Title: Entanglement and the fermion sign problem - Peter Broecker

Date: Feb 24, 2016 03:30 PM

URL: http://pirsa.org/16020116

Abstract: <p>The precise determination of the entanglement of an interacting quantum many-body systems is now appreciated as an indispensable tool to identify the fundamental character of the ground state of such systems. This is particularly true for unconventional ground states harbouring non-local topological order or so-called quantum spin liquids that evade a standard description in terms of correlation functions. With the entanglement entropy emerging as one of the central measures of entanglement, recent progress has focused on a precise characterization of its scaling behaviour, in particular in the determination of (subleading) corrections to the prevalent boundary-law. </p>

<p>In the past years, much progress has been made for certain spin, bosonic, and even fermionic quantum many-body systems. However, a large class of interacting models is thought to be exempt from numerical studies due to the fermion sign problem. At its heart, it occurs when the statistical weights in the simulation are positive and negative resulting in an exponential scaling of the algorithm instead of a polynomial one. In this work, we study the connection of the sign problem and the entanglement entropy using Determinantal Quantum Monte Carlo, the method of choice for unbiased, large-scale simulations of fermionic systems. We show that there is a strong correlation between the behavior of the entanglement entropy and the sign problem and that the particular structure of the â€˜observableâ€™ entanglement entropy to some extent allows to handle the sign problem much better than for usual correlation functions.</p>

Entanglement and the sign problem

An auxiliary field Monte Carlo approach

Peter Bröcker and Simon Trebst

Universität zu Köln

J. Stat. Mech. 2014

arXiv: 1511.02878

A brief history of entanglement

EINSTEIN ATTACKS QUANTUM THEORY

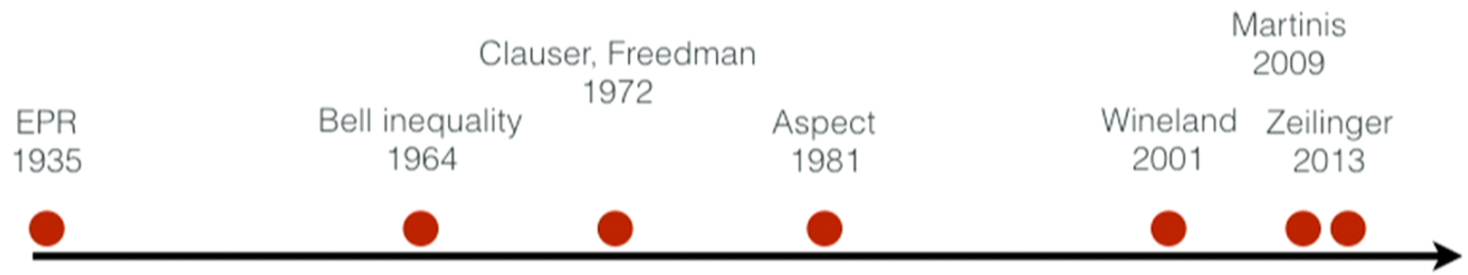
Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

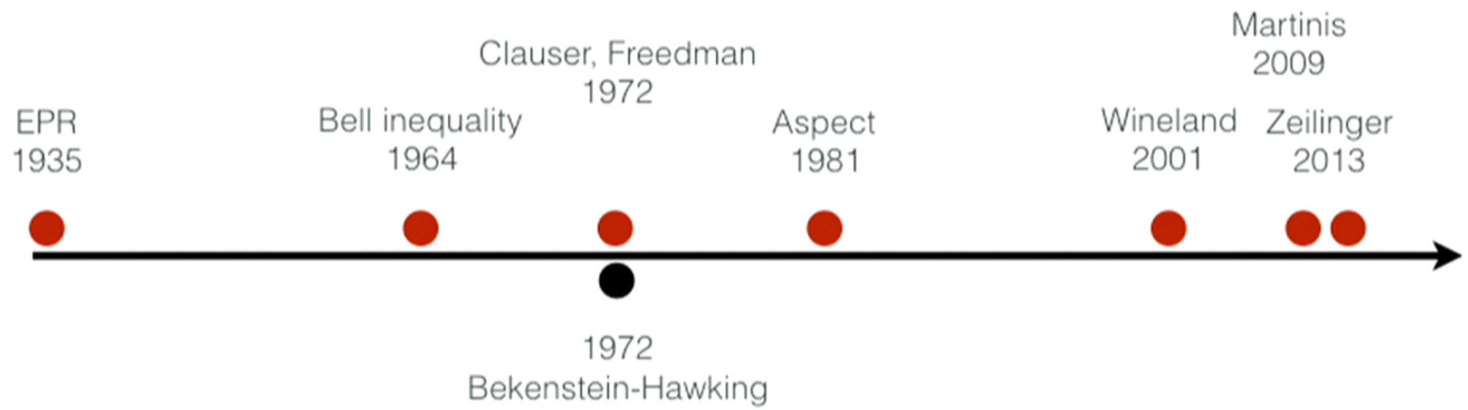
Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

New York Times
May 4th, 1935

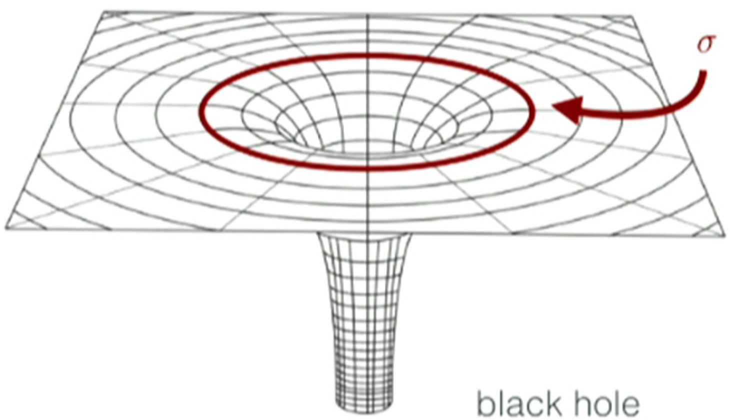
A brief history of entanglement



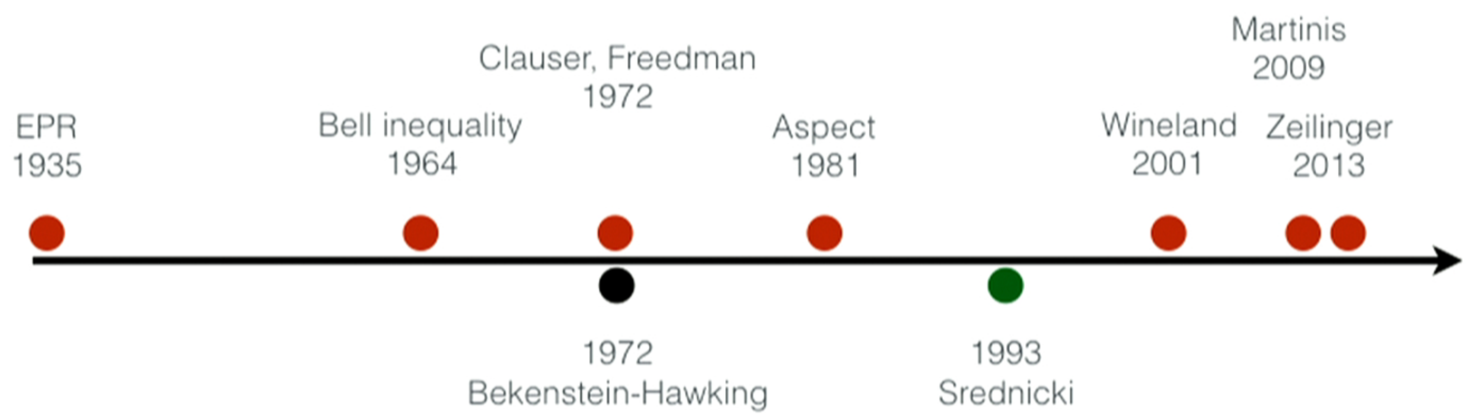
A brief history of entanglement



$$S_{BH} = \frac{c^3}{4G\hbar}\sigma$$



A brief history of entanglement



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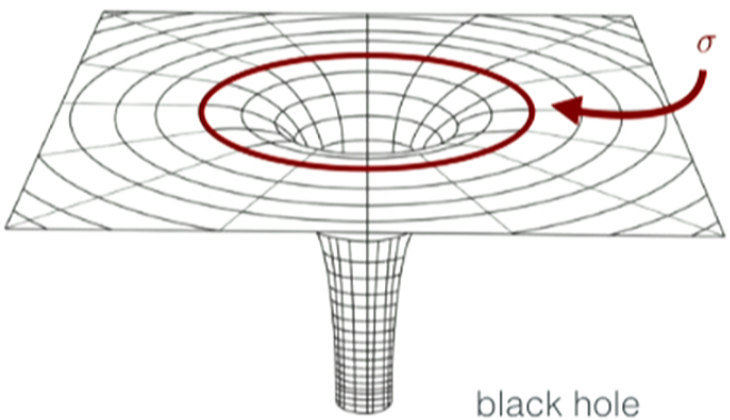
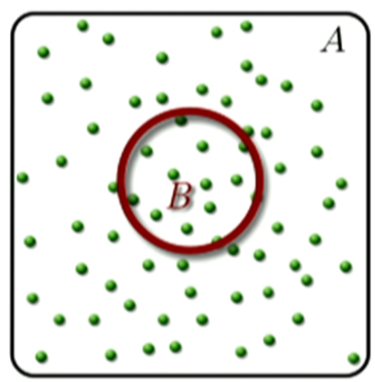


black hole

$$\rho_A = \mathrm{Tr}_B \rho$$



free quantum field

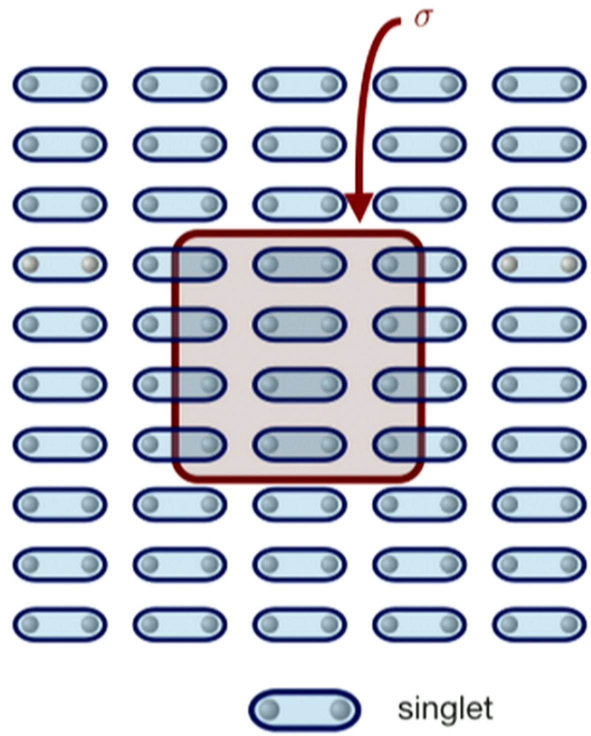
Beyond the Standard Model

classify ground states by contributions
beyond the boundary law

simple ground state

$$S_A(\sigma) = a \cdot \sigma$$

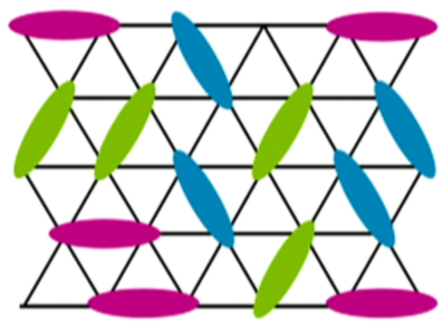
e.g. valence bond crystal



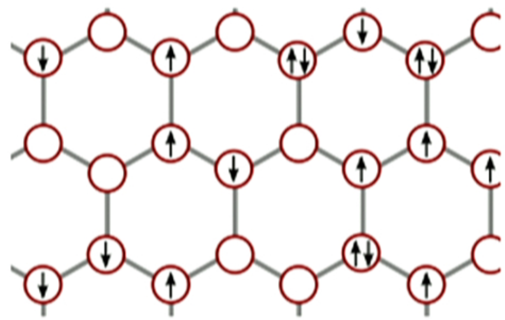
what we are interested in

exotic phases of matter in correlated many fermion systems

topological order or spin liquids

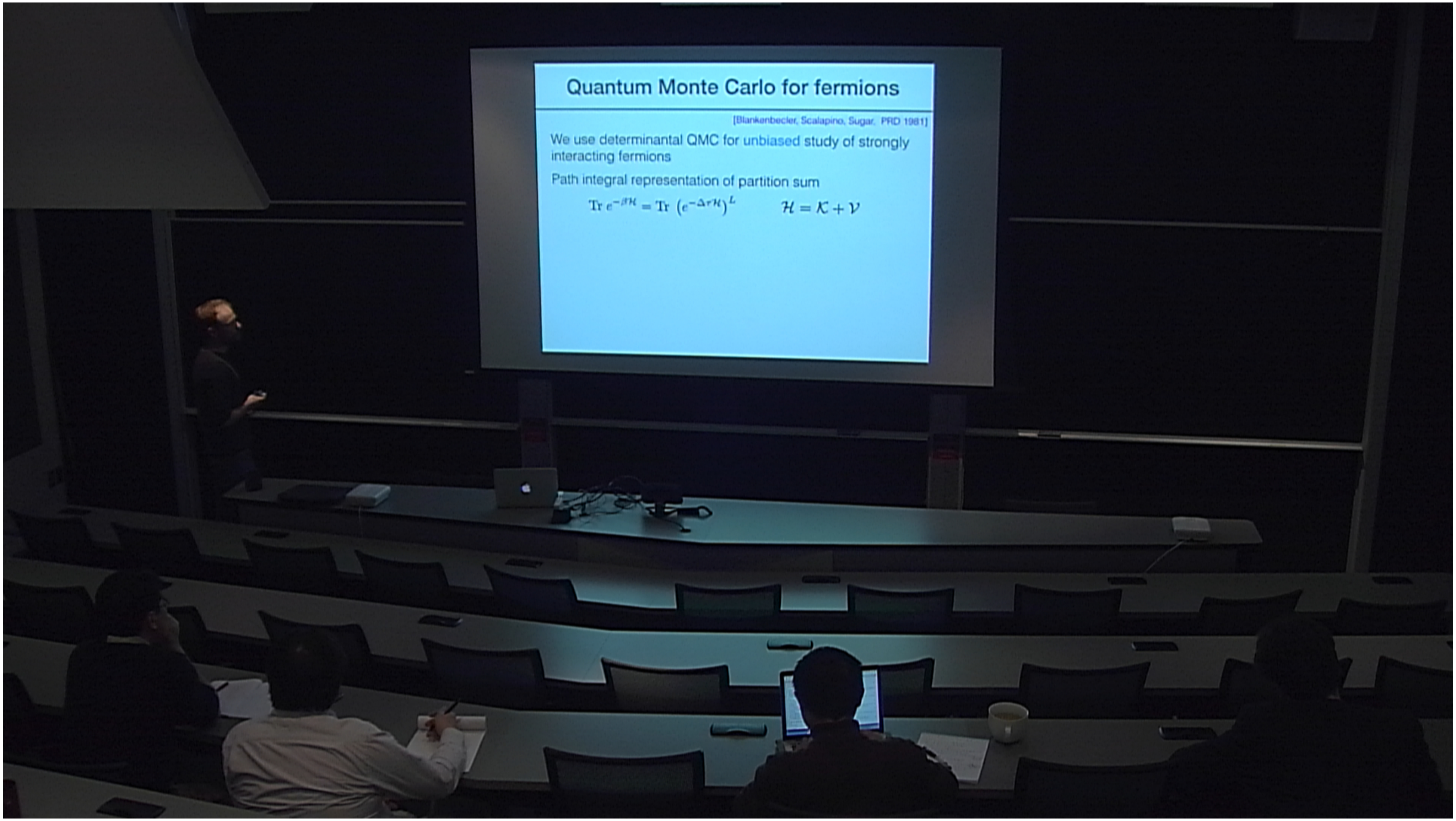


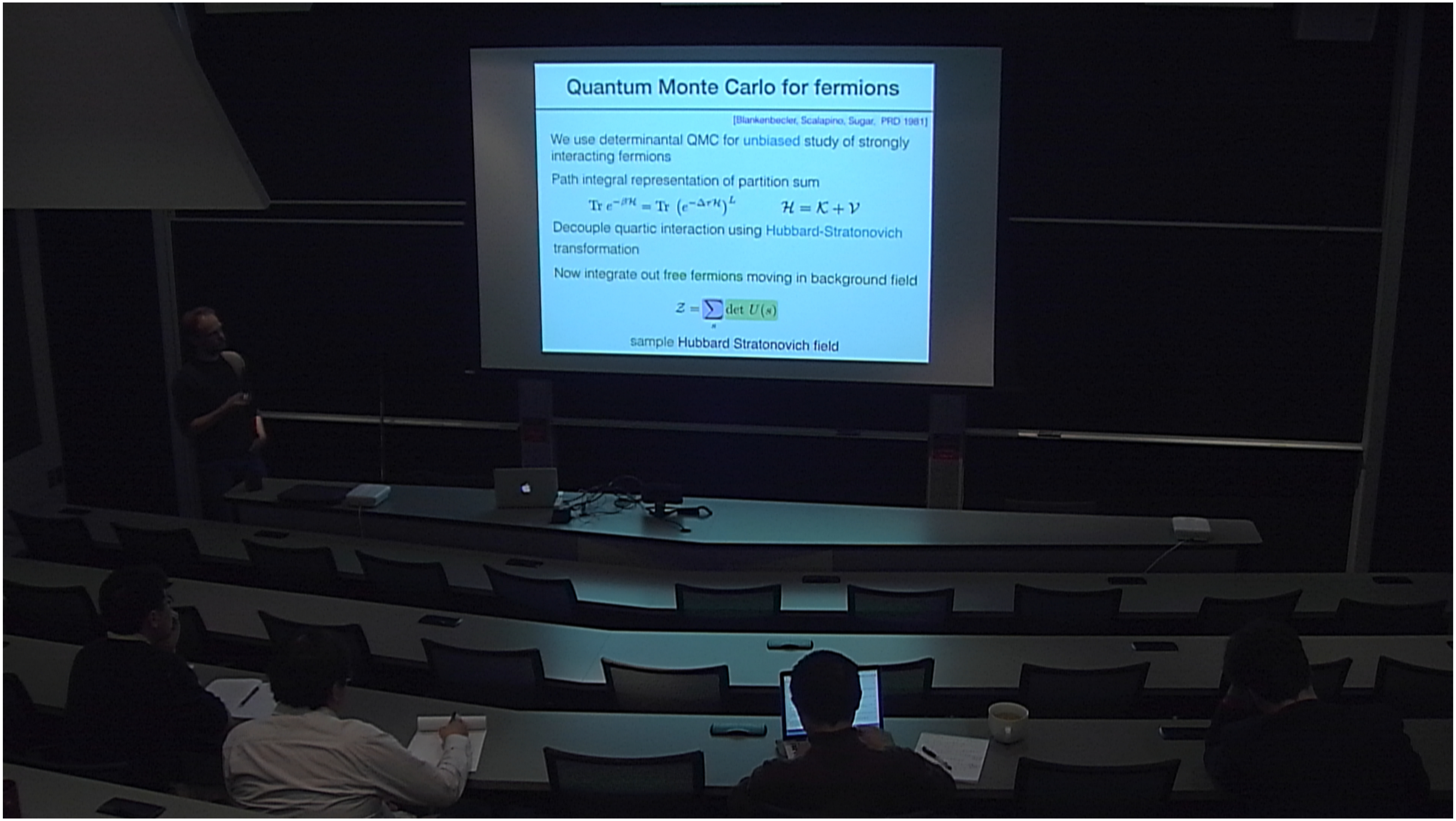
frustrated magnets



doped electron system

classification of states of matter via their entanglement


Quantum Monte Carlo for fermions
[Blankenbecler, Scalapino, Sugar, PRD 1981]
We use determinantal QMC for unbiased study of strongly interacting fermions
Path integral representation of partition sum
$\mathrm{Tr}\, e^{-\beta\mathcal{H}} = \mathrm{Tr}\, \left(e^{-\Delta\tau\mathcal{H}}\right)^L \qquad \mathcal{H} = \mathcal{K} + \mathcal{V}$


Quantum Monte Carlo for fermions
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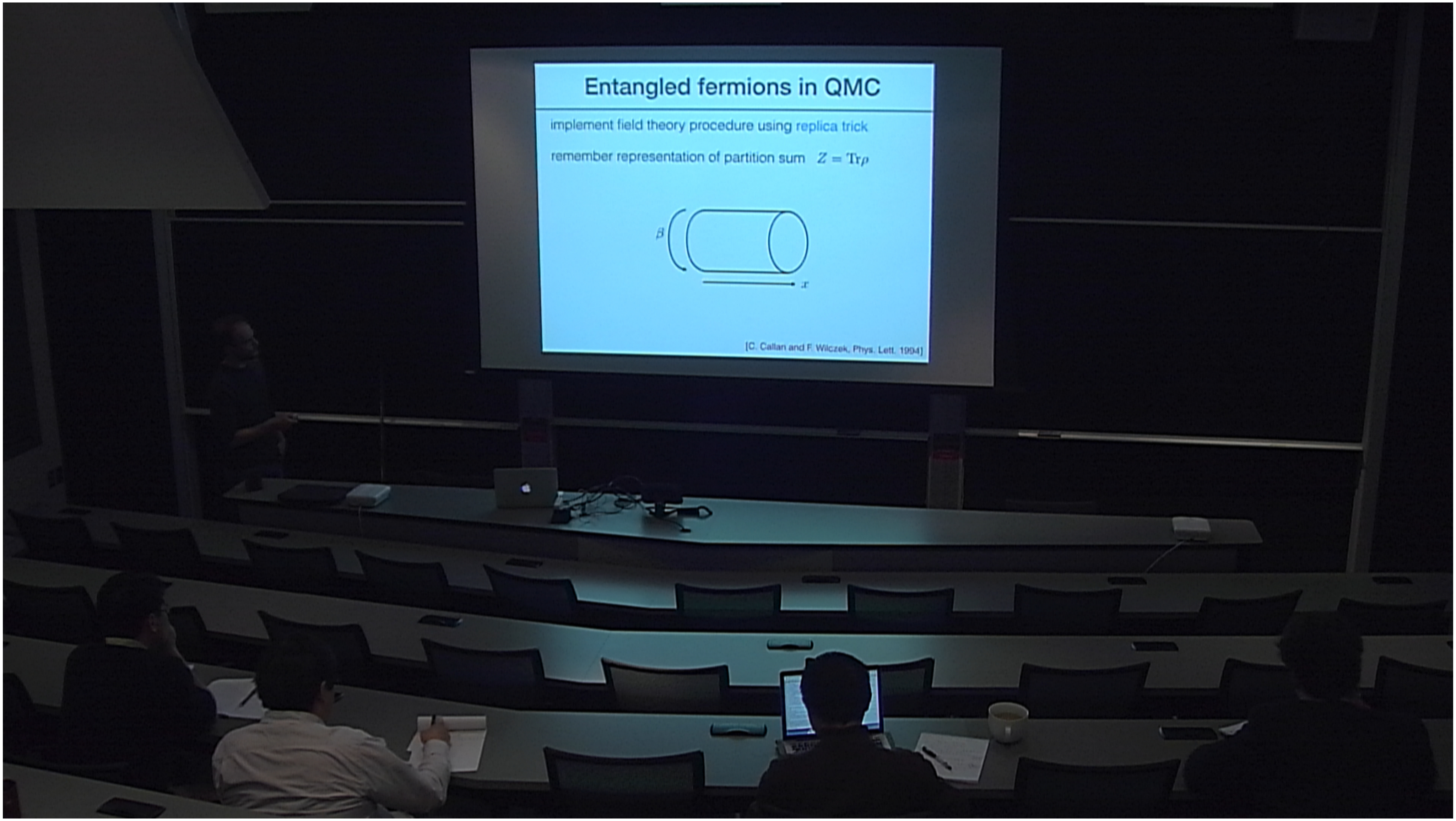
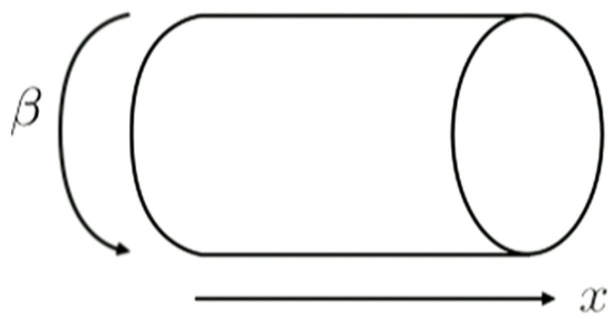
$\mathrm{Tr}\, e^{-\beta\mathcal{H}} = \mathrm{Tr}\, \left(e^{-\Delta\tau\mathcal{H}}\right)^{L}$ $\mathcal{H} = \mathcal{K} + \mathcal{V}$
Decouple quartic interaction using Hubbard-Stratonovich transformation
Now integrate out free fermions moving in background field
$\mathcal{Z} = \sum_{s} \det U(s)$
sample Hubbard Stratonovich field


Entangled fermions in QMC
implement field theory procedure using replica trick
remember representation of partition sum $Z = \mathrm{Tr}\rho$
β
x
[C. Callan and F. Wilczek, Phys. Lett. 1994]

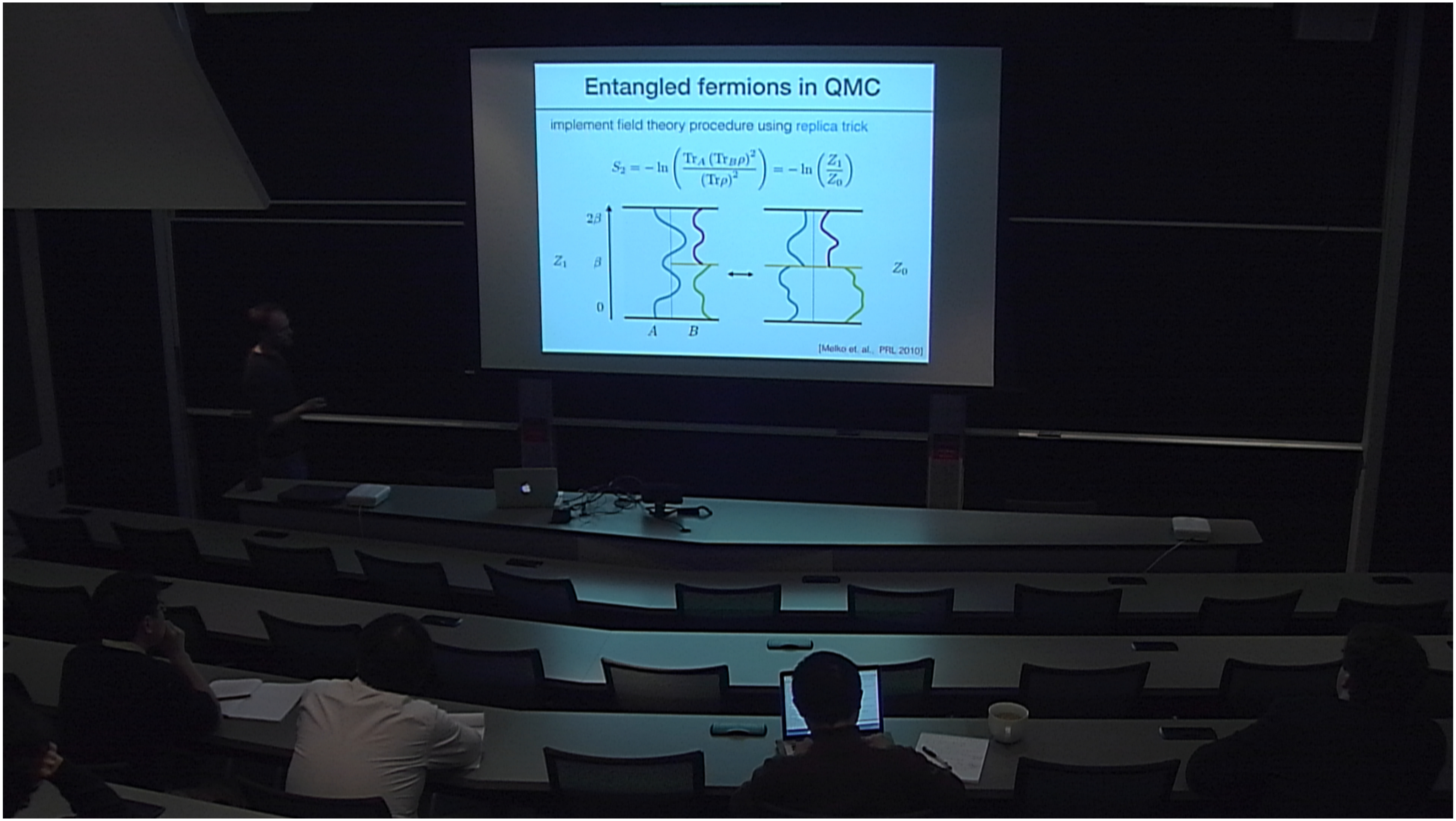
Entangled fermions in QMC

implement field theory procedure using replica trick

remember representation of partition sum $Z = \mathrm{Tr}\rho$

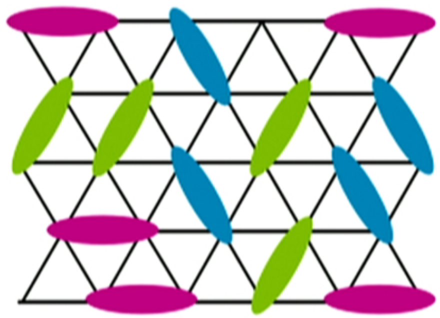


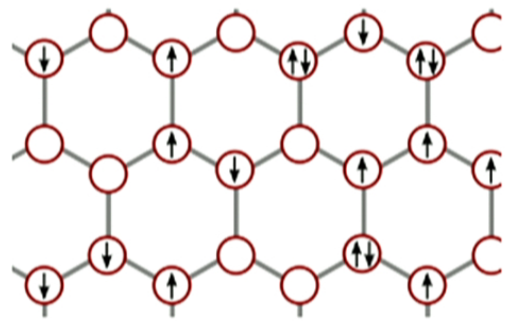
[C. Callan and F. Wilczek, Phys. Lett. 1994]


Entangled fermions in QMC
implement field theory procedure using replica trick
$S_2 = -\ln\left(\frac{\mathrm{Tr}_A(\mathrm{Tr}_B\rho)^2}{(\mathrm{Tr}\rho)^2}\right) = -\ln\left(\frac{Z_1}{Z_0}\right)$
2β
β
0
Z_1
Z_0
A
B
[Melko et. al., PRL 2010]


Entangled fermions in QMC
implement field theory procedure using replica trick
adoption to DQMC not straight-forward
use free fermion property
[Grover, PRL 2013]

Exotic phases and QMC





Unambiguous signatures in the entanglement entropy. ✓

Computational approach to calculate entanglement entropies ✓

Can we cleverly avoid the sign problem?

It is basis dependent

energy eigenbasis ⟷ simulation basis

successful basis changes

- Meron cluster [Wiese et al., PRL 1995]
- Fermion bag [Chandrasekharan, PRD 2009]
- Majorana fermion basis [Yao et al., PRB 2015]

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No general solution - the sign problem is NP hard

[Troyer and Wiese, PRL 2005]

change of perspective

- effective actions [Berg, Metlitski, Sachdev, Science 2012]

Entanglement and the sign problem

With explicit dependence on sign $Z = Z^{||}\langle\sigma\rangle$

$$S_2 = -\log\left(\frac{Z_1}{Z_0}\right) = -\log\left(\frac{Z_1^{||}}{Z_0^{||}}\frac{\langle\sigma_1\rangle}{\langle\sigma_0\rangle}\right)$$

Entanglement and the sign problem

$$S_2 = S_2^{||} + S_2^{\sigma}$$

$$\mathcal{A}\, l \log l + a\, l + b \log l + \gamma + \cdots = S_2^{||} + S_2^{\sigma}$$

Dirac fermions on honeycomb

spinless fermions with next nearest neighbor interactions

$$H = \sum_{\langle i,j \rangle} -t\, c_i^\dagger c_j + V n_i n_j$$

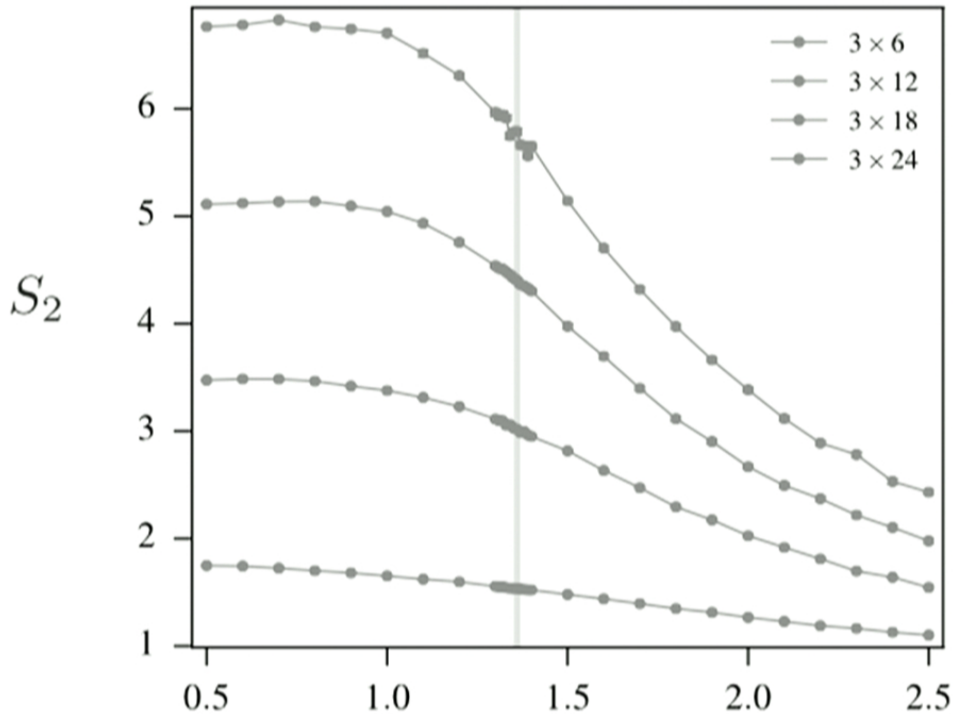
interaction V

Gross Neveu type
fermionic quantum critical point

[Wang, Corboz and Troyer NJP, 2014]
[Motrok, Grushin, de Juan and Pollmann, PRB 2015]
[Capponi and Läuchli, PRB 2015]

Dirac fermions on honeycomb

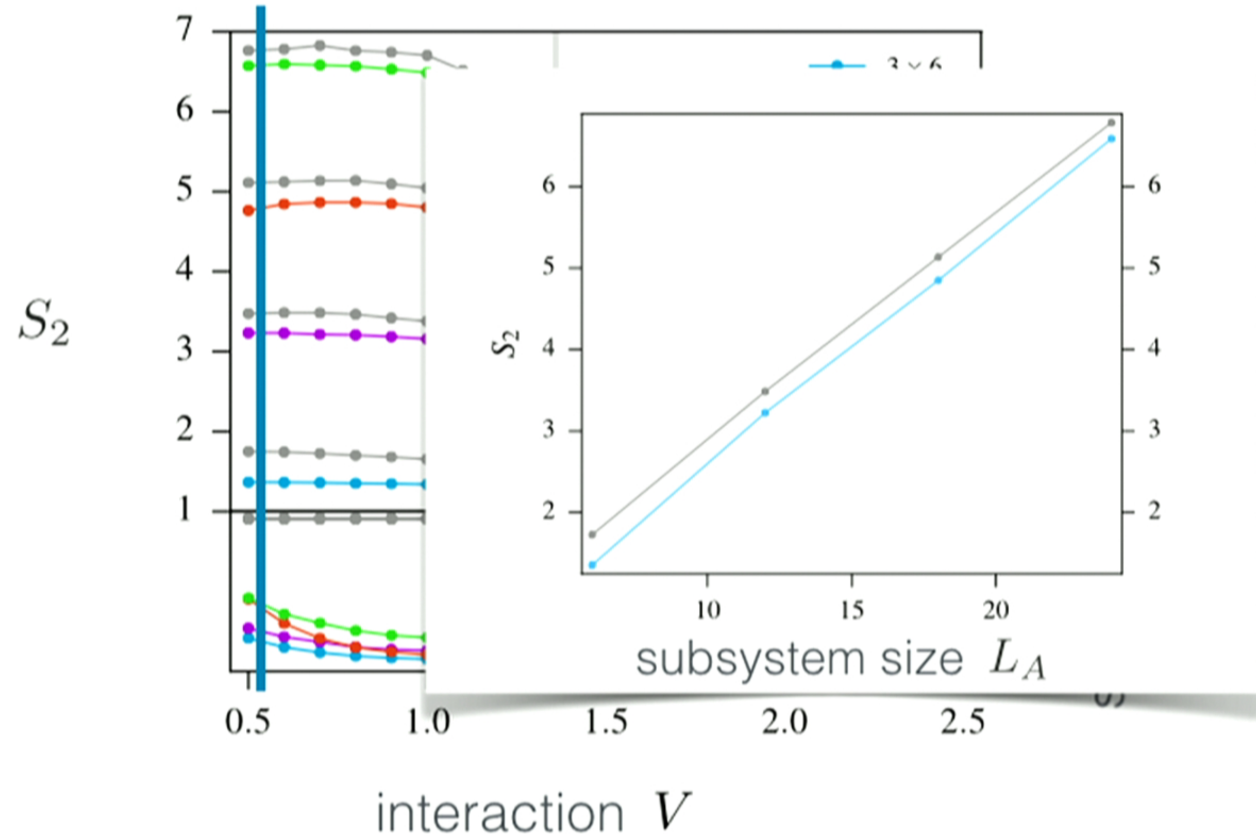
semi-metal → charge density wave



interaction V

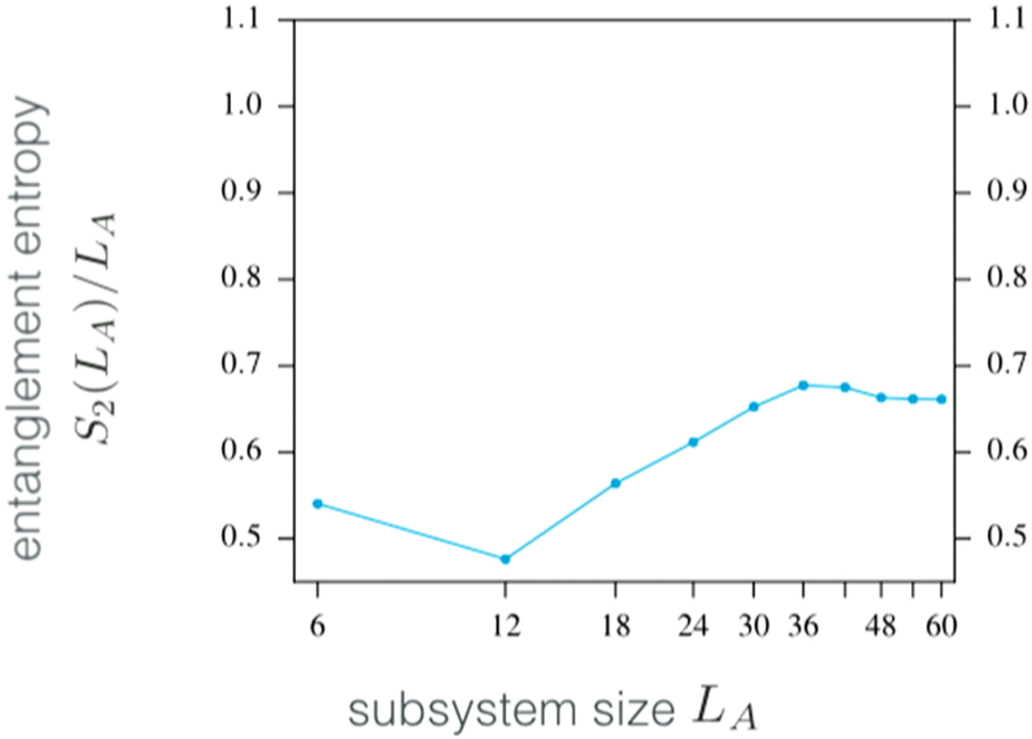
Dirac fermions on honeycomb

Do the correct scaling properties persist?



Entanglement scaling at Dirac point

boundary law $S_2(L_A) \propto a L_A + \dots$



Kagome spin liquids

Heisenberg models on Kagome lattice

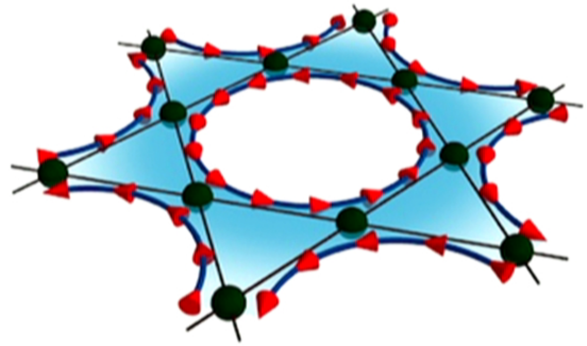
nearest neighbor $H = \sum_{\langle i,j \rangle} S_i S_j$ $\mathbb{Z}_2$ spin liquid?

[Yan et al., Science 2012]
[Jiang et al., Nature 2012]
[Depenbrock et al., PRL 2012]

[Hermele et al., PRB 2008]
[Iqbal et al., PRB 2013]

Kagome spin liquids

Chiral Spin Liquids



microscopic Hamiltonians

realize bosonic analogue of fractional quantum Hall effect

$$H = J_{HB} \sum_{\langle i,j \rangle} S_i S_j + J_\chi \sum_{\Delta(i,j,k)} S_i \cdot (S_j \times S_k)$$

$$H = J_1 \sum_{\langle i,j \rangle} S_i S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i S_j + J_3 \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} S_i S_j$$

[Bauer et al., Nat. Commun. 2014]

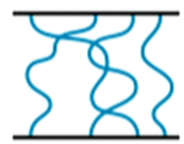
[Gong et al., Sci. Rep. 2014]

[Kumar et al., PRB 2015]

Other QMC flavors

Determinantal approach is special

$$\mathcal{Z} = \sum \det B \longleftarrow \det \left(\quad \right)$$



fermionic nature and propagation protected by determinant