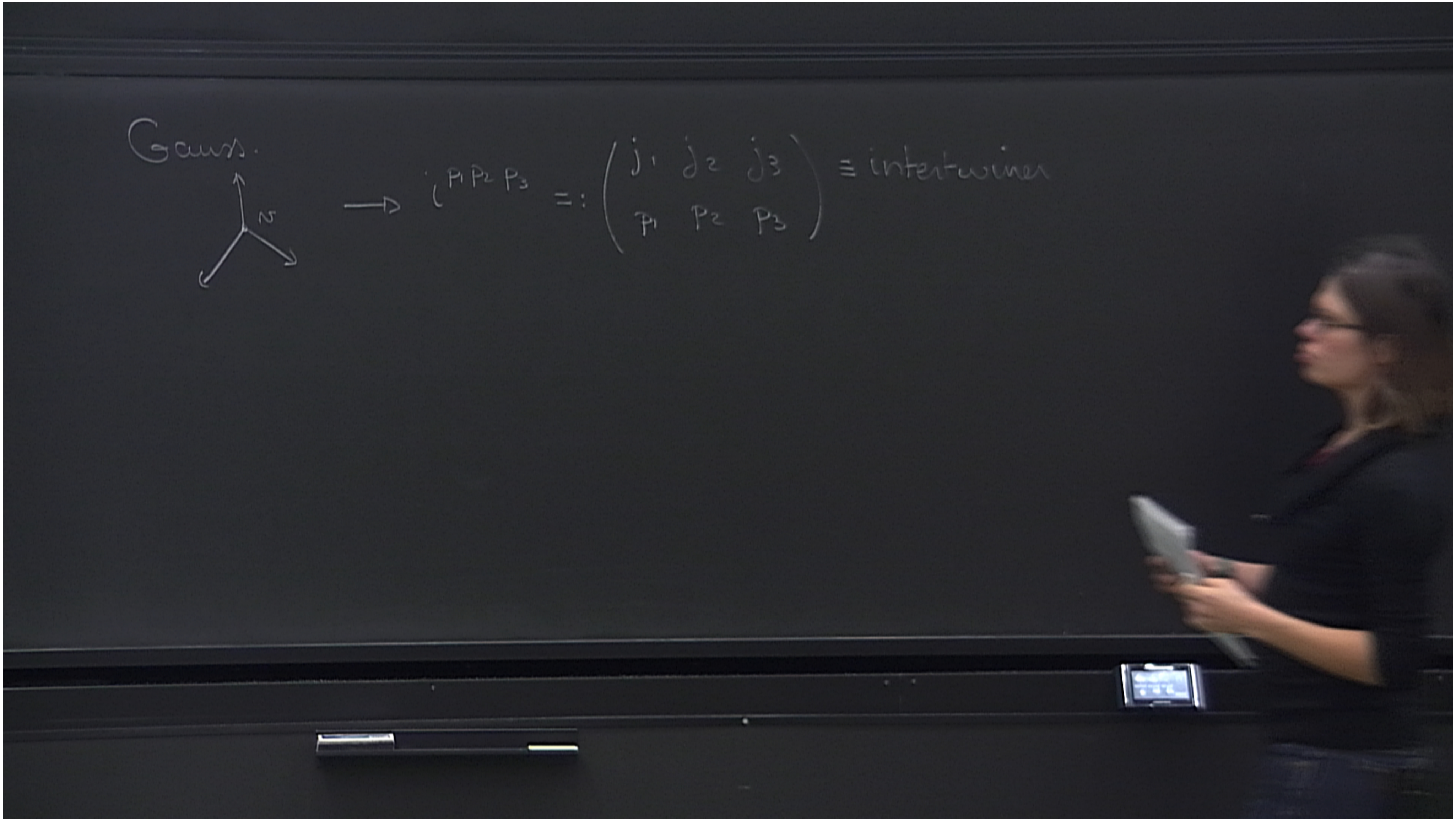
Title: PSI 2015/2016 Quantum Gravity - Lecture 12

Date: Mar 08, 2016 10:15 AM

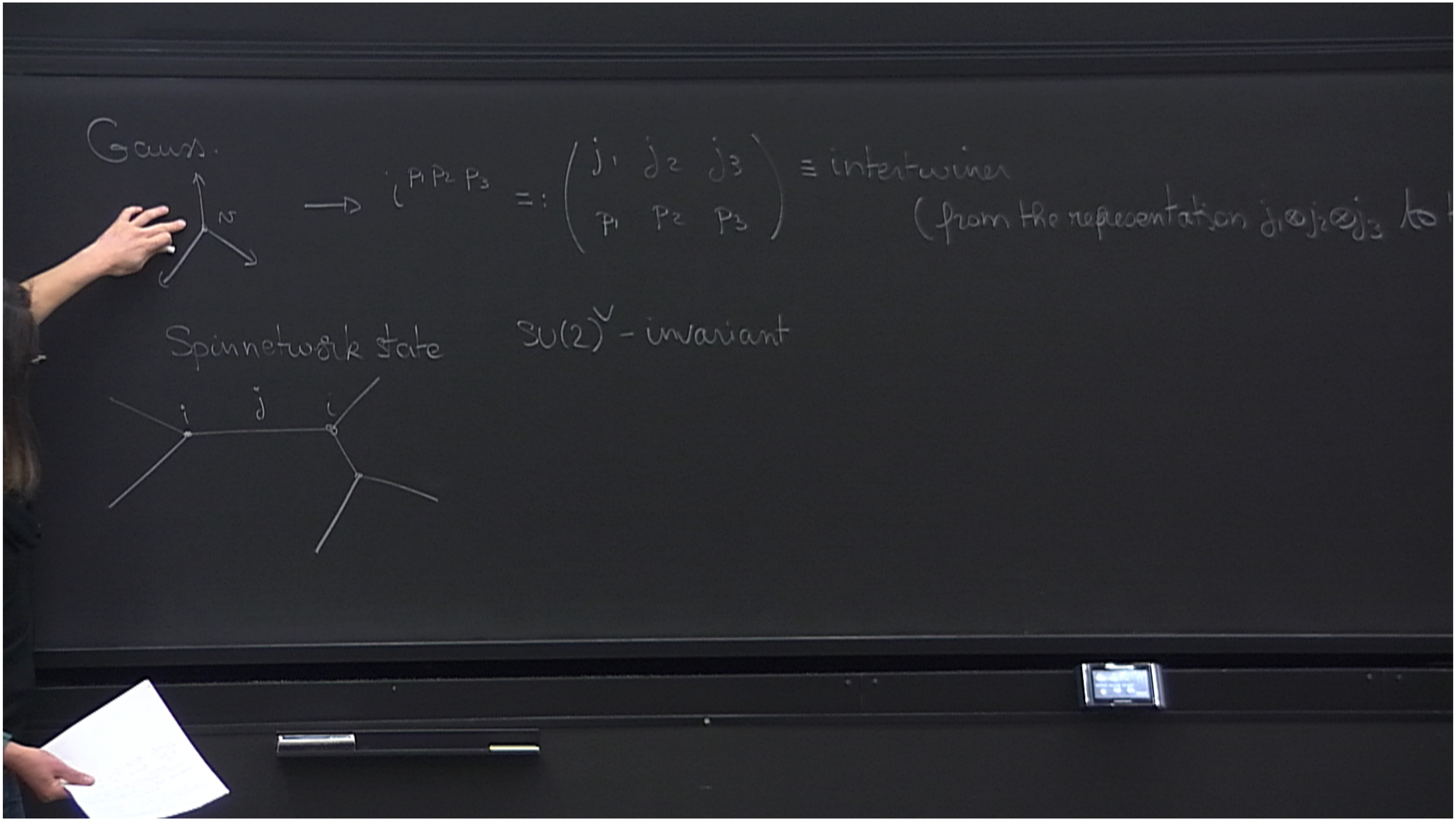
URL: http://pirsa.org/16030050

Abstract:


Gauss.
v
$\longrightarrow \; i^{p_1 p_2 p_3} =: \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \equiv$ intertwiner

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \equiv \text{intertwiner}$$

(from the representation $j_1 \otimes j_2 \otimes j_3$ to the trivial representation)


Gauss.
$\longrightarrow \iota^{p_1 p_2 p_3} =: \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \equiv$ intertwiner
(from the representation $j_1 \otimes j_2 \otimes j_3$ to
Spinnetwork state
$SU(2)^V$ - invariant
i
j
i

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \equiv \text{intertwiner}$$

(from the representation $j_1 \otimes j_2 \otimes j_3$ to the trivial representation)

- invariant functions.

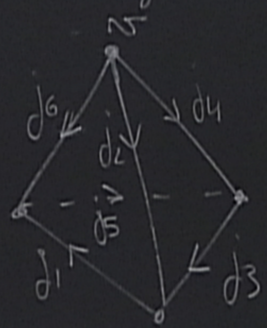
Concrete example   2-sphere triangulated 4 triangles glued together like on the boundary of a tetrahedron.

ntertwiner

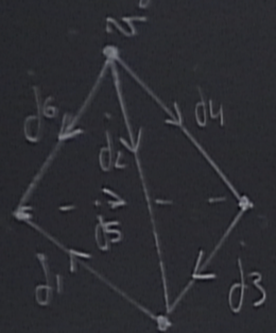
(from the representation $j_1 \otimes j_2 \otimes j_3$ to the trivial representation)

unctions.

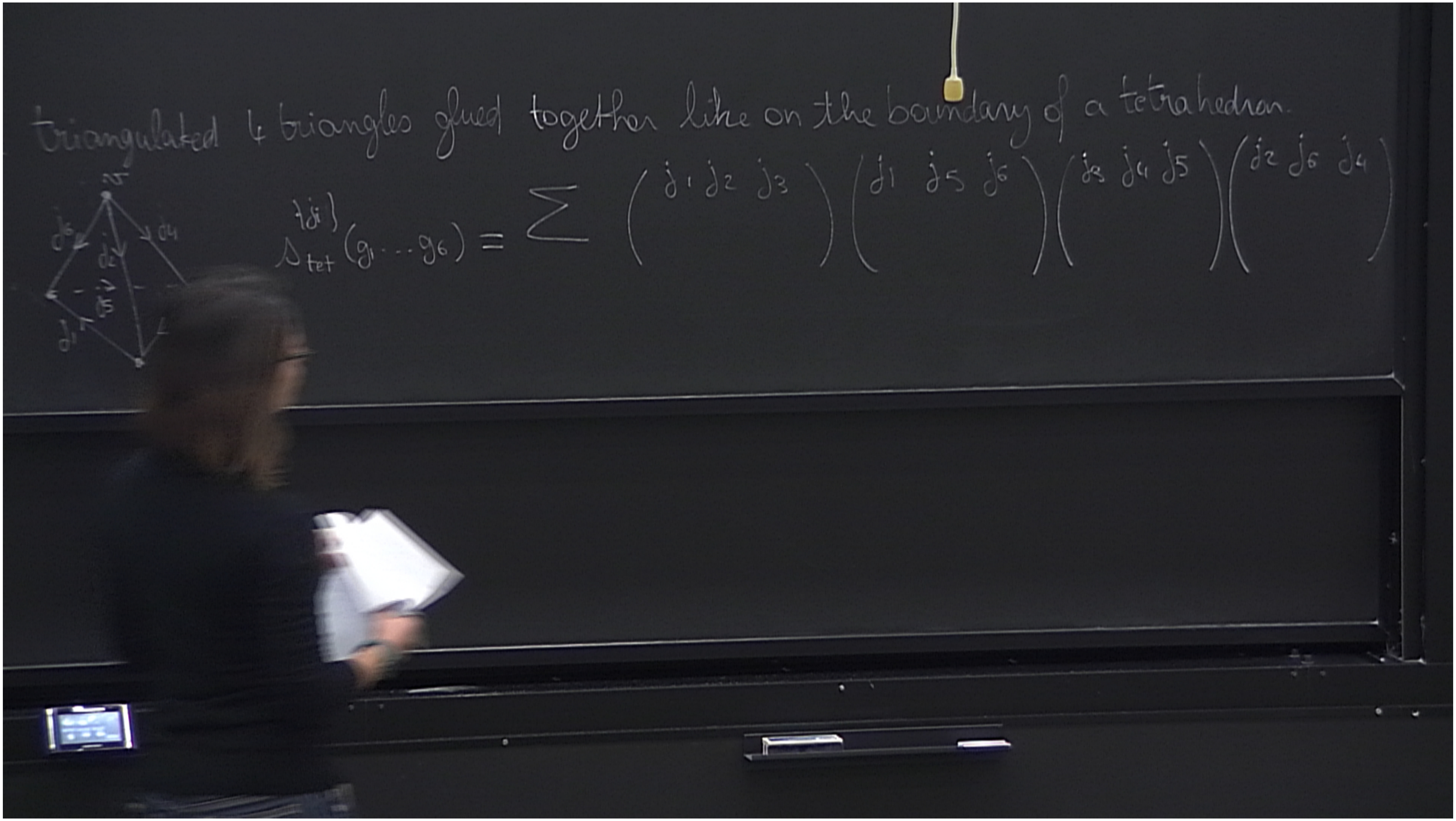
le 2-sphere triangulated 4 triangles glued together like on the boundary

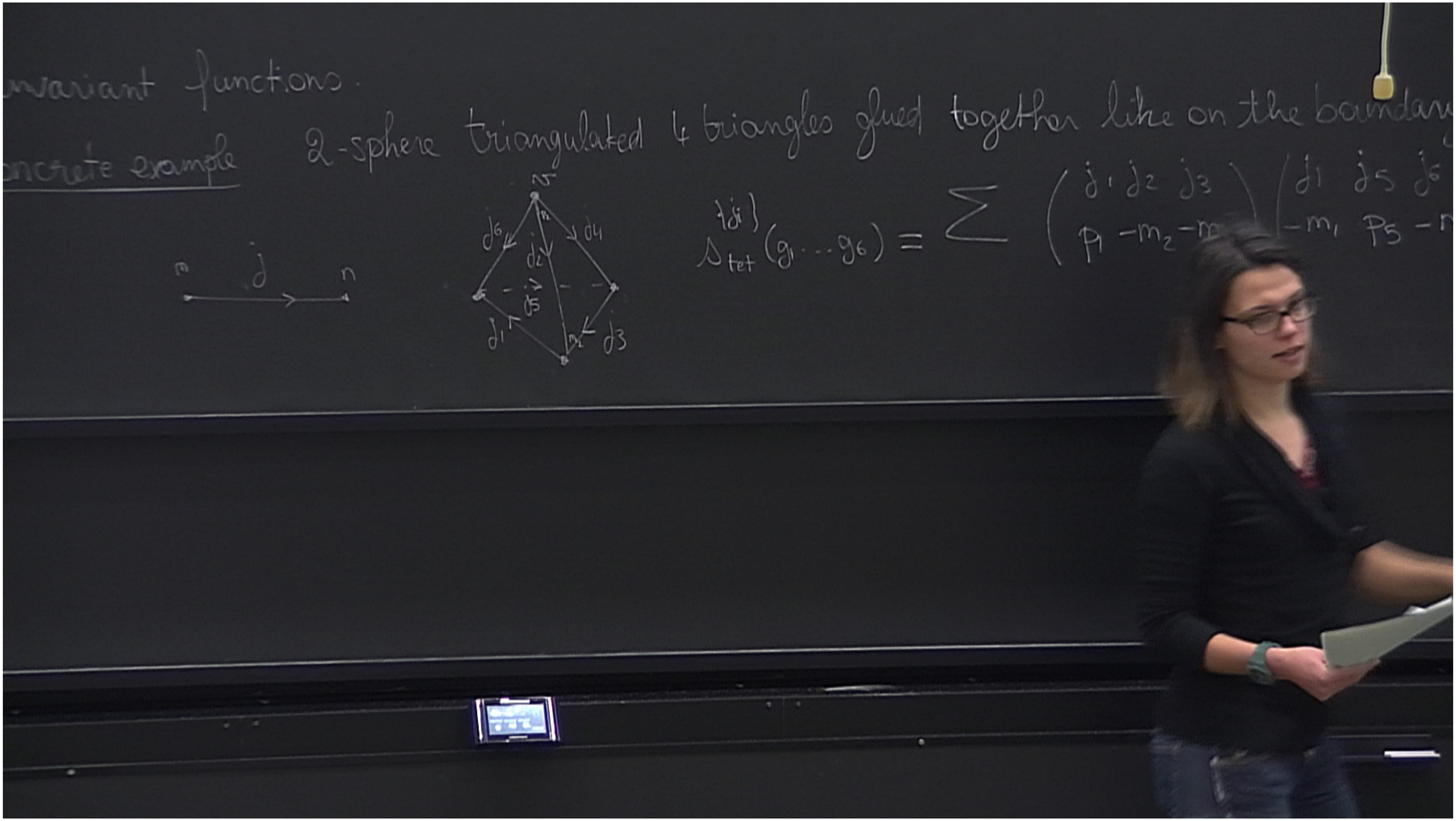


e triangulated 4 triangles glued together like on the boundary of a tetrahed

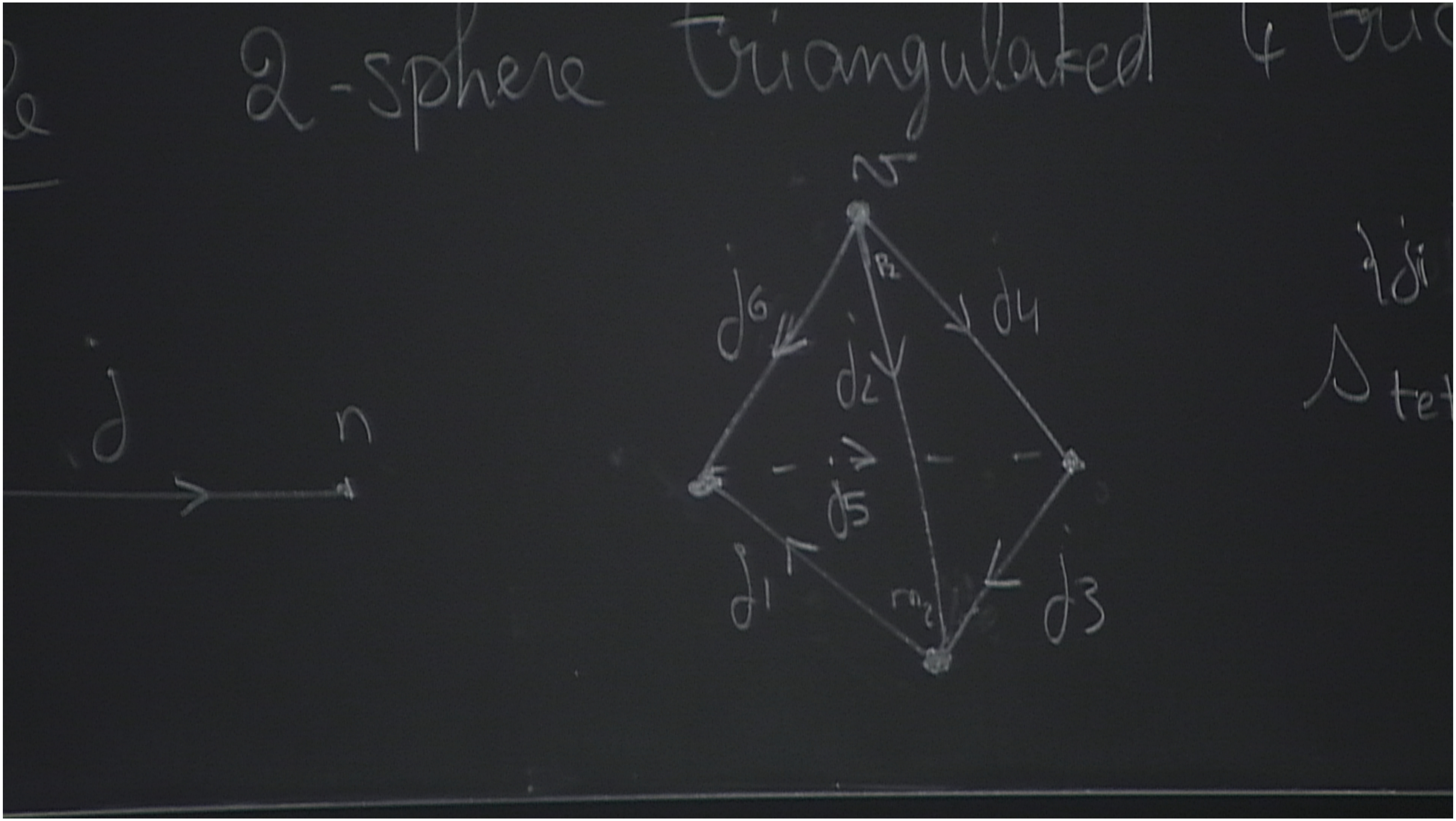


$$\Delta^{\{j_i\}}_{tet}(g_1 \dots g_6) = \sum \begin{pmatrix} j_1 & j_2 & j_3 \\ & & \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ & & \end{pmatrix} \begin{pmatrix} j_3 & j \\ & \end{pmatrix}$$

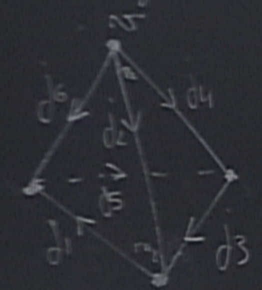

triangulated 4 triangles glued together like on the boundary of a tetrahedron.
$s^{\{j_i\}}_{tet}(g_1 \dots g_6) = \sum \begin{pmatrix} j_1 & j_2 & j_3 \\ & & \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ & & \end{pmatrix} \begin{pmatrix} j_3 & j_4 & j_5 \\ & & \end{pmatrix} \begin{pmatrix} j_2 & j_6 & j_4 \\ & & \end{pmatrix}$
j_6
j_4
j_2
j_5
j_1



invariant functions.
oncrete example
2-sphere triangulated 4 triangles glued together like on the boundary

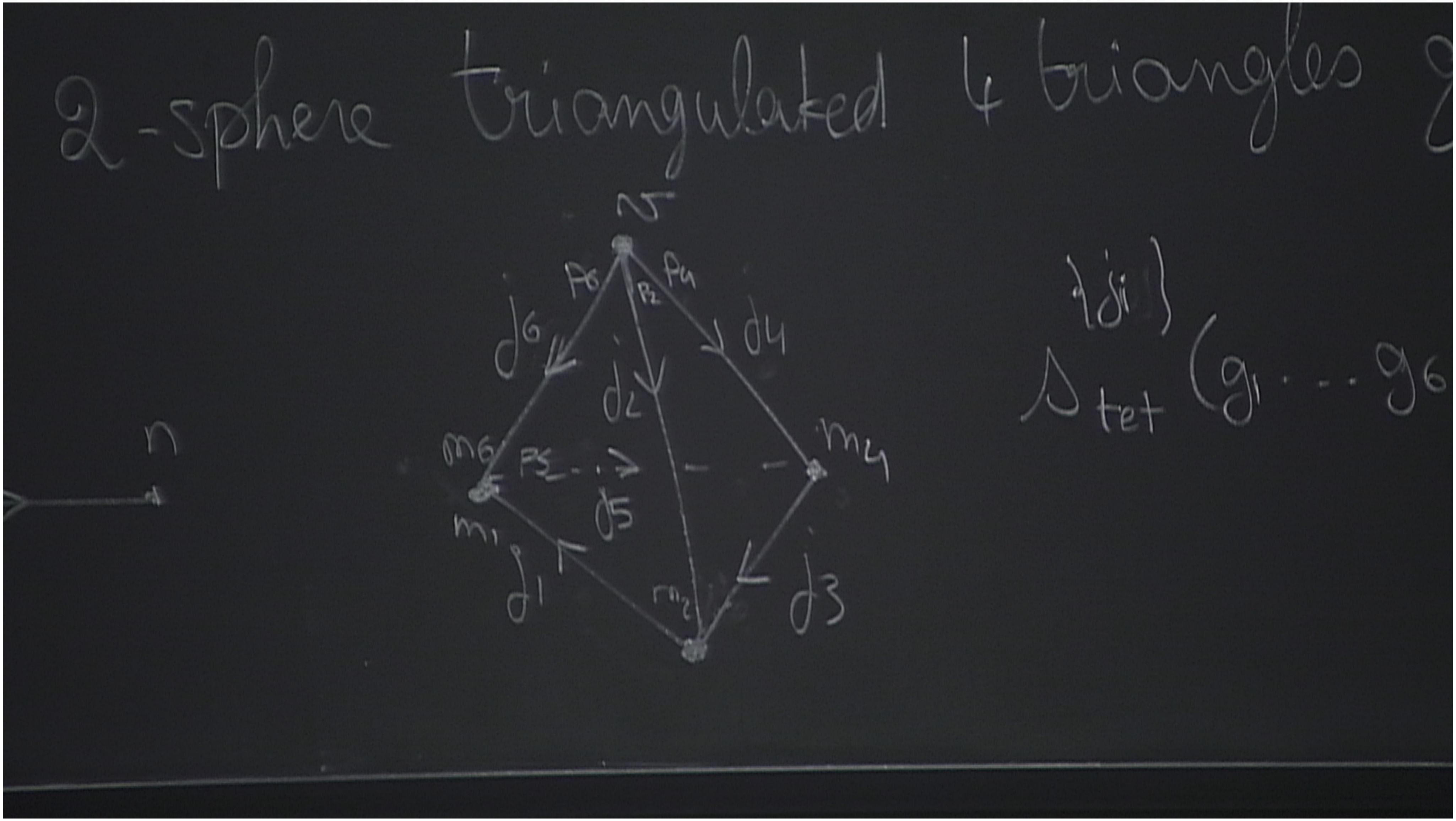

2-sphere triangulated
j
n
j6
j4
j2
j5
j1
j3
tet

...e triangulated 4 triangles glued together like on the boundary of a tetrahedron.

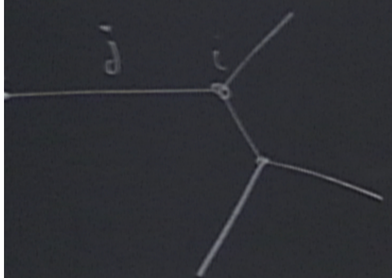


$$\mathcal{S}^{\{j_i\}}_{tet}(g_1 \dots g_6) = \sum \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & -m_2 & -m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ -m_1 & p_5 & -m_6 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & j_5 \\ p_3 & -m_4 & -m_5 \end{pmatrix} \begin{pmatrix} j_2 & j_6 & j_4 \\ p_2 & p_6 & p_4 \end{pmatrix}$$

$$\prod_{e=1}^{6} (-1)^{j_e - m_e} \langle g_e | j_e \; m_e \; p_e \rangle .$$


2-sphere triangulated 4 triangles g
{ji}
Δtet (g1 . . . g6
j6
j4
j2
j5
j1
j3
m6
m4
m1
m2
n

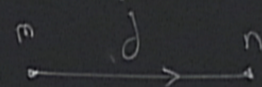
pinnetwork state

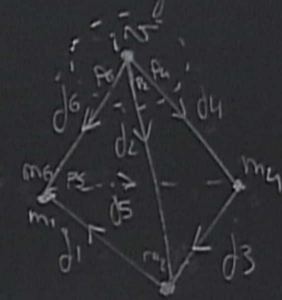


$SU(2)^V$ - invariant functions.

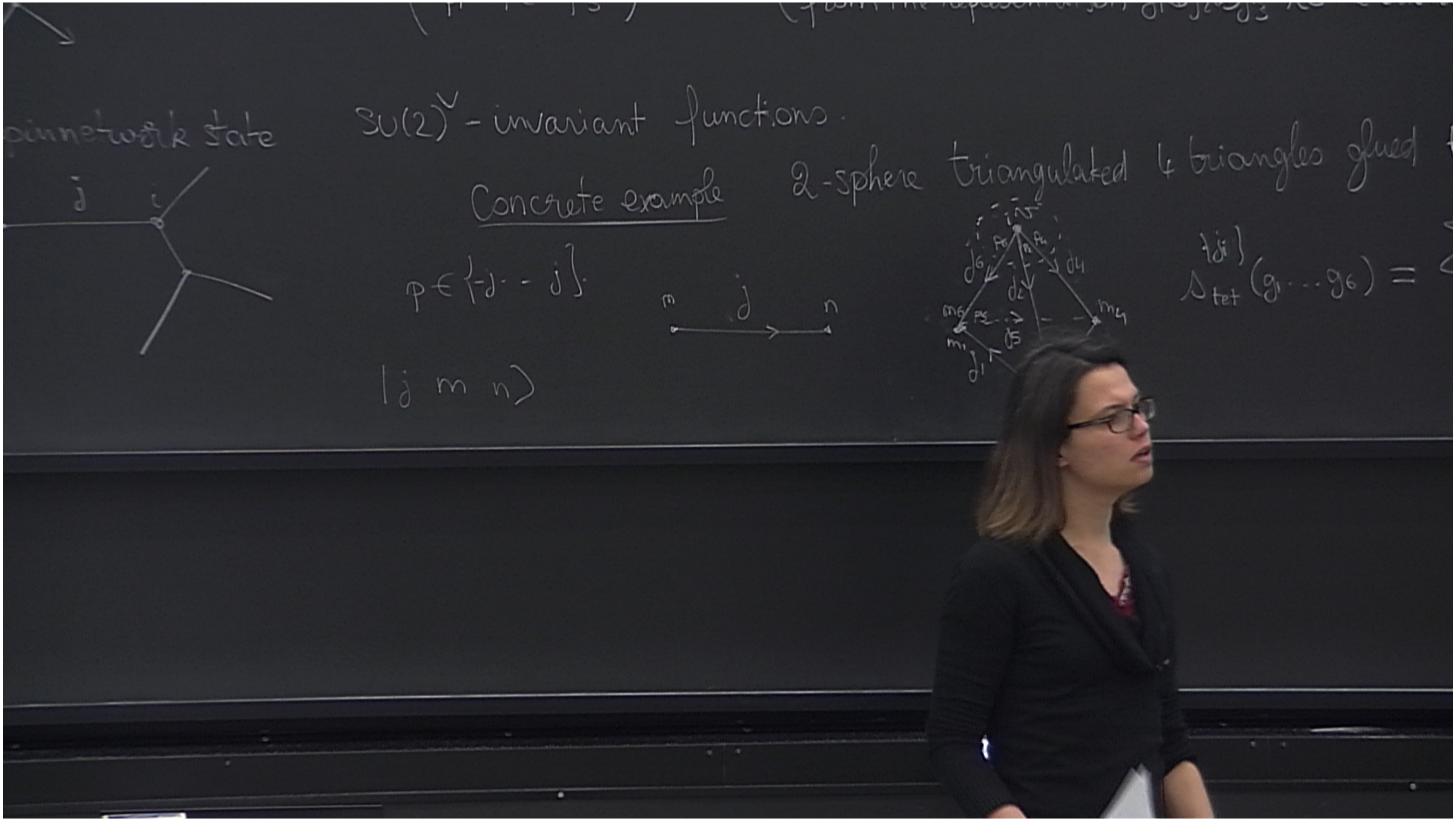
Concrete example 2-sphere triangulated 4 triangles glued

$p \in \{-j \dots j\}$.

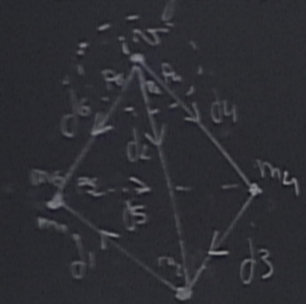




$S^{\{j_i\}}_{tet}(g_1 \dots g_6) =$

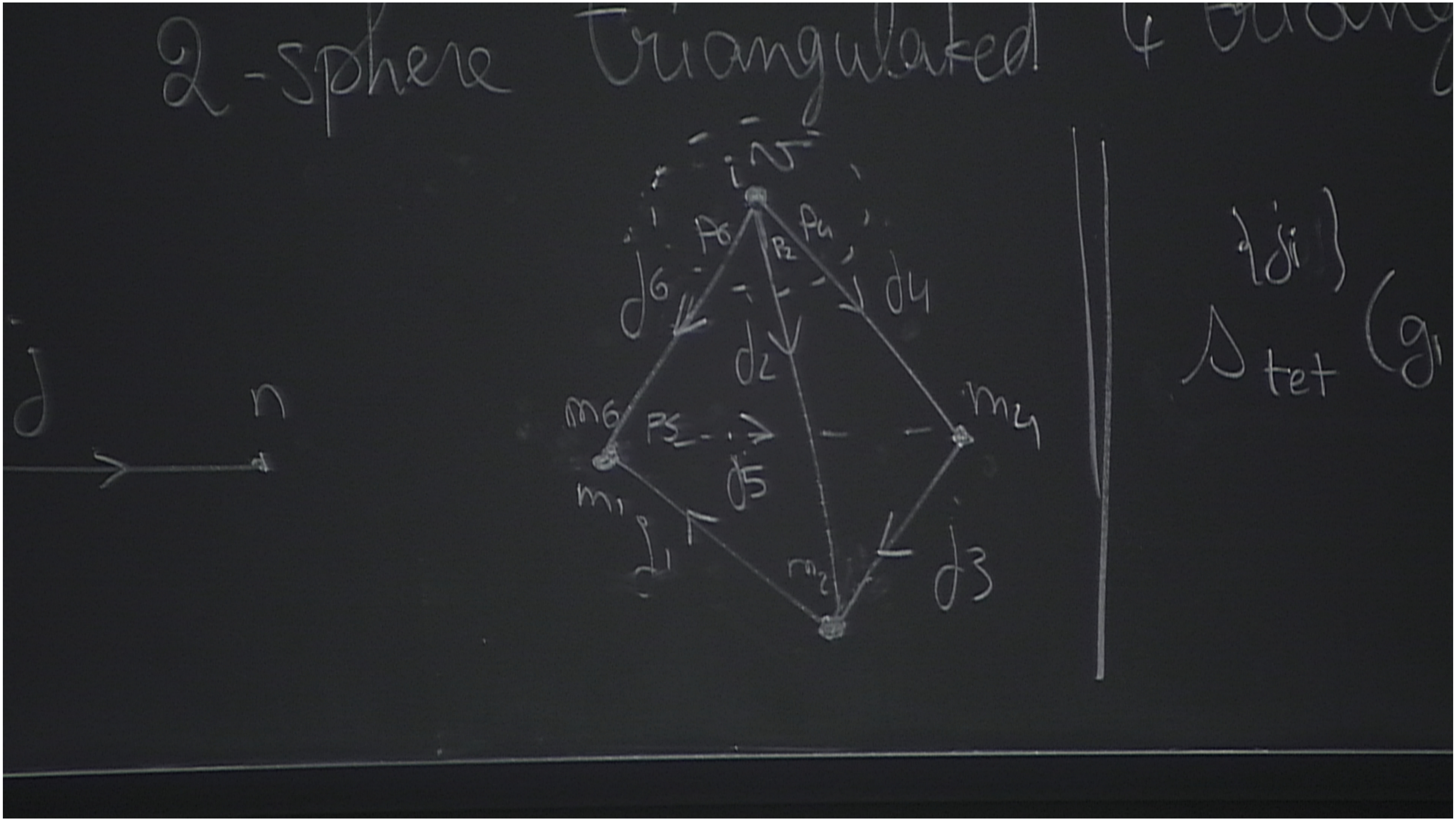

SU(2)-invariant functions.
Concrete example
2-sphere triangulated 4 triangles glued

...e triangulated 4 triangles glued together like on the boundary of a tetrahedron.

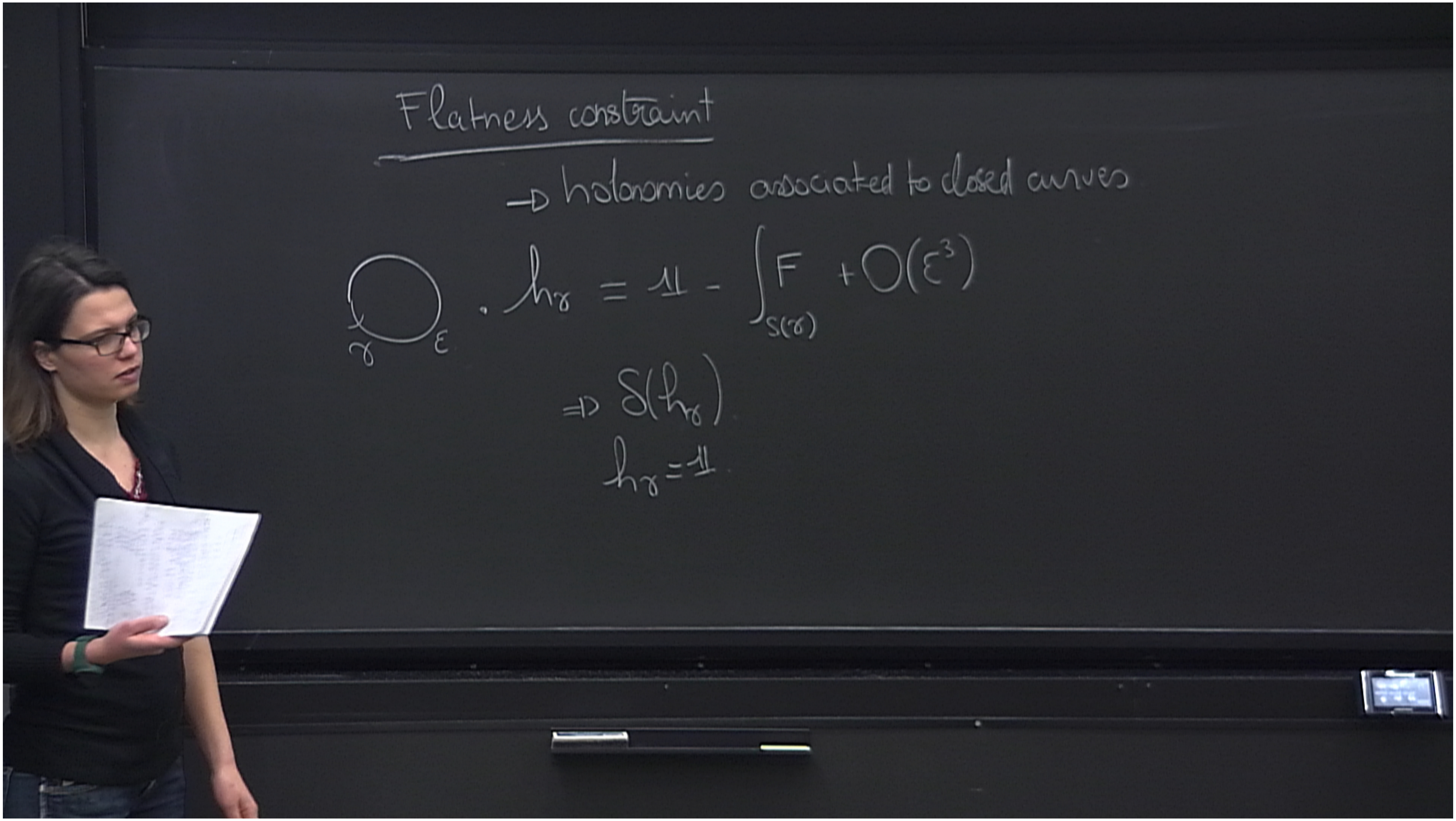


$$\Delta^{\{j_i\}}_{tet}(g_1 \dots g_6) = \sum_{p,m} \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & -m_2 & -m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ -m_1 & p_5 & -m_6 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & j_5 \\ p_3 & -m_4 & -m_5 \end{pmatrix} \begin{pmatrix} j_2 & j_6 & j_4 \\ p_2 & p_6 & p_4 \end{pmatrix}$$

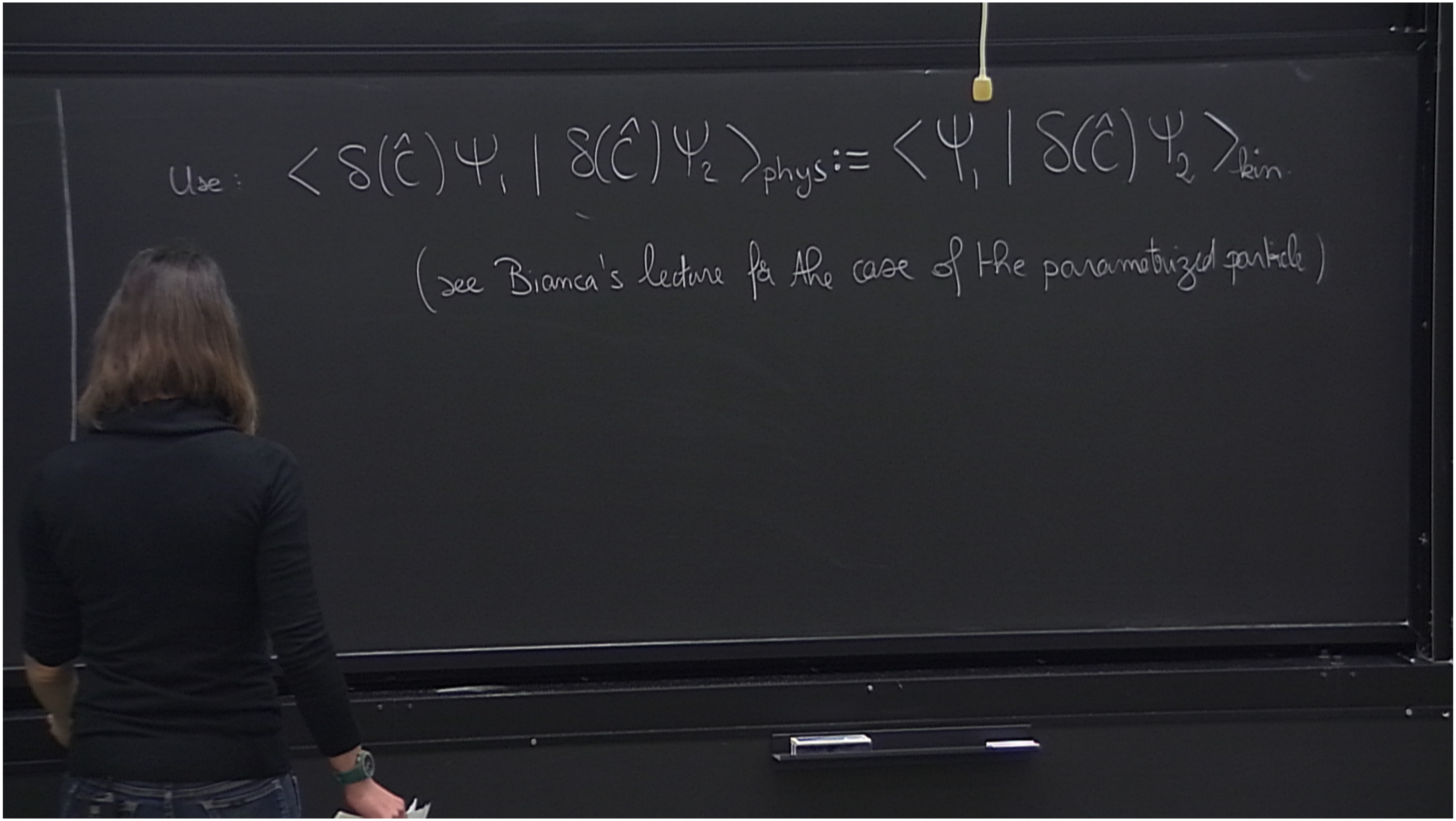
$$\prod_{e=1}^{6} (-1)^{j_e - m_e} \langle g_e | j_e \; m_e \; p_e \rangle .$$



2-sphere triangulated
j
n
j6
j4
j2
j5
j1
j3
m6
m4
m1
m2
{ji}
S tet


Flatness constraint
→ holonomies associated to closed curves
γ
ε
$\cdot\ h_\gamma = 1\!\!1 - \int_{S(\gamma)} F + O(\varepsilon^3)$
$\Rightarrow \delta(h_\gamma)$
$h_\gamma = 1\!\!1$

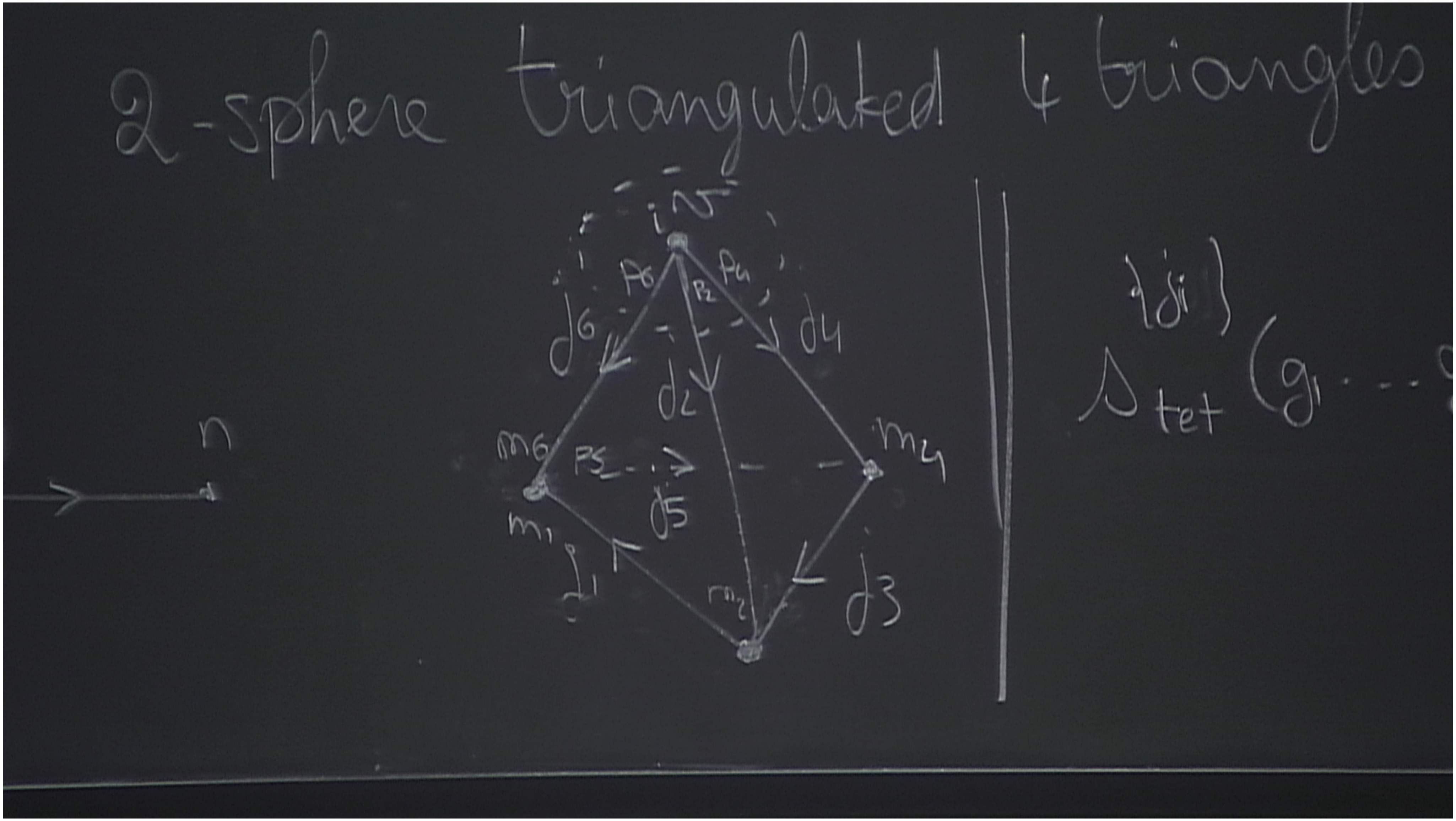
Use: $\langle \delta(\hat{C})\Psi_1 \mid \delta(\hat{C})\Psi_2 \rangle_{phys} := \langle \Psi_1 \mid \delta(\hat{C})\Psi_2 \rangle_{kin}$.


Use : $\langle \delta(\hat{C})\Psi_1 \mid \delta(\hat{C})\Psi_2 \rangle_{phys} := \langle \Psi_1 \mid \delta(\hat{C})\Psi_2 \rangle_{kin}$.
(see Bianca's lecture for the case of the parametrized particle)

Tetrahedral network.

4 loops : $H_1 = g_4^{-1} g_5 g_6$

H


2-sphere triangulated 4 triangles
{ji}
S tet (g1 . . .
n
p6
p4
p2
j6
j4
j2
m6
m4
p5
j5
m1
j1
m2
j3

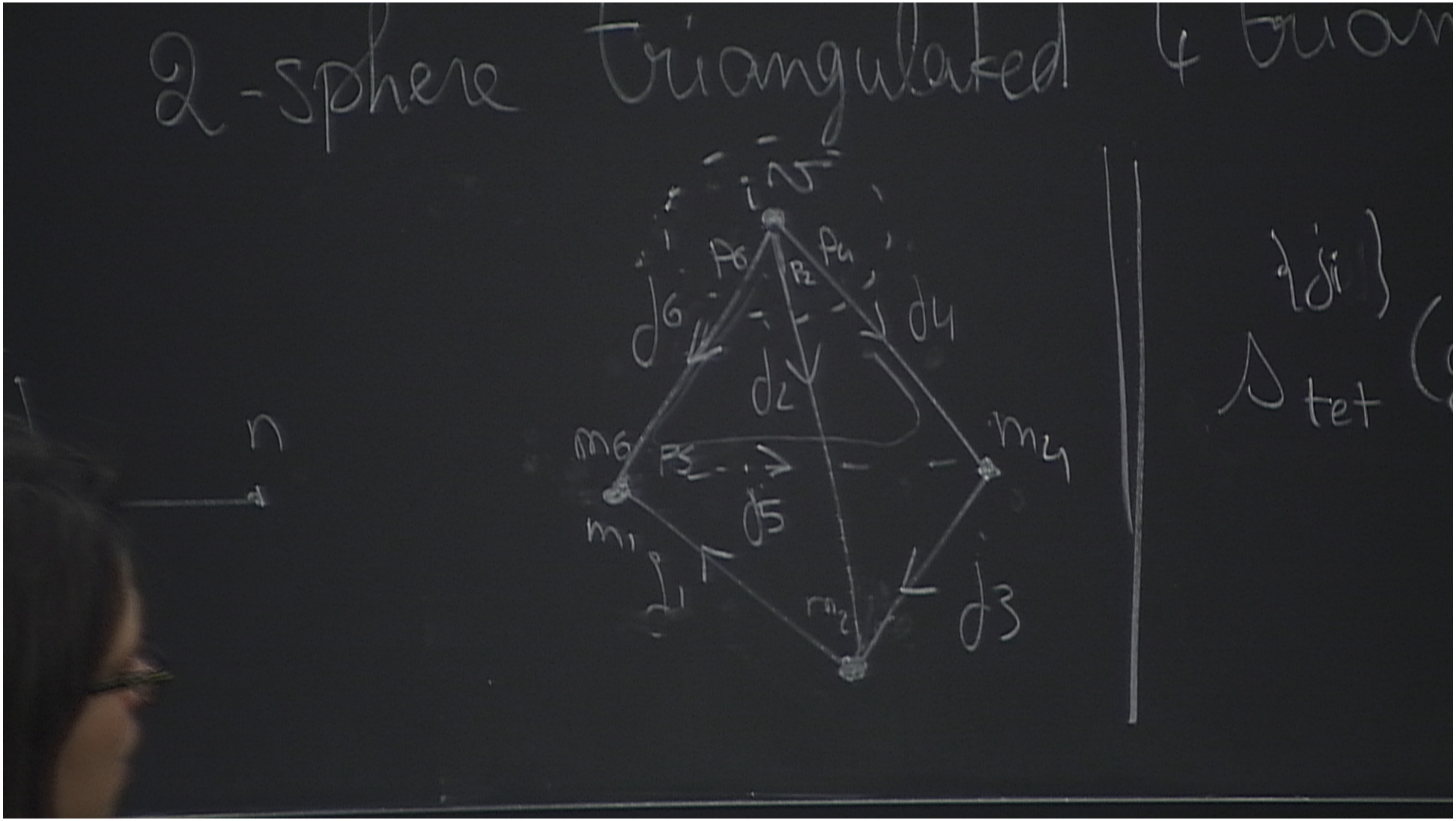
Tetrahedral network.

4 loops : $H_1 = g_4^{-1} g_5 g_6$

$H_2 = g_2^{-1} g_1^{-1} g_6$

$H_3 = g_4^{-1} g_5^{-1} g_2$

$H_4 = g_5^{-1} g_3^{-1} g_1^{-1}$


2-sphere triangulated
j6
j4
j2
m6
m4
j5
m1
j1
m2
j3
n
{ji}
tet

ork.

s : $H_1 = g_4^{-1} g_5 g_6$

$H_2 = g_2^{-1} g_1^{-1} g_6$

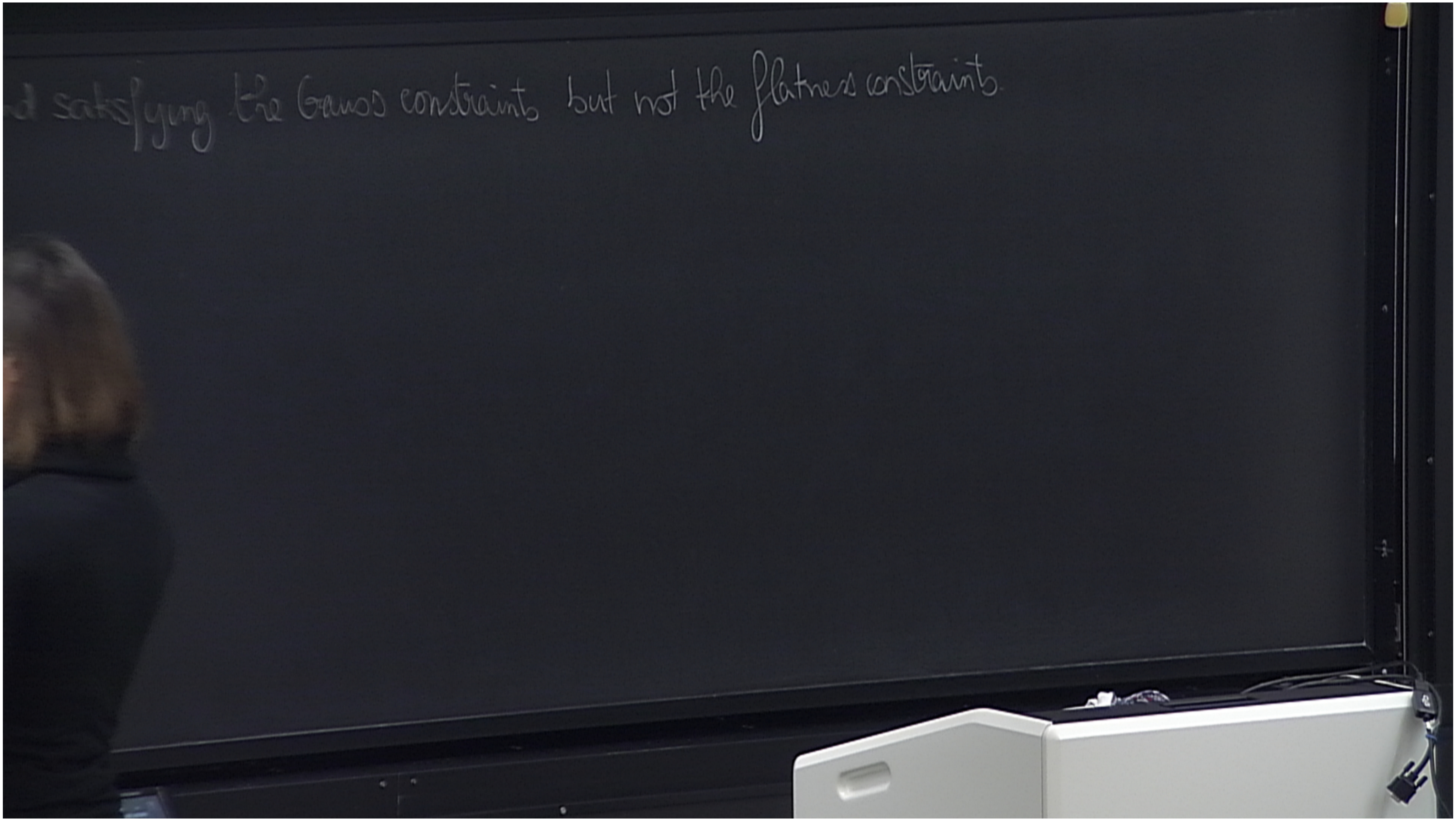
$H_3 = g_4^{-1} g_5^{-1} g_2$

$H_4 = g_5^{-1} g_3^{-1} g_1^{-1}$

Flatness $\longrightarrow$ $\Psi \to \delta(H_1)\,\delta(H_2) \ldots \delta(H_4)\,\Psi.$

$$H_4 = \underbrace{g_6 \underbrace{H_1^{-1} H_3 H_2}_{\text{if } = 1} g_6^{-1}}_{\text{then } = 1}.$$

For 2 kinematical states Ψ_1, Ψ_2 based on the tetrahedral graph and satisfy


satisfying the Gauss constraints but not the flatness constraints.

For 2 kinematical states Ψ_1, Ψ_2 based on the tetrahedral graph and satisfying

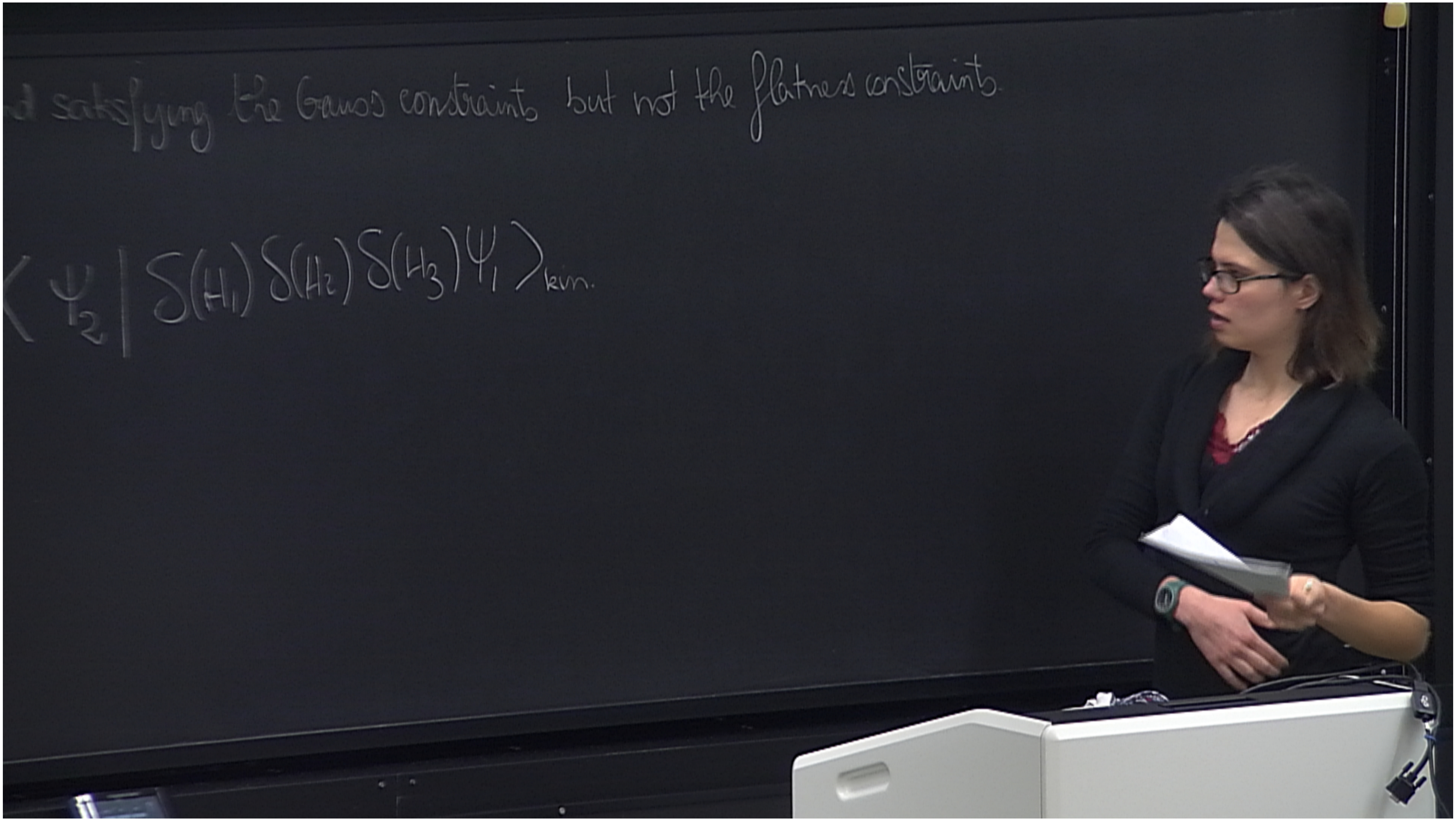
physical inner product given as:

$$< \delta(H_1)\,\delta(H_2)\,\delta(H_3)\,\Psi_2 \,|\, \delta(H_1)\,\delta(H_2)$$

For 2 kinematical states Ψ_1, Ψ_2 based on the tetrahedral graph and satisfying

physical inner product given as.

$$< \delta(H_1)\delta(H_2)\delta(H_3)\Psi_2 \,|\, \delta(H_1)\delta(H_2)\delta(H_3)\Psi_1 >_{phys} = \langle \Psi_2 \,|\, \delta($$

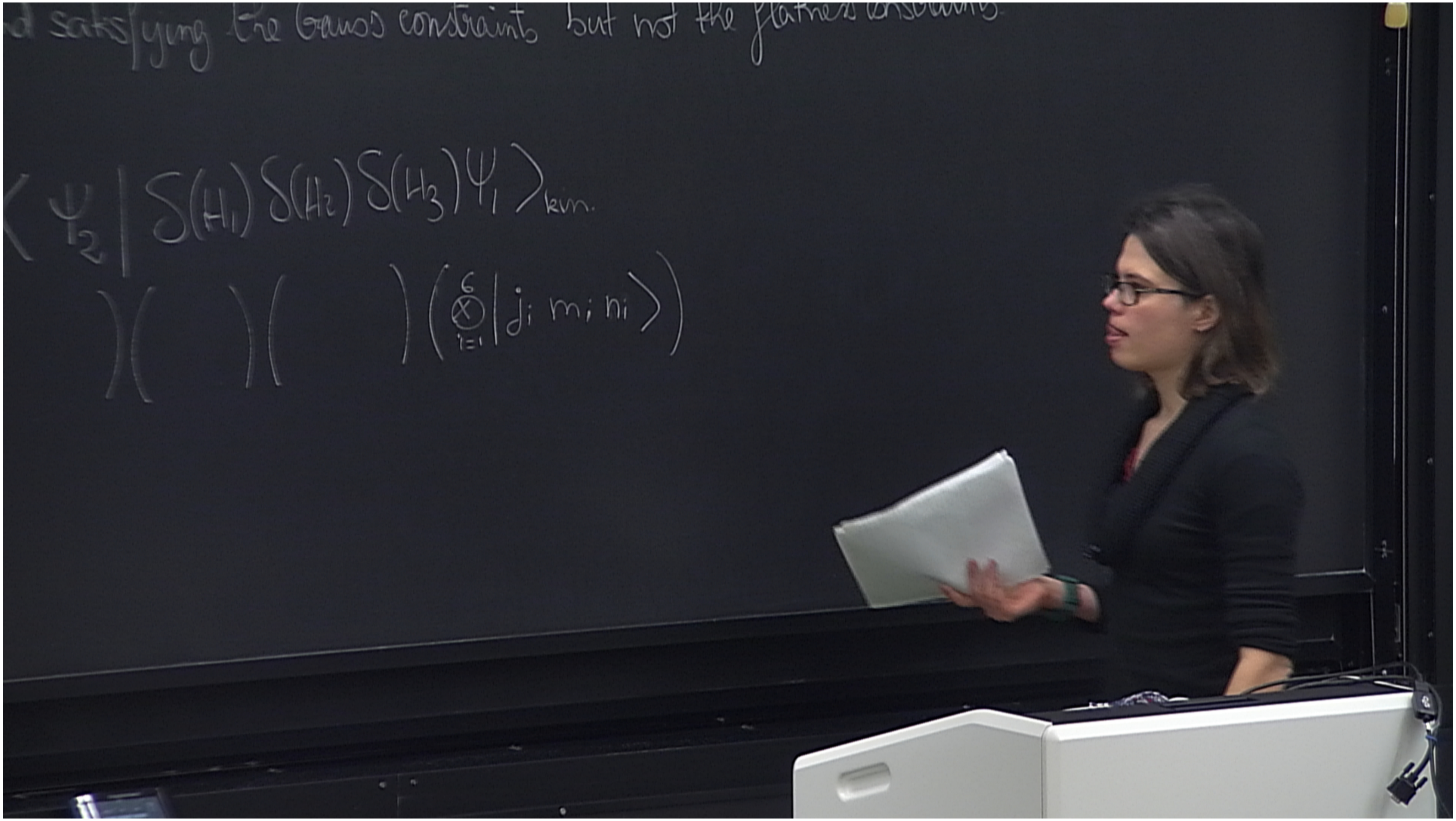

satisfying the Gauss constraints but not the flatness constraints.
$\langle \Psi_2 | \delta(H_1)\, \delta(H_2)\, \delta(H_3)\, \Psi_1 \rangle_{kin.}$

For 2 kinematical states Ψ_1, Ψ_2 based on the tetrahedral graph and satisfying

physical inner product given as.

$$< \delta(H_1)\delta(H_2)\delta(H_3)\Psi_2 \,|\, \delta(H_1)\delta(H_2)\delta(H_3)\Psi_1 >_{phys} := \langle \Psi_2 | \delta($$

$$\Psi_1 = \text{tetra}\begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} = \sum_{m_i, n_i} (-1)^{\sum_{i=1}^{6}(j_i - m_i)} \left(\quad \right)\left(\quad \right)\left(\right.$$


satisfying the Gauss constraints but not the flatness constraints.
⟨ Ψ2 | δ(H1) δ(H2) δ(H3) Ψ1 ⟩kin.
) ( ) ( ) (⊗ i=1 6 | ji mi ni ⟩)

Ψ_2 based on the tetrahedral graph and satisfying the Gauss constraints but not the

product given as.

$$\langle \Psi_2 | \delta(H_1)\delta(H_2)\delta(H_3)\Psi_1 \rangle_{phys} := \langle \Psi_2 | \delta(H_1)\delta(H_2)\delta(H_3)\Psi_1 \rangle_{kin.}$$

$$= \sum_{\ldots, n_i} (-1)^{\sum_{i=1}^{6}(j_i - m_i)} \left(3nj \right) \left(\quad \right) \left(\quad \right) \left(\quad \right) \left(\bigotimes_{i=1}^{6} | j_i\, m_i\, n_i \rangle \right)$$

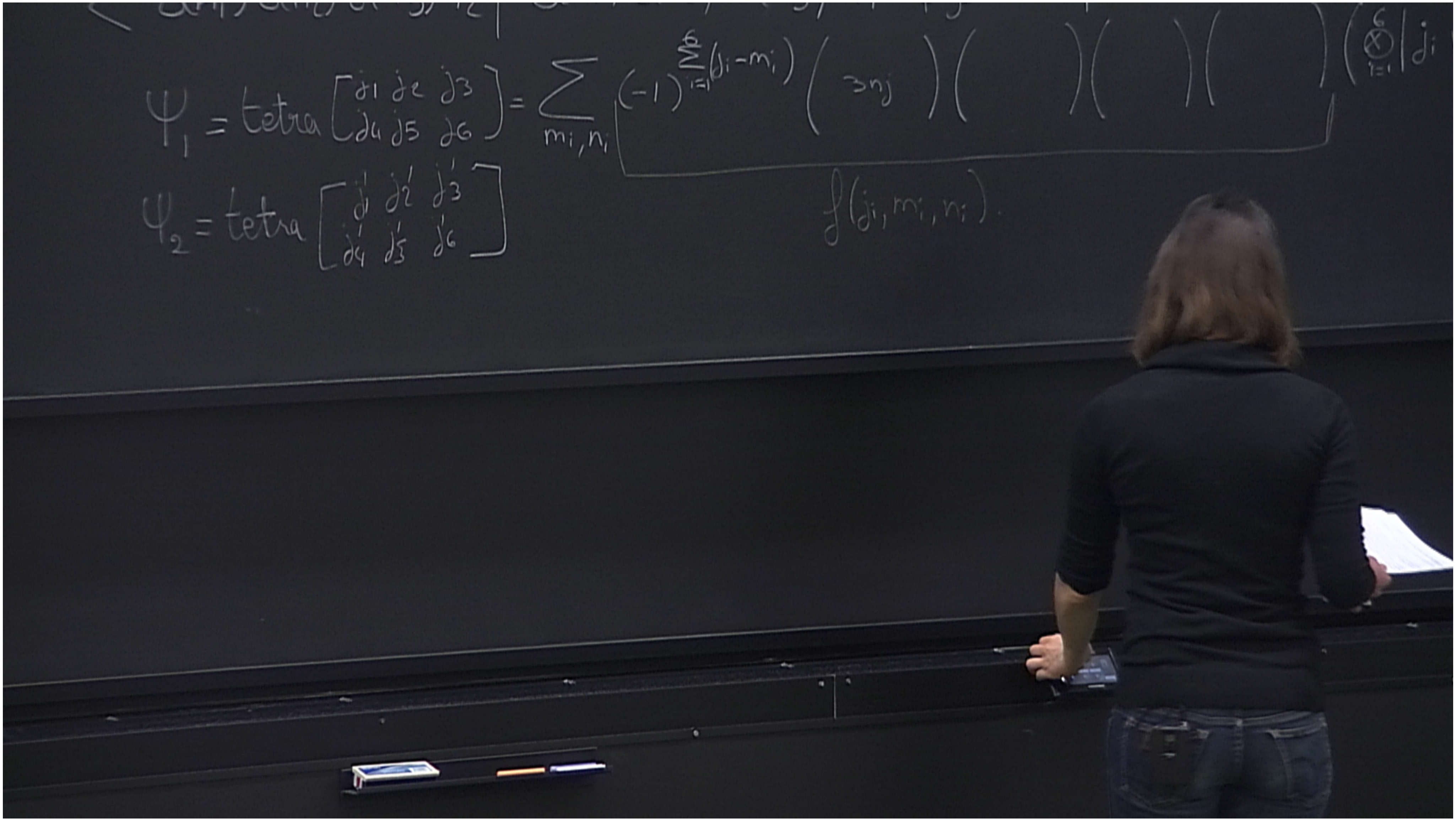
For 2 kinematical states Ψ_1, Ψ_2 based on the tetrahedral graph and satisf

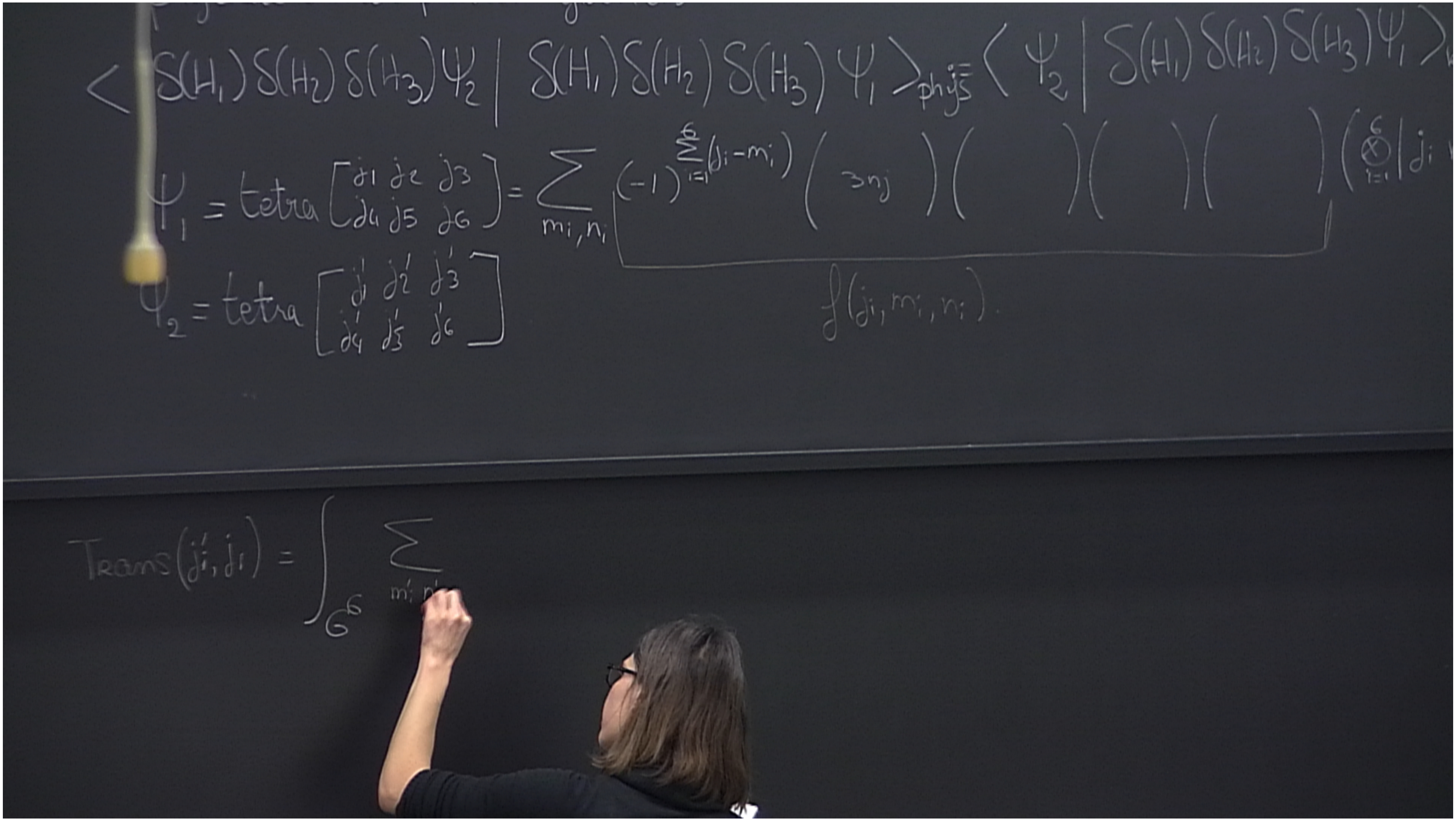
physical inner product given as

$$< \delta(H_1)\delta(H_2)\delta(H_3)\Psi_2 \,|\, \delta(H_1)\delta(H_2)\delta(H_3)\Psi_1 >_{phys} = \langle \Psi_2 |$$

$$\Psi_1 = \text{tetra}\begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} = \sum_{m_i, n_i} (-1)^{\sum_{i=1}^{6}(j_i - m_i)} \begin{pmatrix} 3nj \end{pmatrix} \begin{pmatrix} \quad \end{pmatrix} ($$

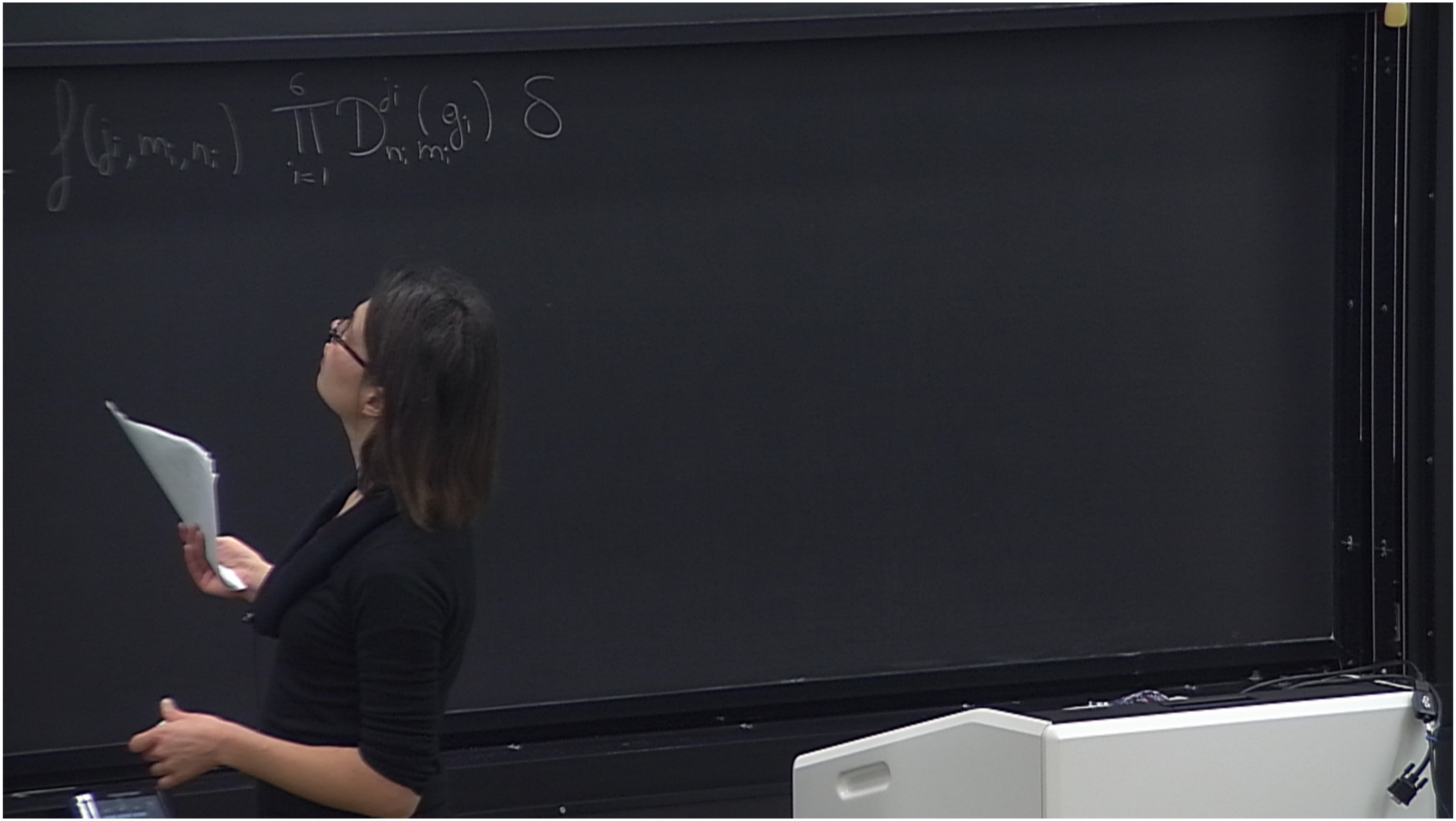
$$\Psi_2 = \text{tetra}\begin{bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{bmatrix}$$

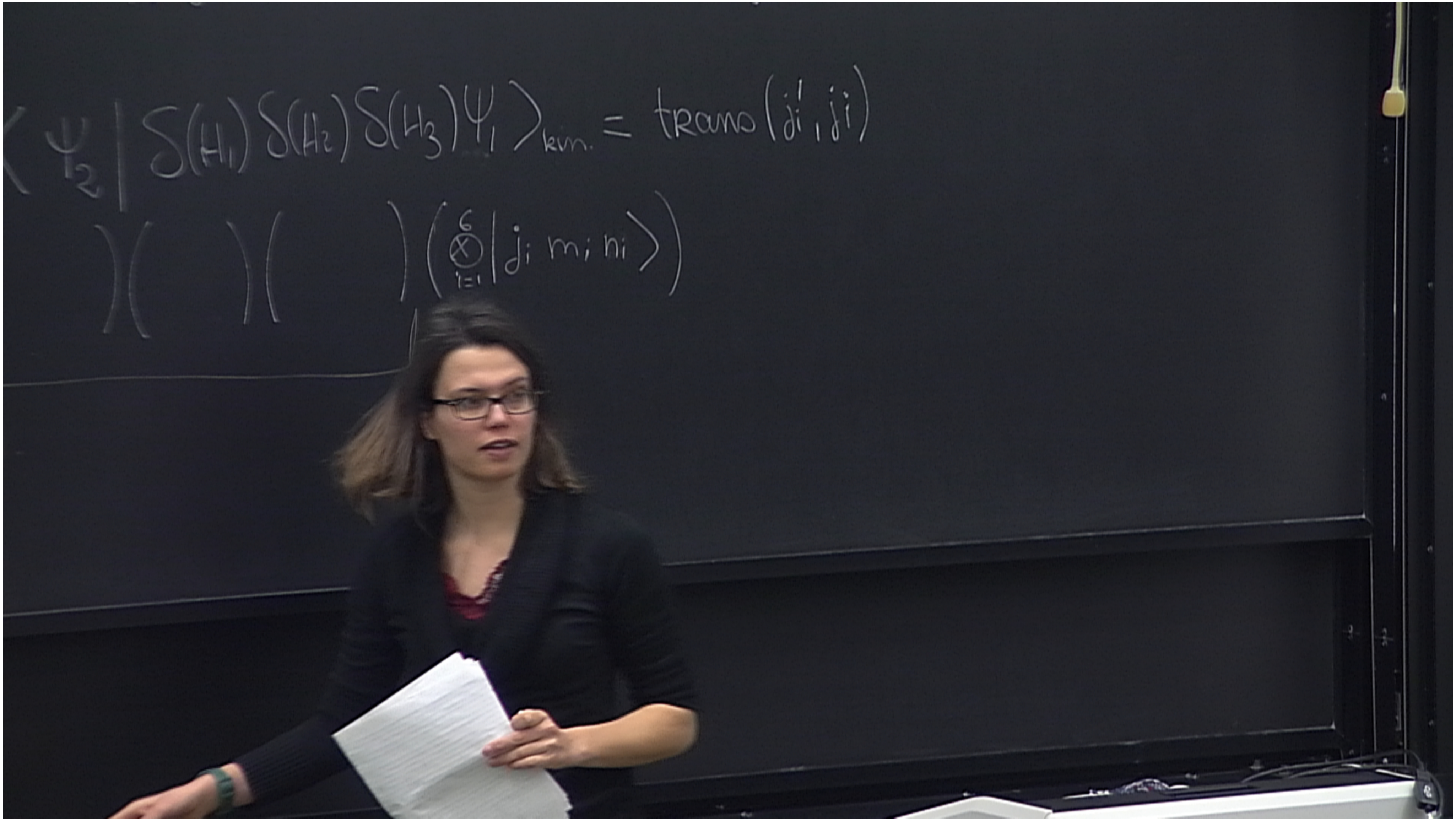

$\Psi_1 = \text{tetra}\begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} = \sum_{m_i, n_i} \underbrace{(-1)^{\sum_{i=1}^{6}(j_i - m_i)} \begin{pmatrix} 3nj \end{pmatrix}\begin{pmatrix} \end{pmatrix}\begin{pmatrix} \end{pmatrix}\begin{pmatrix} \end{pmatrix}}_{f(j_i, m_i, n_i).} \left(\bigotimes_{i=1}^{6} |j_i\right.$
$\Psi_2 = \text{tetra}\begin{bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{bmatrix}$



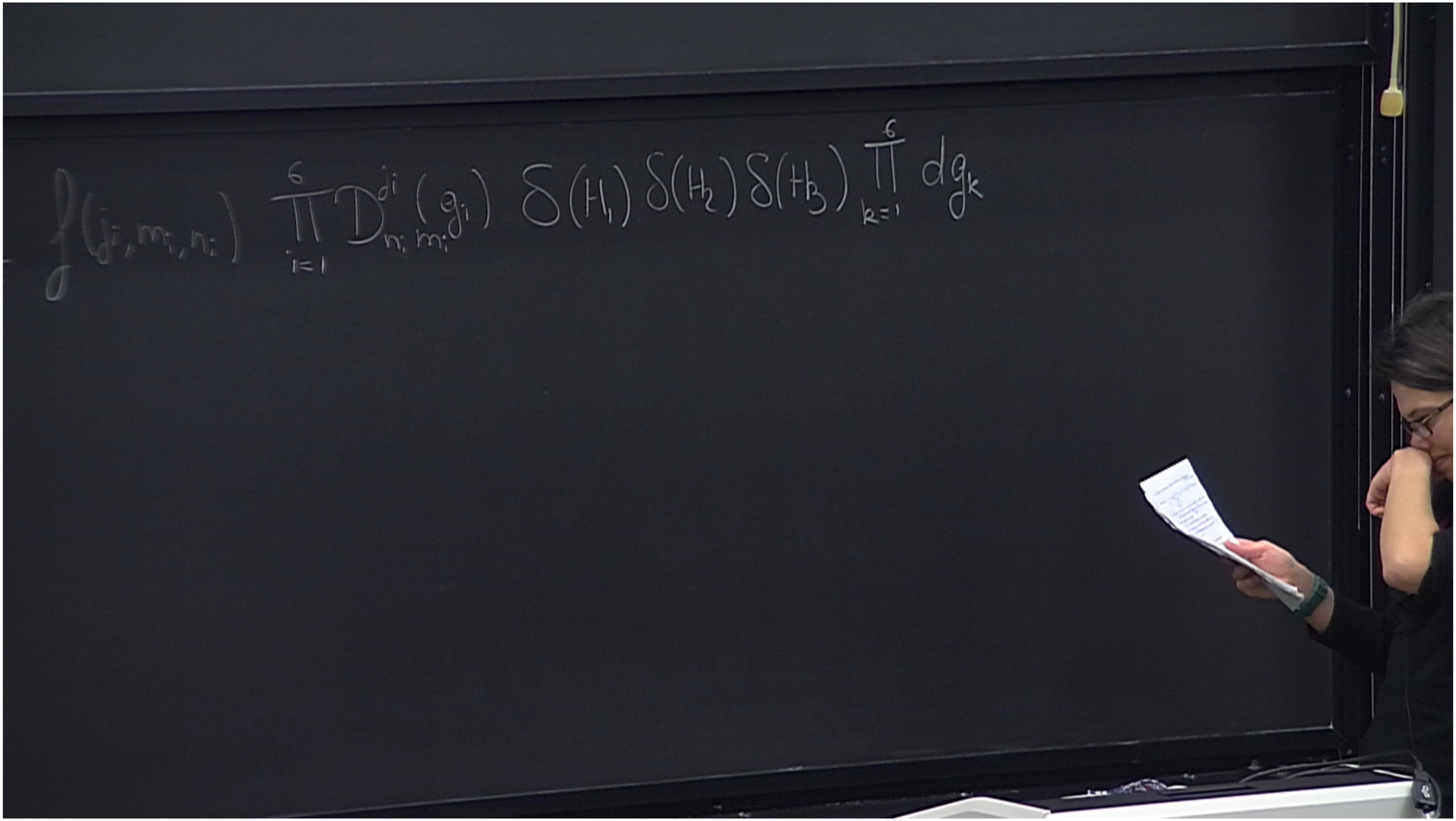
<S(H1)S(H2)S(H3)Ψ2 | S(H1)S(H2)S(H3)Ψ1 >phys := < Ψ2 | S(H1)S(H2)S(H3)Ψ1 >
Ψ1 = tetra [j1 j2 j3 ; j4 j5 j6] = Σ_{mi,ni} (-1)^{Σ_{i=1}^{6}(ji - mi)} (3nj)( )( )( ) (⊗_{i=1}^{6} | ji
f(ji, mi, ni).
Ψ2 = tetra [j1' j2' j3' ; j4' j5' j6']
Trans(ji', ji) = ∫_{G^6} Σ_{mi', ni'}

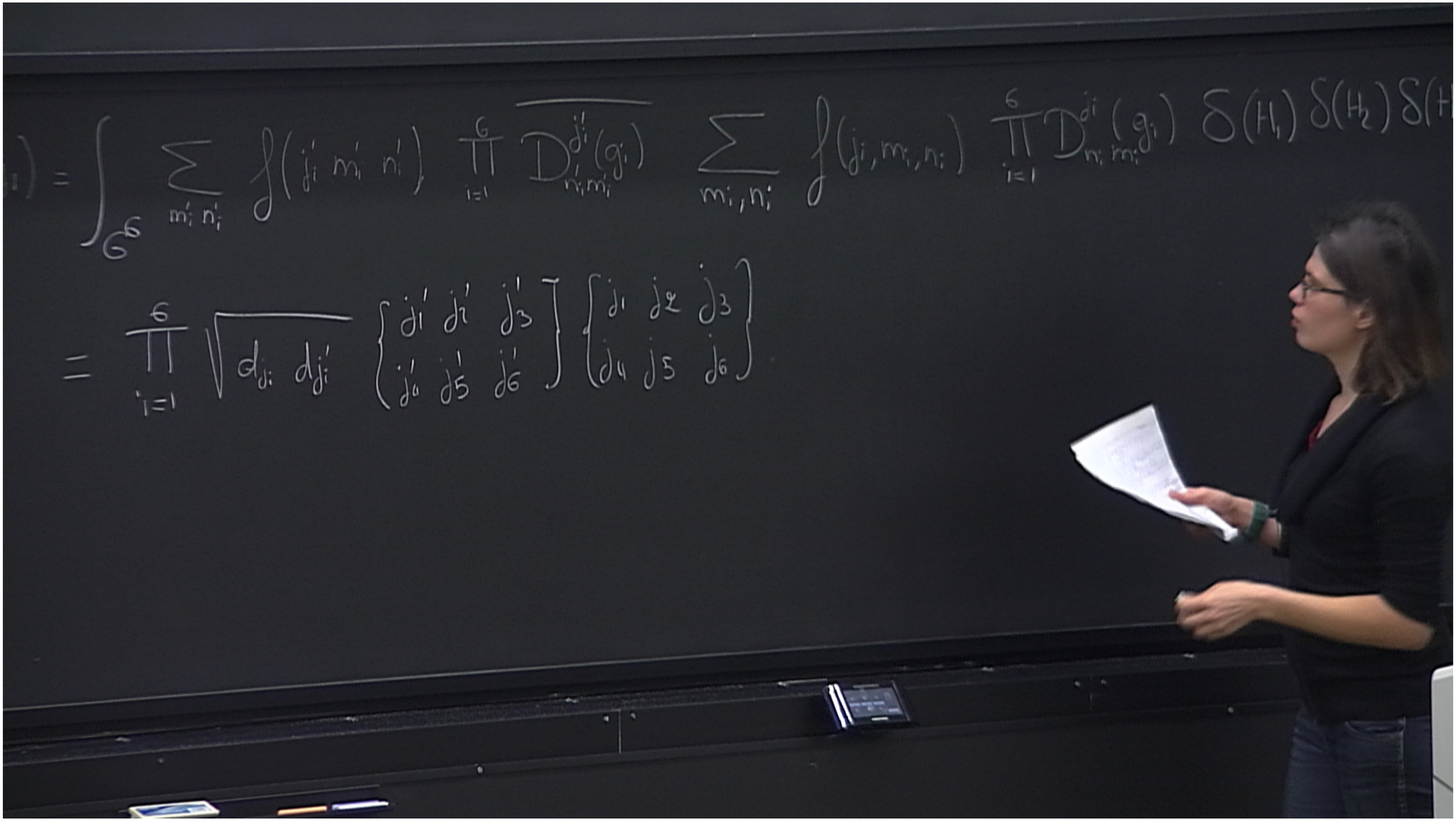
$$\mathrm{Trans}(j_i', j_i) = \int_{G^6} \sum_{m_i'\, n_i'} f(j_i'\, m_i'\, n_i') \prod_{i=1}^{6} \overline{D^{j_i'}_{n_i' m_i'}(g_i)} \sum_{m_i, n_i} f(j_i, m_i, n_i) \prod_{i=1}^{6} D^{j_i}_{n_i m_i}(g$$


$\int f(j_i, m_i, n_i) \prod_{i=1}^{6} D^{j_i}_{n_i m_i}(g_i) \, \delta$



$\langle \psi_2 | \delta(H_1) \delta(H_2) \delta(H_3) \psi_1 \rangle_{kin.} = \mathrm{trans}(j_i', j_i)$
$)(\quad)(\quad)\left(\bigotimes_{i=1}^{6} | j_i \, m_i \, n_i \rangle \right)$


$\int f(j_i, m_i, n_i) \prod_{i=1}^{6} D^{j_i}_{n_i m_i}(g_i)\, \delta(H_1)\,\delta(H_2)\,\delta(H_3) \prod_{k=1}^{6} dg_k$



$) = \int_{G^6} \sum_{m_i' n_i'} f(j_i' m_i' n_i') \prod_{i=1}^{6} \overline{D^{j_i'}_{n_i' m_i'}(g_i)} \sum_{m_i, n_i} f(j_i, m_i, n_i) \prod_{i=1}^{6} D^{j_i}_{n_i m_i}(g_i) \, \delta(H_1)\, \delta(H_2)\, \delta(H$
$= \prod_{i=1}^{6} \sqrt{d_{j_i} \, d_{j_i'}} \begin{Bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{Bmatrix} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$

$$\ldots) = \int_{G^6} \sum_{m_i' n_i'} f(j_i' \, m_i' \, n_i') \prod_{i=1}^{6} \overline{D^{j_i'}_{n_i' m_i'}(g_i)} \sum_{m_i, n_i} f(j_i, m_i, n_i) \prod_{i=1}^{6} D^{j_i}_{n_i m_i}(g_i) \, \delta(H_1) \, \delta(H_2) \, \delta(H \ldots$$

see Bianca's lecture notes

$$= \prod_{i=1}^{6} \sqrt{d_{j_i} \, d_{j_i'}} \begin{Bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{Bmatrix} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

$$\mathrm{ms}(j_i', j_i) = \int_{G^6} \sum_{m_i' n_i'} f(j_i'\, m_i'\, n_i') \prod_{i=1}^{6} \overline{D^{j_i'}_{n_i' m_i'}(g_i)} \sum_{m_i, n_i} f(j_i, m_i, n_i) \prod_{i=1}^{6} D^{j_i}_{n_i m_i}(g_i)\, \delta(H_1)\, \delta(\ldots$$

see Bianca's lecture notes

$$= \prod_{i=1}^{6} \sqrt{d_{j_i}\, d_{j_i'}} \begin{Bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{Bmatrix} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

$$\langle \psi_2 | \psi_1 \rangle = \sqrt{\langle \psi_2 | \psi_2 \rangle} \sqrt{\langle \psi_1 | \psi_1 \rangle}$$

$$\ldots ms(j_i', j_i) = \int_{G^6} \sum_{m_i' n_i'} f(j_i' \, m_i' \, n_i') \prod_{i=1}^{6} \overline{D^{j_i'}_{n_i' m_i'}(g_i)} \sum_{m_i, n_i} f(j_i, m_i, n_i) \prod_{i=1}^{6} D^{j_i}_{n_i m_i}(g_i) \, \delta(H_1) \, \delta(\ldots$$

see Bianca's lecture notes

$$= \prod_{i=1}^{6} \sqrt{d_{j_i} \, d_{j_i'}} \begin{Bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{Bmatrix} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

$$\langle \psi_2 | \psi_1 \rangle = \sqrt{\langle \psi_2 | \psi_2 \rangle} \sqrt{\langle \psi_1 | \psi_1 \rangle} \longrightarrow$$

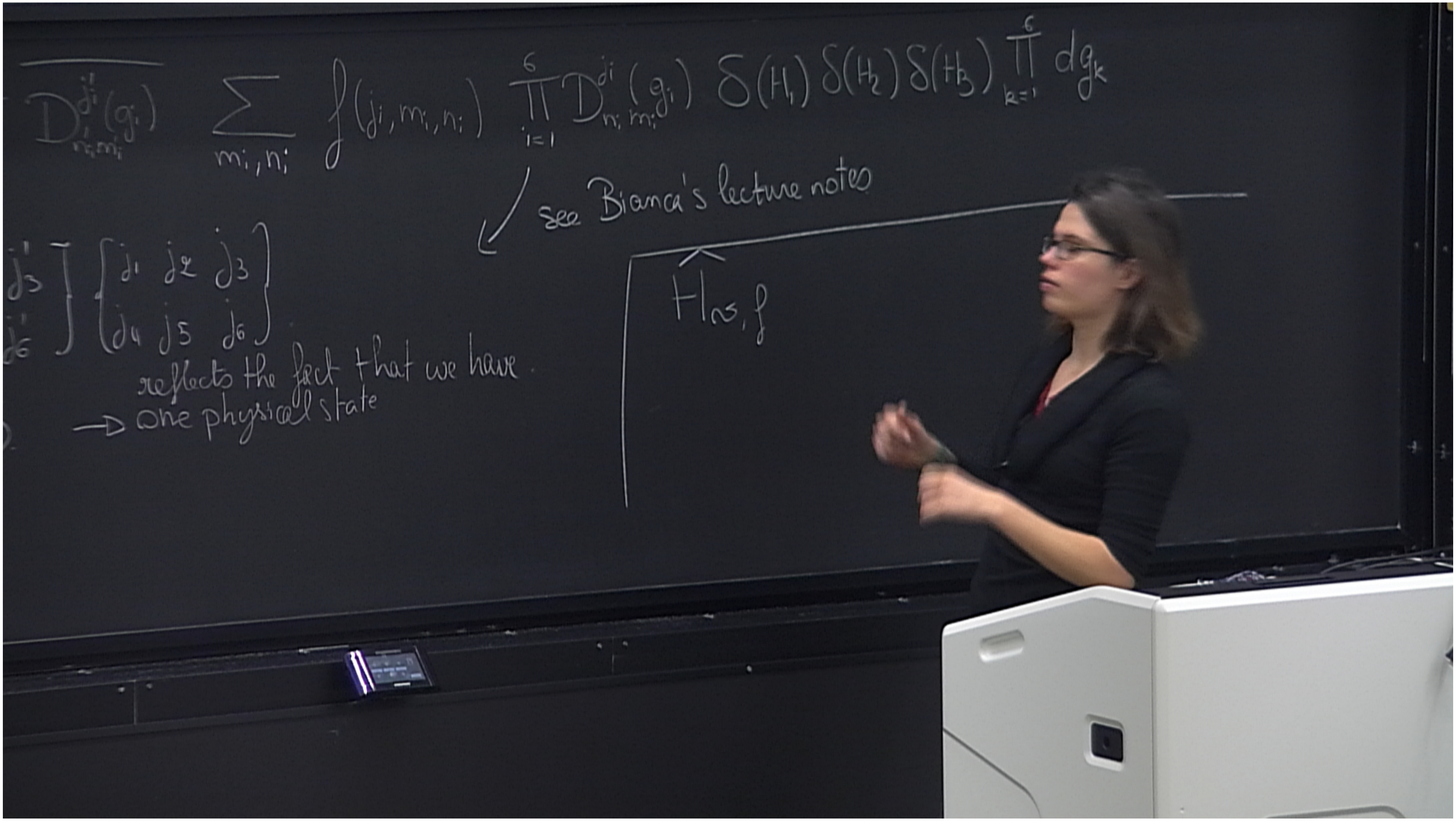
reflects the fact that we have one physical state

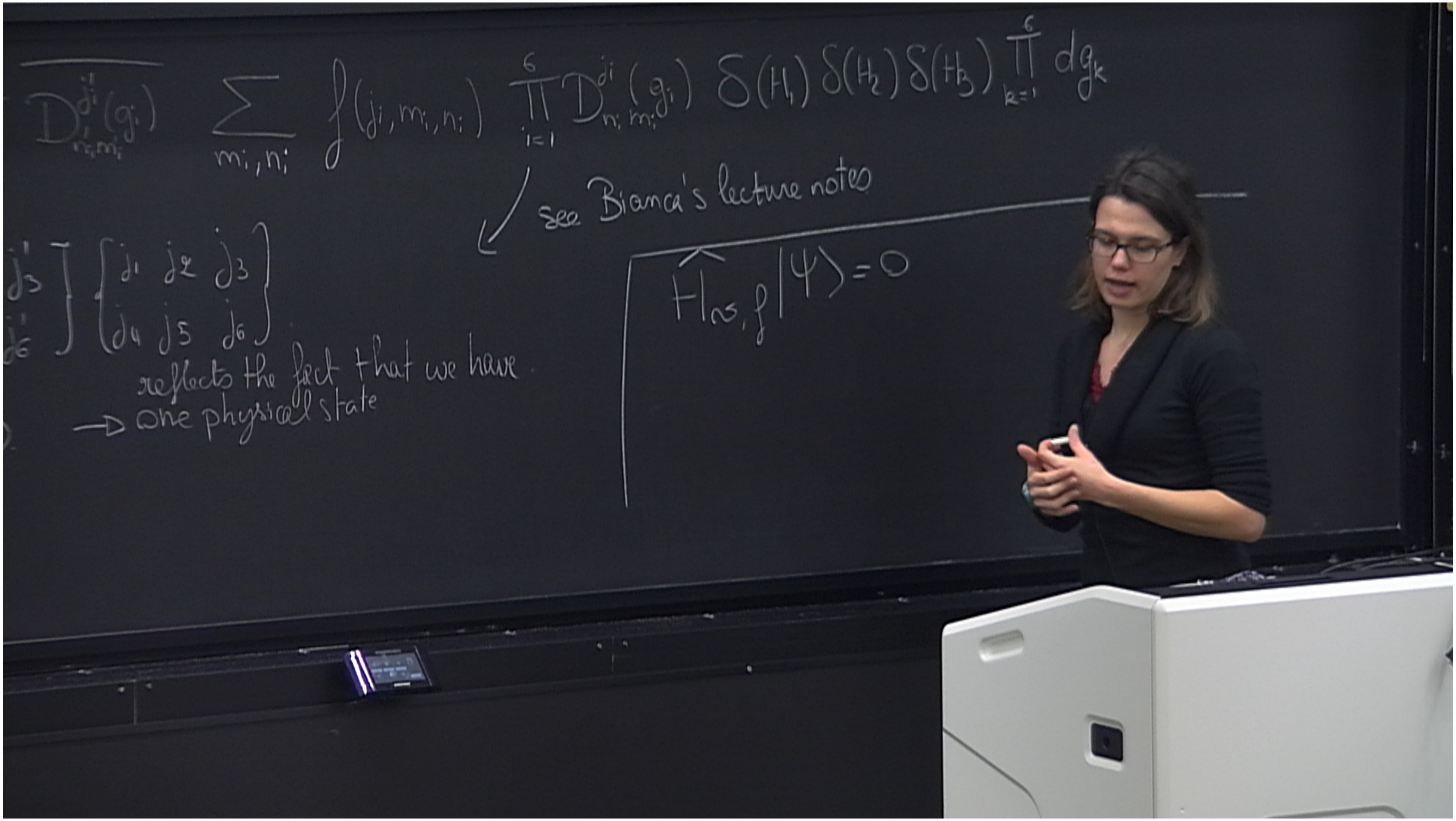
(see Bianca's lecture for the case of the parametrized particle)

$$\Psi_{phy}(j_i) = \left\langle \text{Tetra} \begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} \middle| \delta(H_1)\,\delta(H_2)\,\delta(H_3) \right\rangle$$

$$= \sqrt{d_{j_1} \cdots d_{j_6}}\ \underbrace{\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}}_{6j\text{-symbol}}$$

physical state in the spinnetwork basis for the triangulated 2-sphere.


see Bianca's lecture notes
reflects the fact that we have
one physical state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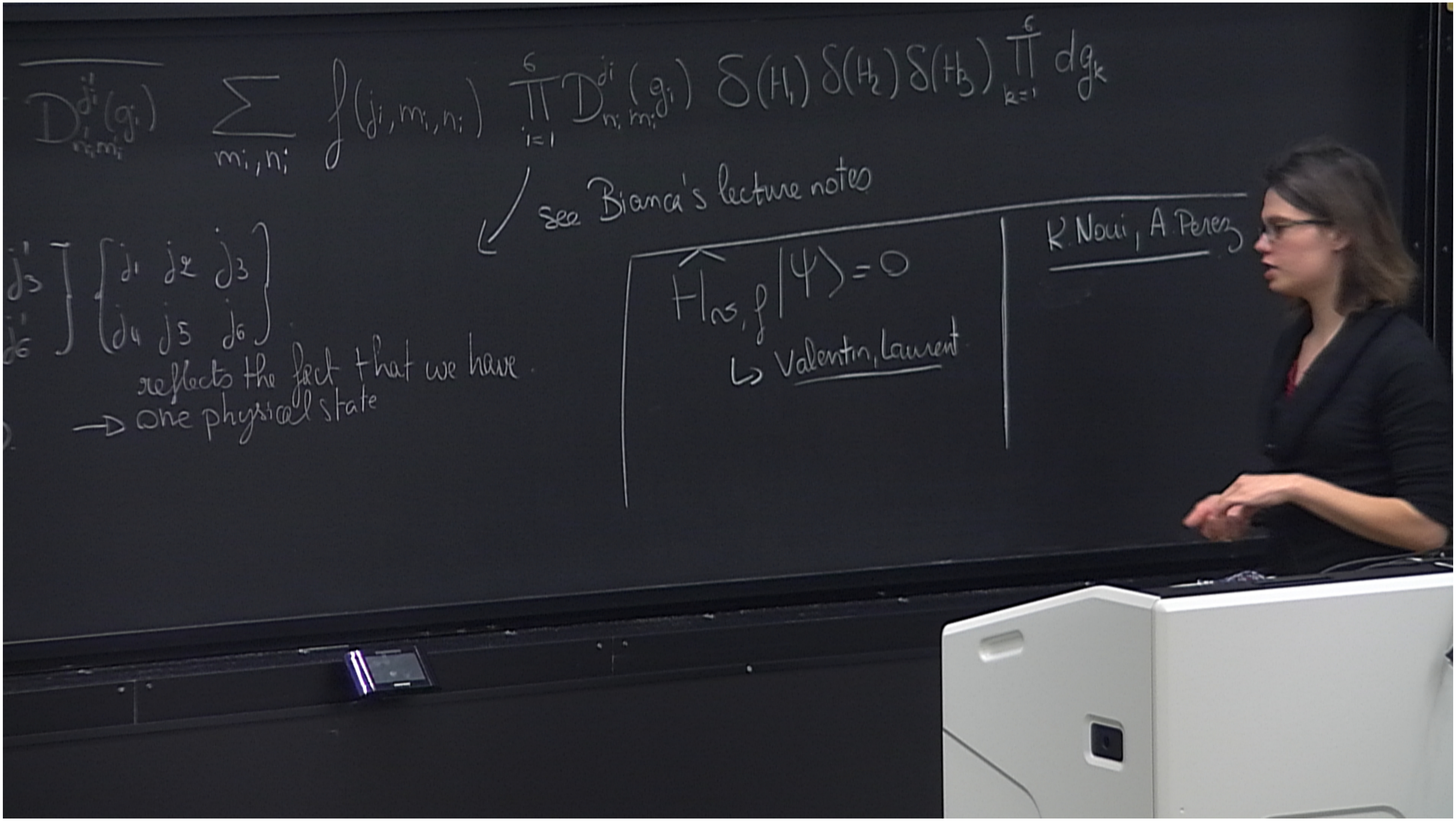
see Bianca's lecture notes
$\hat{H}_{\infty, f} |\Psi\rangle = 0$
Valentin, Laurent
K. Noui, A. Perez
reflects the fact that we have
one physical state