Title: Theoretical Structure and Theoretical Equivalence

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Abstract: <p>Our physical theories often admit multiple formulations or variants. Although these variants are generally empirically indistinguishable, they nonetheless appear to represent the world as having different structures. In this talk, I will discuss several criteria for comparing empirically equivalent theories that may be used to identify (1) when one variant has more structure than another (i.e., when a formulation of a theory has â€œexcess structureâ€•) and (2) when two variants are theoretically equivalent, even though they appear to represent the world differently. I will then discuss where this leaves the philosopher trying to use our empirically successful theories as a guide to the structure of the world.</p>

Functors

Let **C** and **D** be categories. A **functor** $F : \mathbf{C} \to \mathbf{D}$ is a map that:

- Takes objects to objects;
- Takes arrows to arrows;
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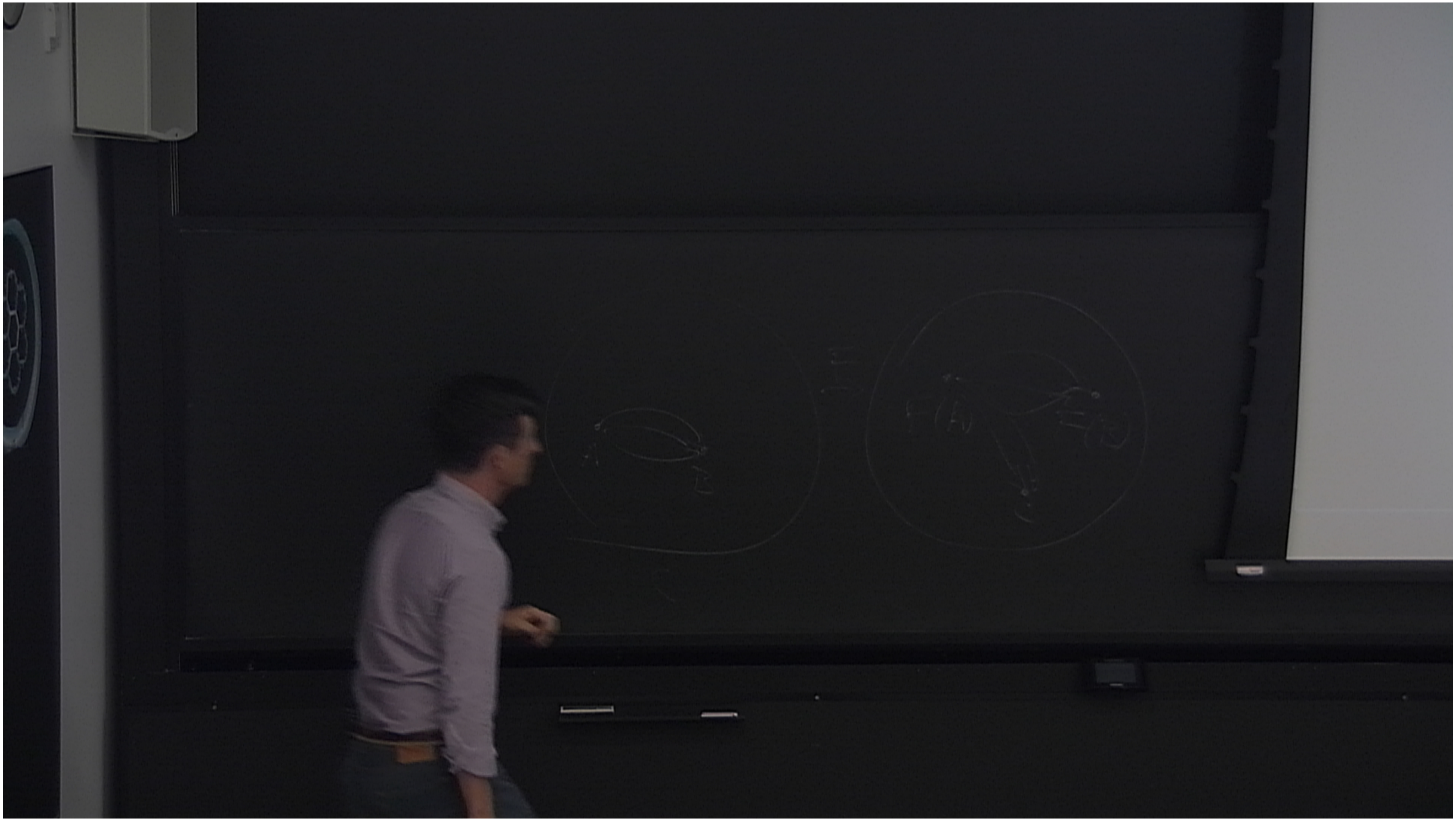
Forgetful functors

A functor $F : \mathbf{C} \to \mathbf{D}$ is **full** if $(f : A \to B) \mapsto (F(f) : F(A) \to F(B))$ is surjective for all A and B.



Forgetful functors

Baez-Dolan-Bartels-Barrett classification:

Forgetful functors

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A functor forgets:

- **Nothing** if it is full, faithful, and essentially surjective. (**Equivalence of categories**)
- Only **structure** if it is faithful and essentially surjective.

Making "surplus structure" precise

The map $A \mapsto dA = F$ determines a functor $G : \mathbf{EM}_2 \to \mathbf{EM}_1$. ($F$ acts trivially on arrows.)

This functor is **essentially surjective** and **faithful** but not **full**.

Aside: a diagnostic tool

This criterion for when structure is “surplus” provides a diagnostic tool.

Rule of Thumb

A theory (or formulation of a theory) has “surplus structure” if and only if there are ***non-isomorphic*** *models that have the same representational capacities.*

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Equivalence regained?

Physicists often take EM_1 and EM_2 to be equivalent.

The status of gauge transformations

In terms of Glymour's criterion: we have a $1-1$ relation between models **up to physical equivalence**.

Equivalence regained

Define a new category.

$\widetilde{\mathbf{EM}}_2$: Objects are 4-vector potentials A; arrows are spacetime symmetries that preserve any gauge-transformed A.

Criteria compared

How different are criteria 1 and 2?

Equivalence regained

Criterion 2

Two theories are theoretically equivalent just in case there exists an equivalence between their categories of models that preserves empirical content.

Criteria compared

Criterion 2 is strictly weaker than criterion 1.

Criteria compared

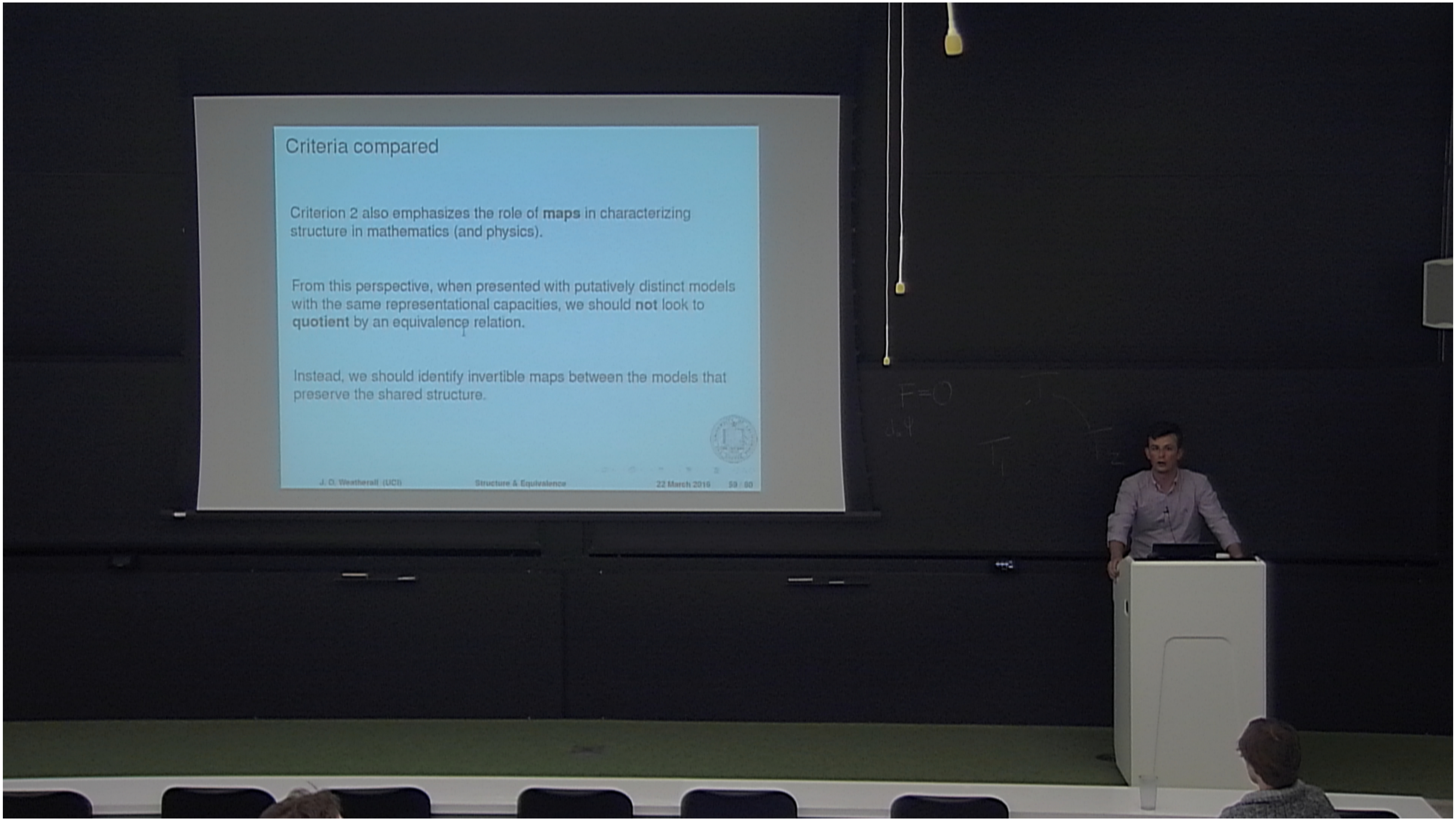
Criterion 2 draws attention to how we use mathematical structures to represent physical situations, and to when two structures may have the same representational capacities.

Criteria compared

Criterion 2 also emphasizes the role of **maps** in characterizing structure in mathematics (and physics).

From this perspective, when presented with putatively distinct models with the same representational capacities, we should **not** look to **quotient** by an equivalence relation.

Instead, we should identify invertible maps between the models that preserve the shared structure.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Structure & Equivalence
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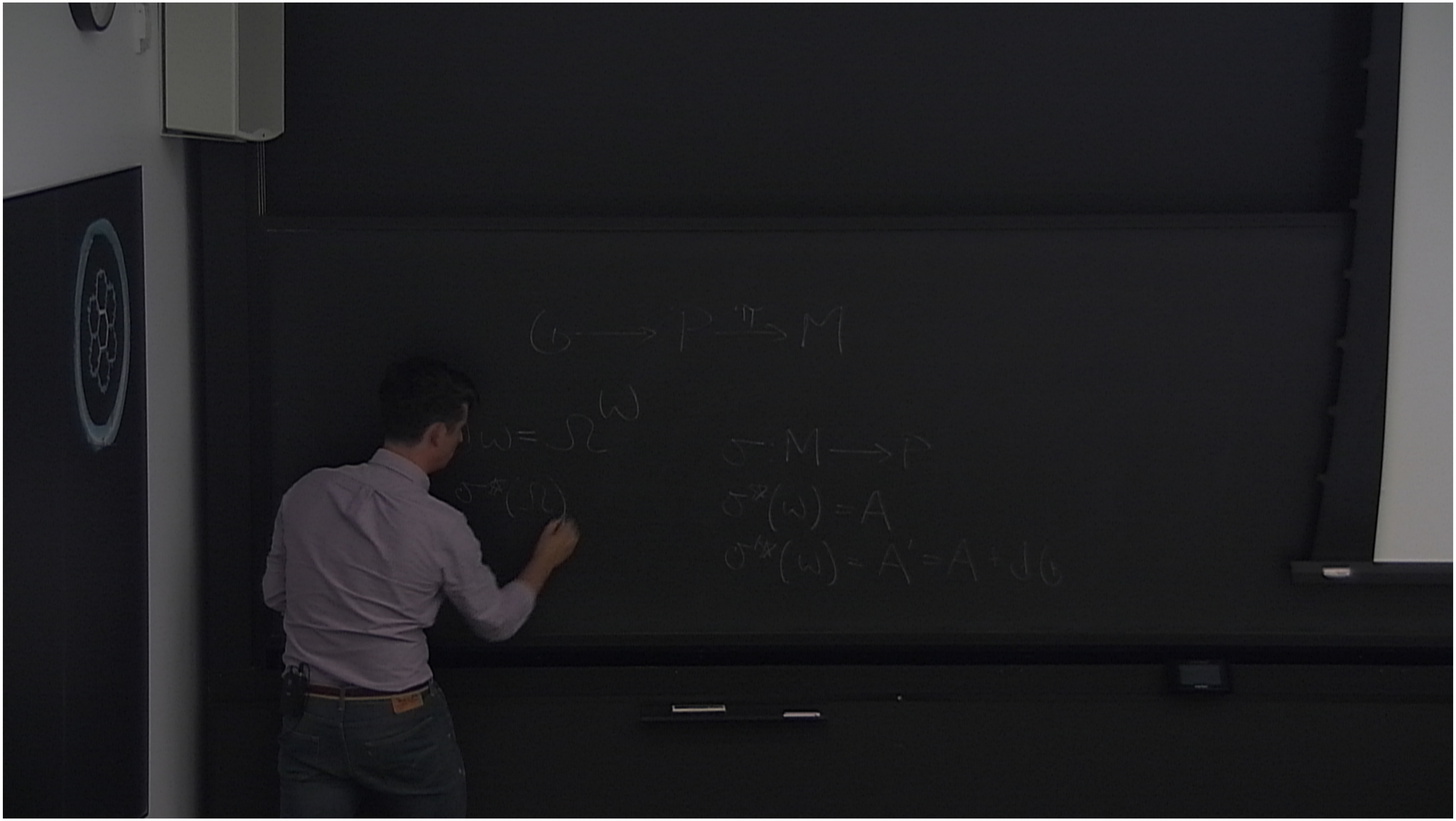
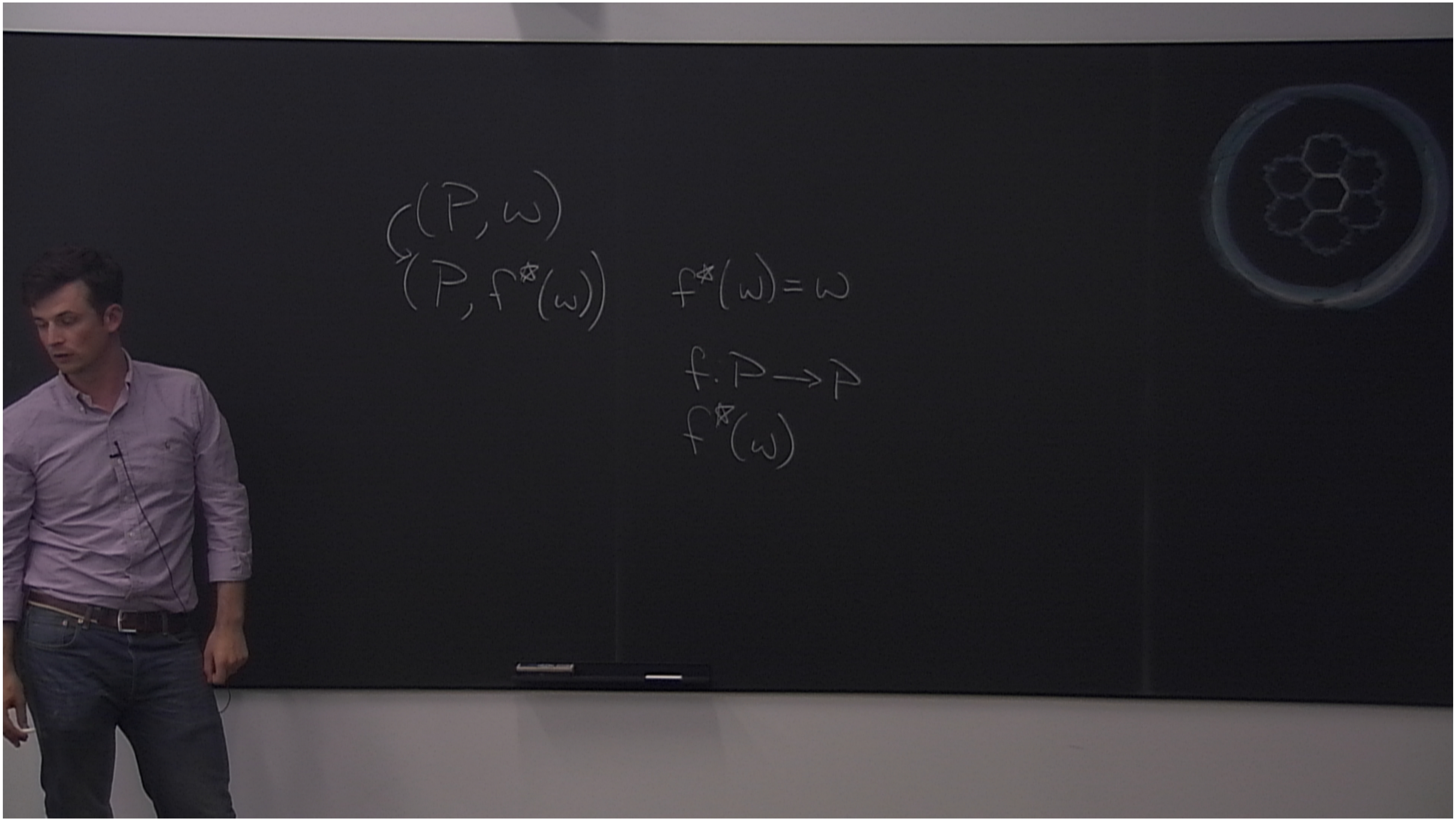

G → P →π M
σ : M → P
σ*(ω) = A
σ'*(ω) = A' = A + dG


(P, ω)
(P, f*(ω))
f*(ω) = ω
f: P → P
f*(ω)