

Title: PHYS 733 - Quantum Many-Body Physics (W2016) - Roger Melko - Lecture 15

Date: Mar 01, 2016 10:00 AM

URL: <http://pirsa.org/16030026>

Abstract:

Perturbation theory

$$S = \underbrace{\int_{\mathbb{R}^d} d^d x}_{S_{\text{I}}} \left\{ \underbrace{\frac{1}{2} [(\partial\phi)^2 + r\phi^2]}_{S_0} + \underbrace{\frac{\lambda}{4!} \phi^4}_{S_{\text{I}}} \right\}$$

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$$\langle \phi(x_1) \phi(x_2) \rangle = \frac{\langle \phi(x_1) \phi(x_2) e^{-S_I(\phi)} \rangle_0}{\langle e^{-S_I(\phi)} \rangle_0}$$

$$\langle \phi(r_1) \phi(r_2) \rangle = \frac{\langle \phi(r_1) \phi(r_2) e^{-S_I(\phi)} \rangle_0}{\langle e^{-S_I(\phi)} \rangle_0} \quad \text{Taylor expand}$$

$$= \frac{\sum_{k=0}^{\infty} \frac{1}{k!} \langle \phi(r_1) \phi(r_2) (-S_I)^k \rangle_0}{\sum_{k=0}^{\infty} \frac{1}{k!} \langle (-S_I)^k \rangle_0}$$

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Set up to evaluate nicely with Wick's theorem

$$\langle \phi_{i_1} \phi_{i_2} \dots \phi_{i_n} \rangle = \sum_{\text{pairings}} G_{j_1, j_2} G_{j_3, j_4} \dots G_{j_{n-1}, j_n}$$

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last time

$$\langle \phi(r_1) \phi(r_2) \rangle = \frac{\mathcal{N}}{\mathcal{D}}$$

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$$N = \langle \phi(r_1) \phi(r_2) \rangle_0 - \frac{4}{4!} \int_r \langle \phi(r_1) \phi(r_2) \phi^4(r) \rangle_0$$

$$\begin{aligned}
 N = \langle \phi(r_1) \phi(r_2) \rangle_0 & - \frac{\lambda}{4!} \int_r \langle \phi(r_1) \phi(r_2) \phi^4(r) \rangle_0 \\
 & + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 \int_r \int_{r'} \langle \phi(r_1) \phi(r_2) \phi^4(r) \phi^4(r') \rangle_0 + \dots
 \end{aligned}$$

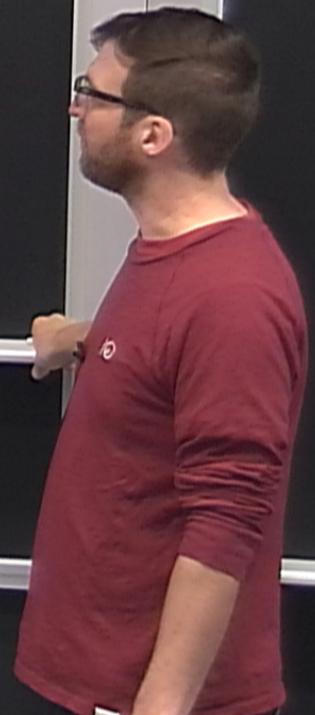
$$N = \langle \phi(r_1) \phi(r_2) \rangle_0 - \frac{\lambda}{4!} \int_r \langle \phi(r_1) \phi(r_2) \phi^4(r) \rangle_0 \leftarrow k=1$$

$$+ \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 \int_r \int_{r'} \langle \phi(r_1) \phi(r_2) \phi^4(r) \phi^4(r') \rangle_0 + \dots$$

Last time: to order $k=1$ we got

$$\langle \phi(r_1) \phi(r_2) \rangle = \dots + 12 \dots + \dots$$

$\leftarrow k=2$



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$\nearrow k=2$

Let's look at the $k=2$ term with coefficient $\frac{1}{2} \left(\frac{2!}{4!} \right)^2$



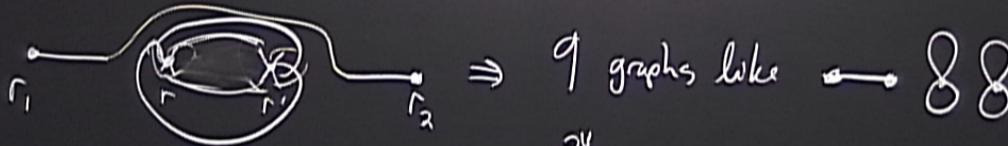
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Diagrams in the Numerator:



$$N = 9 \text{---} 88 + 72 \text{---} \infty\infty + \overset{24}{\vee} \text{---} \text{---}$$



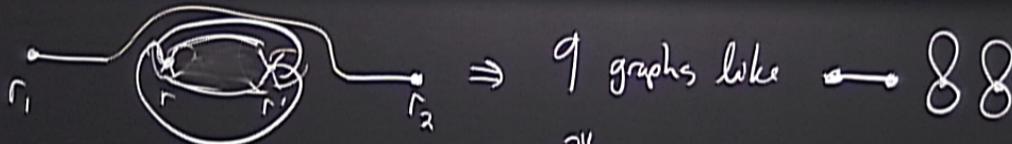
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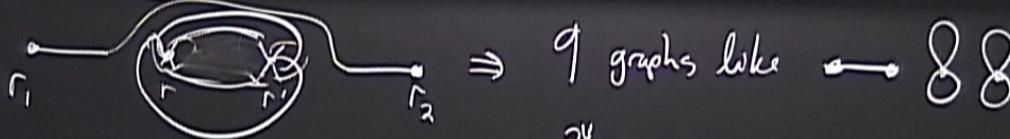
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Diagrams in Numerator:



$$N = 9 \text{---} 88 + 72 \text{---} \infty\infty + \overset{24}{\vee} \text{---} \text{---} \\ \text{for } u^2 + 72 \text{---} 88 + 288 \text{---} 8 + 192 \text{---} \text{---} + 288 \text{---} \text{---}$$

$$D = 1 + \underbrace{38}_u + \underbrace{9\infty + 72\infty\infty + 24 \text{---}}_{u^2} + \dots$$



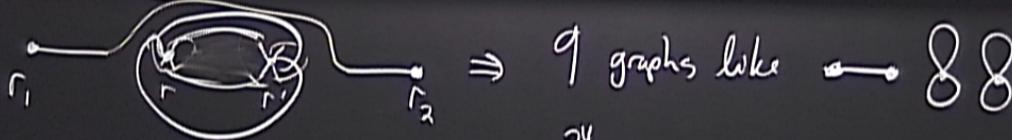
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for u^2

$$D = 1 + \underbrace{38}_u + \underbrace{9 \text{---} \text{---} + 72 \text{---} \text{---}}_{u^2} + 24 \text{---} \text{---} + \dots \text{ (we've done this before)}$$

graphically we get something like

$$\langle \phi(r_1) \phi(r_2) \rangle = \frac{\text{diagrams}}{1 - \epsilon + \frac{1}{2} \left(\text{diagrams} \right) + \dots}$$

The numerator contains diagrams: a pair of vertices connected by a line, minus a pair of vertices connected by a line with a loop on each vertex, plus a pair of vertices connected by a line with a loop on one vertex, plus a pair of vertices connected by a line with a loop on the other vertex, plus a pair of vertices connected by a line with a loop on both vertices, plus a pair of vertices connected by a line with a loop on the line itself, plus a pair of vertices connected by a line with a loop on the line and a loop on one vertex, plus a pair of vertices connected by a line with a loop on the line and a loop on the other vertex, plus a pair of vertices connected by a line with a loop on the line and loops on both vertices.

The denominator contains diagrams: 1 minus epsilon plus half times a diagram of two vertices connected by a line with a loop on one vertex, plus a diagram of two vertices connected by a line with a loop on the other vertex, plus a diagram of two vertices connected by a line with a loop on both vertices, plus a diagram of two vertices connected by a line with a loop on the line itself, plus a diagram of two vertices connected by a line with a loop on the line and a loop on one vertex, plus a diagram of two vertices connected by a line with a loop on the line and a loop on the other vertex, plus a diagram of two vertices connected by a line with a loop on the line and loops on both vertices.

Again, expanding the denominator $\frac{1}{1+\epsilon} \approx 1 - \epsilon$



$$1 - \infty + \frac{1}{2} \left(\frac{\infty}{\infty} + \infty\infty + \text{⊖} \right) + \dots$$

Again, expanding the denominator $\frac{1}{1+\epsilon} \approx 1 - \epsilon$

all the "vacuum" graphs cancel.

A vacuum graph contains

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Thus the contribution to the correlation function $\langle \phi(r_1) \dots \phi(r_n) \rangle$ to k^{th} order is the sum of all excluding vacuum graphs.

• no need to calculate both N and D

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Thus the contribution to the correlation function $\langle \phi(x_1) \dots \phi(x_n) \rangle$ to k^{th} order is the sum of all graphs excluding vacuum graphs.

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$$- \frac{u}{4!} \int_r \langle \phi(r_1) \phi(r_2) \phi(r_3) \phi(r_4) \phi^4(r) \rangle_0 + \dots$$

$$= \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \end{array} \right) - \frac{u}{4!} \left(\begin{array}{c} \text{Diagram 4} \\ \text{Diagram 5} \end{array} \right) + \dots$$

The diagrams are:

- Diagram 1: A vertex labeled r with four external legs labeled r_1, r_2, r_3, r_4 .
- Diagram 2: A vertex labeled r with two external legs labeled r_3, r_4 and two internal legs forming a loop.
- Diagram 3: A vertex labeled r with four external legs labeled r_1, r_2, r_3, r_4 and a central vertex connected to all four.
- Diagram 4: A vertex labeled r with four external legs labeled r_1, r_2, r_3, r_4 and a central vertex connected to all four, with an 'X' over it.
- Diagram 5: A vertex labeled r with four external legs labeled r_1, r_2, r_3, r_4 and a central vertex connected to all four, with a loop on one of the internal lines.

Let's look at the graphs for the u term.

$$- \frac{u}{4!} \int_{\Gamma} \langle \phi(r_1) \phi(r_2) \phi(r_3) \phi(r_4) \phi^4(r) \rangle_0 + \dots$$

$$= \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \end{array} \right) - \frac{u}{4!} \left(\begin{array}{c} \text{Diagram 4} \\ \text{Diagram 5} \end{array} \right) + \dots$$

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$$\Rightarrow \left(\begin{array}{c} \text{Diagram 6} \\ \text{Diagram 7} \\ \text{Diagram 8} \end{array} \right) \left(\text{Diagram 9} \right)$$

$$- \frac{u}{4!} \int_r \langle \phi(r_1) \phi(r_2) \phi(r_3) \phi(r_4) \phi^4(r) \rangle_0 + \dots$$

$$= \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \end{array} \right) - \frac{u}{4!} \left(\begin{array}{c} \text{Diagram 4} \\ \text{Diagram 5} \end{array} \right) + \dots$$

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$$\Rightarrow \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \end{array} \right) \left(\begin{array}{c} \text{Diagram 4} \end{array} \right) \xrightarrow{0} \text{vacuum graph}$$

+ 24



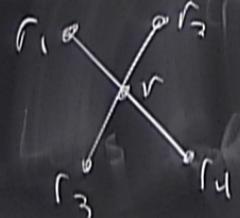
$$+ \left(\begin{array}{c} \text{---} \circ \text{---} \\ \text{---} \circ \text{---} \end{array} + \begin{array}{c} \text{---} \circ \text{---} \\ \text{---} \circ \text{---} \end{array} + \begin{array}{c} \text{---} \circ \text{---} \\ \text{---} \circ \text{---} \end{array} + \begin{array}{c} \text{---} \circ \text{---} \\ \text{---} \circ \text{---} \end{array} + \begin{array}{c} \text{---} \circ \text{---} \\ \text{---} \circ \text{---} \end{array} + \begin{array}{c} \text{---} \circ \text{---} \\ \text{---} \circ \text{---} \end{array} + \begin{array}{c} \text{---} \circ \text{---} \\ \text{---} \circ \text{---} \end{array} \right) \times 12$$

+ 24



$$+ \left(\begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \\ \text{diagram 3} \\ \text{diagram 4} \end{array} + \begin{array}{c} \text{diagram 5} \\ \text{diagram 6} \end{array} \right) \times 12$$

don't forget what you are calculating



$$= 24 \left(\frac{-24}{4!} \right) \int d^d r G_0(r_1-r) G_0(r_2-r) G_0(r_3-r) G_4(r_4-r)$$

$$\psi = 1 + 38 + 100 + 12000 + 24 \text{ (diagram)} + \dots$$

u^2

before)

CAUTION

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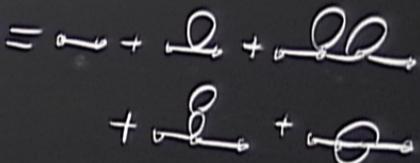


Diagrams in momentum space

(Back to our two-point c.f.) $\langle \phi(r_1) \phi(r_2) \rangle =$

- momentum space: fix some repetition
- simplify the calculation (e.g. we know G_0)

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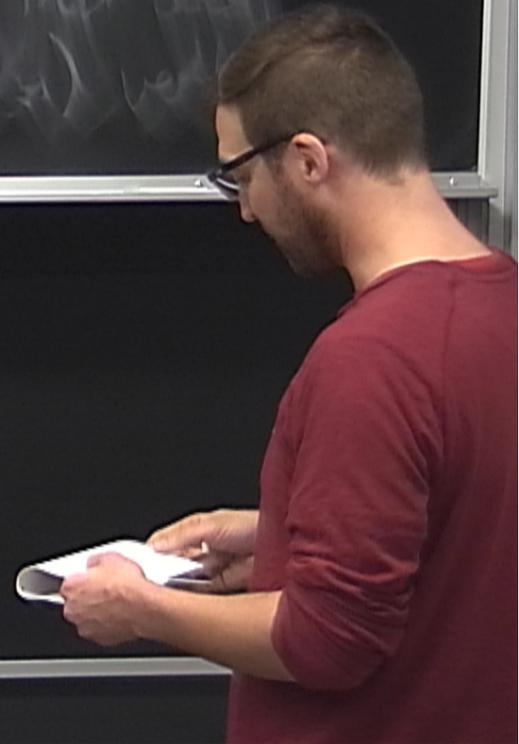
- momentum space: fix some repetition
- simplify the calculation (e.g. we know G_0)

Recall: $G_0(r_1, r_2) = \langle \phi(r_1) \phi(r_2) \rangle_0$ labelled by 

v. l. using $\phi(\vec{r}) = \int \frac{d^d p}{(2\pi)^d} \phi(\vec{p}) e^{i\vec{p}\cdot\vec{r}}$

$$S_0[\phi] = \int d^d r \int \frac{d^d p_1}{(2\pi)^d} \int \frac{d^d p_2}{(2\pi)^d} \frac{1}{2} (\rho^2 + r) \phi(p_1) \phi(p_2) e^{i(\vec{p}_1 + \vec{p}_2) \cdot \vec{r}}$$

using $\int d^d r e^{i(\vec{p}_1 + \vec{p}_2) \cdot \vec{r}} = (2\pi)^d \delta(\vec{p}_1 + \vec{p}_2)$



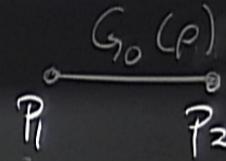
$$\text{using } \int d^d r e^{i(\vec{p}_1 + \vec{p}_2) \cdot r} = (2\pi)^d \delta(\vec{p}_1 + \vec{p}_2)$$

$$\Rightarrow S_0[\phi] = \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \phi(-p) \phi(p) (p^2 + m^2) \phi(p)$$

using $\int d^d r e^{i(\vec{p}_1 + \vec{p}_2) \cdot r} = (2\pi)^d \delta(\vec{p}_1 + \vec{p}_2)$

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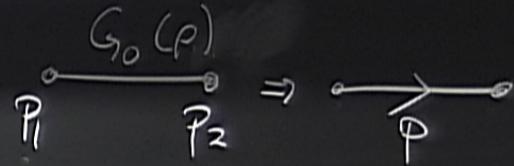
Then $G_0(p) = \langle \phi(p) \phi(-p) \rangle_0 = \frac{1}{p^2 + r}$



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conservation of momentum comes from $\delta(p_1 + p_2) \Rightarrow p_1 = -p_2$

Similarly, the ϕ^4 part of the action can be written in momentum space.

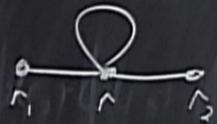
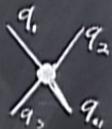
$$S_{\text{I}} = \frac{\lambda}{4!} \int d^d r \phi^4(r) = \frac{\lambda}{4!} \int d^d r \int \frac{d^d q_1}{(2\pi)^d} \dots \int \frac{d^d q_4}{(2\pi)^d} \phi(q_1) \phi(q_2) \phi(q_3) \phi(q_4) e^{i(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4) \cdot r}$$

$p=q$

and $\int d^d r e^{i(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4) \cdot \vec{r}} = (2\pi)^d \delta(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4)$

From this, momentum will be conserved at "internal" (interaction) vertices

e.g. $12 \times \text{diagram} = 12 \left(\frac{-24}{4!}\right) \int d^d r G_0(r-r) G_0(r_1-r) G_0(r-r_2)$

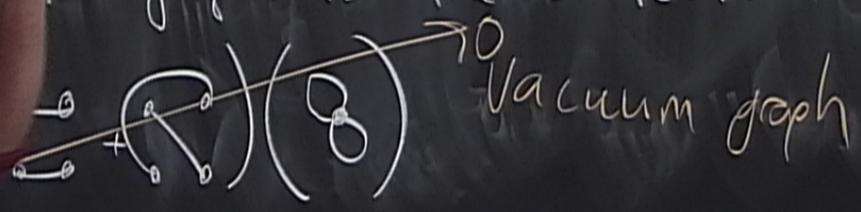


From this, momentum will be conserved at "internal" (interaction) vertices

e.g. $12 \times \text{diagram} = 12 \left(\frac{-4}{4!} \right) \int d^d r G_0(r-r_1) G_0(r-r_2) G_0(r-r_3)$

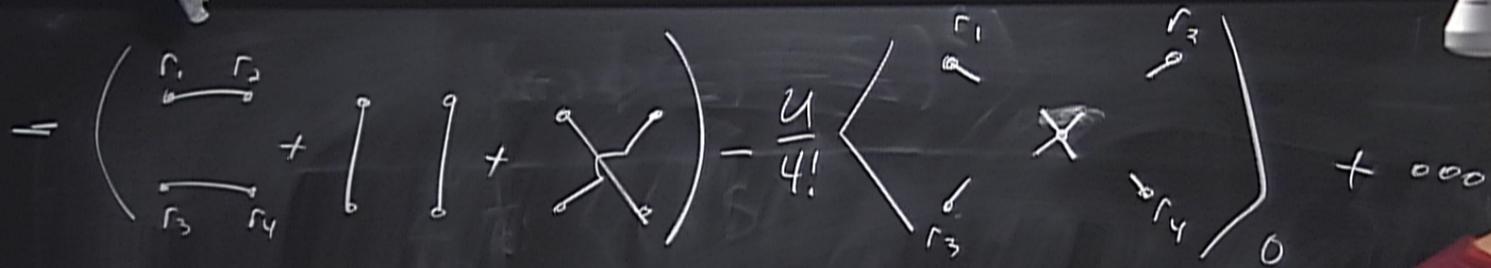
$= \frac{-4}{2} G_0(0) \int d^d r G_0(r-r_1) G_0(r-r_2) + \dots$

Look at the graphs for the U term.

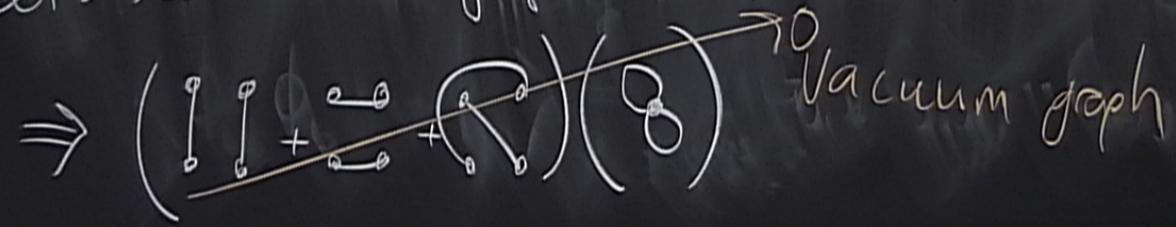


$$= \frac{-4}{2} G_0(0) \int d^d r \underbrace{G_0(r_1-r_2) G_0(r-r_2)}_{(G*G)(r_1-r_2)} + \dots$$

(no r -dependence)



Let's look at the graphs for the u term.



$$= \frac{-4}{2} G_0(0) \int d^d r \underbrace{G_0(r_1-r) G_0(r-r_2)}_{(G*G)(r_1-r_2)} + \dots$$

(no r -dependence)

F.T. of $(G*G)(r_1-r_2)$ exercise

$$= (2\pi)^d \delta(q_1+q_2) [G_0(q_1)]^2$$

$$= \frac{-u}{2} G_0(0) \int d^d r \underbrace{G_0(r_1-r) G_0(r-r_2)}_{(G*G)(r_1-r_2)} \quad (\text{no } r\text{-dependence})$$

F.T. of $(G*G)(r_1-r_2)$ exercise

$$= (2\pi)^d \delta(q_1+q_2) [G_0(q_1)]^2$$

So the F.T. of \circlearrowleft in momentum space is given by

$$(2\pi)^d \delta(q_1+q_2) \left\{ \frac{-u}{2} [G_0(q_1)]^2 \cdot \int \frac{d^d q}{(2\pi)^d} G_0(q) \right\}$$

for \hat{U}^2

$$+ 72 \text{ (loop)} + 288 \text{ (loop)} + 192 \text{ (loop)} + 288 \text{ (loop)}$$

$$D = 1 + 38 + 9 \infty + 72 \infty + 24 \text{ (loop)} + \dots \text{ (we've done this before)}$$

Define a momentum space diagram

$$(2\pi)^d \delta(q_1 + q_2) \left\{ \begin{array}{c} \text{loop} \\ q_1 \quad q_2 \end{array} \right\} = \begin{array}{c} \text{loop} \\ q_1 \quad q_2 \end{array}$$

Define a momentum space diagram

$$(2\pi)^d \delta(q_1 + q_2) \left\{ \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} \right\} = \text{---} \text{---} \text{---}$$

Rules for computing momentum-space diagrams

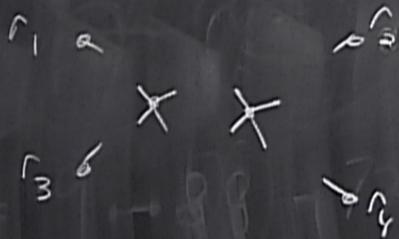
- 1) assign momenta to a particular direction for each line
- 2) Enforce total momentum into internal vertices = 0
- 3) Integrate over all internal momenta

- simplify the calculation (e.g. we know G_0)

Recall: $G_0(r_1, r_2) = \langle \phi(r_1) \phi(r_2) \rangle_0$ labelled by

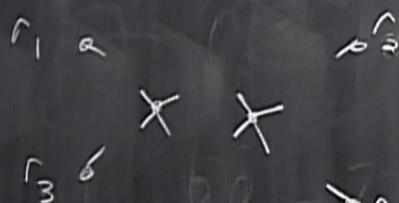
- 1) assign momenta to a particular direction for each line
- 2) Enforce total momentum into internal vertices = 0
- 3) Integrate over all internal momenta

example: a 2nd order contribution to a 4-point correlation function

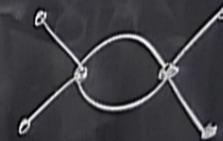


- 1) assign momenta to a particular direction for each line
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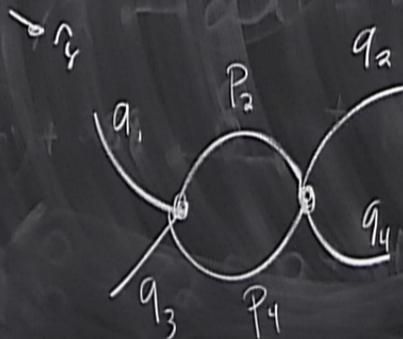
example: a 2nd order contribution to a 4-point correlation function



pick one contribution, say

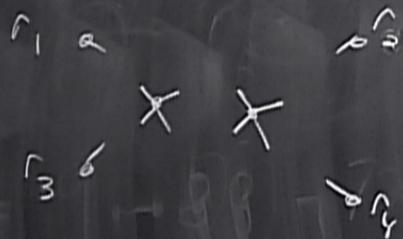


Use the rules:



- 1) assign momenta to a particular direction for each line
- 2) Enforce total momentum into internal vertices = 0
- 3) Integrate over all internal momenta

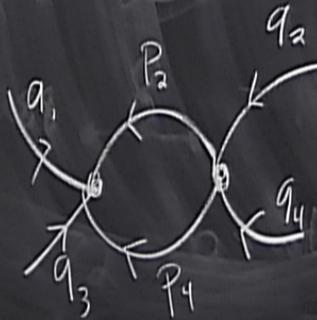
example: a 2nd order contribution to a 4-point correlation function



pick one contribution, say



Use the rules:



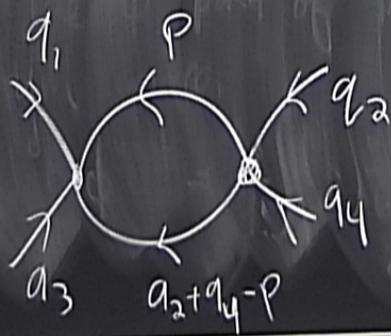
CAUTION
Do not touch the board when the teacher is writing.
Do not talk when the teacher is writing.
Do not drink when the teacher is writing.

$$\delta(q_1 + q_2 + q_3 + q_4)$$

$$p_4 = q_2 + q_4 - p_3$$

$$2\pi^2 G_0(q_1) G_0(q_2) G_0(q_3) G_0(q_4) \delta(q_1 + q_2 + q_3 + q_4)$$

$$\int \frac{d^2 p_3}{(2\pi)^2} G_0(p_3) G_0(q_2 + q_4 - p_3)$$



CAUTION
 PLEASE DO NOT TOUCH THE BOARD
 OR THE BOARD OR THE BOARD

$$\begin{aligned} &= \frac{-4}{2} G_0(0) \int d^d r G_0(r_1-r) G_0(r-r_2) \end{aligned}$$

Python "Enthought Canopy" ← matplotlib

TFIM in 1D OBC

$$= \frac{-4}{2} G_0(0) \int d^d r G_0(r_1-r) G_0(r-r_2) + \dots$$

Python "Enthought Canopy" ← matplotlib

TFIM in 1D OBC Hilbert space 2^N

$$\sigma^z \quad | \uparrow \downarrow \uparrow \downarrow \downarrow \downarrow \uparrow \rangle$$

N

FIM in 1D OBC Hilbert space 2^N

$$\sigma^z \quad |\uparrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\rangle = |100|1001\rangle = |\alpha'\rangle$$

$\underbrace{\quad\quad\quad}_B \quad \underbrace{\quad\quad\quad}_A$

$$\langle \alpha | H | \alpha' \rangle$$

$$\frac{\sigma^z \sigma^z}{\sigma^+ \sigma^-}$$

$$\sigma^+ = \sigma^+ + \sigma^-$$

$$S^+ S^- + S^- S^+$$

$$= (2\pi) \delta(q_1 + q_2) [G_0(q_1)]$$

F.T. of ρ in momentum space is given by

$$\delta(q_1 + q_2) \left\{ -\frac{u}{2} [G_0(q_1)]^2 \cdot \int \frac{dq}{(2\pi)^d} G_0(q) \right\}$$

