

Title: Quantum Field Theory for Cosmology - Achim Kempf - Lecture 18

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Abstract:

PERIMETER  INSTITUTE FOR THEORETICAL PHYSICS

QFT for Cosmology, Achim Kempf, Winter 2016, Lecture 17

Note Title

From the particle picture to the wave picture

So far: Spacetime dynamics can produce particles.

When? When mode oscillators $\omega_k(t)$ changes nonadiabatically fast: $\omega_k'(t)/\omega_k(t)^2 \gg 1$.

In cosmology? No: particles mostly produced conventionally from inflation potential at the end of inflation.

Now: Spacetime dynamics can enhance quantum field fluctuations!

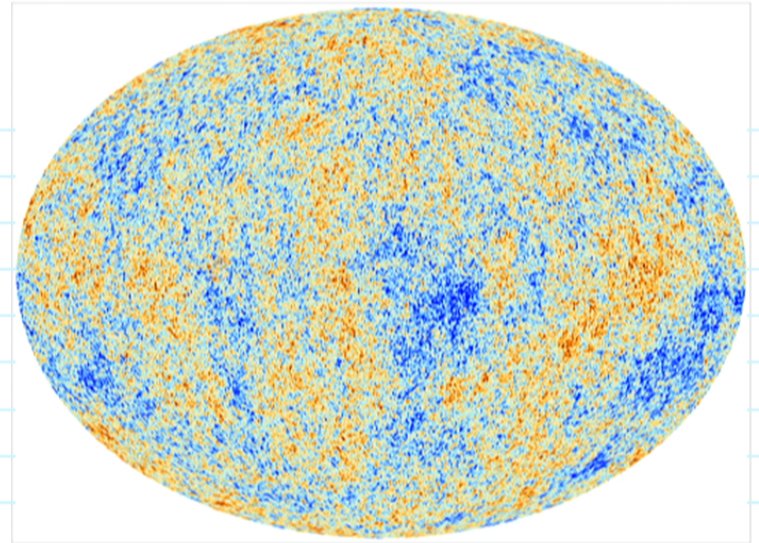
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When? When $\omega_k(\eta)$ becomes imaginary.

Recall: $\omega_k^2(\eta) := k^2 + m^2 a^2(\eta) - \frac{a''(\eta)}{a(\eta)}$

Quantum field fluctuations

Do the amplitudes $\hat{\phi}(x,t)$ of a quantum field necessarily quantum fluctuate?



Yes: \square Consider a real-valued function $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ and a time η_0 .

Expectation value: $\bar{\phi}(x, \gamma_0) = \langle f | \hat{\phi}(x, \gamma_0) | f \rangle$

$$= f(x) \langle f | f \rangle$$

$$= f(x)$$

Variance: $\Delta\phi^2(x, \gamma_0) = \langle f | (\hat{\phi}(x, \gamma_0) - \bar{\phi}(x, \gamma_0))^2 | f \rangle$

$$= \langle f | (f(x) - f(x))^2 | f \rangle$$

$$= 0 \quad \text{i.e. no fluctuations.}$$

But, can such states $|f\rangle$ occur in practice?

No: The reason is that for these states:


$$\langle f | \hat{H}^{(2)}(\gamma_0) | f \rangle = \infty \quad \text{Exercise: Show this.}$$

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Hint: For these states, $\Delta\phi = 0$, and so $\Delta\pi^{(p)} = \infty$

But $\hat{H}^{(p)}$ contains a term $\hat{\pi}^2 \dots$



\Rightarrow * Even the state $|f\rangle$ with $f(x) = 0$ for all x has ∞ energy and is, therefore, not accessible.

* Thus, all finite energy states do possess quantum fluctuations

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How to calculate the amount of quantum field fluctuations?

- It is not realistic to measure all operators $\hat{\phi}(x)$ individually.
- Realistically, we could at most hope to measure an average of the values of $\hat{\phi}$ over regions $B \subset \mathbb{R}^3$ of not too small volume $V = L \times L \times L$.

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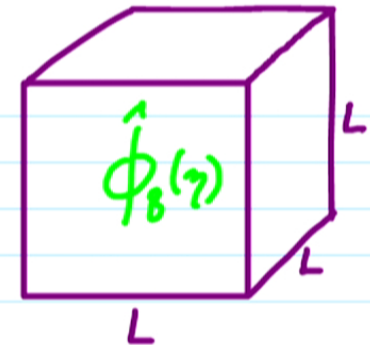
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$$\hat{\phi}_B(\gamma) := \int_{\mathbb{R}^3} \hat{\phi}(x, \gamma) W(x) d^3x$$

with "window function" W

$$W(x) = \begin{cases} \approx 0 & \text{for all } x \notin B \\ \approx V^{-1} & \text{for all } x \in B \end{cases}$$



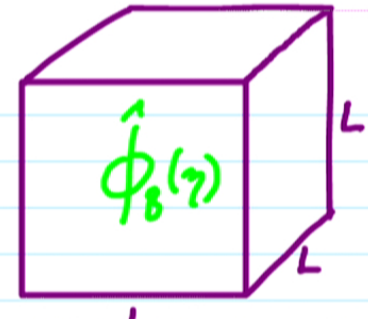
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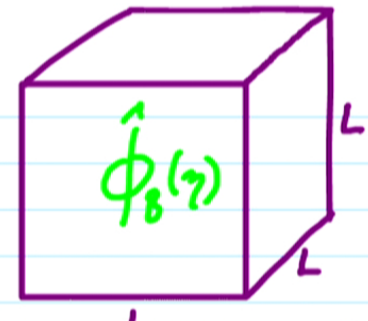
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Quantum field fluctuations in FRW spacetime

□ Choose conformal time η and comoving coordinates x .

□ Choose a region B of size $L \times L \times L$.

Note: In proper coordinates, this is a box of increasing size:

$$a(\eta)L \times a(\eta)L \times a(\eta)L$$

□ Assume that at η_0 the system's state, $|\Omega\rangle$, is the vacuum state:

$$|\Omega\rangle = |\text{vac}_{\eta_0}\rangle$$

□ We choose the mode functions $v_k(\eta)$ so that $|\text{vac}_{\eta_0}\rangle = |0\rangle$ with.

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$$\hat{x}_k(\eta) = \frac{1}{\sqrt{2}} (v_k^*(\eta) a_k + v_k a_{-k}^\dagger) \quad \text{and} \quad a_k |0\rangle = 0$$

→ The expectation value of the region-averaged field at a time $\eta \gg \eta_0$:

$$\bar{\phi}_B(\eta) = \langle \Omega | \hat{\phi}_B(\eta) | \Omega \rangle$$

$$= \langle \text{vac}_{\eta_0} | \hat{\phi}_B(\eta) | \text{vac}_{\eta_0} \rangle$$

$$= \langle 0 | \int_{\mathbb{R}^3} \hat{\phi}(x, \eta) W(x) d^3x | 0 \rangle$$

$$= \langle 0 | \frac{1}{a(\eta)} \int_{\mathbb{R}^3} \hat{\chi}(x, \eta) W(x) d^3x | 0 \rangle$$

$$= \frac{1}{a(\eta)} \int_{\mathbb{R}^3} \langle 0 | \frac{1}{\sqrt{2}} (\cancel{v_k^+(\eta) a_k} + \cancel{v_k(\eta) a_{-k}^+}) | 0 \rangle$$

$$= 0$$

⇒ The average amplitude of $\hat{\phi}_B$ vanishes in the vacuum state.

□ The quantum fluctuations

While $\bar{\Phi}_B(\eta)$ vanishes, measurement outcomes for $\hat{\phi}_B(\eta)$ are not fully predictable because subject to fluctuations around zero with this standard deviation:

$$\begin{aligned}\Delta \phi_B^2(\eta) &= \langle \Omega | (\hat{\phi}_B(\eta) - \bar{\Phi}_B(\eta))^2 | \Omega \rangle \\ &= \langle 0 | \hat{\phi}_B(\eta)^2 | 0 \rangle\end{aligned}$$

$$\Delta \phi_B^2(\eta) = \langle \Omega | (\hat{\phi}_B(\eta) - \bar{\phi}_B(\eta))^2 | \Omega \rangle$$

$$= \langle 0 | \hat{\phi}_B(\eta)^2 | 0 \rangle$$

$$= \frac{1}{a(\eta)^2} \langle 0 | \left(\int_{\mathbb{R}^3} \hat{\chi}(x, \eta) W(x) d^3x \right)^2 | 0 \rangle$$

= ... Exercise: fill in the steps

$$= \frac{1}{2a(\eta)^2} \frac{1}{(2\pi)^3} \int |v_k(\eta)|^2 |\tilde{W}(k)|^2 d^3k$$

$$= \langle 0 | \hat{\phi}_0(\eta)^2 | 0 \rangle$$

$$= \frac{1}{a(\eta)^2} \langle 0 | \left(\int_{\mathbb{R}^3} \hat{\chi}(x, \eta) W(x) d^3x \right)^2 | 0 \rangle$$

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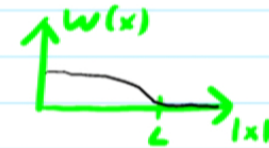
↑
Fourier transform of the
window function $W(x)$.

$$= \frac{1}{2 a(\eta)^2} \frac{1}{(2\pi)^3} \int_0^{\infty} k^2 4\pi |V_k(\eta)|^2 \tilde{W}(k) dk$$

\uparrow $k = \sqrt{k_1^2 + k_2^2 + k_3^2}$

Notice the dimension dependence of the integral's measure!

Approximation: Consider that:



\uparrow typical scale is L

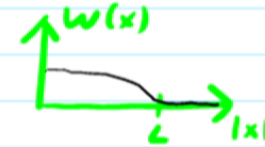
(using Fourier) \Rightarrow We can assume that, roughly:

$$\tilde{W}(k) \sim \frac{1}{k} \text{ for } |k| > \frac{2\pi}{L}$$

$$L k = \sqrt{k_1^2 + k_2^2 + k_3^2}$$

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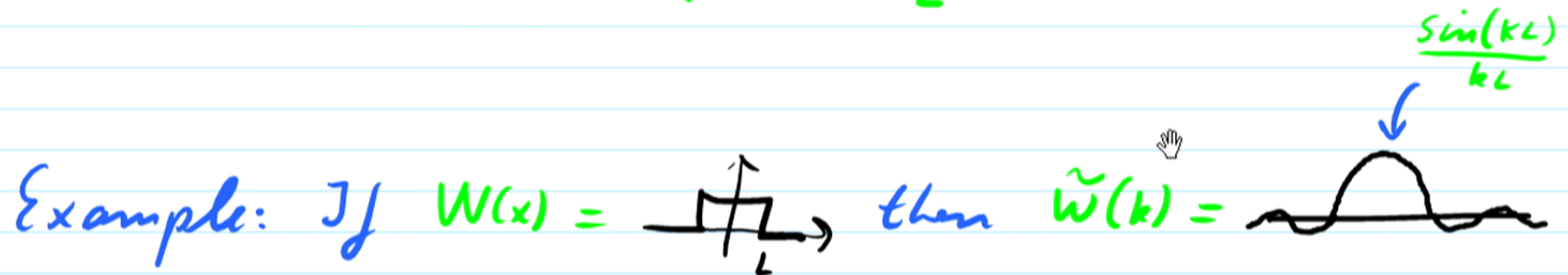
$$\tilde{W}(k) \approx 0 \text{ for } |k| > \frac{2\pi}{L}$$

$$\frac{\sin(kL)}{kL}$$

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Example: If $W(x) = \text{rect}_L(x)$, then $\tilde{W}(k) = \frac{\sin(kL)}{kL}$



and we approximate that $\tilde{W}(k) \approx \text{tri}_{2\pi/L}(k)$



$$\Rightarrow \sum_{n=-\infty}^{\infty} |h^2| \dots \left(\dots \right)^2 \dots$$

In the integral, the values of $|V_k(\eta)|^2$ for small k are suppressed by k^2 .

\Rightarrow Can approximately replace $|V_k(\eta)|$ by its value at $k = 2\pi/L$:

$$\Delta\phi_B^2(\eta) \approx \frac{1}{4\pi^2 a(\eta)^2} \int_0^{2\pi/L} k^2 |V_{\frac{2\pi}{L}}(\eta)|^2 dk$$

$$\Delta\phi_B^2(\eta) \approx \frac{1}{4\pi^2} \frac{(2\pi)^3}{3L^3} \left| \frac{V_{\frac{2\pi}{L}}(\eta)}{a(\eta)} \right|^2$$

$$= \frac{2\pi}{3L^3} |W_{\frac{2\pi}{L}}(\eta)|^2$$

$$\Delta \phi_B^2(\eta) \approx \frac{1}{4\pi^2 a(\eta)^2} \int_0^k k' \left| v_{\frac{2\pi}{L}}(\eta) \right|^2 dk$$

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$$= \frac{2\pi}{3L^3} \left| w_{\frac{2\pi}{L}}(\eta) \right|^2$$

Here, w is the mode function of $\hat{\phi}$, because $\frac{\dot{\chi}(\eta)}{a(\eta)} = \hat{\phi}(\eta)$.

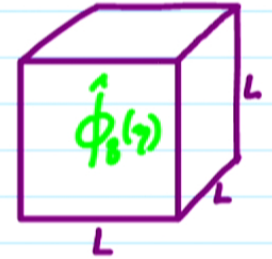
Conclusion:

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Assume that at a time η_0 the vacuum state was the state $|0\rangle$ corresponding to $\{v_k\}$.

Then, the typical amplitude of fluctuations of size L at an arbitrary time η is:

$$\Delta\phi_B^2(\eta) \approx \frac{2\pi}{3L^3} \left| \frac{V_{2\pi}(\eta)}{a(\eta)} \right|^2 = \frac{2\pi}{3L^3} \left| W_{\frac{2\pi}{L}}(\eta) \right|^2$$



Special case: Minkowski space (massless field)

$$\square v_k(\eta) = w_k(\eta)$$

$$\square \eta = t$$

$$\square |v_k(t)|^2 = \frac{1}{L^3} = \frac{1}{L^3} = \frac{1}{L^3}$$

$$\Delta\phi_B \uparrow \sim \frac{1}{L}$$

How to describe field quantum fluctuations using correlators

A primer on classical fluctuations:

all frequencies occur to same amount

□ Assume $n(t)$ is a Ω -bandlimited gaussian white noise signal, i.e., a random signal with gaussian distributed amplitudes, filtered to leave only frequencies in the interval $[-\Omega, \Omega]$.

□ Then, for an ensemble of such noise signals, one can show:

$$\overline{n(t)} = 0 \quad \forall t$$

"2-point correlator": $\overline{n(t)n(t+L)} = c \frac{\sin(\Omega L)}{\Omega L} \quad \forall t$

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How can we see this?

$$\overline{n(t)} = 0$$

VA

"2-point correlator": $\overline{n(t)n(t+L)} = c \frac{\sin(\Omega L)}{\Omega L}$

VA

How can we see this?

This noise is ergodic, i.e. we could instead average over all t :

$$\overline{n(t)n(t+L)} = \int f(t)f(t+L)dt \quad \left(\text{suitably regularize if non-normalizable, e.g. } \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T f(t)f(t+L)dt \right)$$

"Auto-correlator"

$$= \iiint \tilde{f}(\omega) \tilde{f}(\omega') e^{i\omega t} e^{i\omega'(t+L)} dt d\omega d\omega'$$

$$= \iiint \underbrace{e^{it(\omega+\omega')}}_{dt} \tilde{f}(\omega) \tilde{f}(\omega') e^{i\omega' L} d\omega d\omega'$$

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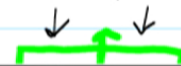
$$= \iiint \underbrace{e^{it(\omega+\omega')}}_{=(2\pi)^{-1}\delta(\omega+\omega')} dt \tilde{f}(\omega)\tilde{f}(\omega')e^{i\omega'L}d\omega d\omega'$$

$$= \frac{1}{2\pi} \int \tilde{f}(\omega)\tilde{f}(\omega)e^{i\omega L}d\omega$$

$$= \frac{1}{2\pi} \int |\tilde{f}(\omega)|^2 e^{i\omega L}d\omega \quad \rightarrow \text{"Spectral power function"}$$

\Rightarrow \square Auto correlation and power spectrum are a Fourier pair!

Recall: flatness of spectrum means noise is "white"



The 2-point correlator in QFT:

$$\langle 0 | \hat{\phi}(\vec{x}, \eta) \hat{\phi}(\vec{x} + \vec{L}, \eta) | 0 \rangle = \frac{1}{a(\eta)^2} \langle 0 | \hat{\chi}(\vec{x}, \eta) \hat{\chi}(\vec{x} + \vec{L}, \eta) | 0 \rangle$$

Exercise: use mode expansion of $\hat{\chi}$ and spherical coordinates to derive:

$$= \frac{1}{a(\eta)^2} \int_0^\infty \frac{k^2 dk}{4\pi^2} \frac{\sin(kL)}{kL} |v_k(\eta)|^2 \quad \text{with } k = |\vec{k}|, L = |\vec{L}|.$$

↑ Notice dimension dependence of the integral's measure!

Observe: $\frac{\sin(kL)}{kL} \approx \begin{cases} 1 & \text{for } k < \frac{1}{L} \\ 0 & \text{for } k > \frac{1}{L} \end{cases}$

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$12\pi^2$

$k = \frac{2\pi}{L}$

Special case: Minkowski space

Mode function: $|V_k|^2 = \frac{1}{|k|}$

$$\Rightarrow \langle 0 | \hat{\phi}(\vec{x}, \eta) \hat{\phi}(\vec{x} + \vec{l}, \eta) | 0 \rangle \approx \frac{1}{3} \frac{1}{L^2}$$

$$\langle 0 | \hat{\phi}(\vec{x}, \eta) \hat{\phi}(\vec{x} + \vec{l}, \eta) | 0 \rangle^{1/2}$$

same as for $\Delta\phi_B$!



We notice: The variance in a box scales like the correlator!
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Definition: We define the so-called Fluctuation Spectrum at time η as a function of k :

$$\delta\phi_k(\eta) := a(\eta)^{-1} k^{3/2} |v_k(\eta)|$$

$$k = \frac{2\pi}{L}$$

Special case Minkowski space with massive field:

□ Scale factor: $a(\eta) = 1$ for all η

□ Mode functions:

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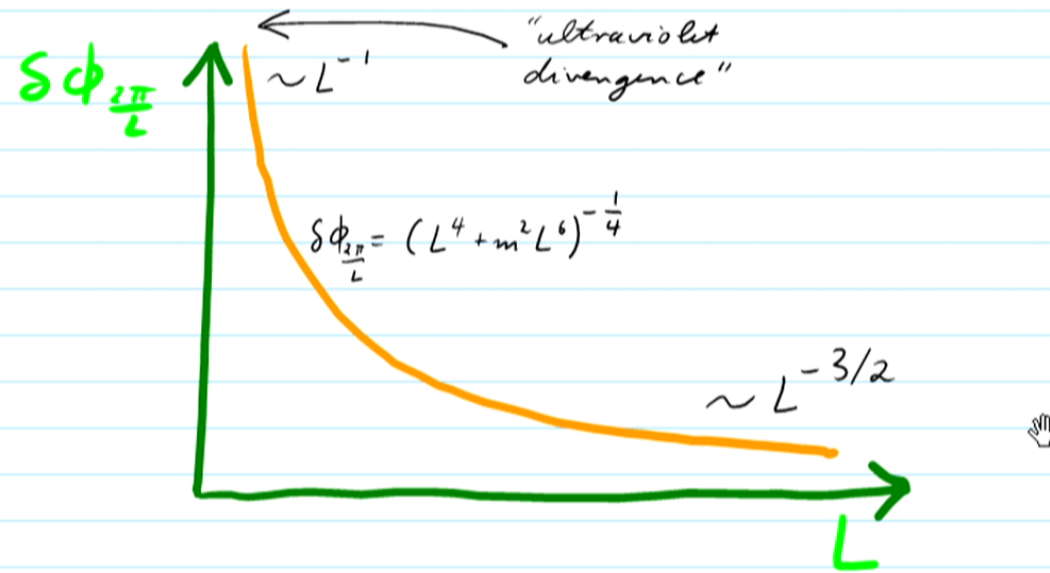
□ Mode functions:

$$v_k(\eta) = \frac{1}{\sqrt{\omega_k}} e^{i\eta\omega_k} \quad \text{with } \omega_k = \sqrt{k^2 + m^2}$$

⇒ The fluctuation spectrum is: (recall: $k = \frac{2\pi}{L}$)

$$\delta\phi_k = \frac{k^{3/2}}{(m^2 + k^2)^{1/4}} \approx \begin{cases} k & \text{for } k \rightarrow \infty \\ \frac{k^{3/2}}{\sqrt{m}} & \text{for } k \rightarrow 0 \end{cases}$$

and as a function of L it is:
$$\delta\phi_{\frac{2\pi}{L}} = \frac{(2\pi)^{3/2} L^{-3/2}}{\left(\frac{4\pi}{L^2} + m^2\right)^{1/4}}$$



Notice: The two different scaling behaviors are not clearly visible in this plot.

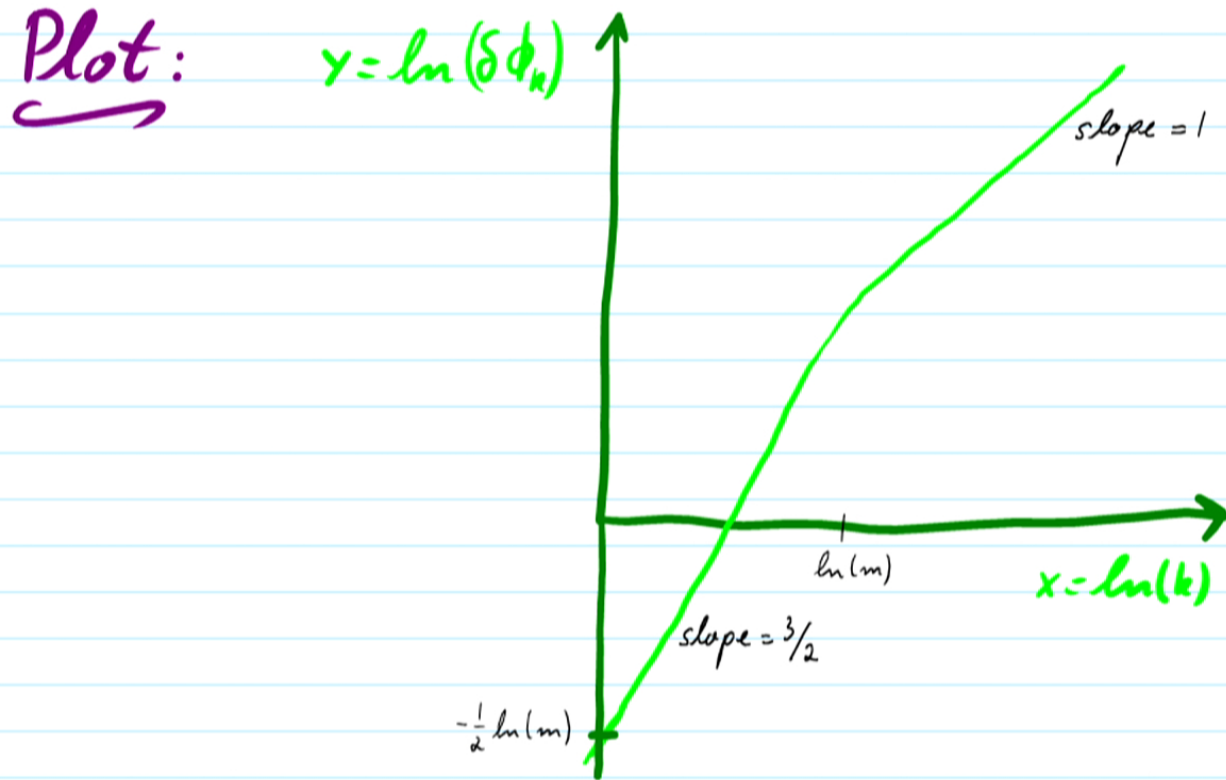
Recall "Log-Log plots":

$$x := \ln(k), \quad y = \ln(\delta\phi_k)$$

Here: $\ln \delta\phi_k = \ln \left(\frac{k^{3/2}}{(m^2+k^2)^{1/4}} \right) \approx \begin{cases} \ln k & \text{for } k \rightarrow \infty \\ \ln \left(\frac{k^{3/2}}{\sqrt{m}} \right) & \text{for } k \rightarrow 0 \end{cases}$

$= -\frac{1}{2} \ln(m) + \frac{3}{2} \ln k$

Thus: $y \approx \begin{cases} x & \text{for } x \rightarrow \infty \\ -\frac{1}{2} \ln(m) + \frac{3}{2} x & \text{for } x \rightarrow -\infty \end{cases}$



□ We notice that, in Minkowski space, large scale

□ We notice that, in Minkowski space, large scale (i.e., large L , small k) fluctuations are strongly suppressed, especially if mass $m \neq 0$.

□ Regarding the Infrared (IR):

- * The mass m does not matter at very short distances, i.e., in the ultraviolet.
- * But, for large L the mass term does help to suppress the quantum fluctuations. ($\delta\phi \sim L^{-3/2}$ vs. $\sim L^{-1}$)
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- * Generally, in phenomena of QFT, the mass of particles tends to play a rôle only in the infrared, but not in the ultraviolet, which is why in studies of UV phenomena the mass can often be neglected (e.g. for "perturbative

- * The mass m does not matter at very short distances, i.e., in the ultraviolet.
- * But, for large L the mass term does help to suppress the quantum fluctuations. ($\delta\phi \sim L^{-3/2}$ vs. $\sim L^{-1}$)
- * Generally, in phenomena of QFT, the mass of particles tends to play a rôle only in the infrared, but not in the ultraviolet, which is why in studies of UV phenomena the mass can often be neglected (e.g. for "perturbative power counting" - a method you encounter in renormalization).

Regarding the Ultraviolet (UV)

- * We see that QFT predicts divergingly large quantum fluctuations to be found in measurements that resolve smaller and smaller regions.
 - * For small enough regions B the fluctuations $\Delta\phi_B$ in ϕ_B would lead to fluctuations in the Klein Gordon energy momentum tensor that are large enough to cause black holes.
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- \Rightarrow At this scale, $\approx 10^{-35} \text{ m}$, the Planck scale, the notion of distance is assumed to break down.
(Accelerators can probe only distances down to about 10^{-19} m)