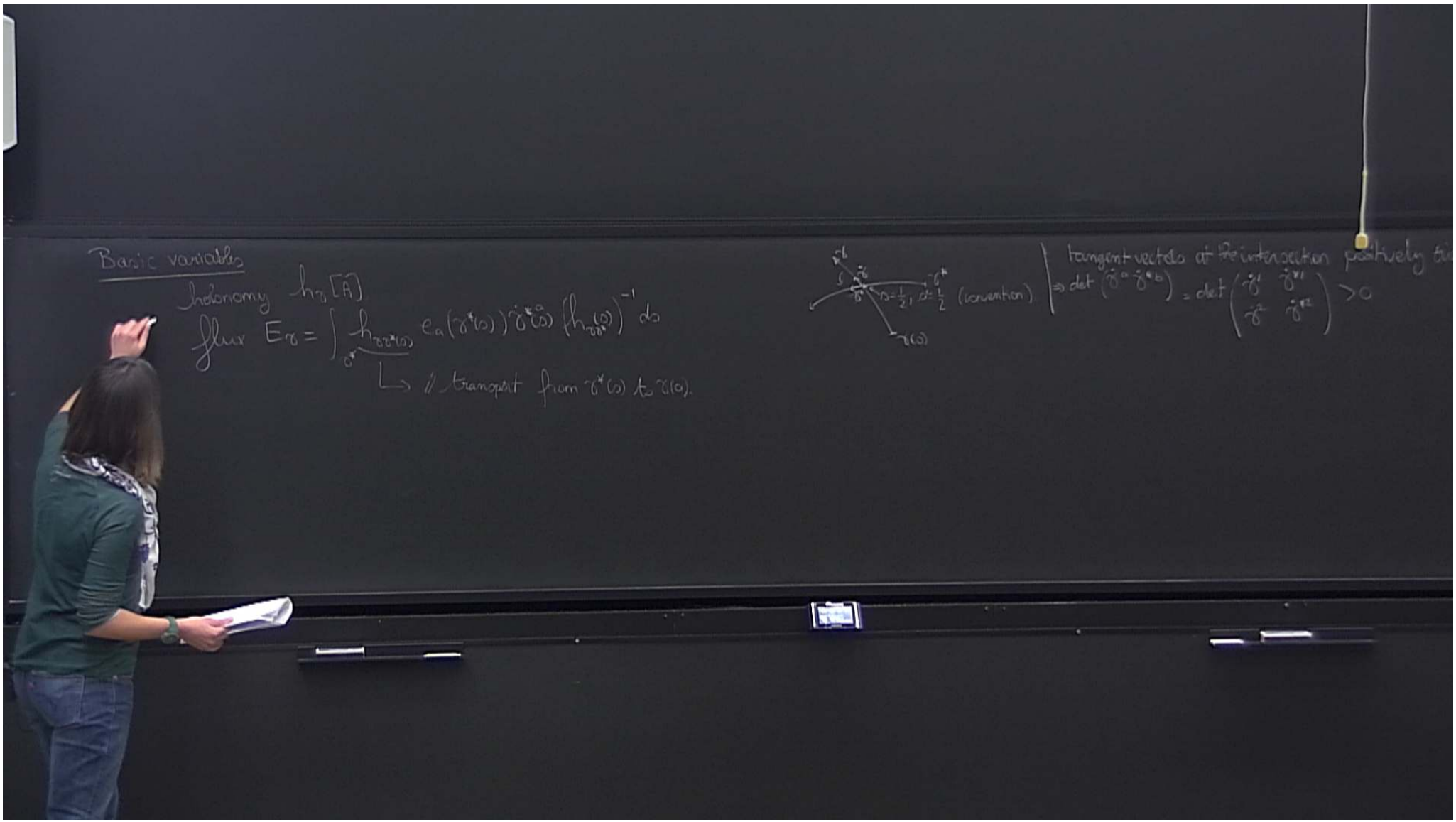
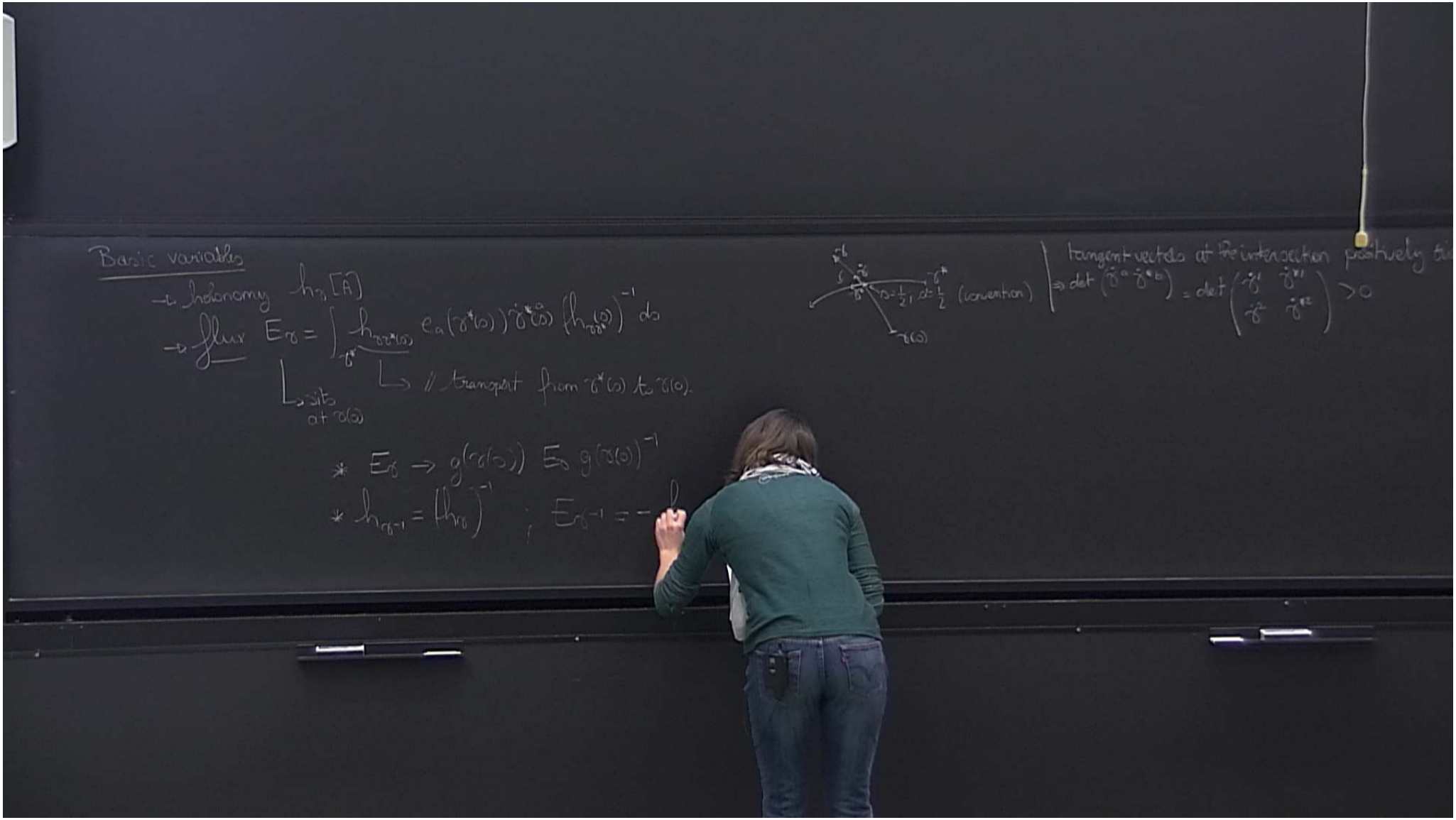
Title: PSI 2015/2016 Quantum Gravity - Lecture 8

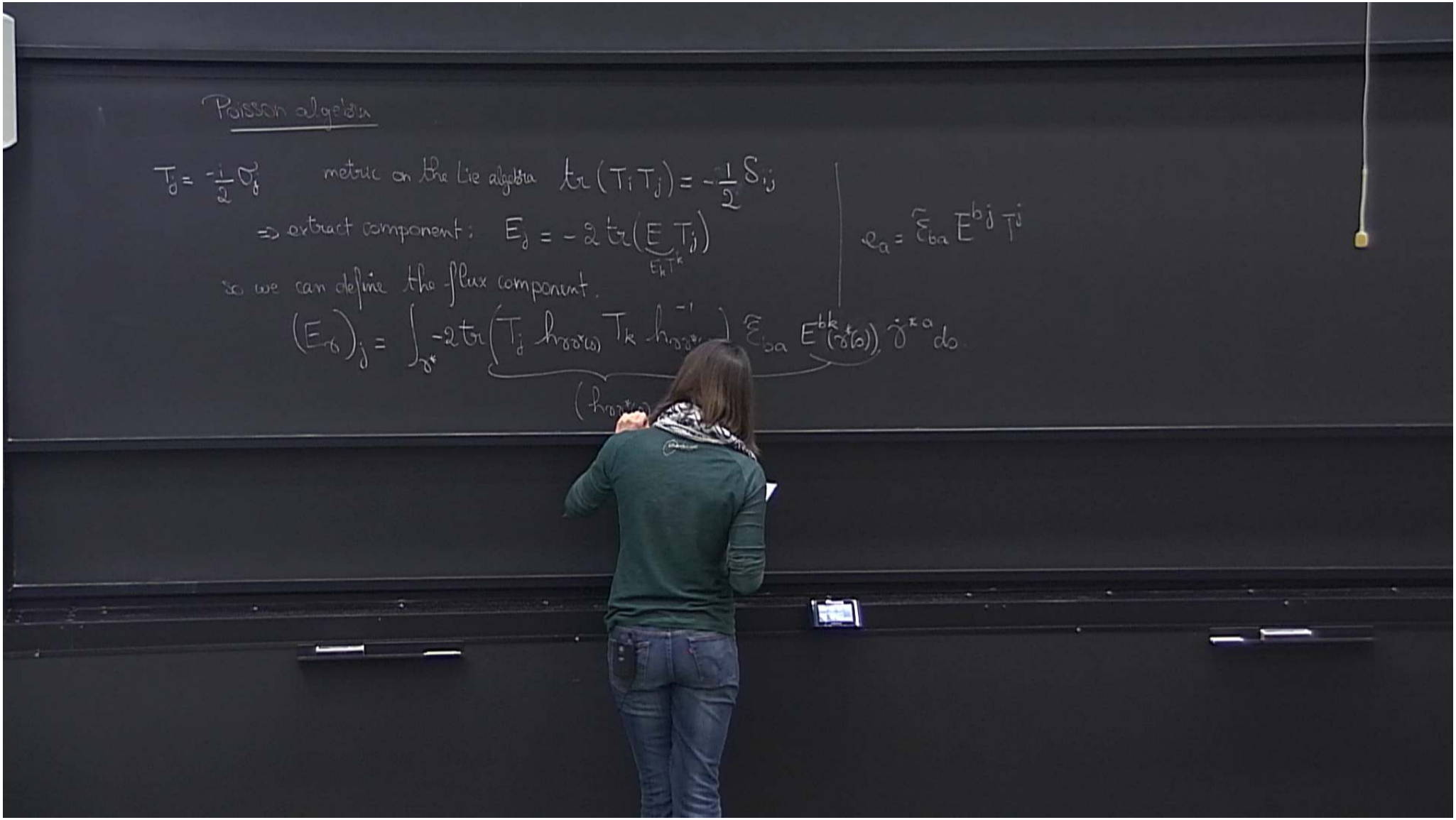
Date: Mar 02, 2016 10:15 AM

URL: http://pirsa.org/16030046

Abstract:


Basic variables
holonomy $h_\gamma[A]$
flux $E_\gamma = \int_{\gamma^*} h_{\gamma\gamma^*(s)}\, e_a(\gamma^*(s))\, \dot\gamma^{*a}(s)\, (h_{\gamma\gamma^*}(s))^{-1}\, ds$
// transport from $\gamma^*(s)$ to $\gamma(0)$.
$s=\frac{1}{2}$, $\alpha=\frac{1}{2}$ (convention)
$\gamma(0)$
tangent vectors at the intersection positively
$\Rightarrow \det(\dot\gamma^a \dot\gamma^{*b}) = \det\begin{pmatrix} \dot\gamma^1 & \dot\gamma^{*1} \\ \dot\gamma^2 & \dot\gamma^{*2} \end{pmatrix} > 0$


Basic variables
→ holonomy $h_\gamma[A]$
→ flux $E_\gamma = \int_{\gamma^*} h_{\gamma\gamma^*(s)}\, e_a(\gamma^*(s))\, \dot\gamma^{*a}(s)\, (h_{\gamma\gamma^*}(s))^{-1}\, ds$
↳ sits at $\gamma(0)$
↳ // transport from $\gamma^*(s)$ to $\gamma(0)$.
* $E_\gamma \to g(\gamma(0))\, E_\gamma\, g(\gamma(0))^{-1}$
* $h_{\gamma^{-1}} = (h_\gamma)^{-1}$; $E_{\gamma^{-1}} = -$
$s=\frac{1}{2}$, $\alpha=\frac{1}{2}$ (convention)
$\gamma(0)$
tangent vectors at the intersection positively
$\Rightarrow \det(\dot\gamma^a \dot\gamma^{*b}) = \det\begin{pmatrix} \dot\gamma^1 & \dot\gamma^{*1} \\ \dot\gamma^2 & \dot\gamma^{*2} \end{pmatrix} > 0$


Poisson algebra
$T_\delta = -\frac{i}{2}\sigma_\delta$ metric on the Lie algebra $\mathrm{tr}(T_i T_j) = -\frac{1}{2}\delta_{ij}$
$\Rightarrow$ extract component: $E_j = -2\,\mathrm{tr}(E\,T_j)$
$E_k T^k$
$e_a = \varepsilon_{ba} E^{bj} \tau^j$
so we can define the flux component.
$(E_\sigma)_j = \int_{\gamma^*} -2\,\mathrm{tr}\left(T_j\, h_{\sigma\gamma(s)}\, T_k\, h^{-1}_{\sigma\gamma^*(s)}\right) \varepsilon_{ba}\, E^{bk}(\sigma(s))\, \dot\gamma^a\, ds$
$(h_{\sigma\gamma^*(s)})$

Poisson algebra

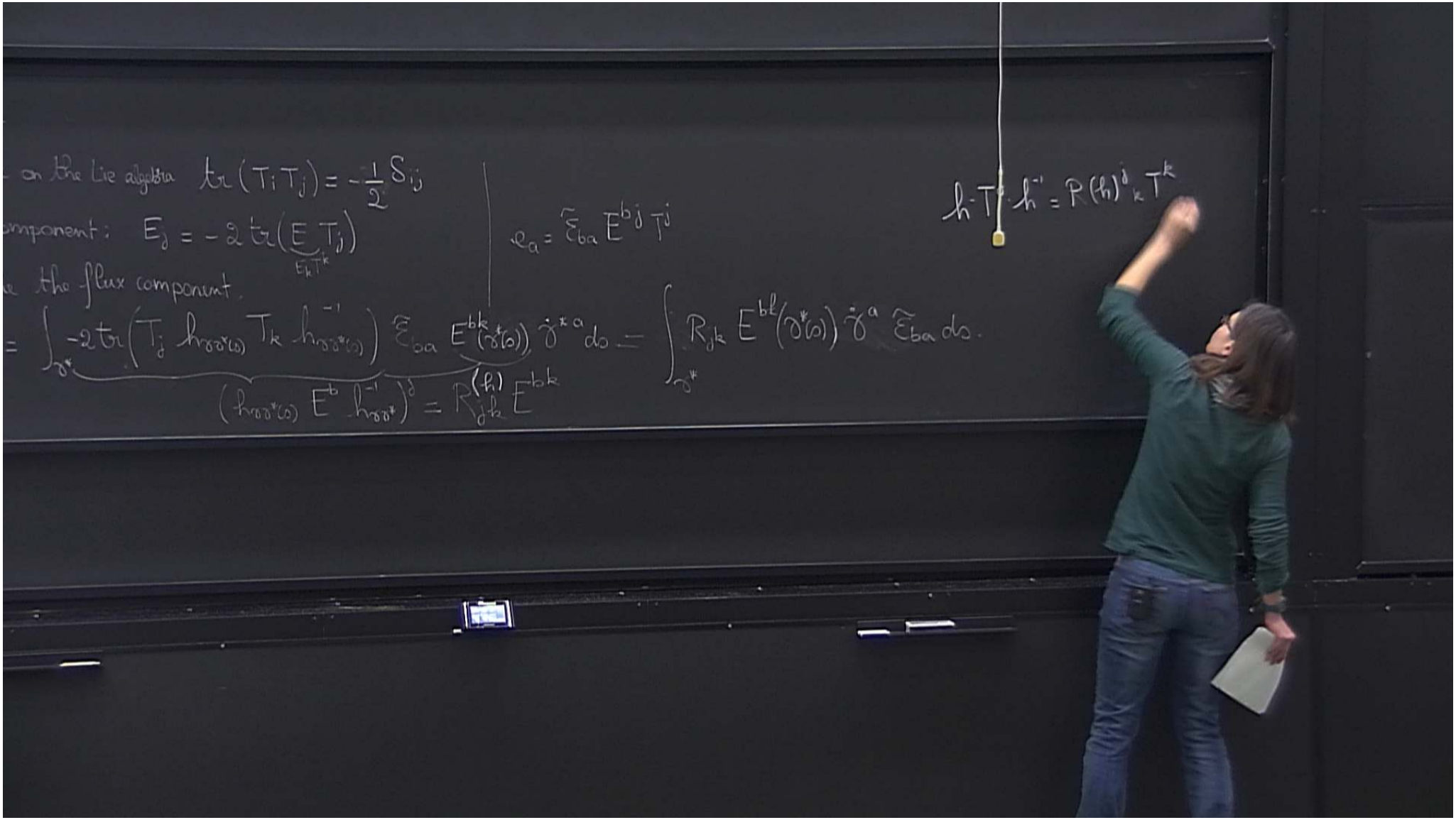
$T_0 = -\frac{i}{2}\sigma_j$ metric on the Lie algebra $\mathrm{tr}(T_i T_j) = -\frac{1}{2}\delta_{ij}$

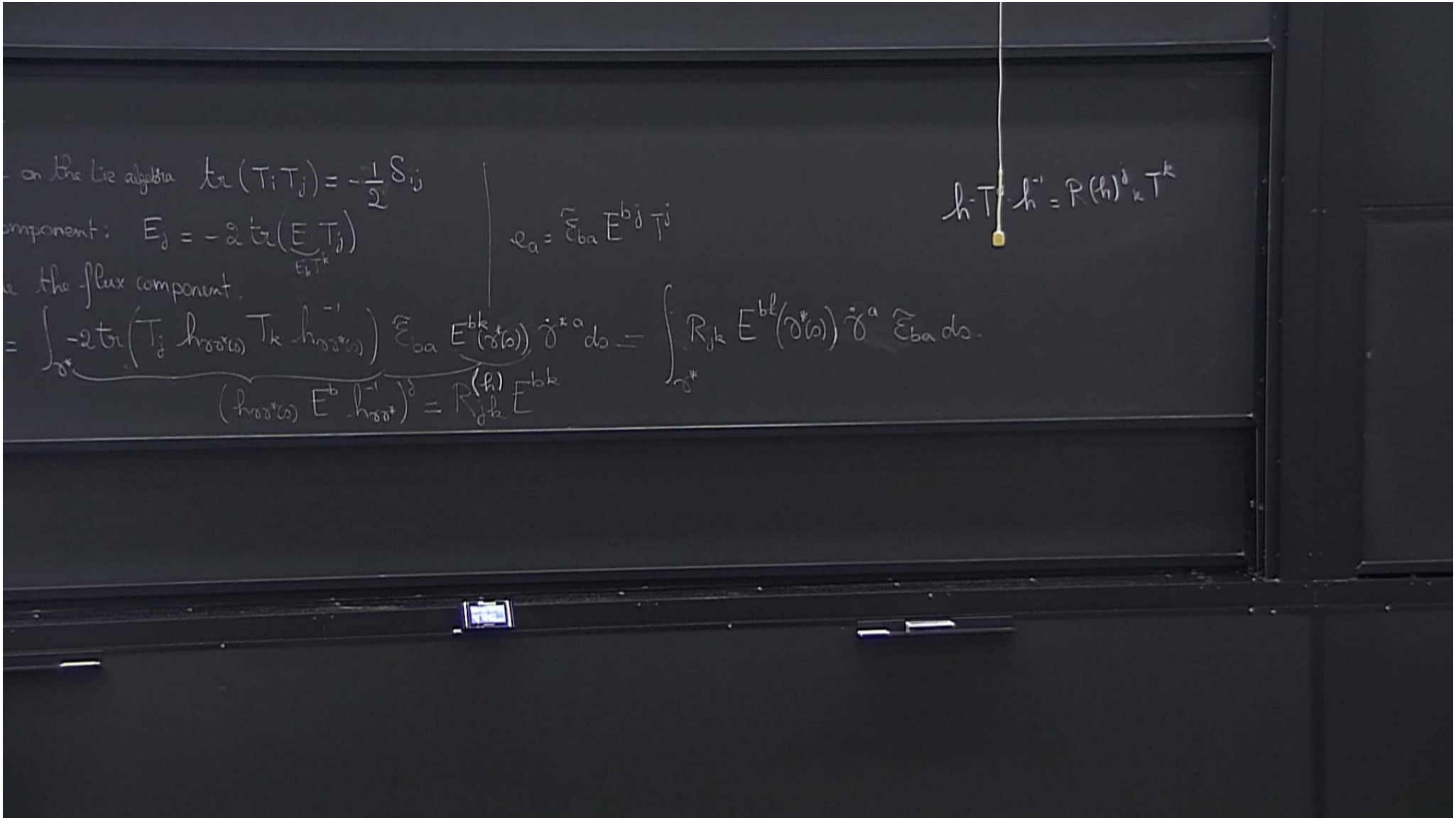
$\Rightarrow$ extract component: $E_j = -2\,\mathrm{tr}(\underbrace{E}_{E_k T^k} T_j)$

$e_a = \tilde{\varepsilon}_{ba} E^{bj} \dot{T}^j$

so we can define the flux component.

$$(E_\sigma)_j = \int_{\gamma^*} \underbrace{-2\,\mathrm{tr}\left(T_j\, h_{\gamma\sigma^*(s)}\, T_k\, h^{-1}_{\gamma\sigma^*(s)}\right)}_{\left(h_{\gamma\sigma^*(s)}\, E^b\, h^{-1}_{\gamma\sigma^*}\right)^j = R_{jk} E^{bk}} \tilde{\varepsilon}_{ba}\, E^{bk}(\sigma^*(s))\, \dot{\gamma}^a\, ds = \int_{\gamma^*} R_{jk}\, E^{bk}(\sigma^*(s))\, \dot{\gamma}^a\, \tilde{\varepsilon}_{ba}\, ds.$$


on the Lie algebra $\mathrm{tr}(T_i T_j) = -\frac{1}{2} S_{ij}$
component: $E_j = -2\,\mathrm{tr}(E_k T^k T_j)$
the flux component
$e_a = \tilde{\varepsilon}_{ba} E^{bj} T^j$
$h \cdot T^j \cdot h^{-1} = R(h)^j{}_k T^k$
$= \int_{\gamma^*} -2\,\mathrm{tr}\left(T_j\, h_{\gamma\sigma^*(\sigma)} T_k h^{-1}_{\gamma\sigma^*(\sigma)}\right) \tilde{\varepsilon}_{oa} E^{bk}(\gamma^*(\sigma))\, \dot{\gamma}^{*a}\, d\sigma = \int_{\gamma^*} R_{jk} E^{bk}(\gamma^*(\sigma))\, \dot{\gamma}^a\, \tilde{\varepsilon}_{ba}\, d\sigma.$
$\left(h_{\gamma\sigma^*(\sigma)} E^b h^{-1}_{\gamma\sigma^*}\right)^j = R^{(h)}_{jk} E^{bk}$



on the Lie algebra $\mathrm{tr}(T_i T_j) = -\frac{1}{2}\delta_{ij}$

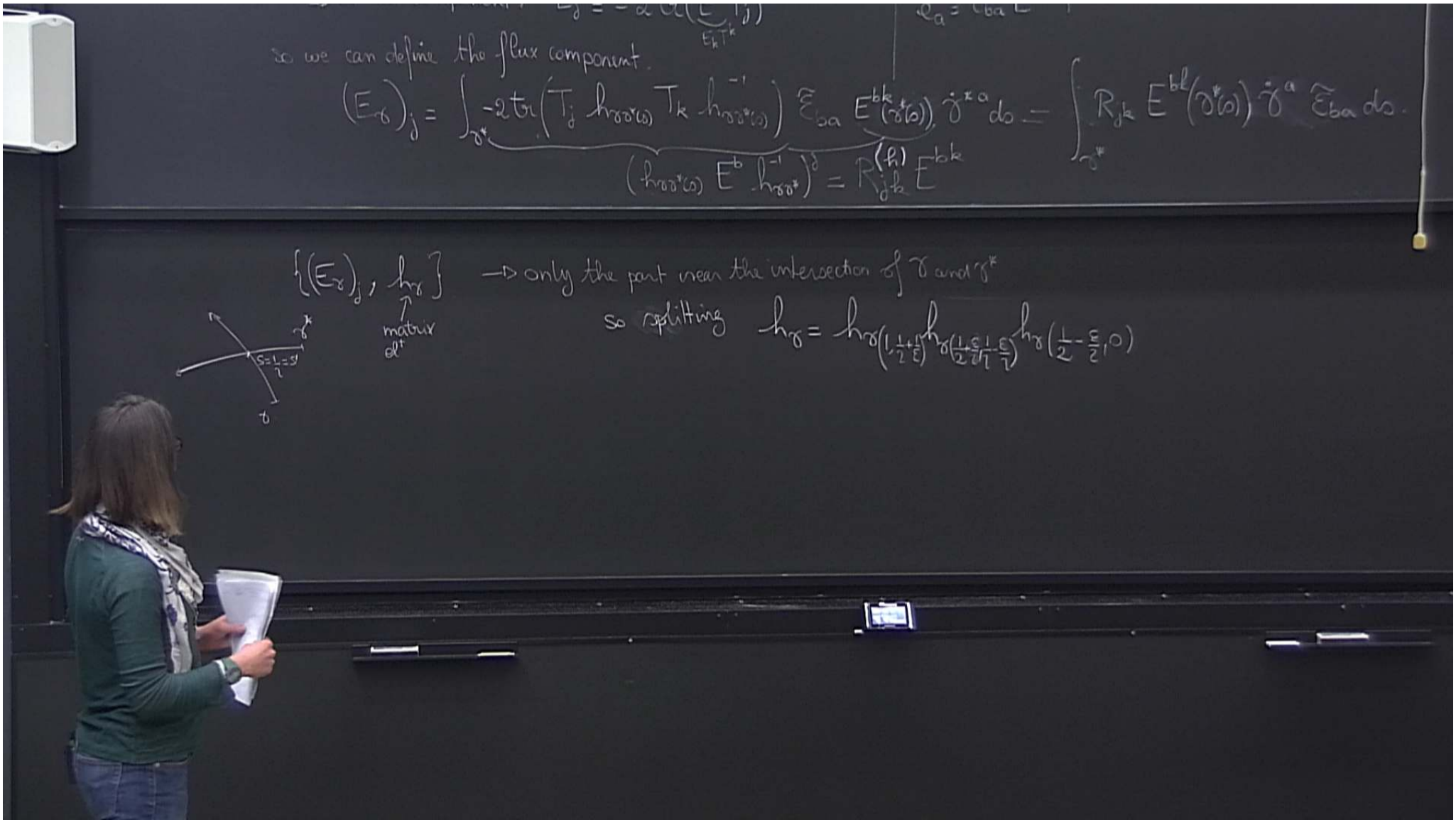
omponent: $E_j = -2\,\mathrm{tr}(\underbrace{E}_{E_k T^k} T_j)$

e the flux component.

$$= \int_{\gamma^*} \underbrace{-2\,\mathrm{tr}\left(T_j\, h_{\gamma\gamma^*(s)}\, T_k\, h^{-1}_{\gamma\gamma^*(s)}\right)}_{\left(h_{\gamma\gamma^*(s)} E^b h^{-1}_{\gamma\gamma^*}\right)^j = R^{(h)}_{jk} E^{bk}} \tilde{\varepsilon}_{ba}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^{*a}\, ds = \int_{\gamma^*} R_{jk}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^a\, \tilde{\varepsilon}_{ba}\, ds.$$

$$e_a = \tilde{\varepsilon}_{ba} E^{bj} T^j$$

$$h \cdot T^j \cdot h^{-1} = R(h)^j{}_k T^k$$


so we can define the flux component.
$(E_\gamma)_j = \int_{\gamma^*} -2\,\mathrm{tr}\left(T_j\, h_{\gamma\gamma^*(s)}\, T_k\, h^{-1}_{\gamma\gamma^*(s)}\right) \tilde{\varepsilon}_{ba}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^{*a}\, ds = \int_{\gamma^*} R_{jk}\, E^{bl}(\gamma^*(s))\, \dot{\gamma}^a\, \tilde{\varepsilon}_{ba}\, ds.$
$(h_{\gamma\gamma^*(s)}\, E^b\, h^{-1}_{\gamma\gamma^*})^j = R^{(h)}_{jk}\, E^{bk}$
$\{(E_\gamma)_j, h_\gamma\}$ → only the part near the intersection of γ and γ^*
matrix el^t
so splitting $h_\gamma = h_{\gamma(1,\frac{1}{2}+\frac{\varepsilon}{2})}\, h_{\gamma(\frac{1}{2}+\frac{\varepsilon}{2},\frac{1}{2}-\frac{\varepsilon}{2})}\, h_{\gamma(\frac{1}{2}-\frac{\varepsilon}{2},0)}$
γ^*
$s=\frac{1}{2}=s'$
γ

so we can define the flux component.

$$(E_\gamma)_j = \int_{\gamma^*} \underbrace{-2\,\mathrm{tr}\left(T_j\, h_{\gamma\gamma^*(s)}\, T_k\, h^{-1}_{\gamma\gamma^*(s)}\right)}_{\left(h_{\gamma\gamma^*(s)} E^b h^{-1}_{\gamma\gamma^*}\right)^j = R^{(h)}_{jk} E^{bk}} \tilde{\varepsilon}_{ba}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^{*a}\, ds = \int_{\gamma^*} R_{jk}\, E^{bl}(\gamma^*(s))\, \dot{\gamma}^a\, \tilde{\varepsilon}_{ba}\, ds.$$

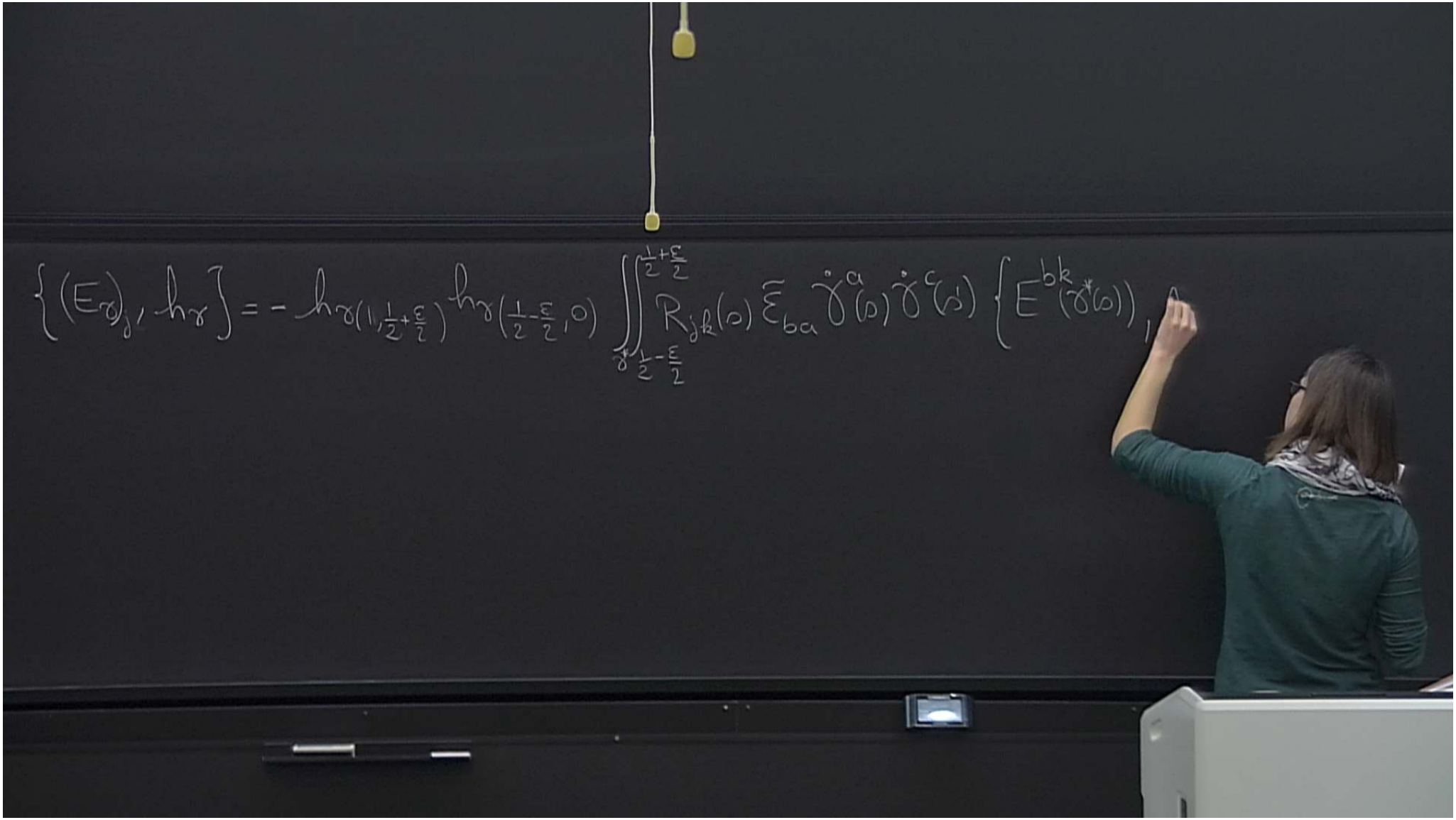
$\{(E_\gamma)_j, h_\gamma\}$ → only the part near the intersection of γ and γ^*

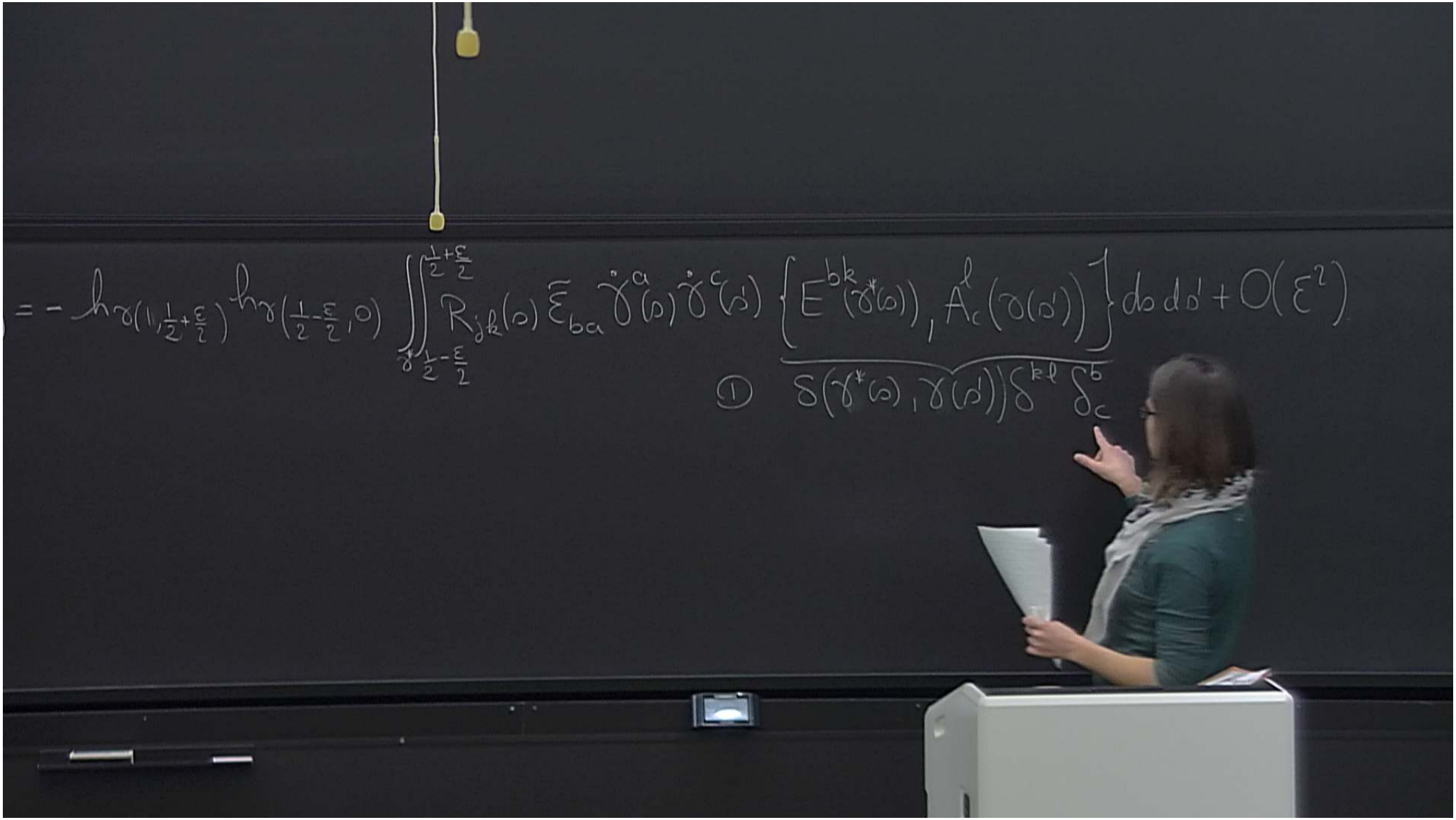
matrix d^+

so splitting $h_\gamma = h_{\gamma(1,\frac{1}{2}+\frac{\varepsilon}{2})}\, h_{\gamma(\frac{1}{2}+\frac{\varepsilon}{2},\frac{1}{2}-\frac{\varepsilon}{2})}\, h_{\gamma(\frac{1}{2}-\frac{\varepsilon}{2},0)}$ -

commute with the flux

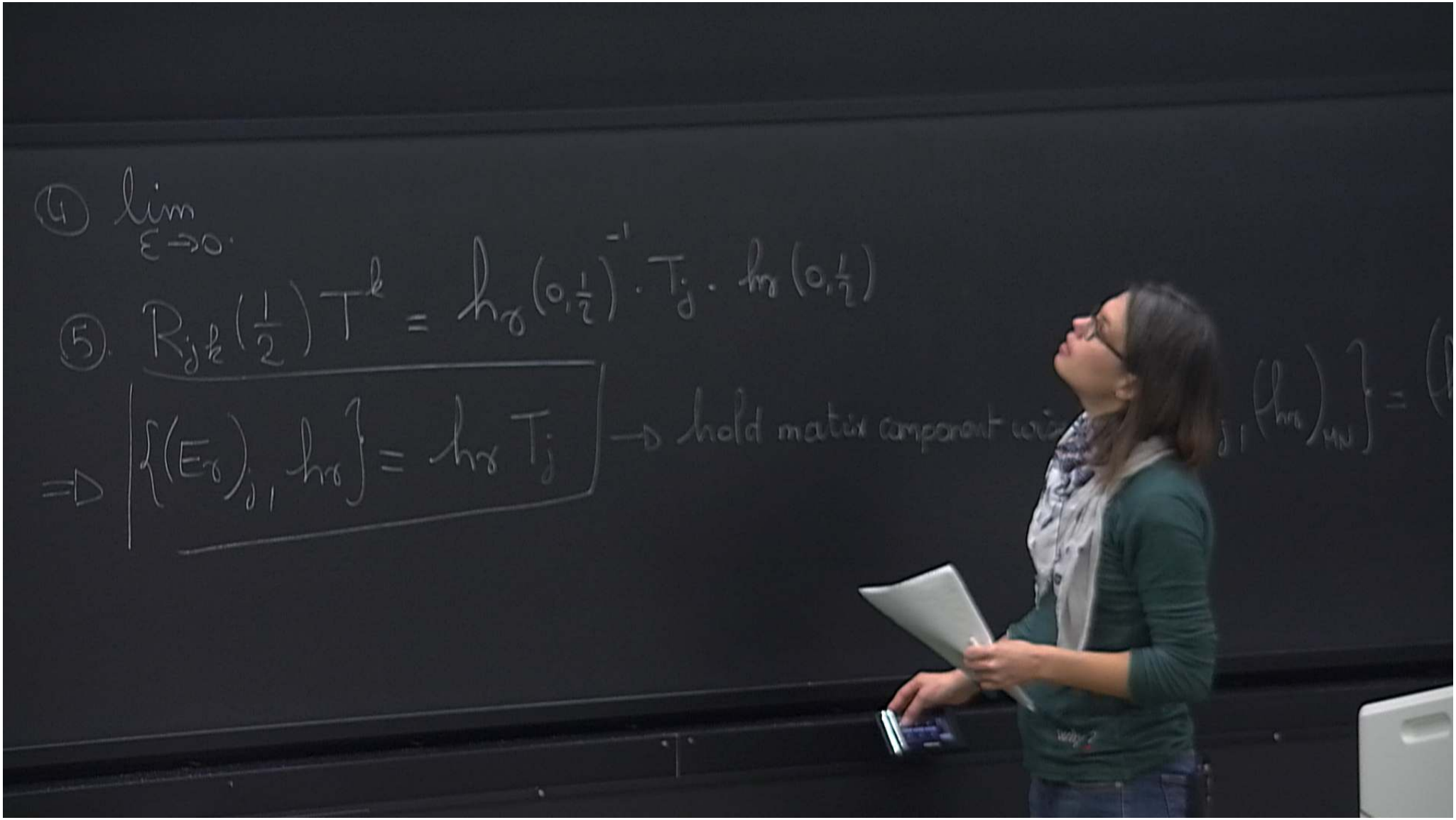
approximation $\mathbb{1} - \int_{\frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} T_l\, \dot{\gamma}^c(s')\, A^l_c(\gamma(s'))\, ds' + O(\varepsilon^2)$

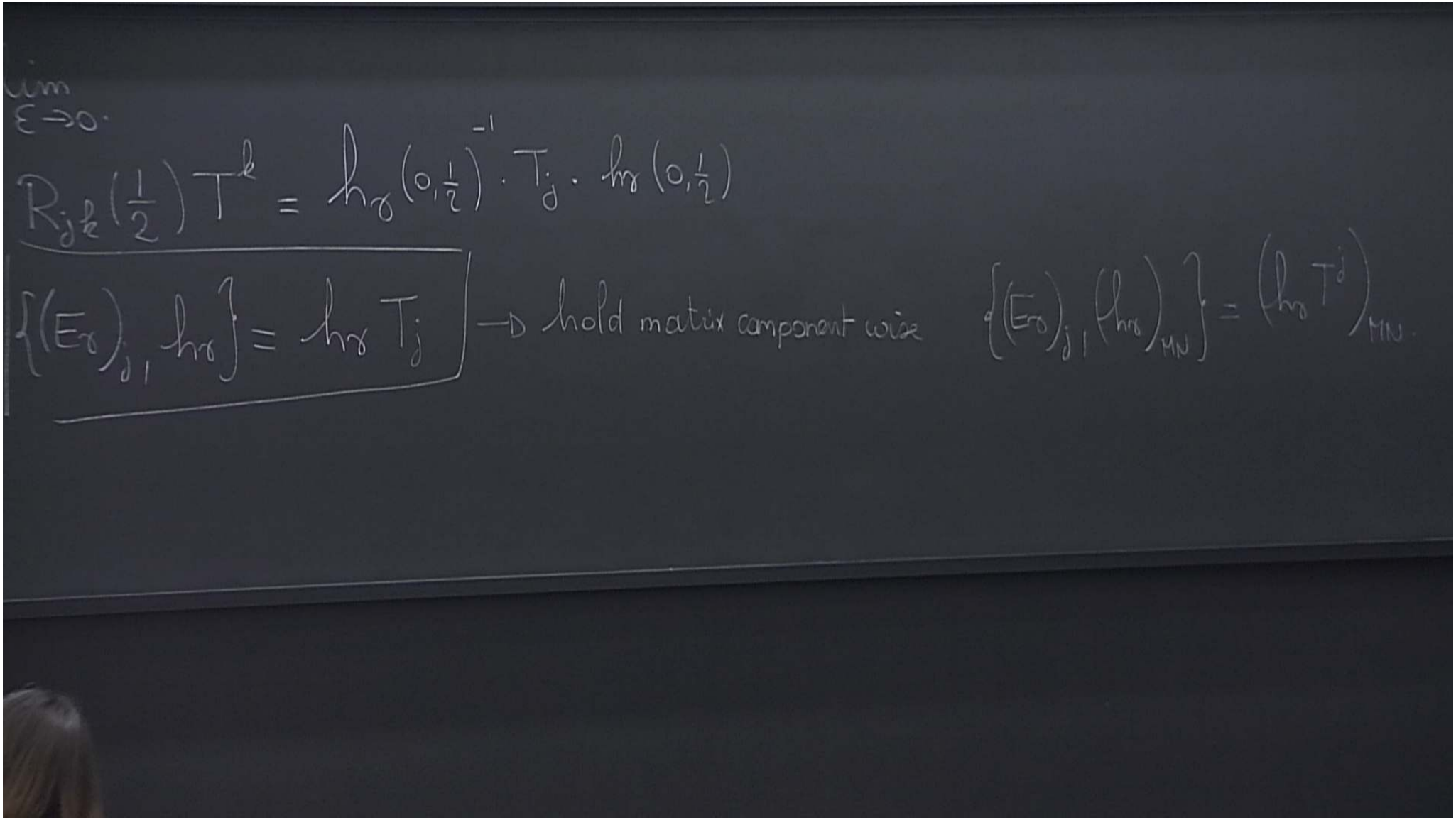

$\{(E_\sigma)_j, h_\gamma\} = -h_{\gamma(1,\frac{1}{2}+\frac{\varepsilon}{2})} h_{\gamma(\frac{1}{2}-\frac{\varepsilon}{2},0)} \int_{\frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} R_{jk}(\sigma)\, \tilde{\varepsilon}_{ba}\, \dot{\gamma}^a(\sigma)\, \dot{\gamma}^c(\sigma) \{E^{bk}(\gamma^*(\sigma)),$

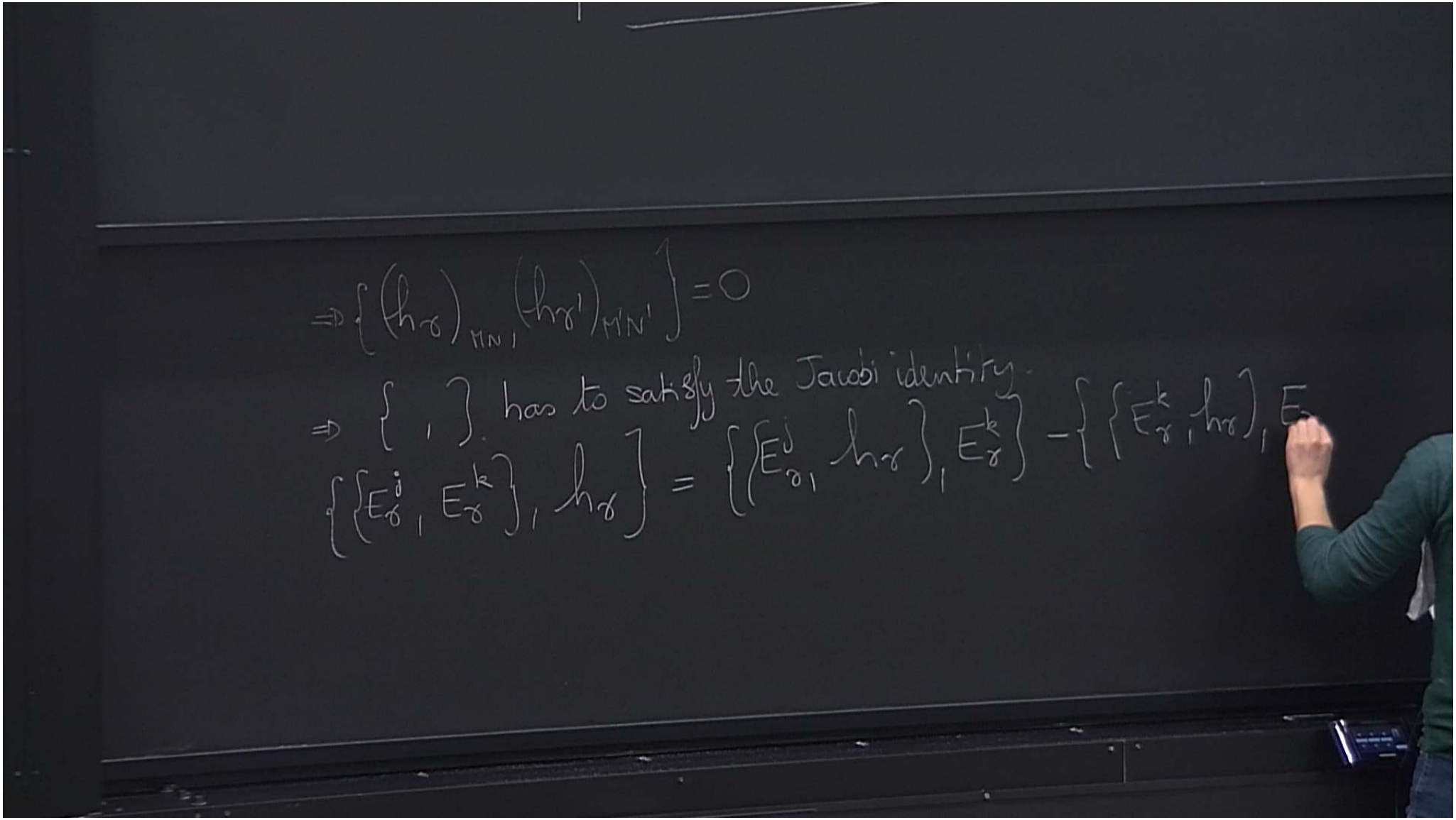

$= -h_{\gamma(1,\frac{1}{2}+\frac{\varepsilon}{2})} h_{\gamma(\frac{1}{2}-\frac{\varepsilon}{2},0)} \int_{\frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} \int_0^1 R_{jk}(\sigma)\, \varepsilon_{ba}\, \dot{\gamma}^a(\sigma)\, \dot{\gamma}^c(\sigma') \underbrace{\left\{ E^{bk}(\gamma^*(\sigma)), A_c^l(\gamma(\sigma')) \right\}}_{① \quad \delta(\gamma^*(\sigma), \gamma(\sigma'))\, \delta^{kl}\, \delta^b_c} d\sigma\, d\sigma' + O(\varepsilon^2)$

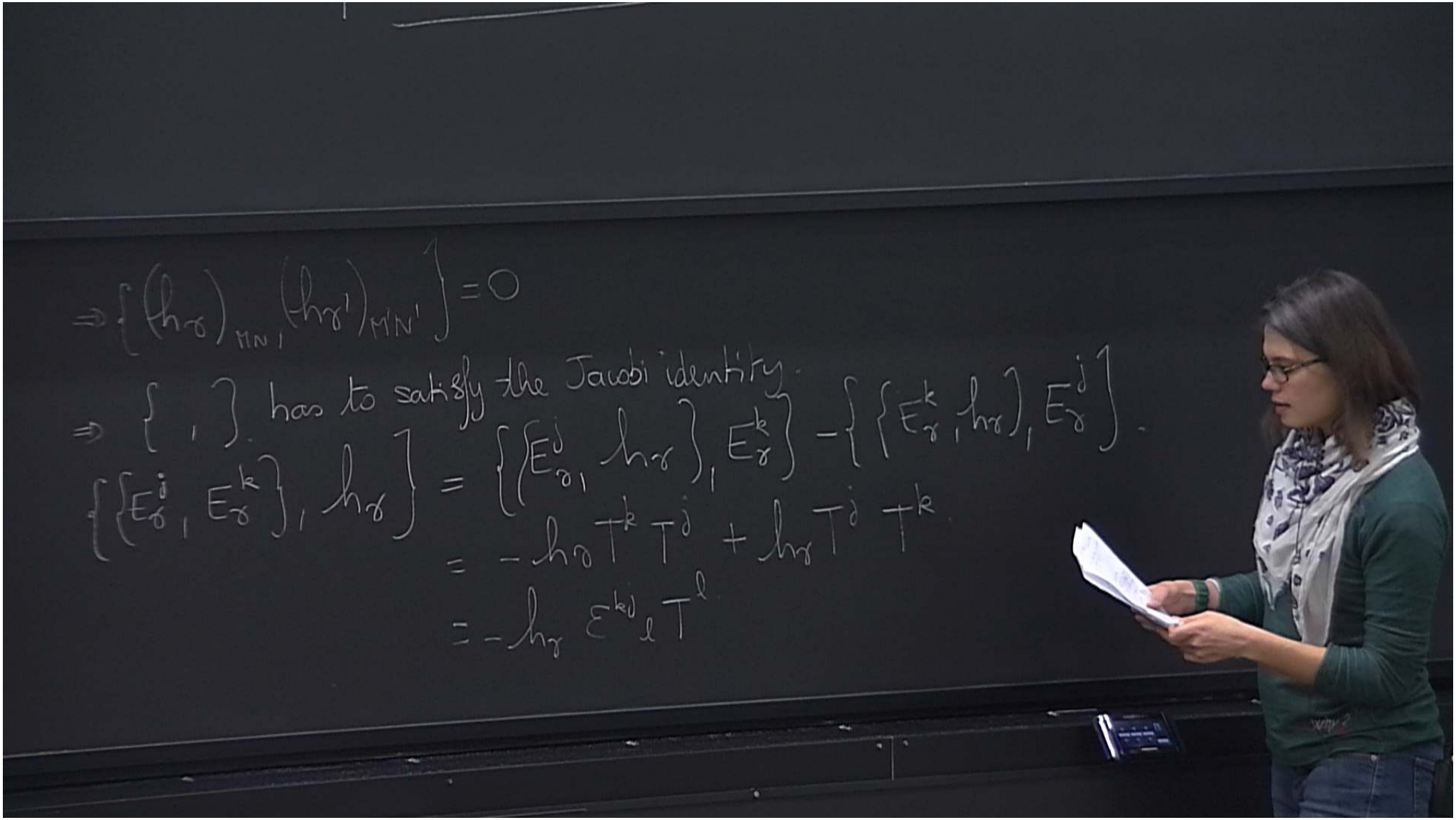
$$= -h_{\gamma(1,\frac{1}{2}+\frac{\varepsilon}{2})} h_{\gamma(\frac{1}{2}-\frac{\varepsilon}{2},0)} \iint_{\gamma^*\frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} R_{jk}(\sigma)\, \varepsilon_{ba}\, \dot{\gamma}^{*a}(\sigma)\, \dot{\gamma}^{c}(\sigma') \underbrace{\left\{ E^{bk}(\gamma^*(\sigma)), A^{l}_{c}(\gamma(\sigma')) \right\}}_{① \;\; \delta(\gamma^*(\sigma), \gamma(\sigma'))\, \delta^{kl}\, \delta^{b}_{c}} d\sigma\, d\sigma' + O(\varepsilon^2)$$

$$② \;\; \varepsilon_{ca}\, \dot{\gamma}^{*a}(\sigma)\, \dot{\gamma}^{c}(\sigma') = \det \begin{pmatrix} \dot{\gamma}^{1}(\sigma') & \dot{\gamma}^{2}(\sigma') \\ \dot{\gamma}^{*1}(\sigma) & \dot{\gamma}^{*2}(\sigma) \end{pmatrix}$$


④ lim ε→0.
⑤. $R_{jk}(\frac{1}{2})T^k = h_\alpha(0,\frac{1}{2})^{-1} \cdot T_j \cdot h_\alpha(0,\frac{1}{2})$
$\Rightarrow \{(E_0)_j, h_\alpha\} = h_\alpha T_j$
→ hold matrix


lim
ε→0.
$R_{jk}(\frac{1}{2})T^k = h_\alpha(0,\frac{1}{2})^{-1} \cdot T_j \cdot h_\alpha(0,\frac{1}{2})$
$\{(E_\alpha)_j, h_\alpha\} = h_\alpha T_j$
→ hold matrix component wise
$\{(E_\alpha)_j, (h_\alpha)_{MN}\} = (h_\alpha T^j)_{MN}$.


$\Rightarrow \{(h_\alpha)_{MN}, (h_{\alpha'})_{M'N'}\} = 0$
$\Rightarrow \{\,,\,\}$ has to satisfy the Jacobi identity.
$\{\{E_\alpha^i, E_\alpha^k\}, h_\alpha\} = \{\{E_\alpha^j, h_\alpha\}, E_\alpha^k\} - \{\{E_\alpha^k, h_\alpha\}, E_\alpha^j\}$


$\Rightarrow \{(h_\alpha)_{MN}, (h_{\alpha'})_{M'N'}\} = 0$
$\Rightarrow \{\,,\,\}$ has to satisfy the Jacobi identity.
$\{\{E_\alpha^i, E_\gamma^k\}, h_\alpha\} = \{\{E_\alpha^i, h_\alpha\}, E_\gamma^k\} - \{\{E_\gamma^k, h_\alpha\}, E_\alpha^i\}$
$= -h_\alpha T^k T^i + h_\alpha T^i T^k$
$= -h_\alpha \varepsilon^{ki}{}_l T^l$

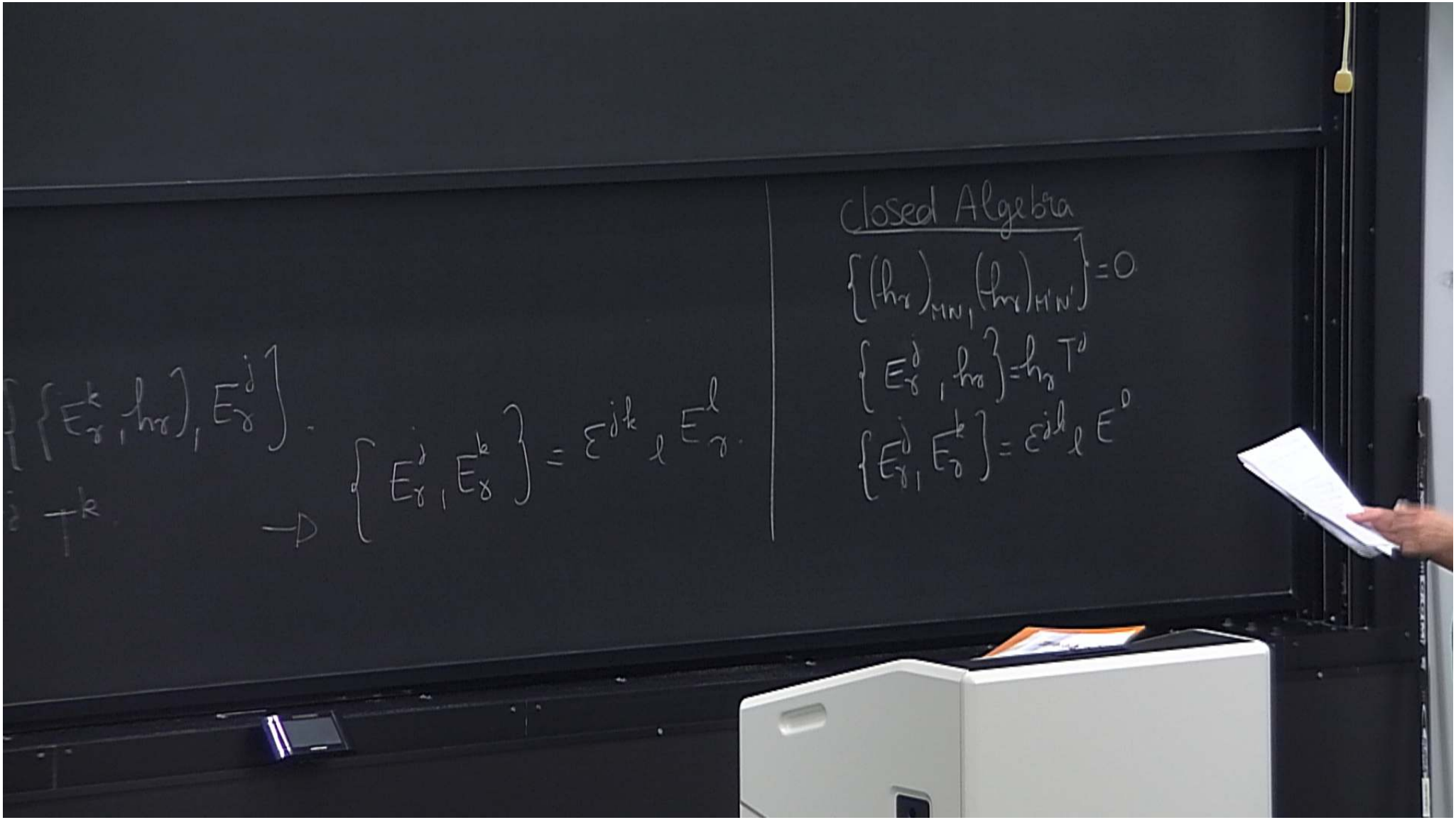
$$\Rightarrow \{(h_\alpha)_{MN}, (h_{\alpha'})_{M'N'}\} = 0$$

$\Rightarrow \{\,,\,\}$ has to satisfy the Jacobi identity.

$$\{\{E_\alpha^i, E_\gamma^k\}, h_\alpha\} = \{\{E_\alpha^i, h_\alpha\}, E_\gamma^k\} - \{\{E_\gamma^k, h_\alpha\}, E_\alpha^i\}$$

$$= -h_\alpha T^k T^i + h_\alpha T^i T^k \quad \rightarrow \{E$$

$$= -h_\alpha \varepsilon^{ki}{}_l T^l$$


Closed Algebra

→ length $l_{e^*} = |E_e|$

dihedral angles $\phi_{ee'}$ between (e^*, e'^*) of a triangle.

is contained in the dot product $E_e, E_{e'}$

$$\cos \phi_{ee'} = - \varepsilon_{ee'} \frac{E_e \cdot E_{e'}}{|E_e||E_{e'}|}$$

$\varepsilon_{ee'} = +1$ if e, e'