

Title: PSI 2015/2016 More/Beyond Standard Model - Lecture 13

Date: Mar 09, 2016 09:00 AM

URL: <http://pirsa.org/16030042>

Abstract:

Extra Dimensions

$$M_{\text{Pl}} \gg m_w$$

"Planck scale we see

M_* = Fundamental Planck scale

$$\rightsquigarrow M_n$$

Extra Dimensions

$$M_{\text{Pl}} \gg m_w$$

"Planck scale we see"

M_* = Fundamental Planck scale

$$\uparrow M_{\text{Pl}}$$

$$d = 4 + N$$

"1+3" "spacelike"

$$\vec{\nabla}^2 \Phi \sim \frac{1}{M_*^{2+N}} \rho$$

$$\Phi(r) \sim \frac{1}{M_*^{2+N}} \frac{M}{r^{1+N}}$$

$$F(r) \sim \frac{1}{M_*^{2+N}} \frac{m_1 m_2}{r^{2+N}}$$

$$x^M =$$

$$10^{15} \text{ cm}, N=1$$

$$1 \text{ mm}, N=2$$

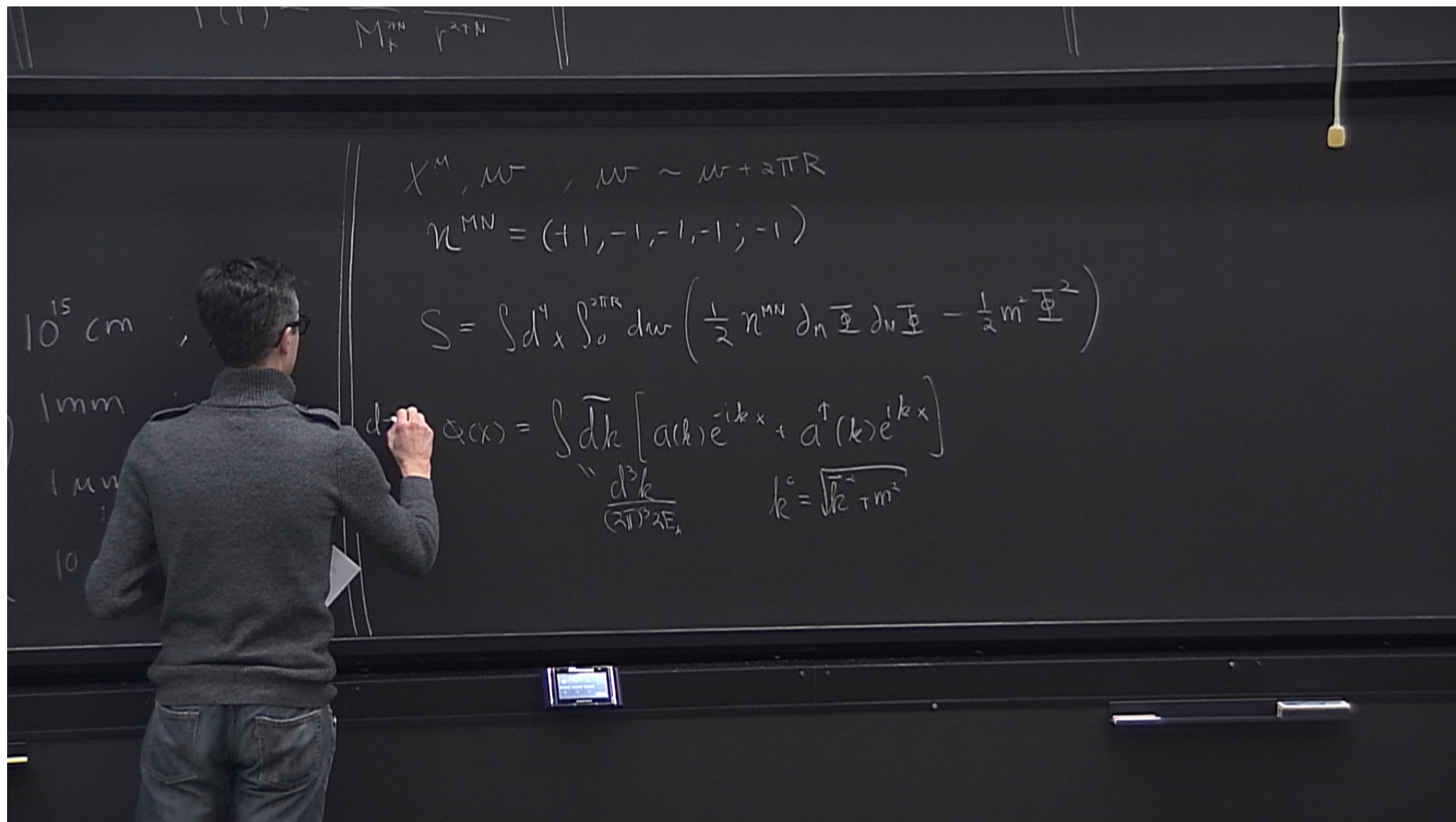
$$1 \text{ } \mu\text{m}, N=3$$

$$10 \text{ fm}, N=6$$

$$X^M, W, W \sim W + 2\pi R$$

$$\eta^{MN} = (+1, -1, -1, -1; -1)$$

$$S = \int d^4x \int_0^{2\pi R} dW \left(\frac{1}{2} \eta^{MN} \partial_M \Phi \partial_N \Phi - \frac{1}{2} m^2 \Phi^2 \right)$$



$$\omega \sim \omega + 2\pi R$$

$$1, -1, -1, -1; -1)$$

$$\int_0^{2\pi R} d\omega \left(\frac{1}{2} \eta^{MN} \partial_N \Phi \partial_M \Phi - \frac{1}{2} \right)$$

$$\frac{d^3 k}{(2\pi)^3 2E_k} \left[a(k) e^{-ikx} + a^\dagger(k) e^{ikx} \right]$$

$$k^0 = \sqrt{k^2 + m^2}$$

$$\Phi(x, \omega) = \sum_n f_n(\omega) \Phi^{(n)}(x)$$

$$S = \int d^4 x \int d\omega \left[\frac{1}{2} \Phi (-\partial^2 + \partial_5^2) \Phi - \frac{1}{2} m^2 \Phi^2 \right]$$

$$\int_0^{2\pi R} d\omega f_n f_m = \delta_{nm}$$

$$f_n(\omega) = \frac{1}{\sqrt{2\pi R}} e^{in\omega/R}, n \in \mathbb{Z}$$

$$\partial_5^2 f_n = -n^2/R^2 f_n$$

$$\left(\begin{array}{c} 10 \text{ fm} \\ N = 6 \end{array} \right)$$

$$\frac{d^3k}{(2\pi)^3 2E_k}$$

$$k^0 = \sqrt{\vec{k}^2 + m^2}$$

$$S = \int d^4x \left(\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi^{(0)} \partial_\nu \phi^{(0)} - \frac{1}{2} m^2 \phi^{(0)2} + \sum_{n=1}^{\infty} \left[\eta^{\mu\nu} \partial_\mu \phi^{(n)\dagger} \partial_\nu \phi^{(n)} - \frac{1}{2} (m^2 + m_n^2) |\phi^{(n)}|^2 \right] \right)$$

$$10 \text{ fm}, N=6$$

$$\frac{d^4 k}{(2\pi)^4 2E_k}, \quad k^0 = \sqrt{k^2 + m^2}$$

$$S = \int d^4 x \left(\underbrace{\frac{1}{2} \eta^{\mu\nu} \partial_\mu \Phi^{(0)} \partial_\nu \Phi^{(0)} - \frac{1}{2} m^2 \Phi^{(0)2}}_{\text{Zero-mode}} + \underbrace{\sum_{n=1}^{\infty} \left[\eta^{\mu\nu} \partial_\mu \Phi^{(n)\dagger} \partial_\nu \Phi^{(n)} - \frac{1}{2} (m^2 + m_n^2) |\Phi^{(n)}|^2 \right]}_{\text{KK modes}} \right)$$

$$M_n = \sqrt{m^2 + m_n^2} = \sqrt{m^2 + n^2/R^2}$$

$$1 \mu m, N=3$$

$$10 \text{ fm}, N=6$$

$$d=4: \phi(x) = \int \frac{d^3k}{(2\pi)^3 2E_k} [a(k) e^{-ikx} + a^\dagger(k) e^{ikx}]$$

$$k^0 = \sqrt{\vec{k}^2 + m^2}$$

$$S = \int d^4x \left(\underbrace{\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi^{(0)} \partial_\nu \phi^{(0)}}_{\text{Zero-mode}} - \frac{1}{2} m^2 \phi^{(0)2} + \underbrace{\sum_{n=1}^{\infty} \left[\eta^{\mu\nu} \partial_\mu \phi^{(n)\dagger} \partial_\nu \phi^{(n)} - \frac{1}{2} (m^2 + m_n^2) |\phi^{(n)}|^2 \right]}_{\text{KK modes}} \right)$$

$$\phi^{(-n)} = \phi^{(n)\dagger}$$

$$- \int d^4x \int d\omega \lambda_5 \bar{\Phi}^4 \rightarrow - \int d^4x \frac{\lambda_5}{2\pi R} \sum_{k,l,m,n} \phi^{(k)} \phi^{(l)} \phi^{(m)} \phi^{(n)} \delta_{k+l+m+n,0}$$

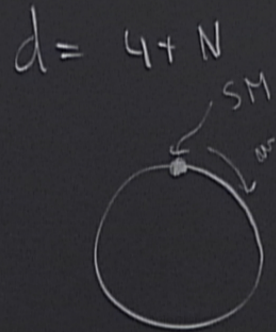
$$M_n = \sqrt{m^2 + m_n^2} = \sqrt{m^2 + n^2/R^2}$$

$$d=4+N \quad f_{\vec{n}}(\vec{u}) = \frac{1}{(2\pi R)^{N/2}} e^{i\vec{n} \cdot \vec{u}/R}, \quad \vec{n} \in \mathbb{Z}_N$$

$$\vec{n} = (n_1, n_2, \dots, n_N)$$

LED = Large Extra Dimensions

" ADD = Arkani-Hamed - Dimopoulos - Dvali

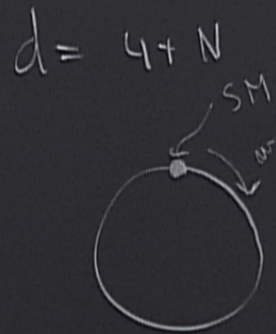


branes

↳ SM at brane at $w=0$

LED = Large Extra Dimensions

" ADD = Arkani-Hamed - Dimopoulos - Dvali



branes

$\hookrightarrow$ SM at brane at $w=0$

$$G_{AB} = \gamma_{AB} + h_{AB}(X, \vec{w}) / M_*^{1+N/2}$$

$$A, B = M, a$$

$$\sim (\partial h)^2$$

$$S_{\text{grav}} = \frac{M_*^{N+2}}{2} \int d^4x \int d^N w \sqrt{|G|} R^{(d)}$$

$$h_{AB}(X, w) = \sum_{\substack{\vec{n} \in \mathbb{Z}_N}} \frac{1}{(2\pi R)^{N/2}} h_{AB}^{(\vec{n})} e^{i\vec{n} \cdot \vec{w} / R}$$

$$w^a \rightarrow w^a + 2\pi R$$

$$G_{AB} \rightarrow G'_{PQ} = \frac{\partial x^A}{\partial y^P} \frac{\partial x^B}{\partial y^Q} G_{AB}$$

$$M^A \rightarrow M^A + 2\pi R$$

$$G_{AB} \rightarrow G'_{PQ} = \frac{\partial x^A}{\partial y^P} \frac{\partial x^B}{\partial y^Q} G_{AB}$$

$$y^A = x^A + \xi^A(x)$$

$$h_{AB} \rightarrow h'_{PQ} = h_{PQ} - \partial_P \xi_Q - \partial_Q \xi_P$$

$$\partial_A h^A_B = \frac{1}{2} \partial_B h^A_A \Rightarrow d \text{ conditions}$$

$$\partial^2 \xi = 0 \Rightarrow d \text{ components}$$

$$\frac{d(d+1)}{2} - 2d = d$$

$$\partial^2 \zeta = 0$$

$= 1$ d component

$$h_{AB}^{(\vec{n})} \rightarrow \left\{ \begin{array}{l} h_{\mu\nu}^{(\vec{n})} \\ h_{\mu a}^{(\vec{n})} = V_{\mu a}^{(\vec{n})} \\ h_{ab}^{(\vec{n})} = S_{ab}^{(\vec{n})} \end{array} \right.$$

$$\vec{n} = \vec{0}$$

$h_{\mu\nu}^{(\vec{0})}$ = massless graviton 2

$V_{\mu a}^{(\vec{0})}$ = N massless vectors: $2N$

$S_{ab}^{(\vec{n})} = \frac{N(N+1)}{2}$ real scalars

$$2 + 2N + \frac{N(N+1)}{2} = \frac{(N^2 + 5N + 4)}{2}$$

$$M_{\vec{n}}^2 = \sqrt{\vec{n}^2}/R^2 = \|\vec{n}\|/R$$

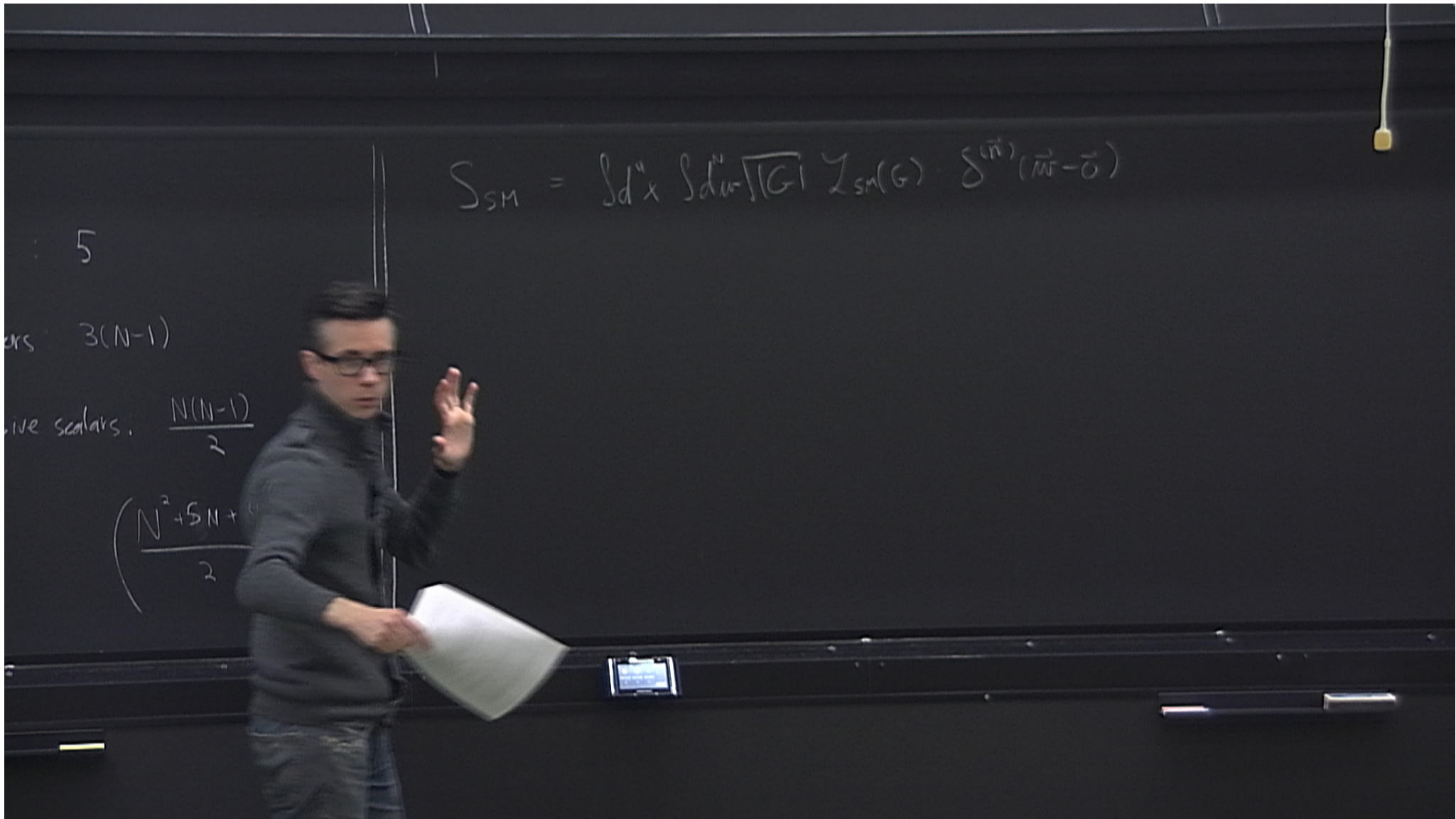
$\vec{n}$: $h_{\mu\nu}^{(\vec{n})}$ = one massive graviton : 5

$V_{\mu a}^{(\vec{n})}$ = $(N-1)$ massive vectors : $3(N-1)$

$S_{ab}^{(\vec{n})}$ = $\frac{N(N+1)}{2} - N$ massive scalars. $\frac{N(N-1)}{2}$

$$\begin{cases} n^a V_{\mu a}^{(\vec{n})} = 0 \\ n^a S_{ab}^{(\vec{n})} = 0 \end{cases}$$

$$\left(\frac{N^2 + 5N + 4}{2} \right)$$



$$S_{SM} = \int d^4x \int d^4w \sqrt{|G|} \mathcal{L}_{SM}(G) \delta^{(n)}(\vec{w} - \vec{0})$$

$$= \int d^4x \sqrt{|g|} \mathcal{L}_{SM}(g)$$

$$g_{\mu\nu}(x) = G_{\mu\nu}(x, \vec{w} = \vec{0})$$

$$S_{SM} = \int d^4x \int d\vec{n} \sqrt{|G|} \mathcal{L}_{SM}(G) \delta^{(\vec{n})}(\vec{w}-\vec{0})$$

$$= \int d^4x \sqrt{|g|} \mathcal{L}_{SM}(g)$$

$$g_{\mu\nu}(x) = G_{\mu\nu}(x, \vec{w}=\vec{0})$$

$$> \int d^4x \frac{1}{M_{*}^{1.5/2}} \frac{1}{\sqrt{|V_N|}} \left[T^{\mu\nu} \sum_{\vec{n}} h_{\mu\nu}^{(\vec{n})} - K T^{\mu}_{\mu} \sum_{\vec{n}} S_a^{(\vec{n})} a \right]$$

$$T^{\mu\nu} = \frac{1}{\sqrt{|g|}} \frac{\delta S_{SM}}{\delta g_{\mu\nu}}$$

$$S_{SM} = \int d^4x \int d\vec{n} \sqrt{|G|} \mathcal{L}_{SM}(G) \delta^{(\vec{n})}(\vec{w}-\vec{0})$$

$$= \int d^4x \sqrt{|g|} \mathcal{L}_{SM}(g)$$

$$g_{\mu\nu}(x) = G_{\mu\nu}(x, \vec{w}=\vec{0})$$

$$> \int d^4x \frac{1}{M_{*}^{1+D/2}} \frac{1}{\sqrt{|V_D|}} \left[\underbrace{T^{\mu\nu} \sum_{\vec{n}} h_{\mu\nu}^{(\vec{n})}}_{\text{usual gravity}} - K T^{\mu}_{\mu} \sum_{\vec{n}} S_a^{(\vec{n})} \right]$$

$$T^{\mu\nu} = \frac{1}{\sqrt{|g|}} \frac{\delta S_{SM}}{\delta g_{\mu\nu}}$$

$$S_{SM} = \int d^4x \int d\vec{w} \sqrt{|G|} \mathcal{L}_{SM}(G) \delta^{(\vec{n})}(\vec{w}-\vec{0})$$

$$= \int d^4x \sqrt{|g|} \mathcal{L}_{SM}(g)$$

$$g_{\mu\nu}(x) = G_{\mu\nu}(x, \vec{w}=\vec{0})$$

$$> \int d^4x \frac{1}{M_*^{4+D/2}} \frac{1}{\sqrt{|V_N|}} \left[\underbrace{T^{\mu\nu} \sum_{\vec{n}} h_{\mu\nu}^{(\vec{n})}}_{\text{usual gravity}} - K T^{\mu}_{\mu} \sum_{\vec{n}} \underbrace{S_a^{(\vec{n})}}_{\parallel}^a \right]$$

$$r^{(\vec{n})} = \sum_a S_a^{(\vec{n})} = \text{radion}$$

$$T^{\mu\nu} = \frac{1}{\sqrt{|g|}} \frac{\delta S_{SM}}{\delta g_{\mu\nu}}$$