

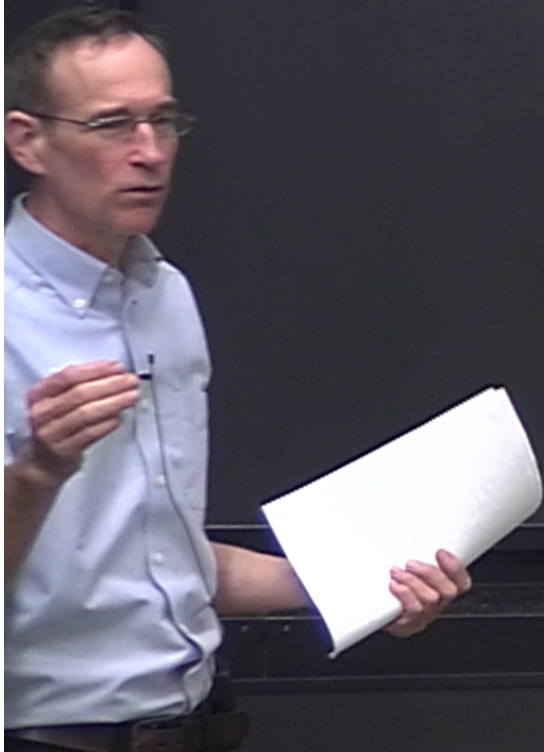
Title: PSI 2015/2016 Quantum Information - Lecture 10

Date: Mar 04, 2016 11:30 AM

URL: <http://pirsa.org/16030057>

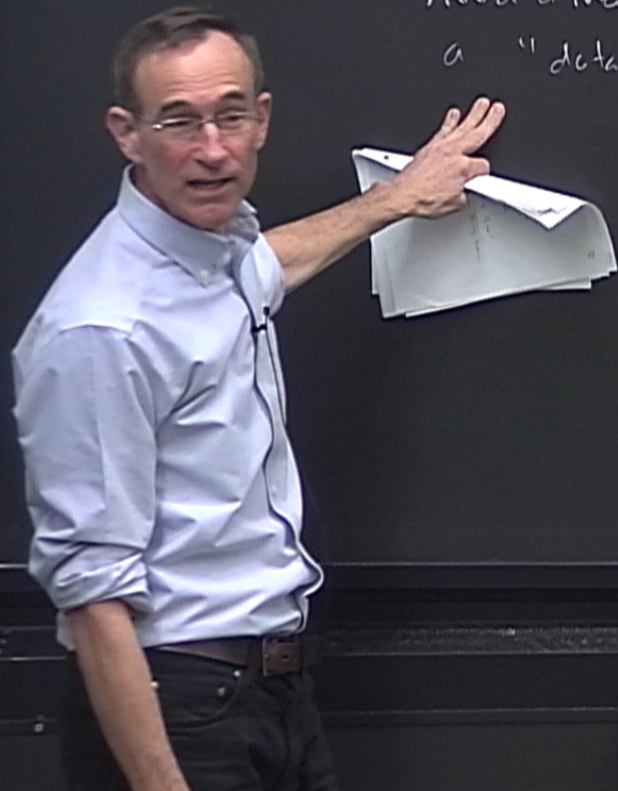
Abstract:

Grover algorithm



Grover algorithm

find a marked element A in
a "database", N , $\sqrt{N}$



Grover algorithm

find a marked element A in
a "database", N , $\sqrt{N}$

- 1) Geometric view
- 2) Grover's, black box
- 3) Phase kick back
- 4) Another view of

Grover algorithm

find a marked element A in
a "database", N , $\log N$

- 1) Geometric view
- 2) Grover's, black box
- 3) Phase kick back
- 4) Another view of
- 5) The algorithm itself

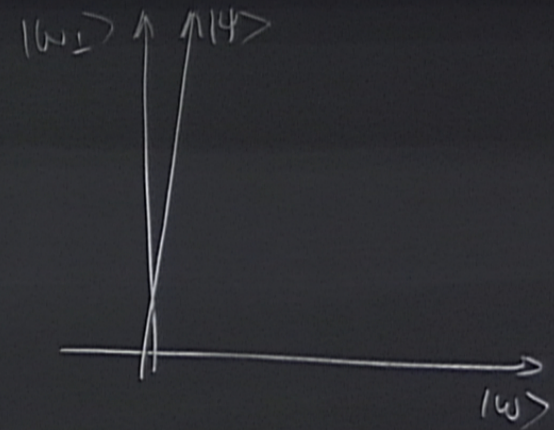
$|w\rangle$

Grover algorithm

find a marked element A in

a "database" N of size N

- 1) Geometric
- 2) Grover's box
- 3) Phase
- 4) Amplitude
- 5, 14



$|w_1\rangle$ $|w_4\rangle$

$U_f: - (\text{reflection around } |w\rangle)$

$|w\rangle$

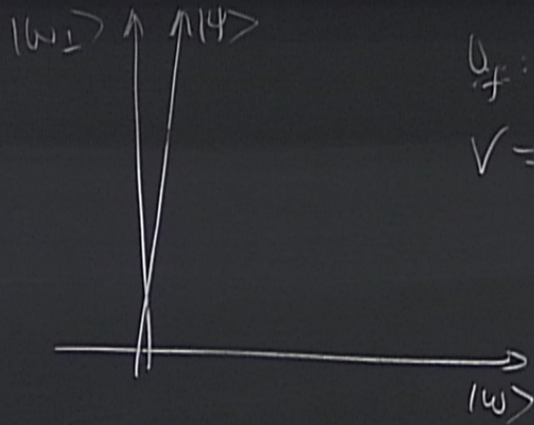
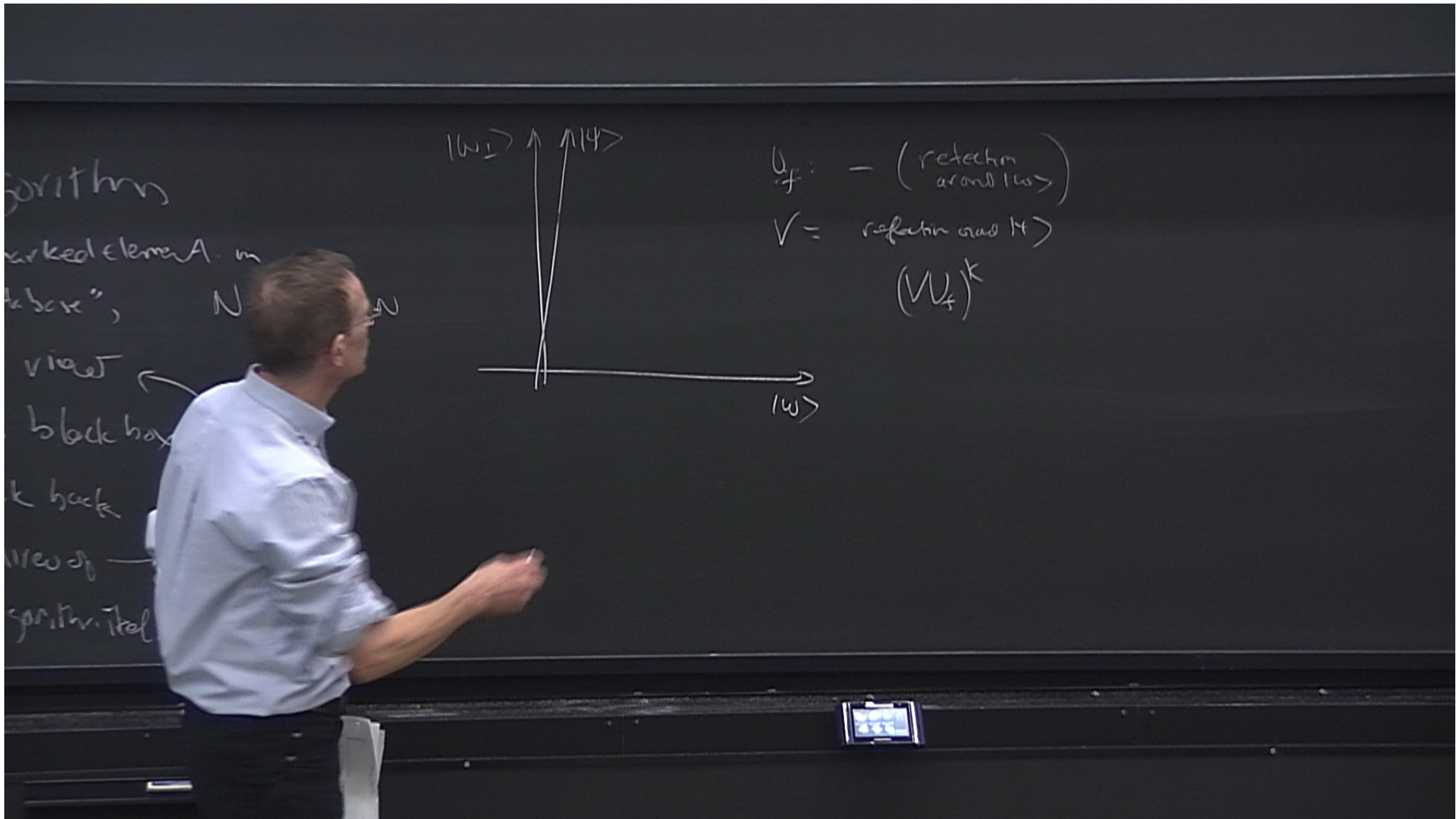
$|w_1\rangle$ $|w_4\rangle$

$U_f: - (\text{reflection around } |w\rangle)$

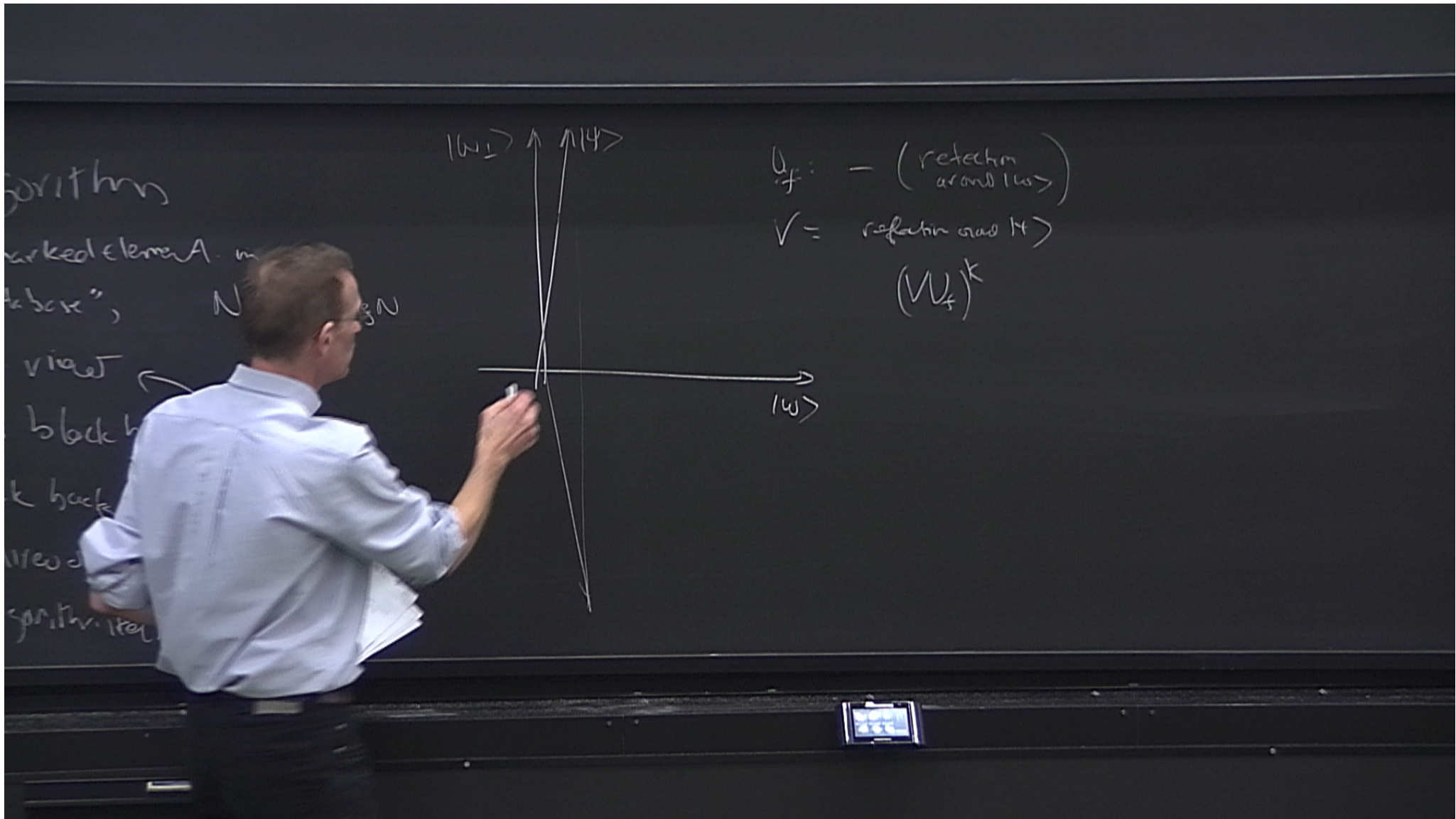
$V = \text{reflection around } |H\rangle$

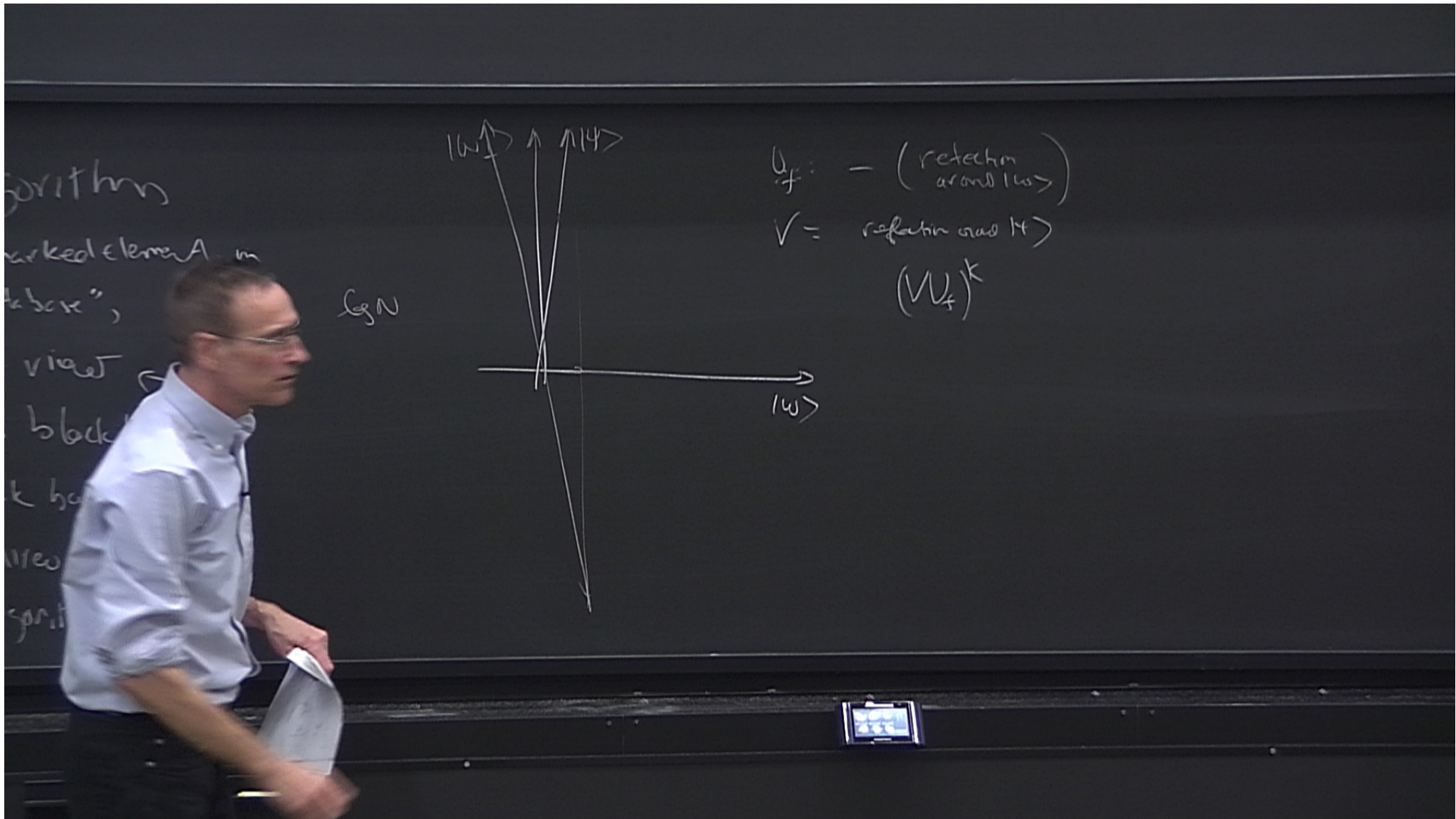
$\frac{1}{\sqrt{2}}$

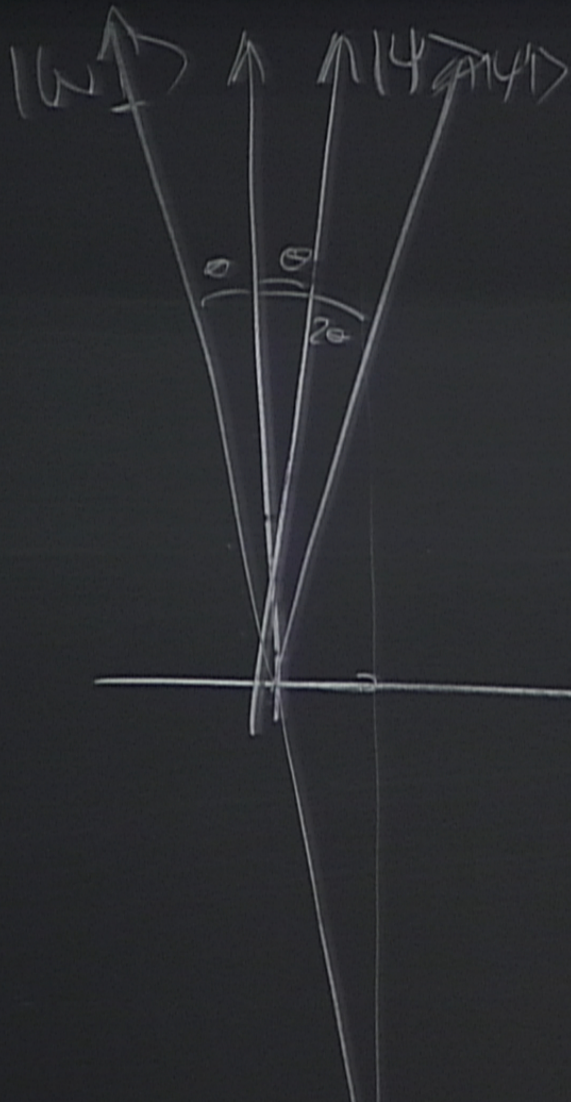
$|w\rangle$



U_f : - (reflection around $|w\rangle$)
 V = reflection around $|w\rangle$
 $(V U_f)^k$



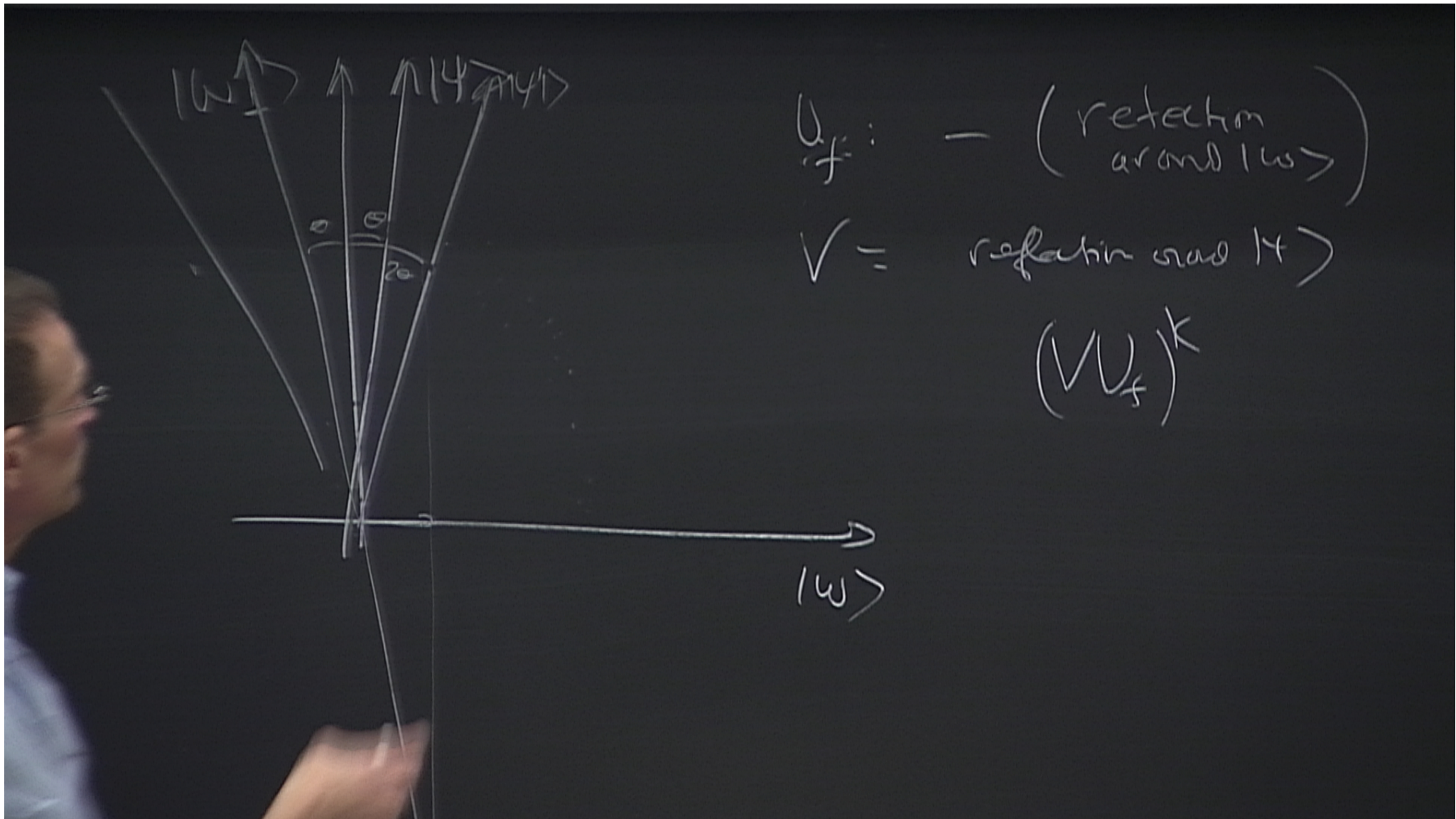


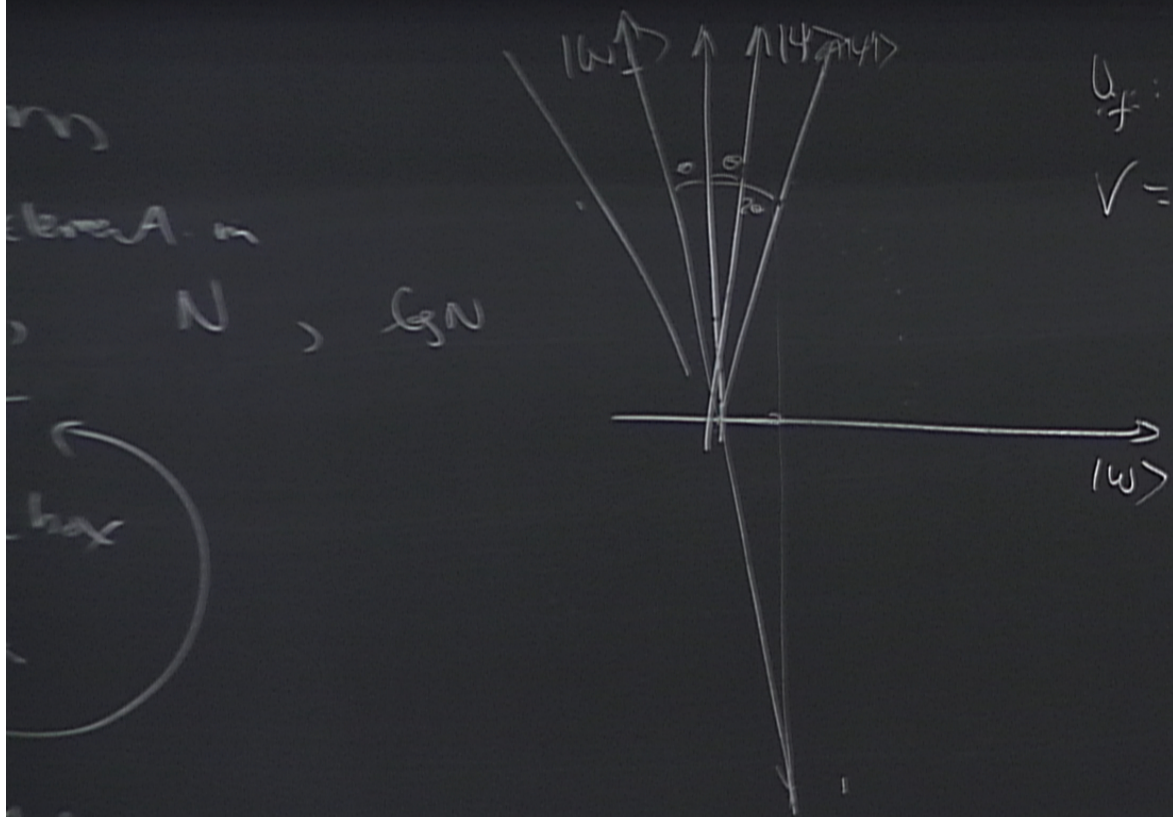


U_f : — (rotation around $|w\rangle$)

$V =$ rotation around $|1/4\rangle$

$$(VU_f)^k$$



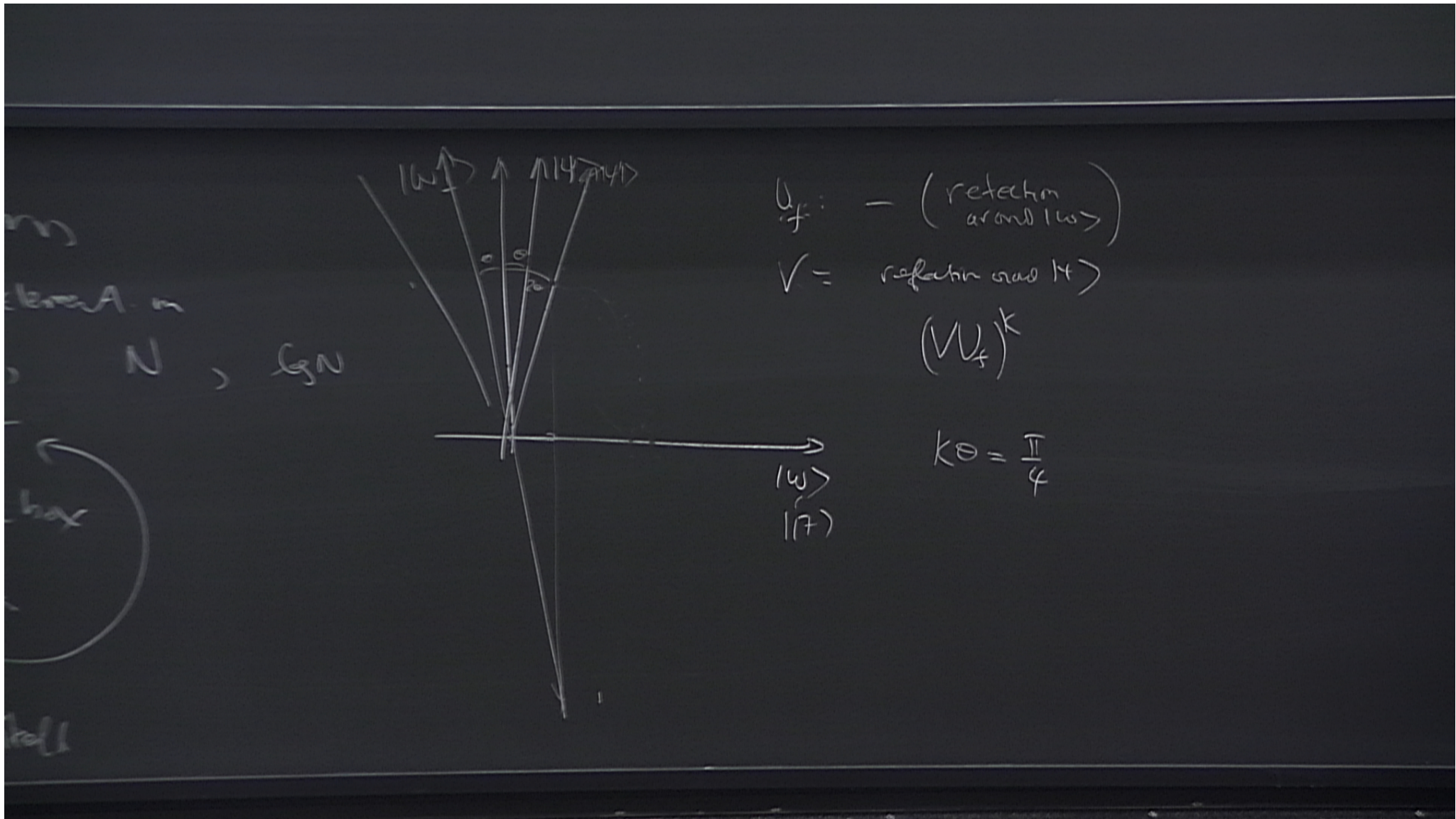


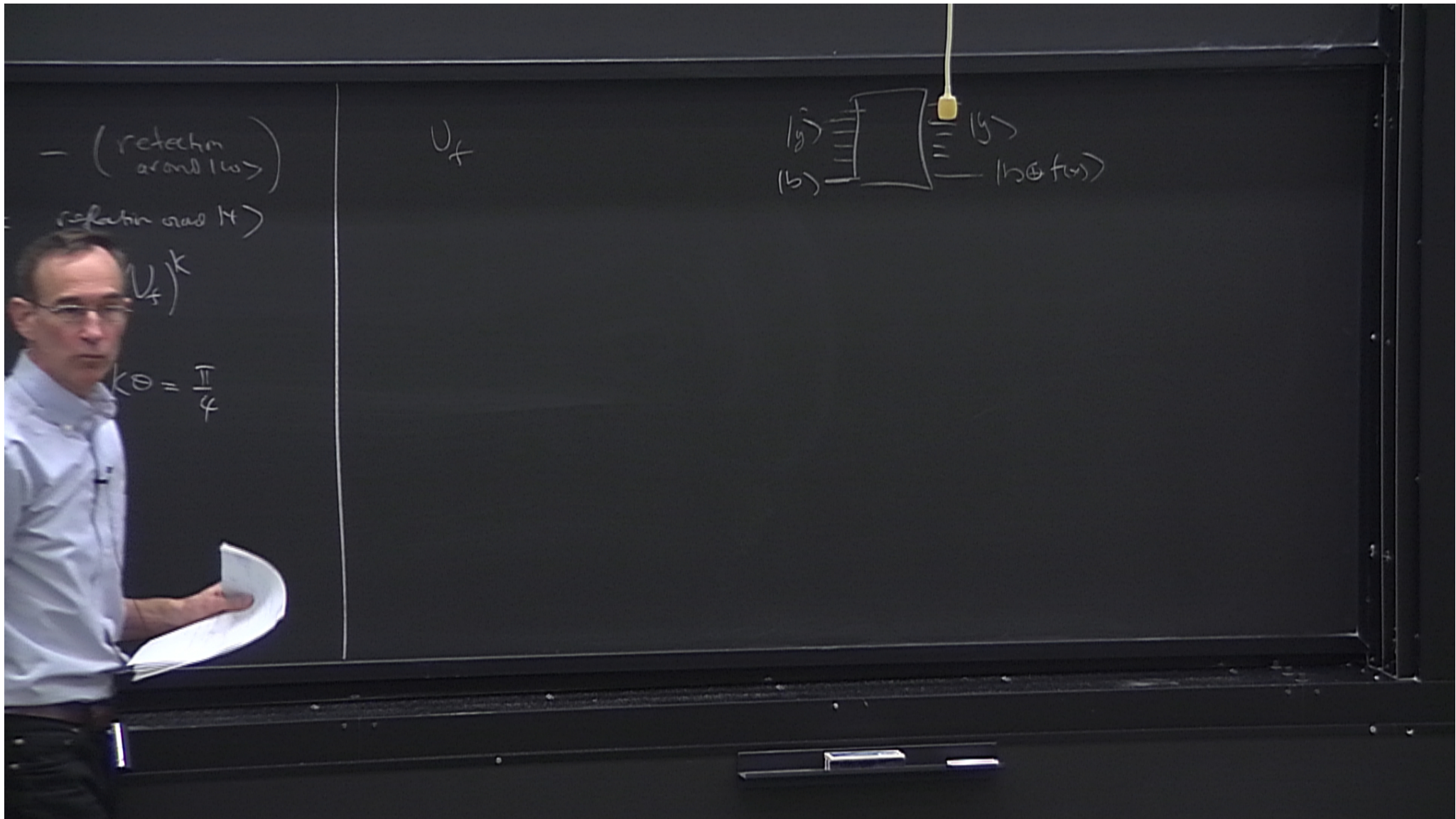
U_f : - (rotation around $|w\rangle$)

V = reflection across H

$$(VU_f)^k$$

$$k\theta = \frac{\pi}{4}$$





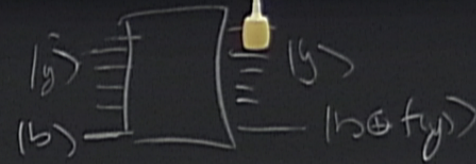
— (retachn
around 1ω)

reflctn over $1\hbar$

$$(U_{\pm})^k$$

$$k\theta = \frac{\pi}{4}$$

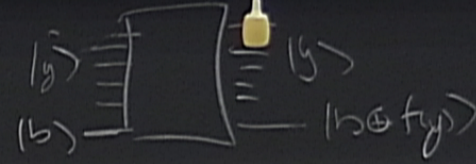
$U_{\pm}$



- (retachn
 around $1w$)
 reflectin over $1t$

$$f(y) = 0 \text{ for } y \neq w$$

$$1 \text{ for } y = w$$



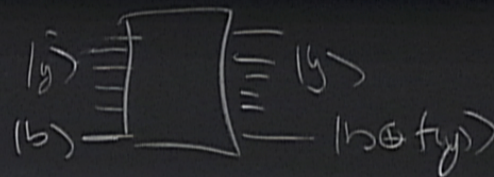
$f = - (\text{reflection around } |w\rangle)$
 $= \text{reflection around } |w\rangle$

$$(V_U)^k$$

$$k\theta = \frac{\pi}{4}$$

$$f(y) = 0 \quad \text{for } y \neq w$$

$$1 \quad \text{for } y = w$$



$$|w\rangle$$

$$|w\rangle = \frac{1}{\sqrt{N-1}} \sum$$

— (retachn
around 1ω)

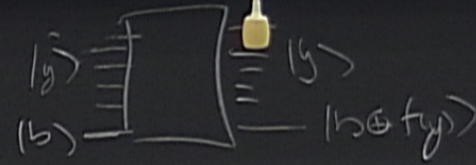
reflexion was 14)

$$(W_4)^k$$

$$k\theta = \frac{\pi}{4}$$

$$f(y) = 0 \quad \text{for } y \neq \omega$$

$$1 \quad \text{for } y = \omega$$



$$|\psi_{\text{good}}\rangle = |\omega\rangle$$

$$|\psi_{\text{bad}}\rangle = \frac{1}{\sqrt{N-1}} \sum_{x \neq \omega} |x\rangle$$

— (retechn
around $|\omega\rangle$)

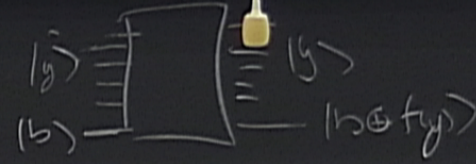
refactor as H

$$(W_4)^k$$

$$k\theta =$$

$$f(y) = 0 \text{ for } y \neq \omega$$

$$1 \text{ for } y = \omega$$



$$|\psi_{\text{good}}\rangle = |\omega\rangle$$

$$|\psi_{\text{bad}}\rangle = \frac{1}{\sqrt{N-1}} \sum_{x \neq \omega} |x\rangle$$

$$|b\rangle \xrightarrow{H} \sum_{x=0}^{N-1} |x\rangle$$

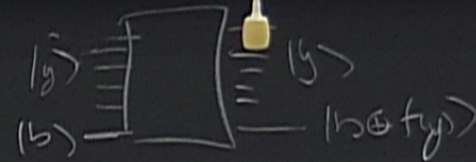
— (retechn
around $|\omega\rangle$)

refactor as H

$(W_{\frac{1}{2}})^k$

$k=0$

$$f(y) = \begin{cases} 0 & \text{for } y \neq \omega \\ 1 & \text{for } y = \omega \end{cases}$$



$$|\psi_{\text{good}}\rangle = |\omega\rangle$$

$$|\psi_{\text{bad}}\rangle = \frac{1}{\sqrt{N-1}} \sum_{x \neq \omega} |x\rangle$$

$$|0\rangle \xrightarrow{H} \sum_{x=0}^{N-1} |x\rangle = \frac{1}{\sqrt{N}} |\omega\rangle + \sqrt{\frac{N-1}{N}} |\psi_{\text{bad}}\rangle$$

— (retachn
around $|w\rangle$)

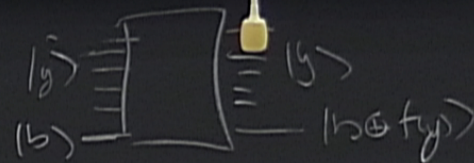
refactor as H)

$$(W_{\frac{\pi}{4}})^k$$

$$k\theta = \frac{\pi}{4}$$

$$f(y) = 0 \text{ for } y \neq w$$

$$1 \text{ for } y = w$$



$$|\psi_{\text{good}}\rangle = |w\rangle$$

$$|\psi_{\text{bad}}\rangle = \frac{1}{\sqrt{N-1}} \sum_{x \neq w} |x\rangle$$

$$|0\rangle \xrightarrow{H} \sum_{x=0}^{N-1} |x\rangle = \frac{1}{\sqrt{N}} |w\rangle + \sqrt{\frac{N-1}{N}} |\psi_{\text{bad}}\rangle = |X\rangle$$

— (retechn
around $|\omega\rangle$)

refactor as H)

(V

$$f(y) = \begin{cases} 0 & \text{for } y \neq \omega \\ 1 & \text{for } y = \omega \end{cases}$$

$$|y\rangle \xrightarrow{U_f} |y \oplus f(y)\rangle$$

$$|\psi_{\text{good}}\rangle = |\omega\rangle$$

$$|\psi_{\text{bad}}\rangle = \frac{1}{\sqrt{N-1}} \sum_{x \neq \omega} |x\rangle$$

$$|0\rangle \xrightarrow{H} \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle = \frac{1}{\sqrt{N}} |\omega\rangle + \sqrt{\frac{N-1}{N}} |\psi_{\text{bad}}\rangle = |K\rangle$$

— (retachn
around $|\omega\rangle$)

reflexion around $|\omega\rangle$

$$(U_f)^k$$

$$k\theta = \frac{\pi}{4}$$

$$f(y) = \begin{cases} 0 & \text{for } y \neq \omega \\ 1 & \text{for } y = \omega \end{cases} \quad \begin{array}{c} |y\rangle \\ \vdots \\ |b\rangle \end{array} \xrightarrow{U_f} \begin{array}{c} |y\rangle \\ \vdots \\ |b \oplus f(y)\rangle \end{array}$$

$$|\psi_{\text{good}}\rangle = |\omega\rangle$$

$$|\psi_{\text{bad}}\rangle = \frac{1}{\sqrt{N-1}} \sum_{x \neq \omega} |x\rangle$$

$$|0\rangle \xrightarrow{H} \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle = \frac{1}{\sqrt{N}} |\omega\rangle + \sqrt{\frac{N-1}{N}} |\psi_{\text{bad}}\rangle = |K\rangle$$

$$U_f |K\rangle = \frac{1}{\sqrt{N}} |\omega\rangle |1\rangle + \sqrt{\frac{N-1}{N}} |\psi_{\text{bad}}\rangle |0\rangle$$

— (retachn
around $|w\rangle$)

reflexion over H

$$(U_f)^k$$

$$k\theta = \frac{\pi}{4}$$

$$f(y) = \begin{cases} 0 & \text{for } y \neq w \\ 1 & \text{for } y = w \end{cases} \quad \begin{array}{c} |g\rangle \\ |b\rangle \end{array} \xrightarrow{U_f} \begin{array}{c} |g\rangle \\ |b \oplus f(y)\rangle \end{array}$$

$$|\psi_{\text{good}}\rangle = |w\rangle$$

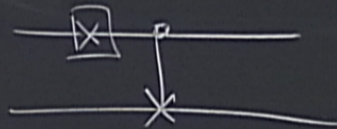
$$|\psi_{\text{bad}}\rangle = \frac{1}{\sqrt{N-1}} \sum_{x \neq w} |x\rangle$$

$$|0\rangle \xrightarrow{H} \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle = \frac{1}{\sqrt{N}} |w\rangle + \sqrt{\frac{N-1}{N}} |\psi_{\text{bad}}\rangle = |X\rangle$$

$$U_f |X\rangle = \frac{1}{\sqrt{N}} |w\rangle |1\rangle + \sqrt{\frac{N-1}{N}} |\psi_{\text{bad}}\rangle |0\rangle$$

$\hookrightarrow \sum_x F(|x\rangle) |b\rangle$

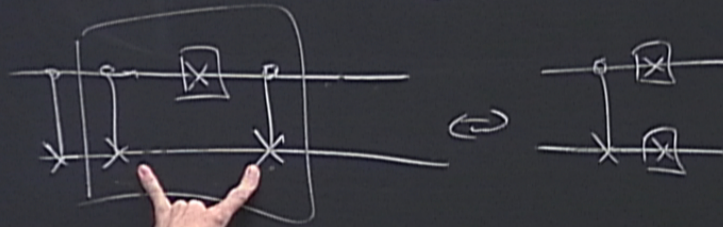
3) Phase kick back



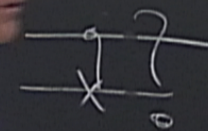
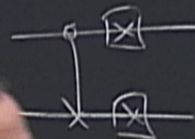
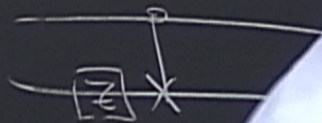
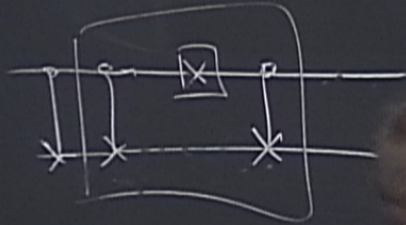
3) Phase kick back



3) Phase kick back



3) Phase kick back



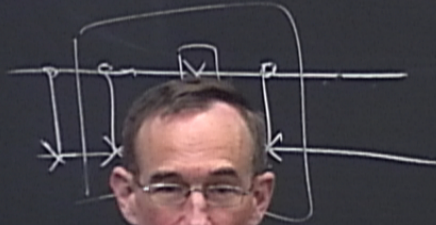
$$(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$\rightarrow Z_2(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

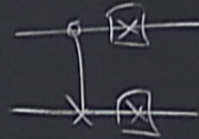
$$\rightarrow (|10\rangle + |11\rangle)(|0\rangle - |1\rangle)$$

$$|00\rangle + |01\rangle + |10\rangle - |11\rangle$$

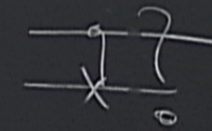
3) Phase kick back



↔



↔



$$(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

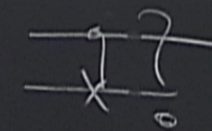
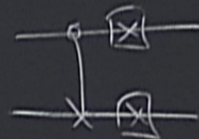
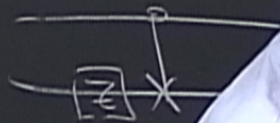
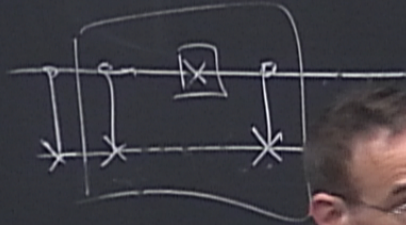
$$\rightarrow Z_2(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$\rightarrow (|0\rangle + |1\rangle)(|0\rangle - |1\rangle)$$

$$|00\rangle + |01\rangle + |10\rangle - |11\rangle$$

$$|00\rangle + |01\rangle + |11\rangle - |10\rangle$$

3) Phase kick back



$$(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$\rightarrow Z_2(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

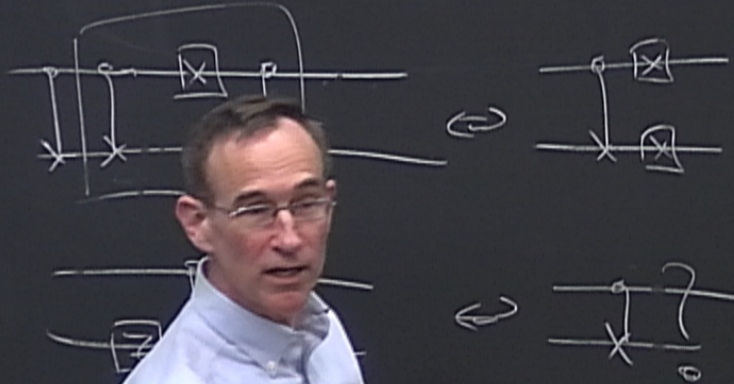
$$\rightarrow (|0\rangle + |1\rangle)(|0\rangle - |1\rangle)$$

$$|00\rangle + |01\rangle - |10\rangle - |11\rangle$$

$$|00\rangle + |01\rangle + |10\rangle - |11\rangle$$

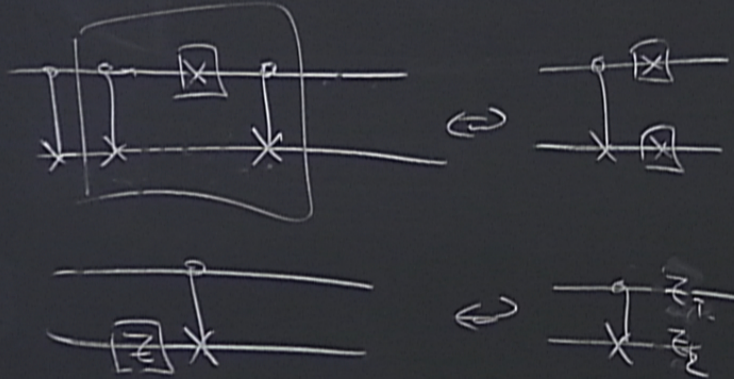
$$Z_1 Z_2$$

3) Phase kick back



$$\begin{aligned}
 & \frac{(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)}{\sqrt{2}} \leftarrow \\
 & \rightarrow Z_2(|0\rangle + |1\rangle)(|0\rangle + |1\rangle) \\
 & \rightarrow (|0\rangle + |1\rangle)(|0\rangle - |1\rangle) \\
 & \quad |00\rangle + |01\rangle - |10\rangle - |11\rangle \\
 & \quad |00\rangle + |01\rangle + |11\rangle - |10\rangle \\
 & Z_1 Z_2(\downarrow)
 \end{aligned}$$

3) Phase kick back



$$(|0\rangle + |1\rangle)(|0\rangle + |1\rangle) \leftarrow$$

$$\rightarrow Z_2(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

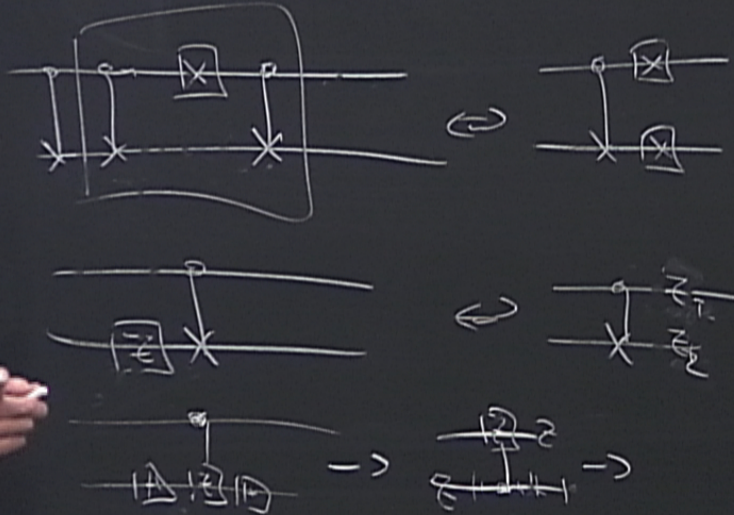
$$\rightarrow (|0\rangle + |1\rangle)(|0\rangle - |1\rangle)$$

$$|00\rangle + |01\rangle - |10\rangle - |11\rangle$$

$$|00\rangle + |01\rangle + |11\rangle - |10\rangle$$

$$Z_1 Z_2(\downarrow)$$

3) Phase kick back



$$(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$\rightarrow Z_2(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

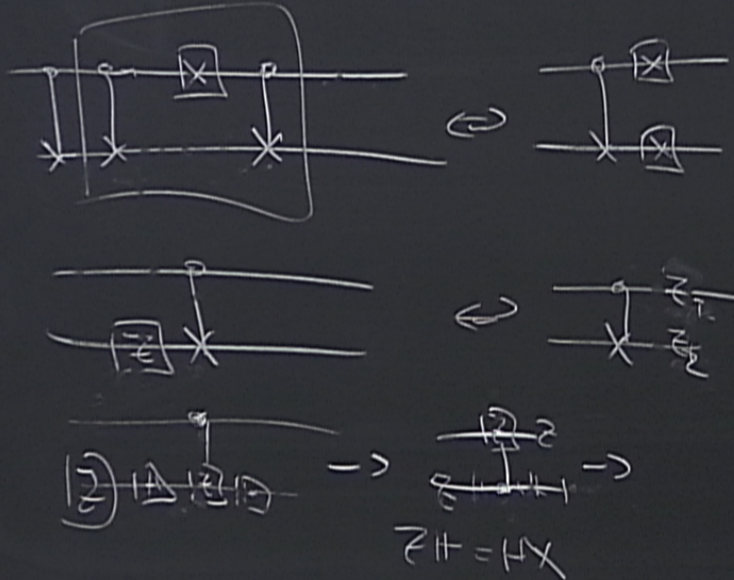
$$\rightarrow (|0\rangle + |1\rangle)(|0\rangle - |1\rangle)$$

$$|00\rangle + |01\rangle + |10\rangle - |11\rangle$$

$$|00\rangle + |01\rangle + |11\rangle - |10\rangle$$

$$Z_1 Z_2(\downarrow)$$

3) Phase kick back



$$(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$\rightarrow Z_2(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$\rightarrow (|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$|00\rangle + |01\rangle + |10\rangle + |11\rangle$$

$$|00\rangle + |01\rangle + |10\rangle + |11\rangle$$

$$Z_1 Z_2$$

$$|0\rangle \xrightarrow{H} \sum_{x=0}^{N-1} |x\rangle = \frac{1}{\sqrt{N}} |w\rangle + \sqrt{\frac{N-1}{N}} |\psi_{\text{BAD}}\rangle = |x\rangle$$

$$U_f |x\rangle |0\rangle = \frac{1}{\sqrt{N}} |w\rangle |1\rangle + \sqrt{\frac{N-1}{N}} |\psi_{\text{BAD}}\rangle |0\rangle$$

$$\rightarrow \sum_x f(x) |x\rangle |b\rangle$$

$$|w\rangle$$

$$|w\rangle|-\rangle \xrightarrow{U_1} (-1)^w |w\rangle|-\rangle$$

$$|\psi_{200}\rangle|-\rangle \xrightarrow{U_+} |\psi_{300}\rangle|-\rangle$$

$$|w\rangle|-\rangle \xrightarrow{U_1} (-1)^w |w\rangle|-\rangle$$

$$|4_{\text{end}}\rangle|-\rangle \xrightarrow{U_4} |4_{\text{end}}\rangle|-\rangle$$

$$|-\rangle \rightarrow |0\rangle - |1\rangle$$

$$|1\rangle - |0\rangle$$



$$|w\rangle |-\rangle \xrightarrow{U_1} (-1) |w\rangle |-\rangle$$

$$|4_{\text{end}}\rangle |-\rangle \xrightarrow{U_4} |4_{\text{end}}\rangle |-\rangle$$

$$|-\rangle \rightarrow |0\rangle - |1\rangle$$

$$|1\rangle - |0\rangle$$

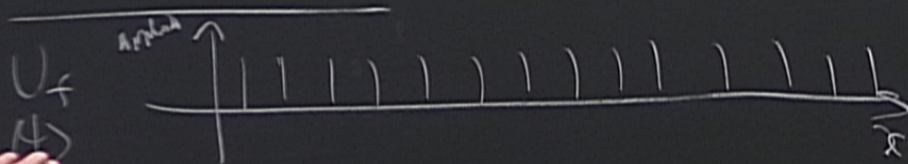


$$|w\rangle \xrightarrow{U_1} (-1) |w\rangle$$

$$|\psi_{\text{2nd}}\rangle \xrightarrow{U_2} |\psi_{\text{3rd}}\rangle$$

$$|-\rangle \rightarrow |0\rangle - |1\rangle$$

$$|1\rangle - |0\rangle$$

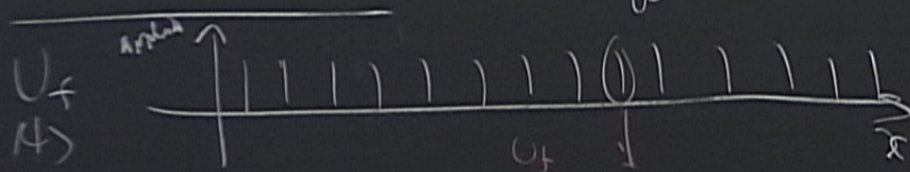


$$|w\rangle \rightarrow \xrightarrow{U_1} (-1) |w\rangle \rightarrow$$

$$|4_{200}\rangle \rightarrow \xrightarrow{U_2} \quad |4_{300}\rangle \rightarrow$$

$$|-\rangle \rightarrow |0\rangle - |1\rangle$$

$$|1\rangle - |0\rangle$$



$$\frac{1}{\sqrt{2}}$$

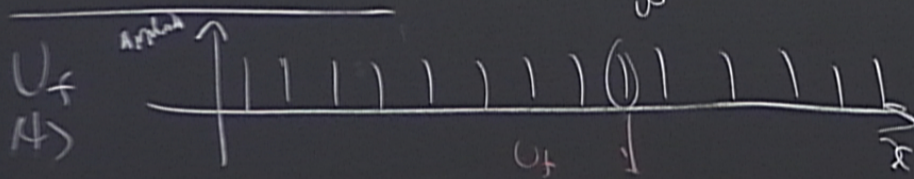
$$\frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle \langle x|$$

$$|w\rangle |-\rangle \xrightarrow{U_f} (-1) |w\rangle |-\rangle$$

$$|\psi_{\text{good}}\rangle |-\rangle \xrightarrow{U_f} |\psi_{\text{good}}\rangle |-\rangle$$

$$|-\rangle \rightarrow |0\rangle - |1\rangle$$

$$|1\rangle - |0\rangle$$



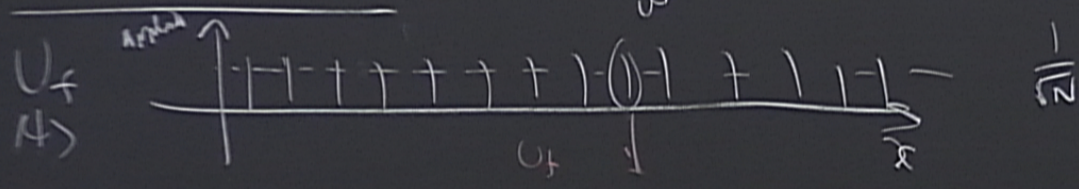
$$\frac{1}{\sqrt{2}}$$

$$U_f = I - 2|w\rangle\langle w|$$

193001-1-1 → 193001-1-1

$$1 \rightarrow 10 \rightarrow -11$$

$$11 \rightarrow -10 \omega$$



$$U_f = I - 2|\omega\rangle\langle\omega|$$

$$\begin{array}{c} \text{---} \\ |2\rangle \text{---} |1\rangle \text{---} |3\rangle \text{---} |0\rangle \\ \text{---} \end{array} \rightarrow \begin{array}{c} \text{---} \\ |2\rangle \text{---} |1\rangle \text{---} |0\rangle \text{---} \\ \text{---} \end{array}$$

$$V = H U_{01} H$$

$$U_{01} \Rightarrow \begin{array}{l} |x\rangle \rightarrow |1-x\rangle \\ |0\rangle \rightarrow |0\rangle \end{array}$$

$$V = 2|4\rangle\langle 4| - I$$

$$\begin{array}{c} \text{---} \\ |2\rangle \text{---} |1\rangle \text{---} |3\rangle \text{---} |4\rangle \\ \text{---} \end{array} \rightarrow \begin{array}{c} \text{---} \\ |2\rangle \text{---} |3\rangle \\ \text{---} \end{array} \rightarrow$$

$$V = \hbar U_{01} \hbar$$

$$U_{01} \Rightarrow \begin{array}{l} |x\rangle \rightarrow |1-x\rangle \\ |0\rangle \rightarrow |0\rangle \end{array}$$

$$V = 2|4\rangle\langle 3| - \hat{I}$$

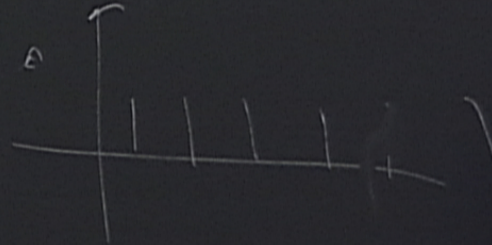
$$\omega = 3$$

$$|4\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle + |3\rangle + |4\rangle)$$

$$U_3 |4\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle - |3\rangle + |4\rangle)$$

$$V U_3 |4\rangle = V \left(\frac{1}{\sqrt{2}} |4\rangle - 2|3\rangle \right)$$

$$= \frac{1}{\sqrt{2}} |4\rangle - 2^{(?) } |3\rangle$$



$$\begin{array}{c} \text{---} \\ |2\rangle \text{---} |1\rangle \text{---} |3\rangle \text{---} |4\rangle \\ \text{---} \end{array} \rightarrow \begin{array}{c} \text{---} \\ |2\rangle \text{---} |3\rangle \\ \text{---} \end{array} \rightarrow$$

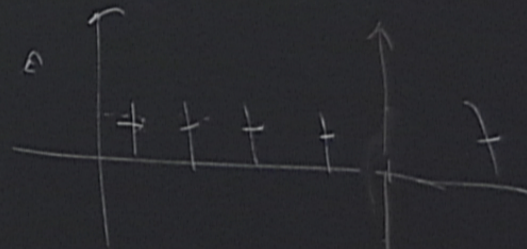
$$V = \hbar U_{01} \hbar$$

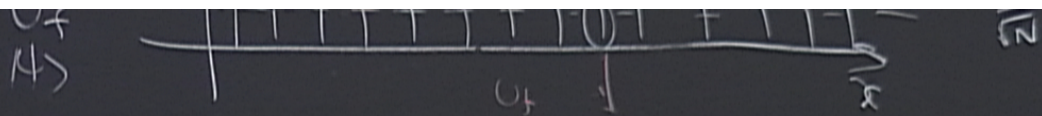
$$U_{01} \Rightarrow \begin{array}{l} |x\rangle \rightarrow |1-x\rangle \\ |0\rangle \rightarrow |0\rangle \end{array}$$

$$V = 2\hbar \left(|1\rangle \langle 1| - |3\rangle \langle 3| \right) \quad \omega = 3 \quad |4\rangle = \frac{1}{2}(|1\rangle + |2\rangle + |3\rangle + |4\rangle)$$

$$U_3 |4\rangle = |1\rangle + |2\rangle - |3\rangle - |4\rangle$$

$$V U_3 |4\rangle = 2(|4\rangle - |3\rangle)$$





$$U_f = \underline{I - 2|\omega\rangle\langle\omega|}$$

$$U_f |4\rangle =$$

$$H = H_X$$

$$V = H U_{01} H$$

$$U_{01} \Rightarrow \begin{matrix} |x\rangle \rightarrow |1-x\rangle \\ |0\rangle \rightarrow |0\rangle \end{matrix}$$

$$V = 2|4\rangle\langle 4| - \mathbb{I}$$

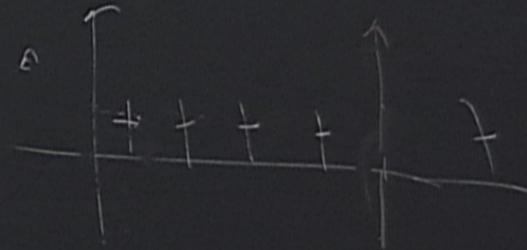
$$\omega = 3$$

$$|4\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle + |3\rangle + |4\rangle)$$

$$U_3 |4\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle - |3\rangle - |4\rangle)$$

$$V U_3 |4\rangle = V \left(\frac{1}{\sqrt{2}} |4\rangle - 2|3\rangle \right)$$

$$= \frac{1}{\sqrt{2}} |4\rangle - 2^{(?) } |3\rangle$$



$$U_3 |4\rangle$$

$$U_f |4\rangle = (I - 2|w\rangle\langle w|) |4\rangle = |4\rangle - \frac{2}{\sqrt{N}} |w\rangle$$

$$V |4\rangle = (2|4\rangle\langle 4| - I) |w\rangle = \frac{2}{\sqrt{N}} |4\rangle - |w\rangle$$

$$V |4\rangle = |4\rangle$$

$$U_f |w\rangle = -|w\rangle$$

$$U_f |4\rangle = (I - 2|w\rangle\langle w|) |4\rangle = |4\rangle - \frac{2}{\sqrt{N}} |w\rangle$$

$$U_f |w\rangle = (2|4\rangle\langle 4| - I) |w\rangle = \frac{2}{\sqrt{N}} |4\rangle - |w\rangle$$

$$U_f |4\rangle = |4\rangle$$

$$U_f |w\rangle = -|w\rangle$$

$$U_f |4\rangle = (I - 2|w\rangle\langle w|) |4\rangle = |4\rangle - \frac{2}{\sqrt{N}} |w\rangle$$

$$\sqrt{N} |w\rangle = (2|4\rangle\langle 4| - I) |w\rangle = \frac{2}{\sqrt{N}} |4\rangle - |w\rangle$$

$$|4\rangle = \frac{1}{\sqrt{N}} |w\rangle + \sqrt{\frac{N-1}{N}} |w^\perp\rangle = \sin\theta |w\rangle + \cos\theta |w^\perp\rangle$$

$$U_f |4\rangle = |4\rangle$$

$$U_f |w\rangle = -|w\rangle$$

$$U_f |4\rangle = (I - 2|w\rangle\langle w|) |4\rangle = |4\rangle - \frac{2}{\sqrt{N}} |w\rangle$$

$$V |w\rangle = (2|4\rangle\langle 4| - I) |w\rangle = \frac{2}{\sqrt{N}} |4\rangle - |w\rangle$$

$$|4\rangle = \frac{1}{\sqrt{N}} |w\rangle + \sqrt{\frac{N-1}{N}} |w^\perp\rangle$$

$$U_f = I - 2|w\rangle\langle w| = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$V = 2|4\rangle\langle 4| - I = 2 \begin{pmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$V |4\rangle = |4\rangle$$

$$V |w\rangle = -|w\rangle$$

$$\sin\theta |w\rangle + \cos\theta |w^\perp\rangle$$

$$2\sin^2\theta - 1 =$$

$$U_f |4\rangle = (I - 2|w\rangle\langle w|) |4\rangle = |4\rangle - \frac{2}{\sqrt{N}} |w\rangle$$

$$V |w\rangle = (2|4\rangle\langle 4| - I) |w\rangle = \frac{2}{\sqrt{N}} |4\rangle - |w\rangle$$

$$|4\rangle = \frac{1}{\sqrt{N}} |w\rangle + \sqrt{\frac{N-1}{N}} |w^\perp\rangle$$

$$\sin\theta |w\rangle + \cos\theta |w^\perp\rangle$$

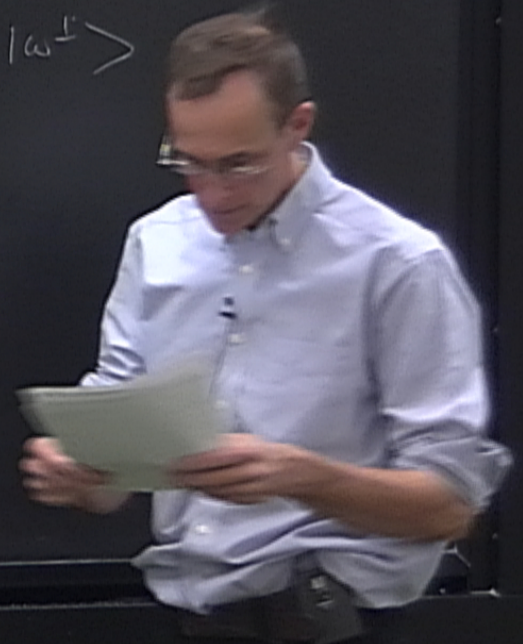
$$2\sin^2\theta - 1 = -\cos 2\theta$$

$$U_f = I - 2|w\rangle\langle w| = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$V = 2|4\rangle\langle 4| - I = 2 \begin{pmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$V|4\rangle = |4\rangle$$

$$V|w\rangle = -|w\rangle$$



$$U_f |4\rangle = (I - 2|w\rangle\langle w|) |4\rangle = |4\rangle - \frac{2}{\sqrt{N}} |w\rangle$$

$$U_f |4\rangle = |4\rangle$$

$$U_f |w\rangle = (2|4\rangle\langle 4| - I) |w\rangle = \frac{2}{\sqrt{N}} |4\rangle - |w\rangle$$

$$U_f |w\rangle = -|w\rangle$$

$$|4\rangle = \frac{1}{\sqrt{N}} |w\rangle + \sqrt{\frac{N-1}{N}} |w^\perp\rangle$$

$$\sin\theta |w\rangle + \cos\theta |w^\perp\rangle$$

$$2\sin^2\theta - 1 = -\cos 2\theta$$

$$U_f = I - 2|w\rangle\langle w| = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U = 2|4\rangle\langle 4| - I = 2 \begin{pmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$U_f |4\rangle = (I - 2|w\rangle\langle w|) |4\rangle = |4\rangle - \frac{2}{\sqrt{N}} |w\rangle$$

$$U_f |4\rangle = |4\rangle$$

$$|w\rangle = (2|4\rangle\langle 4| - I) |w\rangle = \frac{2}{\sqrt{N}} |4\rangle - |w\rangle$$

$$U_f |w\rangle = -|w\rangle$$

$$|4\rangle = \frac{1}{\sqrt{N}} |w\rangle + \sqrt{\frac{N-1}{N}} |w^\perp\rangle$$

$$\sin\theta |w\rangle + \cos\theta |w^\perp\rangle$$

$$2\sin^2\theta - 1 = -\cos 2\theta$$

$$U_f = I - 2|w\rangle\langle w| = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$V = 2|4\rangle\langle 4| - I = 2 \begin{pmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$VU_f = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$U_f |4\rangle = (I - 2|w\rangle\langle w|) |4\rangle = |4\rangle - \frac{2}{\sqrt{N}} |w\rangle$$

$$U_f |4\rangle = |4\rangle$$

$$V |w\rangle = (2|4\rangle\langle 4| - I) |w\rangle = \frac{2}{\sqrt{N}} |4\rangle - |w\rangle$$

$$U_f |w\rangle = -|w\rangle$$

$$|4\rangle = \frac{1}{\sqrt{N}} |w\rangle + \sqrt{\frac{N-1}{N}} |w^\perp\rangle$$

$$\sin\theta |w\rangle + \cos\theta |w^\perp\rangle$$

$$2\sin^2\theta - 1 = -\cos 2\theta$$

$$U_f = I - 2|w\rangle\langle w| = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$2\cos^2\theta - 1 = \cos 2\theta$$

$$V = 2|4\rangle\langle 4| - I = 2 \begin{pmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$VU_f = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$U_f |4\rangle = (I - 2|w\rangle\langle w|) |4\rangle = |4\rangle - \frac{2}{\sqrt{N}} |w\rangle$$

$$U_f |4\rangle = |4\rangle$$

$$V |w\rangle = (2|4\rangle\langle 4| - I) |w\rangle = \frac{2}{\sqrt{N}} |4\rangle - |w\rangle$$

$$U_f |w\rangle = -|w\rangle$$

$$|4\rangle = \frac{1}{\sqrt{N}} |w\rangle + \sqrt{\frac{N-1}{N}} |w^\perp\rangle$$

$$\sin\theta |w\rangle + \cos\theta |w^\perp\rangle$$

$$2\sin^2\theta - 1 = -\cos 2\theta$$

$$U_f = I - 2|w\rangle\langle w| = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$2\cos^2\theta - 1 = \cos 2\theta$$

$$V = 2|4\rangle\langle 4| - I = 2 \begin{pmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$VU_f = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$k\theta = \frac{\pi}{4}$$

$$k = \frac{\pi}{4} \sqrt{N}$$

$$H = H_X$$

$$V = \hat{H} U_{01} \hat{H}$$

$$U_{01} \Rightarrow \begin{matrix} |x\rangle \rightarrow |1-x\rangle \\ |0\rangle \rightarrow |0\rangle \end{matrix}$$

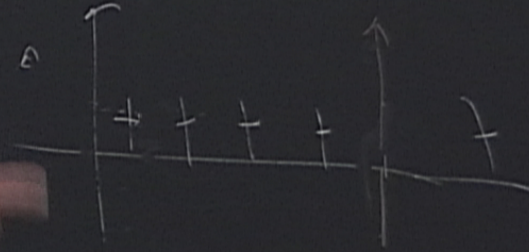
$$V = 2|4\rangle\langle 4| - \hat{I}$$

$$\omega = 3 \quad |4\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle + |3\rangle + |4\rangle)$$

$$U_3 |4\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle + |3\rangle + |4\rangle)$$

$$V U_3 |4\rangle = V \left(\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle + |3\rangle + |4\rangle) \right)$$

$$= \frac{1}{\sqrt{2}} |4\rangle$$



$$U_4 |4\rangle$$

$$V |4\rangle$$

$$|4\rangle$$

$$U_4$$

$$V =$$

$$V U$$

$$H = H_X$$

$$V = \hat{H} U_{01} \hat{H}$$

$$U_{01} \Rightarrow \begin{matrix} |x\rangle \rightarrow -|x\rangle \\ |y\rangle \rightarrow |y\rangle \end{matrix}$$

$$V = 2 \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

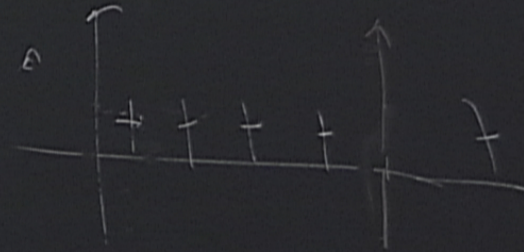
$$\omega = 3$$

$$|4\rangle = \frac{1}{2}(|1\rangle + |2\rangle + |3\rangle + |4\rangle)$$

$$U_3 |4\rangle = \frac{1}{2}(|1\rangle + |2\rangle + |3\rangle + |4\rangle)$$

$$V U_3 |4\rangle = \left(\frac{1}{\sqrt{2}} |4\rangle - 2 |3\rangle \right)$$

$$|3\rangle - 2 |3\rangle$$



$$U_4 |4\rangle$$

$$V |4\rangle$$

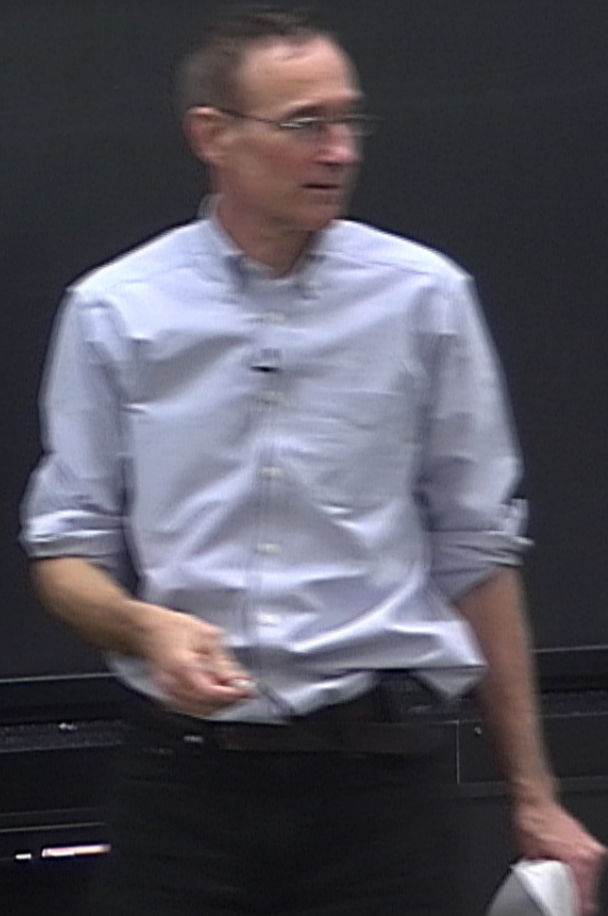
$$|4\rangle$$

$$U_4$$

$$V =$$

$$V U$$

$$e^{-i \frac{\theta}{2} \vec{n} \cdot \vec{\sigma}}$$



$$e^{-i \frac{\theta}{2} \vec{n} \cdot \vec{\sigma}}$$

$$\vec{n} = (\underbrace{n_x, n_y, n_z}_{\text{direction}})$$

