Title: PHYS 733 - Quantum Many-Body Physics (W2016) - Roger Melko - Lecture 22

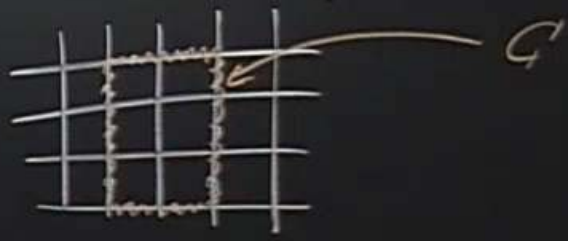
Date: Mar 24, 2016 10:00 AM

URL: http://pirsa.org/16030033

Abstract:

Gauge-invariant observable. "Wilson loop"

$$W_G = \prod_{i \in G} \sigma_i^z$$



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Contour
The expectation value of W_C depends on the characteristics of the contour.
Let's examine the high and low-T series expansions for $\langle W_C \rangle = \langle \prod_{i \in C} \sigma_i^z \rangle$
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Contour

The expectation value of W_C depends on the characteristics of the contour.

Let's examine the high and low-T series expansions for $\langle W_C \rangle = \langle \prod_{i \in C} \sigma_i^z \rangle$

High-T series expansion

$$H = -J \sum_{p} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$

$$e^{\beta \sigma^z \sigma^z \sigma^z \sigma^z} = \cosh\beta + \sigma^z \sigma^z \sigma^z \sigma^z \sinh\beta$$

$$= \cosh\beta \,(1 + \sigma^z \sigma^z \sigma^z \sigma^z \tanh\beta)$$

$$\langle W_C \rangle = \mathrm{Tr}\left[\prod_{i \in C} \sigma_i^z \prod_p (1 + \sigma^z \sigma^z \sigma^z \sigma^z)\right]$$

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of the contour.

Let's examine the high and low-T series expansions for $\langle W_C \rangle = \langle \prod_{i \in C} \sigma_i^z \rangle$

High-T series expansion

$$H = -J \sum_{p=\square} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z$$

$$e^{\beta \sigma^z \sigma^z \sigma^z \sigma^z} = \cosh\beta + \sigma^z \sigma^z \sigma^z \sigma^z \sinh\beta$$

$$= \cosh\beta \left(1 + \sigma^z \sigma^z \sigma^z \sigma^z \tanh\beta\right)$$

$$\langle W_C \rangle = \frac{\mathrm{Tr}\left[\prod_{i \in C} \sigma_i^z \prod_p \left(1 + \sigma^z \sigma^z \sigma^z \sigma^z \tanh\beta\right)\right]}{\mathrm{Tr}\left[\prod_p \left(1 + \sigma^z \sigma^z \sigma^z \sigma^z \tanh\beta\right)\right]}$$

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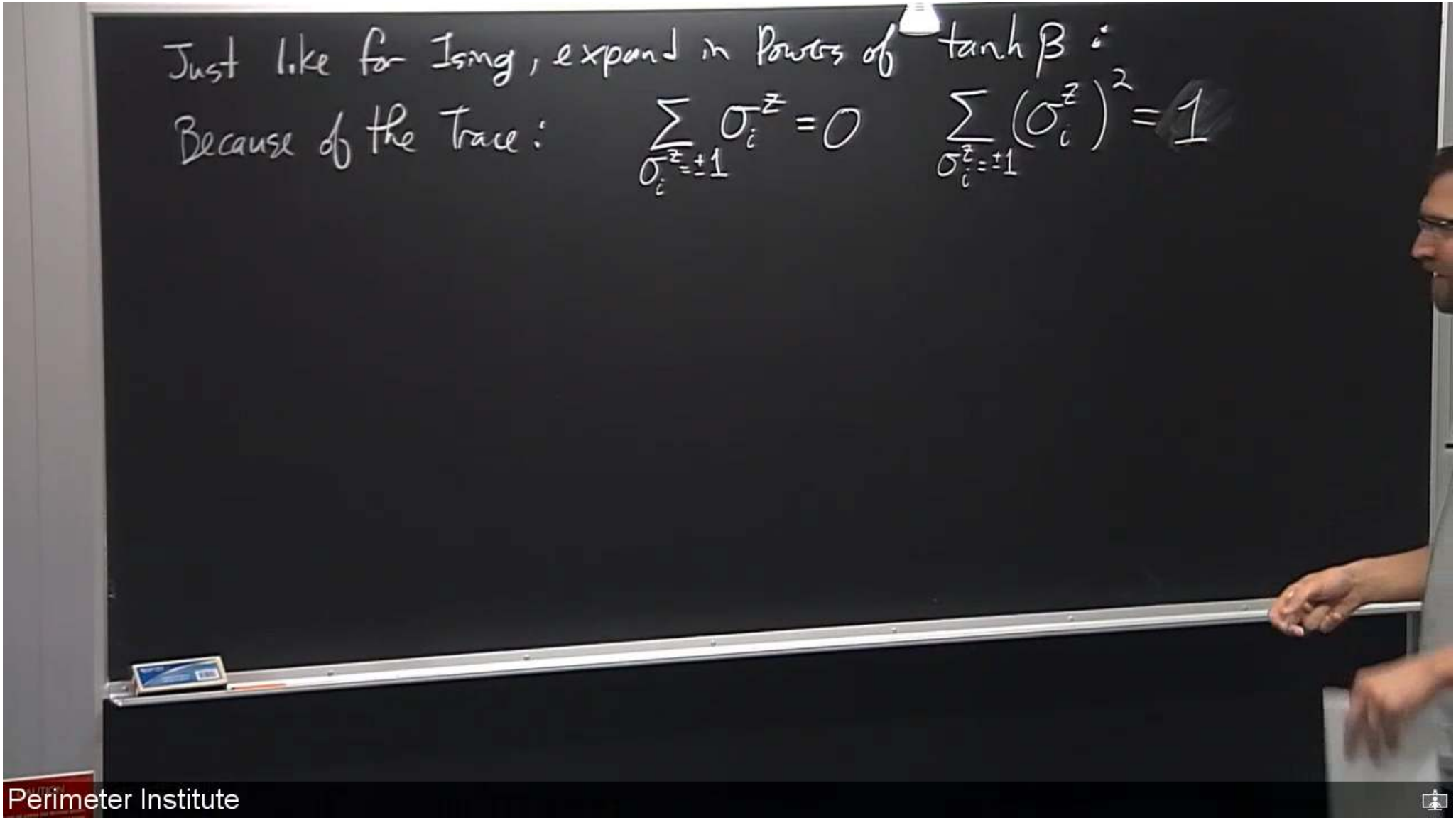
Just like for Ising, expand in Powers of $\tanh\beta$:
Because of the Trace: $\sum_{\sigma_i^z=\pm 1} \sigma_i^z = 0$ $\sum_{\sigma_i^z=\pm 1} (\sigma_i^z)^2 = 1$
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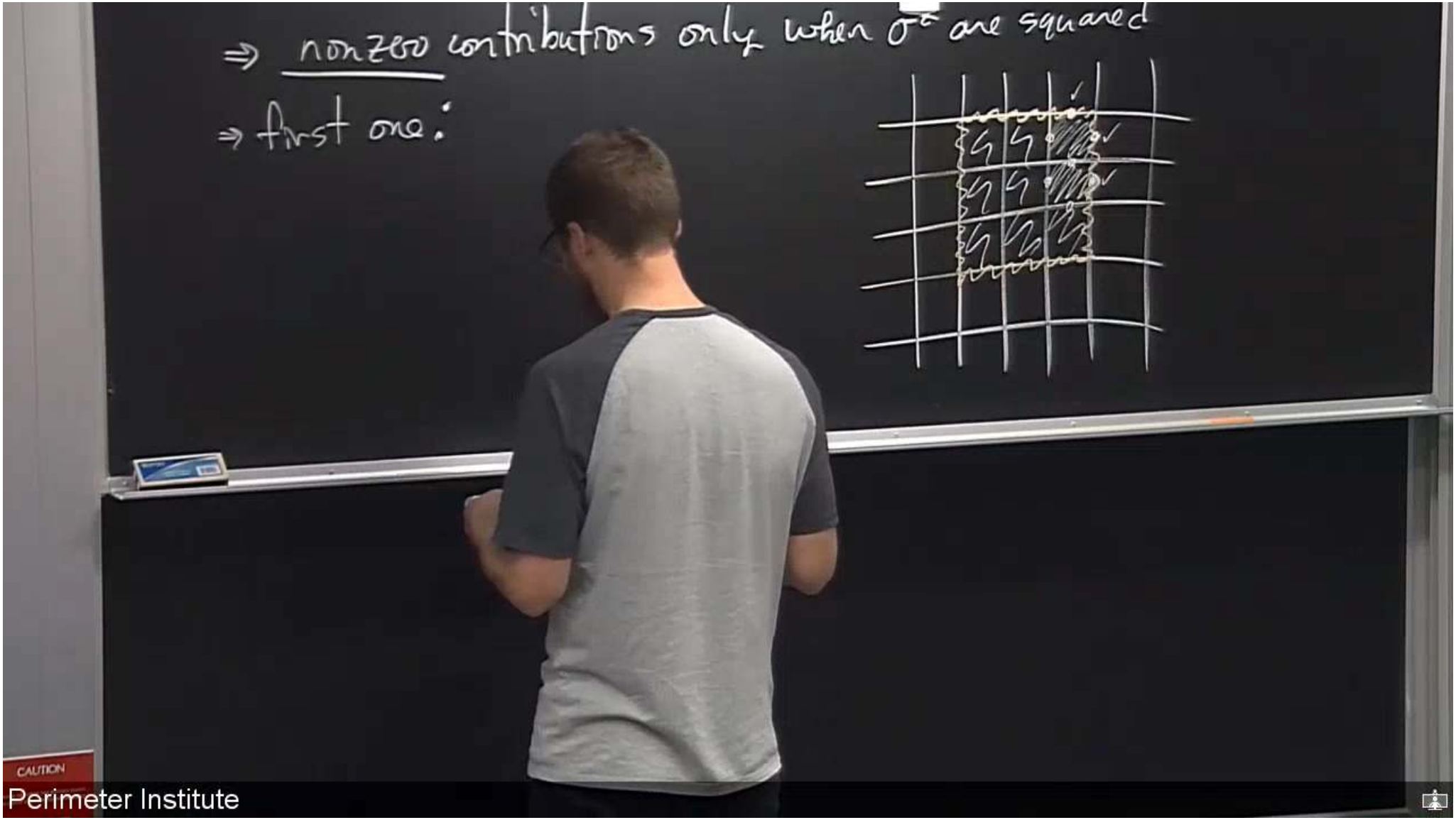
⇒ nonzero contributions only when σ^z are squared
⇒ first one:
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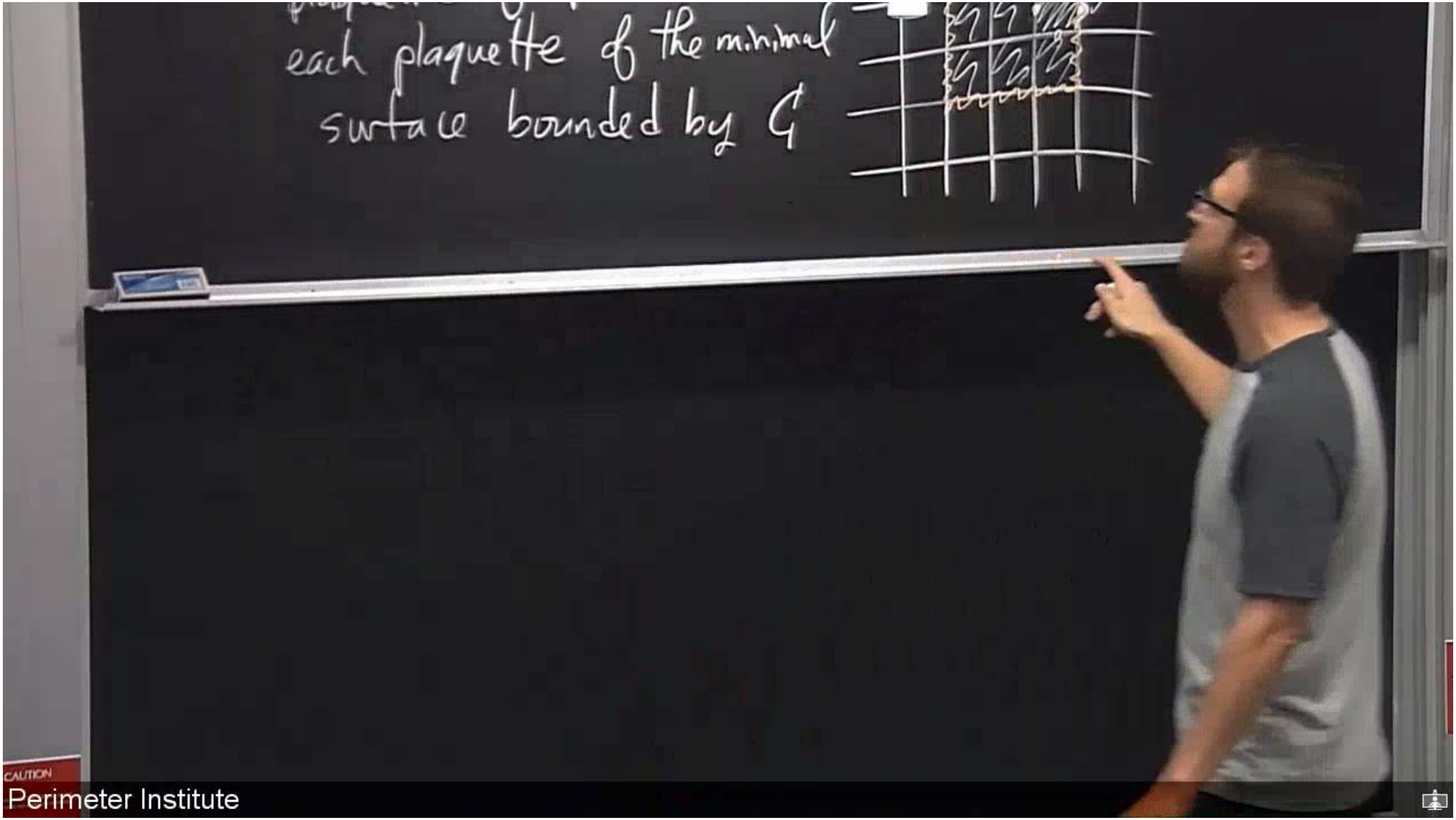
each plaquette of the minimal
surface bounded by G
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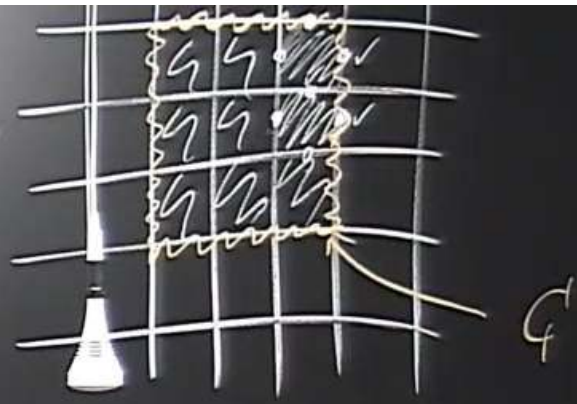
⇒ first one: when there is a plaquette of operators for each plaquette of the minimal surface bounded by $\mathcal{C}$

⇒ call # plaquettes $N_{\mathcal{C}}$



then $\langle W_{\mathcal{C}} \rangle \simeq (\tanh \beta)^{N_{\mathcal{C}}}$

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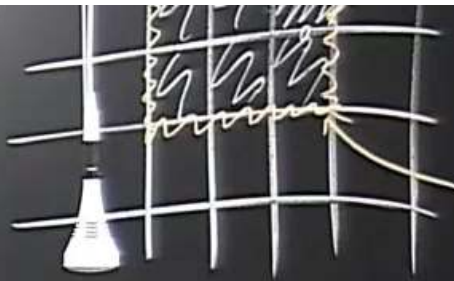
each plaquette of the minimal surface bounded by C

$\Rightarrow$ call # plaquettes N_C



then $\langle W_C \rangle \simeq (\tanh\beta)^{N_C} = e^{N_C \log(\tanh\beta)}$

$= e^{-f(\beta) A}$ true for higher orders in the expansion.

where $f(\beta)$ is some function, and A is the <u>area</u> of C

"Area Law"

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$\Rightarrow$ call # plaquettes N_G

G

then $\langle W_G \rangle \simeq (\tanh\beta)^{N_G} = e^{N_G \log(\tanh\beta)}$

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"Area Law"

Low-T: - choose a representative $T=0$ configuration

- expansion proceeds in a "droplet" picture.

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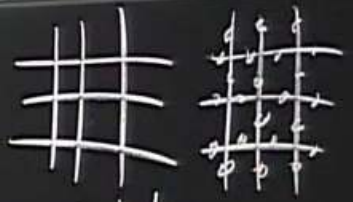
Choose: all spins "down" it is easy to draw 

droplet expansion includes lowest order energy excitations

ie. one flipped spin this "frustrates" 2(d-1) plaquettes.

energy cost is $4(d-1)\beta$

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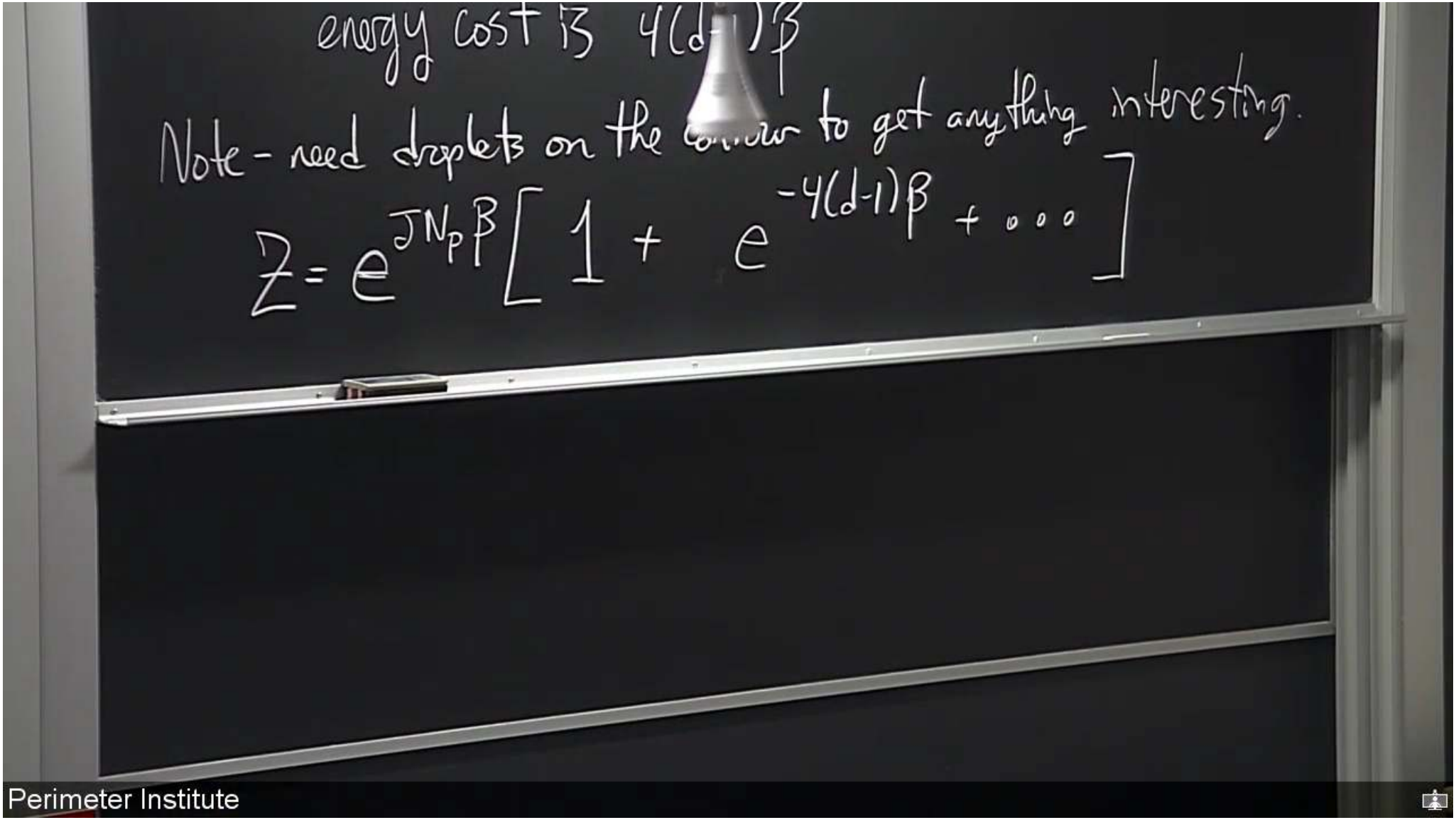
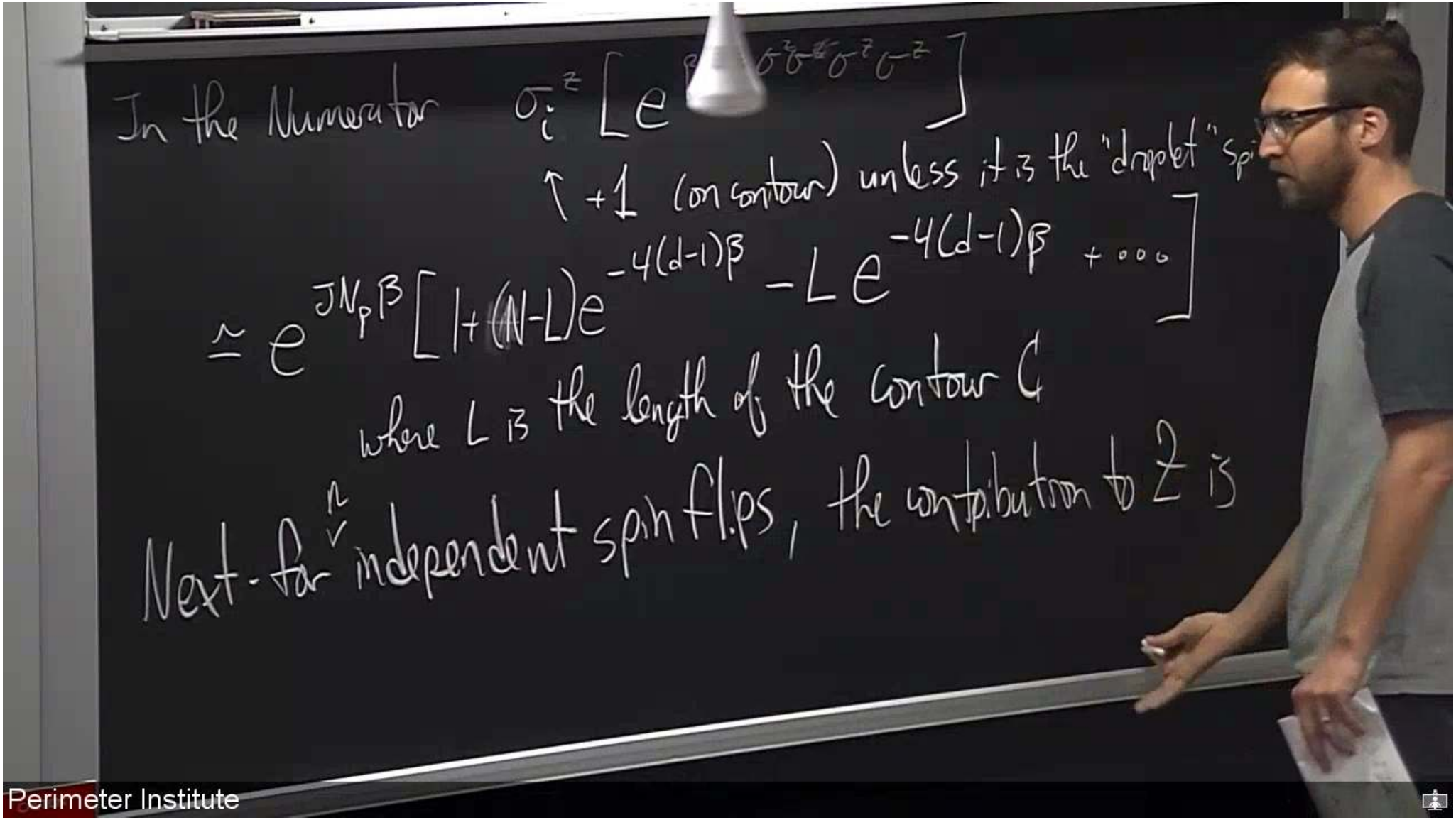
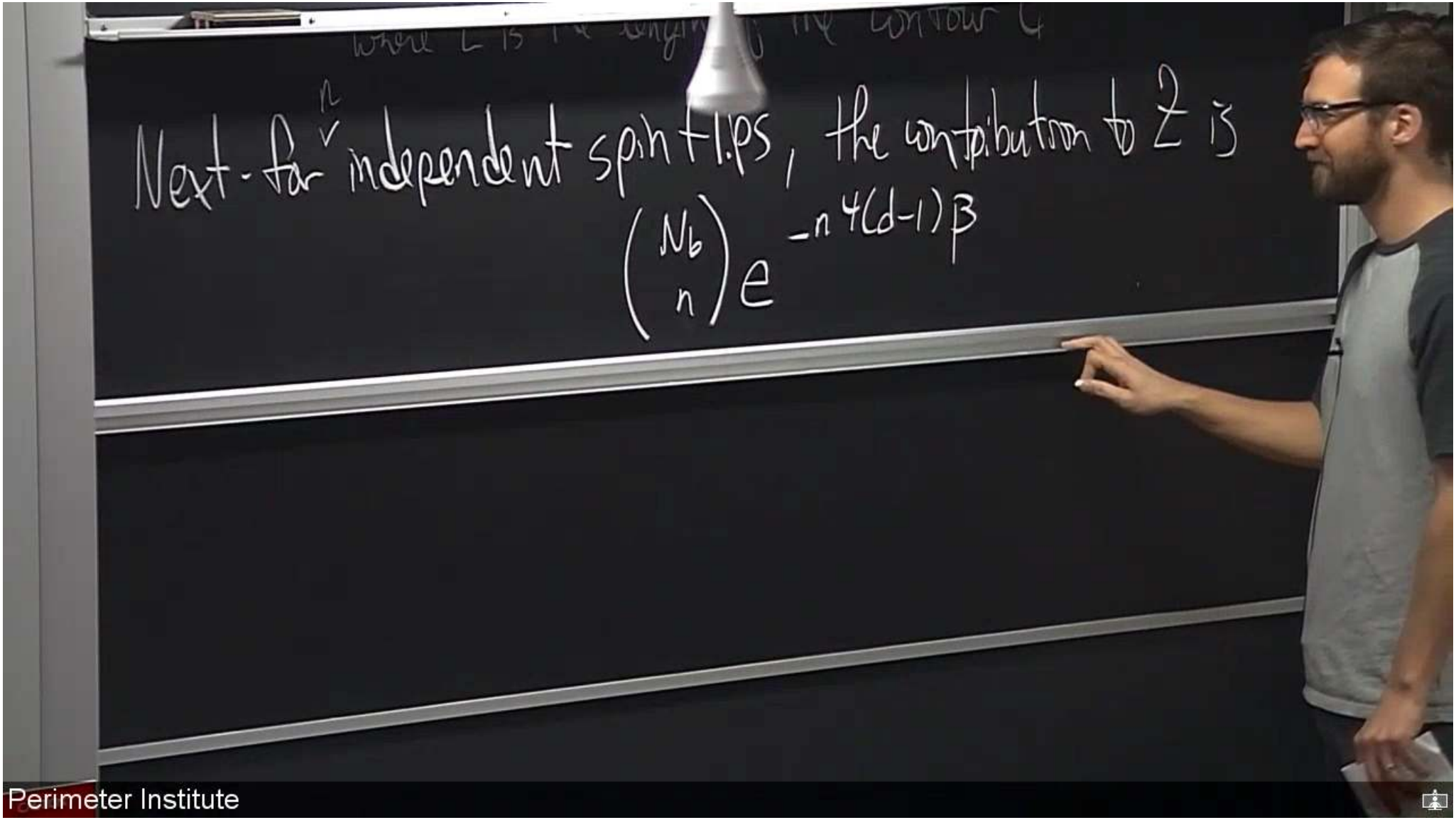

energy cost is 4(d-1)β
Note - need droplets on the contour to get anything interesting.
$Z = e^{\beta N_p \beta}\left[1 + e^{-4(d-1)\beta} + \cdots\right]$


In the Numerator $\sigma_i^z \left[e^{\beta \sigma^z \sigma^z \sigma^z \sigma^z} \right]$
$\uparrow +1$ (on contour) unless it is the "droplet" spi
$\simeq e^{JN_p\beta} \left[1 + (N-L)e^{-4(d-1)\beta} - Le^{-4(d-1)\beta} + \cdots \right]$
where L is the length of the contour C
Next-far independent spin flips, the contribution to Z is


Next-far n independent spin flips, the contribution to Z is
$\binom{N_b}{n} e^{-n\,4(d-1)\beta}$
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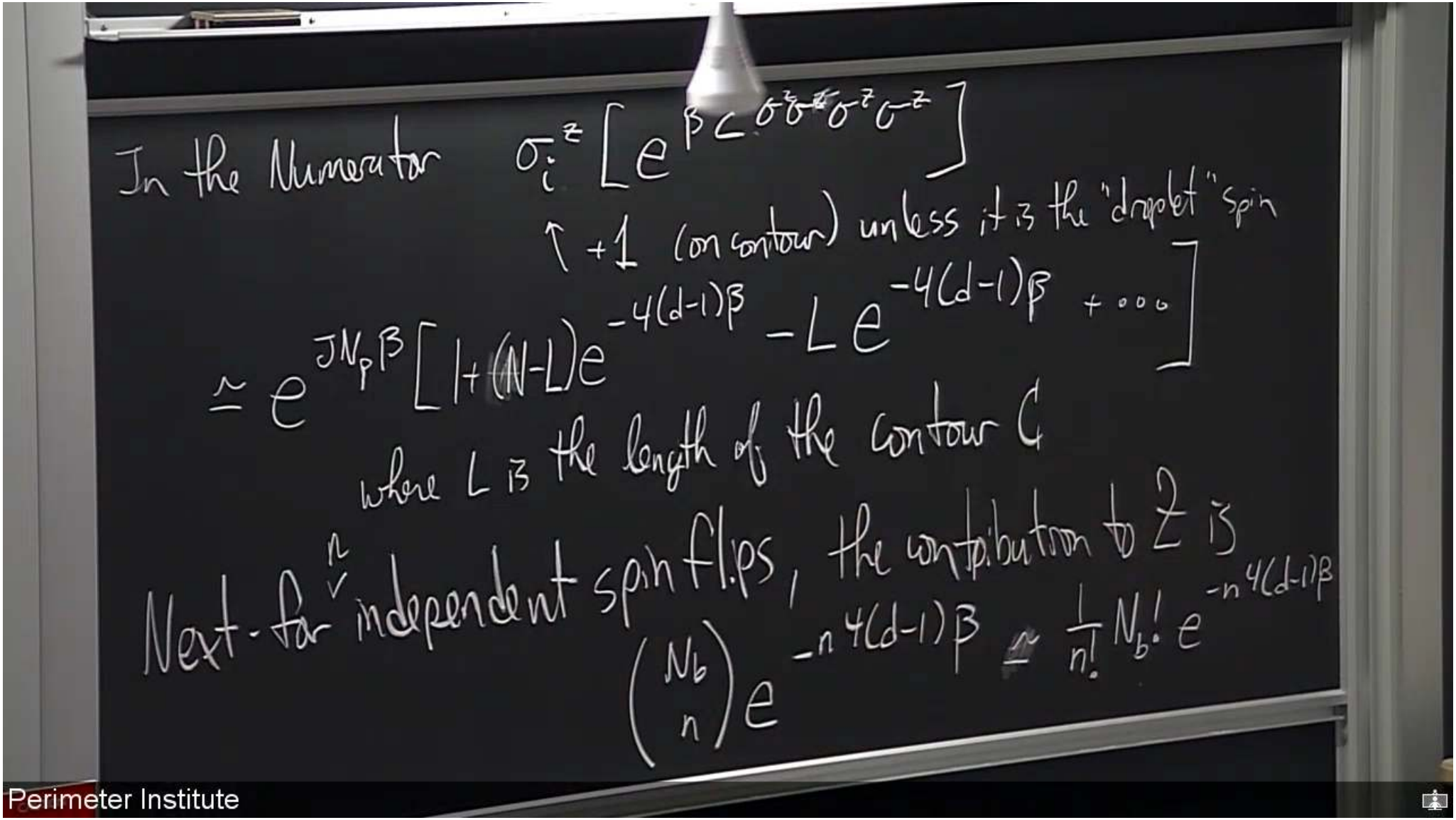
In the Numerator $\sigma_i^z \left[e^{\beta \sum \sigma^z \sigma^z \sigma^z \sigma^z} \right]$
$\uparrow +1$ (on contour) unless it is the "droplet" spin
$\simeq e^{JN_p\beta} \left[1 + (N-L) e^{-4(d-1)\beta} - L e^{-4(d-1)\beta} + \cdots \right]$
where L is the length of the contour C
Next-for n independent spin flips, the contribution to Z is
$\binom{N_b}{n} e^{-n4(d-1)\beta} \leq \frac{1}{n!} N_b! e^{-n4(d-1)\beta}$
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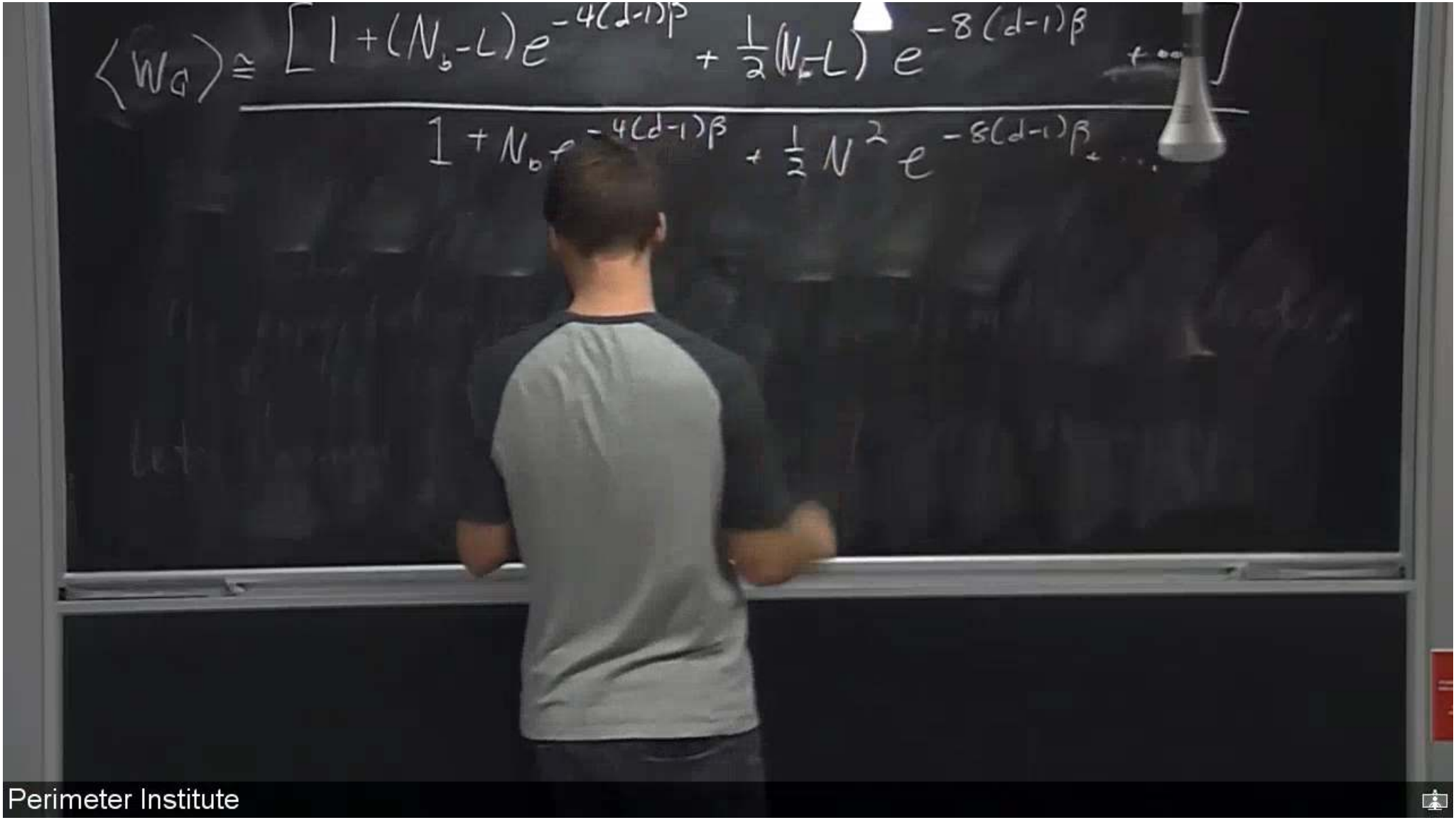
$\langle W_G \rangle \cong \frac{\left[1 + (N_b - L)e^{-4(d-1)\beta} + \frac{1}{2}(N_b - L)^2 e^{-8(d-1)\beta} + \cdots\right]}{1 + N_b e^{-4(d-1)\beta} + \frac{1}{2}N^2 e^{-8(d-1)\beta} + \cdots}$
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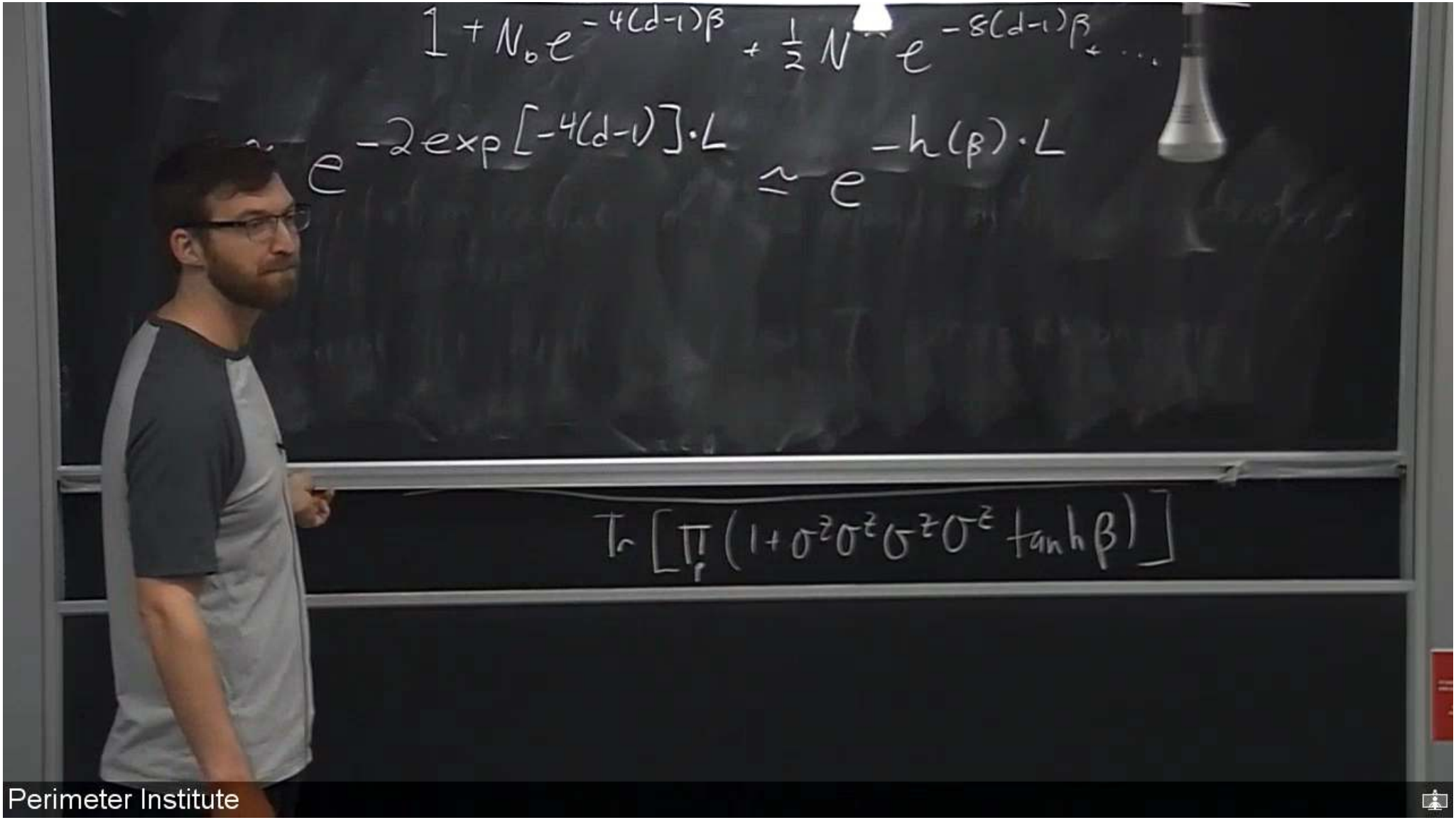
$1 + N_0 e^{-4(d-1)\beta} + \frac{1}{2} N^{\cdots} e^{-8(d-1)\beta} + \cdots$
$e^{-2\exp[-4(d-1)]\cdot L} \simeq e^{-h(\beta)\cdot L}$
$\mathrm{Tr}\left[\prod_p \left(1 + \sigma^z\sigma^z\sigma^z\sigma^z \tanh\beta\right)\right]$
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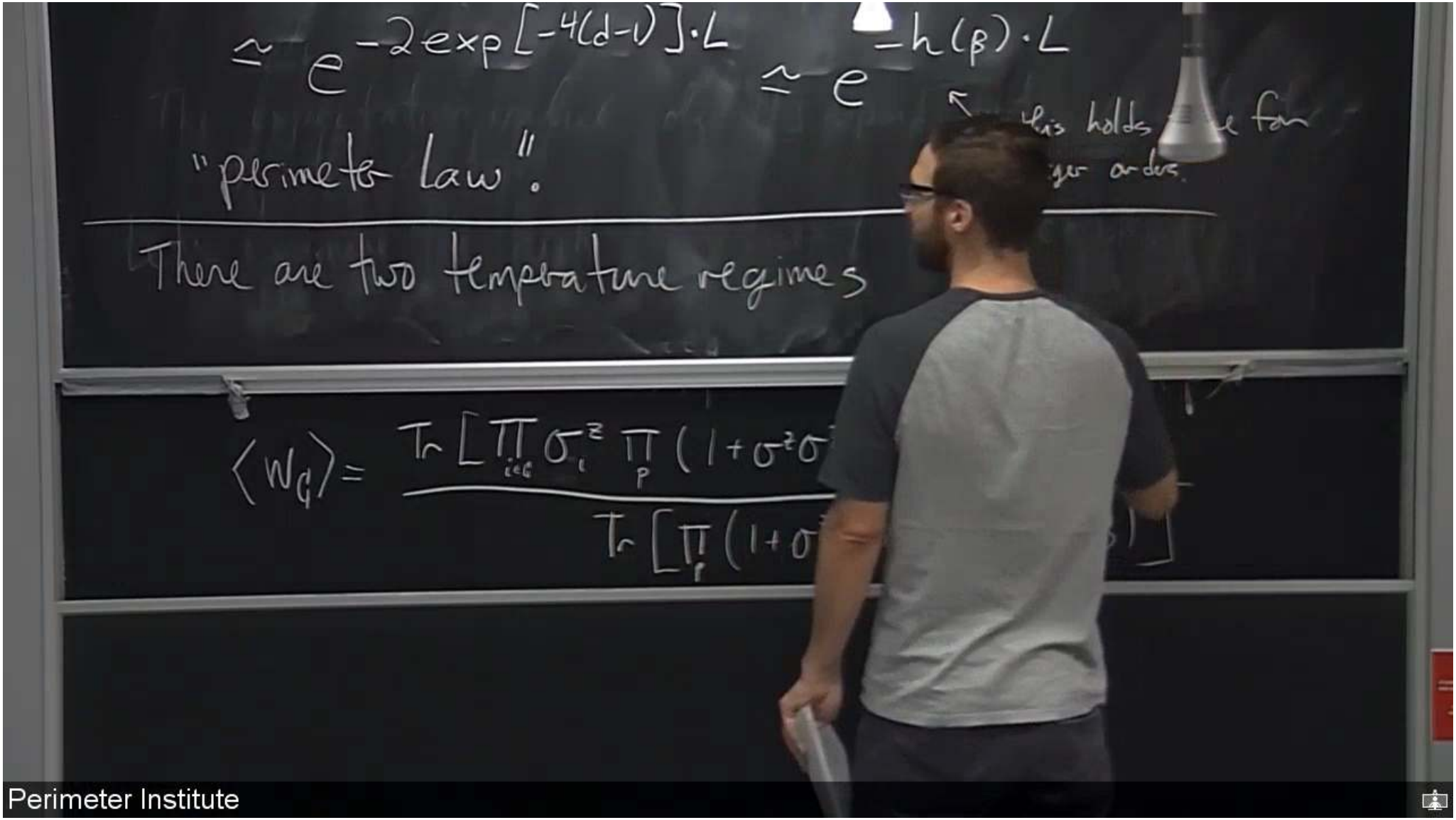
$\simeq e^{-2\exp[-4(d-1)]\cdot L}$
$\simeq e^{-h(\beta)\cdot L}$
"perimeter law".
This holds
There are two temperature regimes
$\langle W_G \rangle =$
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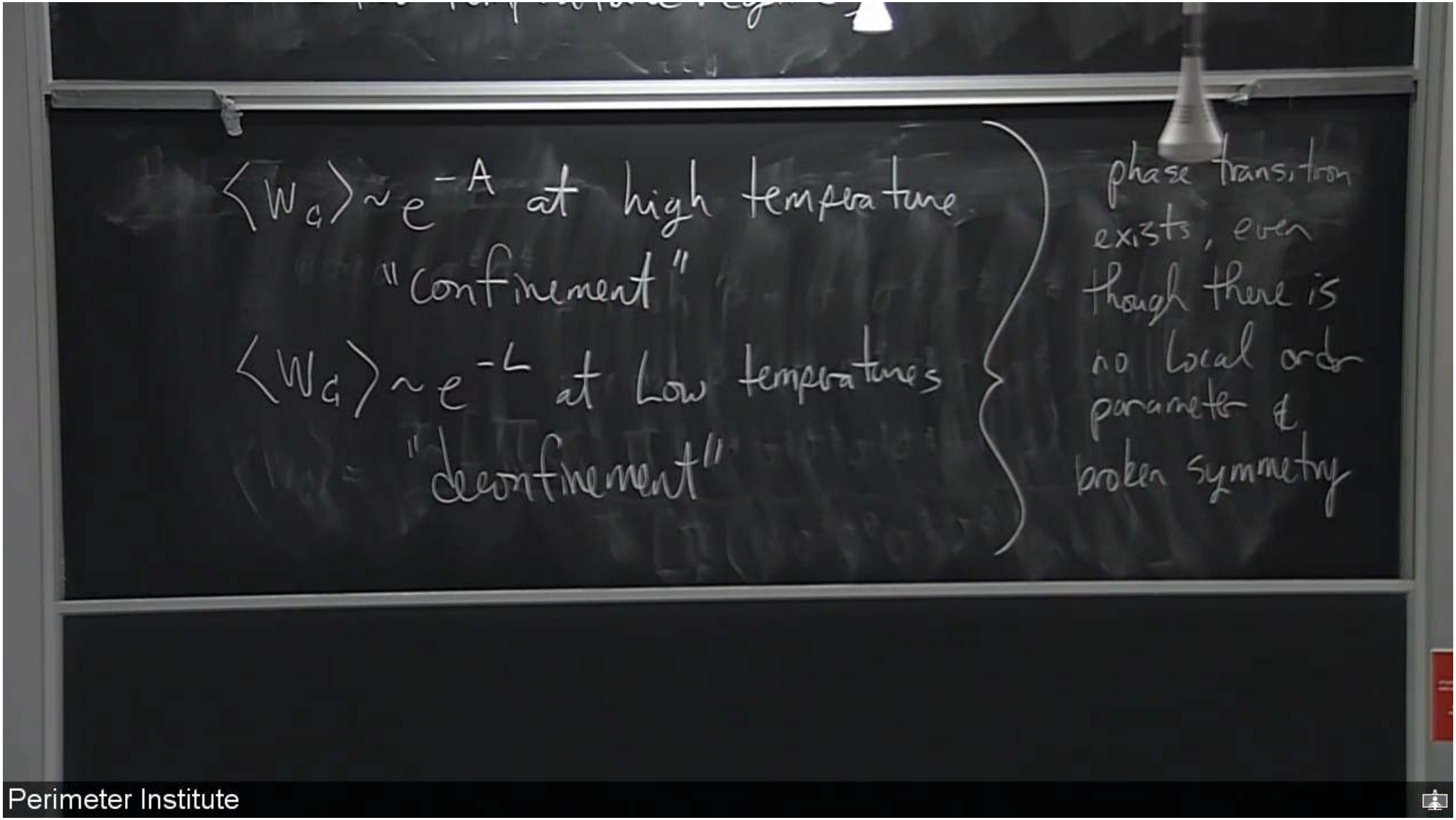
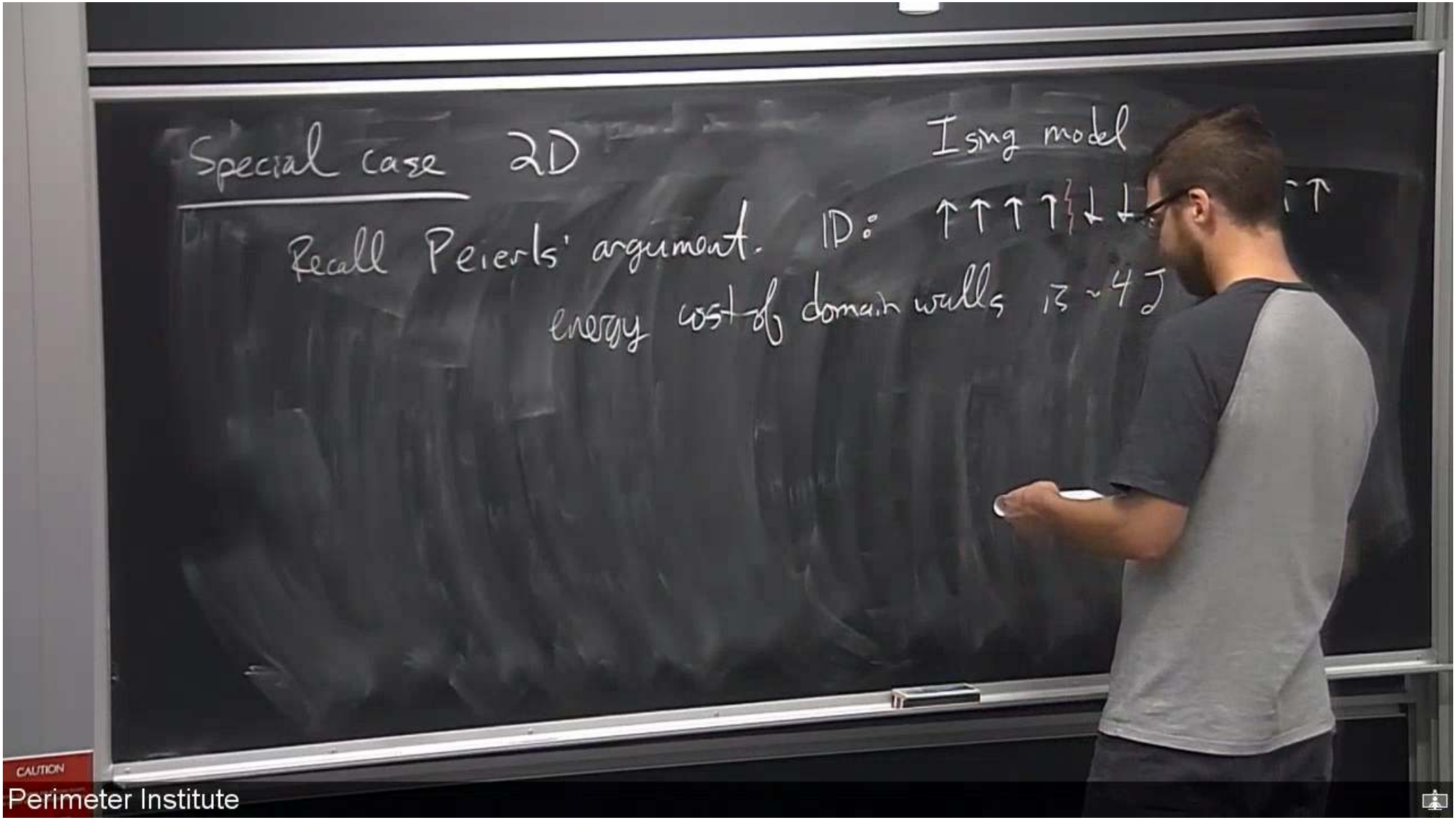

$\langle W_C \rangle \sim e^{-A}$ at high temperature
"confinement"
$\langle W_C \rangle \sim e^{-L}$ at Low temperatures
"deconfinement"
phase transition exists, even though there is no local order parameter & broken symmetry


Special case 2D
Ising model
Recall Peierls' argument. 1D: ↑↑↑↑↓↓
energy cost of domain walls is ~4J
CAUTION
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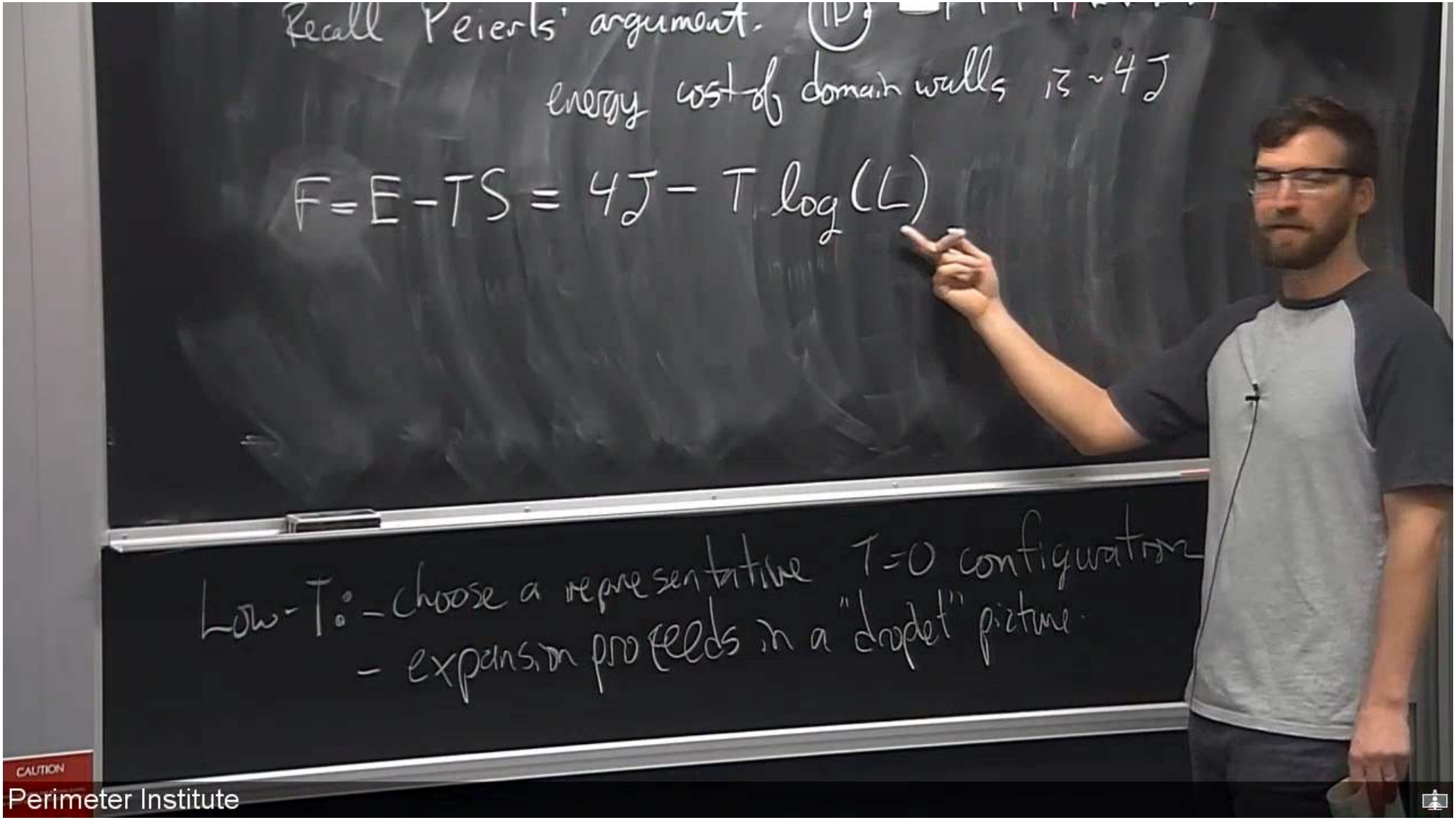
Recall Peierls' argument.
energy cost of domain walls is ~4J
F=E-TS = 4J - T log(L)
Low-T: - choose a representative T=0 configuration
- expansion proceeds in a "droplet" picture.
CAUTION
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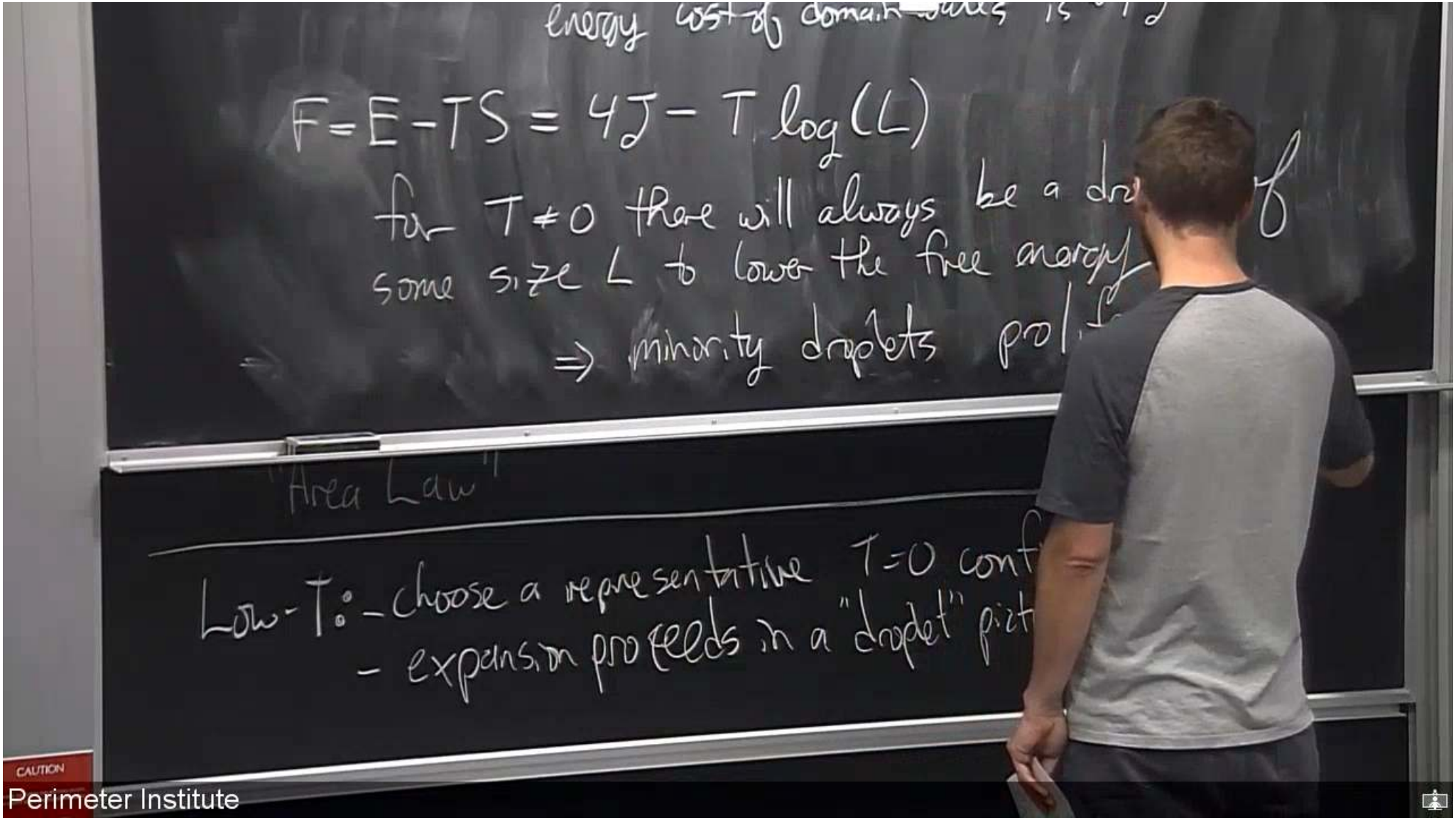
energy cost of domain
$F = E - TS = 4J - T \log(L)$
for $T \neq 0$ there will always be a dro
some size L to lower the free energy
$\Rightarrow$ minority droplets pro
"Area Law"
Low-T:
- choose a representative $T=0$ conf
- expansion proceeds in a "droplet" pict
CAUTION
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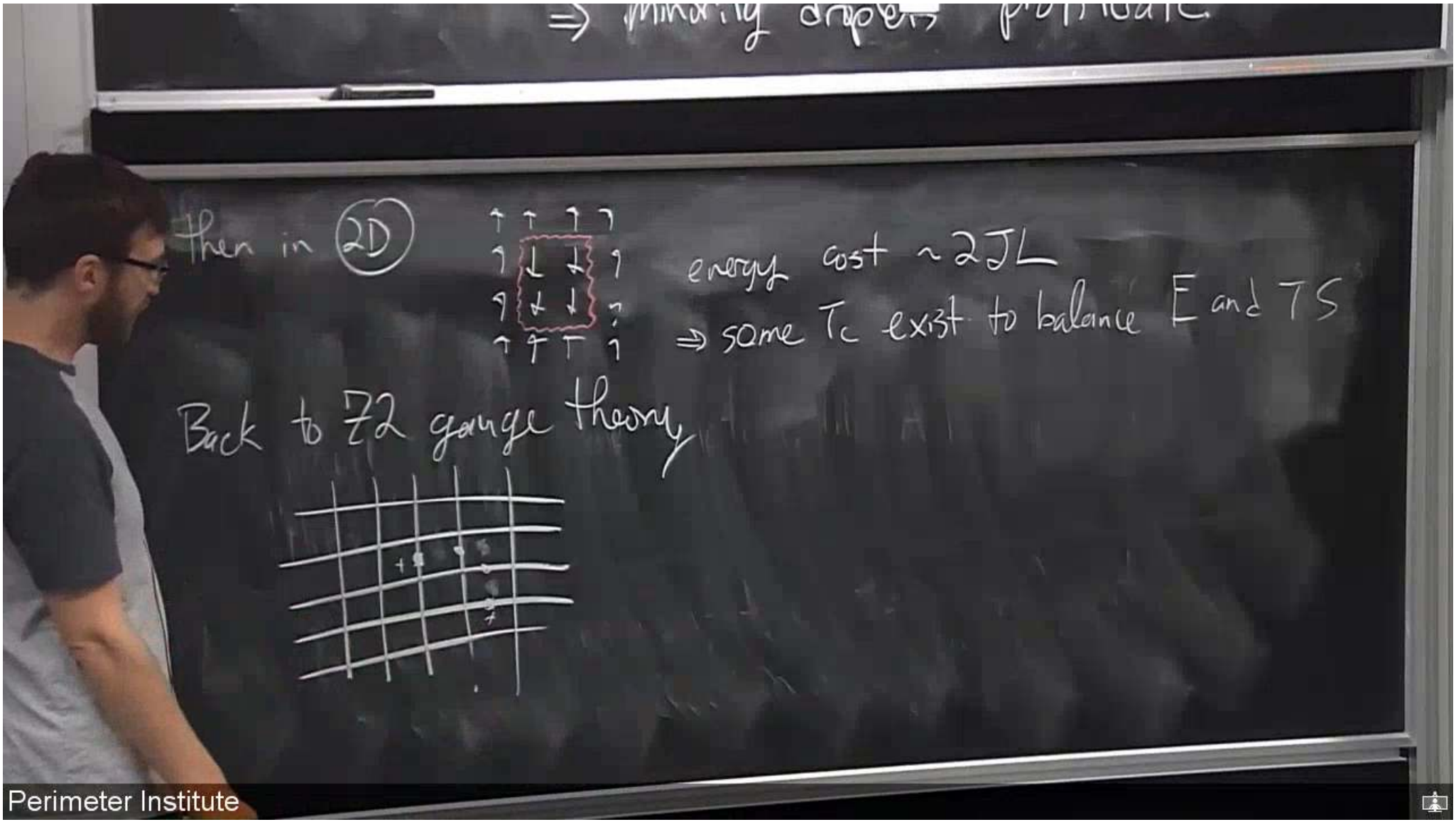
⇒ minority droplets
then in 2D
energy cost ~ 2JL
⇒ some Tc exist to balance E and TS
Back to Z2 gauge theory
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⇒ [illegible]

then in (2D)

energy cost ~ 2JL

⇒ some T_c exist to balance E and TS

Back to Z2 gauge theory

"domain walls" are 0 dimensional

→ entropy will always favour the proliferation of defects

CAUTION

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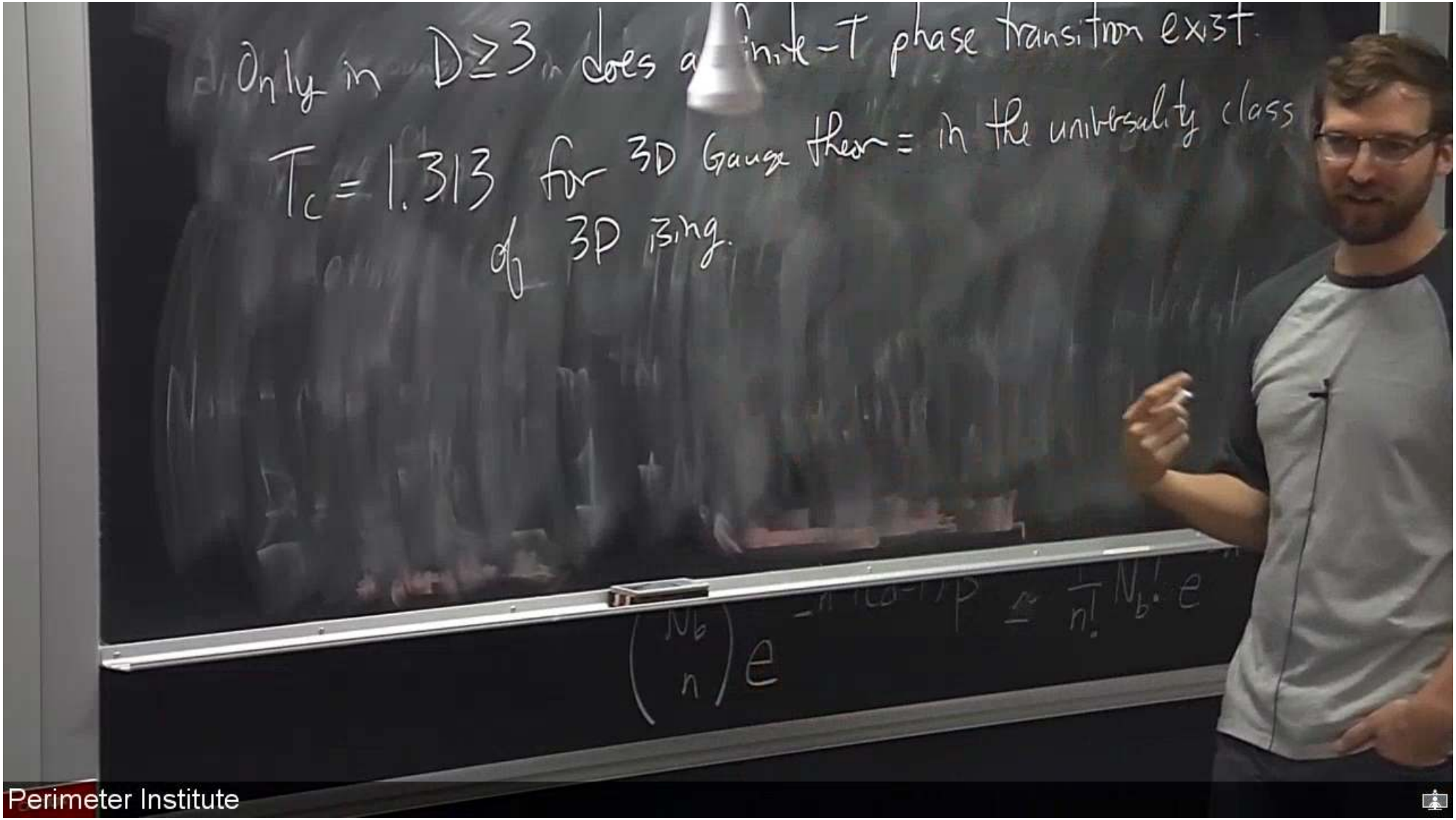
Only in D≥3 does a finite-T phase transition exist.
$T_c = 1.313$ for 3D Gauge theory in the universality class of 3D Ising.
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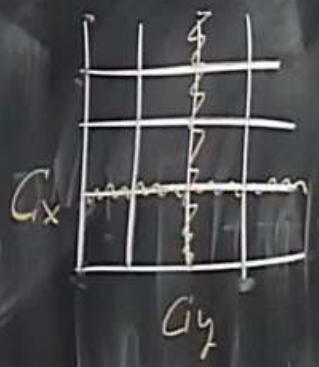
Consider the 2D Gauge Theory on a Torus



define wilson loops that wrap around the PBCs
- non-contractable.

$$W_C = \prod_{i \in C} \sigma_i^z$$

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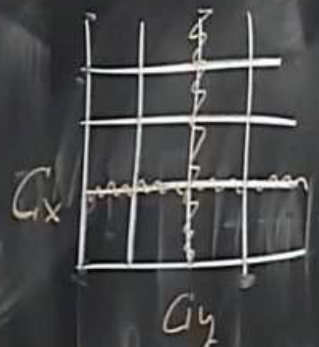
Consider the 2D gauge theory on a Torus



define wilson loops that wrap around the PBCs
- non-contractable.

$$W_C = \prod_{i \in C} \sigma_i^z$$

is invariant under local gauge transformations

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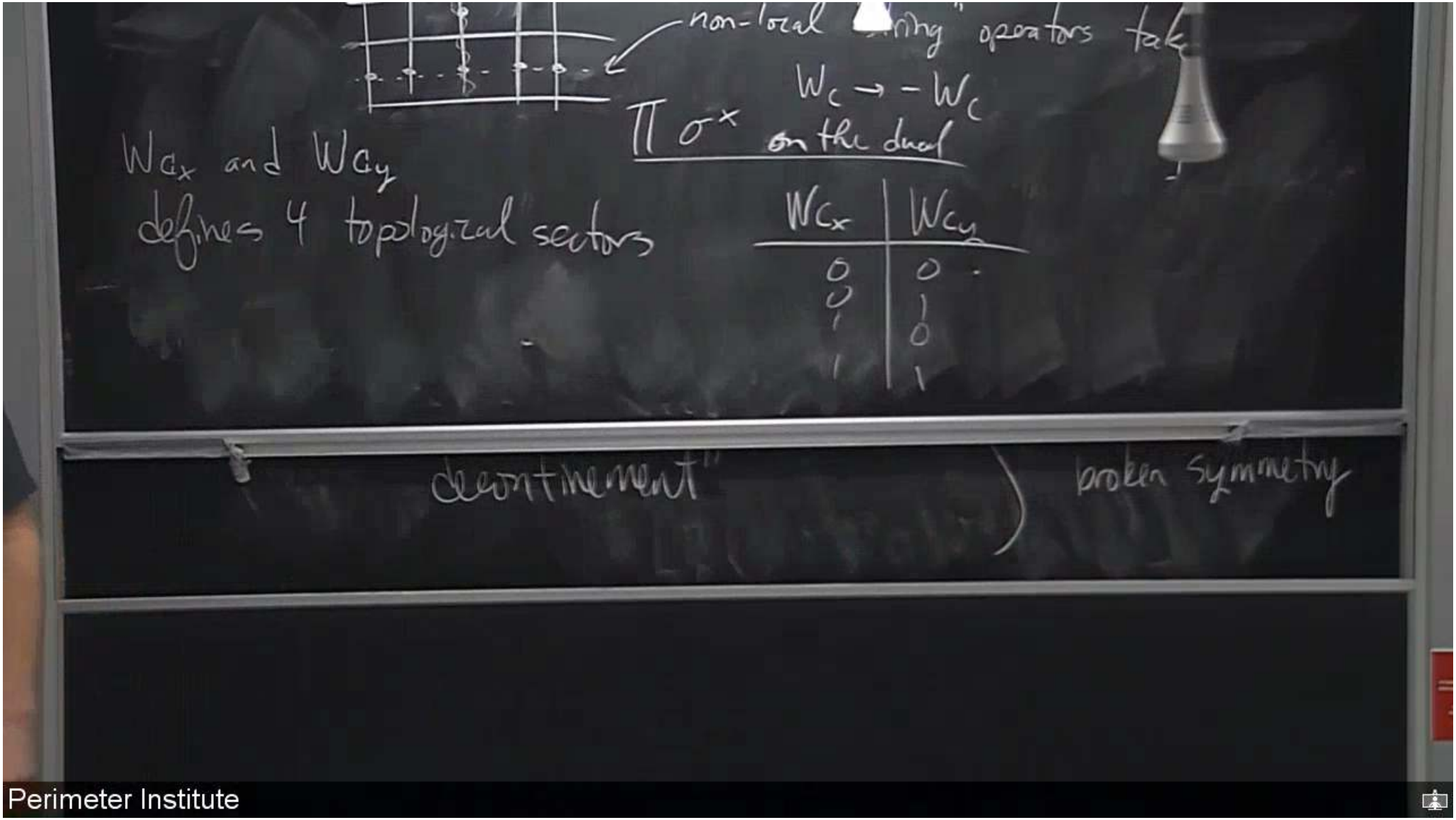
non-local "string" operators take
$W_c \to -W_c$
$\Pi \sigma^x$ on the dual
W_{Cx} and W_{Cy}
defines 4 topological sectors
W_{Cx} | W_{Cy}
0 | 0
0 | 1
1 | 0
1 | 1
deconfinement
broken symmetry
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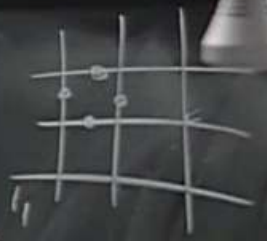
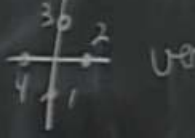
Toric code 2D square lattice mostly



Define "vertex" and "plaquette operators"

$$A(v) = \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x$$



$$B(p) = \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z$$

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→ These operators share 0 or 2 links - Hence they all commute: $[A(v), B(p)] = 0$

→ entropy will always favour the proliferation of defects

CAUTION
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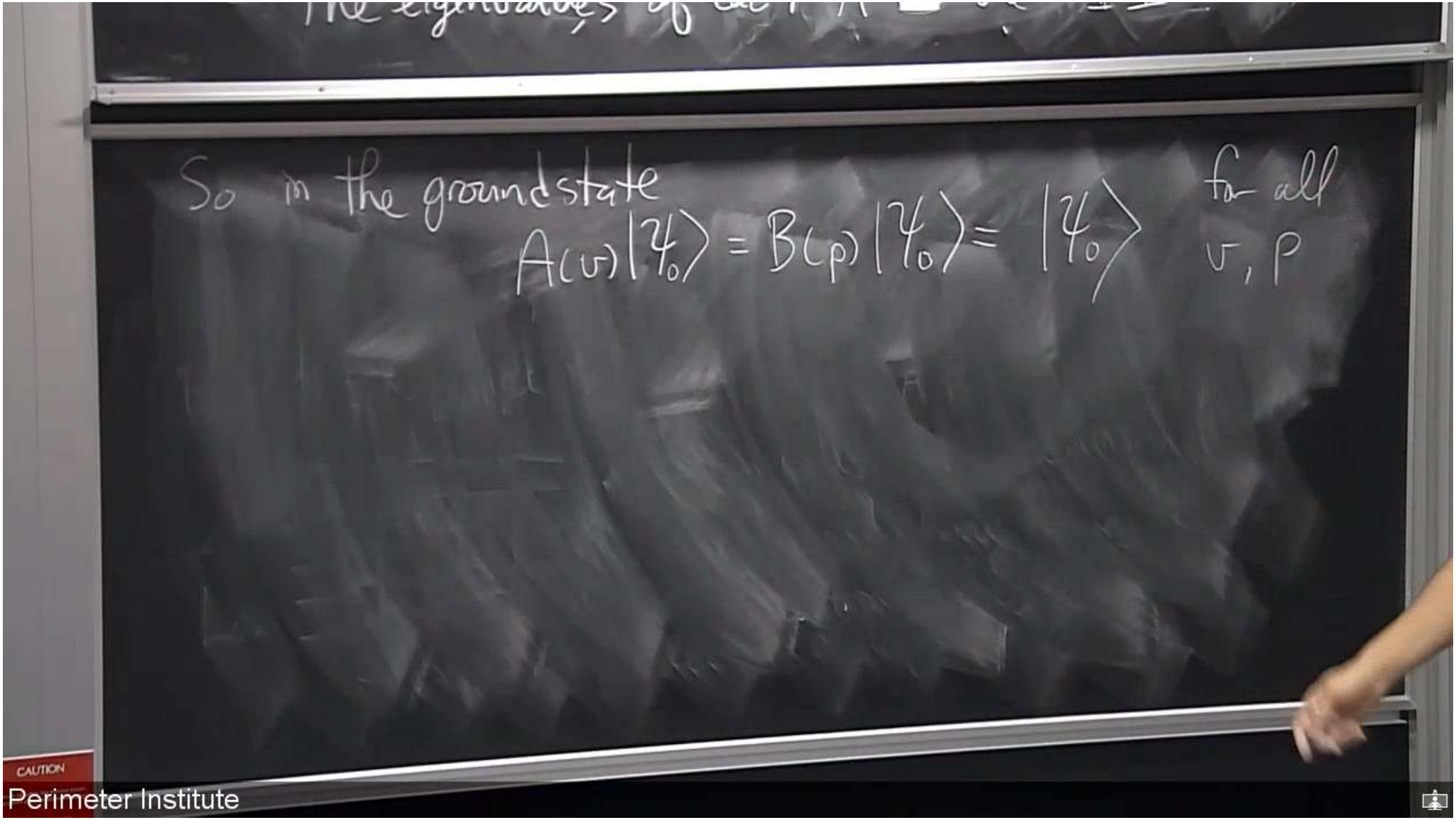
So in the groundstate
$A(v)|\psi_0\rangle = B(p)|\psi_0\rangle = |\psi_0\rangle$ for all v, p
CAUTION
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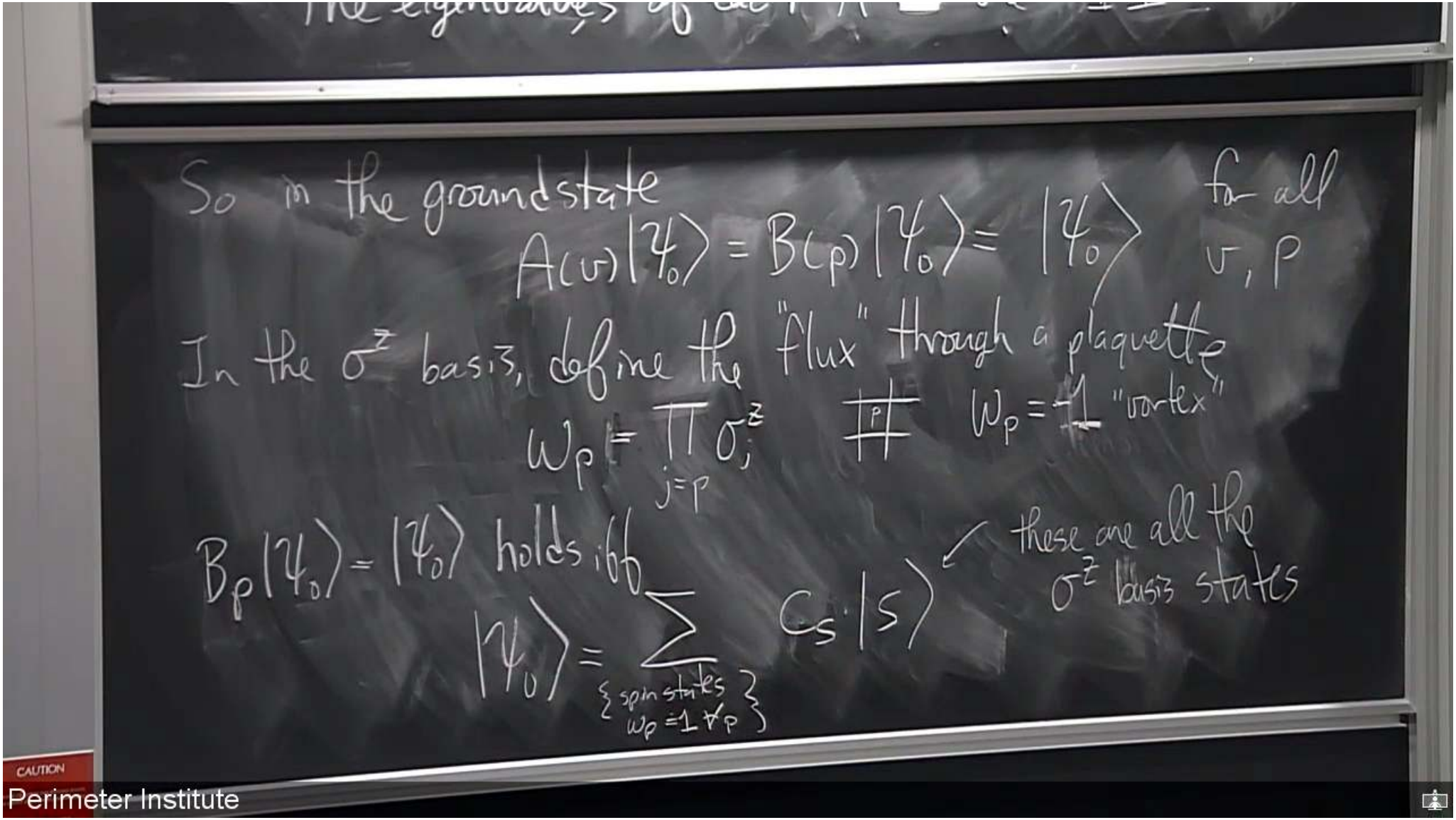

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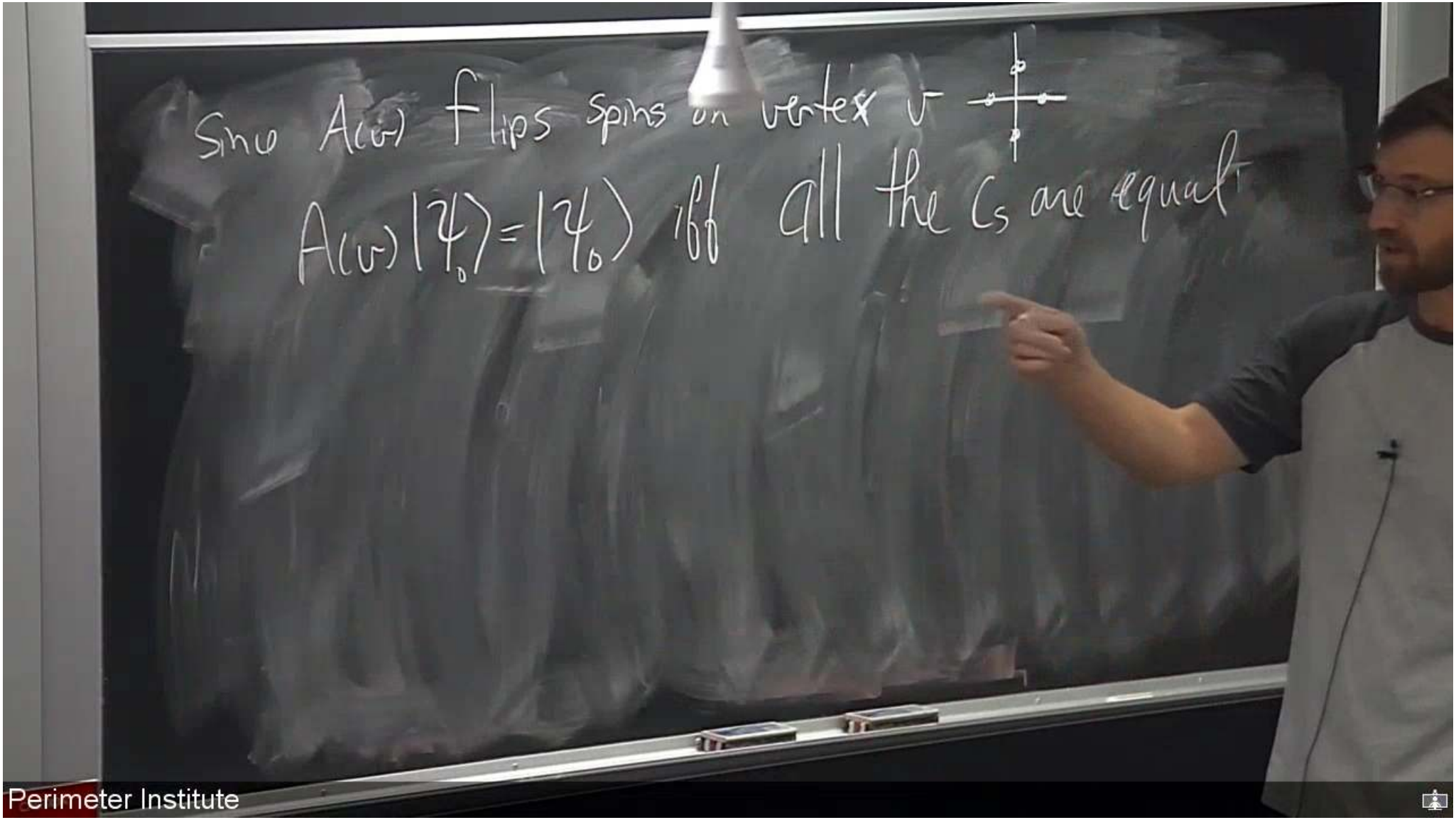
Since A(ω) flips spins on vertex v
A(ω)|ψ0⟩=|ψ0⟩ iff all the Cs are equal
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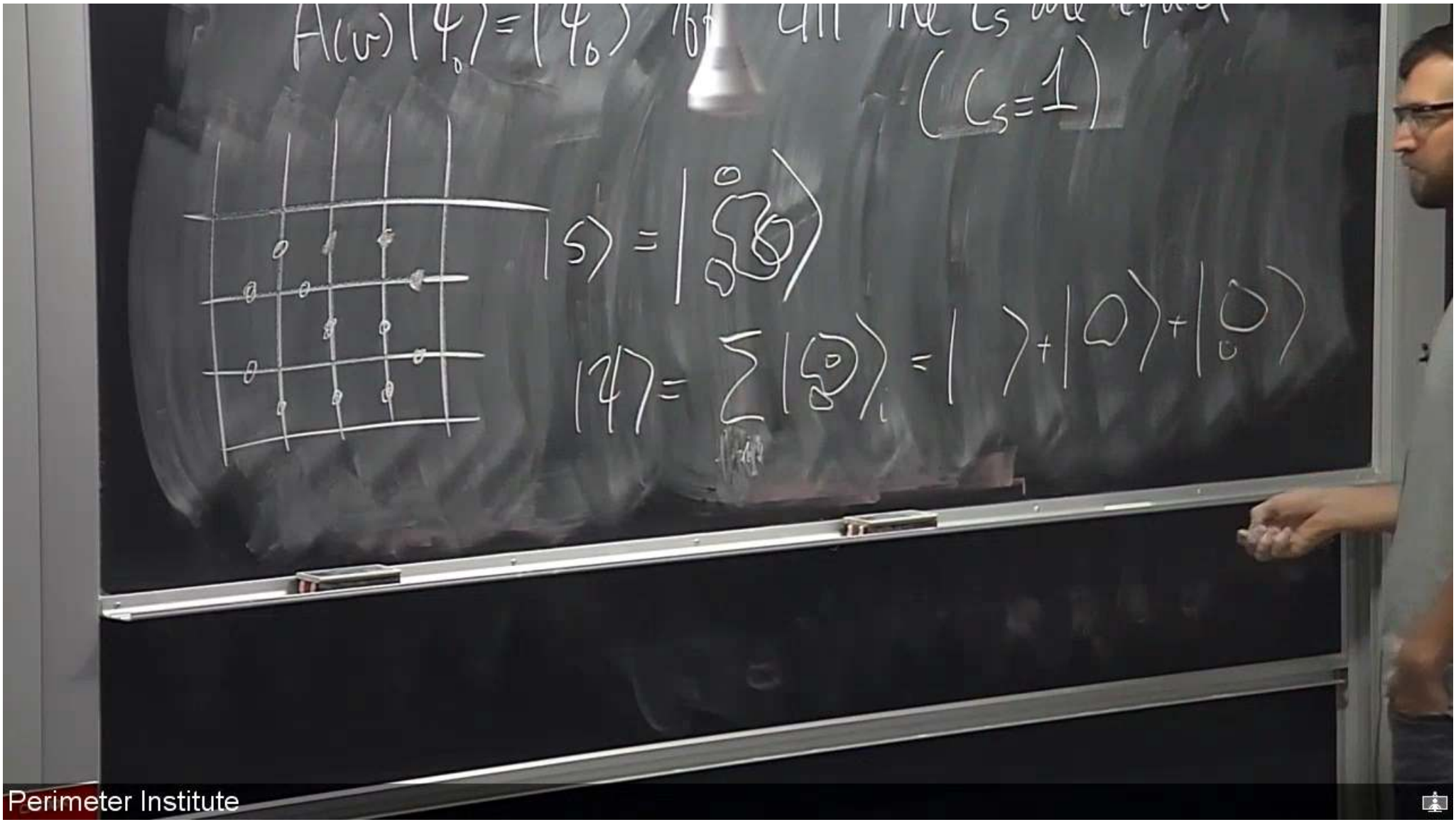
$A(\omega)|\psi_0\rangle = |\psi_0\rangle$
$(c_s = 1)$
$|s\rangle = |$
$|\psi\rangle = \sum |$
$\rangle + |$
$\rangle + |$
$\rangle$
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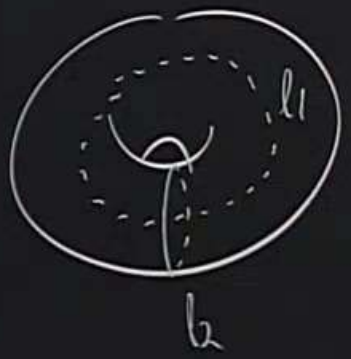
So the groundstate is uniquely determined.

But: there is a "topological" degeneracy.

→ for the non contractable wilson loops. (imagine a torus)



← eigenvalues $W_{l_1}, W_{l_2} = \pm 1$

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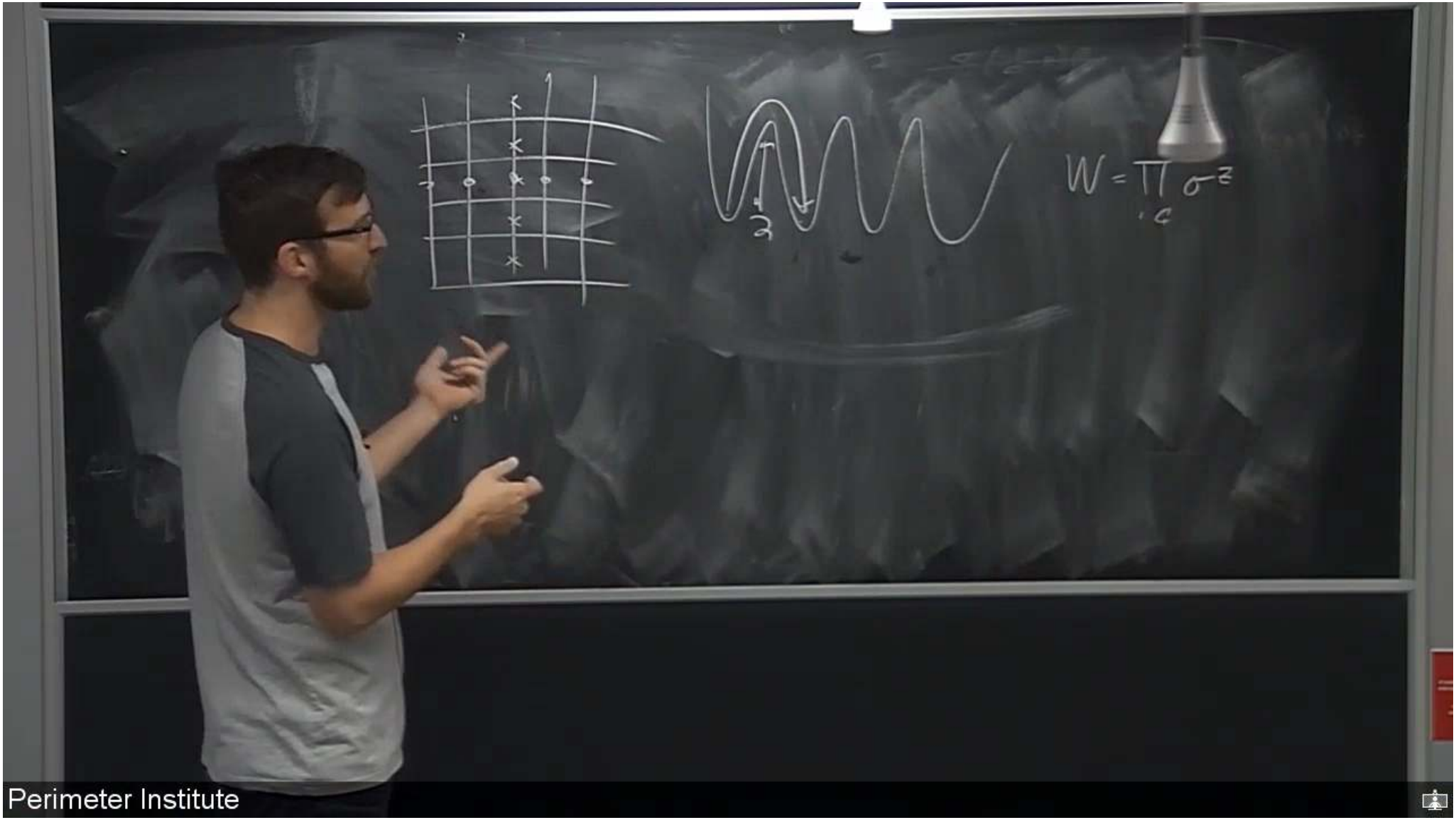
W = Π σ
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