Title: PSI 2015/2016 Quantum Gravity - Lecture 8

Date: Mar 02, 2016 10:15 AM

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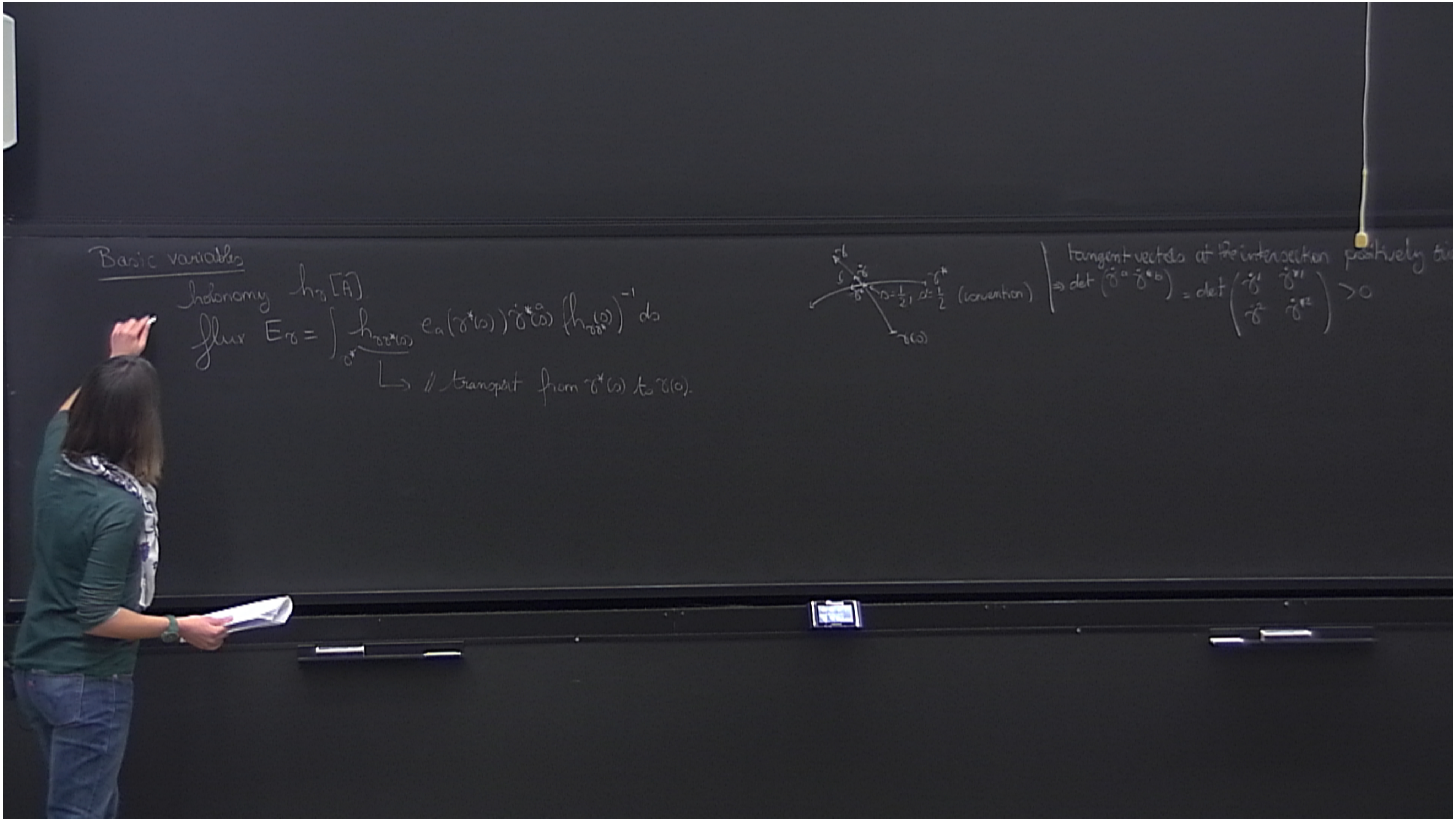
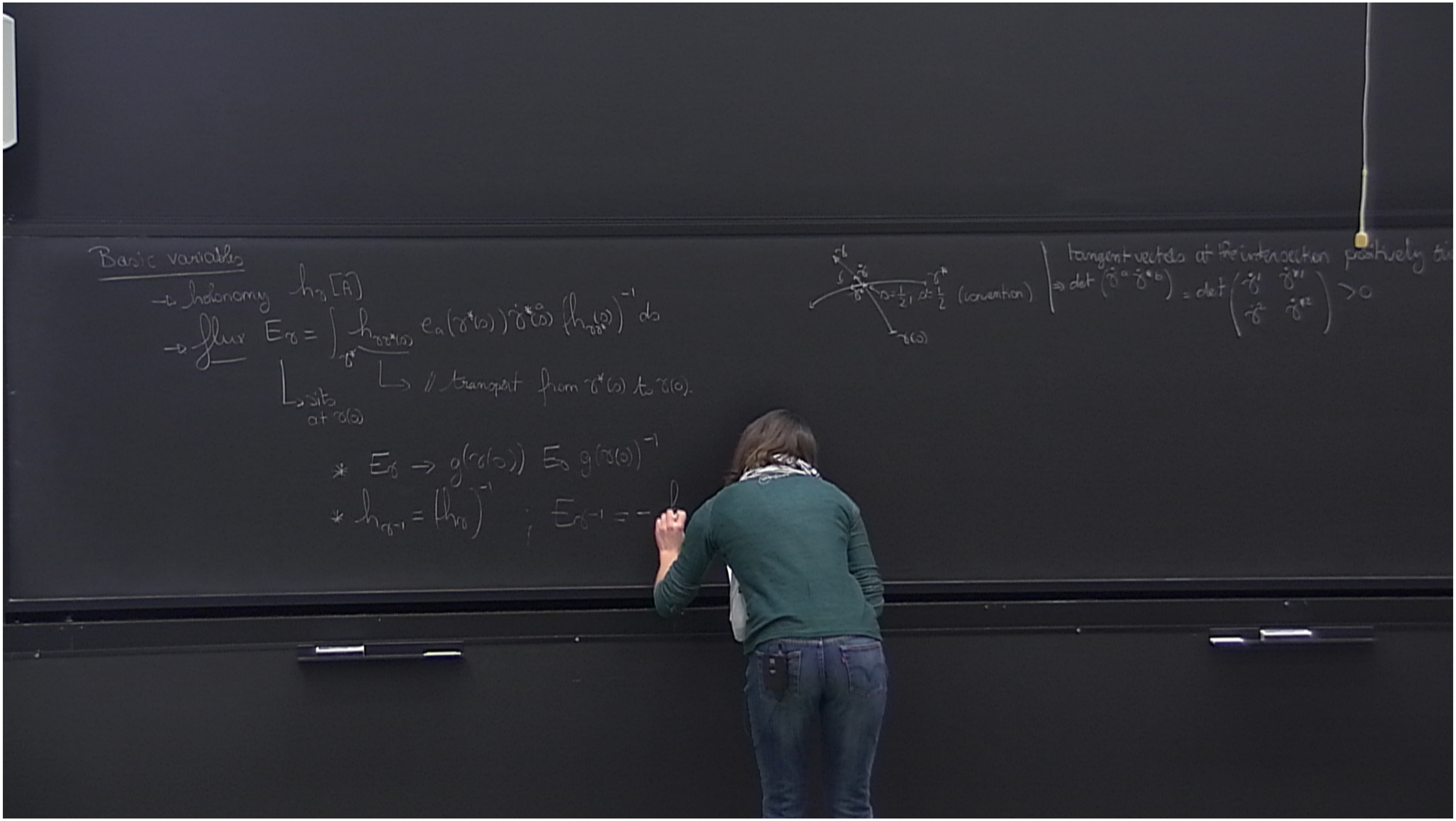
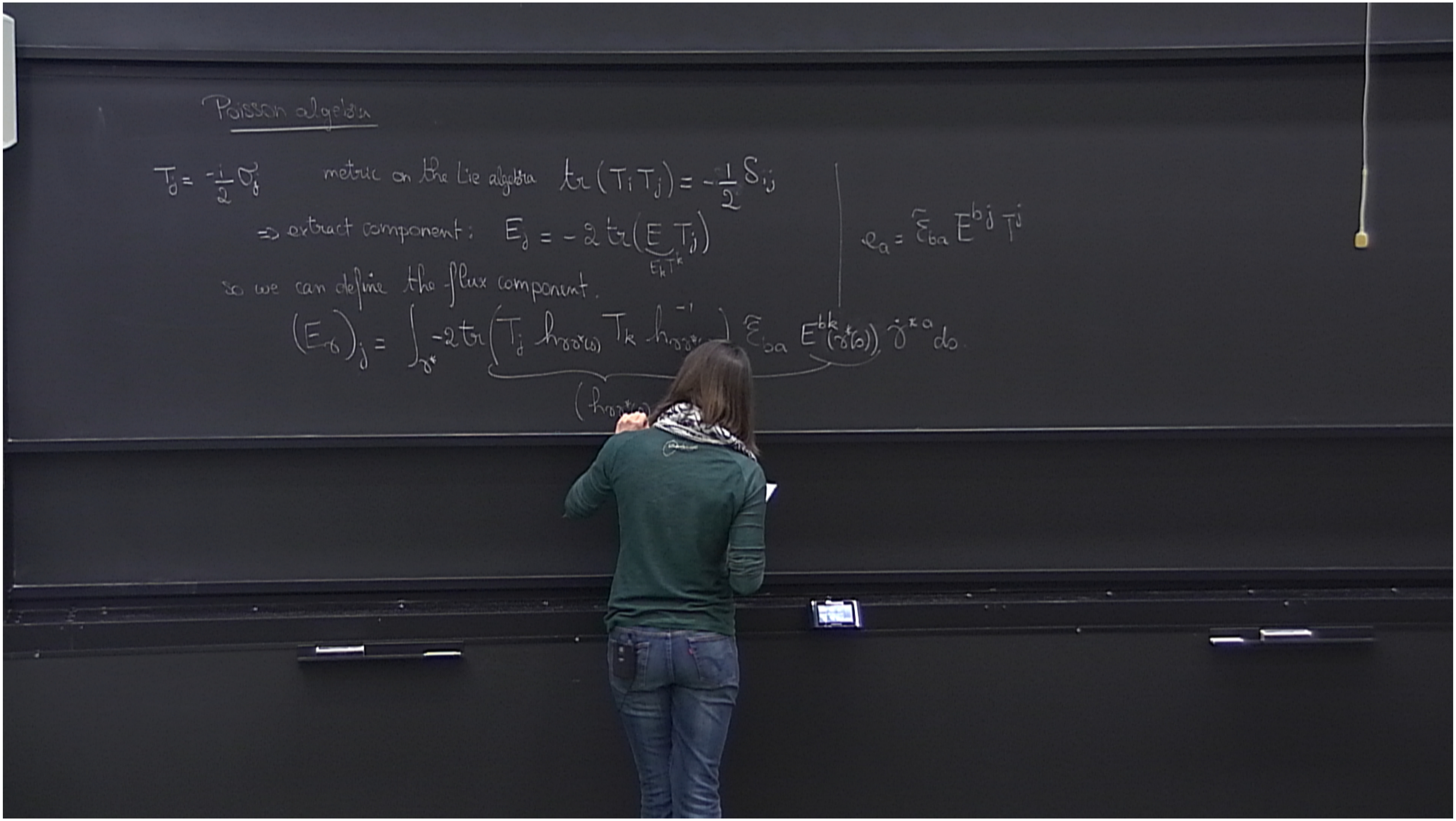
Abstract:



Basic variables
holonomy $h_\gamma[A]$
flux $E_\gamma = \int_{\gamma^*} h_{\gamma\gamma^*(s)}\, e_a(\gamma^*(s))\, \dot{\gamma}^{*a}(s)\, (h_{\gamma\gamma^*}(s))^{-1}\, ds$
↳ // transport from $\gamma^*(s)$ to $\gamma(0)$.
$s = \frac{1}{2}$, $t = \frac{1}{2}$ (convention)
γ^*
$\gamma(0)$
tangent vectors at the intersection positively ti
$\Rightarrow \det(\dot{\gamma}^a\ \dot{\gamma}^{*b}) = \det\begin{pmatrix} \dot{\gamma}^1 & \dot{\gamma}^{*1} \\ \dot{\gamma}^2 & \dot{\gamma}^{*2} \end{pmatrix} > 0$


Basic variables
→ holonomy $h_\gamma[A]$
→ flux $E_\gamma = \int_{\gamma^*} h_{\gamma\gamma^*(s)} \, e_a(\gamma^*(s)) \, \dot{\gamma}^{*a}(s) \, (h_{\gamma\gamma^*(s)})^{-1} \, ds$
sits at $\gamma(0)$
// transport from $\gamma^*(s)$ to $\gamma(0)$.
$*$ $E_\gamma \to g(\gamma(0)) \, E_\gamma \, g(\gamma(0))^{-1}$
$*$ $h_{\gamma^{-1}} = (h_\gamma)^{-1}$; $E_{\gamma^{-1}} = -$
$s = \frac{1}{2}$, $s' = \frac{1}{2}$ (convention)
$\gamma(0)$
tangent vectors at the intersection positively oriented
$\Rightarrow \det(\dot{\gamma}^a \; \dot{\gamma}^{*b}) = \det\begin{pmatrix} \dot{\gamma}^1 & \dot{\gamma}^{*1} \\ \dot{\gamma}^2 & \dot{\gamma}^{*2} \end{pmatrix} > 0$


Poisson algebra
$T_j = -\frac{i}{2}\sigma_j$
metric on the Lie algebra $tr(T_i T_j) = -\frac{1}{2}\delta_{ij}$
$\Rightarrow$ extract component: $E_j = -2\, tr(\underbrace{E}_{E_k T^k} T_j)$
so we can define the flux component.
$(E_\gamma)_j = \int_{\gamma^*} -2\, tr(T_j\, h_{\gamma\gamma^*(\sigma)}\, T_k\, h^{-1}_{\gamma\gamma^*})\, \varepsilon_{ba}\, E^{bk}(\gamma^*(\sigma))\, \dot{\gamma}^{*a}\, d\sigma$
$e_a = \varepsilon_{ba} E^{bj} T^j$

Poisson algebra

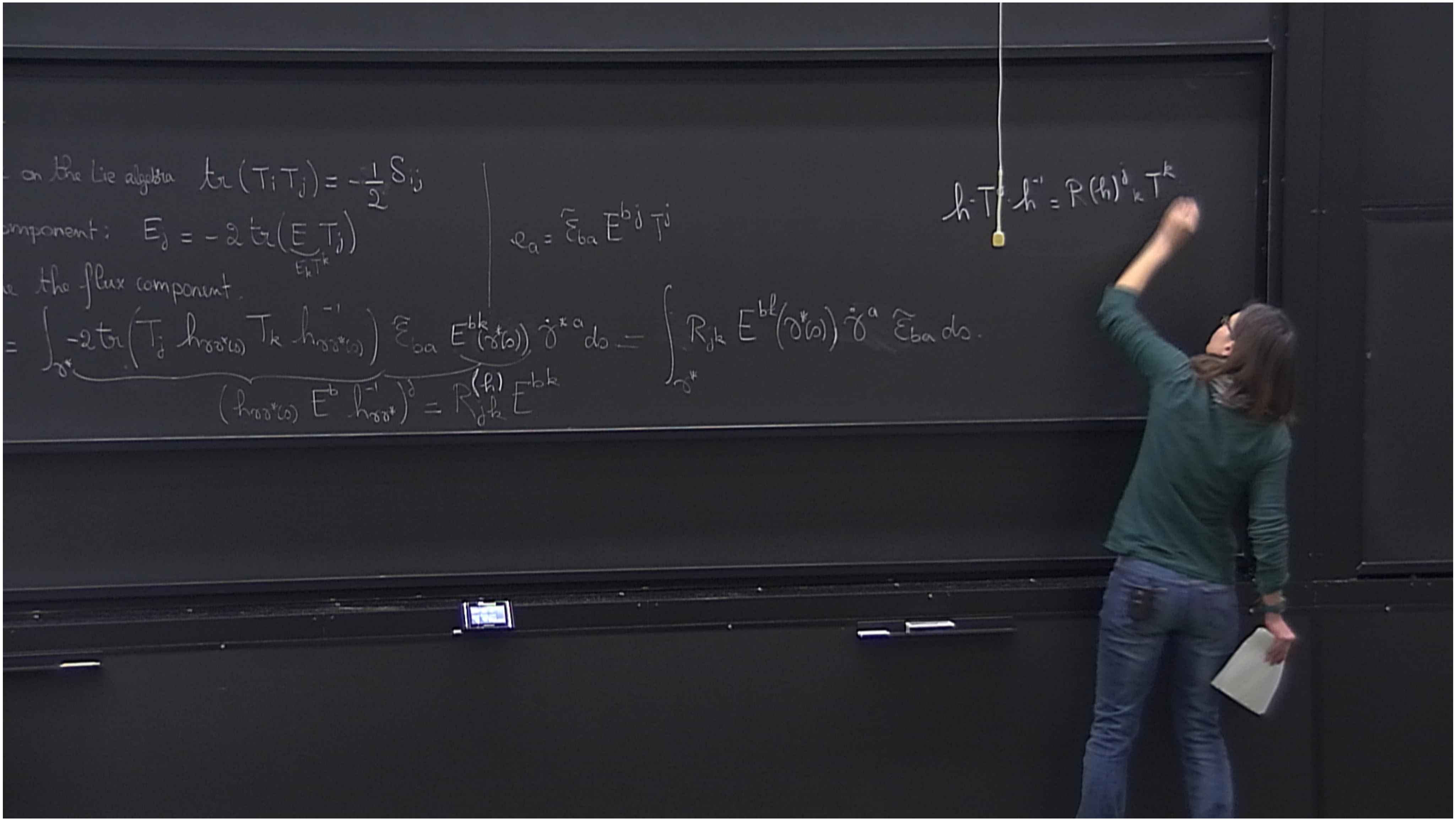
$T_j = -\frac{i}{2}\sigma_j$ metric on the Lie algebra $tr(T_i T_j) = -\frac{1}{2}\delta_{ij}$

$\Rightarrow$ extract component: $E_j = -2\, tr(\underbrace{E}_{E_k T^k} T_j)$

so we can define the flux component.

$e_a = \tilde{\varepsilon}_{ba} E^{bj} T^j$

$$(E_\gamma)_j = \int_{\gamma^*} \underbrace{-2\, tr\left(T_j\, h_{\gamma\gamma^*(s)}\, T_k\, h^{-1}_{\gamma\gamma^*(s)}\right) \tilde{\varepsilon}_{ba}\, E^{bk}}_{\left(h_{\gamma\gamma^*(s)} E^b h^{-1}_{\gamma\gamma^*}\right)^j = R_{jk} E^{bk}}(\gamma^*(s))\, \dot{\gamma}^{*a}\, ds = \int_{\gamma^*} R_{jk}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^a\, \tilde{\varepsilon}_{ba}\, ds.$$


on the Lie algebra tr(T_i T_j) = -1/2 δ_ij
omponent: E_j = -2 tr(E T_j)
E_k T^k
e_a = ε̃_ba E^bj T^j
the flux component.
h T^j h^-1 = R(h)^j_k T^k

... on the Lie algebra $tr(T_i T_j) = -\frac{1}{2}\delta_{ij}$

...mponent: $E_j = -2\, tr(\underbrace{E}_{E_k T^k} T_j)$

...e the flux component.

$e_a = \tilde{\varepsilon}_{ba} E^{bj} T^j$

$h \cdot T^j \cdot h^{-1} = R(h)^j{}_k T^k$

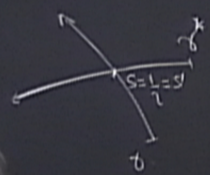
$$= \int_{\gamma^*} \underbrace{-2\, tr\left(T_j\, h_{\gamma\gamma^*(s)} T_k\, h^{-1}_{\gamma\gamma^*(s)}\right)}_{\left(h_{\gamma\gamma^*(s)} E^b h^{-1}_{\gamma\gamma^*}\right)^j = R^{(h)}_{jk} E^{bk}} \tilde{\varepsilon}_{ba}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^{*a}\, ds = \int_{\gamma^*} R_{jk}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^a\, \tilde{\varepsilon}_{ba}\, ds.$$

$$E_j = -2\,\mathrm{tr}(\underbrace{E}_{E_k T^k}\, T_j) \qquad e_a = \epsilon_{ba} E^b$$

so we can define the flux component.

$$(E_\gamma)_j = \int_{\gamma^*} \underbrace{-2\,\mathrm{tr}\left(T_j\, h_{\gamma\gamma^*(s)}\, T_k\, h^{-1}_{\gamma\gamma^*(s)}\right) \tilde{\epsilon}_{ba}\, E^{bk}(\gamma^*(s))}_{\left(h_{\gamma\gamma^*(s)}\, E^b\, h^{-1}_{\gamma\gamma^*}\right)^j = R^{(h)}_{jk} E^{bk}}\, \dot{\gamma}^{*a}\, ds = \int_{\gamma^*} R_{jk}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^a\, \tilde{\epsilon}_{ba}\, ds.$$

$$\{(E_\gamma)_j,\ \underbrace{h_\gamma}_{\text{matrix}}\}$$ → only the part near the intersection of γ and γ^*



so splitting $h_\gamma = h_\gamma\left(1, \frac{1}{2}+\frac{\epsilon}{2}\right) h_\gamma\left(\frac{1}{2}+\frac{\epsilon}{2}, \frac{1}{2}-\frac{\epsilon}{2}\right) h_\gamma\left(\frac{1}{2}-\frac{\epsilon}{2}, 0\right)$

$E_k T^*$

so we can define the flux component.

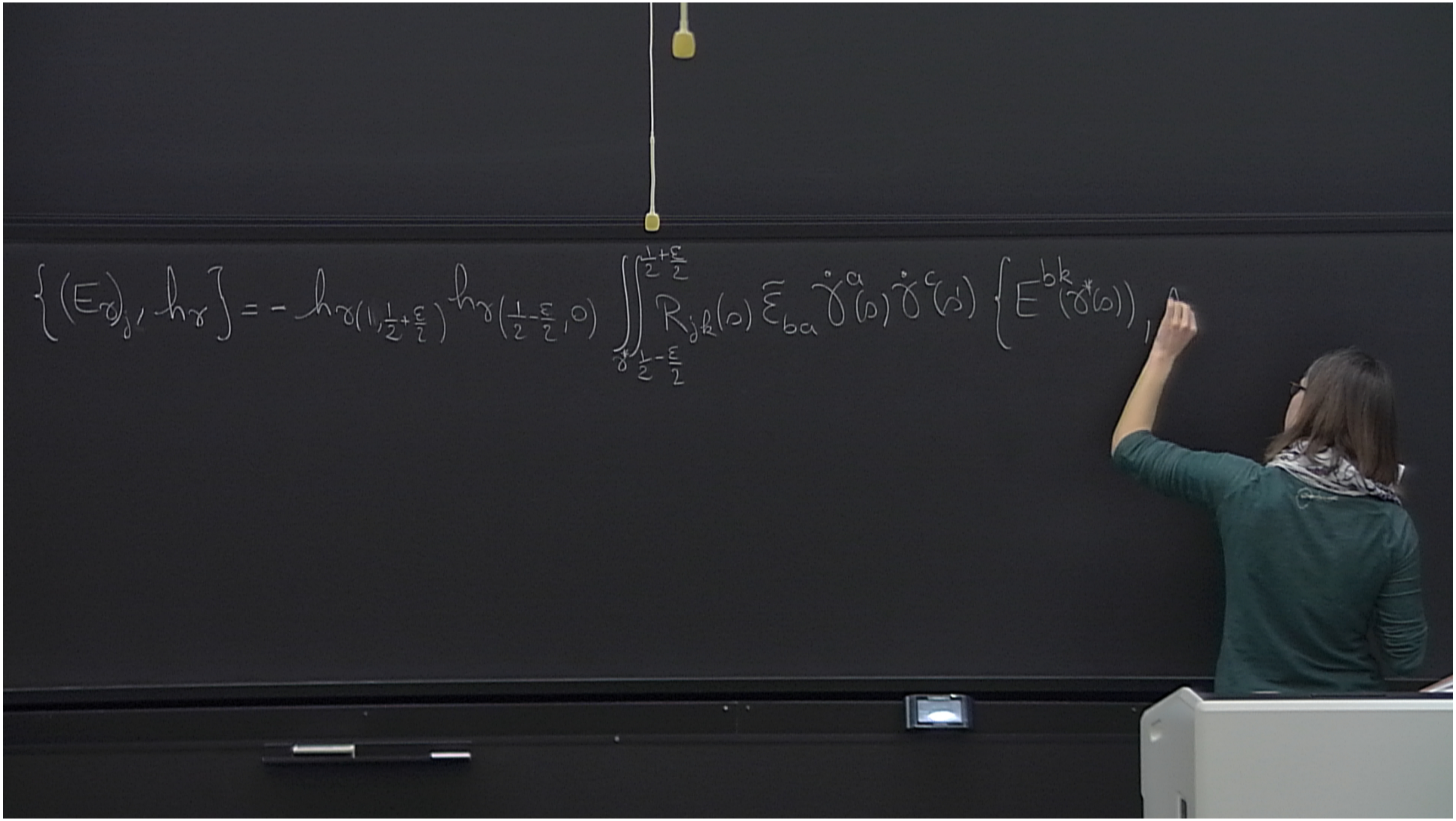
$$(E_\gamma)_j = \int_{\gamma^*} \underbrace{-2\,\mathrm{tr}\left(T_j\, h_{\gamma\gamma^*(s)}\, T_k\, h^{-1}_{\gamma\gamma^*(s)}\right) \tilde{\varepsilon}_{ba}\, E^{bk}(\gamma^*(s))}_{\left(h_{\gamma\gamma^*(s)}\, E^b\, h^{-1}_{\gamma\gamma^*}\right)^j = R^{(h)}_{jk} E^{bk}}\, \dot{\gamma}^{*a}\, ds = \int_{\gamma^*} R_{jk}\, E^{bl}(\gamma^*(s))\, \dot{\gamma}^a\, \tilde{\varepsilon}_{ba}\, ds.$$

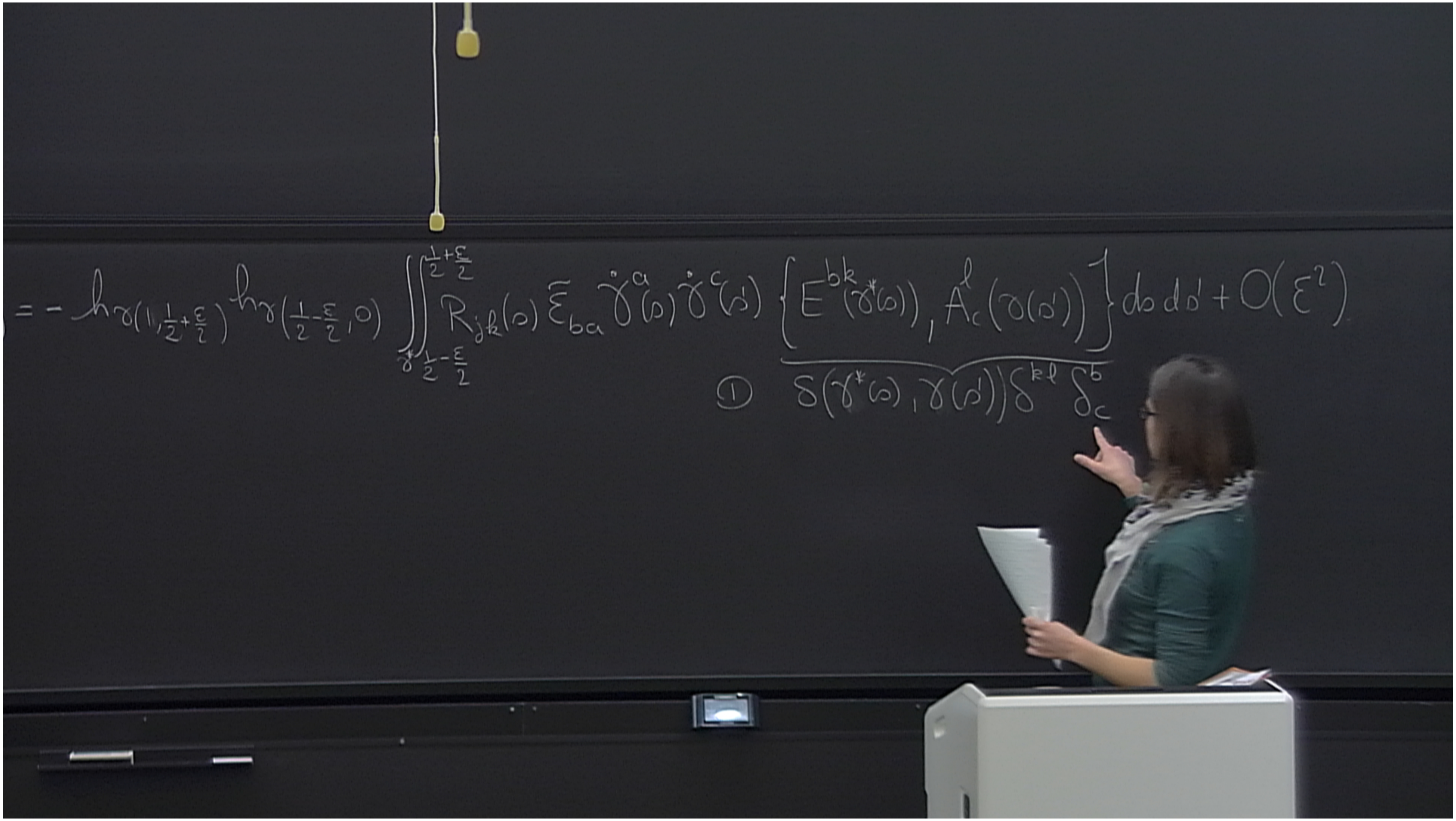
$\{(E_\gamma)_j, \underset{\substack{\uparrow \\ \text{matrix} \\ \mathfrak{sl}^+}}{h_\gamma}\}$ $\longrightarrow$ only the part near the intersection of γ and γ^*

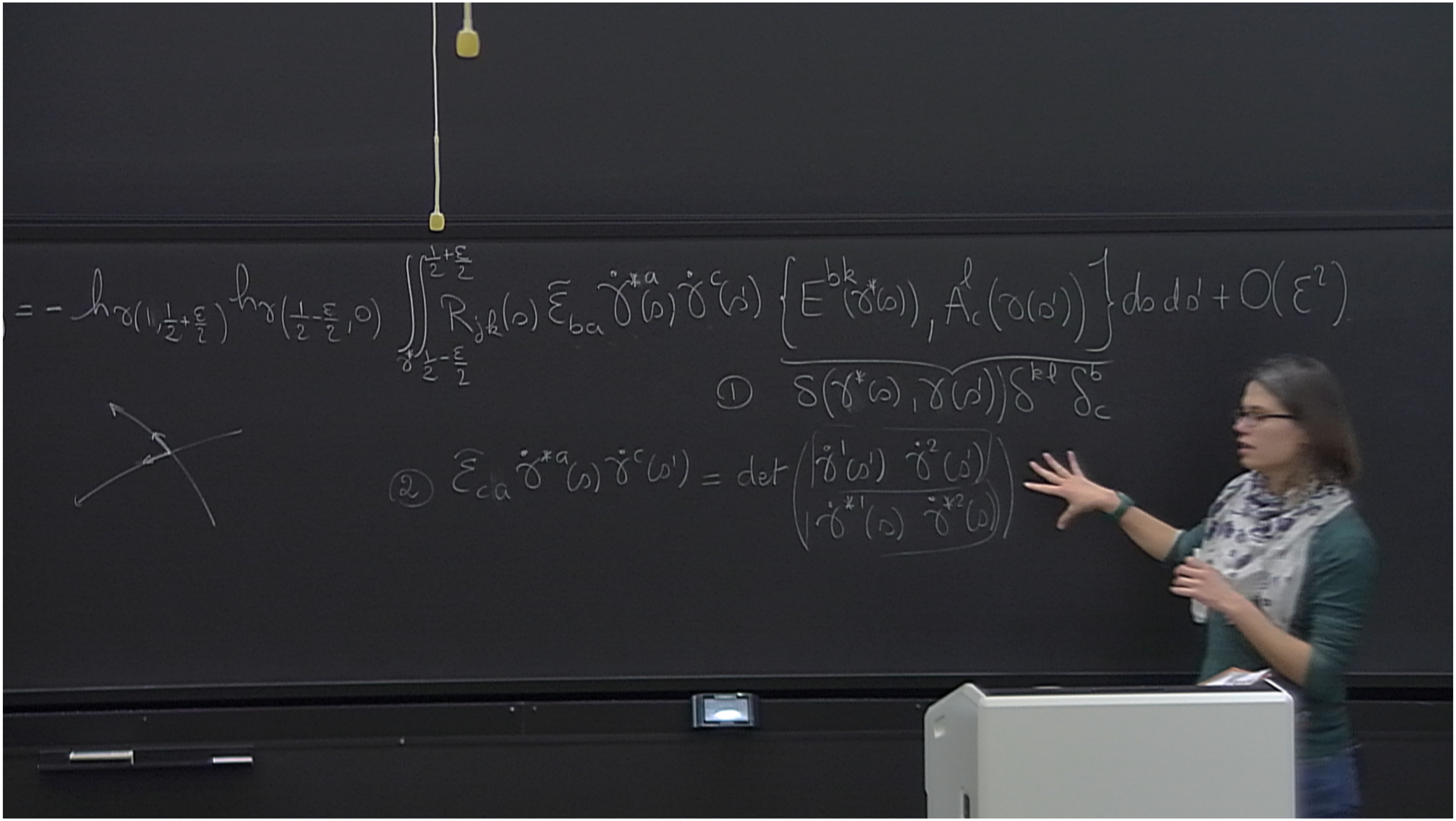
so splitting $h_\gamma = h_{\gamma(1,\frac{1}{2}+\frac{\varepsilon}{2})}\, h_{\gamma(\frac{1}{2}+\frac{\varepsilon}{2},\frac{1}{2}-\frac{\varepsilon}{2})}\, h_{\gamma(\frac{1}{2}-\frac{\varepsilon}{2},0)}$ -

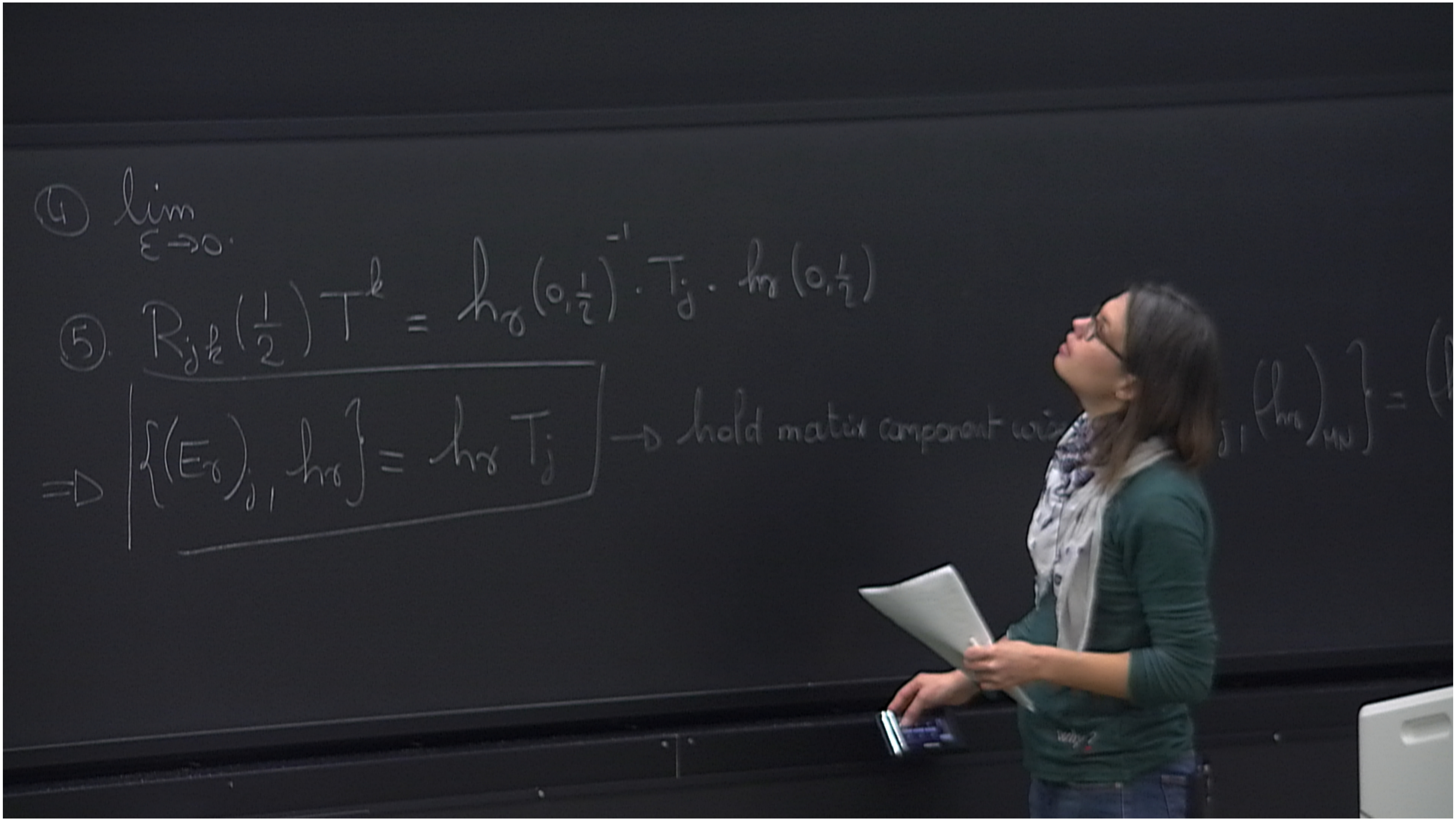
commute with the flux

approximation $\mathbb{1} - \int_{\frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} T_l\, \dot{\gamma}^c(s')\, A^l_c(\gamma(s'))\, ds' + O(\varepsilon^2)$


$\{(E_\gamma)_j, h_\gamma\} = - h_{\gamma(1, \frac{1}{2}+\frac{\varepsilon}{2})} h_{\gamma(\frac{1}{2}-\frac{\varepsilon}{2}, 0)} \int\int_{\gamma^* \frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} R_{jk}(s)\, \varepsilon_{ba}\, \dot\gamma^a(s)\, \dot\gamma^c(s') \{E^{bk}(\gamma^*(s)),$


$= - h_{\gamma}(1, \tfrac{1}{2}+\tfrac{\varepsilon}{2})\, h_{\gamma}(\tfrac{1}{2}-\tfrac{\varepsilon}{2}, 0) \iint_{\gamma^*\, \frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} R_{jk}(s)\, \varepsilon_{ba}\, \dot{\gamma}^a(s)\, \dot{\gamma}^c(s') \overbrace{\{ E^{bk}(\gamma^*(s)), A^l_c(\gamma(s')) \}}^{\delta(\gamma^*(s), \gamma(s'))\, \delta^{kl}\, \delta^b_c}\, ds\, ds' + O(\varepsilon^2)$
① $\delta(\gamma^*(s), \gamma(s'))\, \delta^{kl}\, \delta^b_c$


$= - h_{\gamma(1,\frac{1}{2}+\frac{\varepsilon}{2})} h_{\gamma(\frac{1}{2}-\frac{\varepsilon}{2},0)} \iint_{\gamma^* \frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} R_{jk}(s)\, \varepsilon_{ba}\, \dot{\gamma}^{*a}(s)\, \dot{\gamma}^{c}(s') \underbrace{\left\{ E^{bk}(\gamma^*(s)), A^{l}_{c}(\gamma(s')) \right\}}_{\delta(\gamma^*(s),\gamma(s'))\delta^{kl}\delta^{b}_{c}} ds\, ds' + O(\varepsilon^2)$
① $\delta(\gamma^*(s),\gamma(s'))\delta^{kl}\delta^{b}_{c}$
② $\varepsilon_{ca}\dot{\gamma}^{*a}(s)\dot{\gamma}^{c}(s') = \det\begin{pmatrix} \dot{\gamma}^{1}(s') & \dot{\gamma}^{2}(s') \\ \dot{\gamma}^{*1}(s) & \dot{\gamma}^{*2}(s) \end{pmatrix}$



④ $\lim_{\varepsilon \to 0}$
⑤. $R_{jk}(\frac{1}{2})\, T^k = h_\gamma(0,\frac{1}{2})^{-1} \cdot T_j \cdot h_\gamma(0,\frac{1}{2})$
$\Rightarrow \boxed{\{(E_\gamma)_j, h_\gamma\} = h_\gamma T_j}$
→ hold matrix component wis

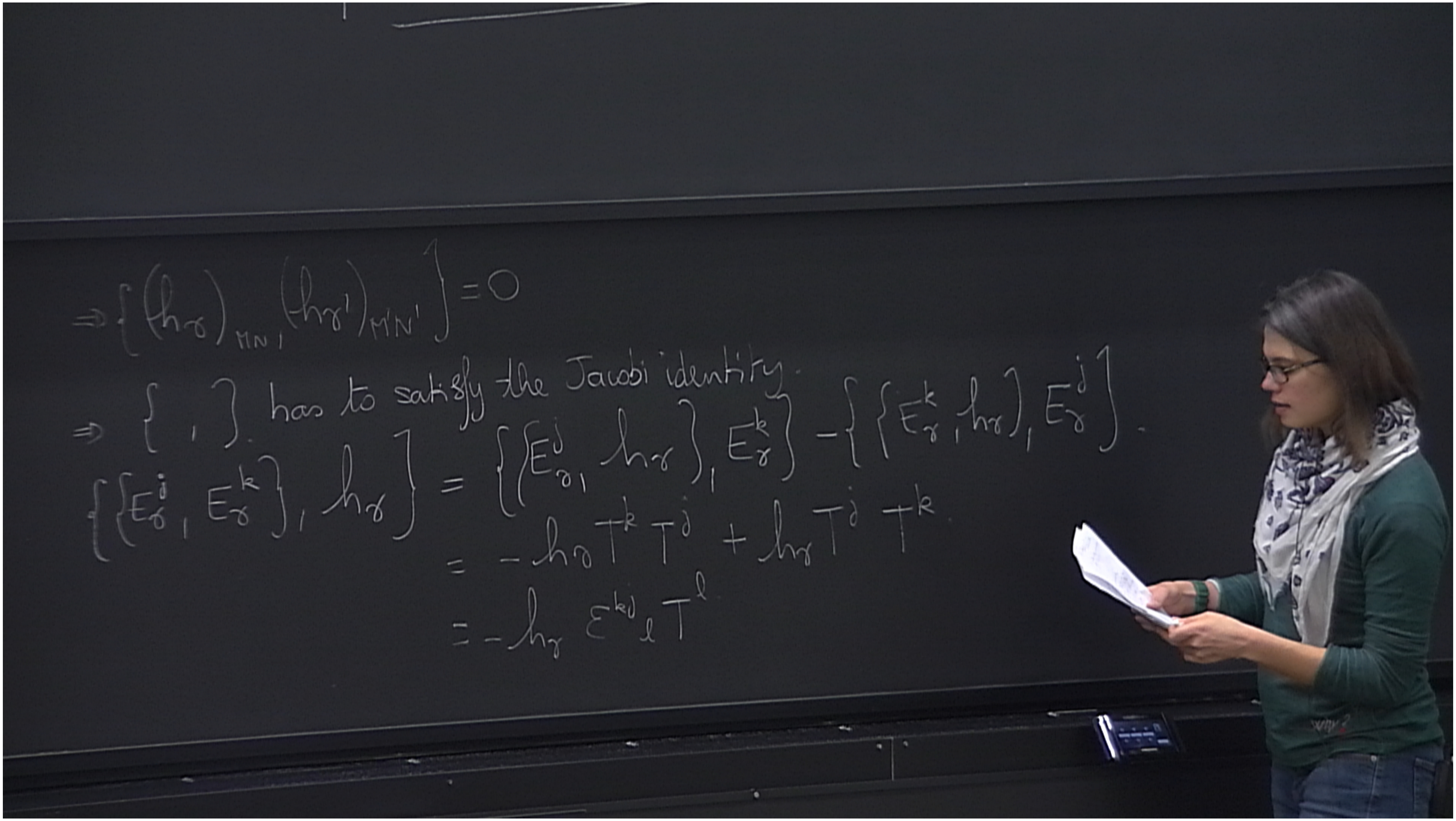
$$\lim_{\varepsilon \to 0} \; R_{jk}\left(\tfrac{1}{2}\right) T^{k} = h_\gamma\left(0,\tfrac{1}{2}\right)^{-1} \cdot T_j \cdot h_\gamma\left(0,\tfrac{1}{2}\right)$$

$$\boxed{\left\{(E_\gamma)_j , h_\gamma\right\} = h_\gamma T_j}$$ → hold matrix component wise $\left\{(E_\gamma)_j , (h_\gamma)_{MN}\right\} = \left(h_\gamma T^j\right)_{MN}$.

$$\Rightarrow \{ (h_\gamma)_{MN}, (h_{\gamma'})_{M'N'} \} = 0$$

$\{\ ,\ \}$ has to satisfy the Jacobi identity.

$$\Rightarrow \{ \{ E^j_\gamma, E^k_\gamma \}, h_\gamma \} = \{ \{ E^j_\gamma, h_\gamma \}, E^k_\gamma \} - \{ \{ E^k_\gamma, h_\gamma \}, E$$


$\Rightarrow \{(h_\gamma)_{MN}, (h_{\gamma'})_{M'N'}\} = 0$
$\Rightarrow \{\ ,\ \}$ has to satisfy the Jacobi identity.
$\{\{E^j_\gamma, E^k_\gamma\}, h_\gamma\} = \{\{E^j_\gamma, h_\gamma\}, E^k_\gamma\} - \{\{E^k_\gamma, h_\gamma\}, E^j_\gamma\}.$
$= -h_\gamma T^k T^j + h_\gamma T^j T^k.$
$= -h_\gamma \varepsilon^{kj}{}_l T^l$

$$\Rightarrow \{(h_\gamma)_{MN}, (h_{\gamma'})_{M'N'}\} = 0$$

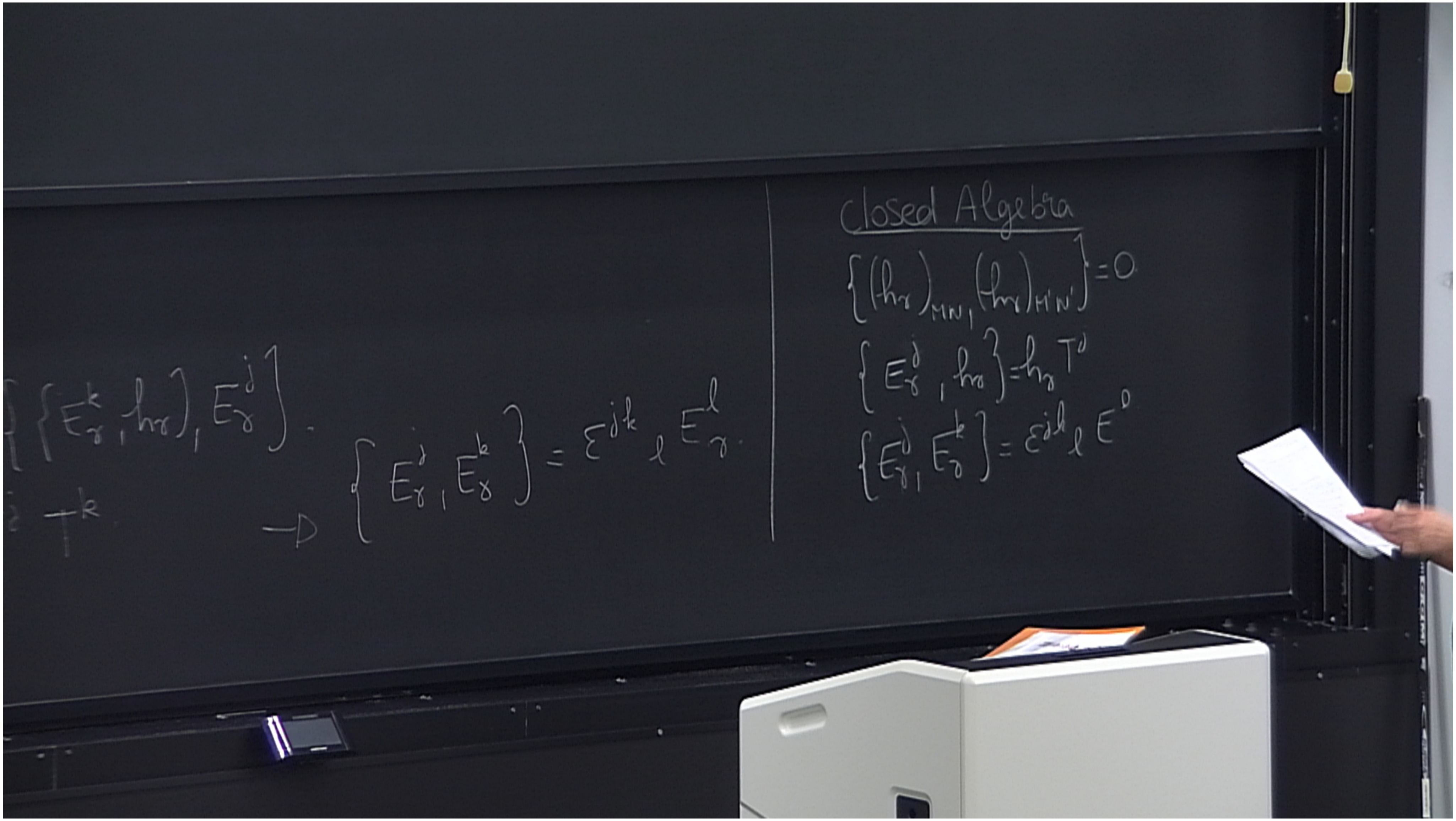
$\Rightarrow$ $\{\ ,\ \}$ has to satisfy the Jacobi identity.

$$\{\{E^j_\gamma, E^k_\gamma\}, h_\gamma\} = \{\{E^j_\gamma, h_\gamma\}, E^k_\gamma\} - \{\{E^k_\gamma, h_\gamma\}, E^j_\gamma\}$$

$$= -h_\gamma T^k T^j + h_\gamma T^j T^k$$

$$= -h_\gamma \epsilon^{kj}{}_l T^l$$

$\rightarrow \{E$


$\{\{E^k_\gamma, h_\gamma\}, E^j_\gamma\}$.
$\to \{E^j_\gamma, E^k_\gamma\} = \varepsilon^{jk}{}_l E^l_\gamma$.
Closed Algebra
$\{(h_\gamma)_{MN}, (h_\gamma)_{M'N'}\} = 0$
$\{E^j_\gamma, h_\gamma\} = h_\gamma T^j$
$\{E^j_\gamma, E^k_\gamma\} = \varepsilon^{jk}{}_l E^l$

$\to$ length $\quad l_{e^*} = |E_e|$

dihedral angles $\phi_{ee'}$ between (e^*, e'^*) of a triangle.

is contained in the dot product $E_e, E_{e'}$

$$\cos \phi_{ee'} = -\varepsilon_{ee'} \frac{E_e \cdot E_{e'}}{|E_e||E_{e'}|} \qquad \varepsilon_{ee'} = +1 \text{ if } e, e'$$