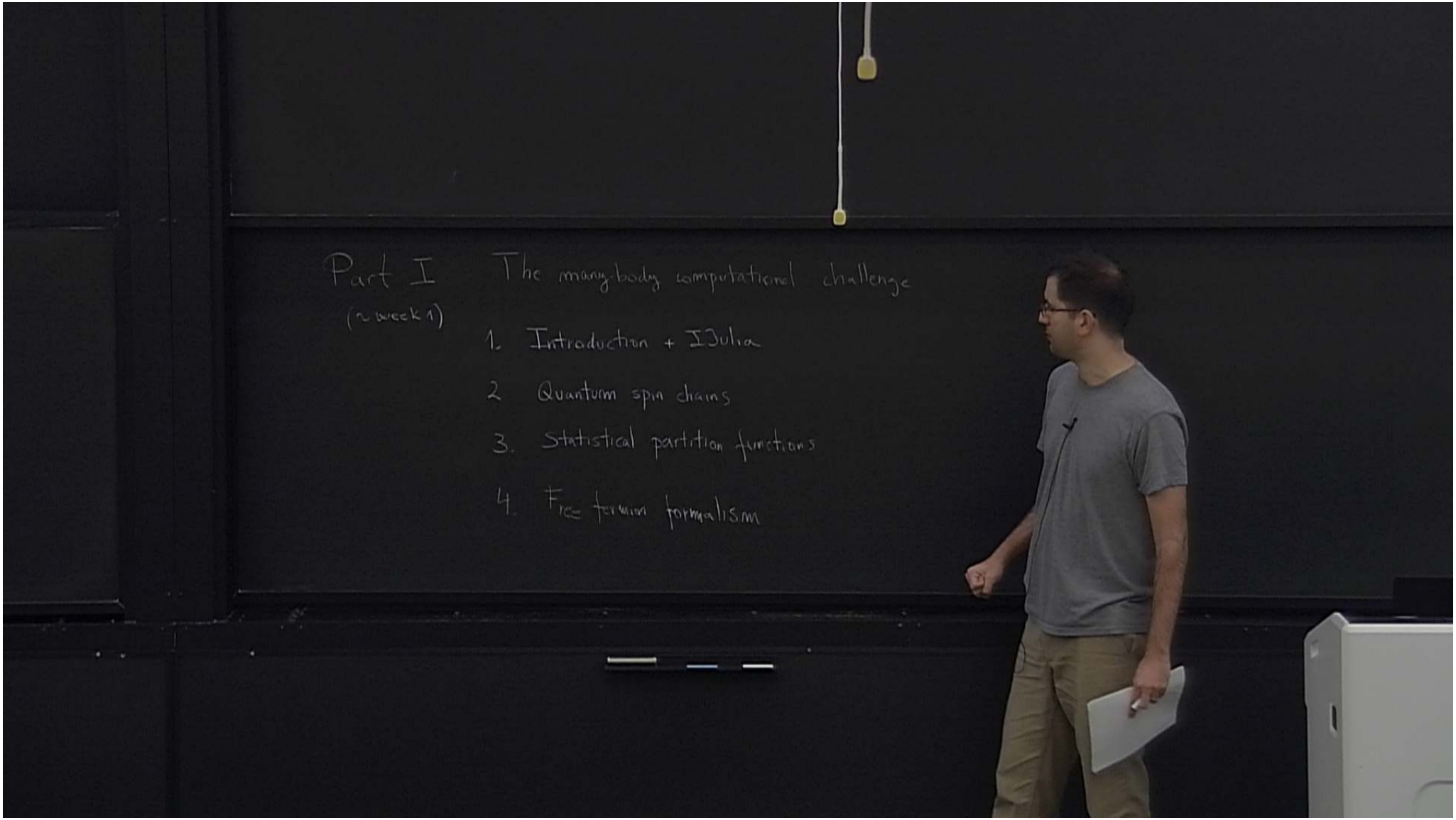
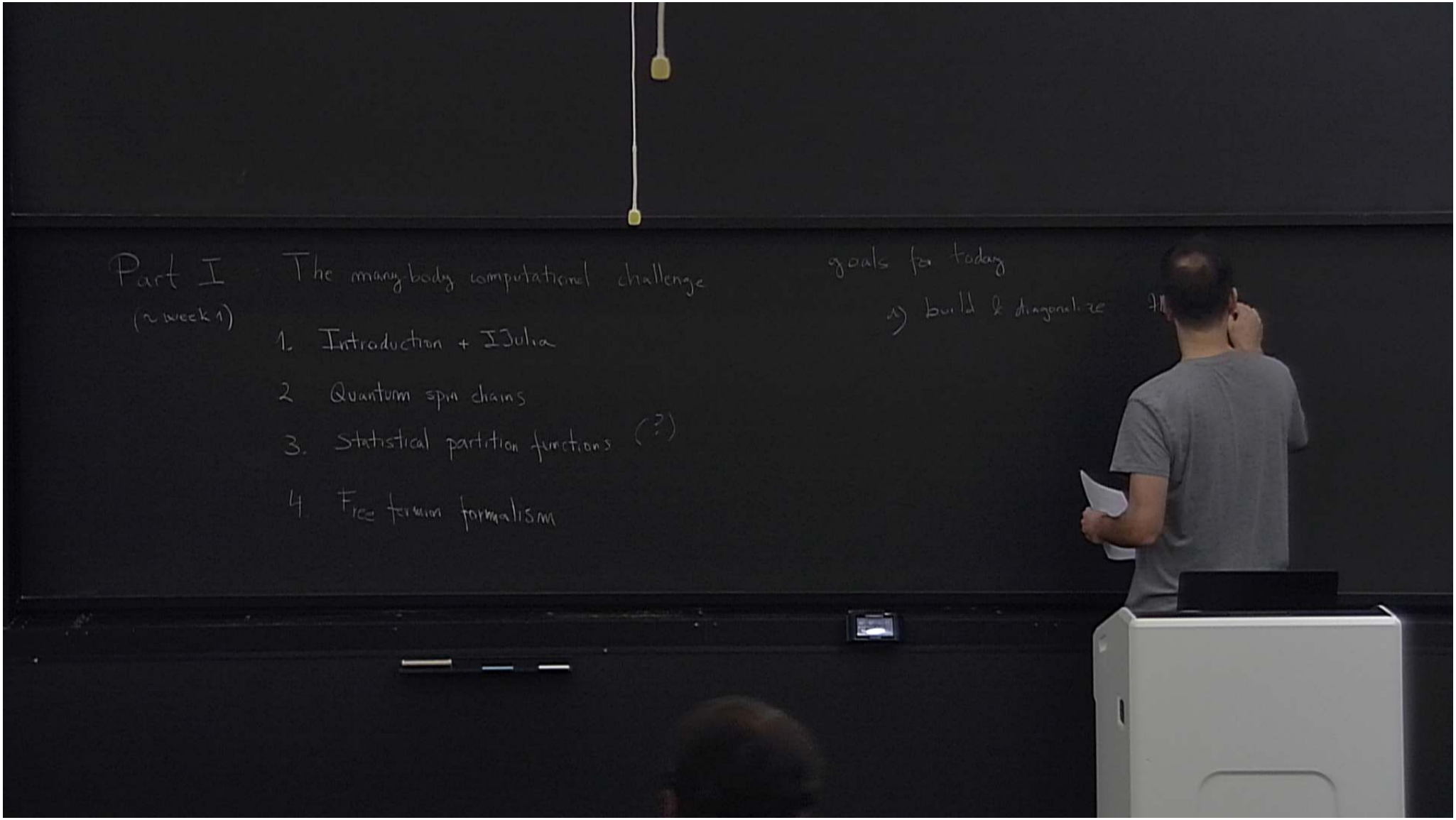
Title: PSI 2015/2016 Explorations in Condensed Matter - Guifre Vidal - 2

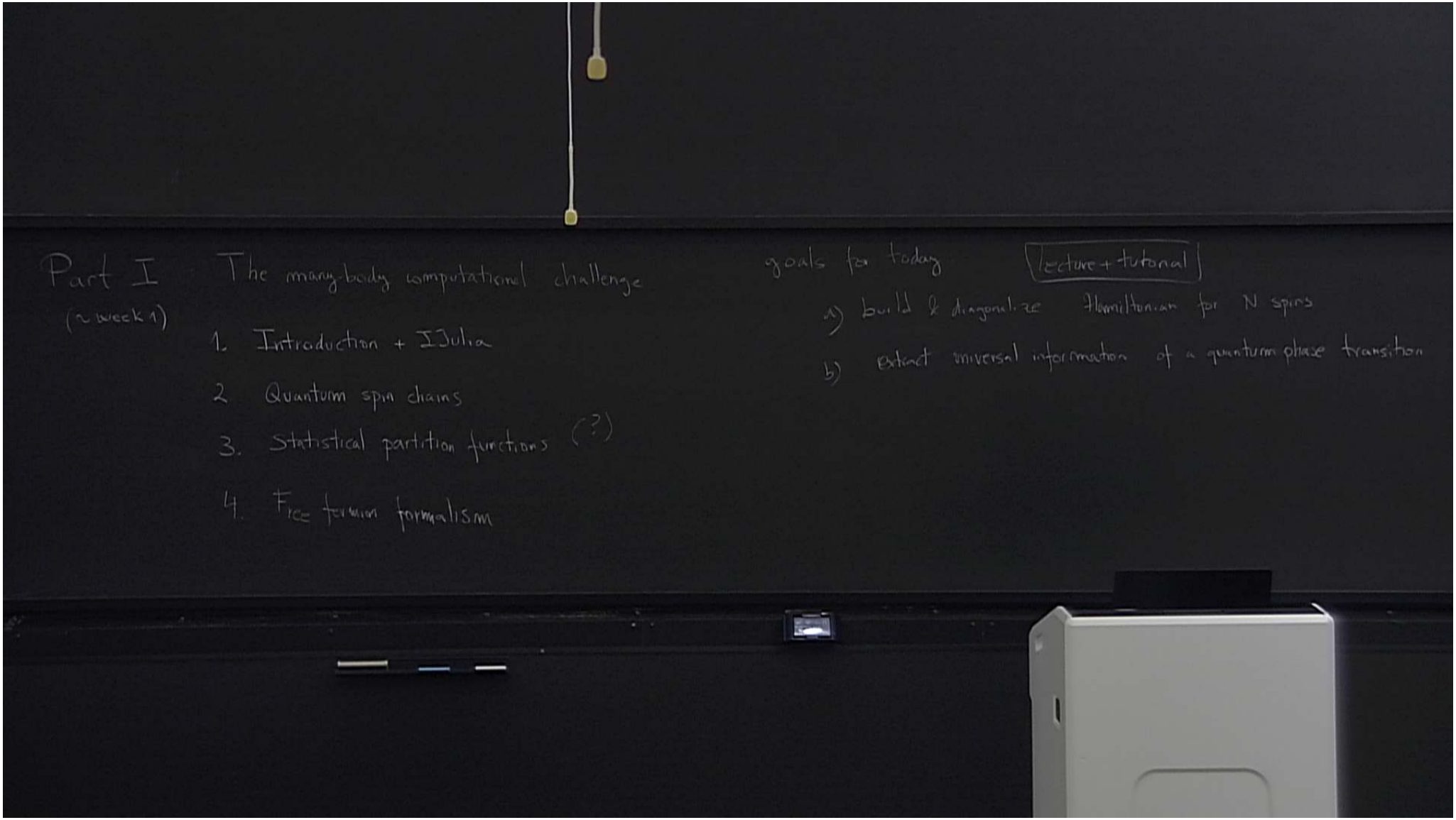
Date: Mar 22, 2016 10:15 AM

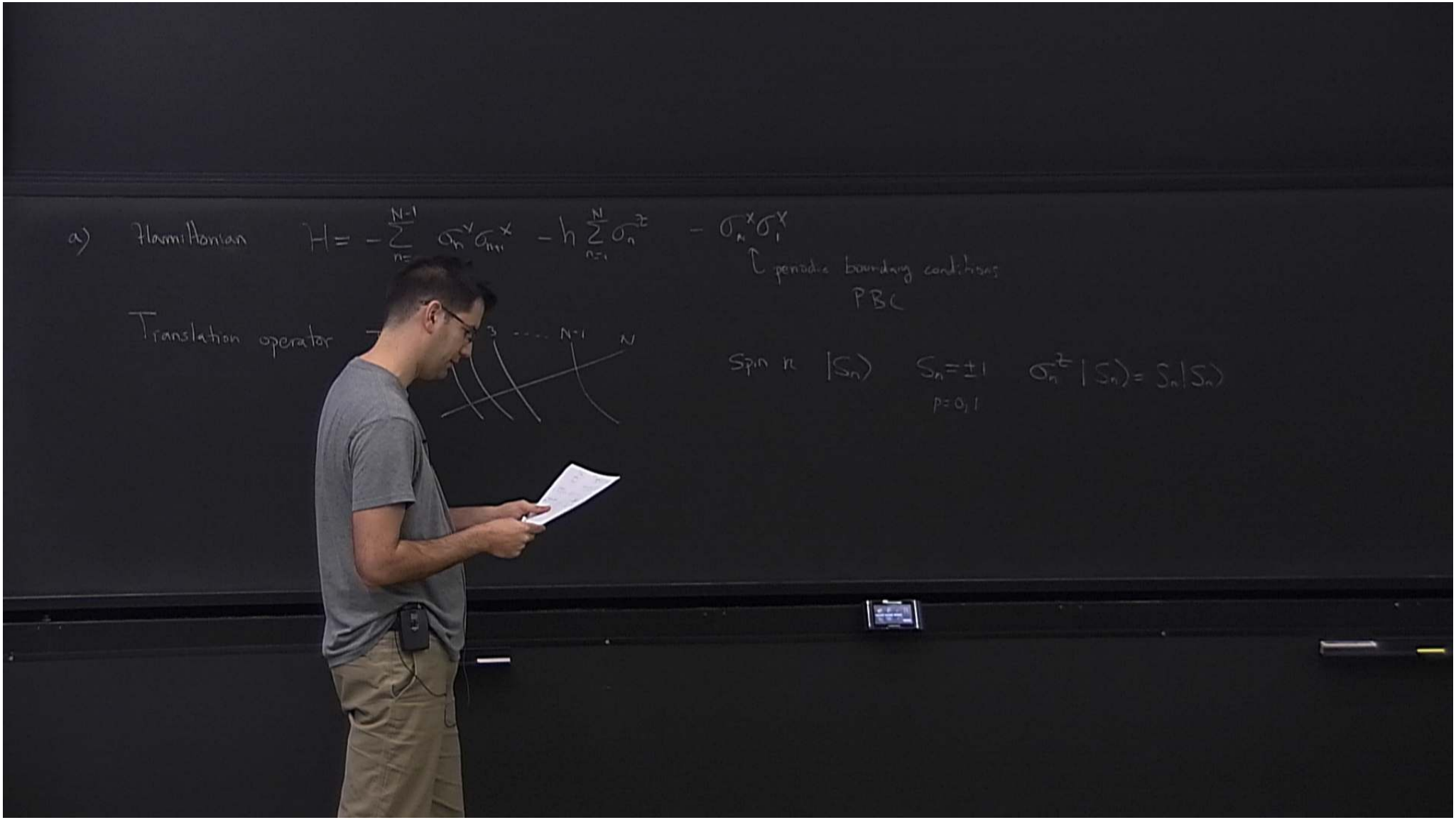
URL: http://pirsa.org/16030009

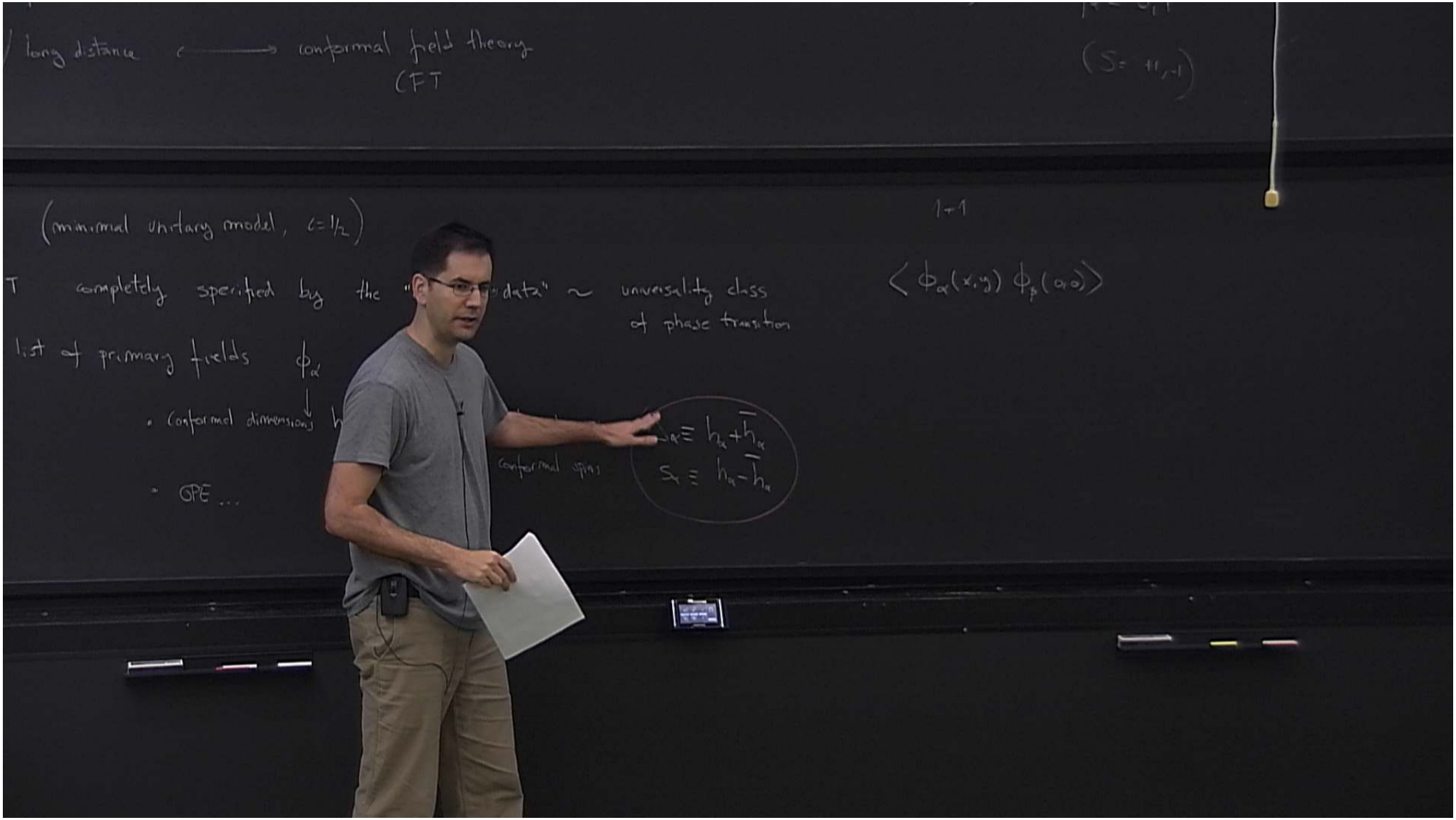
Abstract:

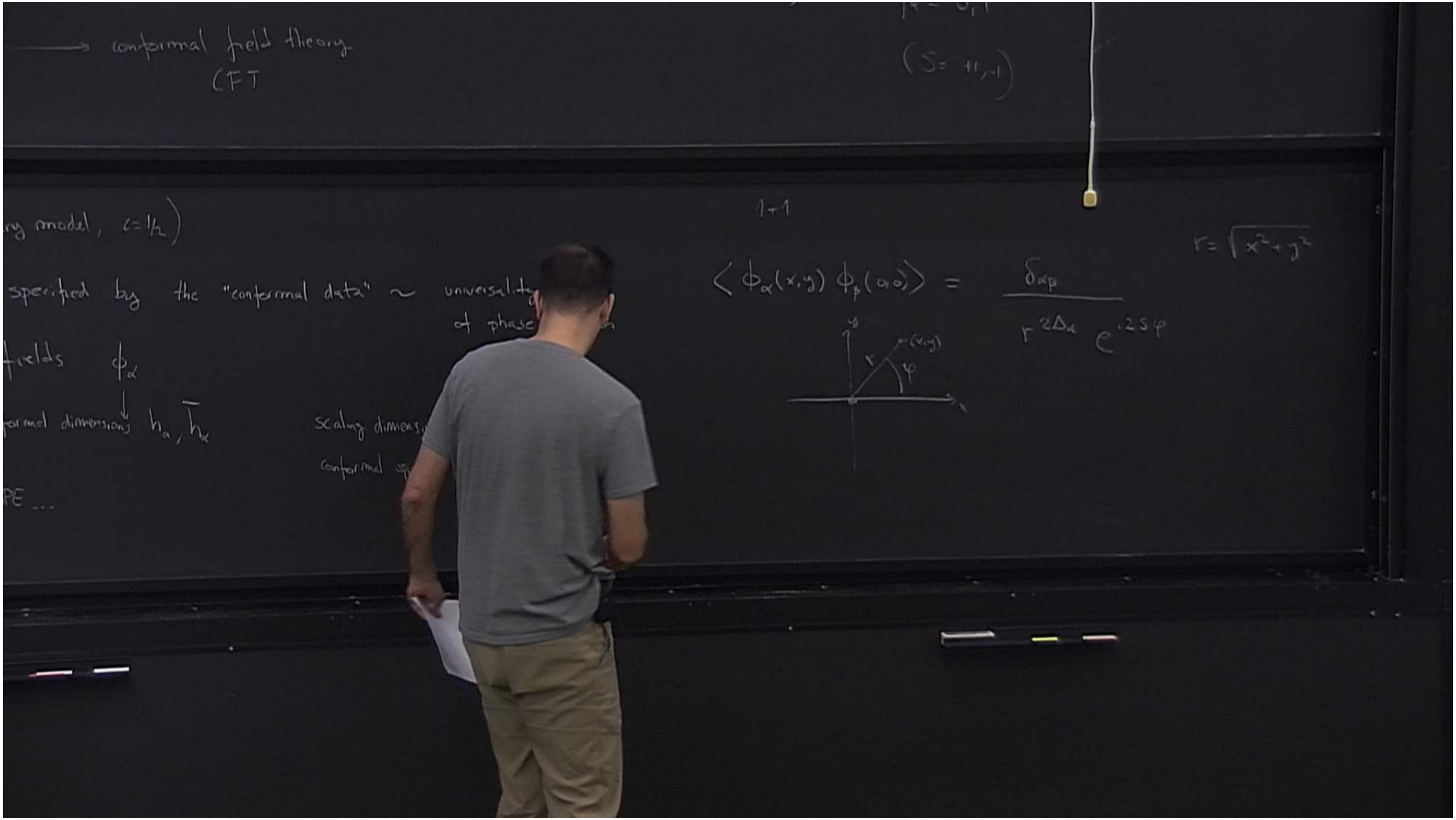

Part I
(~ week 1)
The many-body computational challenge
1. Introduction + IJulia
2. Quantum spin chains
3. Statistical partition functions
4. Free fermion formalism

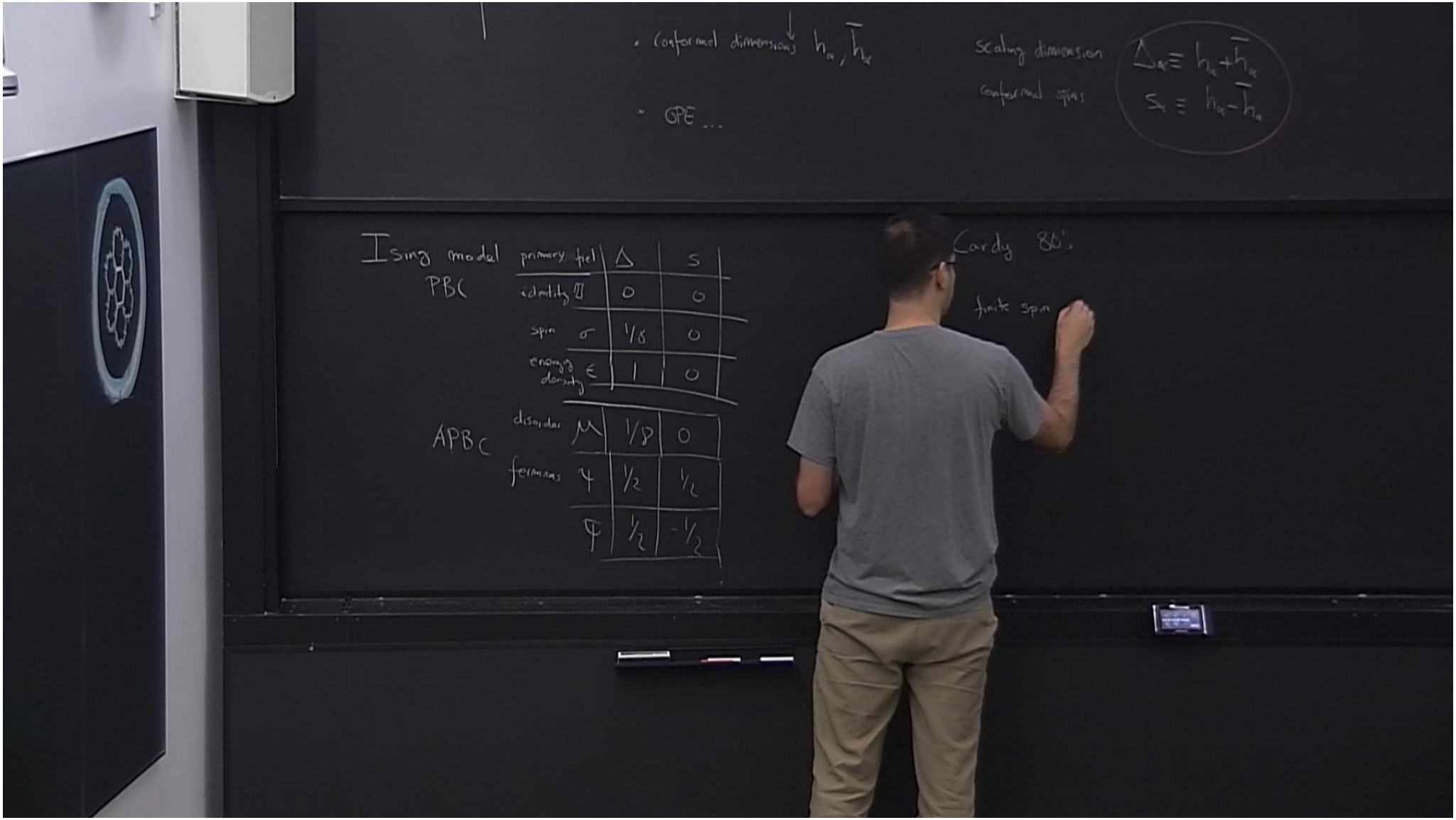

Part I
(~ week 1)
The many body computational challenge
1. Introduction + IJulia
2 Quantum spin chains
3. Statistical partition functions (?)
4 Free fermion formalism
goals for today
1) build & diagonalize

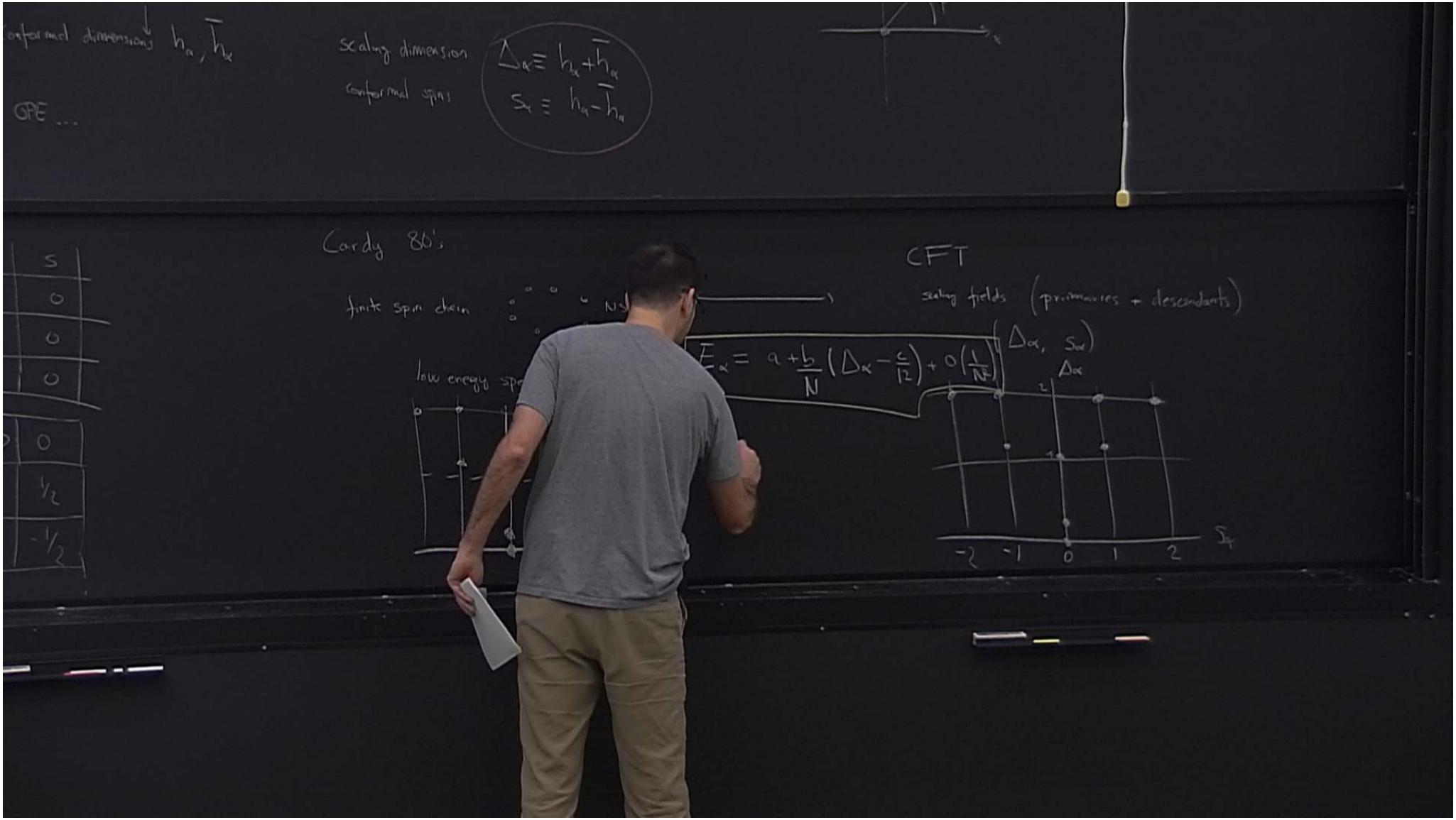

Part I
(~ week 1)
The many-body computational challenge
1. Introduction + IJulia
2 Quantum spin chains
3. Statistical partition functions (?)
4. Free fermion formalism
goals for today
lecture + tutorial
a) build & diagonalize Hamiltonian for N spins
b) Extract universal information of a quantum phase transition


a) Hamiltonian $H = -\sum_{n=1}^{N-1} \sigma_n^y \sigma_{n+1}^x - h \sum_{n=1}^{N} \sigma_n^z - \sigma_N^x \sigma_1^y$
periodic boundary conditions
PBC
Translation operator
1 3 .... N-1 N
Spin n $|S_n\rangle$ $S_n = \pm 1$ $p = 0, 1$ $\sigma_n^z |S_n\rangle = S_n |S_n\rangle$


long distance ⟷ conformal field theory
CFT
(S = +1, -1)
(minimal unitary model, c = 1/2)
1+1
T completely specified by the "data" ~ universality class of phase transition
$\langle \phi_\alpha(x,y)\, \phi_\beta(0,0) \rangle$
list of primary fields ϕ_α
• Conformal dimensions h
conformal spins
$s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
• OPE ...


conformal field theory
CFT
(S = +1, -1)
1+1
specified by the "conformal data" ~ universal
fields ϕ_α
$\langle \phi_\alpha(x,y)\, \phi_\beta(0,0) \rangle = \frac{\delta_{\alpha\beta}}{r^{2\Delta_\alpha}}\, e^{\pm 2 S \varphi}$
$r = \sqrt{x^2+y^2}$
scaling dimens
conformal


• Conformal dimensions $h_\alpha, \bar{h}_\alpha$
• OPE ...
scaling dimension $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$
conformal spins $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
Ising model
PBC
primary field | Δ | s
identity 𝕀 | 0 | 0
spin σ | 1/8 | 0
energy density ε | 1 | 0
APBC
disorder μ | 1/8 | 0
fermions ψ | 1/2 | 1/2
ψ̄ | 1/2 | -1/2
Cardy 86':
finite spin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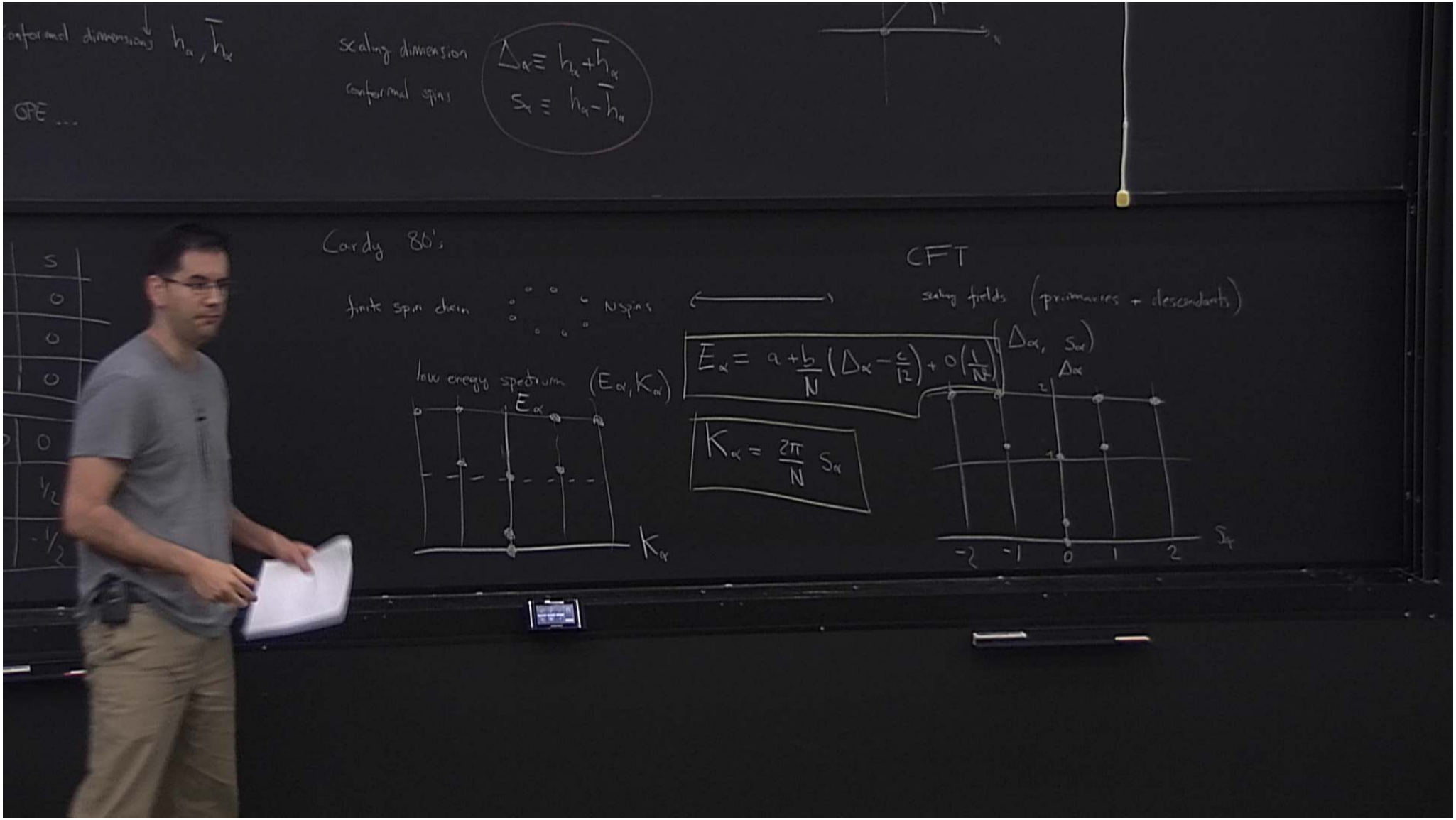
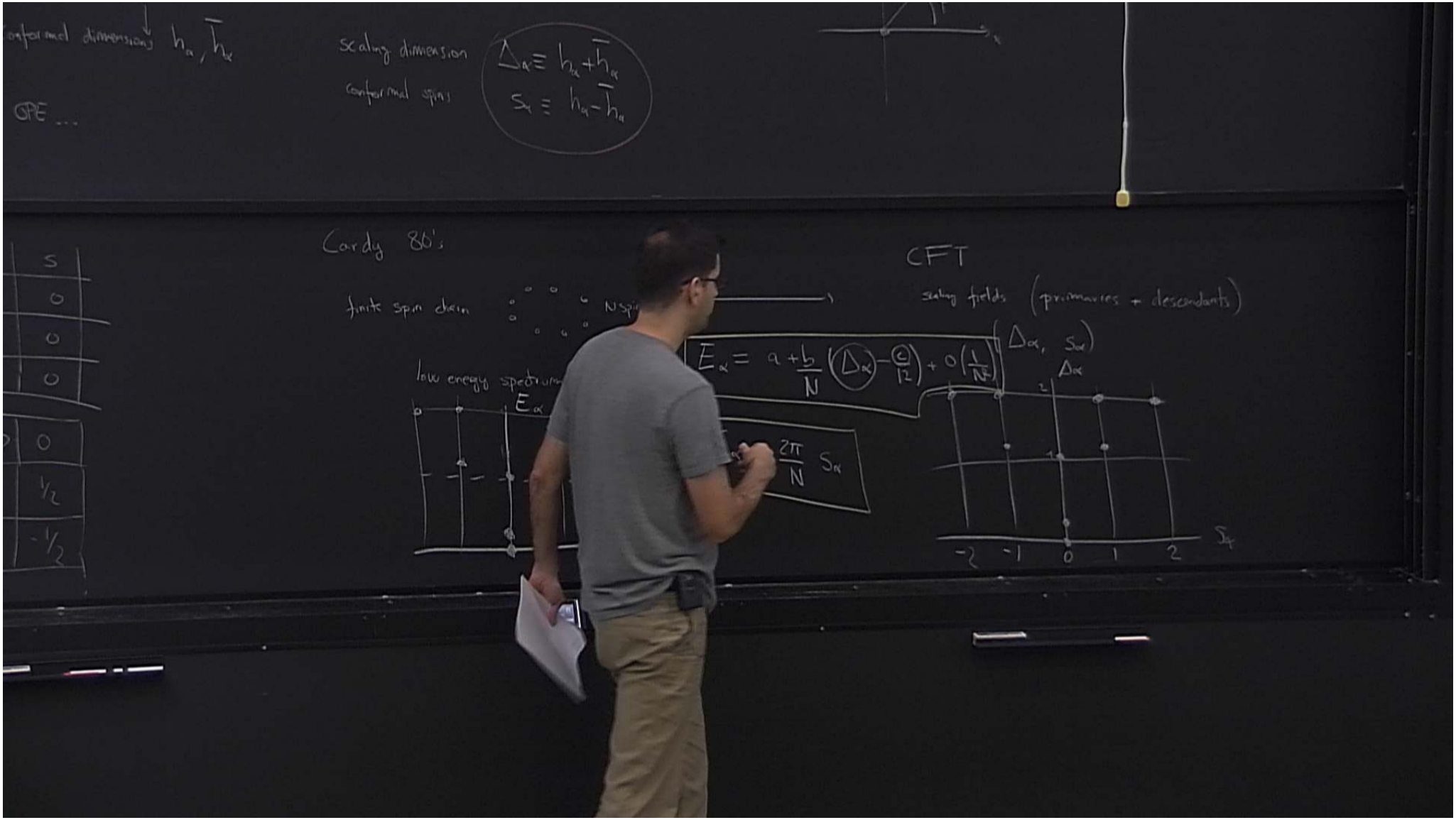
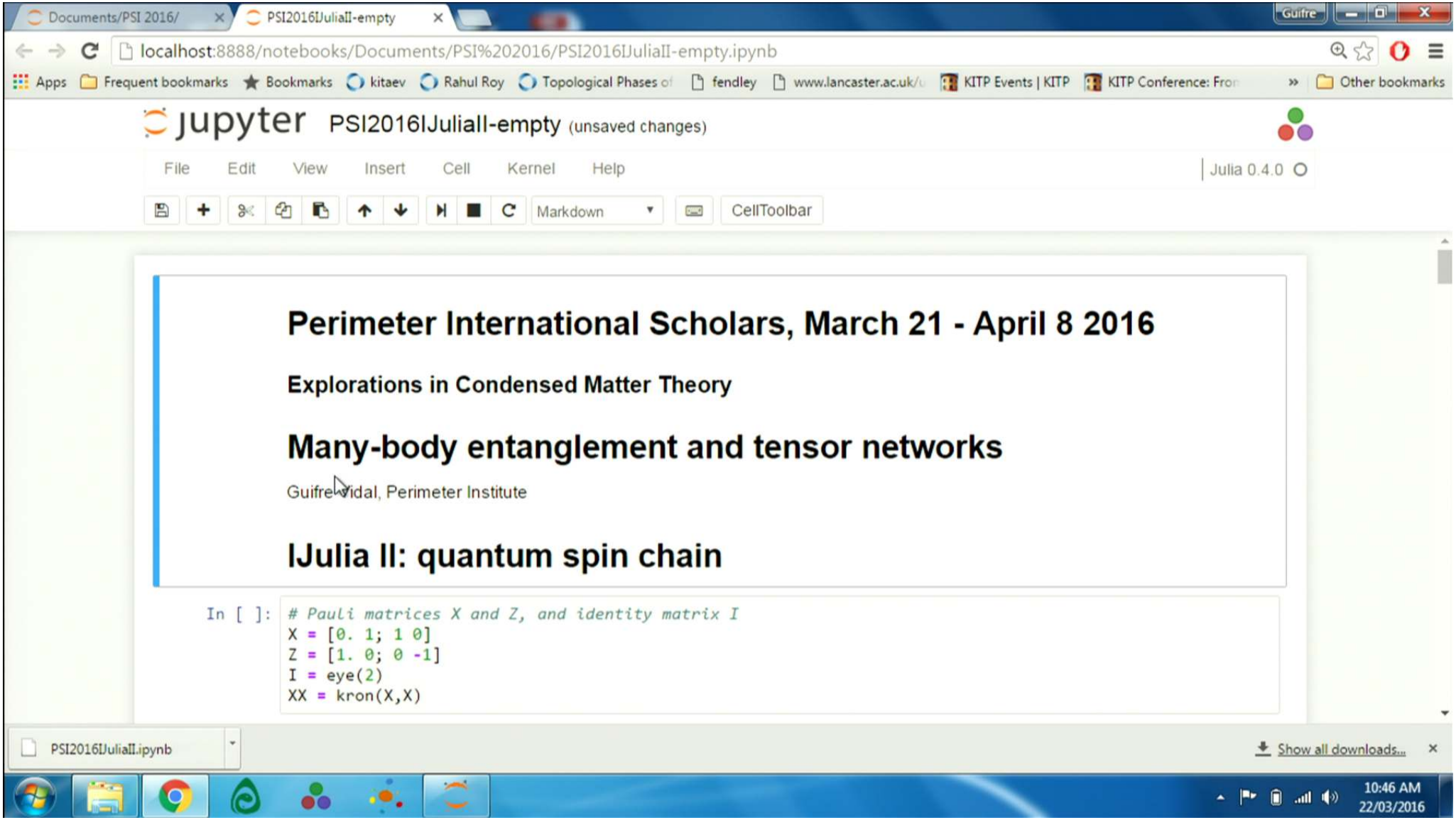
conformal spins $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
OPE ...
Cardy 80's
finite spin chain
low energy spe
$E_\alpha = a + \frac{b}{N}\left(\Delta_\alpha - \frac{c}{12}\right) + O\left(\frac{1}{N^2}\right)$
CFT
scaling fields (primaries + descendants)
$(\Delta_\alpha, s_\alpha)$
Δ_α
s_α
-2 -1 0 1 2
s
0
0
0
0
1/2
-1/2


conformal dimensions $h_\alpha, \bar{h}_\alpha$
scaling dimension $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$
conformal spins $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
OPE ...
Cardy 80's:
finite spin chain
N spins
low energy spectrum (E_α, K_α)
E_α
K_α
$E_\alpha = a + \frac{b}{N}\left(\Delta_\alpha - \frac{c}{12}\right) + O\left(\frac{1}{N^2}\right)$
$K_\alpha = \frac{2\pi}{N} s_\alpha$
CFT
scaling fields (primaries + descendants)
$(\Delta_\alpha, s_\alpha)$
Δ_α
s_α
-2 -1 0 1 2
s
0
0
0
0
1/2
-1/2


conformal dimensions $h_\alpha, \bar{h}_\alpha$
scaling dimension $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$
conformal spins $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
OPE ...
Cardy 80's
finite spin chain
low energy spectrum
$E_\alpha = a + \frac{b}{N}\left(\Delta_\alpha - \frac{c}{12}\right) + o\left(\frac{1}{N}\right)$
$\frac{2\pi}{N} s_\alpha$
CFT
scaling fields (primaries + descendants)
$(\Delta_\alpha, s_\alpha)$
Δ_α
s_α
-2 -1 0 1 2
s
0
0
0
0
1/2
-1/2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Perimeter International Scholars, March 21 - April 8 2016
Explorations in Condensed Matter Theory
Many-body entanglement and tensor networks
Guifre Vidal, Perimeter Institute
IJulia II: quantum spin chain
In [ ]:
# Pauli matrices X and Z, and identity matrix I
X = [0. 1; 1 0]
Z = [1. 0; 0 -1]
I = eye(2)
XX = kron(X,X)
PSI2016IJuliaII.ipynb
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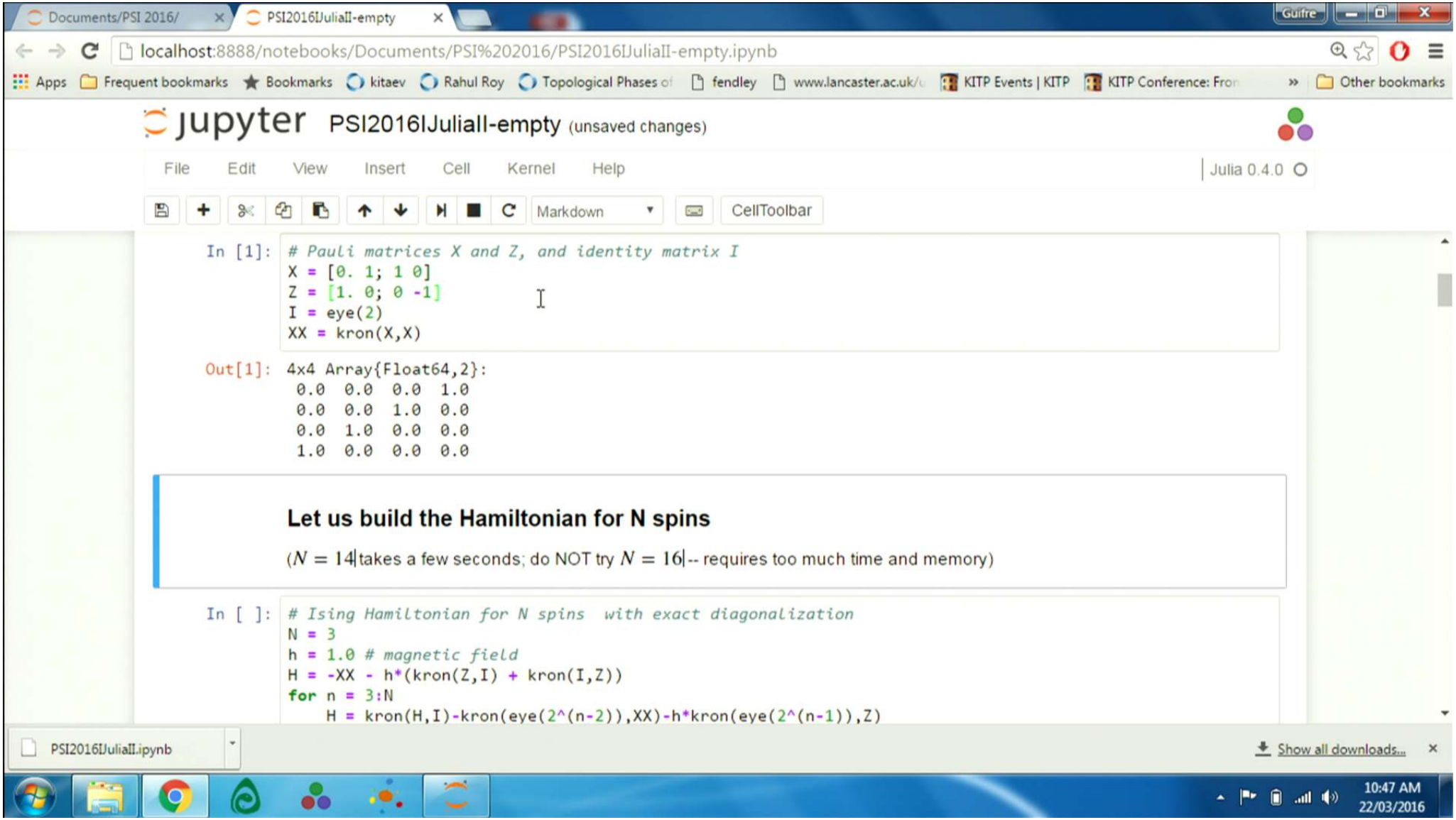
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Julia 0.4.0
Markdown
CellToolbar
In [1]:
# Pauli matrices X and Z, and identity matrix I
X = [0. 1; 1 0]
Z = [1. 0; 0 -1]
I = eye(2)
XX = kron(X,X)
Out[1]:
4x4 Array{Float64,2}:
0.0 0.0 0.0 1.0
0.0 0.0 1.0 0.0
0.0 1.0 0.0 0.0
1.0 0.0 0.0 0.0
Let us build the Hamiltonian for N spins
(N = 14 takes a few seconds; do NOT try N = 16 -- requires too much time and memory)
In [ ]:
# Ising Hamiltonian for N spins with exact diagonalization
N = 3
h = 1.0 # magnetic field
H = -XX - h*(kron(Z,I) + kron(I,Z))
for n = 3:N
H = kron(H,I)-kron(eye(2^(n-2)),XX)-h*kron(eye(2^(n-1)),Z)
PSI2016IJuliaII.ipynb
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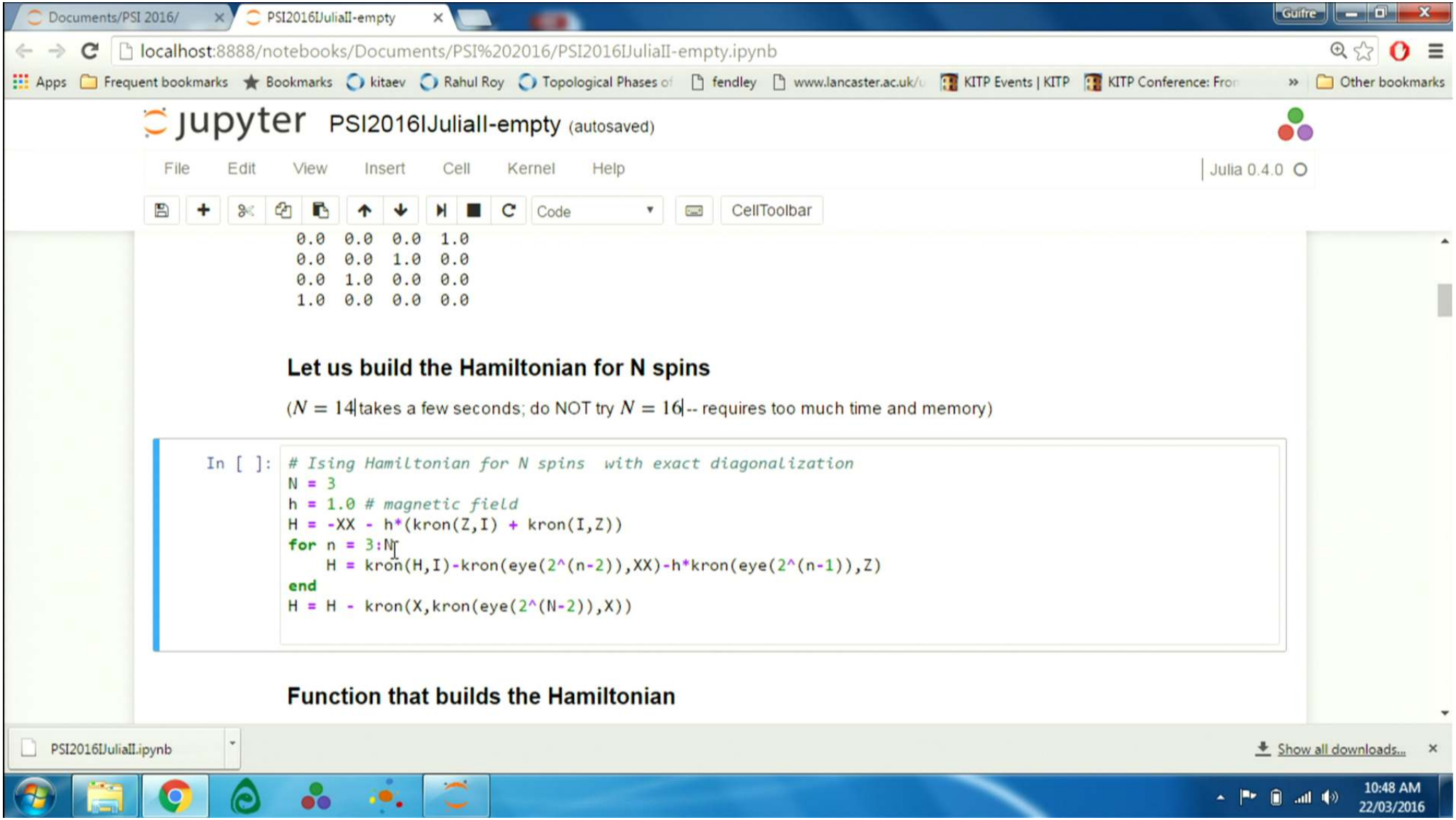
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jupyter PSI2016IJuliaII-empty (autosaved)
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Julia 0.4.0
Code
CellToolbar
0.0 0.0 0.0 1.0
0.0 0.0 1.0 0.0
0.0 1.0 0.0 0.0
1.0 0.0 0.0 0.0
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N = 3
h = 1.0 # magnetic field
H = -XX - h*(kron(Z,I) + kron(I,Z))
for n = 3:N
H = kron(H,I)-kron(eye(2^(n-2)),XX)-h*kron(eye(2^(n-1)),Z)
end
H = H - kron(X,kron(eye(2^(N-2)),X))
Function that builds the Hamiltonian
PSI2016IJuliaII.ipynb
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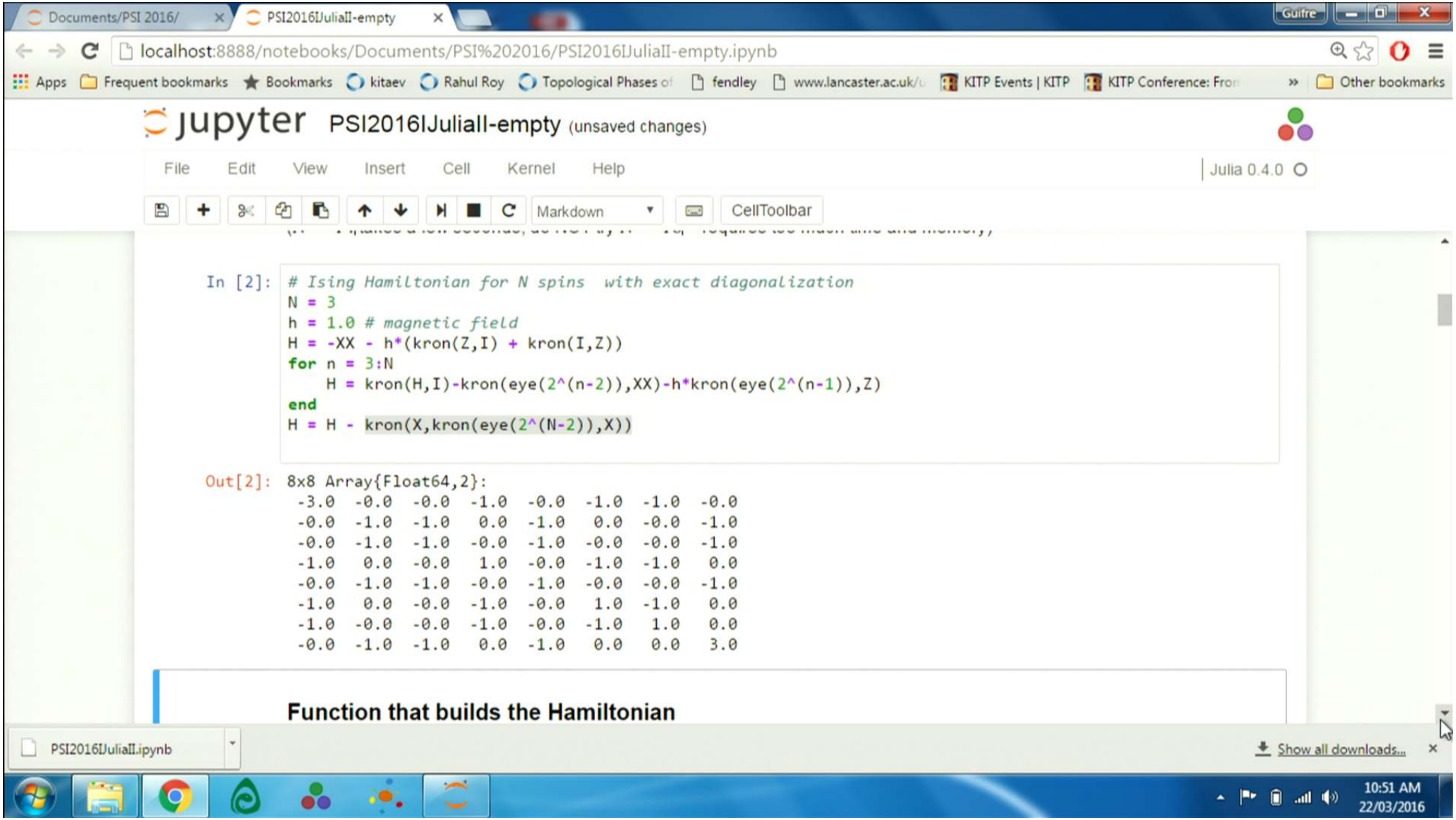
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Julia 0.4.0
Markdown
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In [2]:
# Ising Hamiltonian for N spins with exact diagonalization
N = 3
h = 1.0 # magnetic field
H = -XX - h*(kron(Z,I) + kron(I,Z))
for n = 3:N
H = kron(H,I)-kron(eye(2^(n-2)),XX)-h*kron(eye(2^(n-1)),Z)
end
H = H - kron(X,kron(eye(2^(N-2)),X))
Out[2]:
8x8 Array{Float64,2}:
-3.0 -0.0 -0.0 -1.0 -0.0 -1.0 -1.0 -0.0
-0.0 -1.0 -1.0 0.0 -1.0 0.0 -0.0 -1.0
-0.0 -1.0 -1.0 -0.0 -1.0 -0.0 -0.0 -1.0
-1.0 0.0 -0.0 1.0 -0.0 -1.0 -1.0 0.0
-0.0 -1.0 -1.0 -0.0 -1.0 -0.0 -0.0 -1.0
-1.0 0.0 -0.0 -1.0 -0.0 1.0 -1.0 0.0
-1.0 -0.0 -0.0 -1.0 -0.0 -1.0 1.0 0.0
-0.0 -1.0 -1.0 0.0 -1.0 0.0 0.0 3.0
Function that builds the Hamiltonian
PSI2016IJuliaII.ipynb
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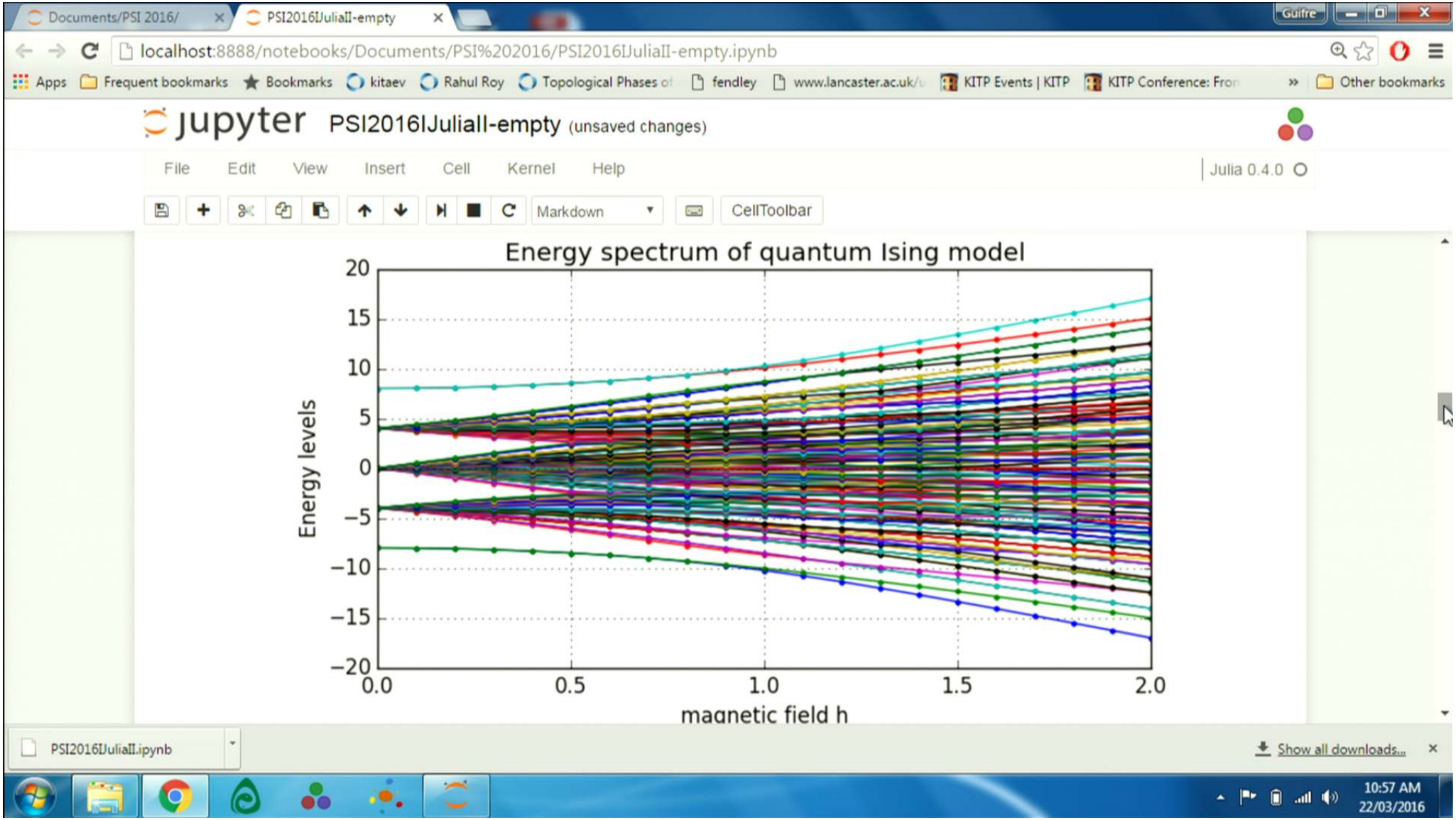
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Julia 0.4.0
Markdown
CellToolbar
Energy spectrum of quantum Ising model
Energy levels
magnetic field h
20
15
10
5
0
−5
−10
−15
−20
0.0
0.5
1.0
1.5
2.0
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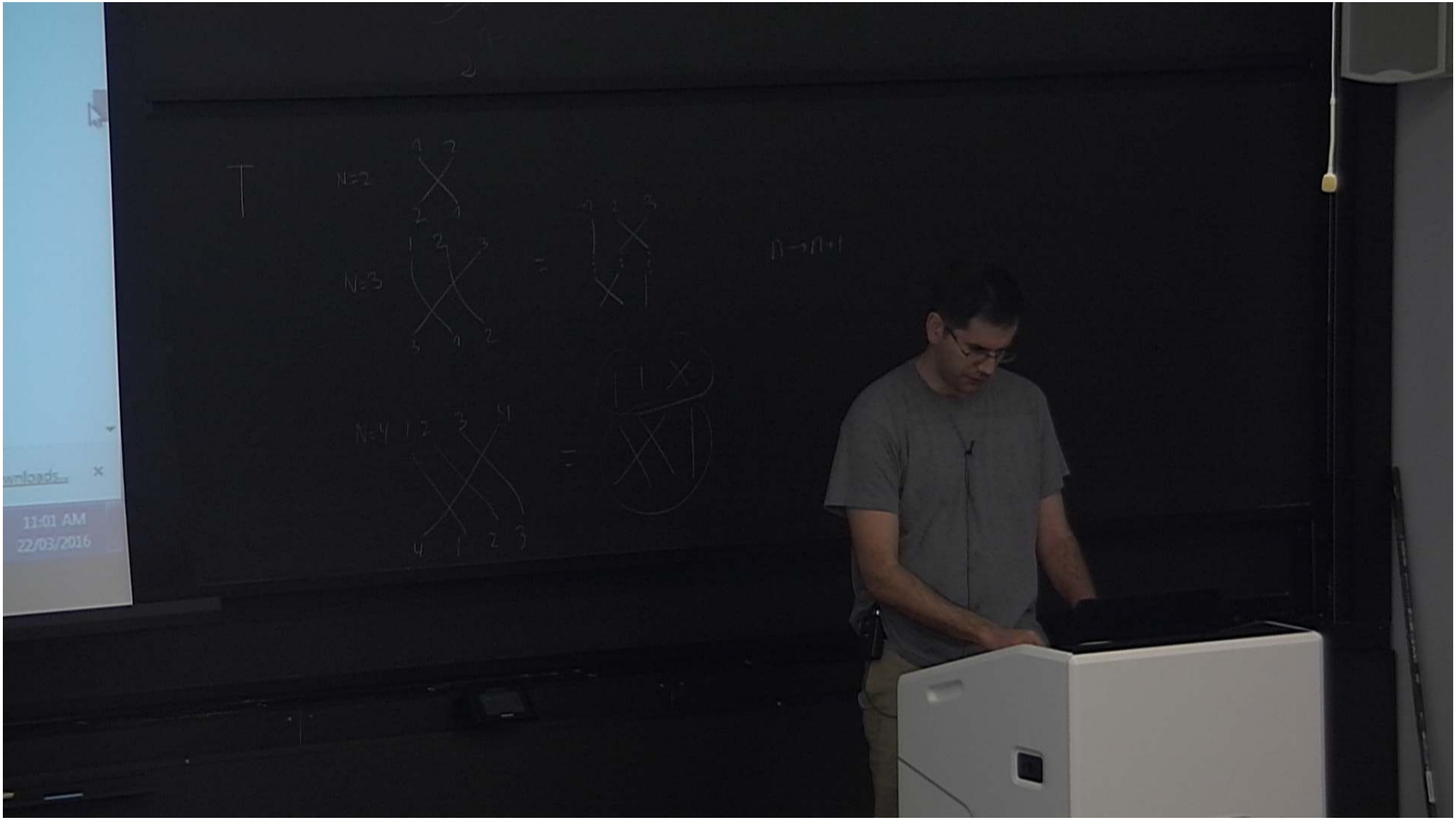
T
N=2
N=3
N=4
n→n+1
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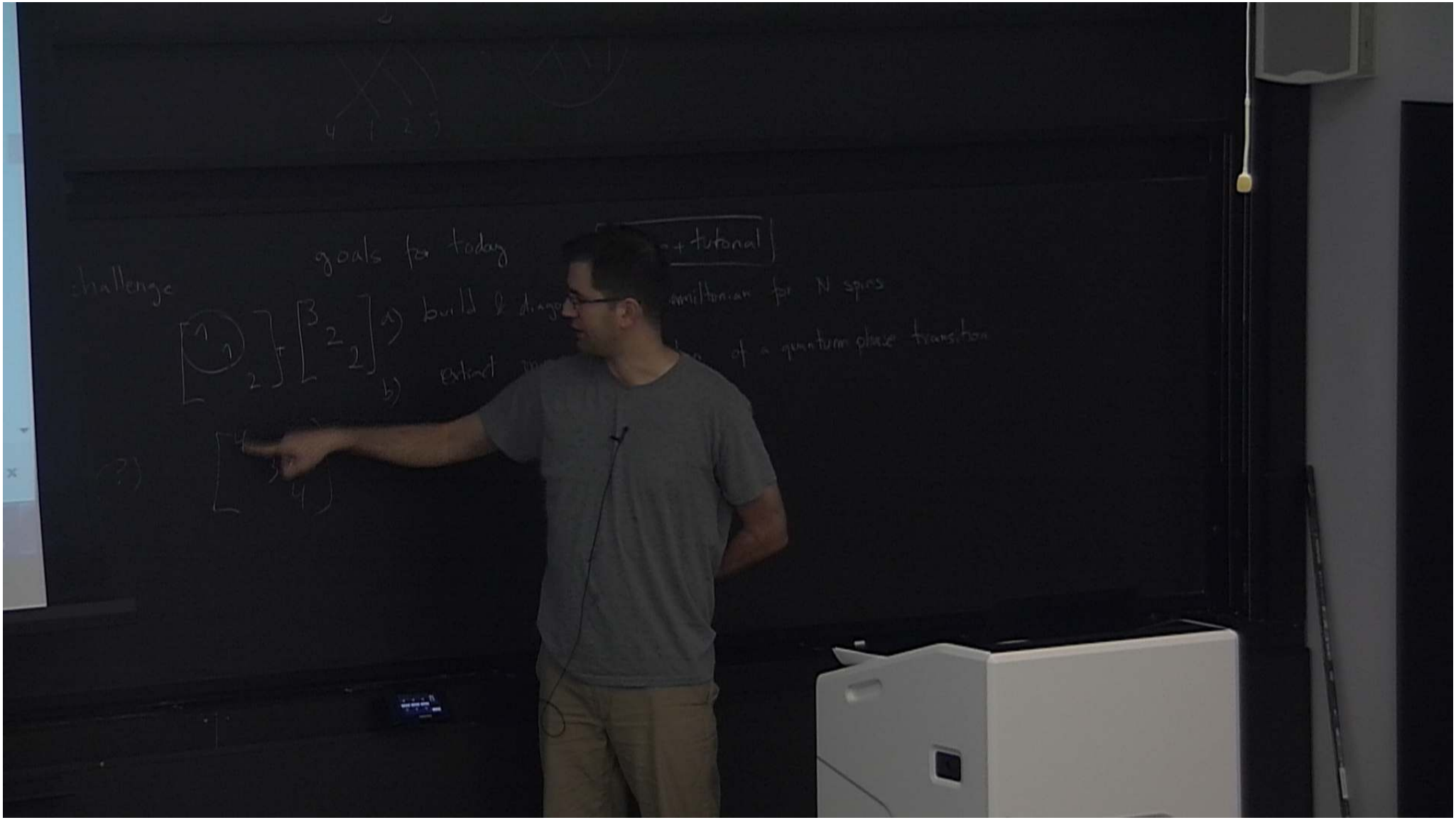
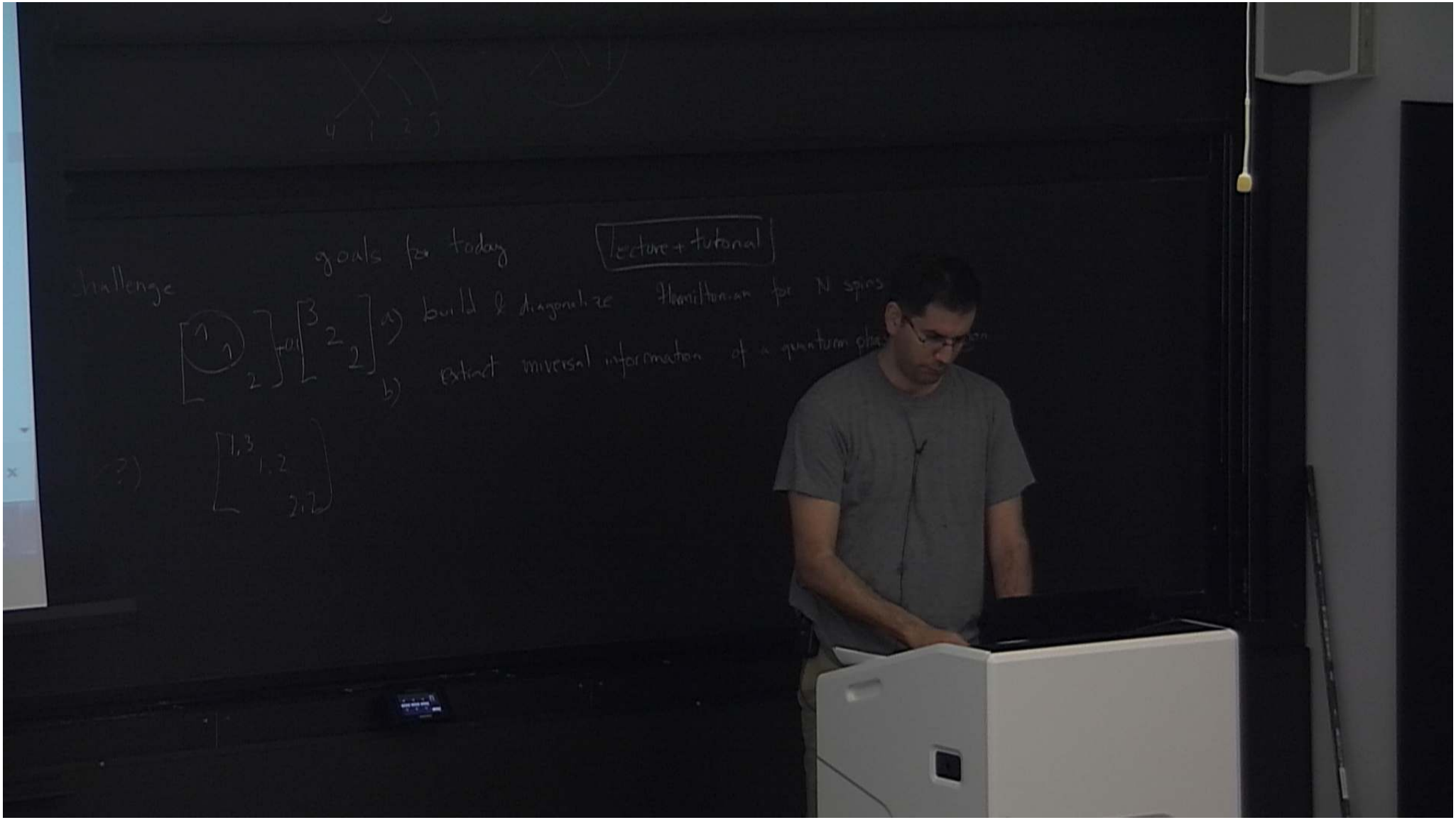
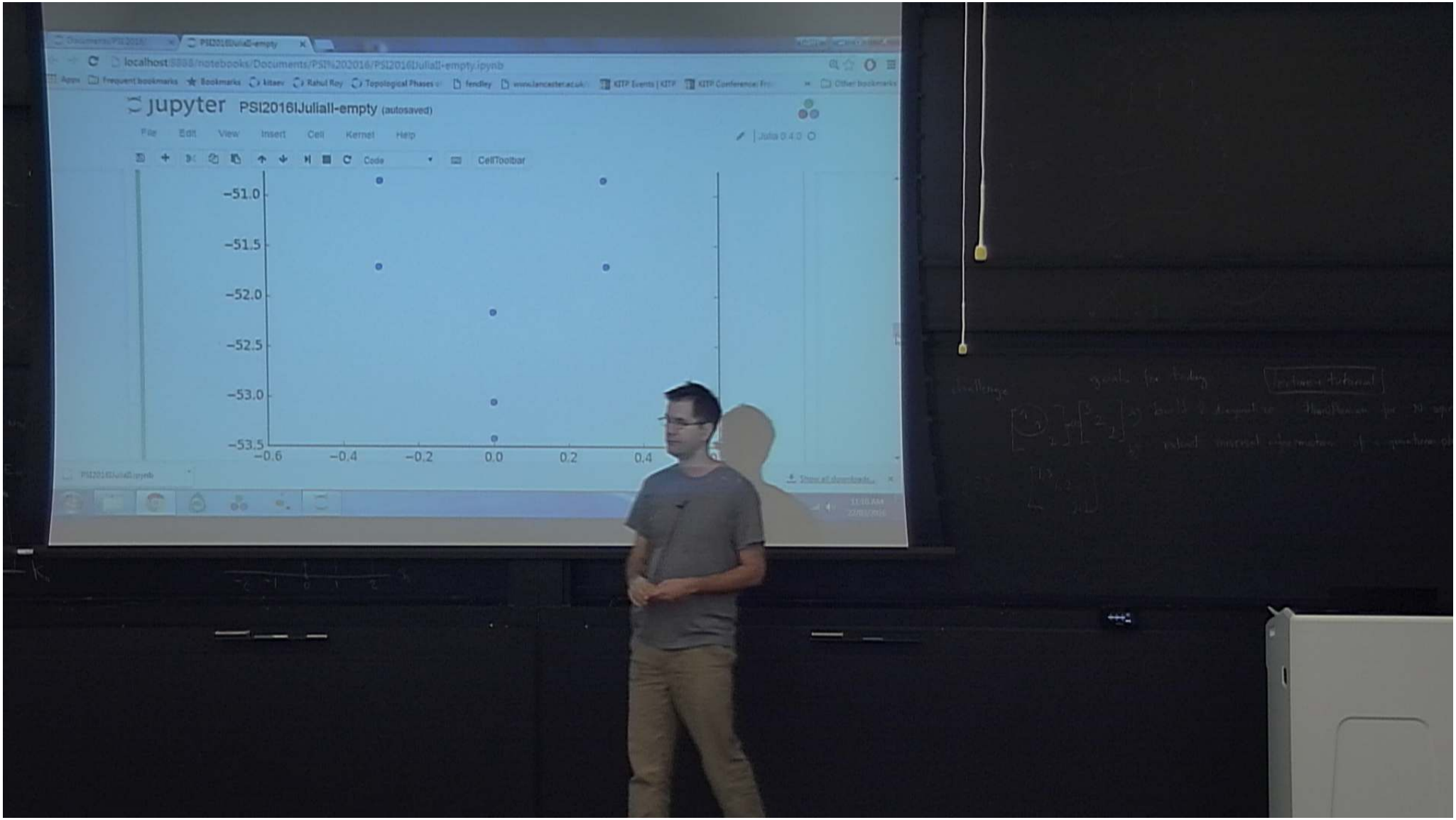

goals for today
challenge
tutorial
build & diago
Hamiltonian for N spins
of a quantum phase transition


goals for today
lecture + tutorial
challenge
a) build & diagonalize Hamiltonian for N spins
b) extract universal information of a quantum phase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−51.5
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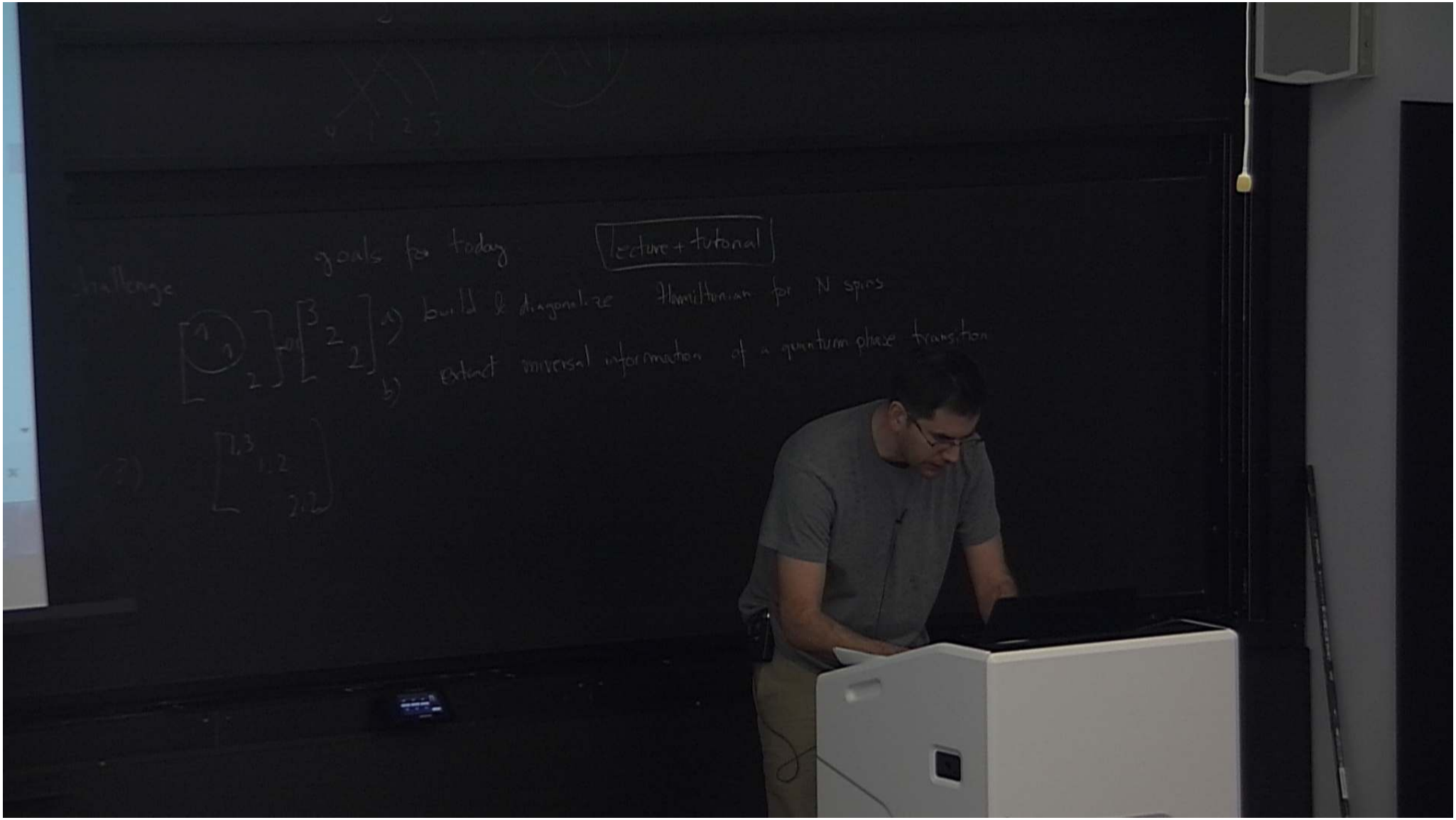
−53.5
−0.6 −0.4 −0.2 0.0 0.2 0.4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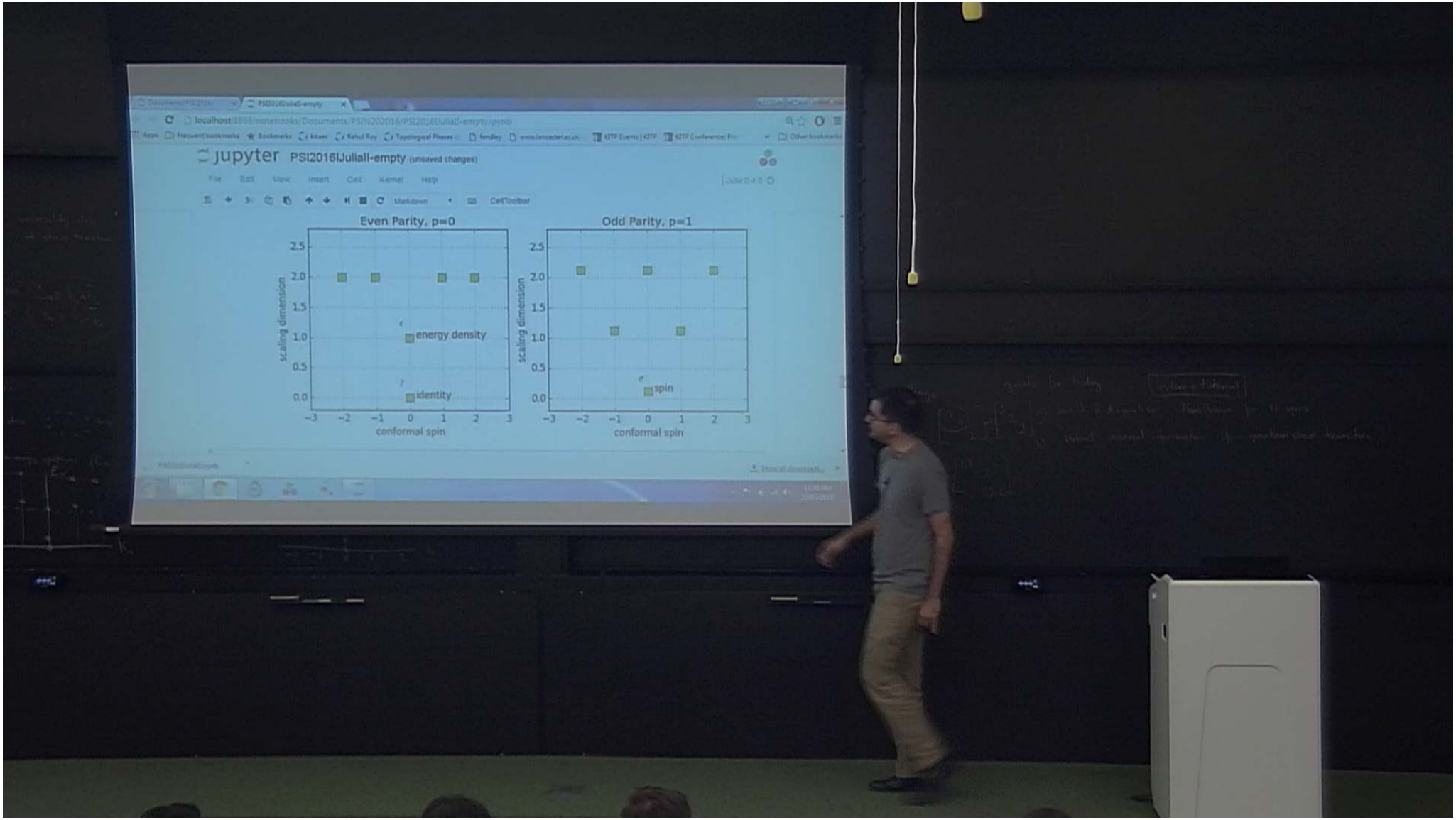
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Even Parity, p=0
Odd Parity, p=1
scaling dimension
conformal spin
energy density
identity
spin