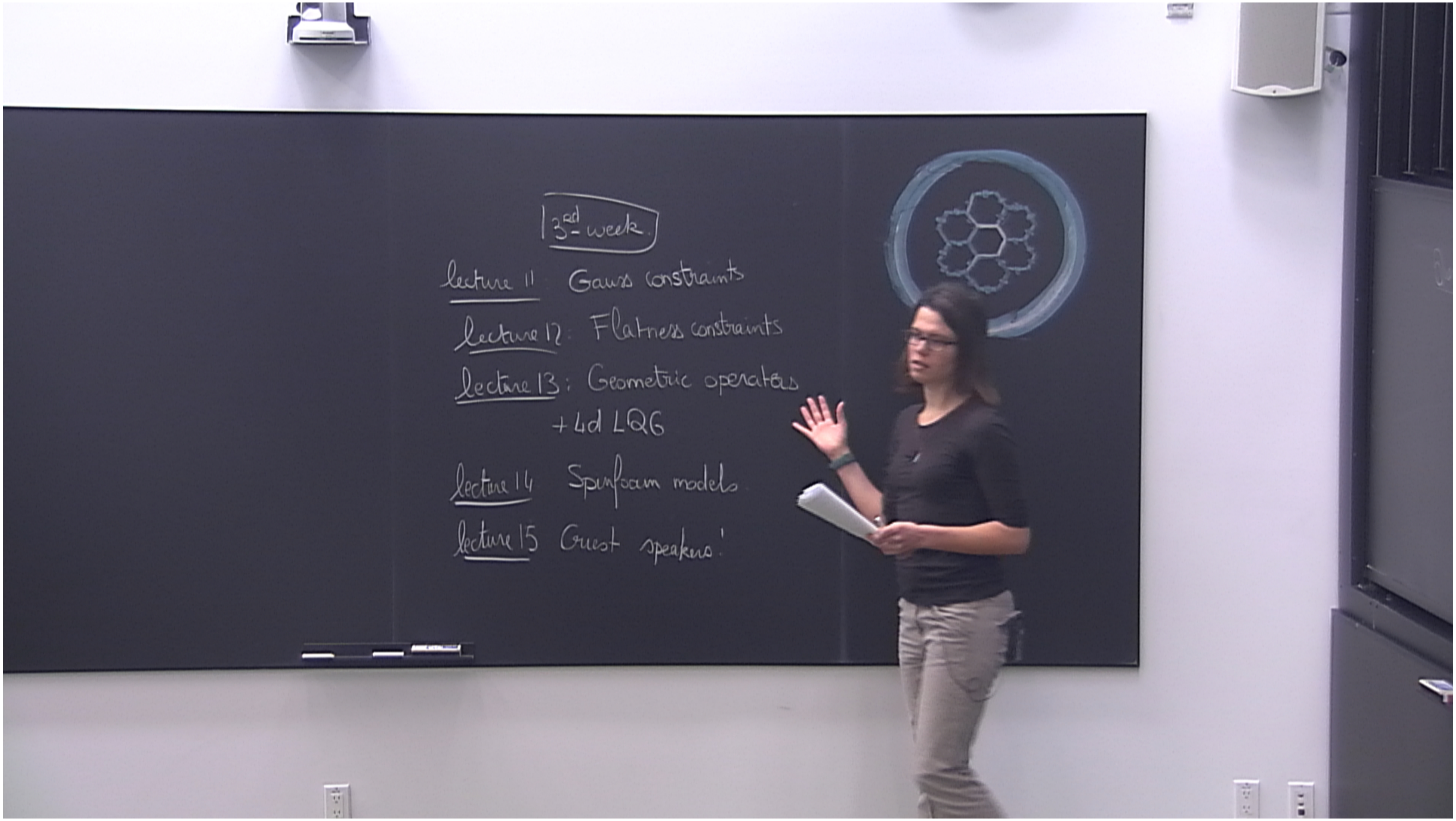
Title: PSI 2015/2016 Quantum Gravity - Lecture 11

Date: Mar 07, 2016 10:15 AM

URL: http://pirsa.org/16030049

Abstract:


3rd week
lecture 11 Gauss constraints
lecture 12: Flatness constraints
lecture 13: Geometric operators
+ 4d LQG
lecture 14 Spinfoam models
lecture 15 Guest speakers!

$\rightarrow$ fluxes act as derivatives...

· Inner product $\rightarrow$ Haar measure.

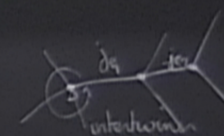
· ONB $\rightarrow$ matrix el^{ts} of irreducible rep of SU(2)

basis states $|j, m\, n\rangle$ $\qquad \langle g | j\, m\, n\rangle = D^j_{mn}(g)\sqrt{d_j}$

Left and Right action

Right · $(R_h D^j_{mn})(g) = D^j_{mn}(gh) = \sum_p D^j_{mp}(g)\, D^j_{pn}(h)$

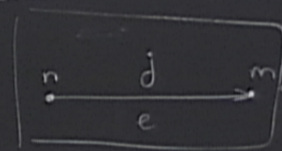
$\Rightarrow R_h |j\, m\, n\rangle = \sum_p D^j_{pn}(h)\, |j\, m\, p\rangle \rightarrow R_h$ acts as D^j on $\{|j\, m, p\rangle \,/\, p = -j \ldots j\}$



Left · $L_h |j\, m\, n\rangle = \sum_q D^j_{mq}(h^{-1})\, |j\, q\, n\rangle \rightarrow$ action on the dual representation

$(D^j_{mn})^*(g) = D^j_{nm}(g^{-1})$

= one edge of a spinnetwork



$\leftarrow$ Angular momentum associated to both ends of the edge

$\Rightarrow$ Hilbert space associated to one edge $\mathcal{H}_e = \mathcal{L}^2(G, d_H g) = \bigoplus_j (V_j \otimes V_j^*)$

Left and Right action

Right $\cdot\ (R_h D^j_{mn})(g) = D^j_{mn}(gh) = \sum_p D^j_{mp}(g)\, D^j_{pn}(h)$

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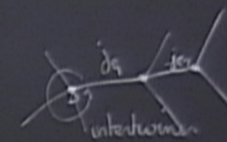

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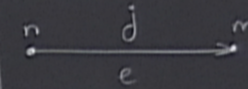


= one edge of a spinnetwork

Angular momentum associated to both ends of the edge

$\mathcal{H}_\Gamma$

$A \rightarrow h_e$: generalized connection : assignment of $h_e \in SU(2)$ to any path $e \subset \Sigma$

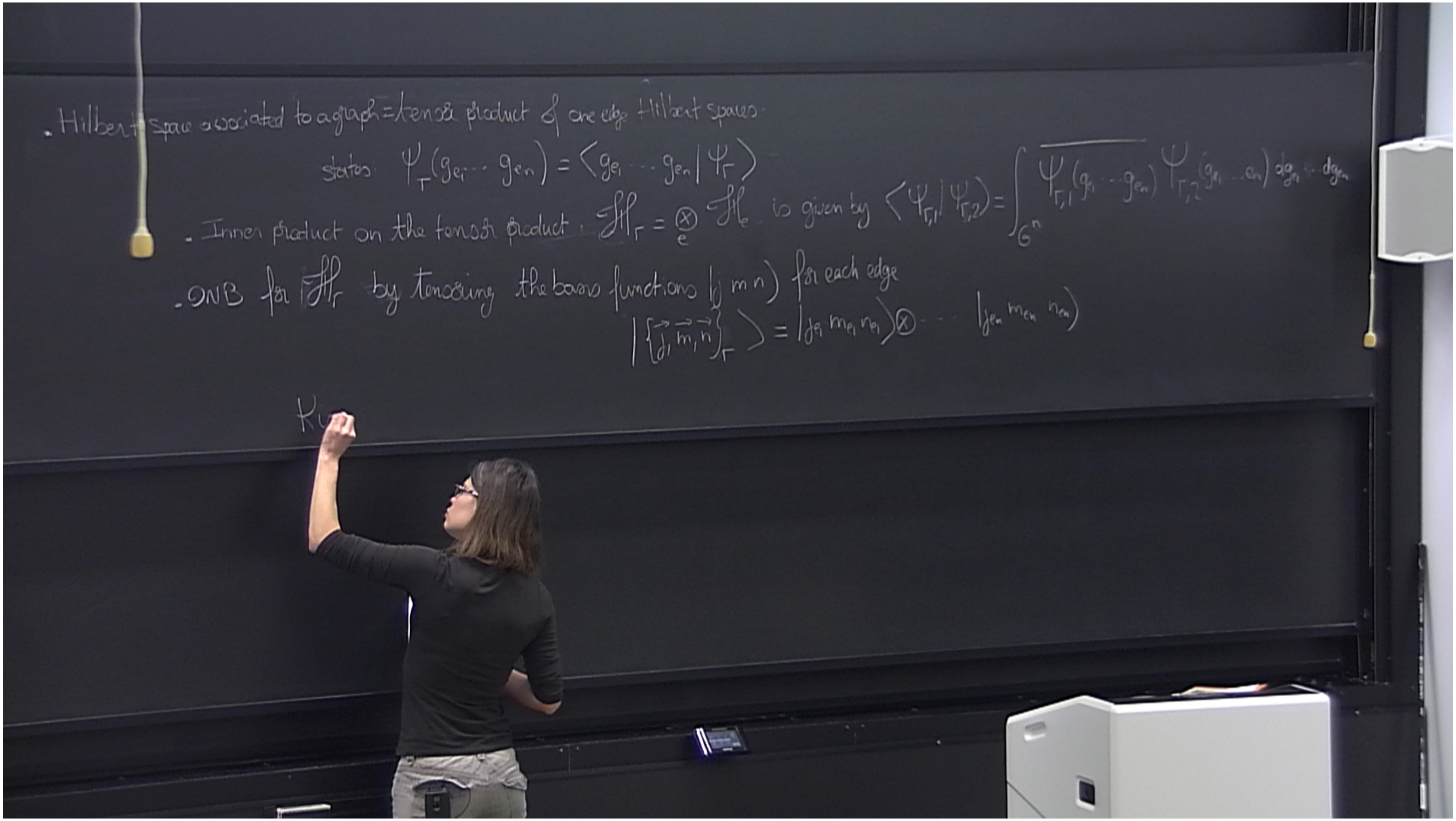
i.e. fundamental variable is the holonomy itself and not its relationship to a smooth connection

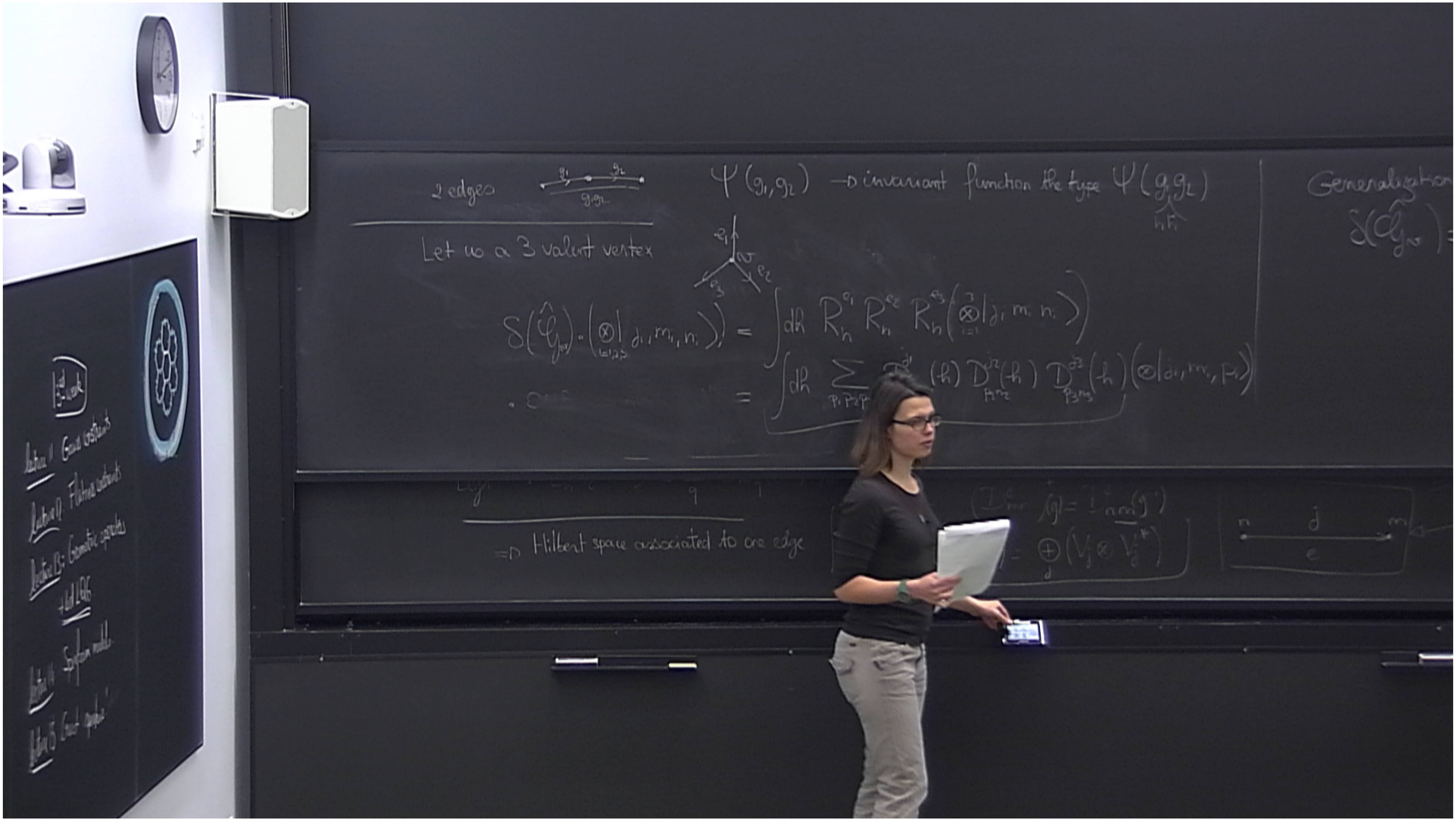
Cylindrical functions of generalized connections Cyl

$Cyl = \bigcup_\Gamma Cyl_\Gamma \qquad \psi_{\Gamma, f} \in Cyl \qquad f$ smooth function: $SU(2)^n \rightarrow \mathbb{C}$

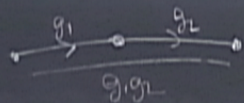
$\Rightarrow$ functional of the connection is defined as $\qquad n = \#$ of edges of Γ

$\psi_{\Gamma, f}[A] = f(h_{e_1}[A], \ldots h_{e_n}[A])$


. Hilbert space associated to a graph = tensor product of one edge Hilbert spaces.
states $\Psi_\Gamma(g_{e_1}\cdots g_{e_n}) = \langle g_{e_1}\cdots g_{e_n}|\Psi_\Gamma\rangle$
. Inner product on the tensor product: $\mathcal{H}_\Gamma = \bigotimes_e \mathcal{H}_e$ is given by $\langle\Psi_{\Gamma,1}|\Psi_{\Gamma,2}\rangle = \int_{G^n} \overline{\Psi_{\Gamma,1}(g_{e_1}\cdots g_{e_n})}\,\Psi_{\Gamma,2}(g_{e_1}\cdots g_{e_n})\, dg_{e_1}\cdots dg_{e_n}$
. ONB for $\mathcal{H}_\Gamma$ by tensoring the basis functions $|j\,m\,n)$ for each edge
$|\{\vec{j},\vec{m},\vec{n}\}_\Gamma\rangle = |j_{e_1} m_{e_1} n_{e_1})\otimes\cdots|j_{e_n} m_{e_n} n_{e_n})$
K


2 edges
→ invariant function the type
Let us a 3 valent vertex
⇒ Hilbert space associated to one edge
Generalization

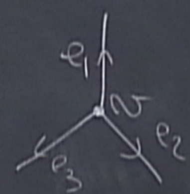
2 edges



$\Psi(g_1, g_2) \to$ invariant function the type $\Psi(g_1 \, g_2)$

$h\,h^{-1}$

Let us a 3 valent vertex



$$\delta(\hat{\varphi}_{v}) \circ \left(\bigotimes_{i=1,2,3} |j_i, m_i, n_i\rangle\right) = \int dh\, R_h^{e_1} R_h^{e_2} R_h^{e_3} \left(\bigotimes_{i=1}^{3} |j_i\, m_i\, n_i\rangle\right)$$

$$= \underbrace{\int dh \sum_{p_1 p_2 p_3} D^{j_1}_{p_1 n_1}(h)\, D^{j_2}_{p_2 n_2}(h)\, D^{j_3}_{p_3 n_3}(h)}\left(\otimes |j_i, m_i, p_i\rangle\right)$$

$g_L)$

$, m_i, p_i\rangle)$

Generalization of ⊛

$$\delta(\widetilde{\mathcal{G}}_v) = \int_G \Big(\overset{\text{in}}{\bigotimes_{e/v=v_t(e)}} L_h^{e}\Big) \otimes \Big(\overset{\text{out}}{\bigotimes_{e'/v=v_s(e')}} R_h^{e'}\Big)(dh)$$

v_s target

source

<u>State</u> $\langle h_e | \Psi_e \rangle = \Psi_e(h_e)$ so state transforms as $\Psi(g(v_t)\, h_e\, g^{-1}(v_s)) = \langle h_e | L_{g^{-1}(v_t)} R_{g^{-1}(v_s)} | \Psi_e \rangle$

→ Gauss constraint: a projector on the subspace of states satisfying the Gauss constraints by averaging over the group SU(2) for both ends of the edge

(Group averaging)

State based on one edge

$$\boxed{\delta(\hat{G}) | \Psi_e \rangle = \int L_{g_1} R_{g_2} | \Psi_e \rangle \, dg_1 \, dg_2} \overset{\circledast}{=} \text{cst}$$

$$\int_{G \times G} L_{g_1} R_{g_2} \Psi(h_e)\, dg_1 \, dg_2 = \int_{G \times G} \Psi(g_1^{-1} \underbrace{h_e g_2}_{g_2'})\, dg_1 \, dg_2 = \int_{G \times G} \Psi(g_1^{-1} g_2')\, dg_1 \, dg_2' = \text{cst}$$

- Hilbert space associated to a graph = tensor product of one edge Hilbert spaces.

states $\Psi_\Gamma(g_{e_1} \cdots g_{e_n}) = \langle g_{e_1} \cdots g_{e_n} | \Psi_\Gamma \rangle$

- Inner product on the tensor product. $\mathcal{H}_\Gamma = \bigotimes_e \mathcal{H}_e$ is given by $\langle \Psi_{\Gamma,1} | \Psi_{\Gamma,2} \rangle = \int_{G^n} \overline{\Psi_{\Gamma,1}(g_{e_1} \cdots g_{e_n})}\, \Psi_{\Gamma,2}(g_{e_1} \cdots g_{e_n})\, dg_{e_1} \cdots dg_{e_n}$

- ONB for $\mathcal{H}_\Gamma$ by tensoring the basis functions $|j\, m\, n\rangle$ for each edge

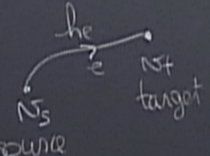
$$\left| \{\vec{j}, \vec{m}, \vec{n}\}_\Gamma \right\rangle = |j_{e_1} m_{e_1} n_{e_1}\rangle \otimes \cdots |j_{e_n} m_{e_n} n_{e_n}\rangle$$

Kinematical Hilbert space $\mathcal{H}_\Gamma = \bigotimes_e \mathcal{H}_e = \bigotimes_e \mathcal{L}^2(G) \cong \mathcal{L}^2(G^n)$

- $(G \sim \partial E + \varepsilon A E)$

- action $\Rightarrow$ generates internal rotation

On a holonomy h_e

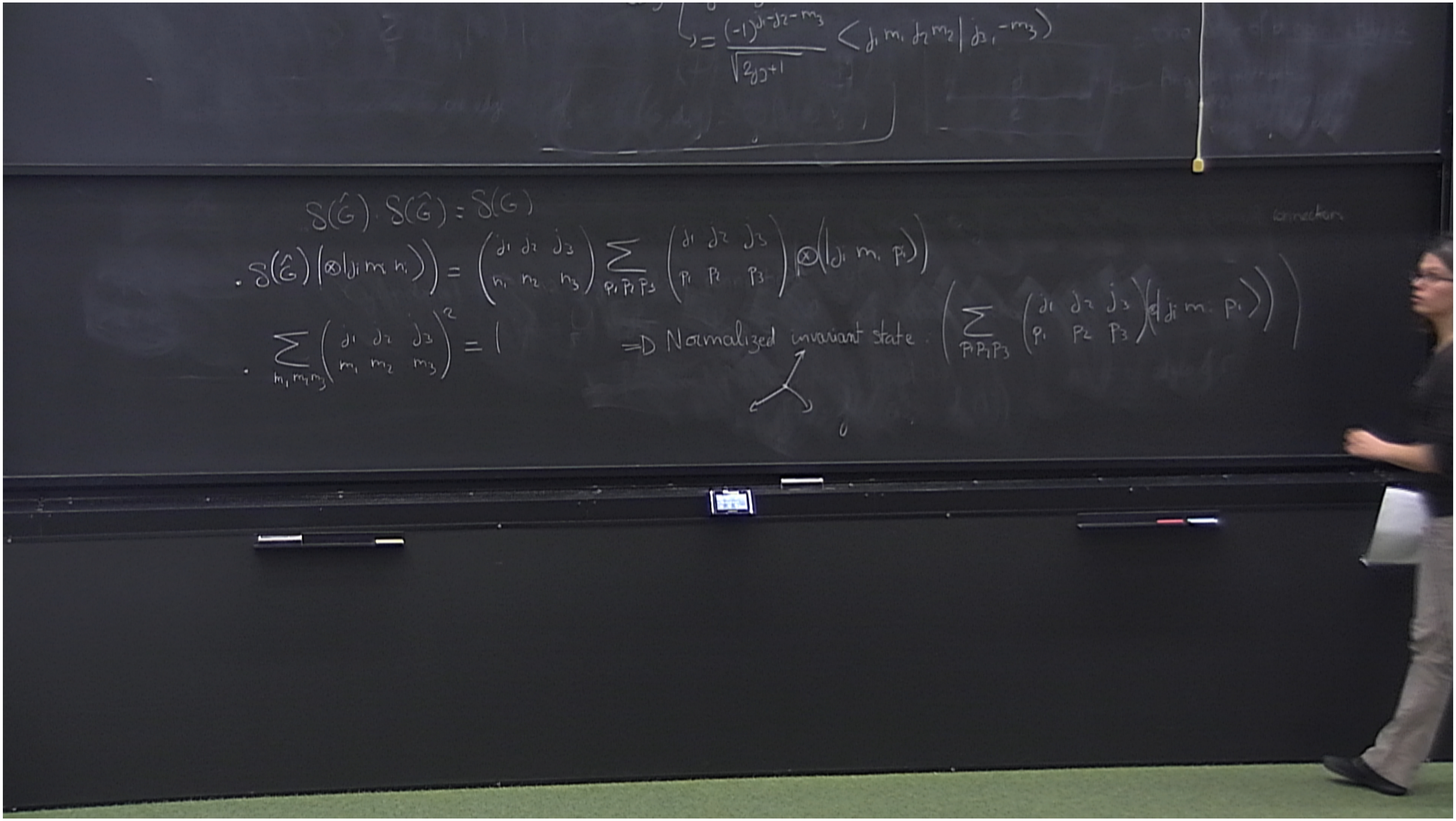


$h_e \to g(v_t)\, h_e\, g^{-1}(v_s)$

State $\langle h_e | \Psi_e \rangle = \Psi_e(h_e)$ so state transforms as $\Psi(g(v_t)\, h_e\, g^{-1}(v_s)) = \langle h_e | L_{g^{-1}(v_t)} R_{g^{-1}(v_s)} | \Psi_e \rangle$

... projector on the subspace of states satisfying the Gauss constraints by averaging over the group SU(2)

... both end of the edge


$= \frac{(-1)^{j_1-j_2-m_3}}{\sqrt{2j_3+1}} \langle j_1 m_1 j_2 m_2 | j_3, -m_3 \rangle$
$S(\hat{G}) \cdot S(\hat{G}) = S(G)$
$S(\hat{G}) \left(\otimes |j_i m_i n_i \rangle \right) = \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix} \sum_{p_1 p_2 p_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \otimes \left(|j_i m_i p_i \rangle \right)$
$\sum_{m_1 m_2 m_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}^2 = 1$
$\Rightarrow$ Normalized invariant state
$\left(\sum_{p_1 p_2 p_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \left(\otimes |j_i m_i p_i \rangle \right) \right)$
connection