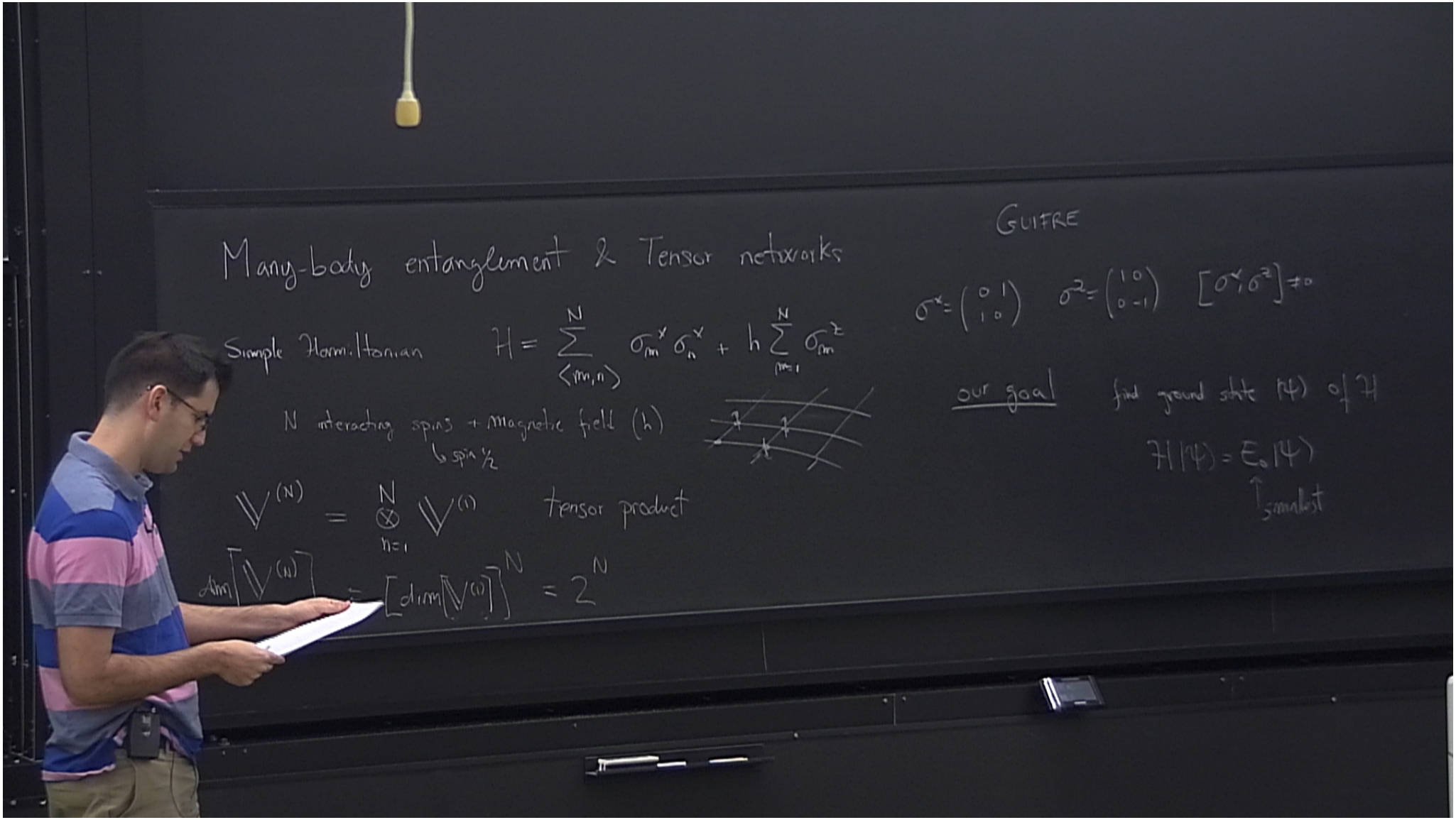
Title: PSI 2015/2016 Explorations in Condensed Matter - Guifre Vidal - 1

Date: Mar 21, 2016 10:15 AM

URL: http://pirsa.org/16030008

Abstract:


Many-body entanglement & Tensor networks
GUIFRE
Simple Hamiltonian $H = \sum_{\langle m,n \rangle} \sigma_m^x \sigma_n^x + h \sum_{m=1}^{N} \sigma_m^z$
$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $[\sigma^x, \sigma^z] \neq 0$
N interacting spins + magnetic field (h)
↳ spin ½
our goal find ground state $|\psi\rangle$ of H
$H|\psi\rangle = E_0|\psi\rangle$
↑ smallest
$\mathbb{V}^{(N)} = \bigotimes_{n=1}^{N} \mathbb{V}^{(1)}$ tensor product
$\dim[\mathbb{V}^{(N)}] = [\dim[\mathbb{V}^{(1)}]]^N = 2^N$

Many-body entanglement & Tensor networks

GUIFRE

Simple Hamiltonian $H = \sum_{\langle m,n \rangle}^{N} \sigma_m^x \sigma_n^x + h \sum_{m=1}^{N} \sigma_m^z$

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$\dim\left[\mathbb{V}^{(N)}\right] = \left[\dim\left[\mathbb{V}^{(1)}\right]\right]^N = 2^N$

Observations:

1) $\mathcal{H}$ is a crude approximation { - degrees of freedom; - interaction }

Possible strategies

2) Generically, we do NOT know how to solve $\mathcal{H}$ (e.g. find $|\Psi\rangle$)

a) Exact diagonalization

cost ~ exp N → $N \lesssim 40$ spins



b) Perturbation the

exact + pertu

b) Perturbation theory

exact solution + perturbation → only perturbative

(e.g. free theory)

~~strong interactions~~

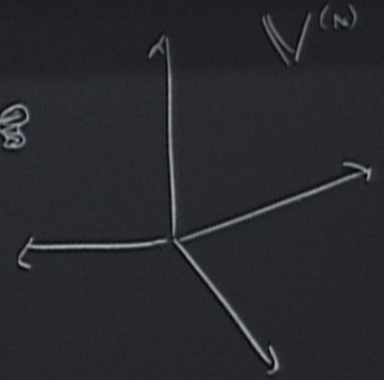
c) Monte Carlo sampling → "sign problem" for fermions; frustrated magnets, etc.

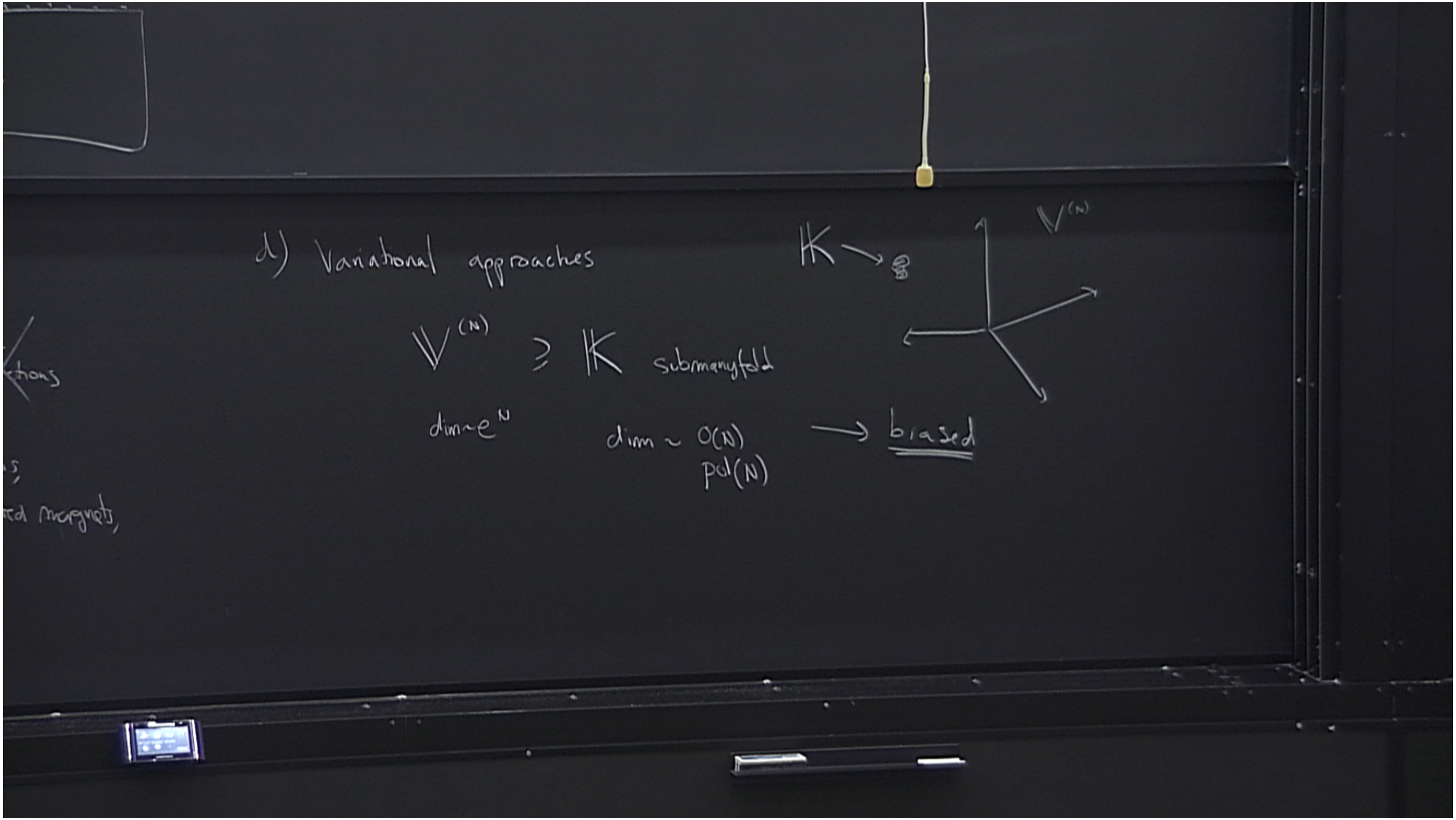
d) Variational approaches

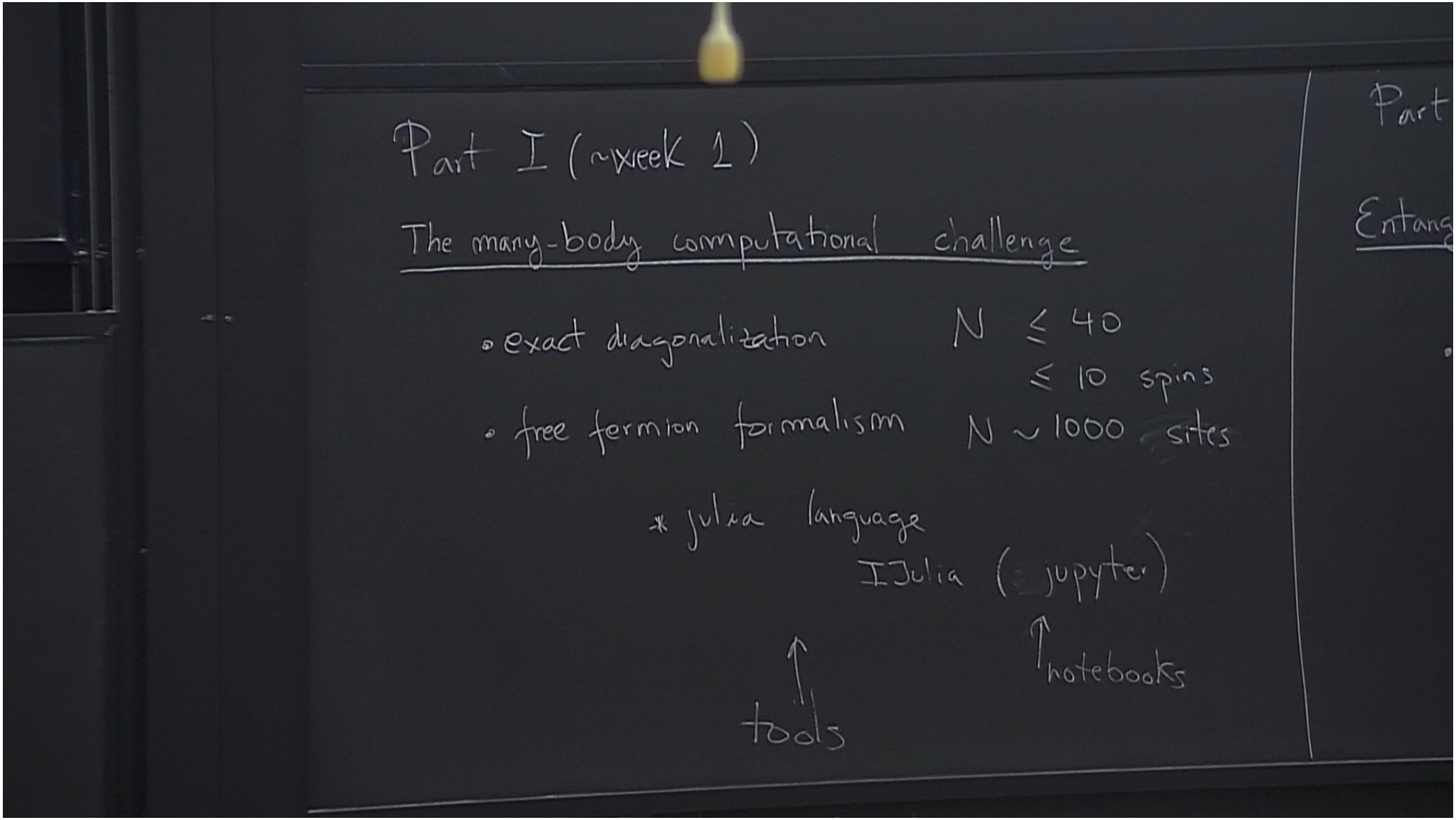
$\mathbb{V}^{(N)} \supseteq \mathbb{K}$ submanyfold

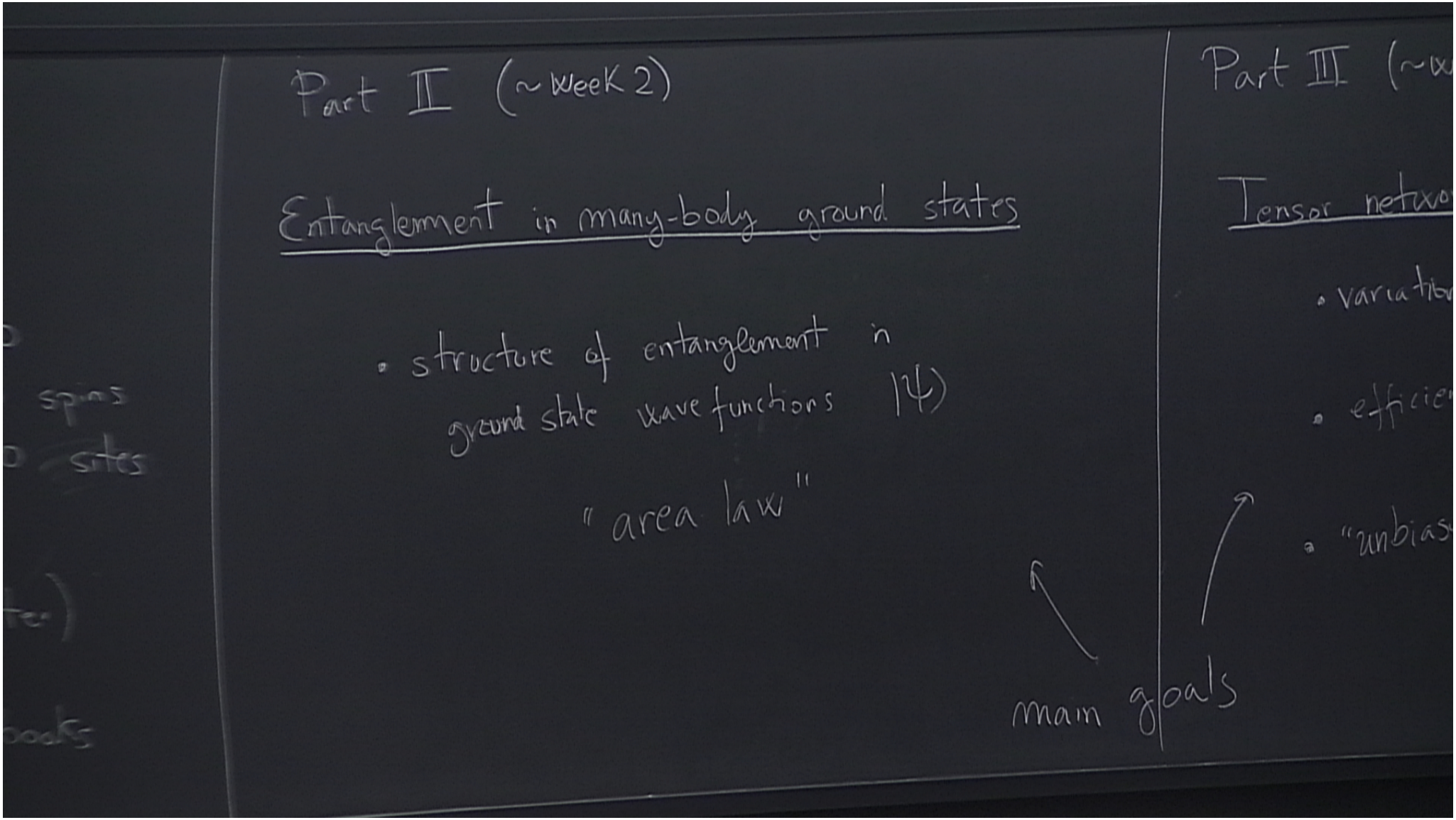
$\dim \sim e^N$

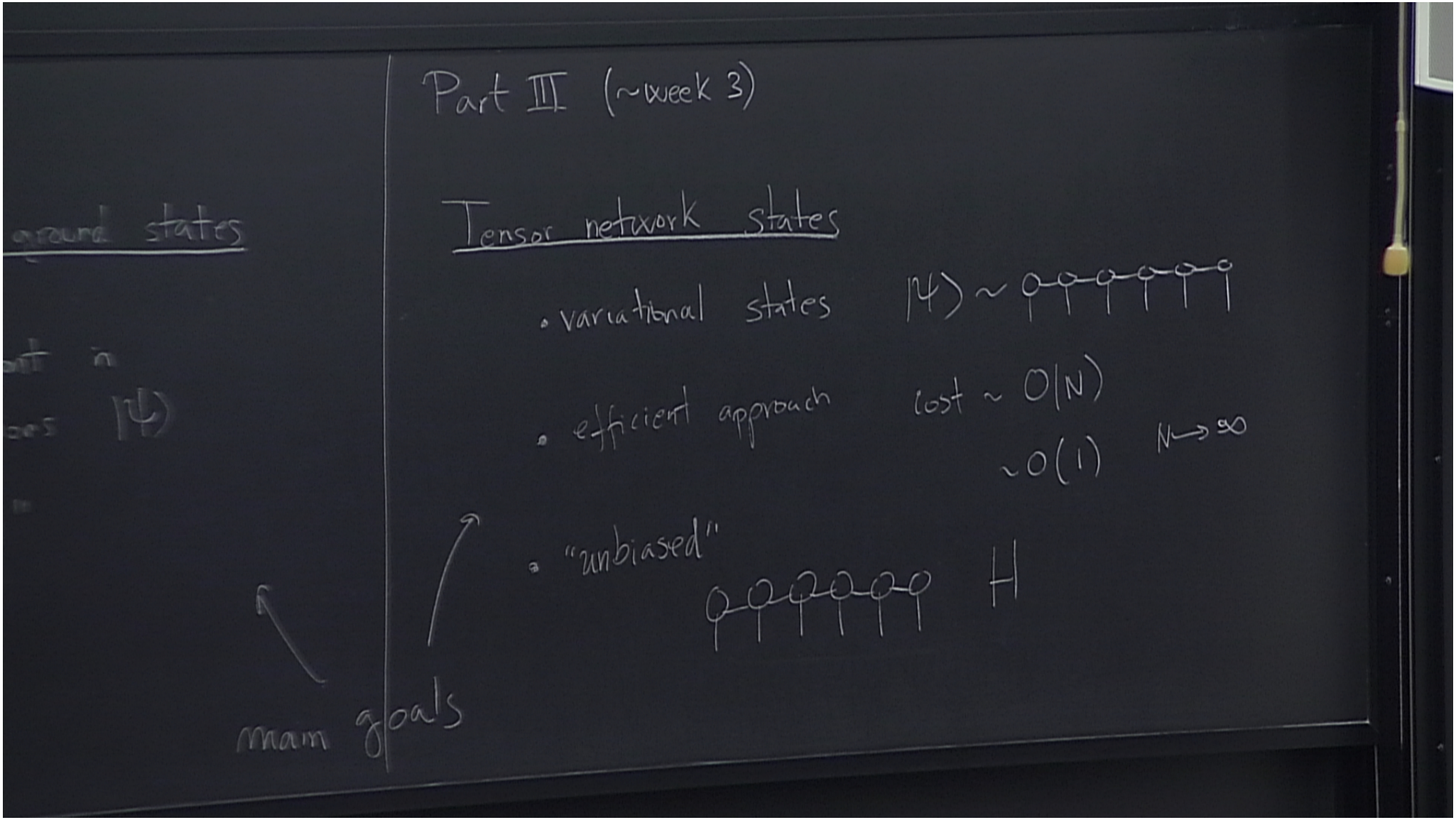
$\dim \sim O(N)$
$pol(N)$

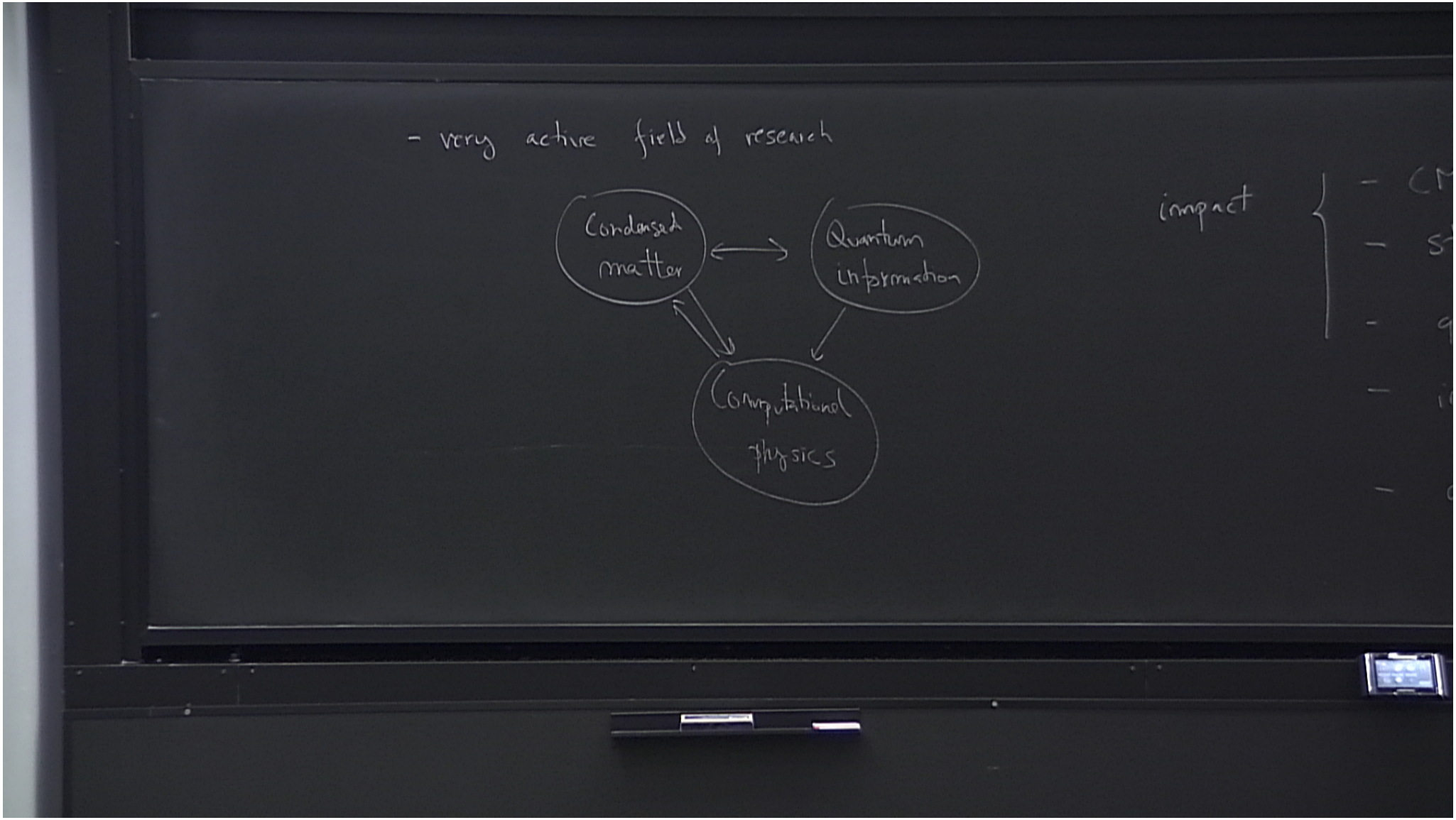


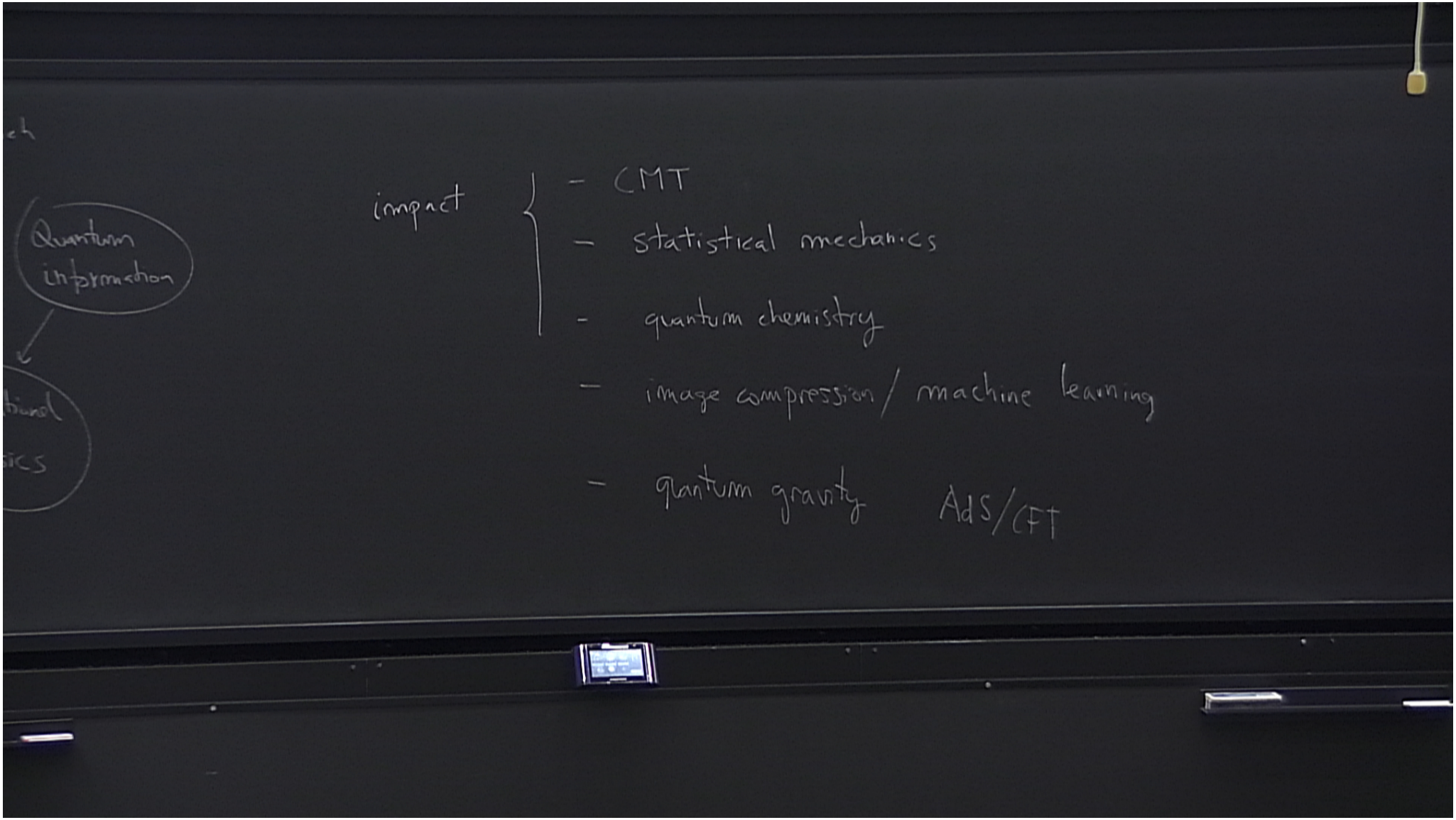

d) Variational approaches
$\mathbb{V}^{(N)} \supseteq \mathbb{K}$ submanyfold
$\dim \sim e^N$
$\dim \sim O(N)$
$\mathrm{pol}(N)$
biased
$\mathbb{V}^{(N)}$

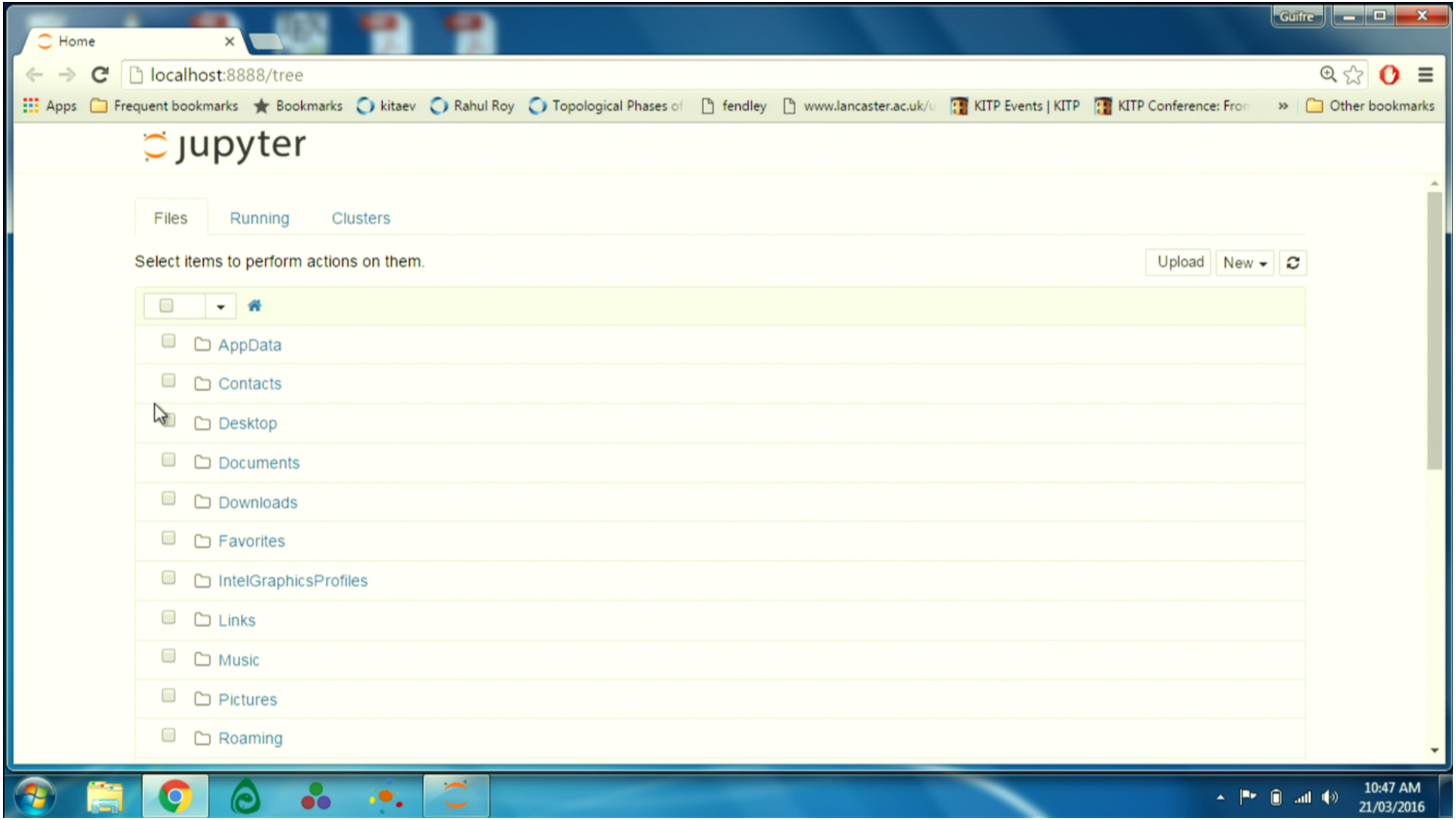

Part I (~week 1)
The many-body computational challenge
• exact diagonalization
N ≤ 40
≤ 10 spins
• free fermion formalism
N ~ 1000 sites
* julia language
IJulia (jupyter)
notebooks
tools
Part
Entang

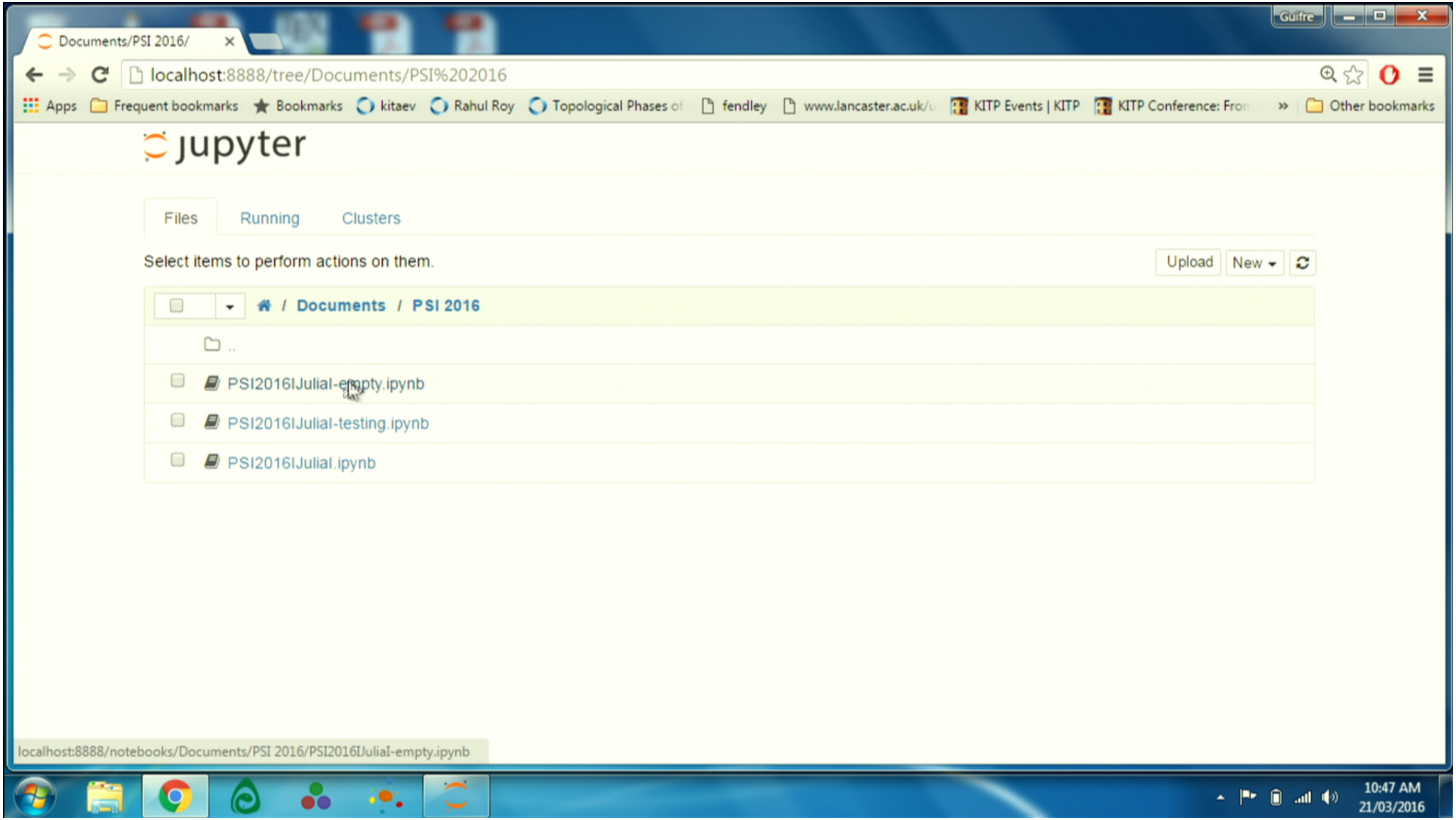

Part II (~ Week 2)
Entanglement in many-body ground states
• structure of entanglement in ground state wave functions |ψ⟩
"area law"
Part II (~
Tensor netwo
• variatio
• efficie
• "unbias
main goals

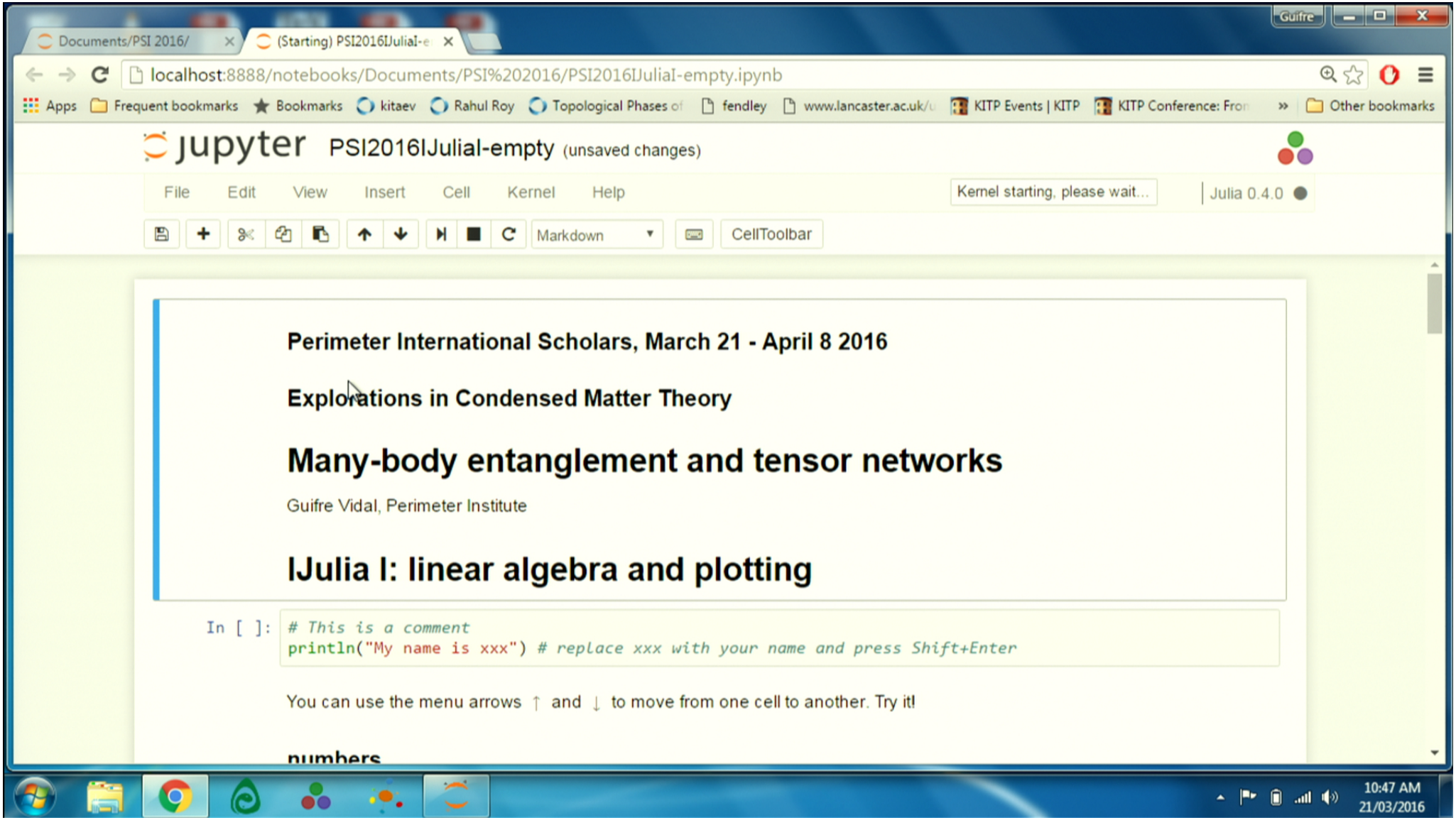

Part II (~week 3)
Tensor network states
• variational states
• efficient approach
cost ~ O(N)
~O(1) N→∞
• "unbiased"
main goals

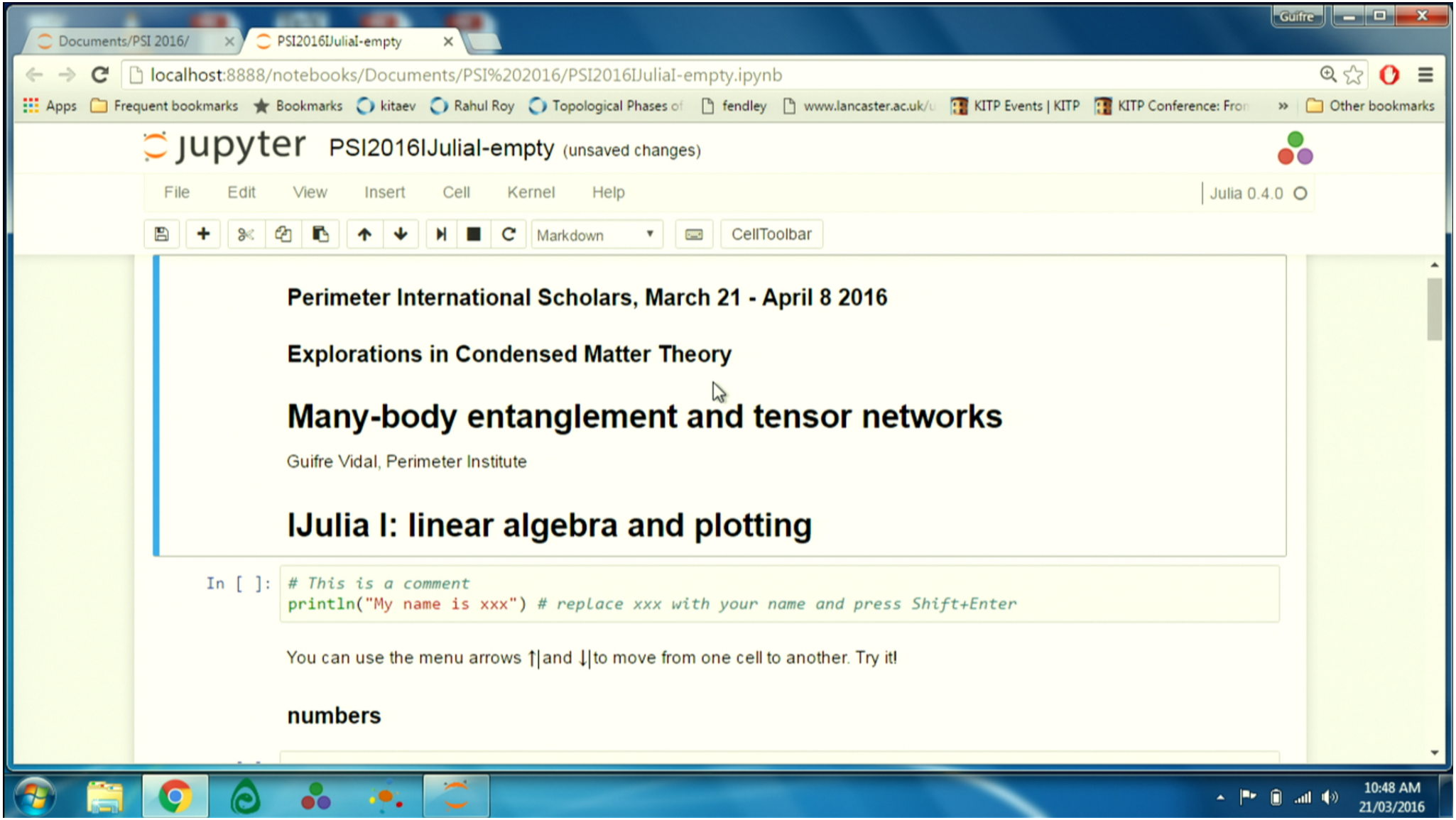

- very active field of research
Condensed matter
Quantum information
Computational physics
impact

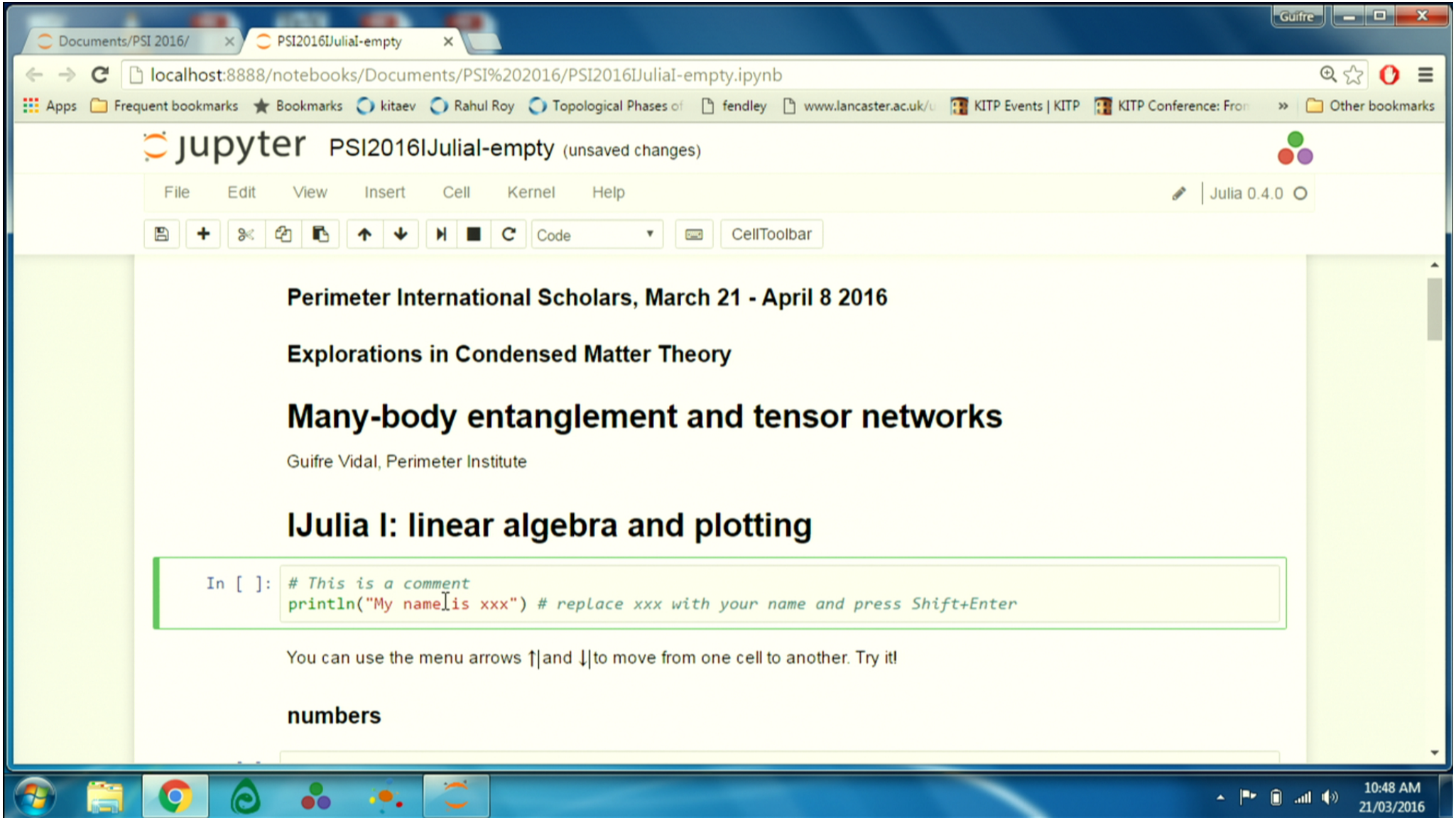

Quantum information
impact
- CMT
- statistical mechanics
- quantum chemistry
- image compression / machine learning
- quantum gravity AdS/CFT

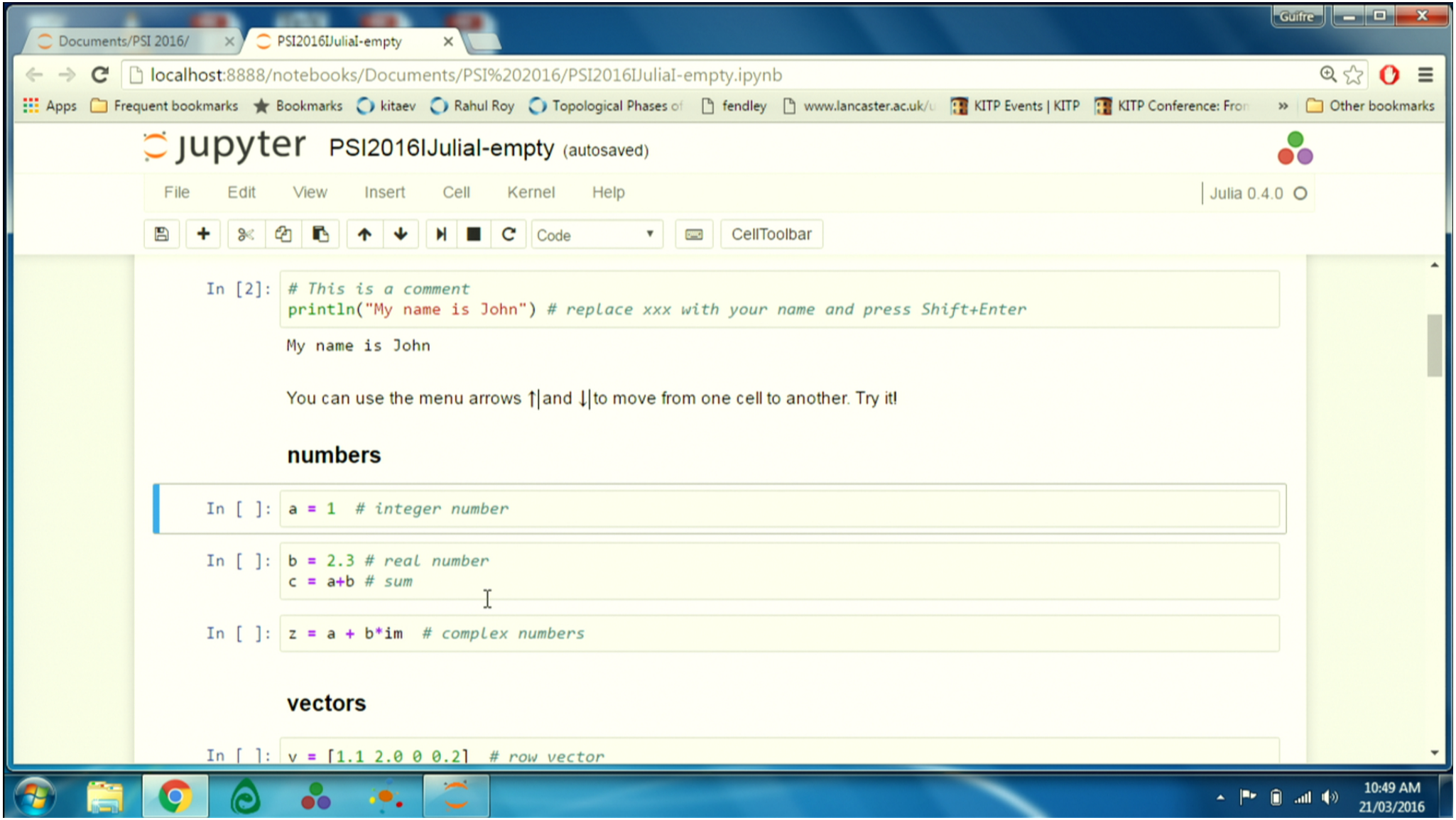

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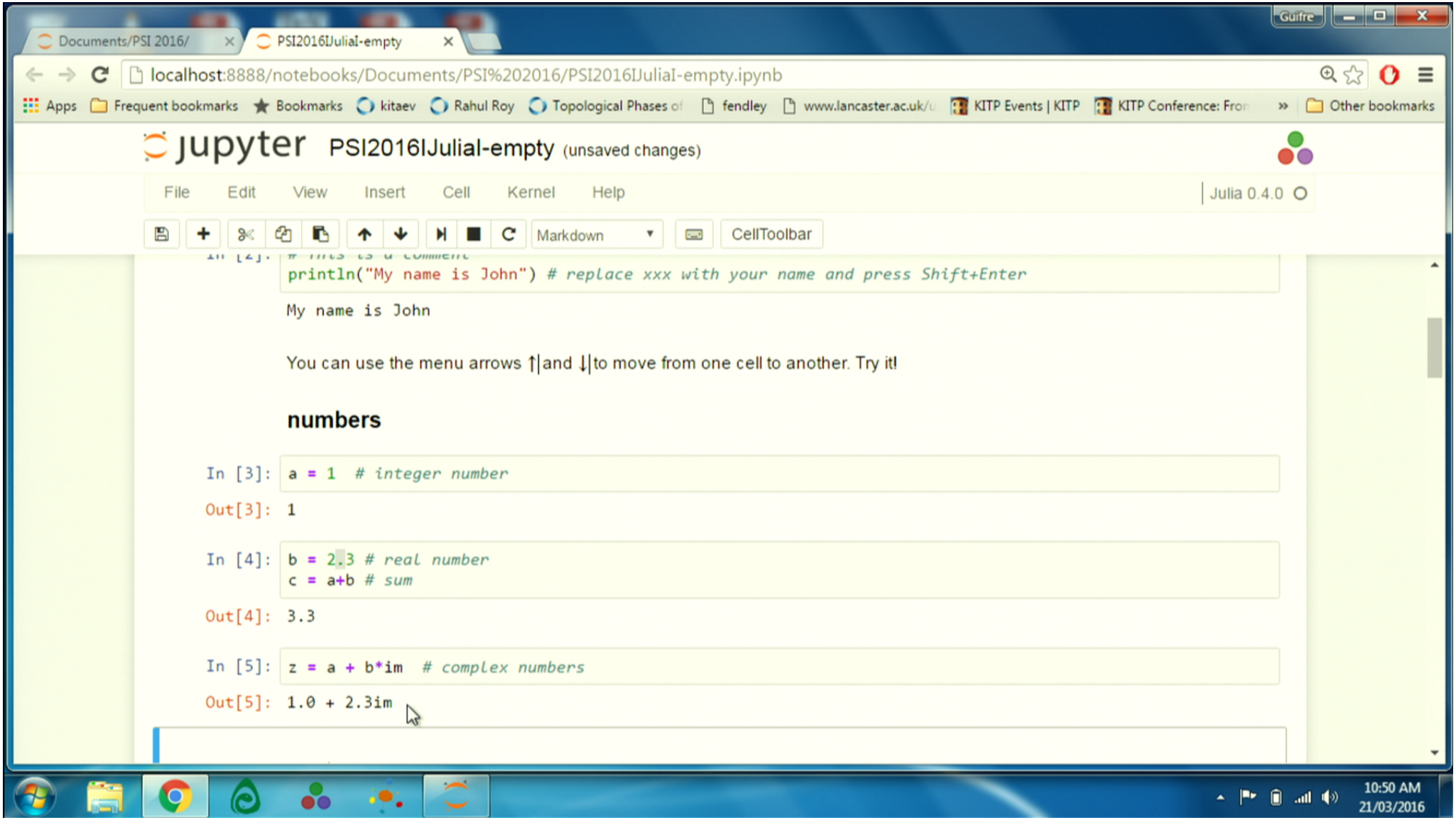

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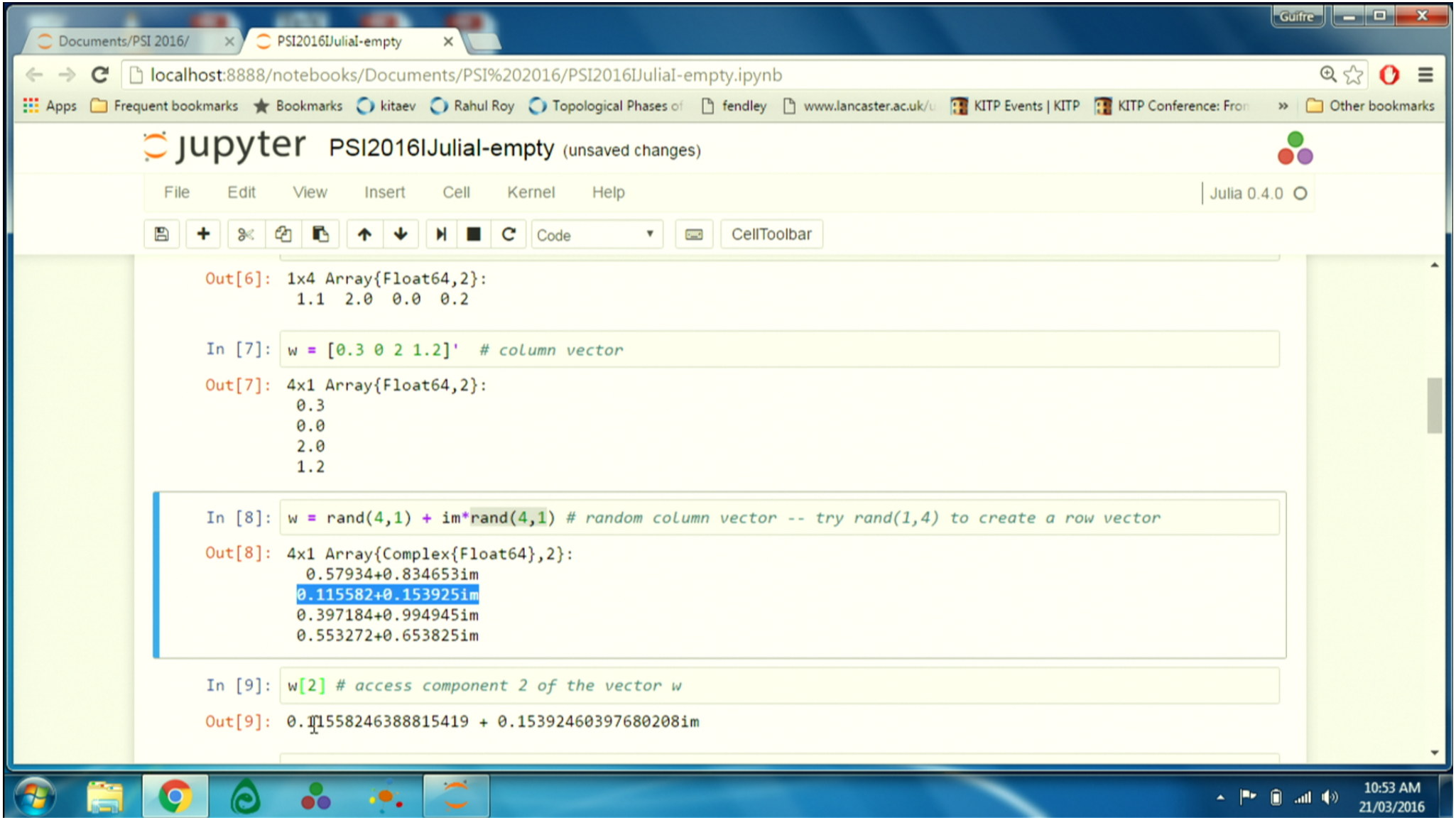

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Markdown
CellToolbar
Perimeter International Scholars, March 21 - April 8 2016
Explorations in Condensed Matter Theory
Many-body entanglement and tensor networks
Guifre Vidal, Perimeter Institute
IJulia I: linear algebra and plotting
In [ ]:
This is a comment
println("My name is xxx") # replace xxx with your name and press Shift+Enter
You can use the menu arrows ↑ and ↓ to move from one cell to another. Try it!
numbers
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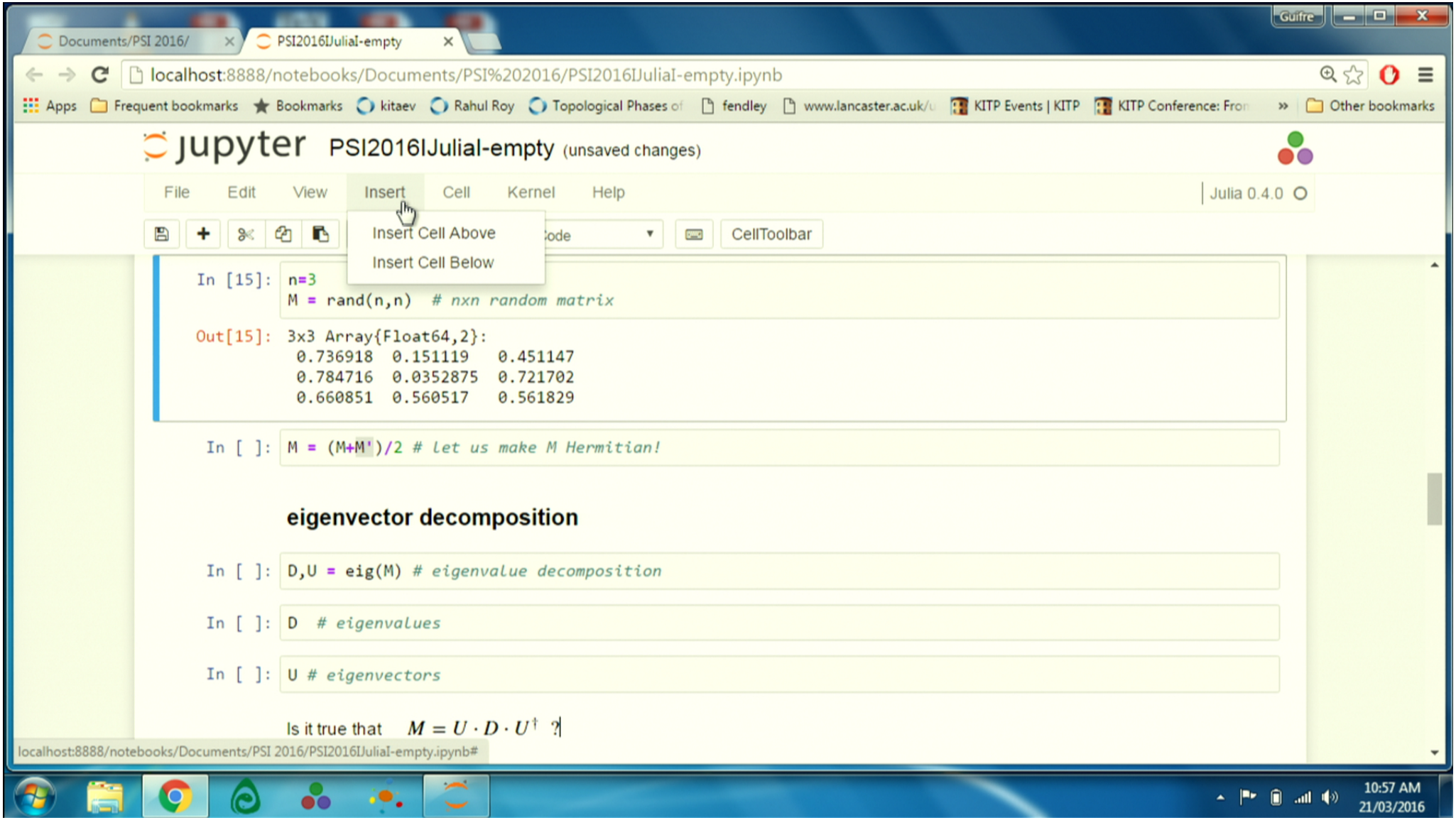

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In [ ]:
This is a comment
println("My name is xxx") # replace xxx with your name and press Shift+Enter
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numbers
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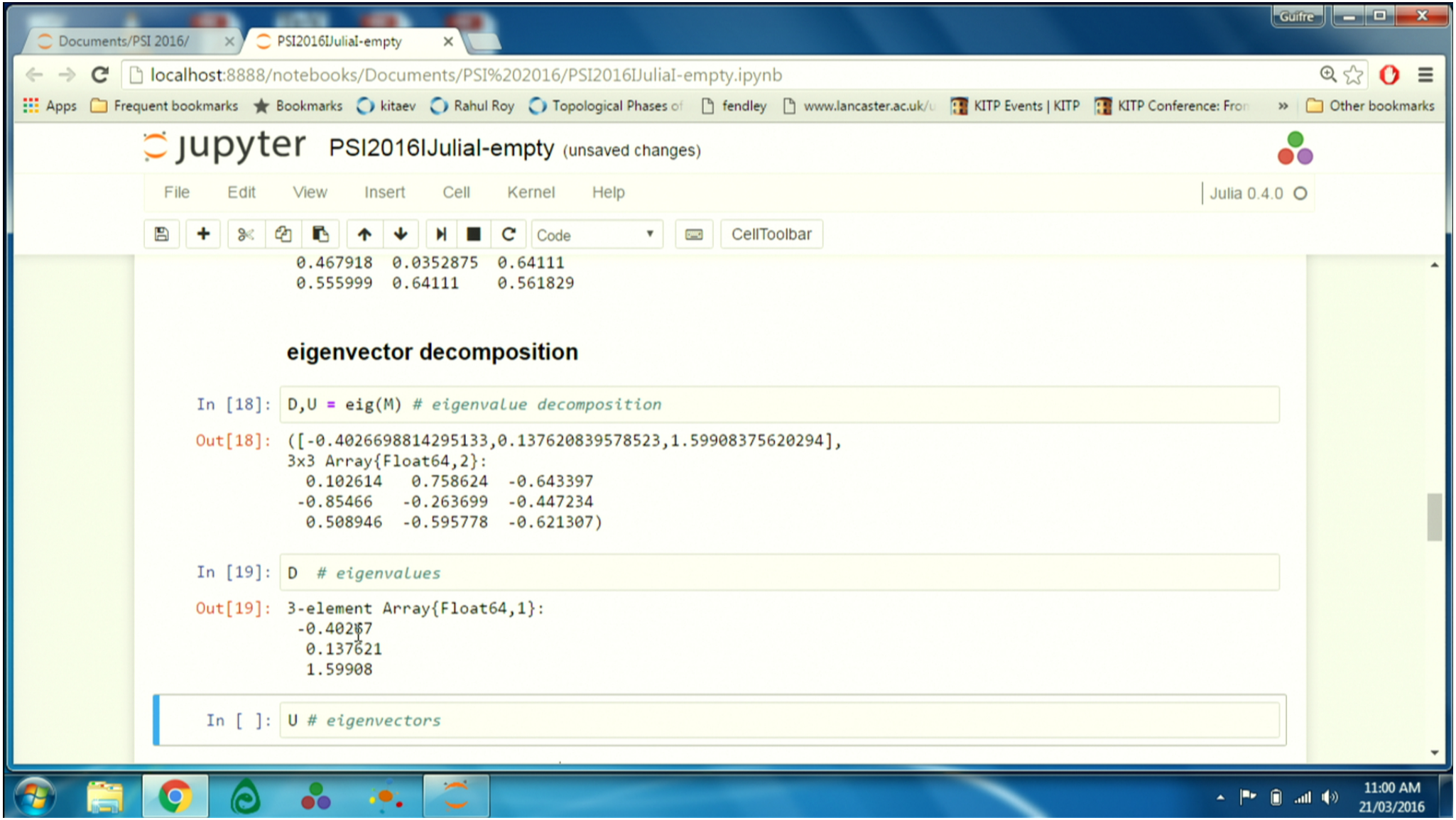

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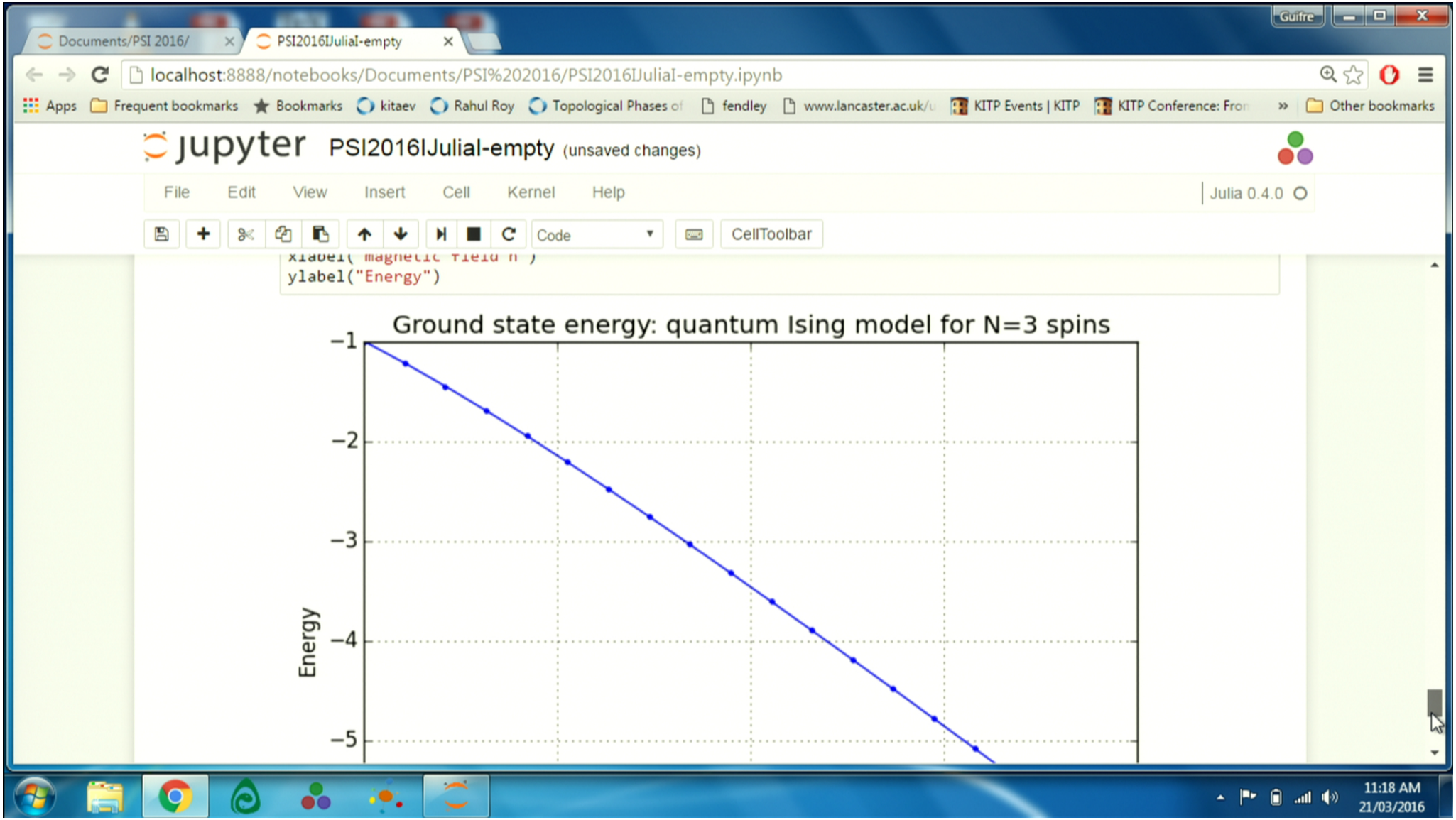

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Julia 0.4.0
Code
CellToolbar
In [2]:
This is a comment
println("My name is John") # replace xxx with your name and press Shift+Enter
My name is John
You can use the menu arrows ↑ and ↓ to move from one cell to another. Try it!
numbers
In [ ]:
a = 1 # integer number
In [ ]:
b = 2.3 # real number
c = a+b # sum
In [ ]:
z = a + b*im # complex numbers
vectors
In [ ]:
v = [1.1 2.0 0 0.2] # row vector
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Julia 0.4.0
Markdown
CellToolbar
println("My name is John") # replace xxx with your name and press Shift+Enter
My name is John
You can use the menu arrows ↑ and ↓ to move from one cell to another. Try it!
numbers
In [3]: a = 1 # integer number
Out[3]: 1
In [4]: b = 2.3 # real number
c = a+b # sum
Out[4]: 3.3
In [5]: z = a + b*im # complex numbers
Out[5]: 1.0 + 2.3im
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Code
CellToolbar
Out[6]: 1x4 Array{Float64,2}:
1.1 2.0 0.0 0.2
In [7]: w = [0.3 0 2 1.2]' # column vector
Out[7]: 4x1 Array{Float64,2}:
0.3
0.0
2.0
1.2
In [8]: w = rand(4,1) + im*rand(4,1) # random column vector -- try rand(1,4) to create a row vector
Out[8]: 4x1 Array{Complex{Float64},2}:
0.57934+0.834653im
0.115582+0.153925im
0.397184+0.994945im
0.553272+0.653825im
In [9]: w[2] # access component 2 of the vector w
Out[9]: 0.115582463888154 19 + 0.15392460397680208im
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Julia 0.4.0
Insert Cell Above
Insert Cell Below
CellToolbar
In [15]: n=3
M = rand(n,n) # nxn random matrix
Out[15]: 3x3 Array{Float64,2}:
0.736918 0.151119 0.451147
0.784716 0.0352875 0.721702
0.660851 0.560517 0.561829
In [ ]: M = (M+M')/2 # let us make M Hermitian!
eigenvector decomposition
In [ ]: D,U = eig(M) # eigenvalue decomposition
In [ ]: D # eigenvalues
In [ ]: U # eigenvectors
Is it true that $M = U \cdot D \cdot U^\dagger$?
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Julia 0.4.0
Code
CellToolbar
0.467918 0.0352875 0.64111
0.555999 0.64111 0.561829
eigenvector decomposition
In [18]: D,U = eig(M) # eigenvalue decomposition
Out[18]: ([-0.4026698814295133,0.137620839578523,1.59908375620294],
3x3 Array{Float64,2}:
0.102614 0.758624 -0.643397
-0.85466 -0.263699 -0.447234
0.508946 -0.595778 -0.621307)
In [19]: D # eigenvalues
Out[19]: 3-element Array{Float64,1}:
-0.40267
0.137621
1.59908
In [ ]: U # eigenvectors
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Julia 0.4.0
Code
CellToolbar
ylabel("Energy")
Ground state energy: quantum Ising model for N=3 spins
Energy
−1
−2
−3
−4
−5
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