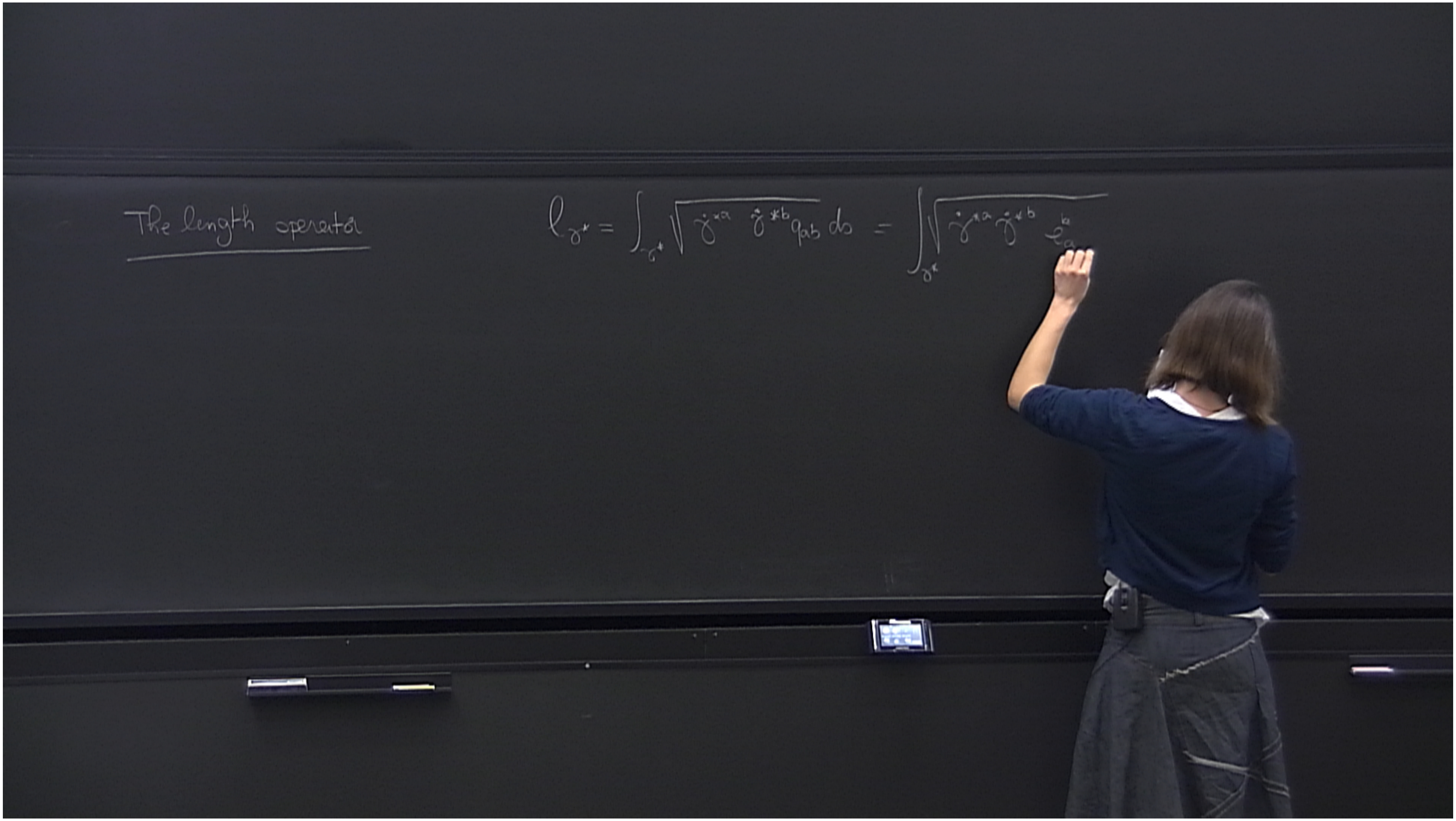
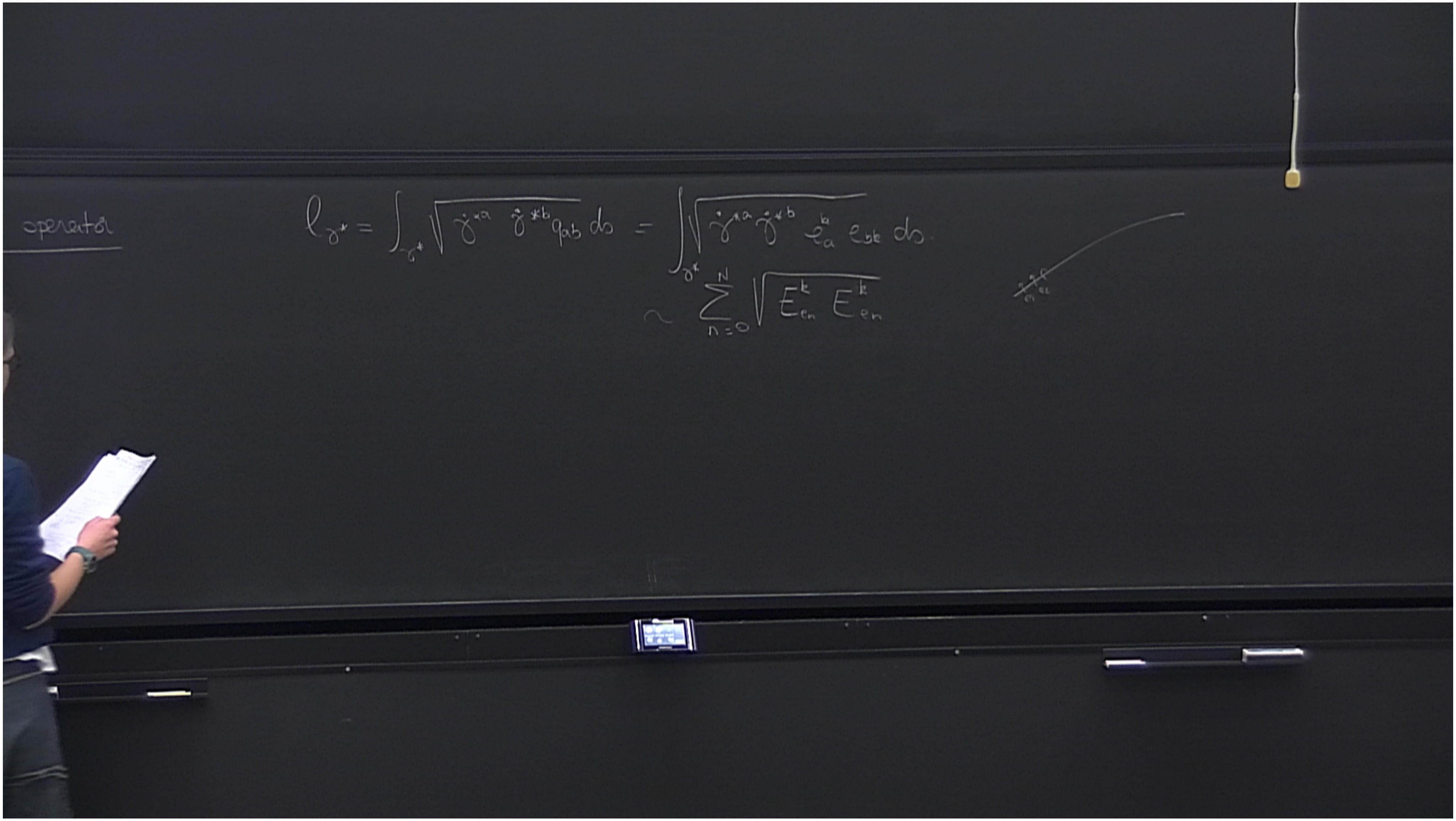
Title: PSI 2015/2016 Quantum Gravity - Lecture 13

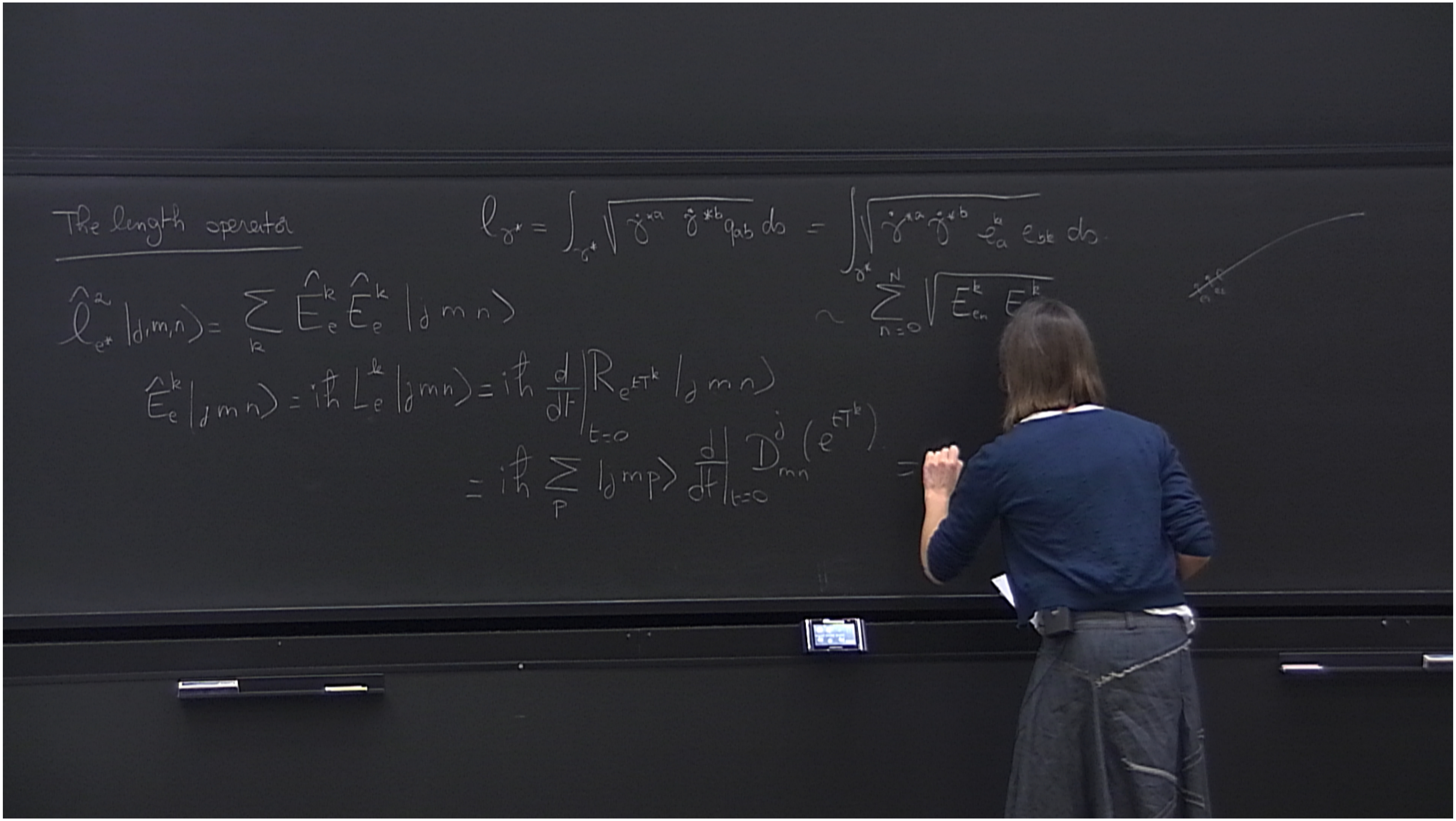
Date: Mar 09, 2016 10:15 AM

URL: http://pirsa.org/16030051

Abstract:


The length operator
$l_{\gamma^*} = \int_{\gamma^*} \sqrt{\dot{\gamma}^{*a} \dot{\gamma}^{*b} q_{ab}}\, ds = \int_{\gamma^*} \sqrt{\dot{\gamma}^{*a} \dot{\gamma}^{*b} e^{b}_{a}}$


operator
$l_{\gamma^*} = \int_{\gamma^*} \sqrt{\dot{\gamma}^{*a} \dot{\gamma}^{*b} q_{ab}}\, ds = \int_{\gamma^*} \sqrt{\dot{\gamma}^{*a} \dot{\gamma}^{*b} e^k_a e_{bk}}\, ds.$
$\sim \sum_{n=0}^{N} \sqrt{E^k_{e_n} E^k_{e_n}}$
e_1 e_2


The length operator
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$\sim \sum_{n=0}^{N} \sqrt{E^k_{e_n} E^k}$
$\hat{l}^2_{e^*} |j,m,n\rangle = \sum_k \hat{E}^k_e \hat{E}^k_e |j\, m\, n\rangle$
$\hat{E}^k_e |j\, m\, n\rangle = i\hbar L^k_e |j\, m\, n\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT^k}} |j\, m\, n\rangle$
$= i\hbar \sum_p |j\, m\, p\rangle \frac{d}{dt}\Big|_{t=0} D^j_{mn}(e^{tT^k}) =$

The length operator

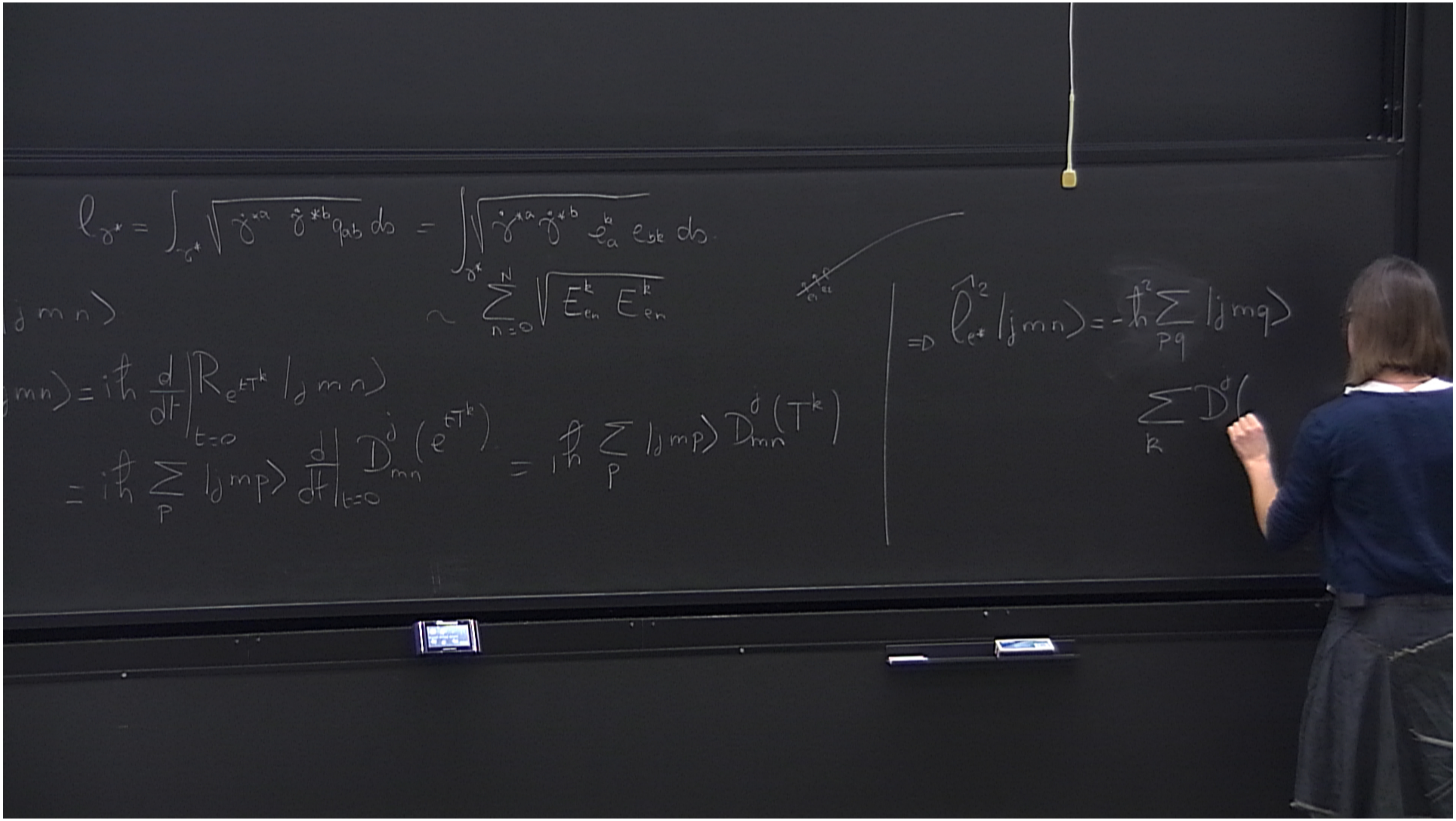
$$\ell_{\gamma^*} = \int_{\gamma^*} \sqrt{\dot\gamma^{*a}\,\dot\gamma^{*b} q_{ab}}\, ds = \int_{\gamma^*} \sqrt{\dot\gamma^{*a}\dot\gamma^{*b}\, e^k_a\, e_{bk}}\, ds$$

$$\sim \sum_{n=0}^{N} \sqrt{E^k_{e_n} E^k_{e_n}}$$

$$\hat\ell^2_{e^*}|j,m,n\rangle = \sum_k \hat E^k_e \hat E^k_e |j\, m\, n\rangle$$

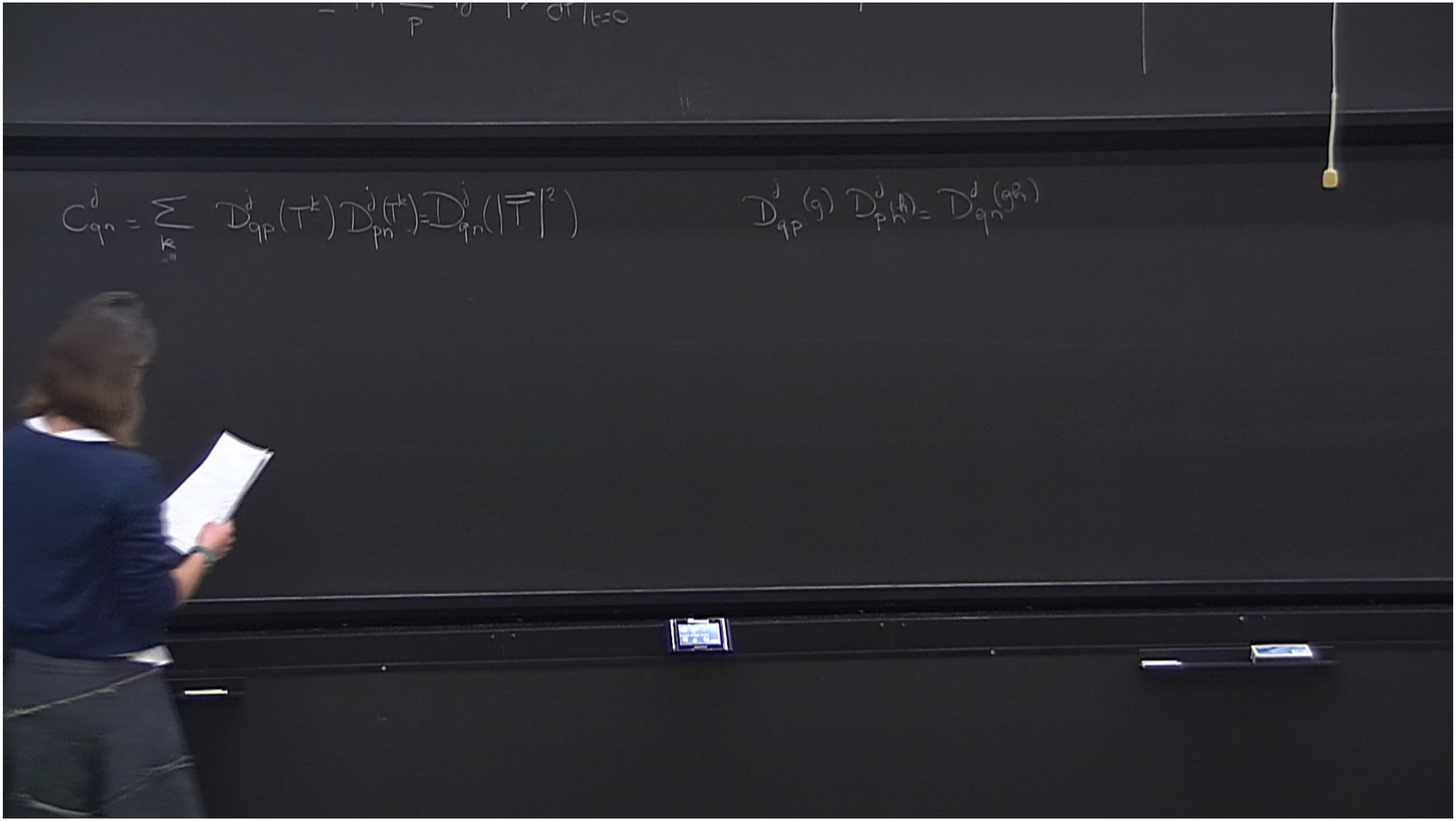
$$\hat E^k_e |j\, m\, n\rangle = i\hbar\, L^k_e |j\, m\, n\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT^k}} |j\, m\, n\rangle$$

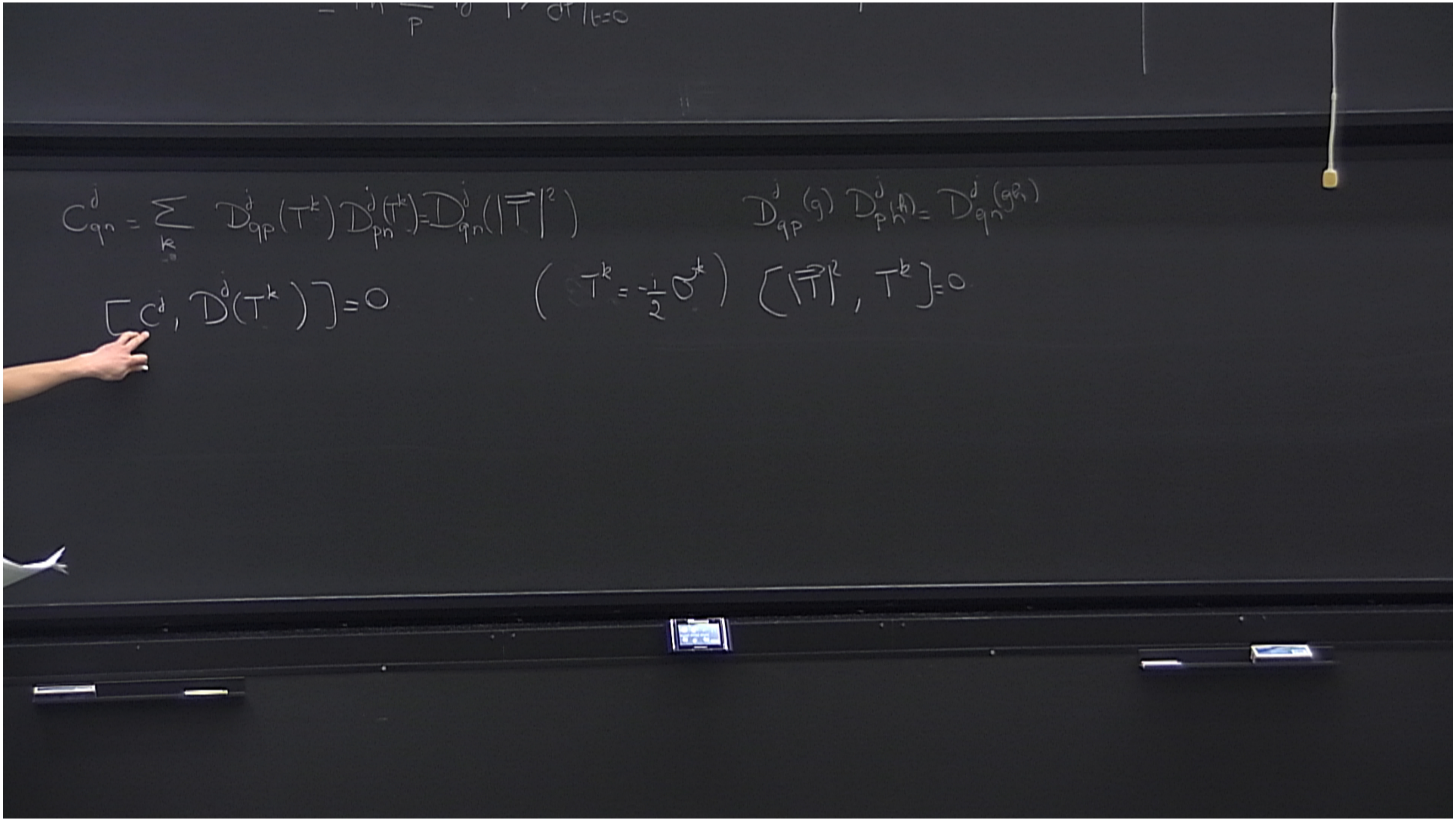
$$= i\hbar \sum_p |j\, m\, p\rangle \frac{d}{dt}\Big|_{t=0} D^j_{mn}(e^{tT^k}) = i\hbar \sum_p |j\, m\, p\rangle D^j_{mn}(T^k)$$

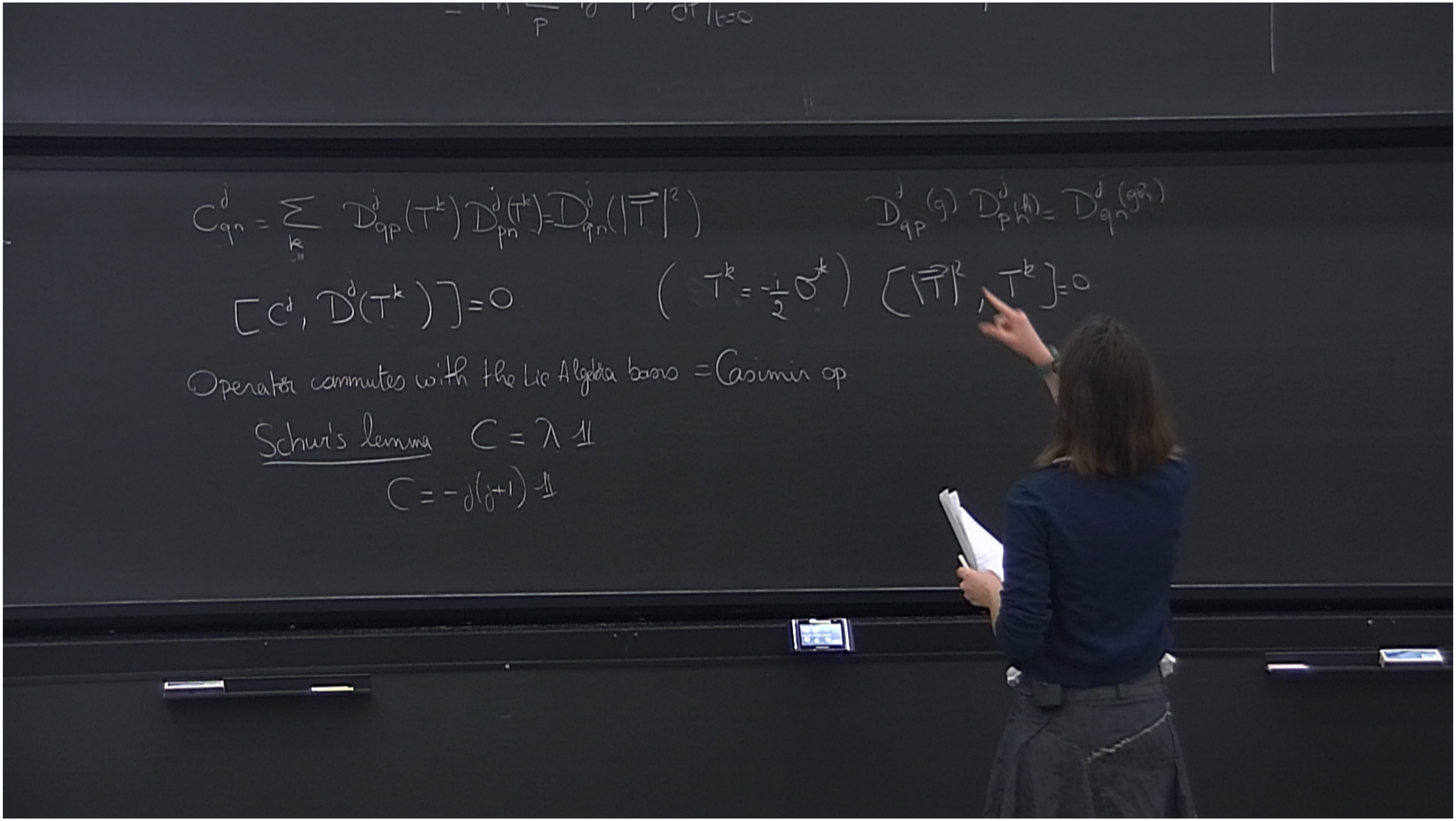


$\ell_{\gamma^*} = \int_{\gamma^*} \sqrt{\dot\gamma^{*a} \dot\gamma^{*b} q_{ab}}\, ds = \int_{\gamma^*} \sqrt{\dot\gamma^{*a} \dot\gamma^{*b} e^k_a e_{bk}}\, ds$
$\sim \sum_{n=0}^{N} \sqrt{E^k_{e_n} E^k_{e_n}}$
$|j m n\rangle$
$j m n\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT^k}} |j m n\rangle$
$= i\hbar \sum_p |j m p\rangle \frac{d}{dt}\Big|_{t=0} D^j_{mn}(e^{tT^k}) = i\hbar \sum_p |j m p\rangle D^j_{mn}(T^k)$
$\Rightarrow \hat{\ell}^2_{e^*} |j m n\rangle = -\hbar^2 \sum_{pq} |j m q\rangle$
$\sum_k D^j($

$$C^j_{qn} = \sum_k D^j_{qp}(T^k)\, D^j_{pn} = D^j_{qn}(|\vec{T}|^2)$$


$C^j_{qn} = \sum_k D^j_{qp}(T^k)\, D^j_{pn}(T^k) = D^j_{qn}(|T|^2)$
$D^j_{qp}(g)\, D^j_{pn}(h) = D^j_{qn}(gh)$


$C^j_{qn} = \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k) = D^j_{qn}(|\vec{T}|^2)$
$D^j_{qp}(g) D^j_{pn}(h) = D^j_{qn}(gh)$
$[C^j, D^j(T^k)] = 0$
$(T^k = -\frac{1}{2}\sigma^k)$
$[|\vec{T}|^2, T^k] = 0$


$C^j_{qn} = \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k) = D^j_{qn}(|\vec{T}|^2)$
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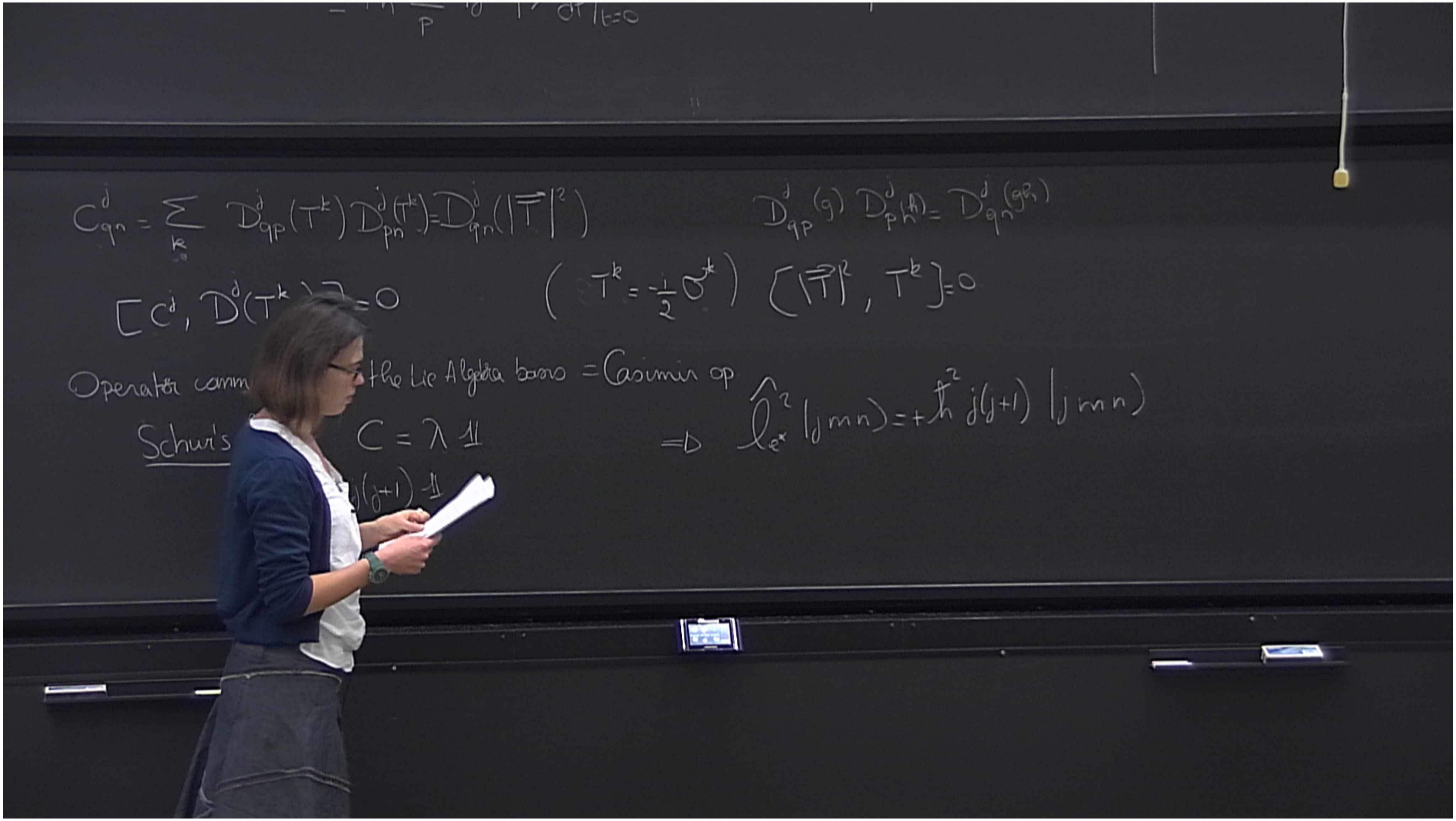
$[C^j, D^j(T^k)] = 0$
$(T^k = -\frac{i}{2}\sigma^k)$
$[|\vec{T}|^2, T^k] = 0$
Operator commutes with the Lie Algebra basis = Casimir op
Schur's lemma $C = \lambda \mathbb{1}$
$C = -j(j+1) \mathbb{1}$


Operator comm ... the Lie Algebra basis = Casimir op
Schur's
C = λ 1
⟹

$)= D^j_{qn}(|\vec{T}|^2)$ $\quad D^j_{qp}(g)\, D^j_{pn}(h) = D^j_{qn}(gh)$

$\left(T^k = -\frac{1}{2}\sigma^k \right) \quad [|\vec{T}|^2, T^k] = 0$

$\rightarrow$ put back units

Lie Algebra basis = Casimir op

$\lambda \mathbb{1}$ $\quad \Rightarrow \quad \hat{l}^2_{e^*} |j m n) = +\hbar^2 j(j+1) |j m n)$

$\mathbb{1}$

spectrum $+\hbar^2 j(j+1) \quad j \in \mathbb{N}/2$

$|t=0$

$) = D^j_{qn}(|\vec{T}|^2)$ $D^j_{qp}(g)\, D^j_{pn}(h) = D^j_{qn}(gh)$

$\left(T^k = -\frac{1}{2}\sigma^k \right) \quad [|\vec{T}|^2, T^k] = 0$

Lie Algebra basis = Casimir op

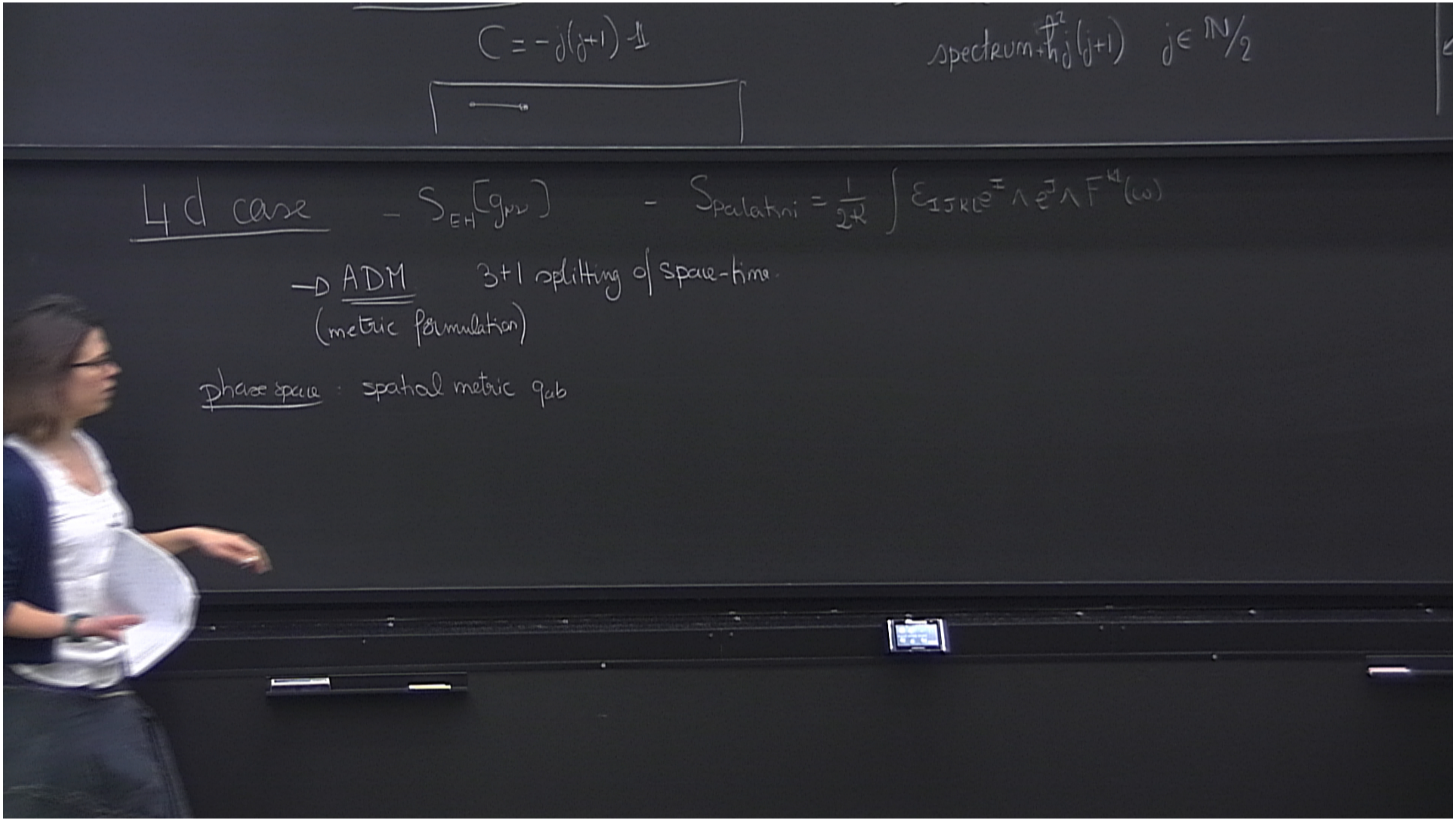
$\lambda \mathbb{1}$ $\Rightarrow \hat{l}^2_{e^*} |j m n) = +\hbar^2 j(j+1) |j m n)$

$\mathbb{1}$ spectrum $\hbar^2 j(j+1)$ $\quad j \in \mathbb{N}/2$

→ put back units for the lenght

$\underbrace{\hbar k^{3D}}_{} \sqrt{j(j+1)}$

$l_p^{3D} = \hbar k^{3D}$


$C = -j(j+1)\,\mathbb{1}$
spectrum $\hbar^2 j(j+1)$ $j \in \mathbb{N}/2$
4d case
$- S_{EH}[g_{\mu\nu}]$
$- S_{Palatini} = \frac{1}{2\kappa}\int \varepsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}(\omega)$
→ ADM 3+1 splitting of space-time.
(metric formulation)
phase space : spatial metric q_{ab}

$C = -j(j+1)\mathbb{1}$

spectrum $\hbar^2 j(j+1)$ $\quad j \in \mathbb{N}/2$

$\hat{E}$

4d case

- $S_{EH}[g_{\mu\nu}]$
- $S_{Palatini} = \frac{1}{2\kappa}\int \varepsilon_{IJKL}\, e^I \wedge e^J \wedge F^{KL}(\omega)$

$\rightarrow$ ADM (metric formulation) 3+1 splitting of space-time $\mathcal{M} = \Sigma \times \mathbb{R}$.

phase space:
- spatial metric q_{ab} on surface Σ
- its conjugated momenta $\pi^{ab} = \frac{\partial L_{EH}}{\partial \dot{q}_{ab}}$

+ constraints $\rightarrow$ V^a (vector constraint $\rightarrow$ diffeo on

$= -j(j+1)\mathbb{1}$

spectrum $\hbar^2 j(j+1)$ $\quad j \in \mathbb{N}/2$

$\hat{E}_1 + \hat{E}_2 + \hat{E}_3 = 0$

$g_{\mu\nu}]$ $\quad - \quad S_{Palatini} = \frac{1}{2k} \int \varepsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}(\omega)$

3+1 splitting of space-time $\mathcal{M} = \Sigma \times \mathbb{R}$

tion)

ric q_{ab} on surface Σ

momenta $\pi^{ab} = \frac{\partial L_{EH}}{\partial \dot{q}_{ab}}$

+ constraints $\rightarrow V^a$ (vector constraint $\rightarrow$ diffeo of the spatial hypersurface)

$\rightarrow H$ (scalar " $\rightarrow$ time reparametrization)

$H^\mu = (S, V^a)$

$C = -j(j+1)\,\mathbb{1}$

spectrum $\hbar^2 j(j+1)$ $\quad j \in \mathbb{N}/2$

$\hat{E}_1 + \hat{E}_2 + \hat{E}_3 = 0$

$- \; S_{EH}[g_{\mu\nu}] \qquad - \; S_{Palatini} = \frac{1}{2\kappa}\int \epsilon_{IJKL}\, e^I \wedge e^J \wedge F^{KL}(\omega)$

ADM $\quad$ 3+1 splitting of space-time $\mathcal{M} = \Sigma \times \mathbb{R}$

(…tric formulation)

… spatial metric q_{ab} on surface Σ

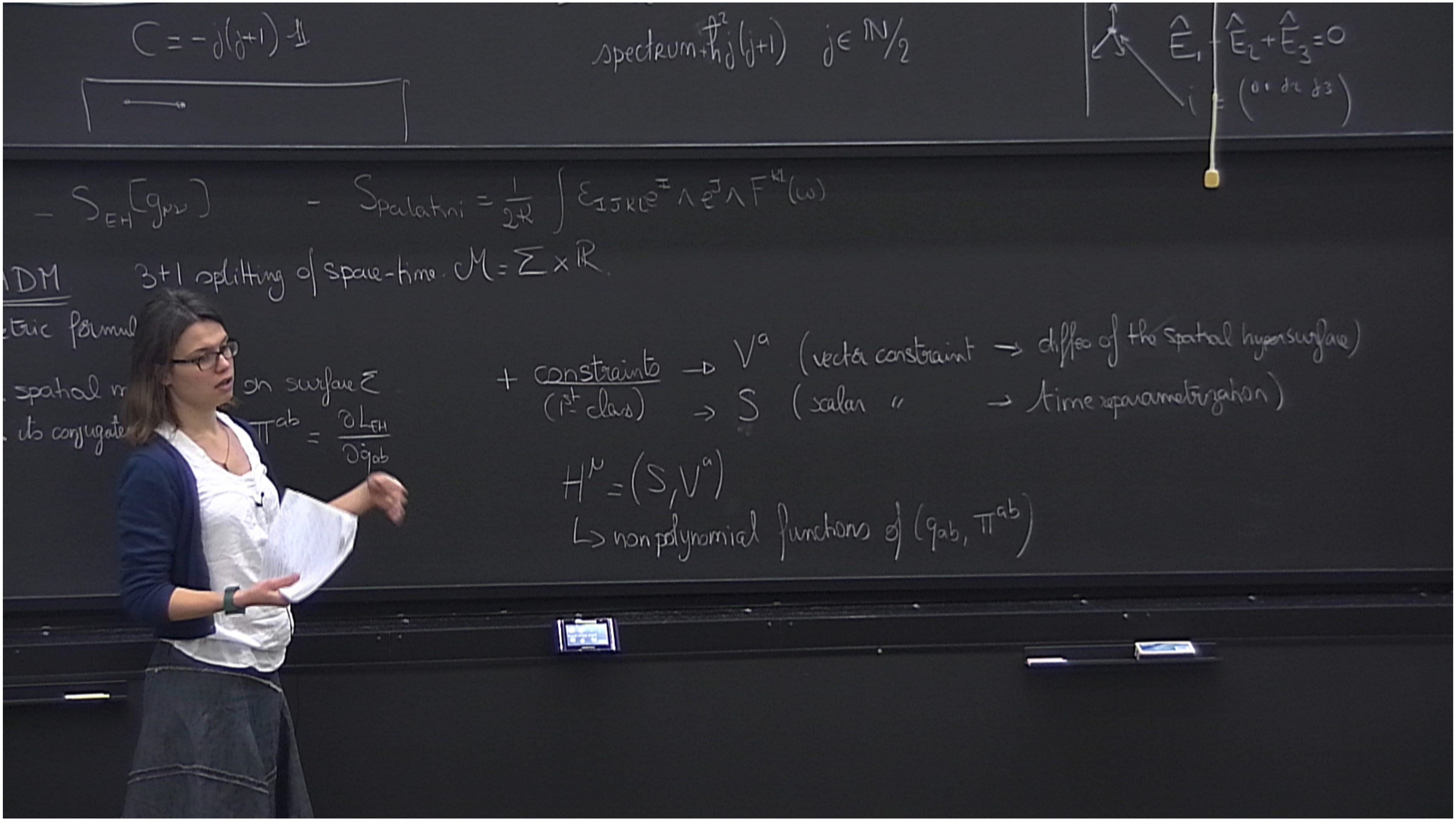
… its conjugated momenta $\pi^{ab} = \frac{\partial L_{EH}}{\partial \dot{q}_{ab}}$

+ constraints (1st class) $\to V^a$ (vector constraint $\to$ diffeo of the spatial hypersurface)

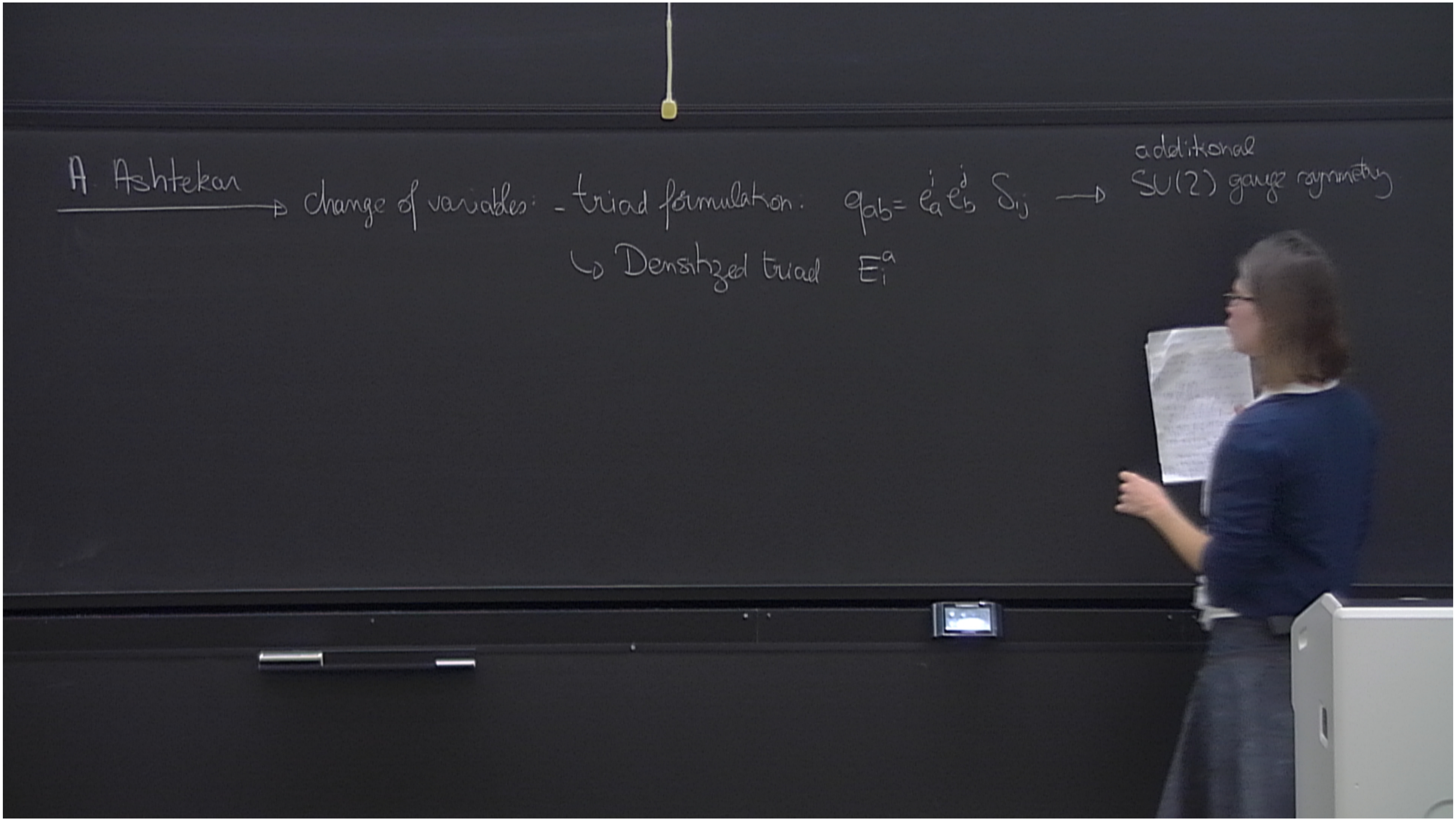
$\to S$ (scalar " $\to$ time reparametrization)

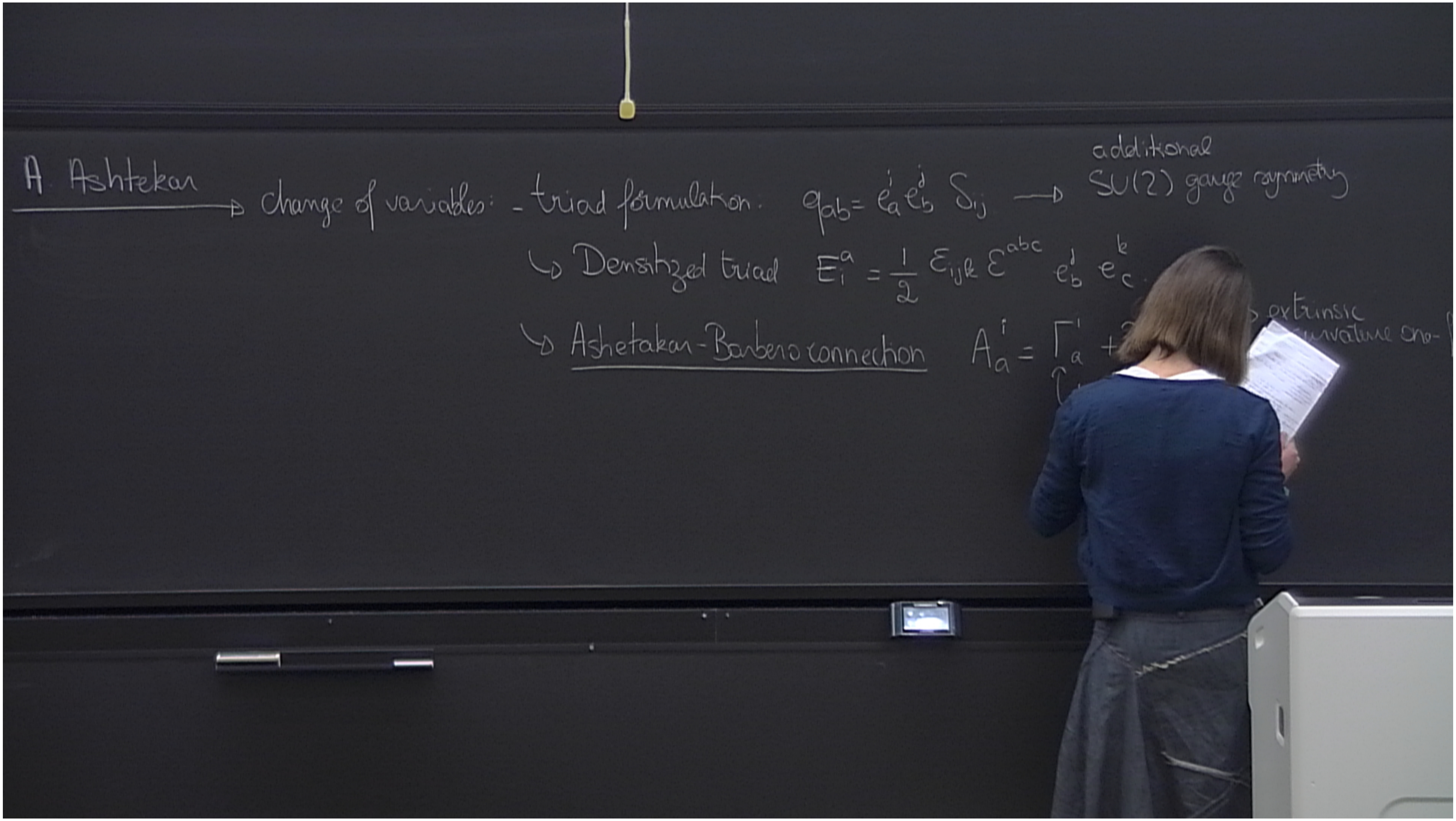
$H^\mu = (S, V^a)$

$\hookrightarrow$ non polynomial functions of (q_{ab}, π^{ab})

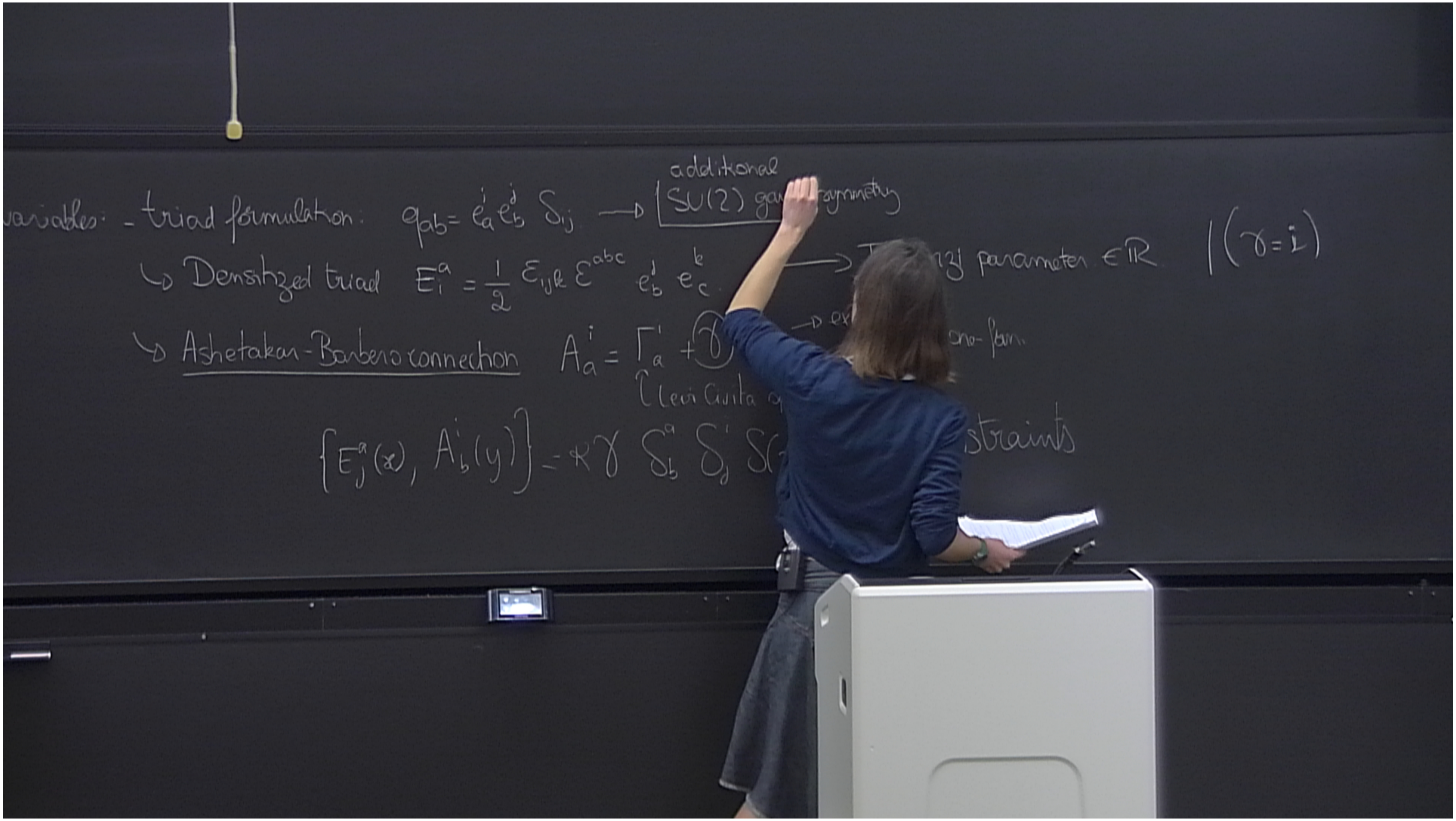

C = -j(j+1) 1
spectrum ħ² j(j+1) j ∈ N/2
Ê₁ + Ê₂ + Ê₃ = 0
- S_EH[g_μν]
- S_Palatini = 1/2κ ∫ ε_IJKL e^I ∧ e^J ∧ F^KL(ω)
ADM
3+1 splitting of space-time M = Σ × R
etric formul
spatial m on surface Σ
its conjugate π^ab = ∂L_EH/∂q̇_ab
+ constraints (1st class) → V^a (vector constraint → diffeo of the spatial hypersurface)
→ S (scalar " → time reparametrization)
H^μ = (S, V^a)
↳ non polynomial functions of (q_ab, π^ab)

A. Ashtekar $\longrightarrow$ change of variables: - triad formulation: $q_{ab} = e_a^i e_b^j \delta_{ij}$

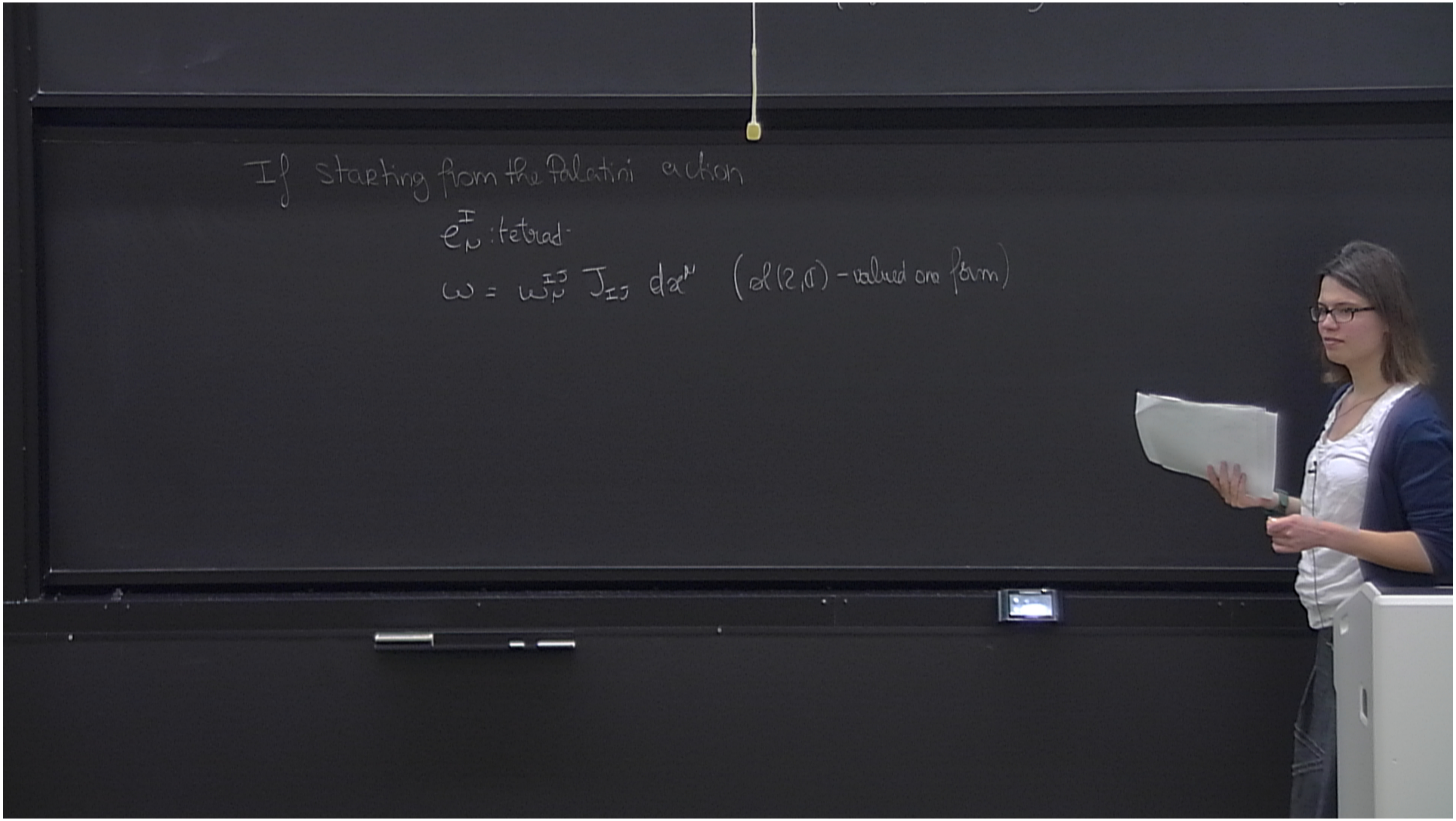

A. Ashtekar
change of variables: - triad formulation: $q_{ab} = e^i_a e^j_b \delta_{ij}$
additional SU(2) gauge symmetry
Densitized triad E^a_i

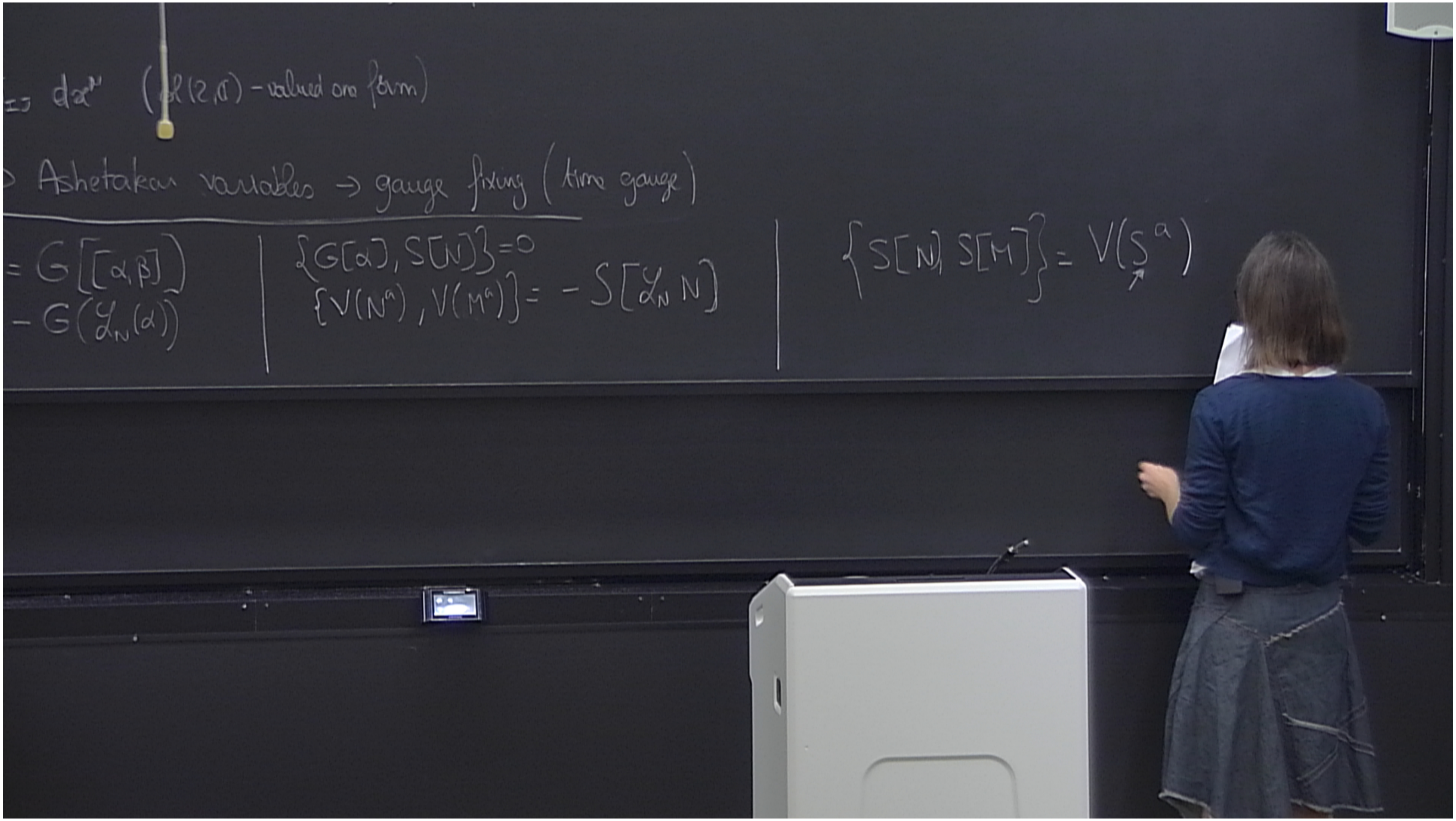


A. Ashtekar
change of variables: - triad formulation. $q_{ab} = e_a^i e_b^j \delta_{ij}$ → additional SU(2) gauge symmetry
↳ Densitized triad $E_i^a = \frac{1}{2} \varepsilon_{ijk} \varepsilon^{abc} e_b^j e_c^k$
↳ Ashetakar-Barbero connection $A_a^i = \Gamma_a^i +$
extrinsic curvature one-



additional
variables: - triad formulation: $q_{ab} = e_a^i e_b^j \delta_{ij}$ → SU(2)
↳ Densitized triad $E_i^a = \frac{1}{2} \varepsilon_{ijk} \varepsilon^{abc} e_b^j e_c^k$
parameter $\in \mathbb{R}$
$(\gamma = i)$
↳ Ashetakar-Barbero connection $A_a^i = \Gamma_a^i + \gamma$
Levi Civita
$\{E_j^a(x), A_b^i(y)\} = \kappa\gamma \, \delta_b^a \, \delta_j^i$


If starting from the Palatini action
e^I_μ : tetrad.
$\omega = \omega_\mu^{IJ} J_{IJ} \, dx^\mu$ ($sl(2,C)$ - valued one form)



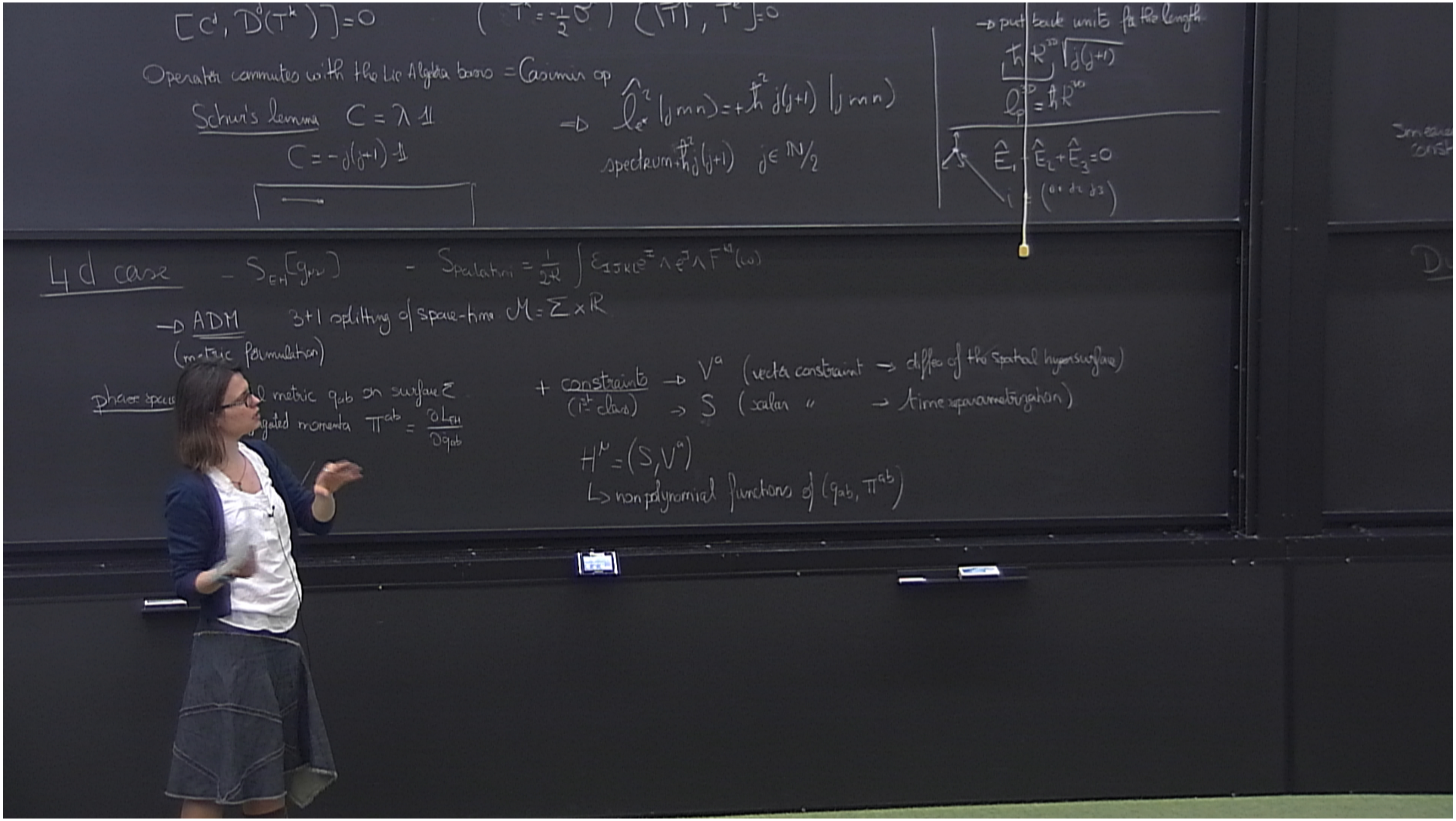
$_{IJ}\, dx^\mu$ ($sl(2,\mathbb{C})$ - valued one form)
Ashetakar variables → gauge fixing (time gauge)
$= G[[\alpha,\beta]]$
$- G(\mathcal{L}_N(\alpha))$
$\{G[\alpha], S[N]\} = 0$
$\{V(N^a), V(M^a)\} = - S[\mathcal{L}_N N]$
$\{S[N], S[M]\} = V(S^a)$

$$\{G[\alpha], V(N^a)\} = -G(\mathcal{L}_N(\alpha))$$

$$\{V(N^a), V(M^a)\} = -V[\mathcal{L}_N N]$$

<u>Dirac's program</u> $\rightarrow$ Kinematical Hilbert space.

* $A \rightarrow h_e \in SU(2)$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H^μ = (S, V^a)
L> non polynomial functions of (q_ab, π^ab)