Title: PSI 2015/2016 Quantum Gravity - Lecture 14

Date: Mar 10, 2016 10:15 AM

URL: http://pirsa.org/16030052

Abstract:

Lid LQG

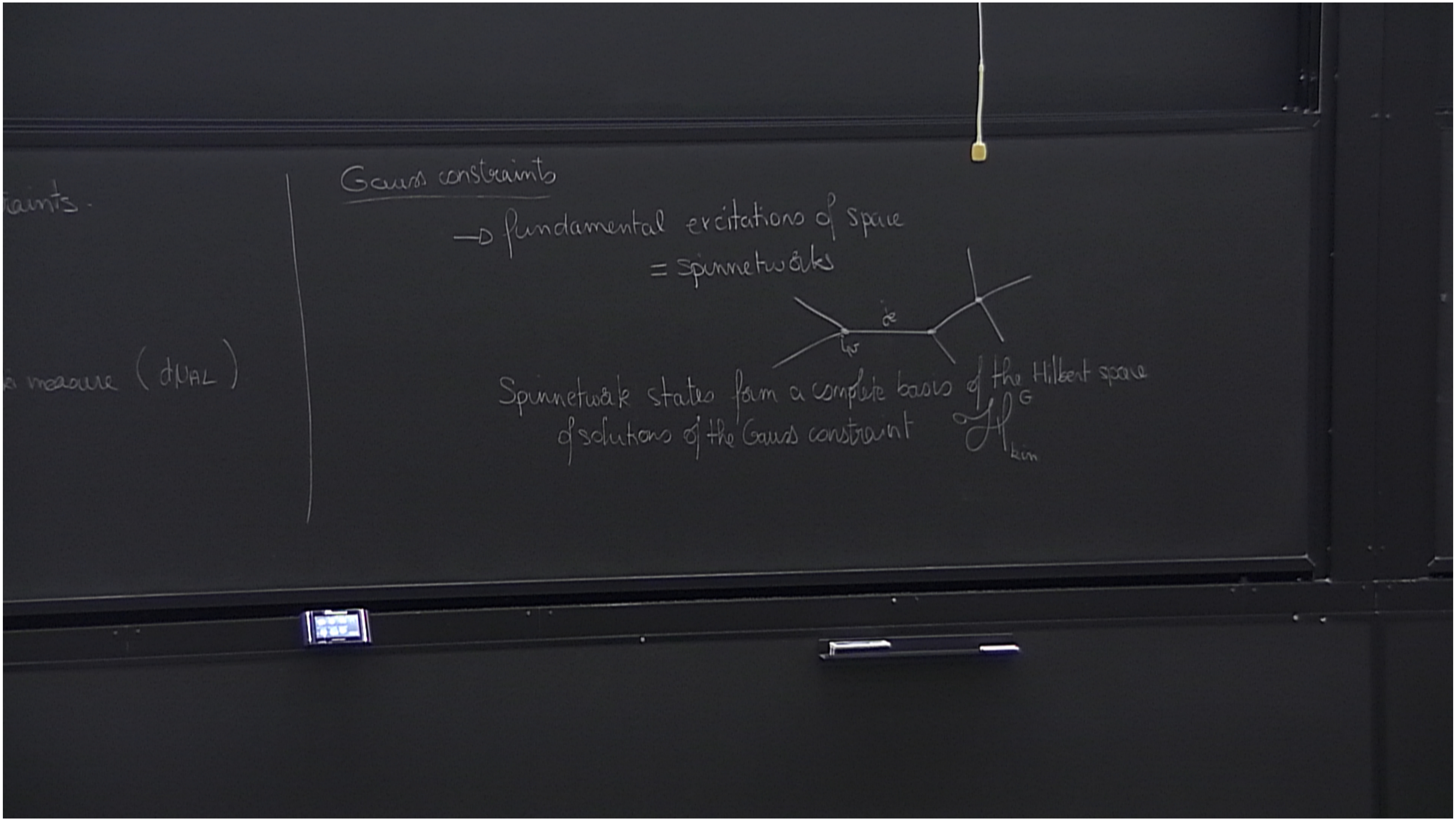
$\to (A^i_a, E^b_j)$. + 1^{st} class constraints.

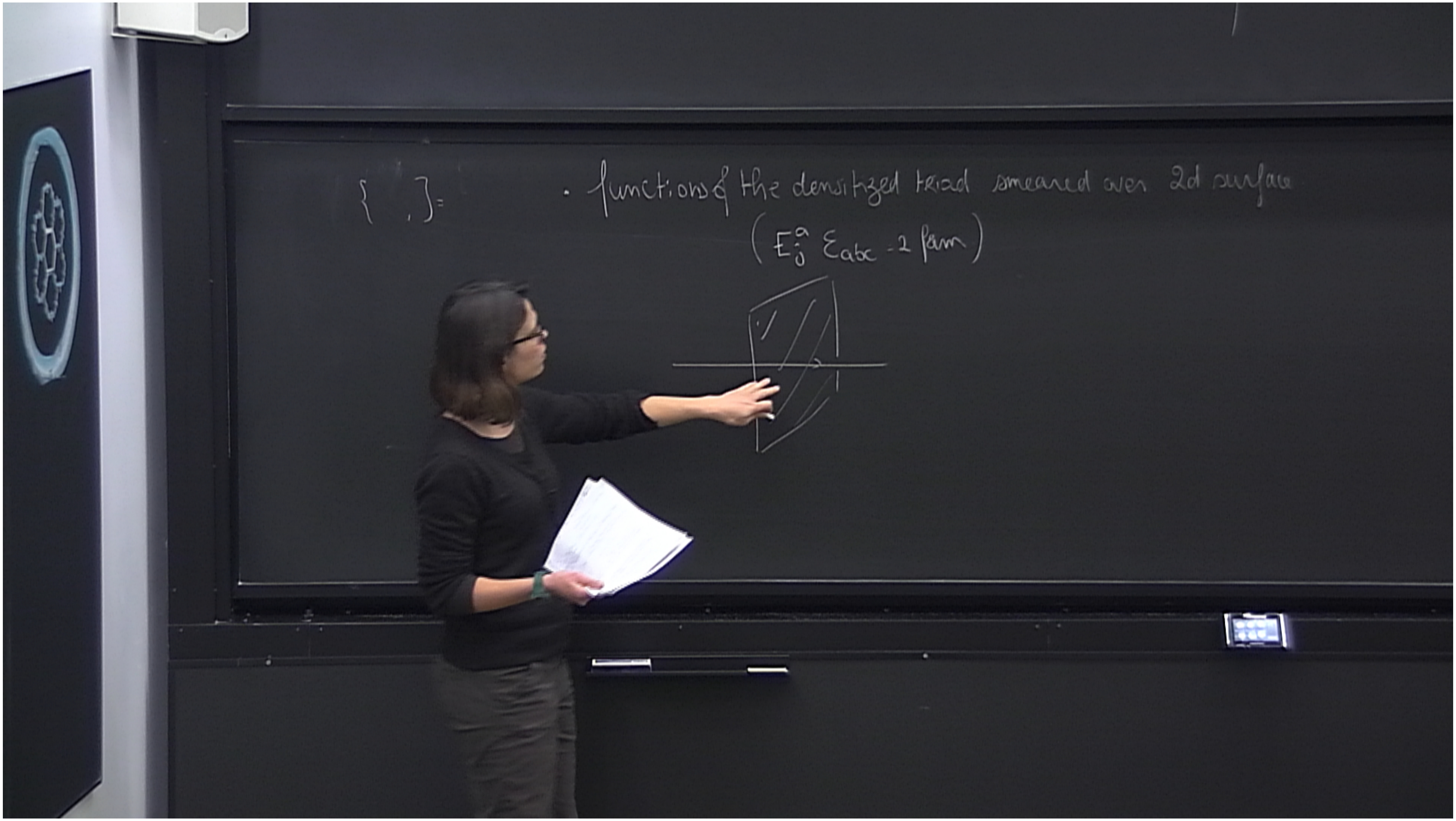
Kinematical Hilbert space

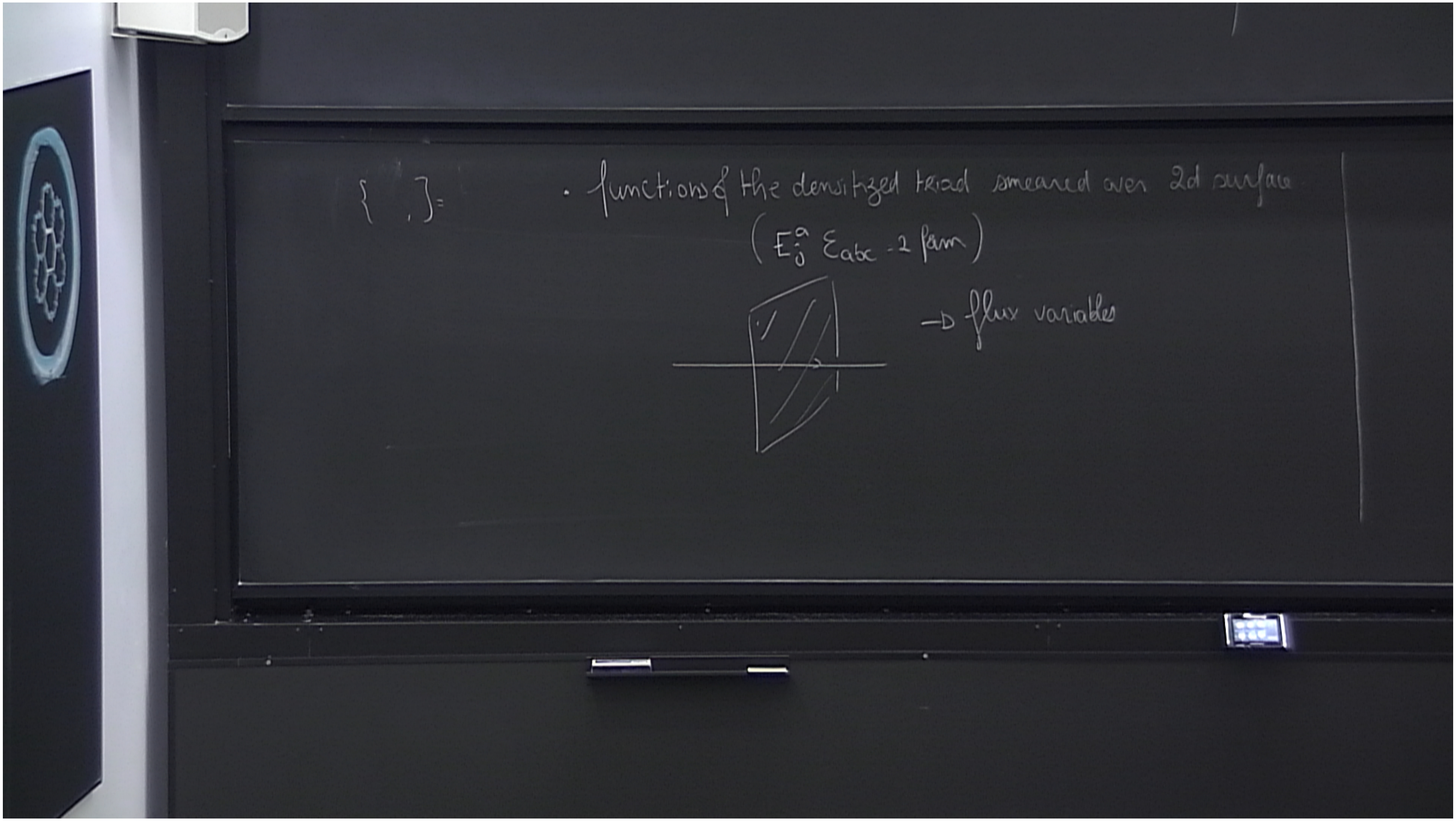
$\to$ holonomies, graphs.

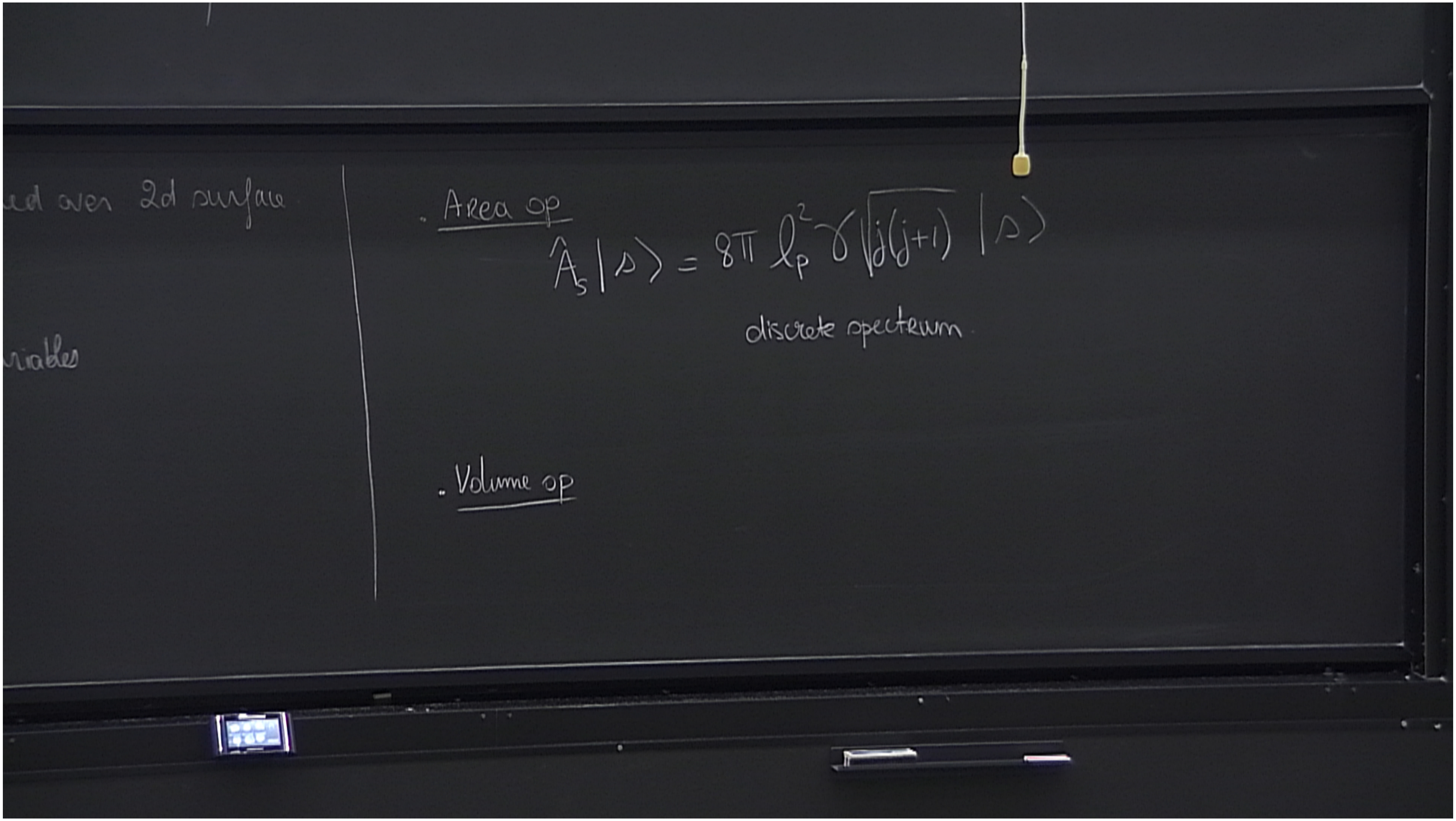
$\to$ Ashtekar-Lewandowski measure ($d\mu_{AL}$)

$\to$ ONB. $|j\, m\, n\rangle$.


measure (dμAL)
Gauss constraints
→ fundamental excitations of space
= spinnetworks
j_e
i_v
Spinnetwork states form a complete basis of the Hilbert space $\mathcal{H}^G_{kin}$
of solutions of the Gauss constraint


{ , }=
· functions of the densitized triad smeared over 2d surface
(E^a_j ε_abc - 2 form)


{ , } =
· functions of the densitized triad smeared over 2d surface
($E^a_j \, \epsilon_{abc}$ - 2 form)
→ flux variables


ed over 2d surface
riables
Area op
$\hat{A}_s |s\rangle = 8\pi l_p^2 \gamma \sqrt{j(j+1)} \, |s\rangle$
discrete spectrum
Volume op

- Volume op

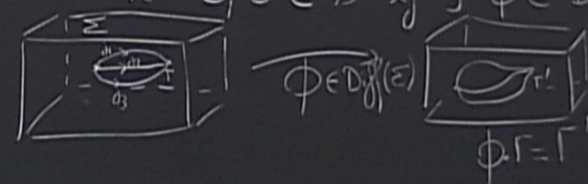
Next step in the Dirac's program

$\hat{V}^a |s\rangle = 0$

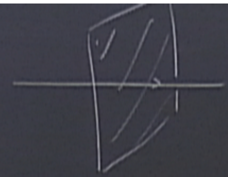
natural basis. s-knots or abstract spinnetworks

↳ an equivalence class s of embedded spinnetworks under the action of diffeo $Diff(\Sigma)$

i.e. $S, S' \in s$ if $\exists\ \phi \in Diffeo(\Sigma)\ /\ S' = \phi \circ S$



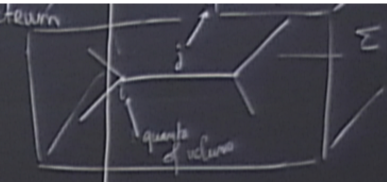
$\{\Gamma\} = \{\Gamma'\}$



→ flux variables

· Volume op

discrete spectrum



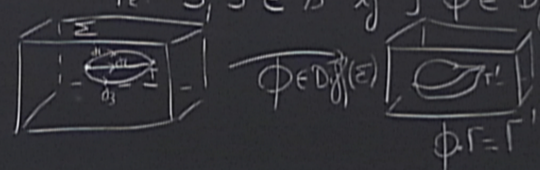
Next step in the Dirac's program

$\hat{V}^a \, |s\rangle = 0$

natural basis: s-knots & abstract spinnetworks

↳ an equivalence class s of embedded spinnetworks under the action of diffeo $Diff(\Sigma)$

i.e. $S, S' \in s$ if $\exists \; \phi \in Diff_0(\Sigma) \, / \, S' = \phi \circ S$ (knot theory)



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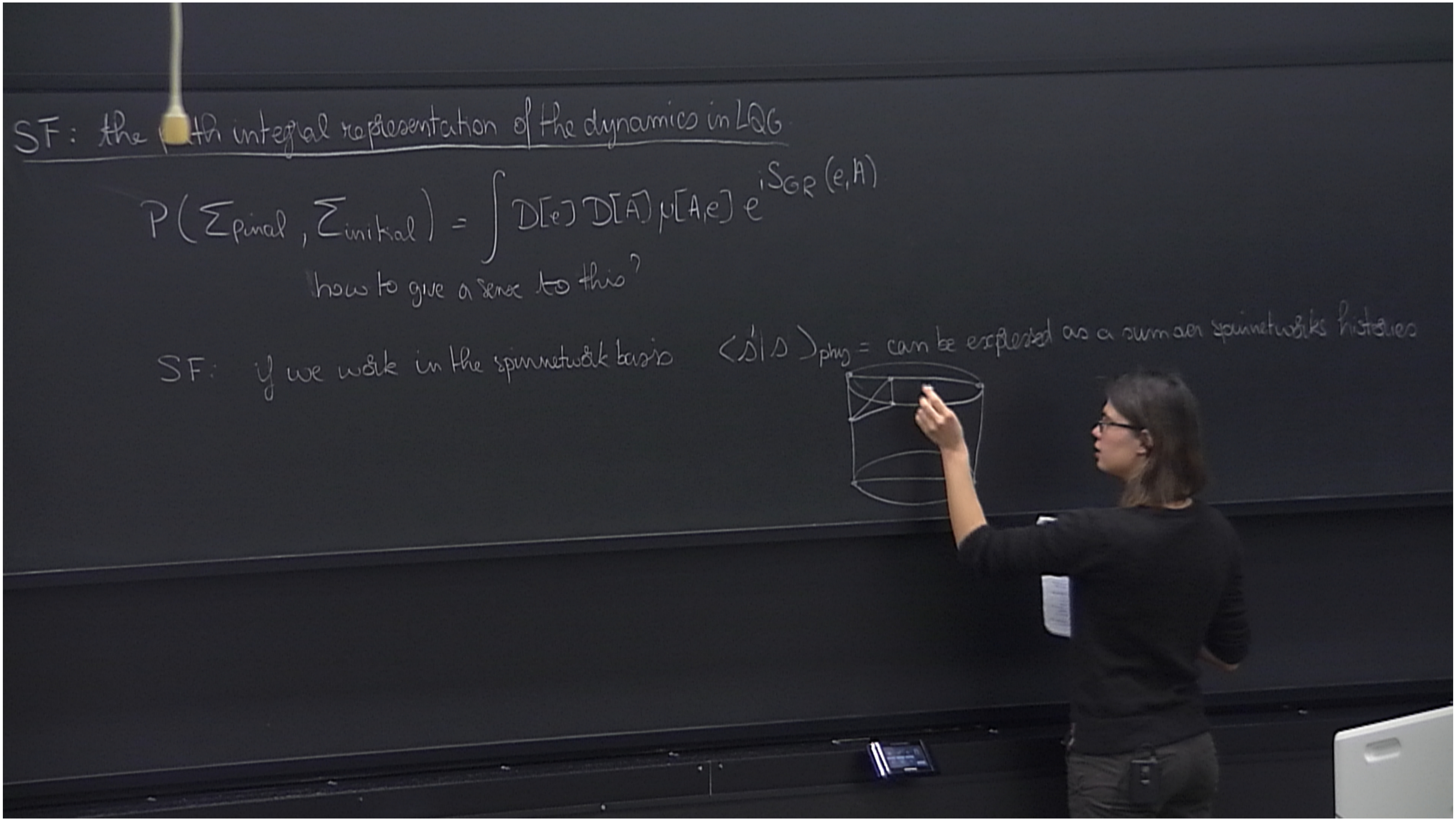
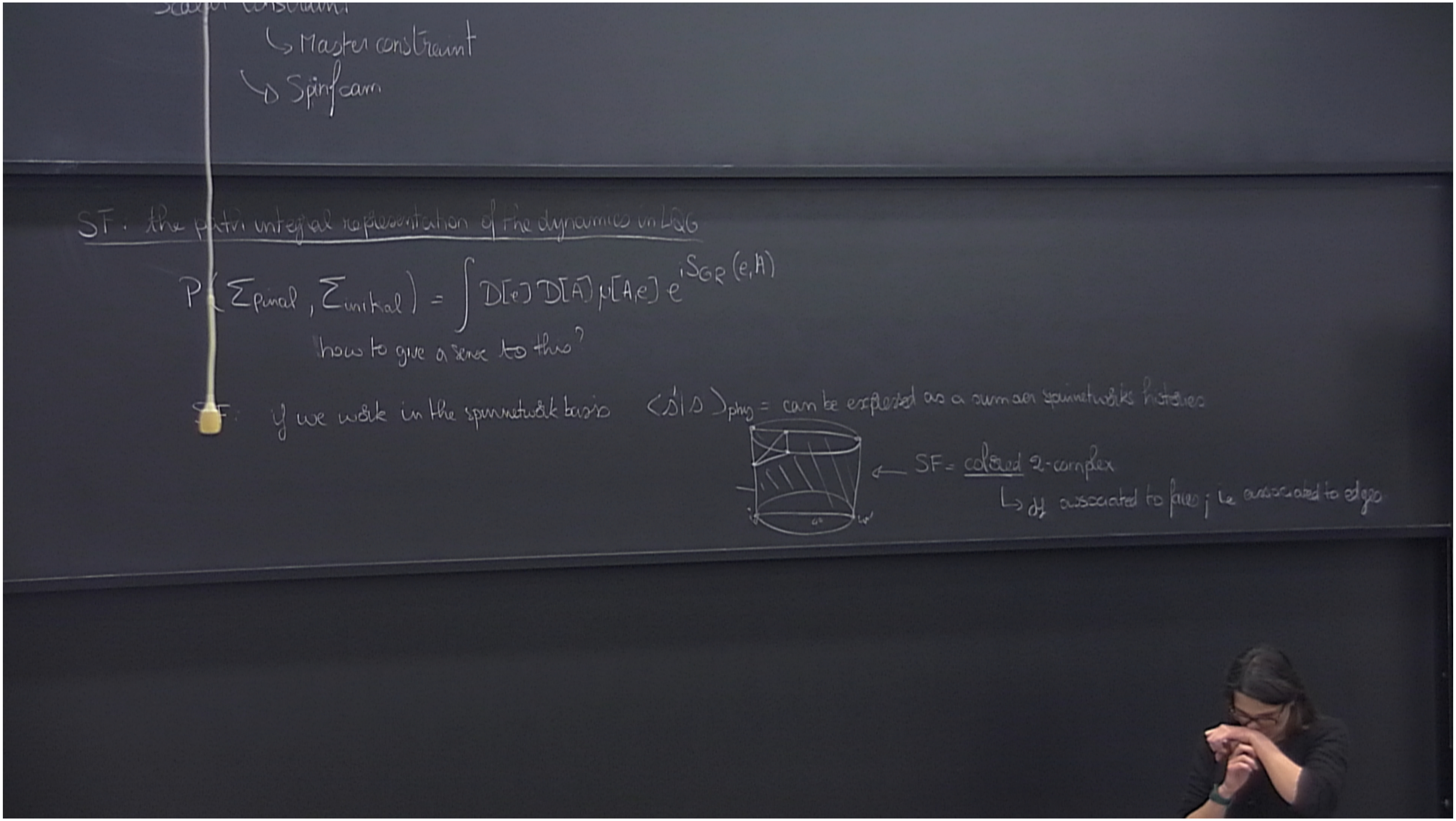

SF: the path integral representation of the dynamics in LQG
$P(\Sigma_{final}, \Sigma_{initial}) = \int D[e]\, D[A]\, \mu[A,e]\, e^{iS_{GR}(e,A)}$
how to give a sense to this?
SF: if we work in the spinnetwork basis $\langle s'|s\rangle_{phys}$ = can be expressed as a sum over spinnetworks histories


↳ Master constraint
↳ Spinfoam
SF: the path integral representation of the dynamics in LQG
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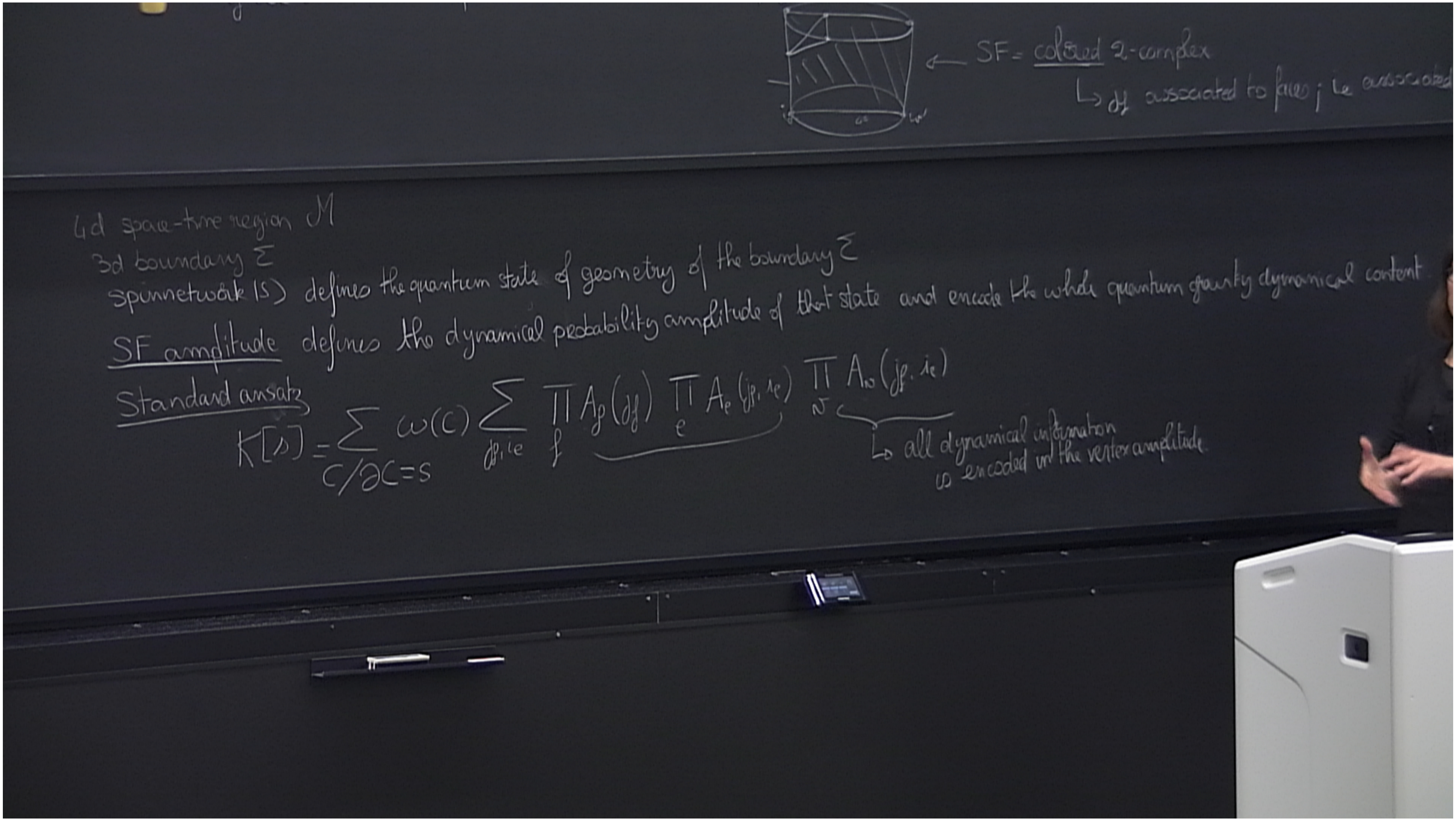
SF: if we work in the spinnetwork basis ⟨s'|s⟩phys = can be expressed as a sum over spinnetworks histories
SF = colored 2-complex
↳ jf associated to faces; ie associated to edges


SF = coloured 2-complex
↳ j_f associated to faces; i_e associated
4d space-time region $\mathcal{M}$
3d boundary Σ
spinnetwork $|s\rangle$ defines the quantum state of geometry of the boundary Σ
SF amplitude defines the dynamical probability amplitude of that state and encode the whole quantum gravity dynamical content
Standard ansatz
$K[s] = \sum_{C/\partial C = s} \omega(C) \sum_{j_f, i_e} \underbrace{\prod_f A_f(j_f) \prod_e A_e(j_f, i_e)} \prod_v \underbrace{A_v(j_f, i_e)}$
↳ all dynamical information is encoded in the vertex amplitude

. 3D Ponzano-Regge model

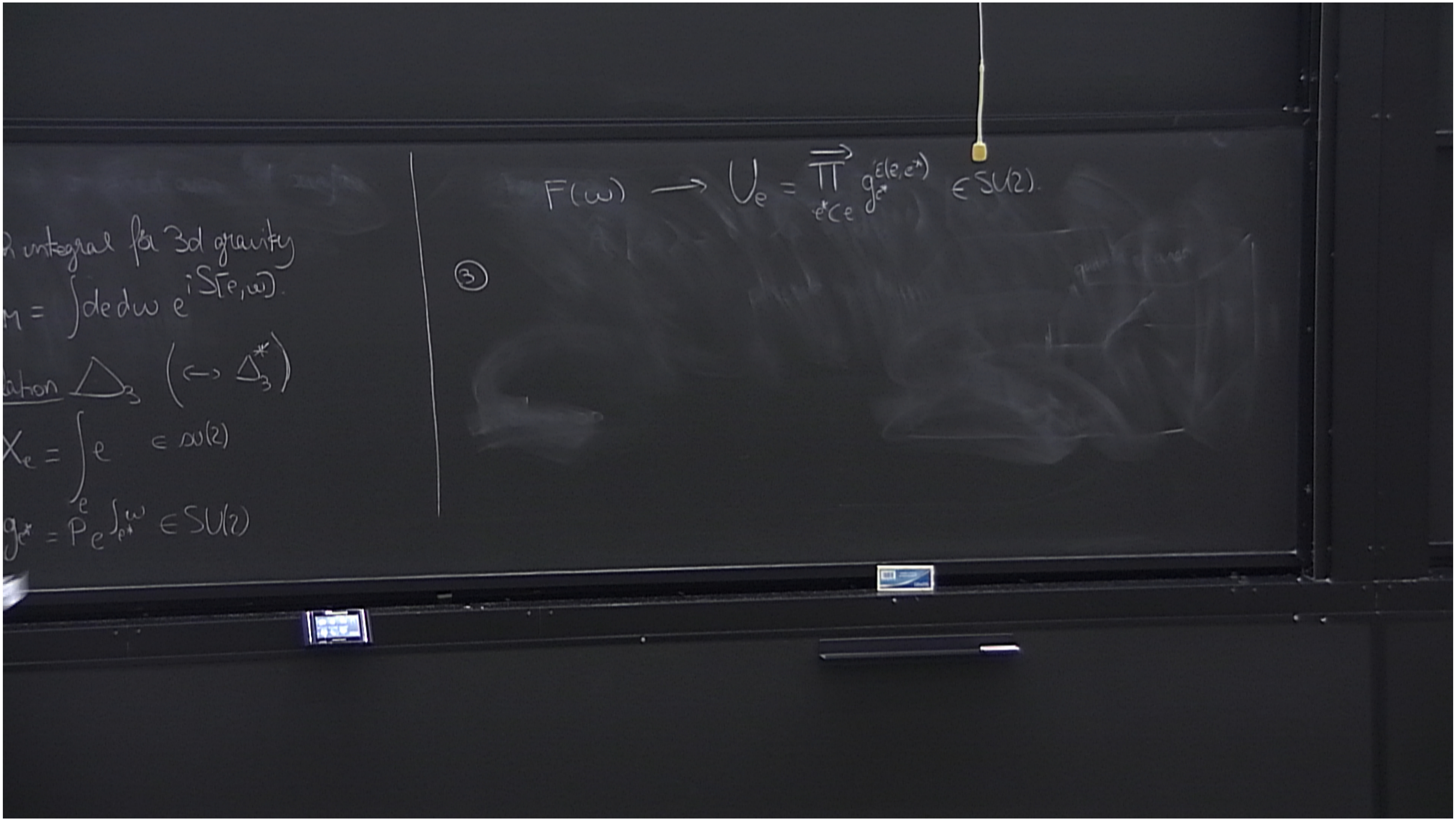
$$S[e,\omega] = \int tr\left(e \wedge F(\omega)\right)$$ $\longrightarrow$ Formal path integral for 3d gravity

is $$Z_M = \int de\, d\omega\; e^{iS[e,\omega]}$$

① discretization of $\mathcal{M}$ $\longrightarrow$ triangulation Δ_3 $\left(\leftrightarrow \Delta_3^*\right)$

② Discretization of variables $\quad e \to X_e = \int_e e \quad \in su(2)$

$$\omega \longrightarrow g_{e^*} = P\, e^{\int_{e^*} \omega} \in SU(2)$$



h integral for 3d gravity
M = ∫ de dω e^{iS[e,ω]}
lation Δ_3 (↔ Δ_3^*)
X_e = ∫_e e ∈ su(2)
g_{e*} = P e^{∫_{e*} ω} ∈ SU(2)
F(ω) → U_e = ∏_{e*⊂e} g_{e*}^{ε(e,e*)} ∈ SU(2).
③

① discretization of $\mathcal{M}$ $\longrightarrow$ triangulation Δ_3 $(\leftrightarrow \Delta_3^*)$

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Discrete path integral integral

$$Z_{\Delta_3} = \left(\prod_{e \in \Delta_3} \int_{su(2)} d^3 X_e \right) \left(\prod_{e^* \in \Delta_3^*} \int_{SU(2)} dg_{e^*} \right) e^{iS[X_e, g_{e^*}]}$$

$$Z_{\Delta_3} = \sum_{\{j_e\}} \prod_e (-1)^{2j_e} d_{j_e} \prod_t \begin{Bmatrix} j_1^t & j_2^t & j_3^t \\ j_4^t & j_5^t & j_6^t \end{Bmatrix}_N$$

4D case $\quad$ Plebanski action $\quad S_{\text{Plebanski}}[B, \omega, \lambda] = \int_{\mathcal{M}} B^{IJ} \wedge F_{IJ}(\omega) - \frac{1}{2} \lambda_{IJKL} B^{KL} \wedge B^{IJ}$

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$\frac{\delta S}{\delta \lambda} \longrightarrow$ constraints . simplicity constraints

$$B^{IJ} = \pm e^I \wedge e^J$$

$$B^{IJ} = \mp \varepsilon^{IJ}{}_{KL}\, e^K \wedge e^L$$

$$\omega \longrightarrow g_{e^*} = P e^{\int_{e^*}^{e} \omega} \in SU(2)$$

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4D case

Plebanski action

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1. discretize the classical theory by putting a cellular decomposition
2. Quantize the topological BF part
3. Impose the simplicity constraints at the quantum level

$\frac{\delta S}{\delta \lambda} \longrightarrow$ constraints simplicity constraints

$$B^{IJ} = \pm e^I \wedge e^J$$

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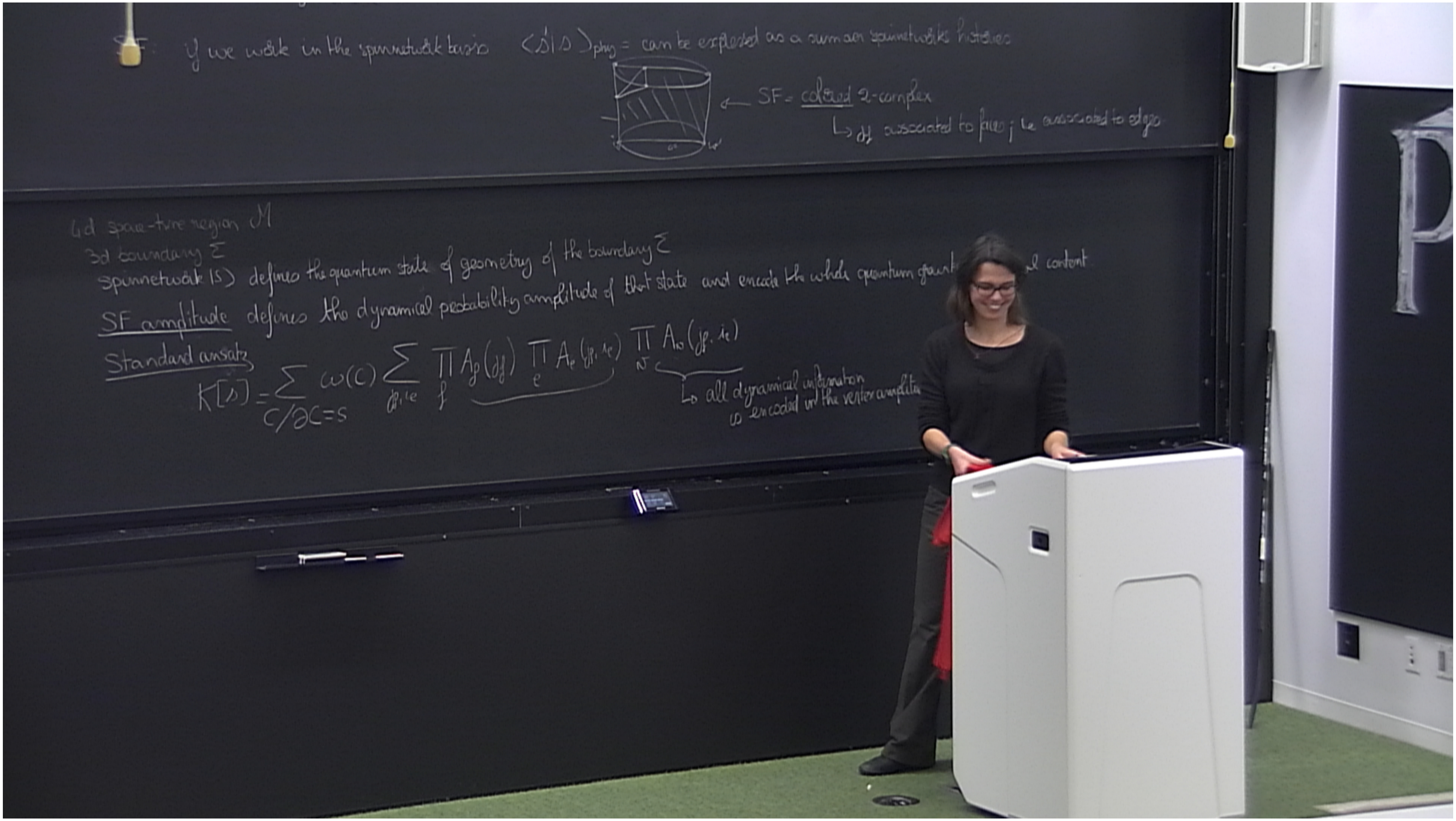
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