = 1/(2√2) (|1⟩ + |2⟩ + |3⟩ + |4⟩ + |5⟩ + |6⟩ + |7⟩ + |8⟩)
U_f|ψ⟩ =
V U_f|ψ⟩ = (2|ψ⟩⟨ψ| − I) U_f|ψ⟩
= 2 (1/(2√2))² (7−1)/(2√2) |ψ⟩ − 1/2 ( )
= 3/(4√2) |ψ⟩ − 1/(2√2) ( )
1/(4√2)
5/(4√2)
Physics Simulat

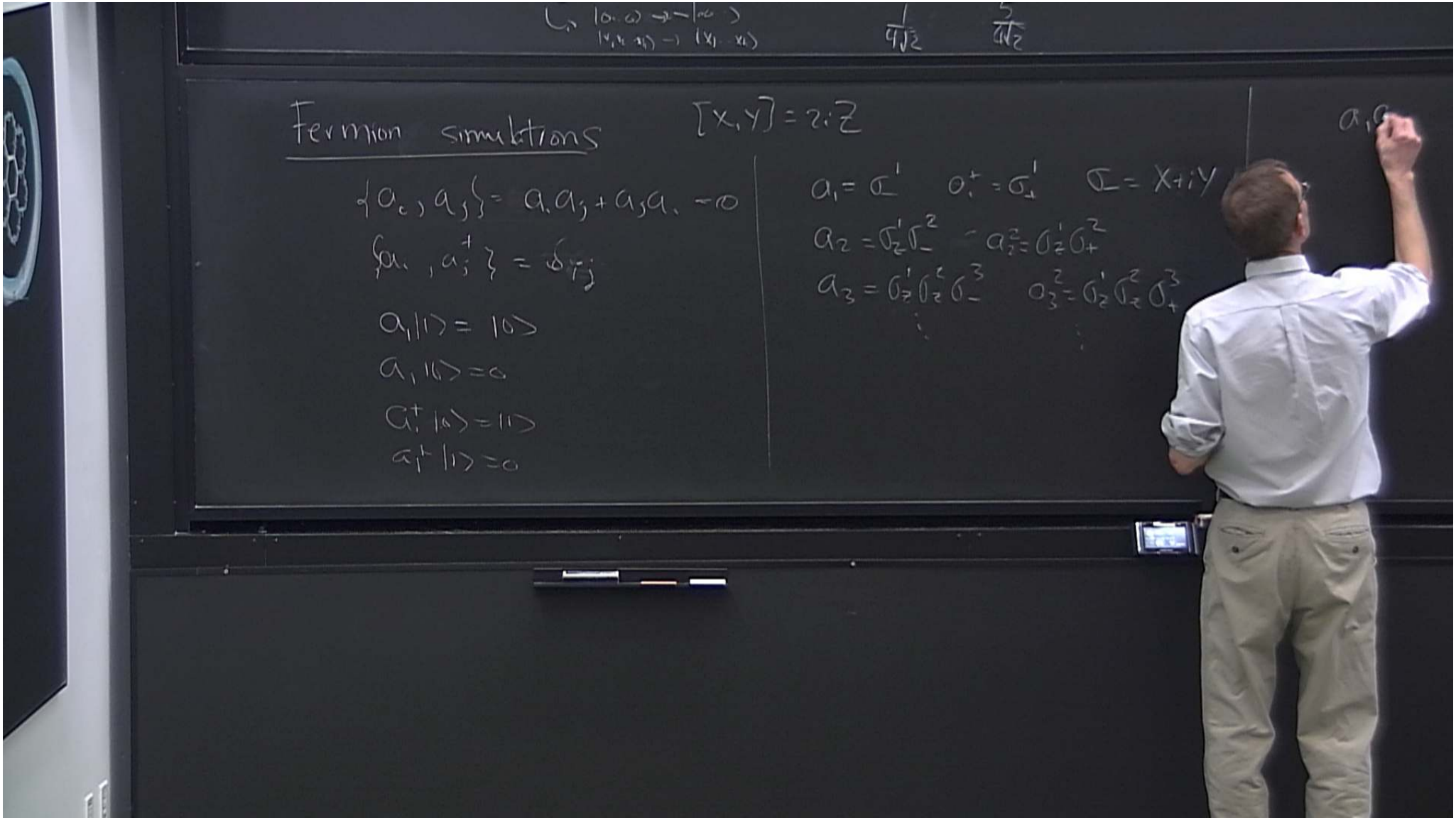

Physics Simulation
$-i\frac{d|\psi\rangle}{dt} = H|\psi\rangle$
$|\psi_t\rangle = U|\psi\rangle = e^{-i\int H\,dt}|\psi_0\rangle$
$= e^{-i\Delta t H/\hbar}\, e^{-i\Delta t H/\hbar}\, e^{-i\Delta t H/\hbar} \cdots |\psi\rangle$

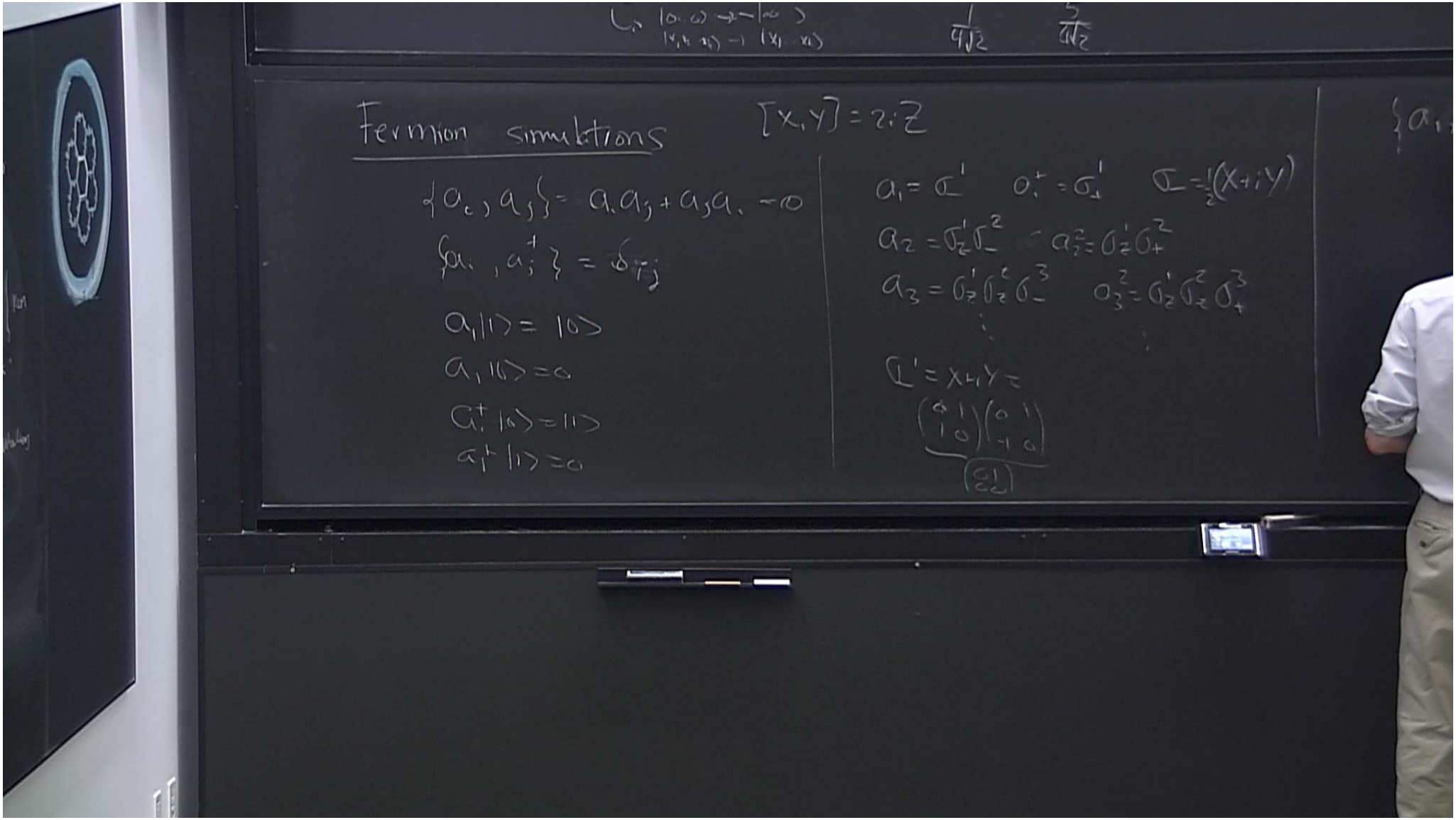

$\frac{5}{\sqrt{2}}$
$H_5 = \frac{Z_1 Z_2 Z_3}{Z_1 Z_3}$
$Z_1 Z_2 \mathbb{1}_3 \xrightarrow{R_2(Y, 90°)} Z \times \mathbb{1} \xrightarrow{e^{-\frac{\pi}{4} Z_1 Z_3}}$
$e^{-\frac{\pi}{4} Z_1 Z_3} Z \times \mathbb{1} \; e^{+\frac{\pi}{4} Z_2 Z_3}$
$\frac{1}{\sqrt{2}}(\mathbb{1} - Z_2 Z_3) Z_1 X_2 \mathbb{1}_3 (\quad + \quad)$
$Z_1 Y_2 Z_3$

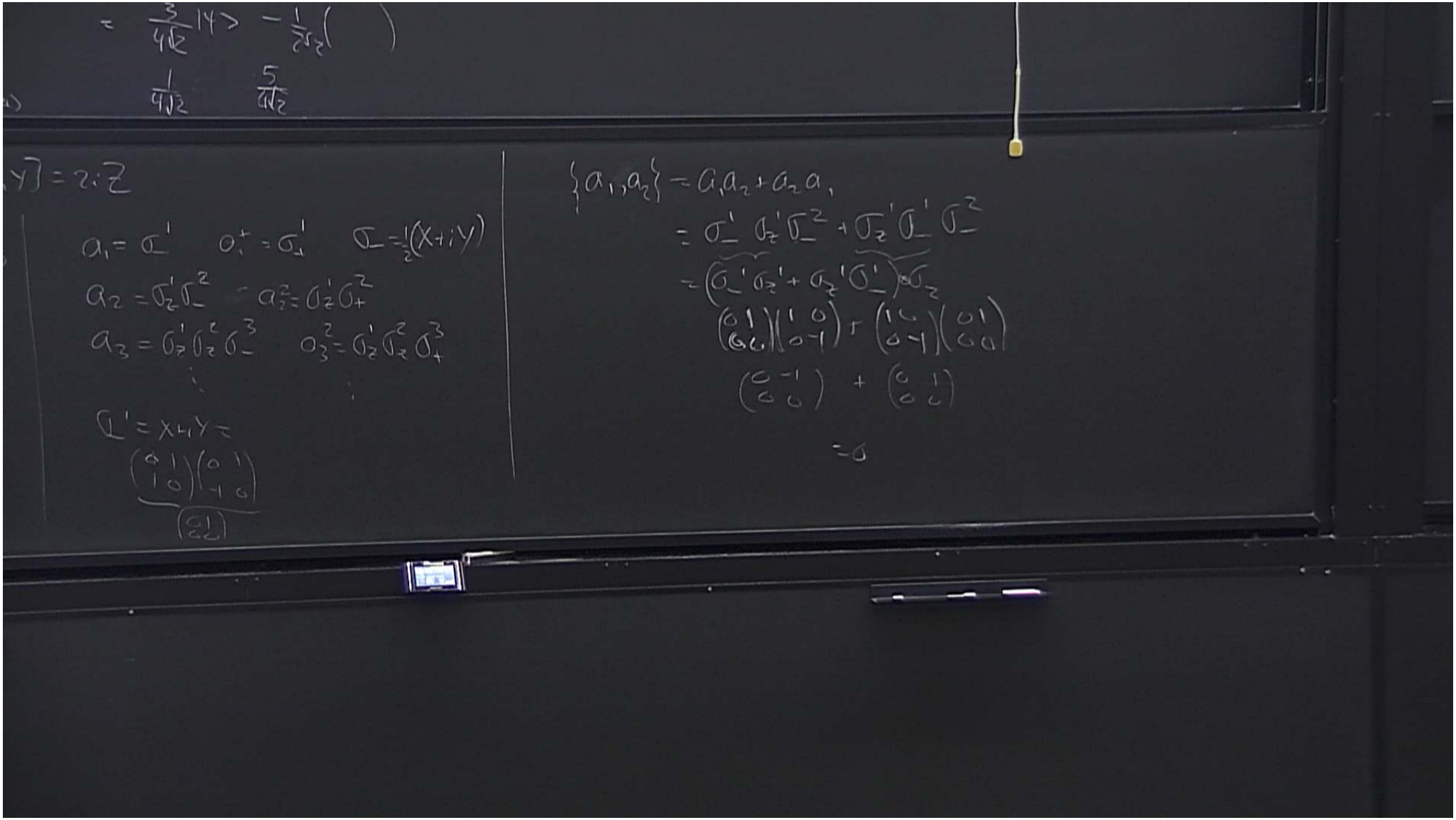

$H_5 = \frac{Z_1 Z_2 Z_3}{Z_1 Z_3}$
$Z_1 Z_2 \mathbb{1}_3 \xrightarrow{R_2(Y,90°)} Z X \mathbb{1}$
$Z_1 Y_2 Z_3$
$\frac{1}{\sqrt{2}}(\mathbb{1} - Z_2 Z_3) Z_1 X_2 \mathbb{1}_3$
$Z_1 Y_2 Z_3$



$= \frac{3}{4\sqrt{2}}|\psi\rangle - \frac{1}{2\sqrt{2}}(\quad)$
$\frac{1}{4\sqrt{2}}$ $\frac{5}{4\sqrt{2}}$
$H_s = \frac{Z_1 Z_2 Z_3}{Z_i Z_j}$
$Z_1 Z_2 \mathbb{1}_3 \xrightarrow{R_2(Y,90^\circ)} ZX\mathbb{1} \xrightarrow{e^{-i\frac{\pi}{4}Z_2Z_3}} Z_1 Y_2 Z_3 \xrightarrow{e^{-i\frac{\pi}{4}X_2}} Z_1 Z_2 Z_3$
$e^{-i\frac{\pi}{4}Z_2Z_3}\, ZX\mathbb{1}\, e^{+i\frac{\pi}{4}Z_2Z_3}$
$\frac{1}{\sqrt{2}}(\mathbb{1} - Z_2Z_3)\, Z_1X_2\mathbb{1}_3 (\quad + \quad)$
$Z_1 Y_2 Z_3$
$H\,dt$
$|\psi_0\rangle$
$e^{-i\Delta t H/\hbar}$ $|\psi_0\rangle$
$e^{-i\Delta t Z_1Z_2Z_3} = e^{-i\frac{\pi}{4}X_2} e^{-i\frac{\pi}{4}Z_2Z_3} e^{-i\frac{\pi}{4}Y_2} e^{-i\Delta t Z_1Z_2} e^{+i\frac{\pi}{4}Y_2} e^{+i\frac{\pi}{4}Z_2Z_3} e^{+i\frac{\pi}{4}X_2}$


Fermion simulations
$[X,Y] = 2iZ$
$\{a_i, a_j\} = a_i a_j + a_j a_i = 0$
$\{a_i, a_j^\dagger\} = \delta_{ij}$
$a_i|1\rangle = |0\rangle$
$a_i|0\rangle = 0$
$a_i^\dagger|0\rangle = |1\rangle$
$a_i^\dagger|1\rangle = 0$
$a_1 = \sigma_-^1$ $a_1^\dagger = \sigma_+^1$ $\sigma = X + iY$
$a_2 = \sigma_z^1 \sigma_-^2$ $a_2^\dagger = \sigma_z^1 \sigma_+^2$
$a_3 = \sigma_z^1 \sigma_z^2 \sigma_-^3$ $a_3^\dagger = \sigma_z^1 \sigma_z^2 \sigma_+^3$


Fermion simulations
[X, Y] = 2iZ
{a_i, a_j} = a_i a_j + a_j a_i = 0
{a_i, a_j^†} = δ_ij
a_1|1> = |0>
a_1|0> = 0
a_1^† |0> = |1>
a_1^† |1> = 0
a_1 = σ_-^1 a_1^† = σ_+^1 σ_- = ½(X+iY)
a_2 = σ_z^1 σ_-^2 a_2^2 = σ_z^1 σ_+^2
a_3 = σ_z^1 σ_z^2 σ_-^3 a_3^2 = σ_z^1 σ_z^2 σ_+^3
σ_-^1 = X+iY =


$= \frac{3}{4\sqrt{2}}|4\rangle - \frac{1}{2\sqrt{2}}(\quad)$
$\frac{1}{\sqrt{2}} \quad \frac{5}{4\sqrt{2}}$
$Y] = 2iZ$
$a_1 = \sigma_1^- \quad a_1^+ = \sigma_1^+ \quad \sigma_- = \frac{1}{2}(X + iY)$
$a_2 = \sigma_z^1 \sigma_-^2 \quad a_2^+ = \sigma_z^1 \sigma_+^2$
$a_3 = \sigma_z^1 \sigma_z^2 \sigma_-^3 \quad a_3^+ = \sigma_z^1 \sigma_z^2 \sigma_+^3$
$\sigma_-^1 = X + iY =$
$\{a_1, a_2\} = a_1 a_2 + a_2 a_1$
$= \sigma_-^1 \sigma_z^1 \sigma_-^2 + \sigma_z^1 \sigma_-^1 \sigma_-^2$
$= (\sigma_-^1 \sigma_z^1 + \sigma_z^1 \sigma_-^1)\sigma_-^2$
$= 0$

$U_+ = I - 2|\omega\rangle\langle\omega|$

$V = 2|\psi\rangle\langle\psi| - I$

$(= H U_{0\perp} H$

$\hookrightarrow |0\ 0\rangle \to -|00\rangle$

$|x_1 x_2\rangle \to |x_1\ x_2\rangle$

$= 2\left(\frac{1}{2\sqrt{2}}\right)^2 \frac{(7-1)}{2\sqrt{2}}|\psi\rangle - \frac{1}{2}\left(\quad \right)$

$= \frac{3}{4\sqrt{2}}|\psi\rangle - \frac{1}{2\sqrt{2}}\left(\quad \right)$

$\frac{1}{4\sqrt{2}} \qquad \frac{5}{4\sqrt{2}}$

Fermion simulations

$[X, Y] = 2iZ$

$\{a_i, a_j\} = a_i a_j + a_j a_i = 0$

$\{a_i, a_j^\dagger\} = \delta_{ij}$

$a_1|1\rangle = |0\rangle$

$a_1|0\rangle = 0$

$a_1^\dagger|0\rangle = |1\rangle$

$a_1^\dagger|1\rangle = 0$

$a_1 = \sigma_-^1 \quad a_1^\dagger = \sigma_+^1 \quad \sigma_- = \frac{1}{2}(X + iY)$

$a_2 = \sigma_z^1\sigma_-^2 \quad a_2^2 = \sigma_z^1\sigma_+^2$

$a_3 = \sigma_z^1\sigma_z^2\sigma_-^3 \quad a_3^2 = \sigma_z^1\sigma_z^2\sigma_+^3$

$\vdots \qquad \vdots$

$\sigma_-^1 = X + iY =$

$\underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}$

$\{a_1, a_2\} = a_1 a_2 + a_2 a_1$

$= \sigma_-^1 \sigma_z^1 \sigma_-^2 + \sigma_z^1 \sigma_-^1 \sigma_-^2$

$= (\sigma_-^1\sigma_z^1 + \sigma_z^1\sigma_-^1)\sigma_z$

$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$= 0$

$Z_2 Z_3$

$X_3 \to I_1 Z_2 Y_3$

$\downarrow$

$Z_1 Z_2 X_3$

Quantum operations

A q. operation is a map sends density matrices to density matrices

$\mathcal{E}$:
- be linear $\mathcal{E}(p\rho + q\sigma) = p\mathcal{E}(\rho) + q\mathcal{E}(\sigma)$
- preserve the trace $tr[\mathcal{E}(\rho)] = tr[\rho]$
- preserve positivity $\rho \geq 0$ $\mathcal{E}(\rho) \geq 0$
- completely positive $(\mathcal{E} \otimes I)\rho \geq 0$

Quantum operations

A q. operation is a map sends density matrices to density matrices

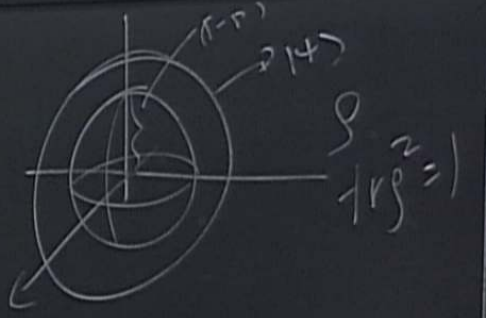
$\mathcal{E}$:
- be linear $\mathcal{E}(p\rho + q\sigma) = p\mathcal{E}(\rho) + q\mathcal{E}(\sigma)$
- preserve the trace $tr[\mathcal{E}(\rho)] = tr[\rho]$
- preserve positivity $\rho \geq 0 \quad \mathcal{E}(\rho) \geq 0$
- completely positive $(\mathcal{E} \otimes I)\rho \geq 0$

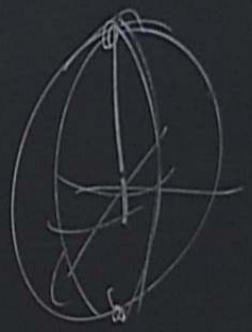
$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Preserve positivity $\rho \geq 0 \quad \mathcal{E}(\rho) \geq 0$

i) Depolarizing channel

$$\rho = (1-p)\rho + \frac{p}{3}\left[X\rho X + Y\rho Y + Z\rho Z\right]$$

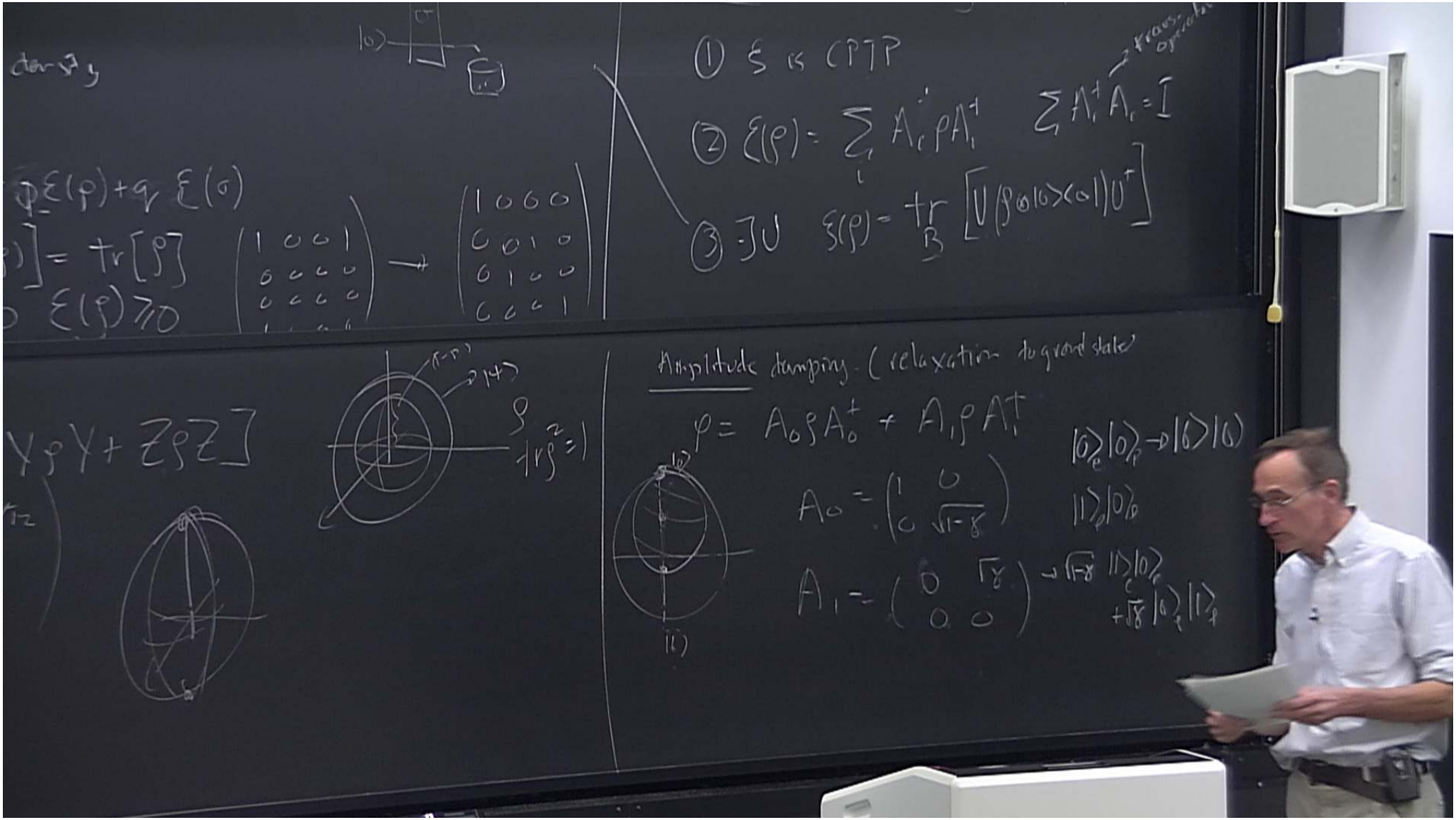




ii) Dephasing

$$\rho = (1-p)\rho + Z\rho Z$$

$$\begin{pmatrix} \alpha\alpha^* & \alpha\beta^* e^{-t/T_2} \\ \alpha^*\beta e^{-t/T_2} & \beta\beta^* \end{pmatrix}$$


① $\mathcal{E}$ is CPTP
② $\mathcal{E}(\rho) = \sum_i A_i \rho A_i^\dagger$ $\sum A_i^\dagger A_i = I$
③ $\exists U$ $\mathcal{E}(\rho) = \mathrm{tr}_B [U(\rho \otimes |0\rangle\langle 0|)U^\dagger]$
Amplitude damping (relaxation to ground state)
$\rho = A_0 \rho A_0^\dagger + A_1 \rho A_1^\dagger$
$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}$
$A_1 = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix}$
$|0\rangle_e|0\rangle_f \to |0\rangle|0\rangle$
$|1\rangle_e|0\rangle_f$
$\to \sqrt{1-\gamma}\,|1\rangle_e|0\rangle_f + \sqrt{\gamma}\,|0\rangle_e|1\rangle_f$