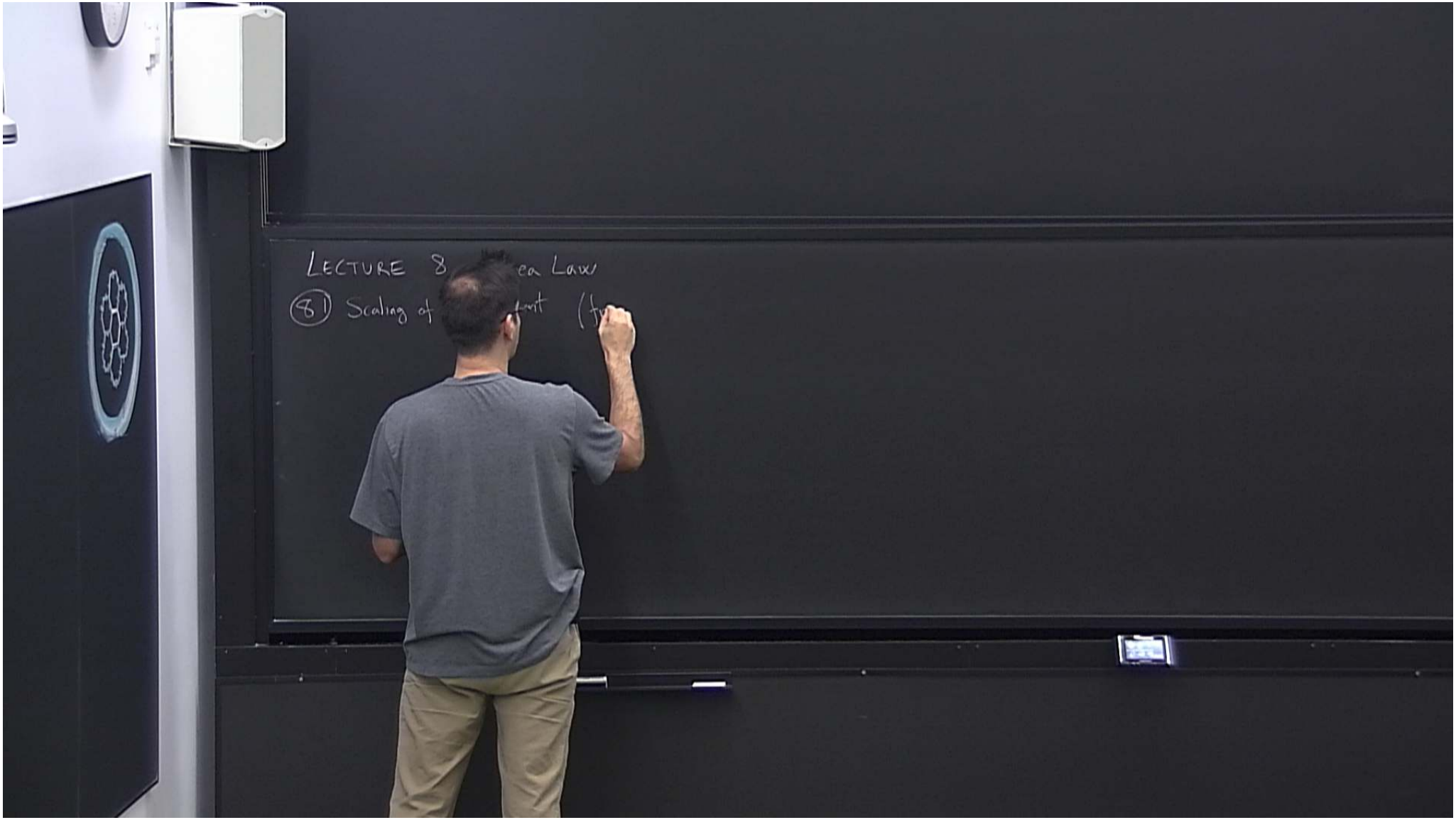
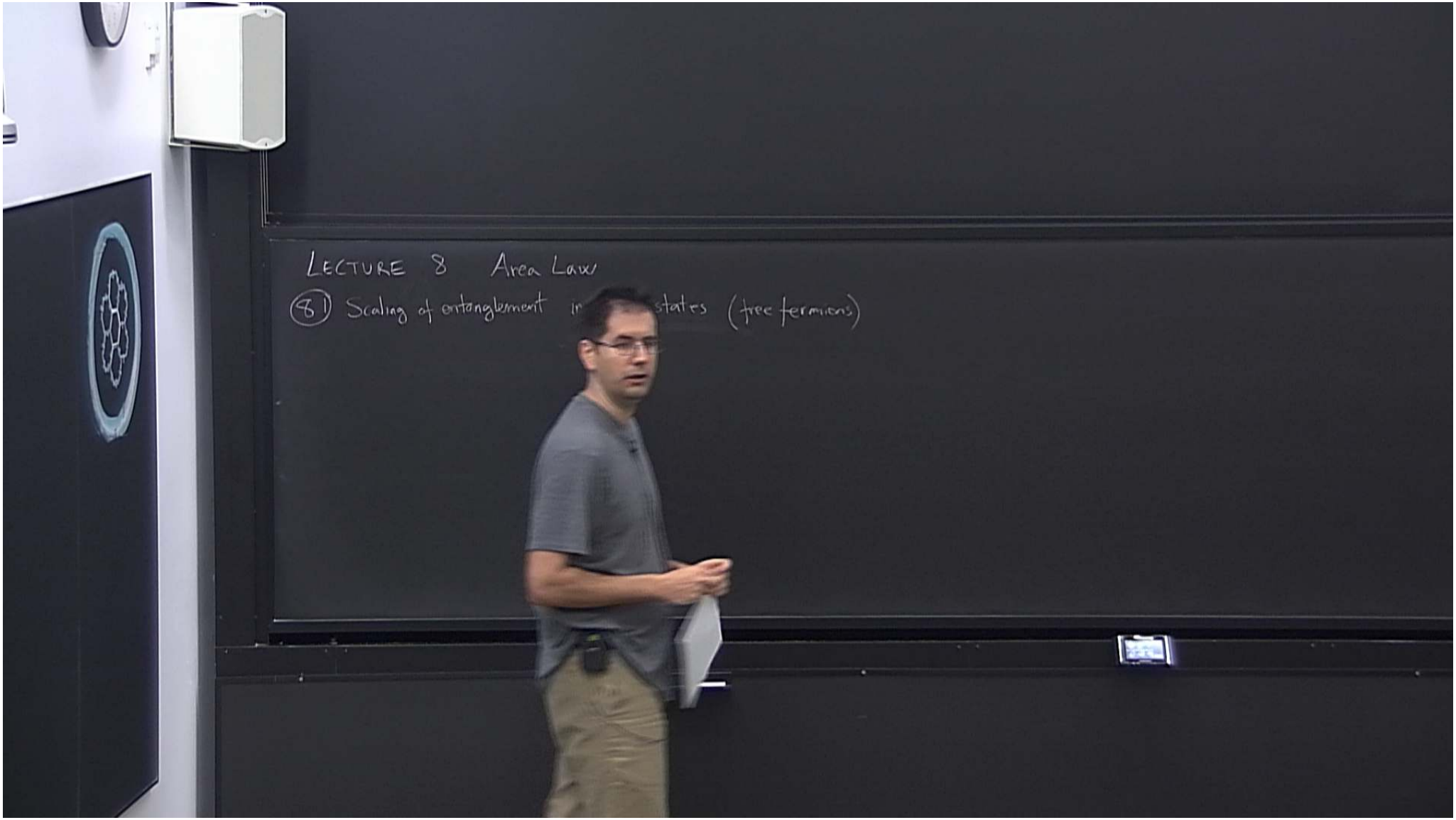
Title: PSI 2015/2016 Explorations in Condensed Matter - Guifre Vidal - 8

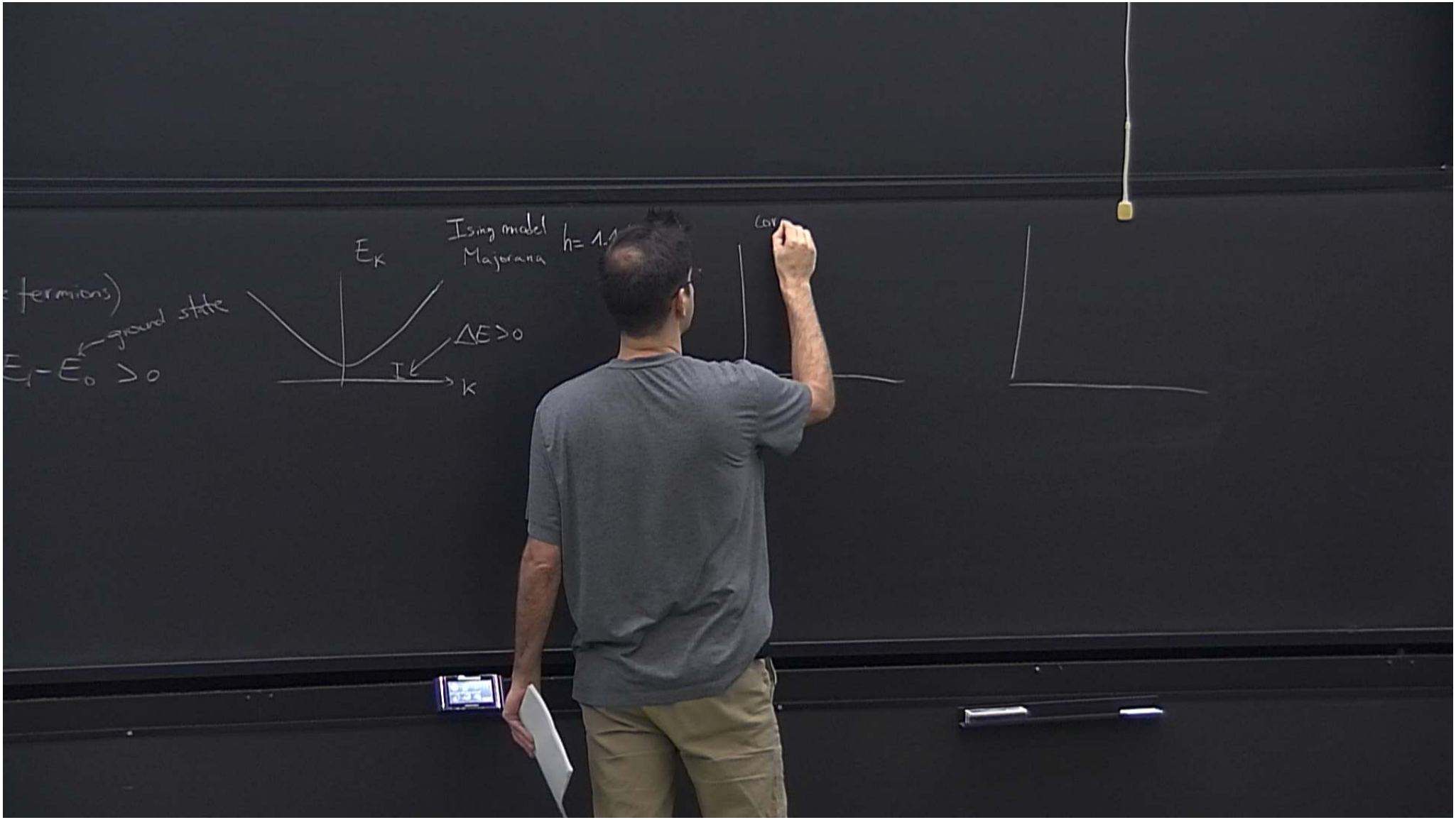
Date: Mar 31, 2016 10:15 AM

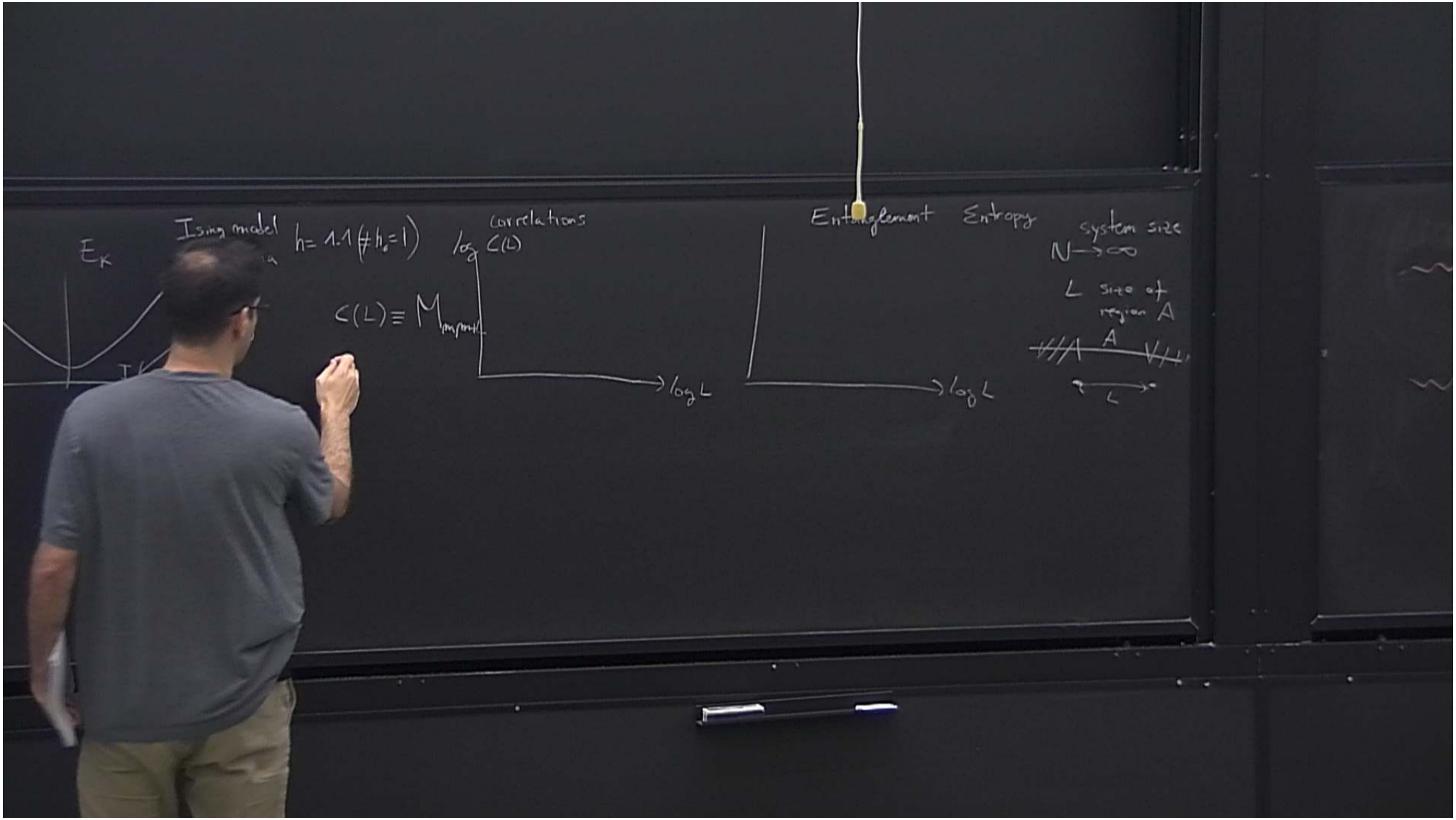
URL: http://pirsa.org/16030015

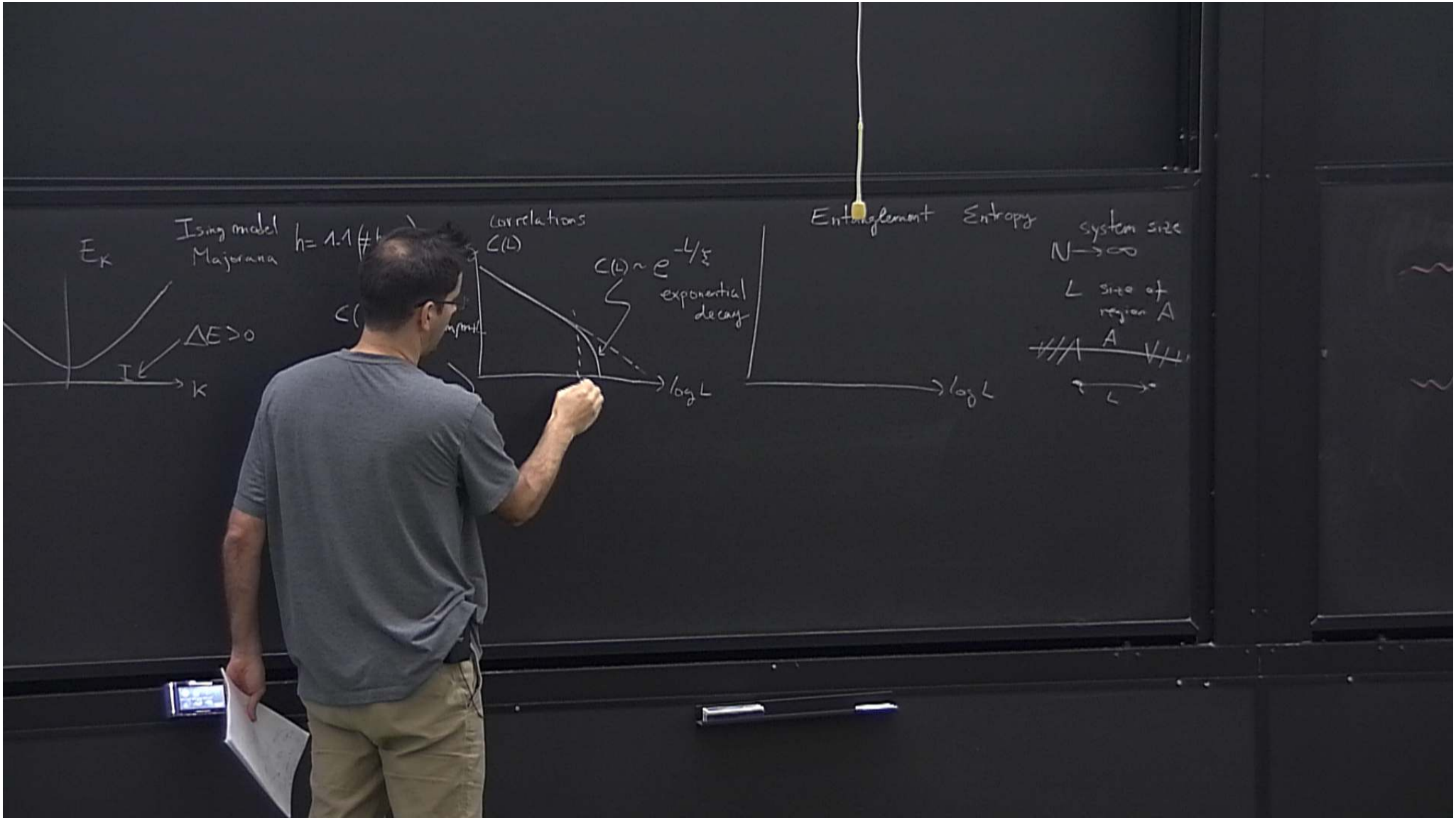
Abstract:

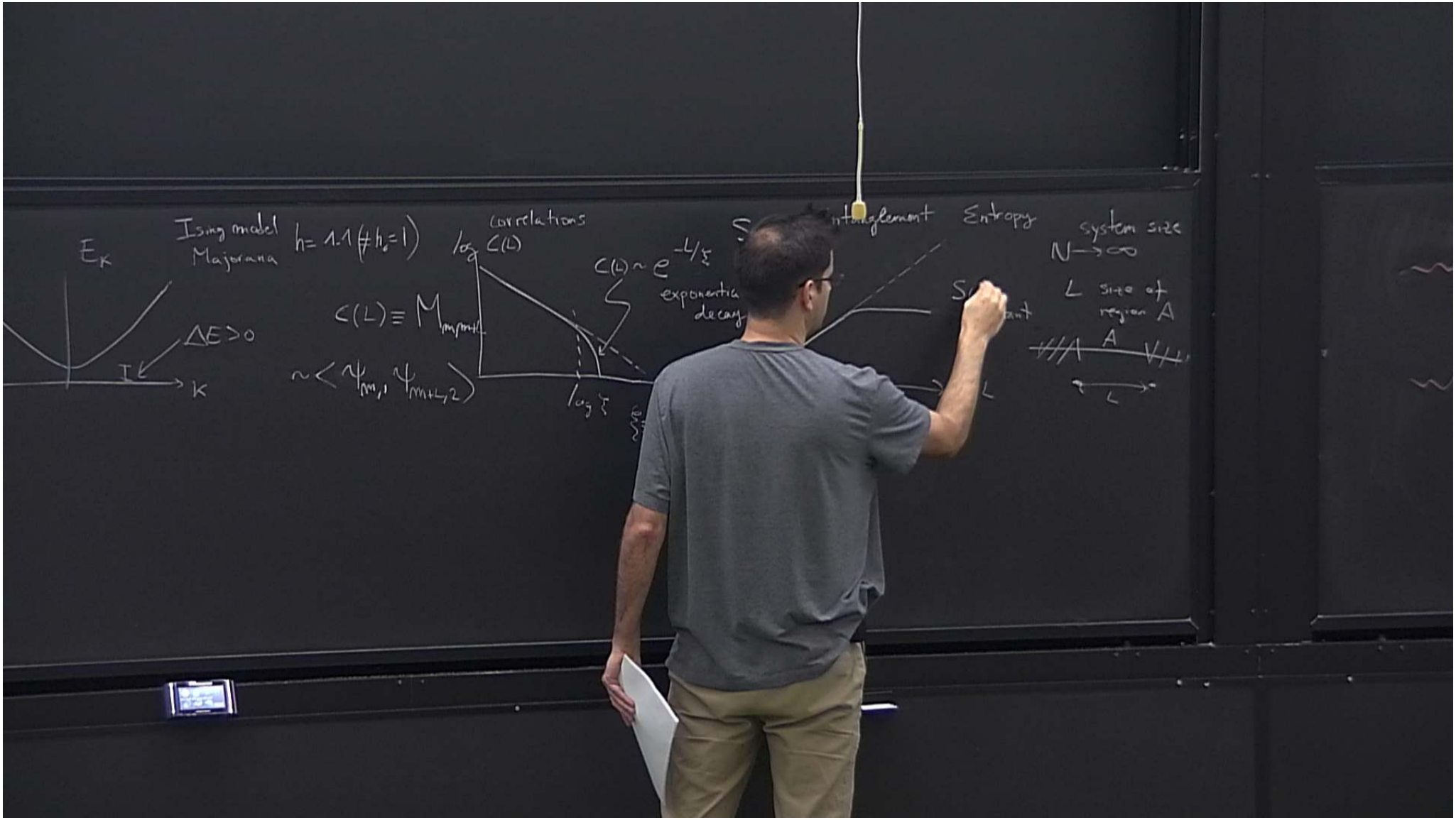

LECTURE 8
ea Laws
(8.1) Scaling of

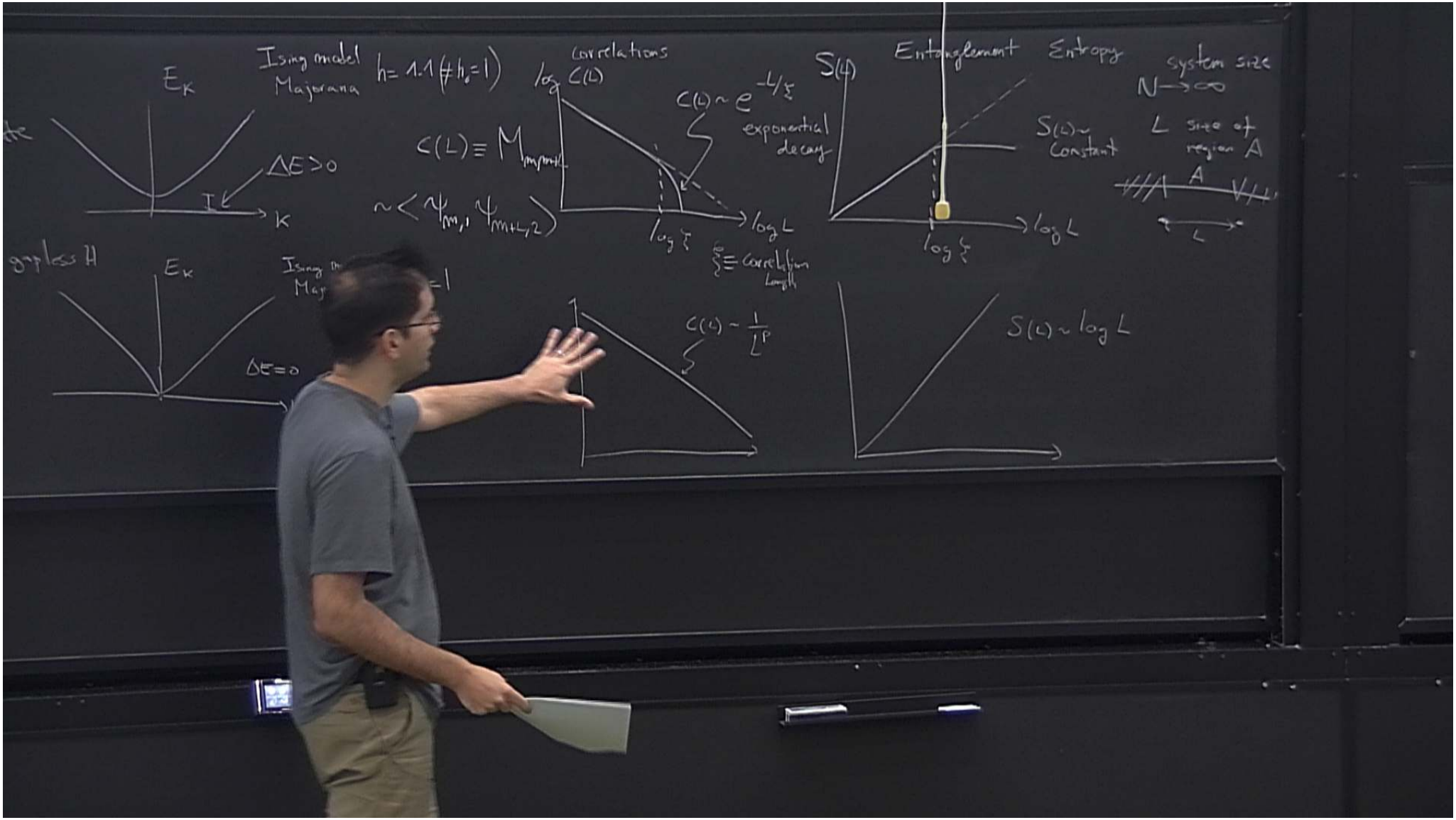

LECTURE 8 Area Law
(8.1) Scaling of entanglement in states (free fermions)

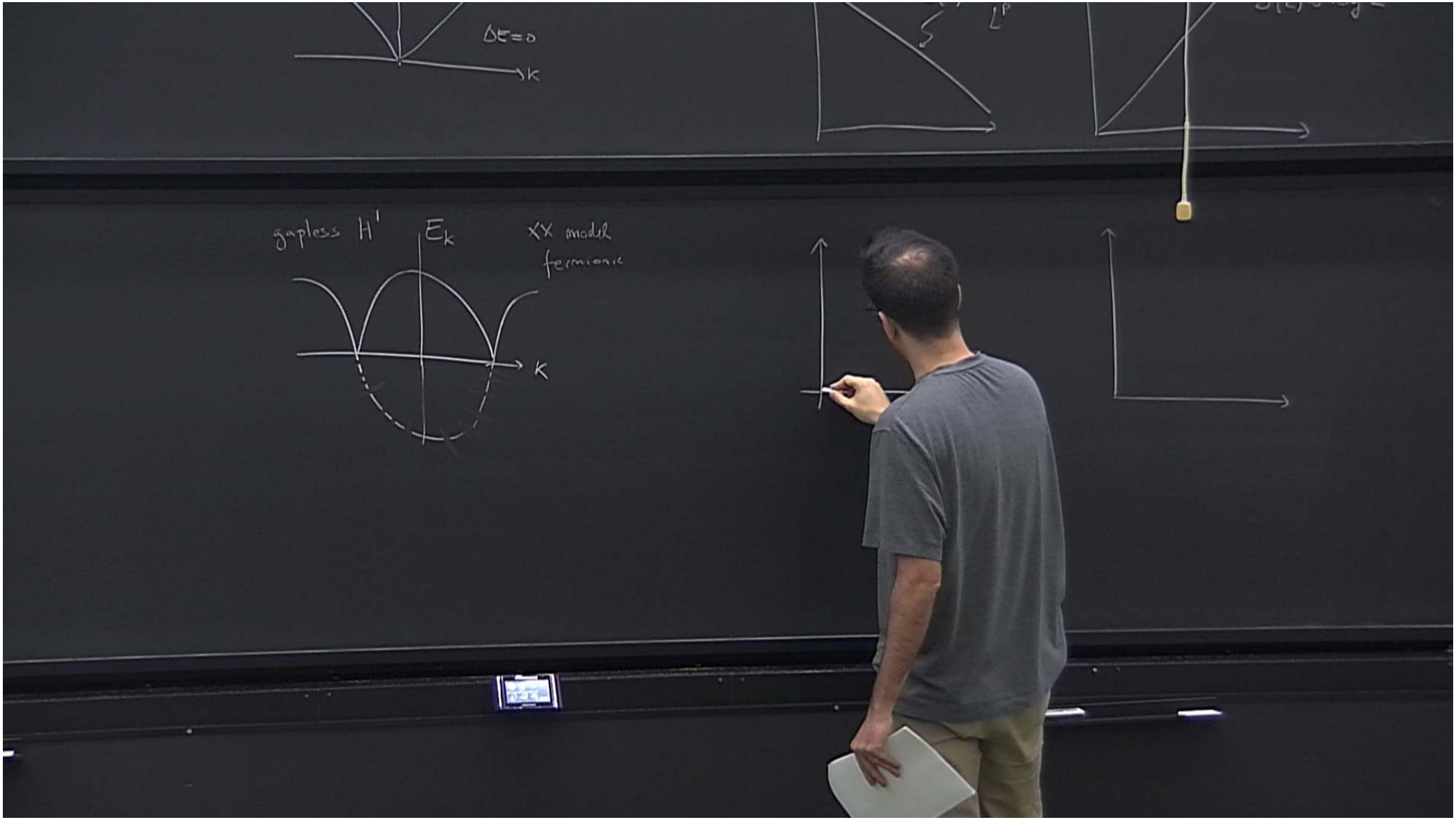

Ising model
Majorana
E_K
ground state
$\Delta E > 0$
$E_1 - E_0 > 0$
k

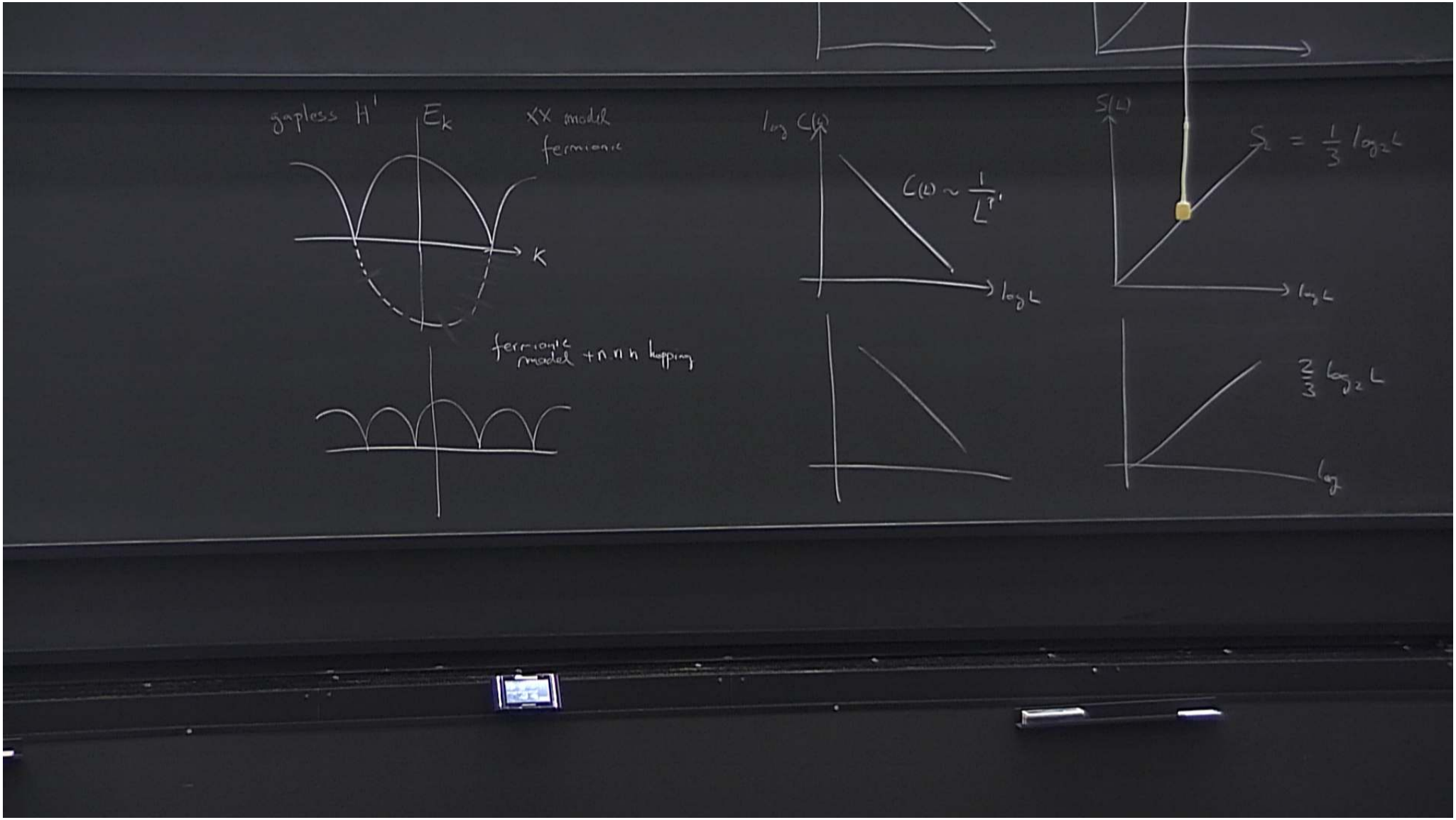

Ising model
h = 1.1 (≠ h_c = 1)
Correlations
log C(L)
C(L) ≡ M
log L
Entanglement Entropy
system size
N → ∞
L size of region A
A
L
log L

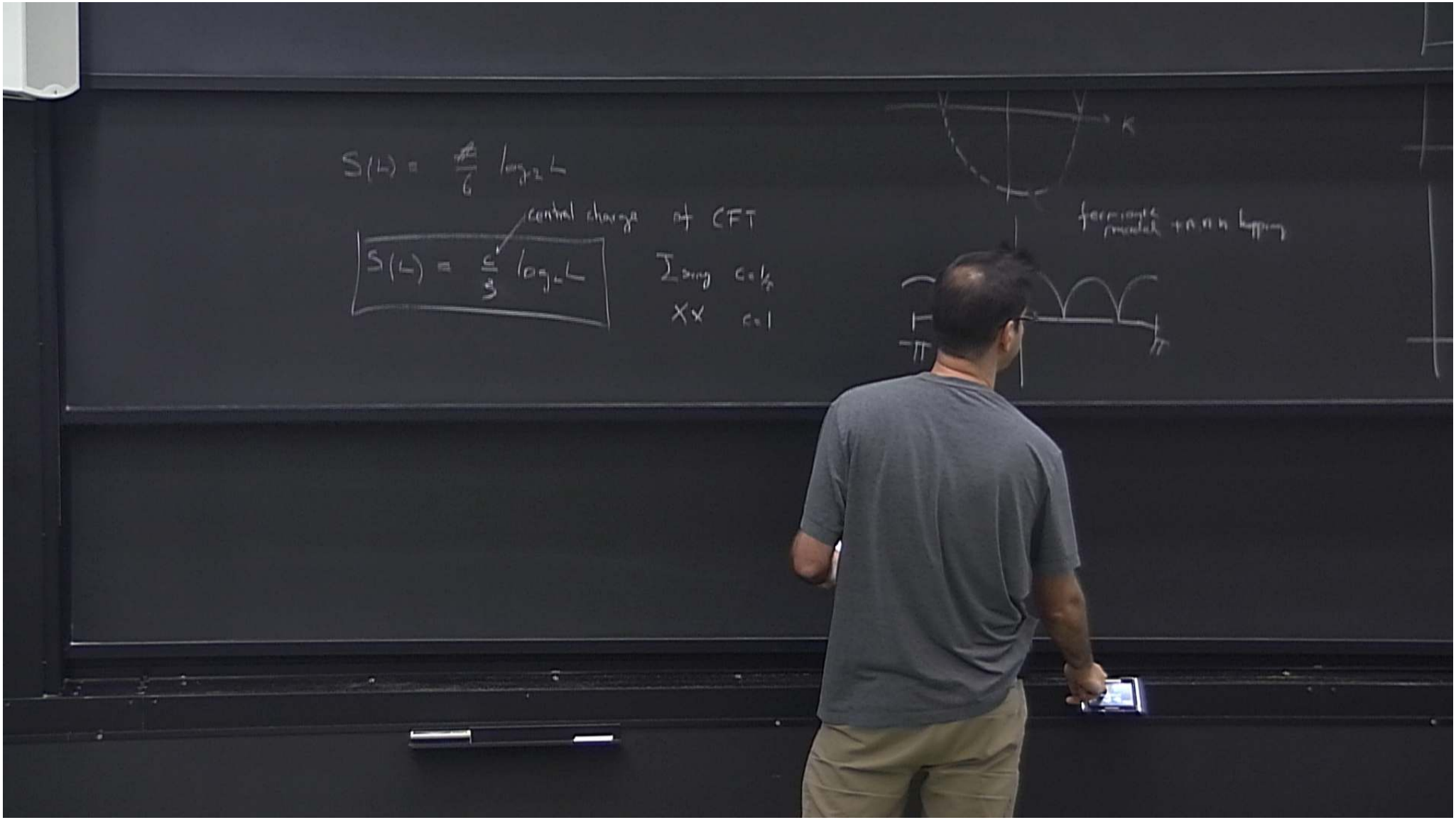

Ising model
Majorana
Correlations
C(L)
C(L) ~ e^{-L/ξ}
exponential decay
Entanglement Entropy
system size
N → ∞
L size of region A
A
ΔE > 0
k
log L
log L

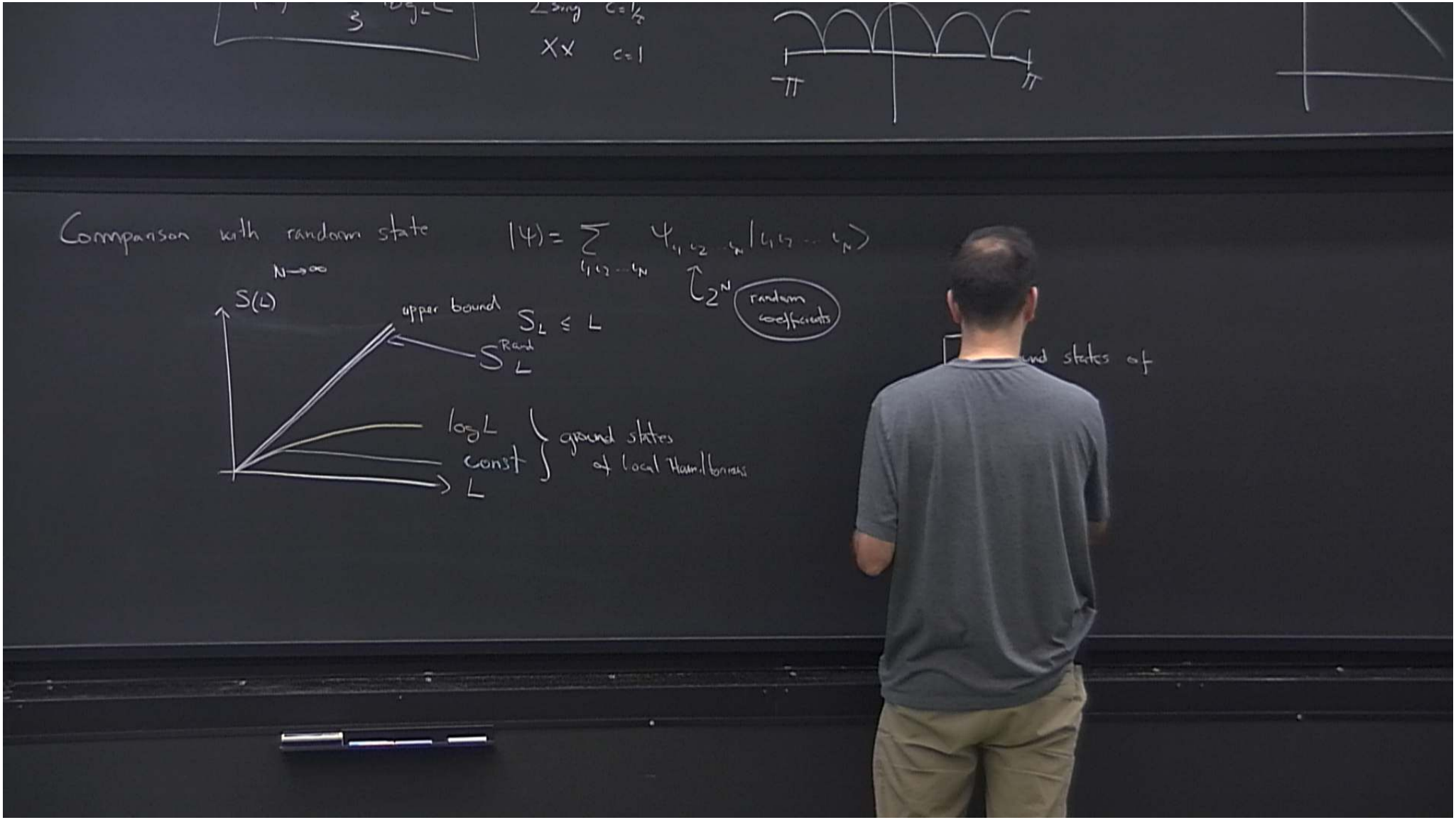

Ising model
Majorana
h = 1.1 (≠ h_c = 1)
Correlations
log C(L)
C(L) ~ e^{-L/ξ}
exponential decay
C(L) ≡ M_impurity
~ ⟨ψ_{m,1} ψ_{m+L,2}⟩
log ξ
E_K
ΔE > 0
k
Entanglement Entropy
system size
N → ∞
L site of region A
A
L

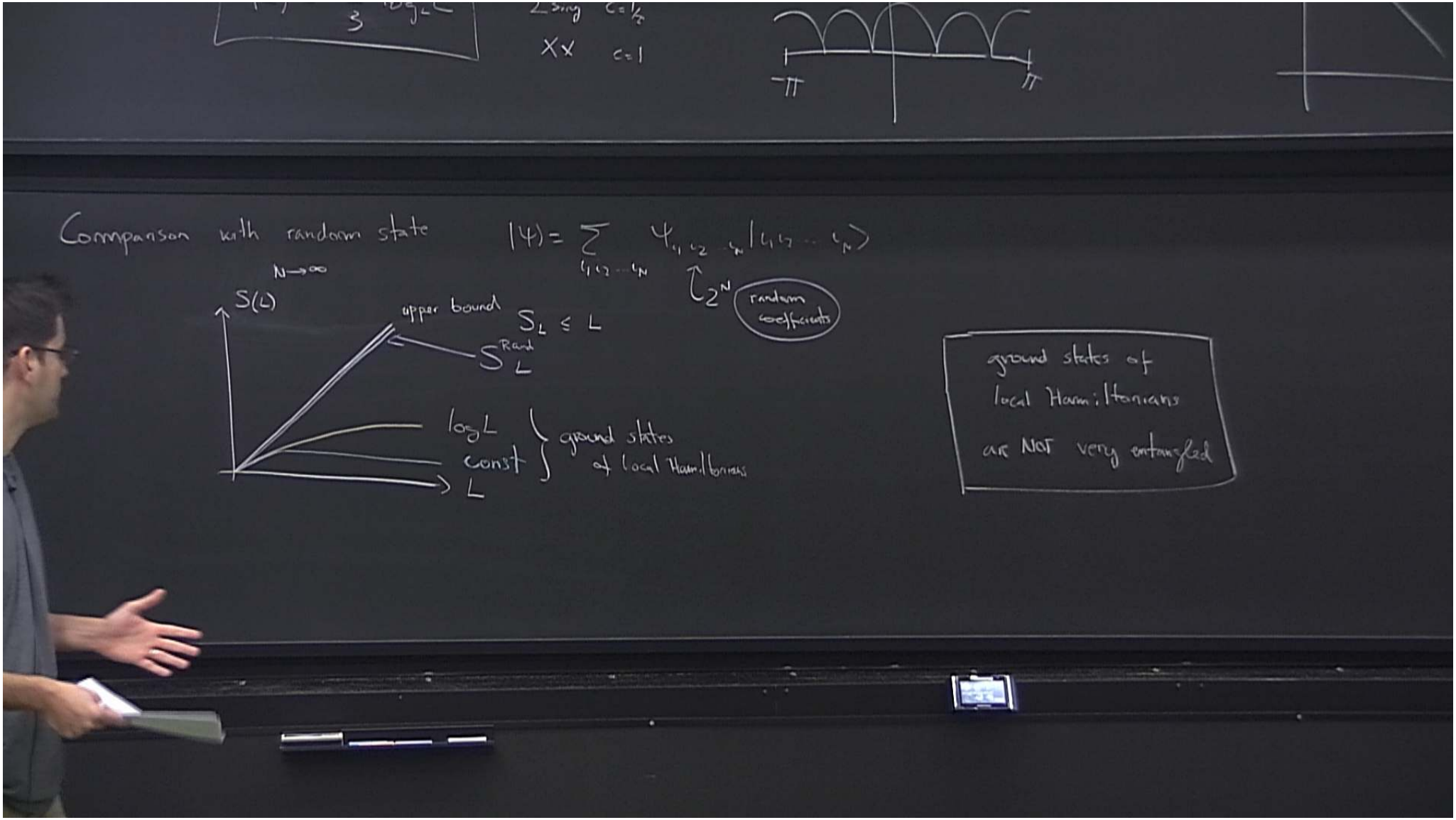

Ising model
Majorana
h = 1.1 (≠ h_c = 1)
correlations
log C(L)
C(L) ~ e^{-L/ξ}
exponential decay
C(L) ≡ M_{m,m+L}
~ < ψ_{m,1} ψ_{m+L,2} >
log ξ
log L
ξ ≡ correlation length
E_k
k
ΔE > 0
gapless H
ΔE = 0
C(L) ~ 1/L^p
Entanglement Entropy
S(L)
S(L) ~ Constant
S(L) ~ log L
system size
N → ∞
L size of region A
A
L

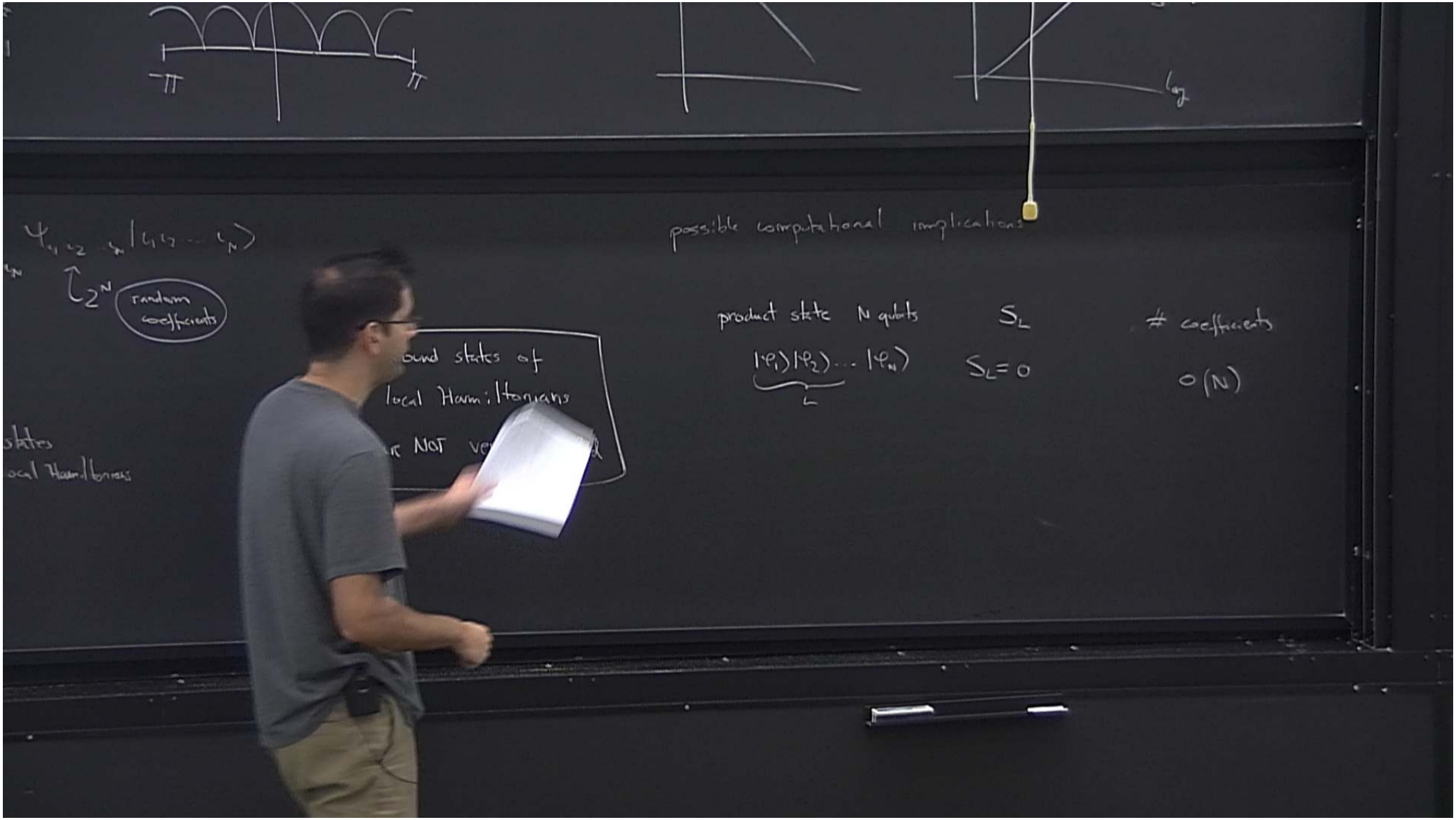

ΔE=0
k
gapless H'
E_k
XX model
fermionic
k

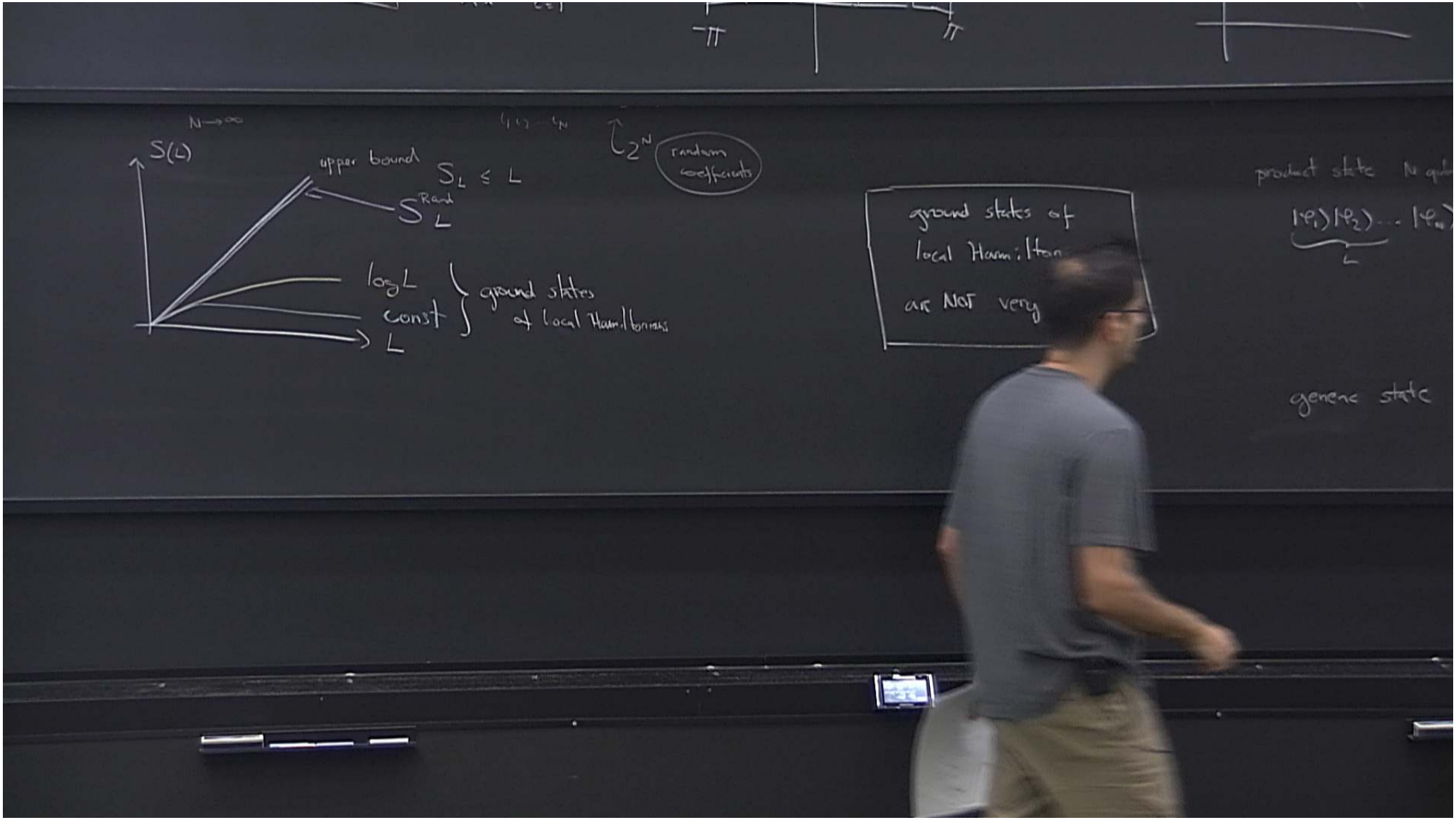

gapless H'
E_k
XX model
fermionic
k
fermionic model + n.n.n hopping
log C(L)
$C(L) \sim \frac{1}{L^{p'}}$
log L
S(L)
$S_L = \frac{1}{3} \log_2 L$
log L
$\frac{2}{3} \log_2 L$
log

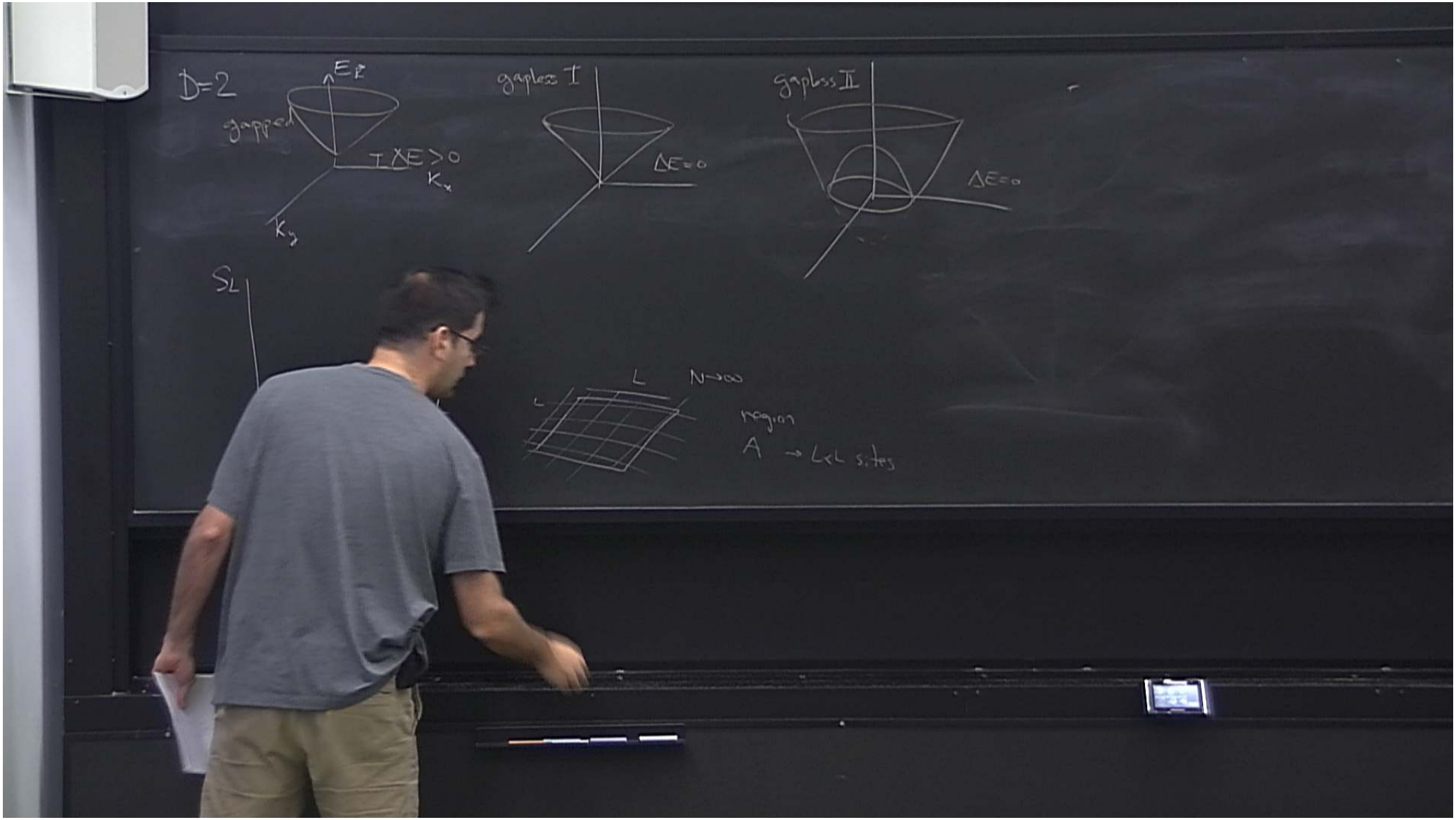

central charge of CFT
XX c=1

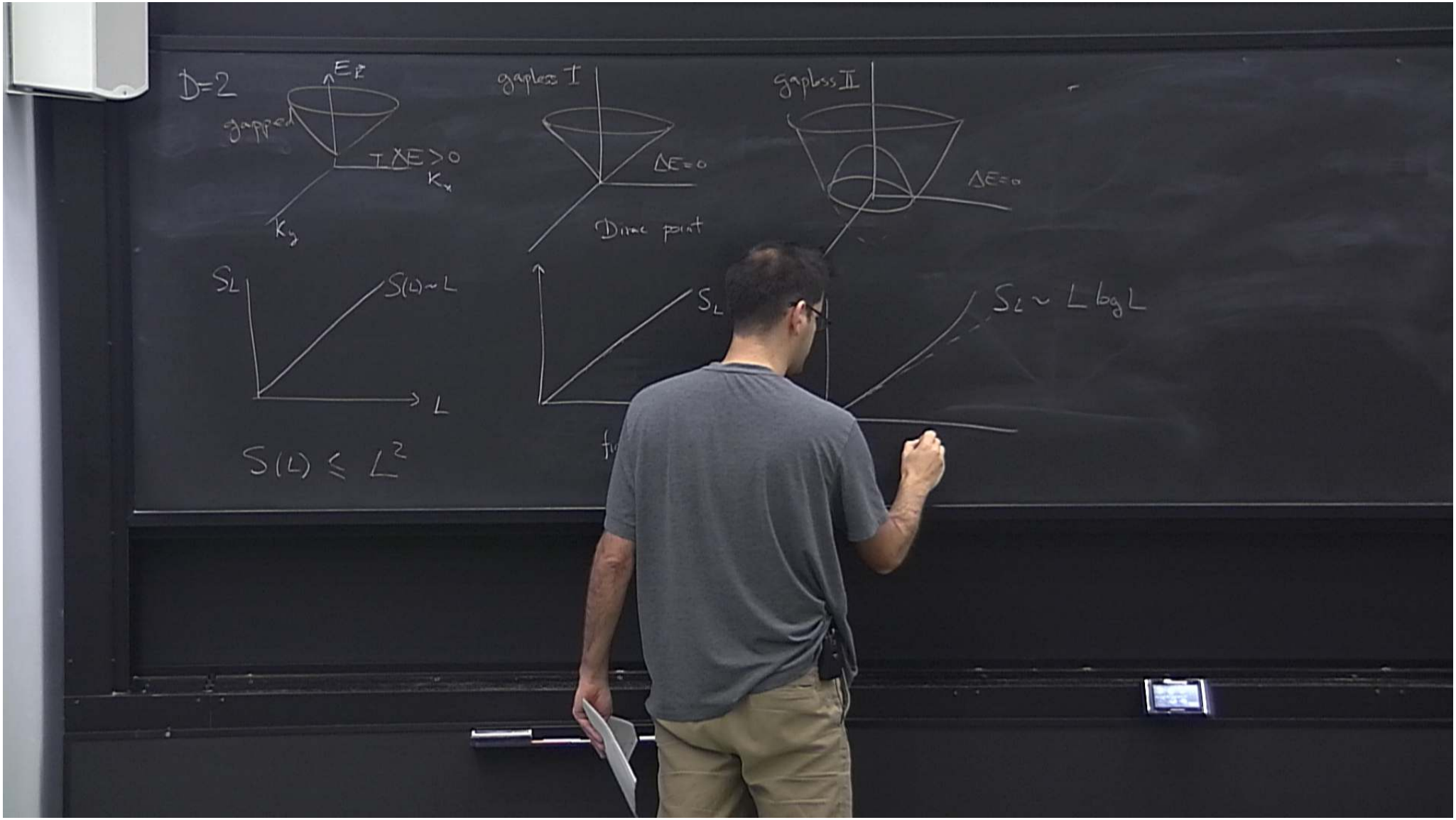

Comparison with random state
$N \to \infty$
$|\psi\rangle = \sum_{i_1 i_2 \cdots i_N} \psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$
2^N
random coefficients
S(L)
upper bound
$S_L \leq L$
S_L^{Rand}
$\log L$
const
L
ground states of Local Hamiltonians

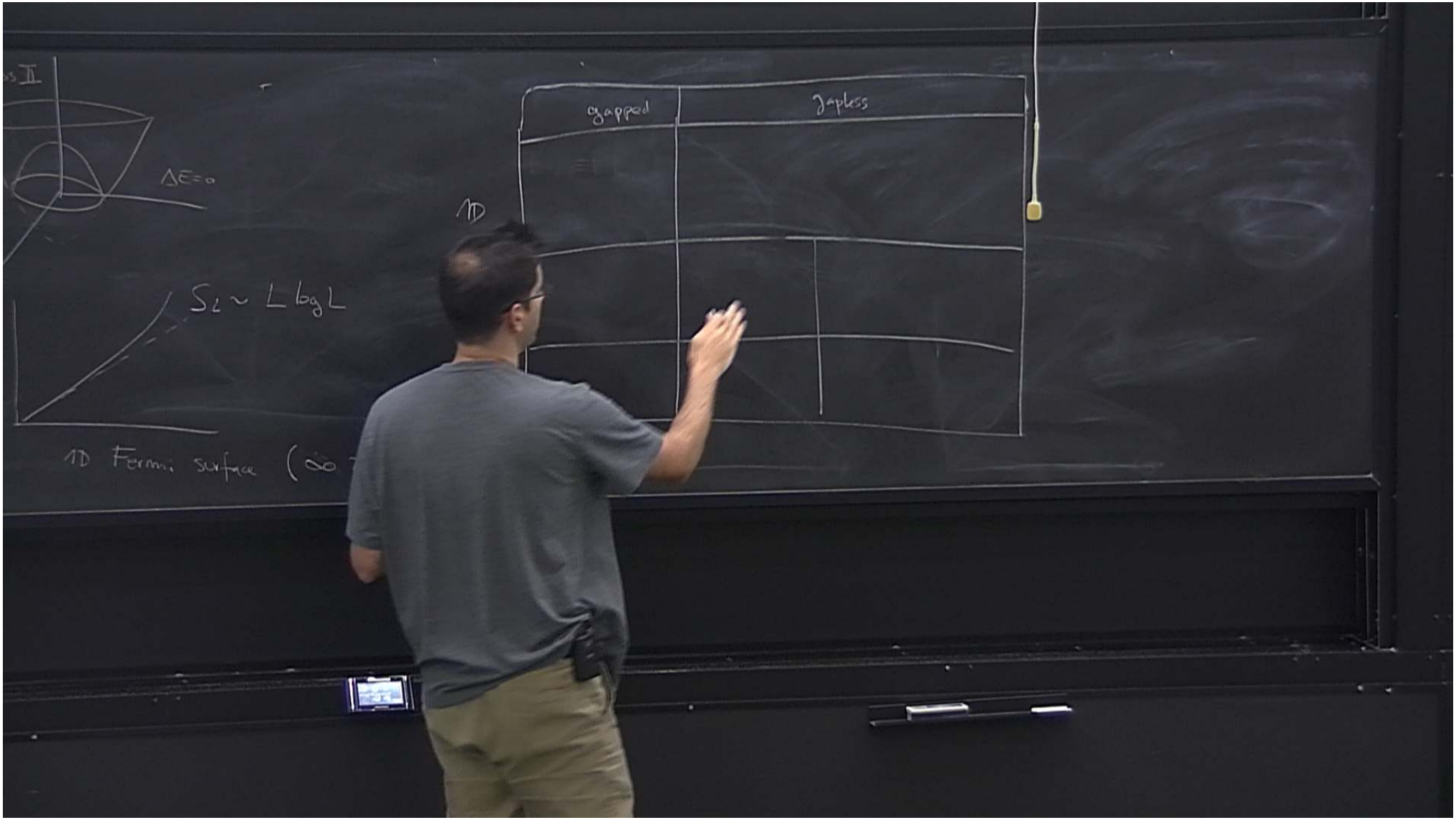

Comparison with random state
$|\psi\rangle = \sum_{i_1 i_2 \cdots i_N} \psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$
$N \to \infty$
2^N
random coefficients
S(L)
upper bound
$S_L \leq L$
S_L^{Rand}
$\log L$
const
L
ground states of local Hamiltonians
ground states of local Hamiltonians are NOT very entangled

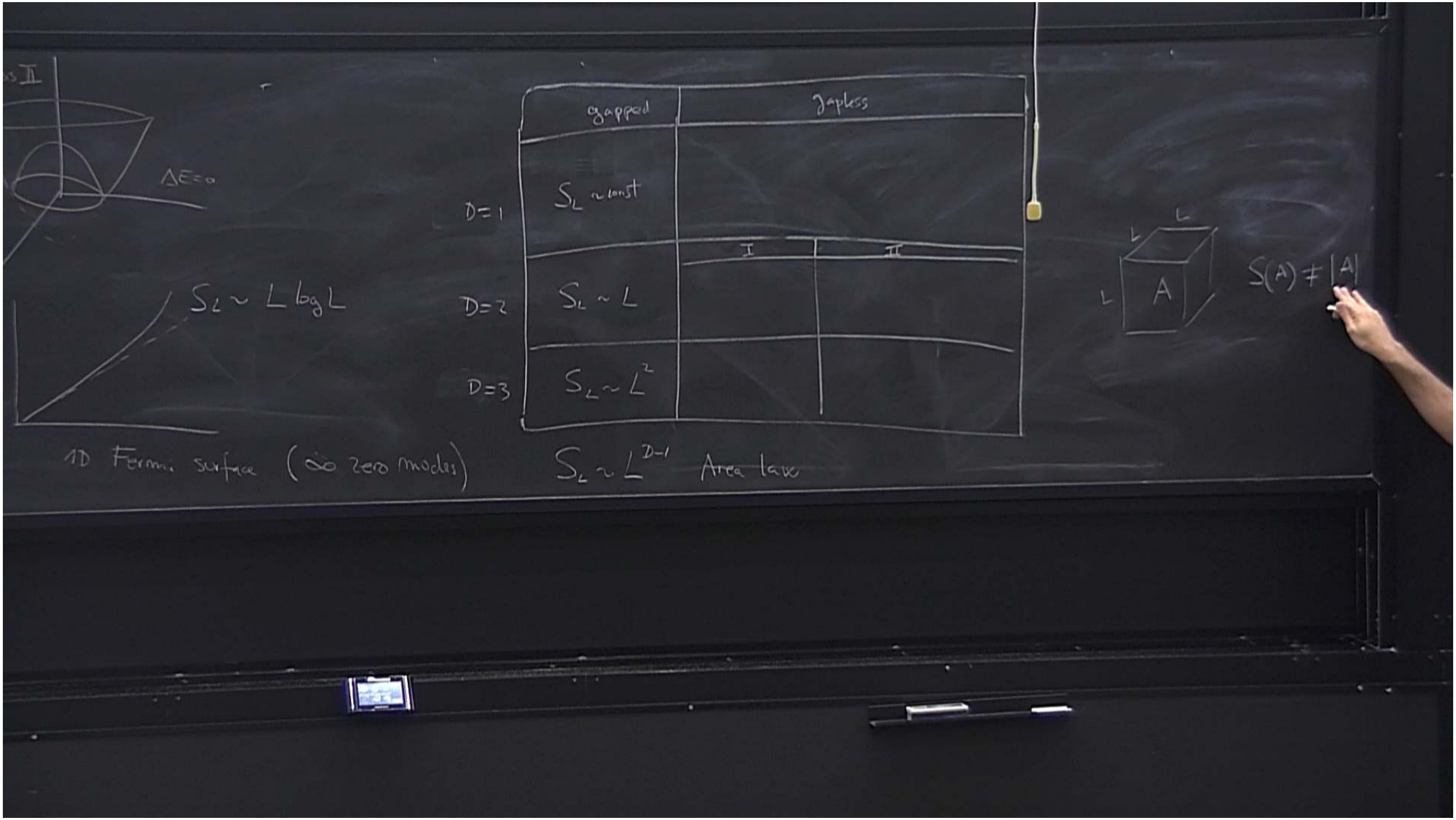

possible computational implications
random coefficients
product state N qubits
S_L
coefficients
$|\varphi_1\rangle|\varphi_2\rangle\ldots|\varphi_N\rangle$
$S_L = 0$
$O(N)$
local Hamiltonians


N → ∞
S(L)
upper bound
$S_L \leq L$
S_L^{Rand}
log L
const
L
ground states of local Hamiltonians
random coefficients
ground states of
local Hamiltonian
are NOT very
product state
genene state


D=2
gapped
gapless I
gapless II
ΔE=0
SL
L
N→∞
region
A → L×L sites


D=2
gapped
gapless I
gapless II
Dirac point
S(L)~L
S(L) ≤ L²
S_L ~ L log L


gapped
gapless
ΔE=0
1D
$S_L \sim L \log L$
1D Fermi surface


II
ΔE=0
$S_L \sim L \log L$
1D Fermi surface (∞ zero modes)
gapped
gapless
D=1
$S_L \sim$ const
I
II
D=2
$S_L \sim L$
D=3
$S_L \sim L^2$
$S_L \sim L^{D-1}$ Area law
L
L
L
A
$S(A) \neq |A|$

	gapped	gapless	
D=1	$S_L \sim \text{const}$	$\log L$	
		I "small" F.S.	II "large" F.S.
D=2	$S_L \sim L$	L	$L \log L$
D=3	$S_L \sim L^2$	L^2	$L^2 \log L$

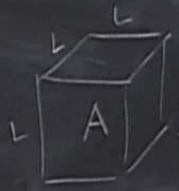
$L \log L$

(∞ zero modes)

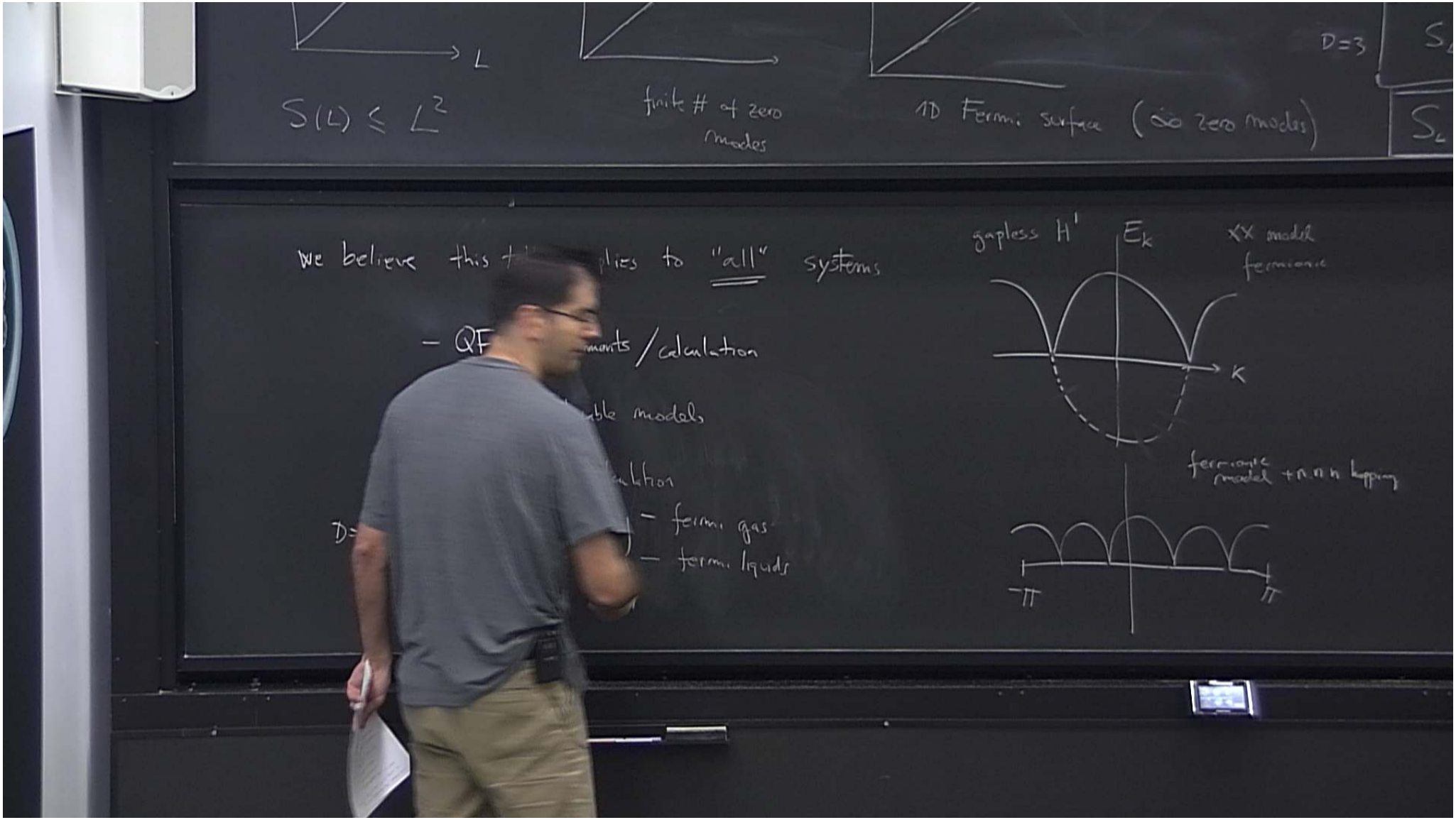
$S_L \sim L^{D-1}$ Area law

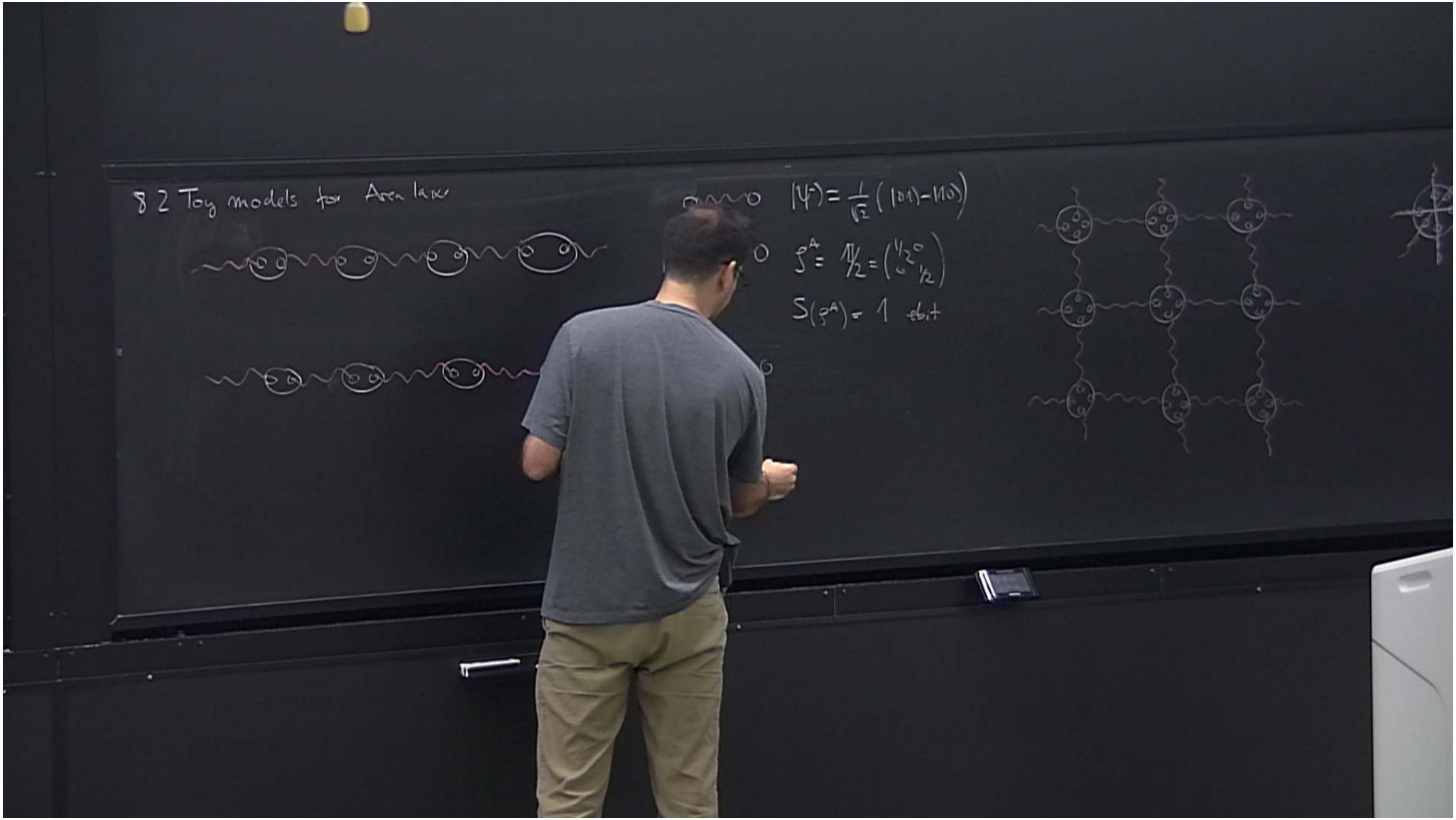
Volume law

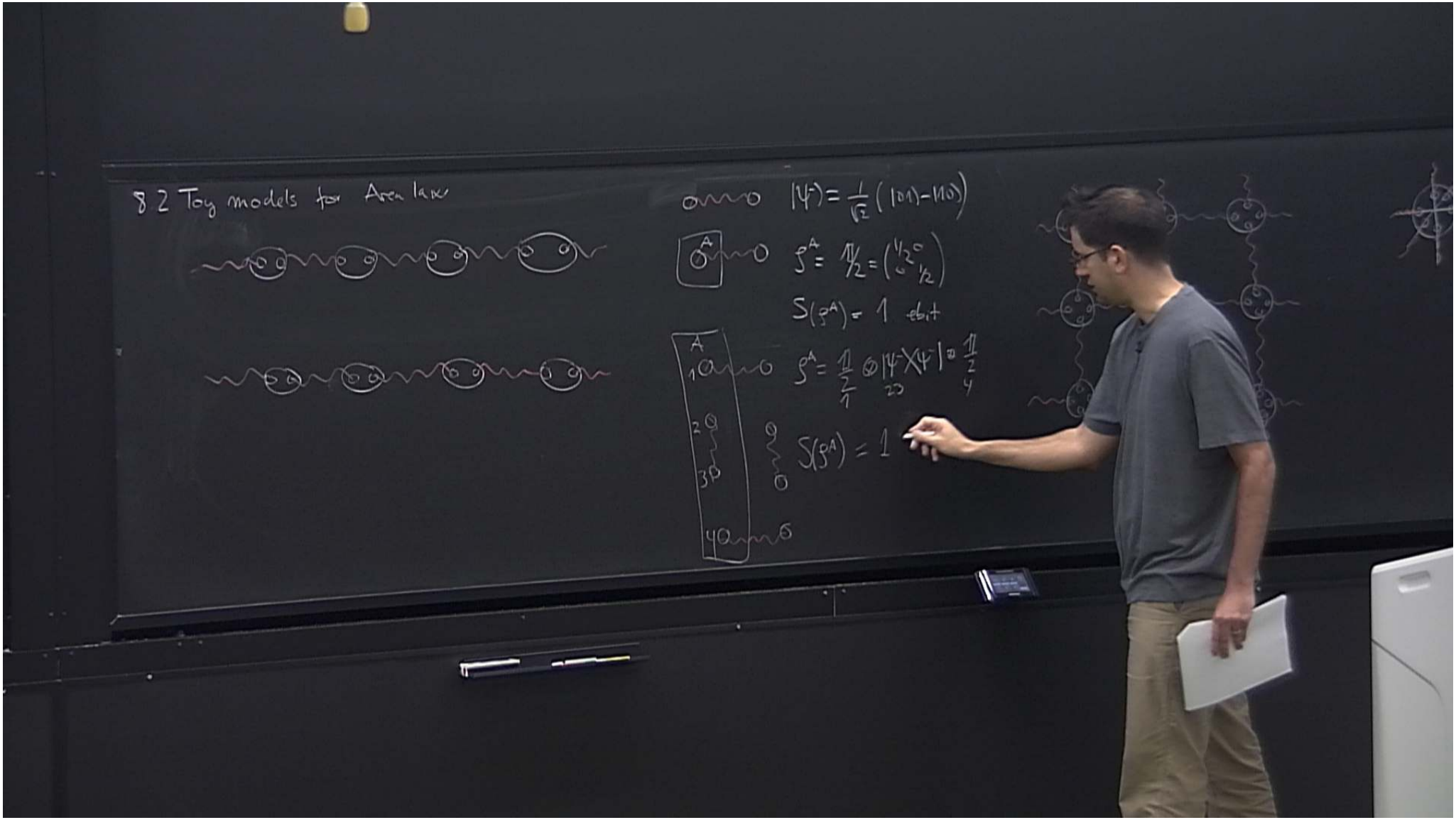
$S^{\text{rand}}(A) = L^3 = |A|$

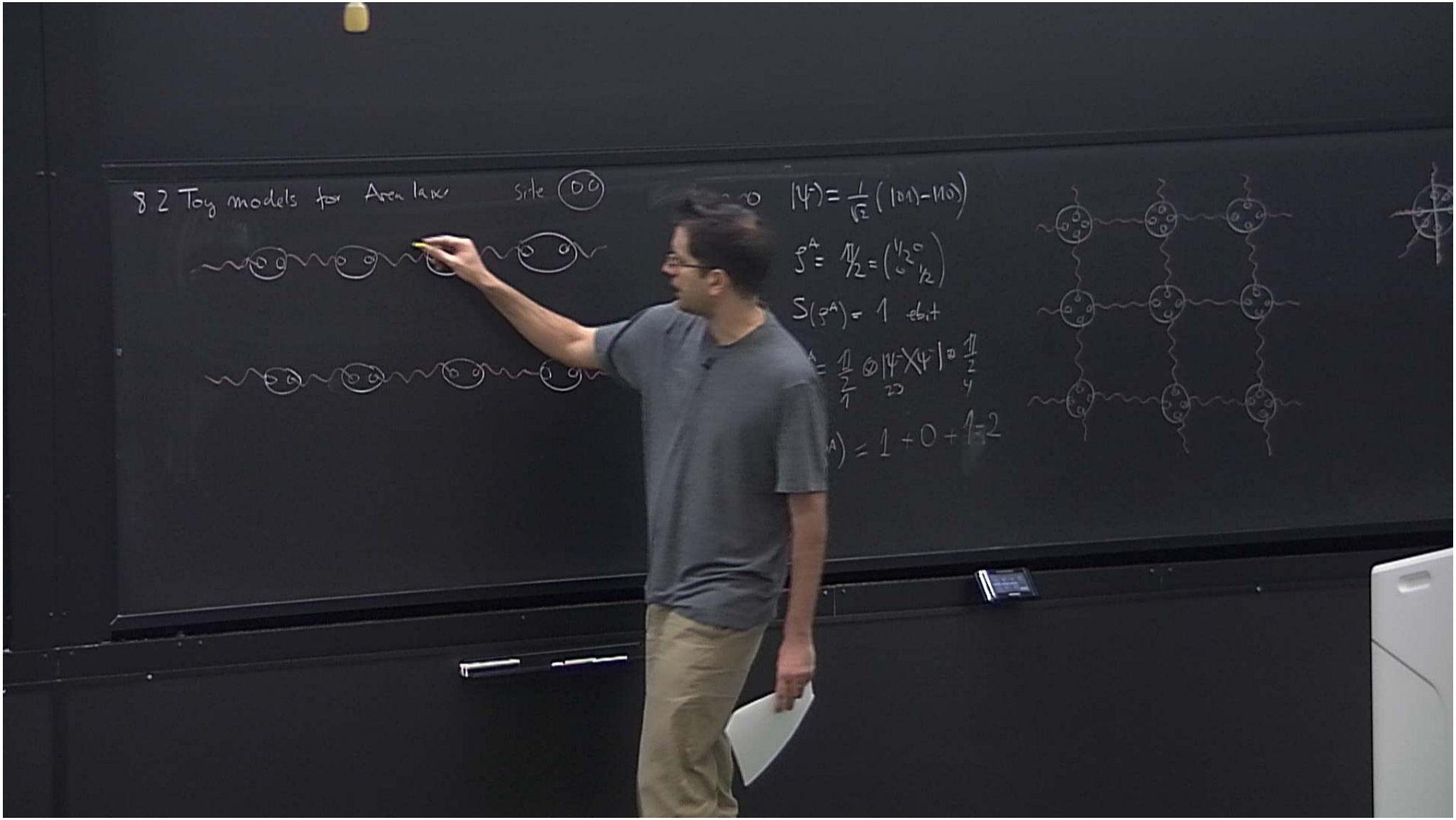


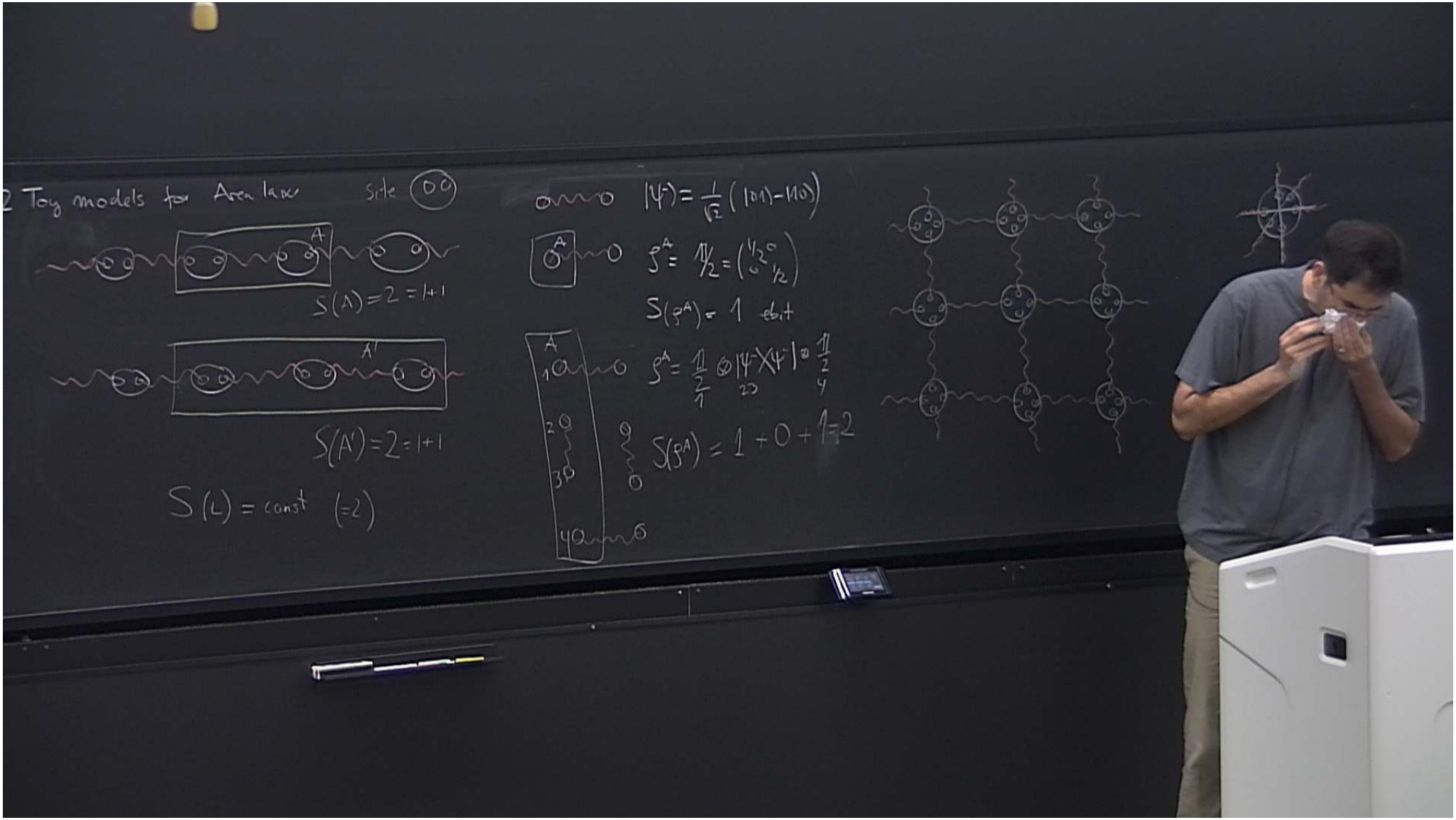
$S^{\text{ground state}}(A) = |\partial A|$

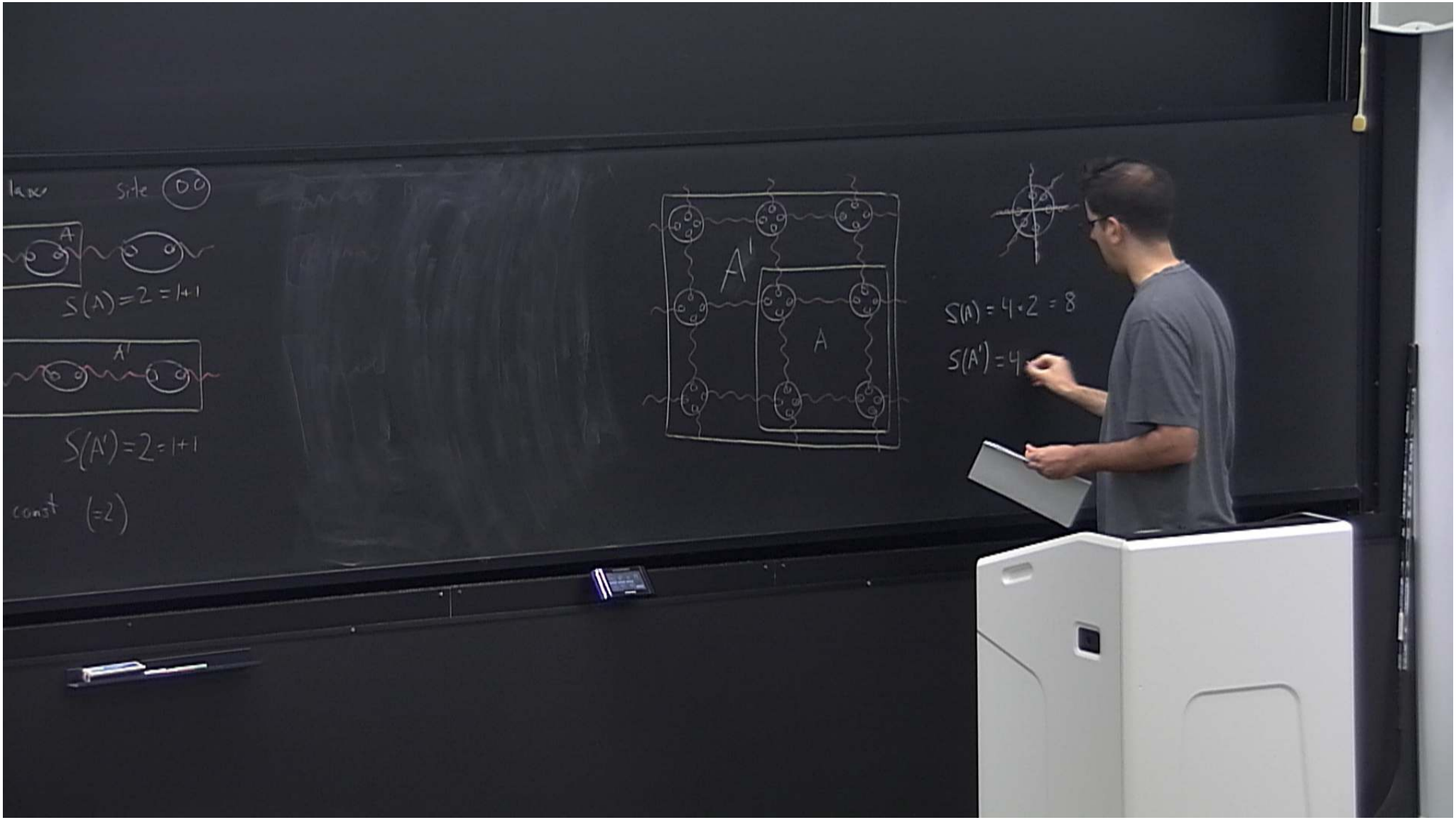

L
S(L) ≤ L²
finite # of zero modes
1D Fermi surface (∞ zero modes)
D=3
we believe this
to "all" systems
– QF
/calculation
models
– fermi gas
– fermi liquids
gapless H'
E_k
XX model
fermionic
k
fermionic model + n.n.n hopping
−π
π

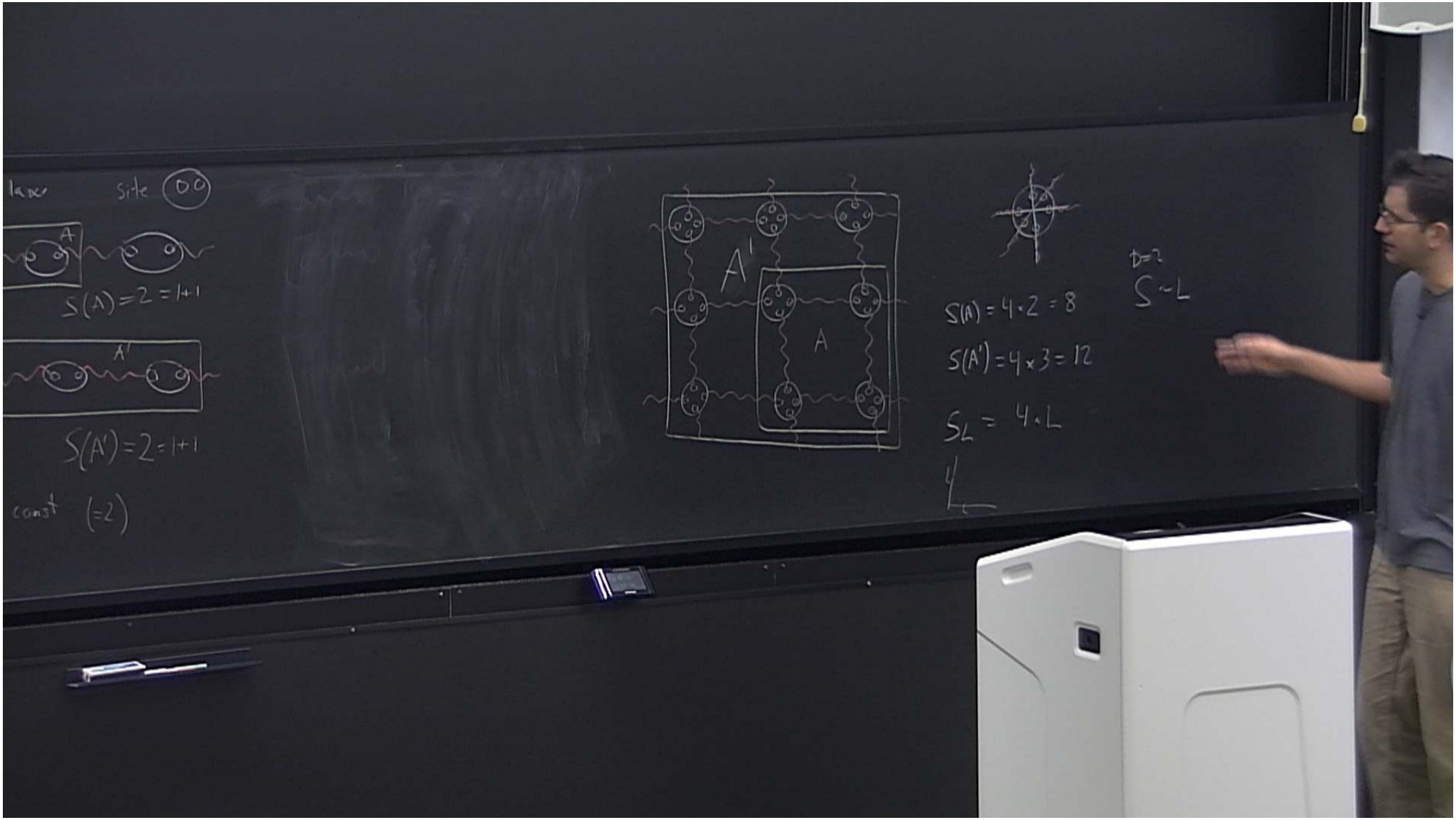

8.2 Toy models for Area laws
$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$
$\rho^A = \mathbb{1}/2 = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$
$S(\rho^A) = 1$ ebit

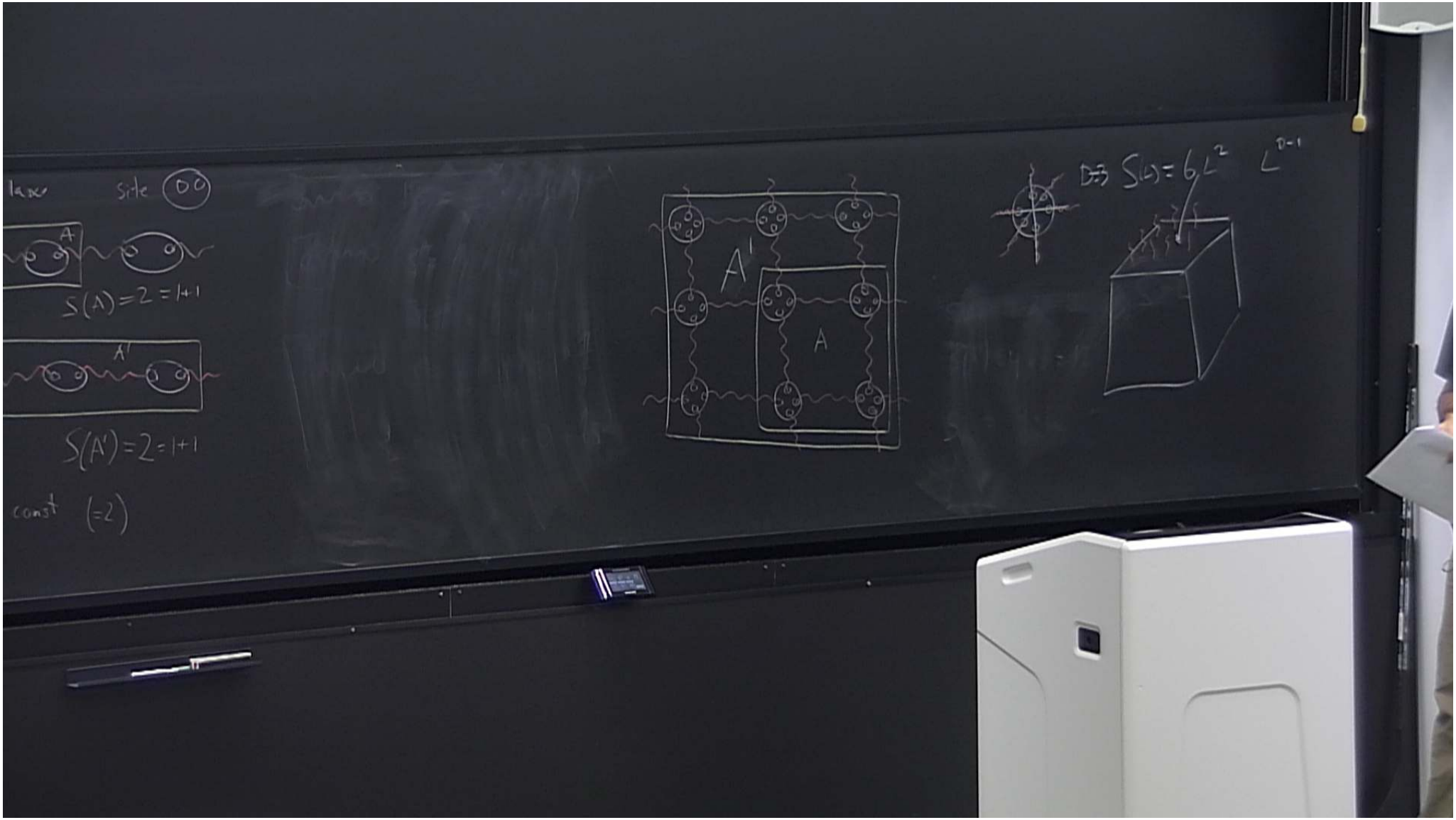

8.2 Toy models for Area laws
$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$
A
$\rho^A = \mathbb{1}/2 = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$
$S(\rho^A) = 1$ ebit
A
1
2
3
4
$\rho^A = \frac{\mathbb{1}}{2} \otimes |\psi\rangle\langle\psi|$
$S(\rho^A) = 1$

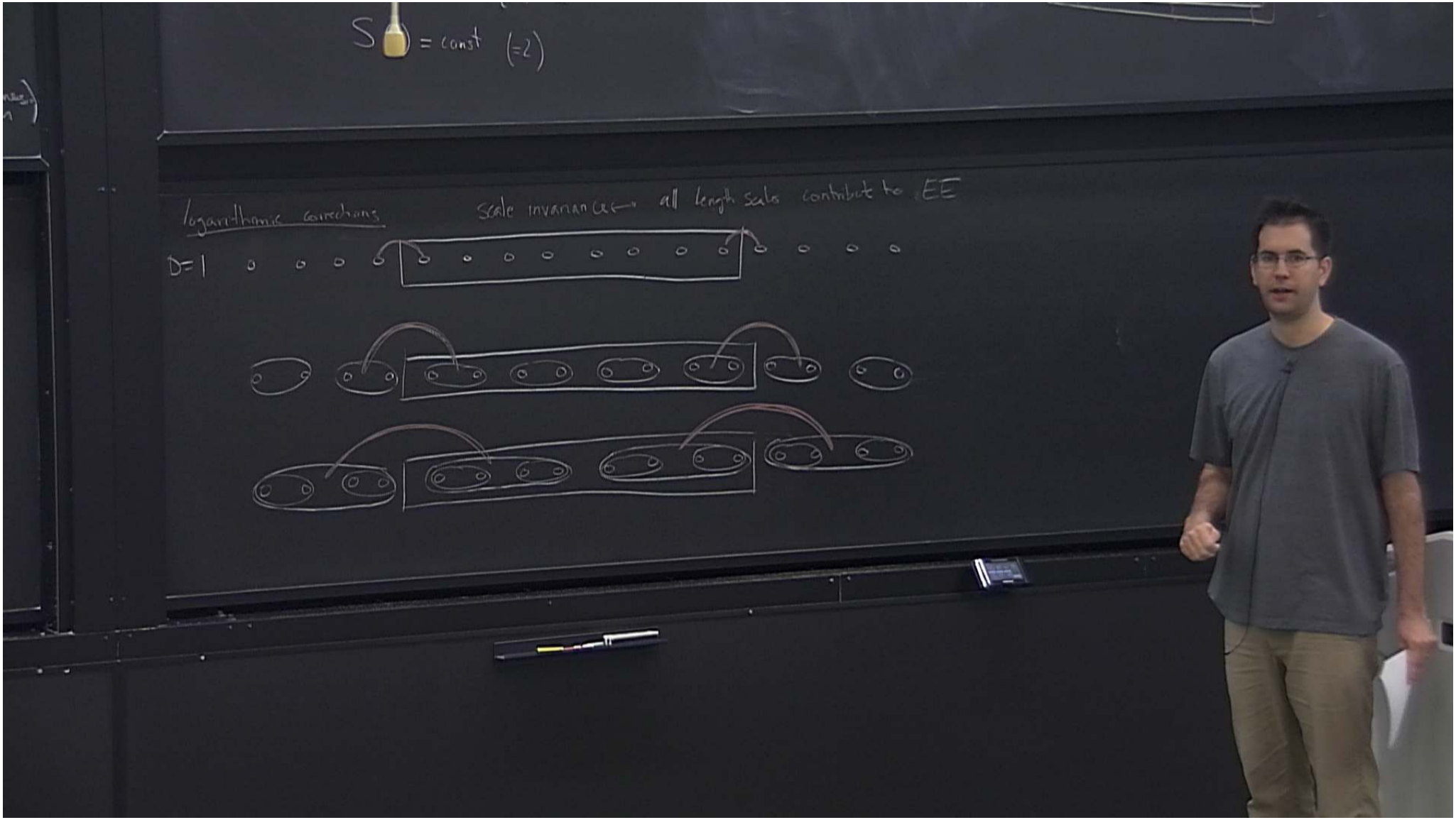

8 2 Toy models for Area laws
site
$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$
$\rho^A = \mathbb{1}/2 = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$
$S(\rho^A) = 1$ ebit
$= \frac{\mathbb{1}}{2} \otimes |\psi\rangle\langle\psi| = \frac{\mathbb{1}}{2}$
$= 1 + 0 + 1 = 2$

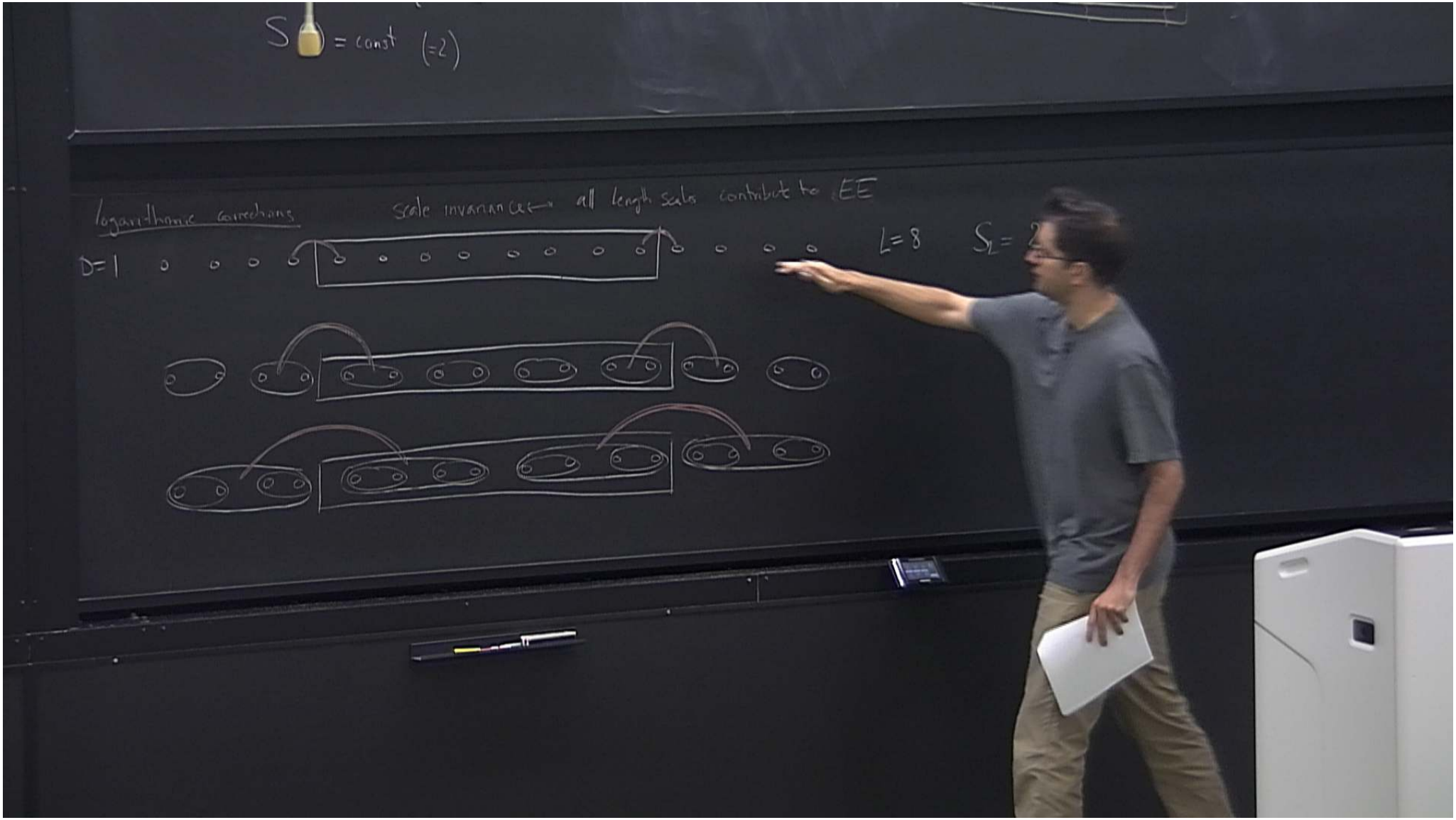

2 Toy models for Area law
site
A
S(A) = 2 = 1+1
A'
S(A') = 2 = 1+1
S(L) = const (=2)
|ψ⁻⟩ = 1/√2 (|01⟩ - |10⟩)
ρ^A = 1/2 = (1/2 0 / 0 1/2)
S(ρ^A) = 1 ebit
ρ^A = 1/2 ⊗ |ψ⁻⟩⟨ψ⁻| ⊗ 1/2
S(ρ^A) = 1 + 0 + 1 = 2
1
2
3
4

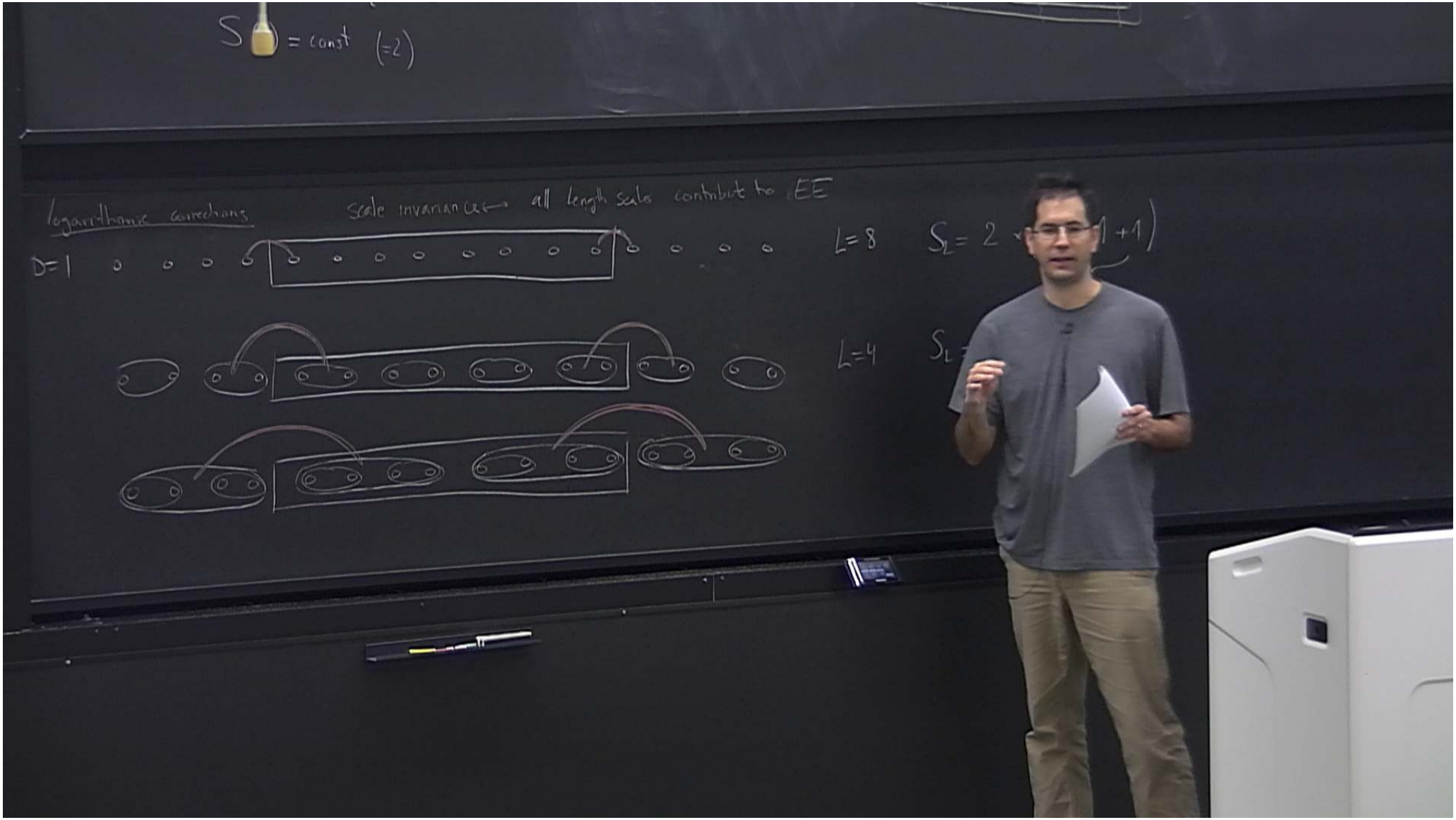

lane
site
A
S(A) = 2 = 1+1
A'
S(A') = 2 = 1+1
const (=2)
A'
A
S(A) = 4·2 = 8
S(A') = 4

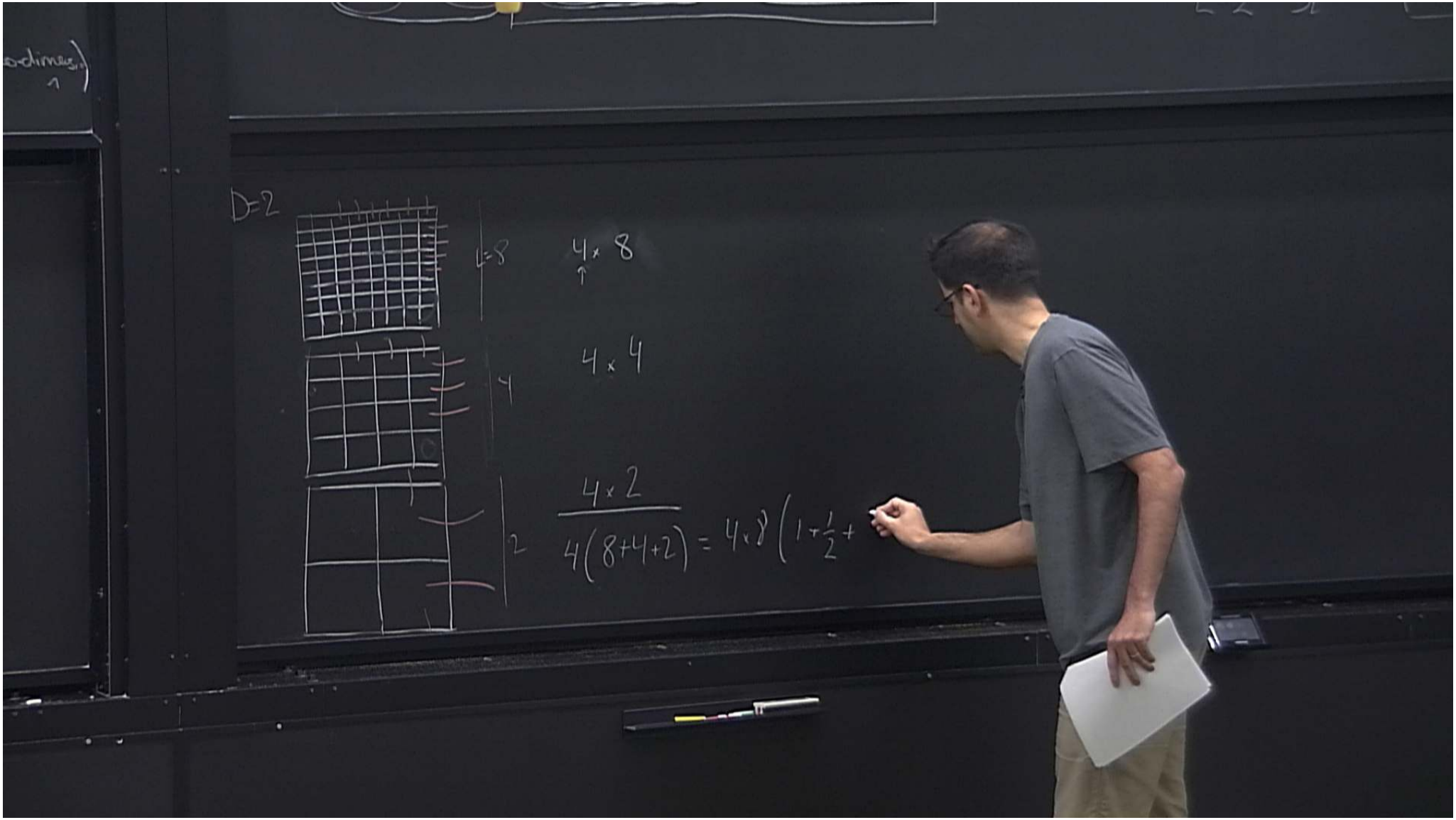

lane
site
A
S(A) = 2 = 1+1
A'
S(A') = 2 = 1+1
const (=2)
A'
A
S(A) = 4·2 = 8
S(A') = 4×3 = 12
S_L = 4·L
L
D=2
S ~ L

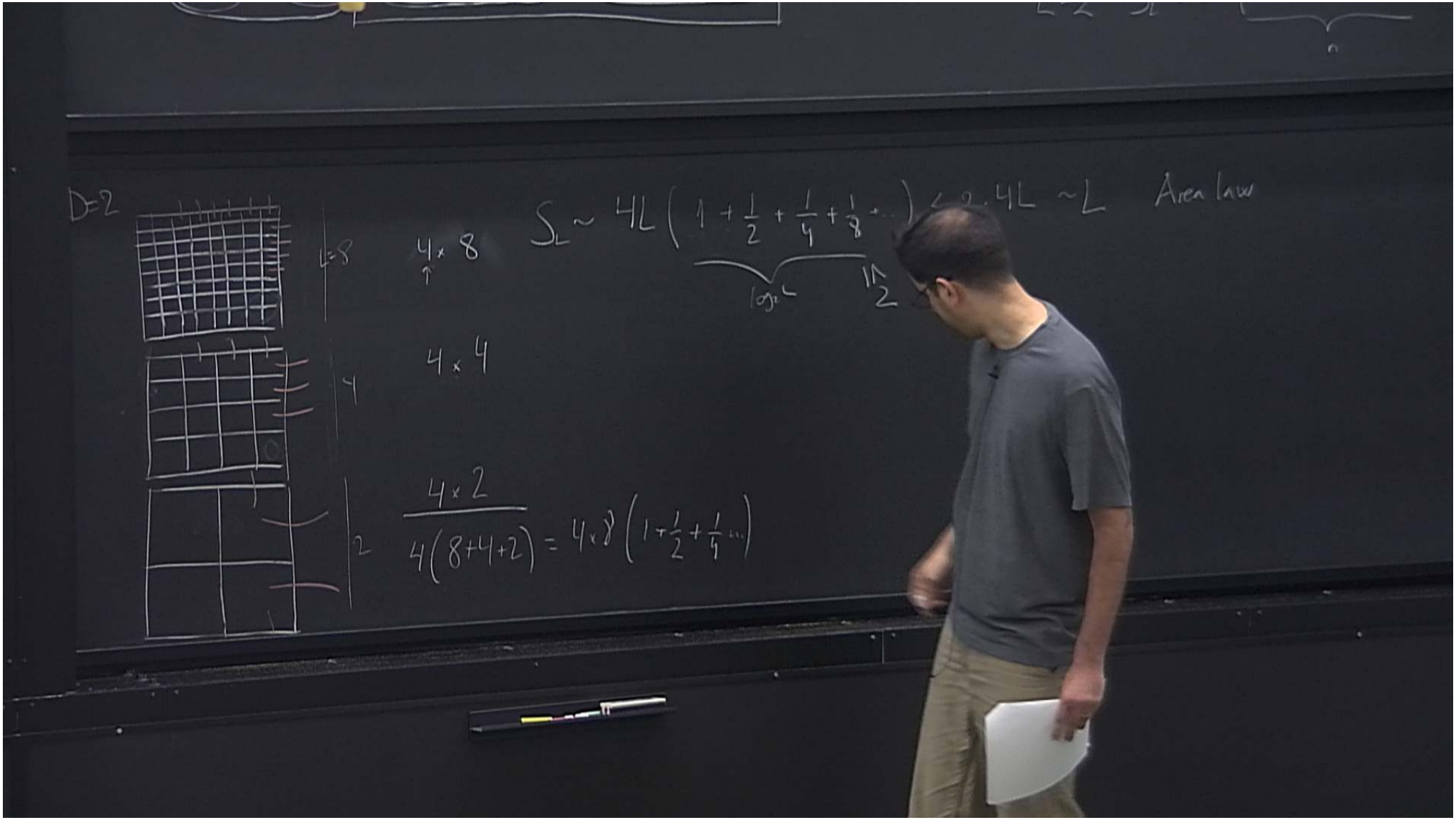

lane
site
A
S(A)=2=1+1
A'
S(A')=2=1+1
const (=2)
A'
A
S(A)=6L^2
L^(D-1)

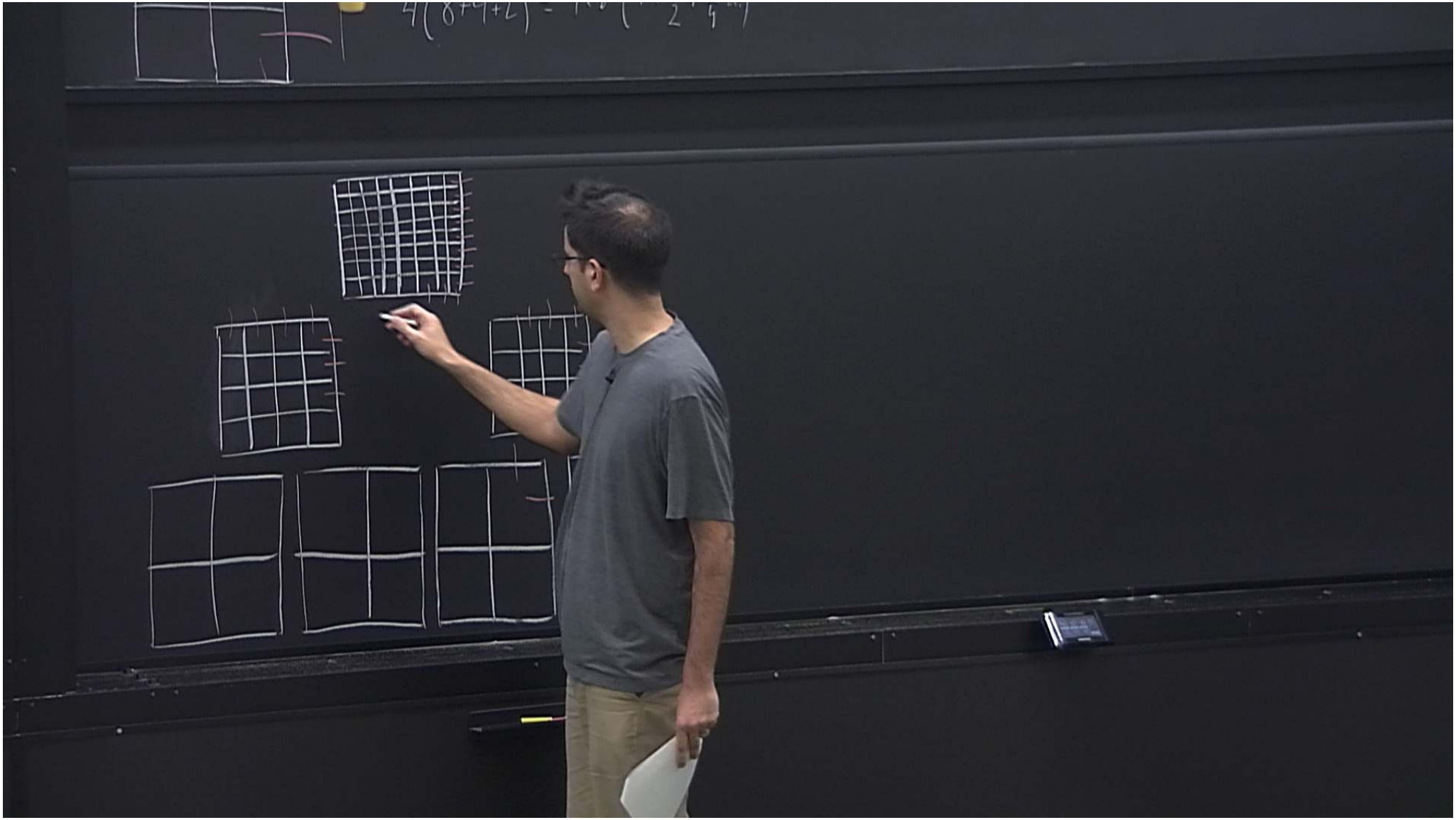

S = const (=2)
logarithmic corrections
scale invariance → all length scales contribute to EE
D=1

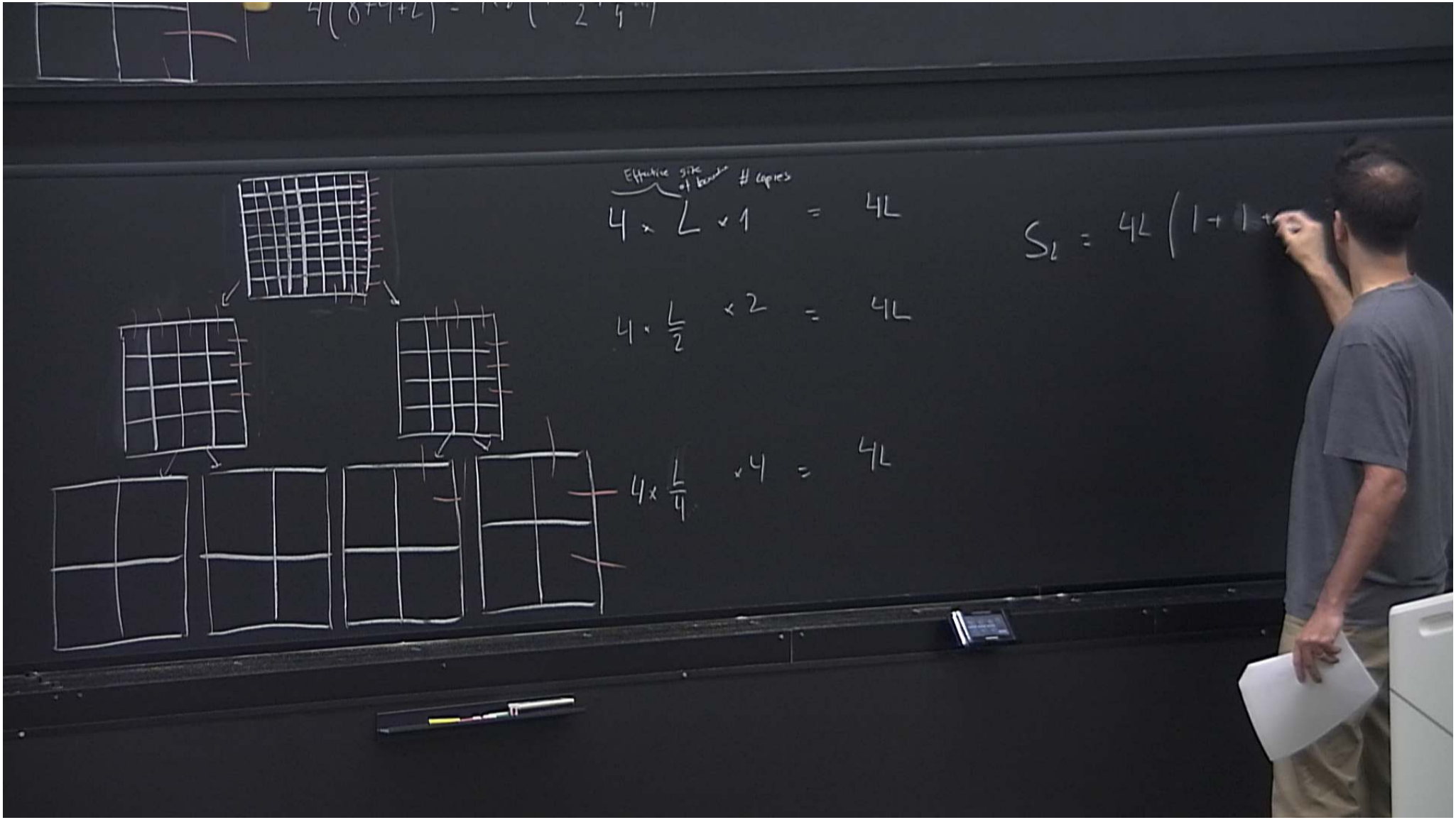

S = const (=2)
logarithmic corrections
scale invariance → all length scales contribute to EE
D=1
L=8

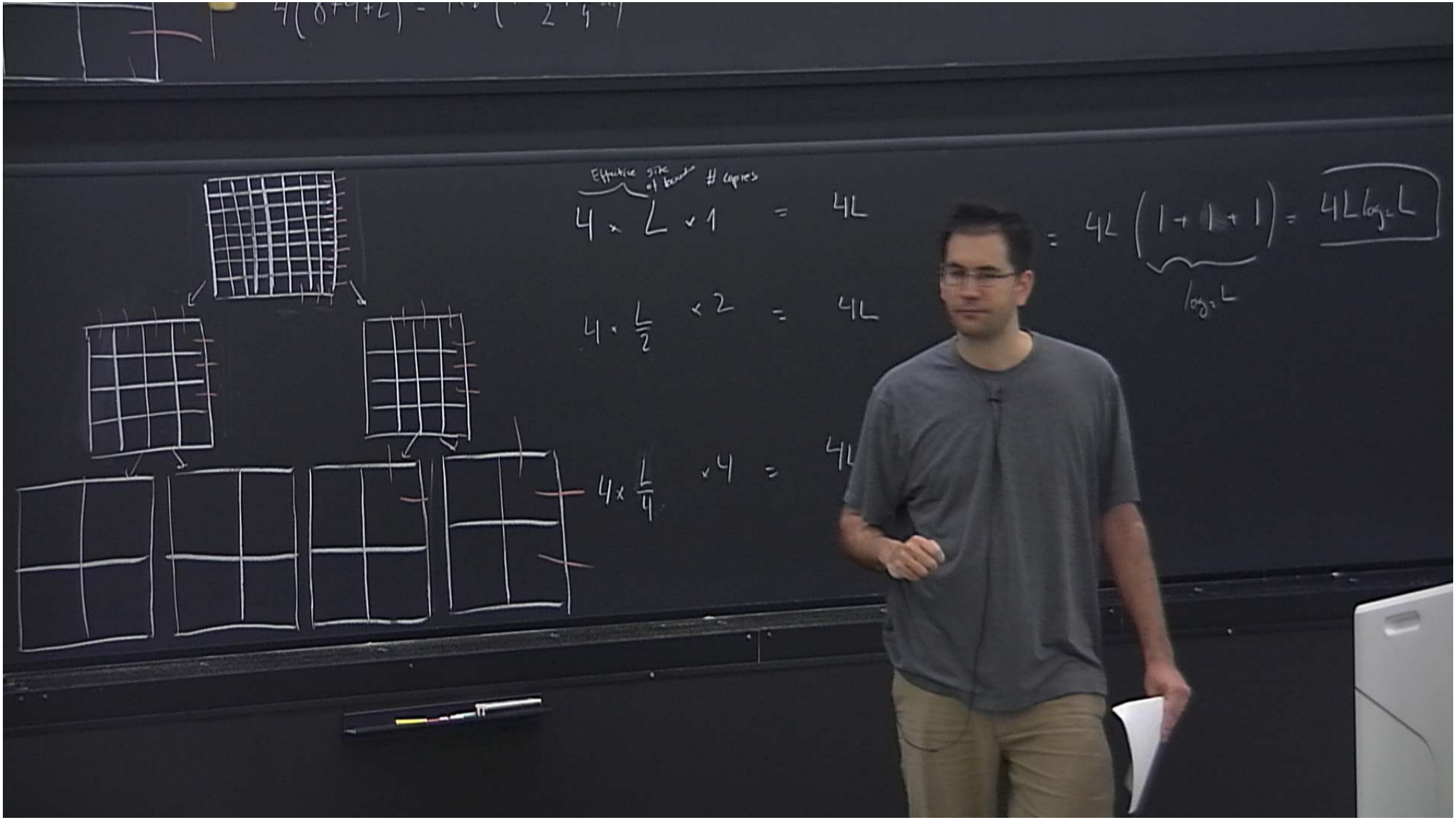

S = const (=2)
logarithmic corrections
scale invariance → all length scale contribute to EE
D=1
L=8
L=4


D=2
4=8
4×8
4
4×4
2
4×2
4(8+4+2) = 4×8(1+1/2+


D=2
L=8
4×8
4×4
4×2
4(8+4+2) = 4×8(1+1/2+1/4+...)
S_L ~ 4L(1+1/2+1/4+1/8+...) < 2·4L ~ L
Area law
log L
1/2




Effective size of boundary
copies
$4 \times L \times 1 = 4L$
$4 \times \frac{L}{2} \times 2 = 4L$
$4 \times \frac{L}{4} \times 4 = 4L$
$S_L = 4L \left(1 + 1 +$


Effective size of boundary
copies
4 × L × 1 = 4L
4 × L/2 × 2 = 4L
4 × L/4 × 4 =
= 4L (1 + 1 + 1) = 4L log₂ L
log₂ L