Title: Towards a cluster structure on the trigonometric zastava - Michael Finkelberg

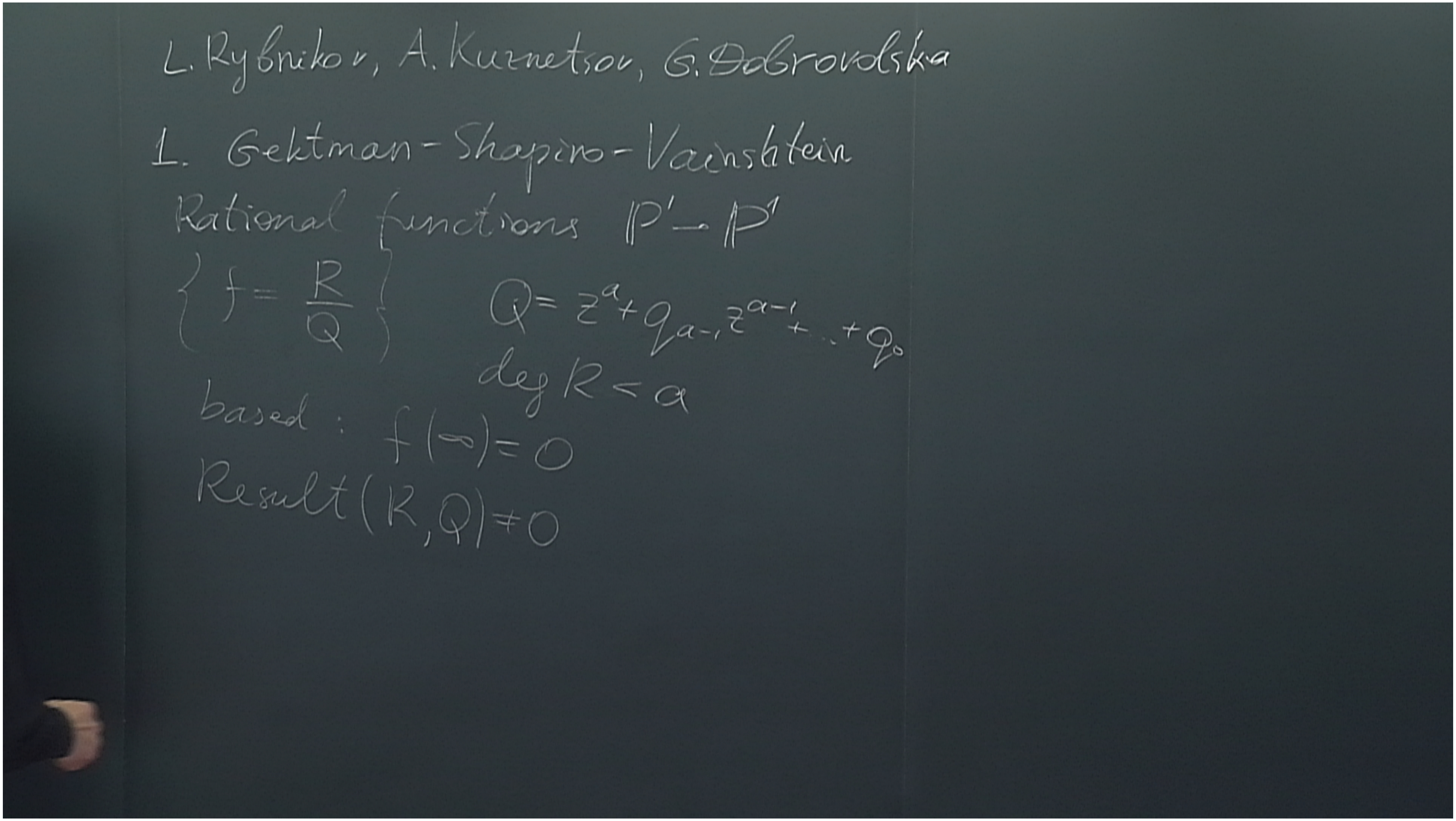
Date: Feb 11, 2016 02:00 PM

URL: http://pirsa.org/16020113

Abstract: <p>This is a joint work with A.Kuznetsov and L.Rybnikov.</p>

<p>We study a moduli problem on a nodal curve of arithmetic genus 1, whose solution is an open subscheme in the zastava space for projective line. This moduli space is equipped with a natural Poisson structure, and we compute it in a natural coordinate system. We compare this Poisson structure with the trigonometric Poisson structure on the transversal slices in an affine flag variety.</p>

<p>We conjecture that certain generalized minors give rise to a cluster structure on the trigonometric zastava.</p>



L. Rybnikov, A. Kuznetsov, G. Dobrovolska

1. Gektman-Shapiro-Vainshtein

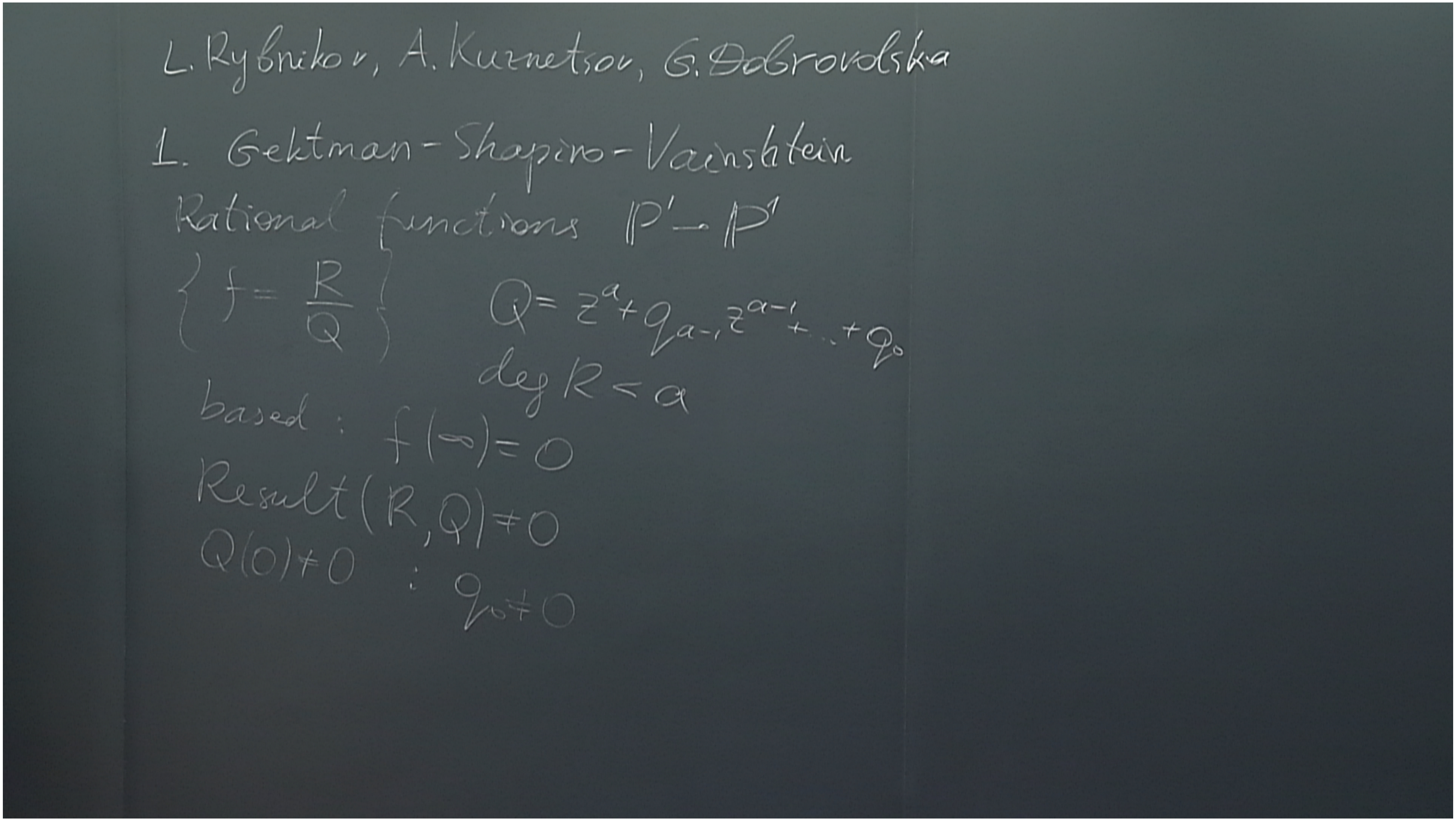
Rational functions $\mathbb{P}^1 \to \mathbb{P}^1$

$\left\{ f = \frac{R}{Q} \right\}$ $\quad Q = z^a + q_{a-1} z^{a-1} + \dots + q_0$

$\deg R < a$

based: $f(\infty) = 0$

$\mathrm{Result}(R, Q) \neq 0$



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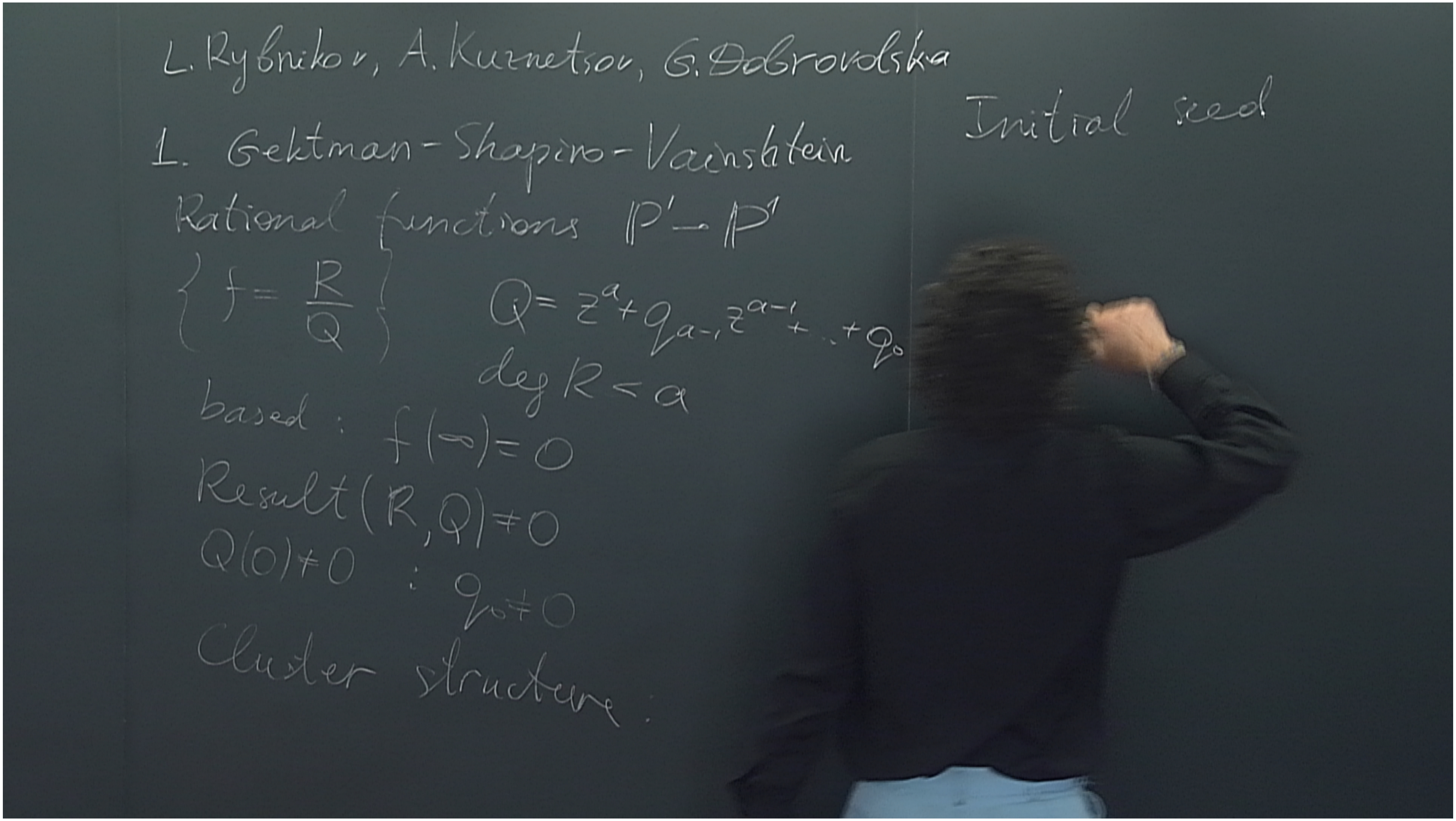

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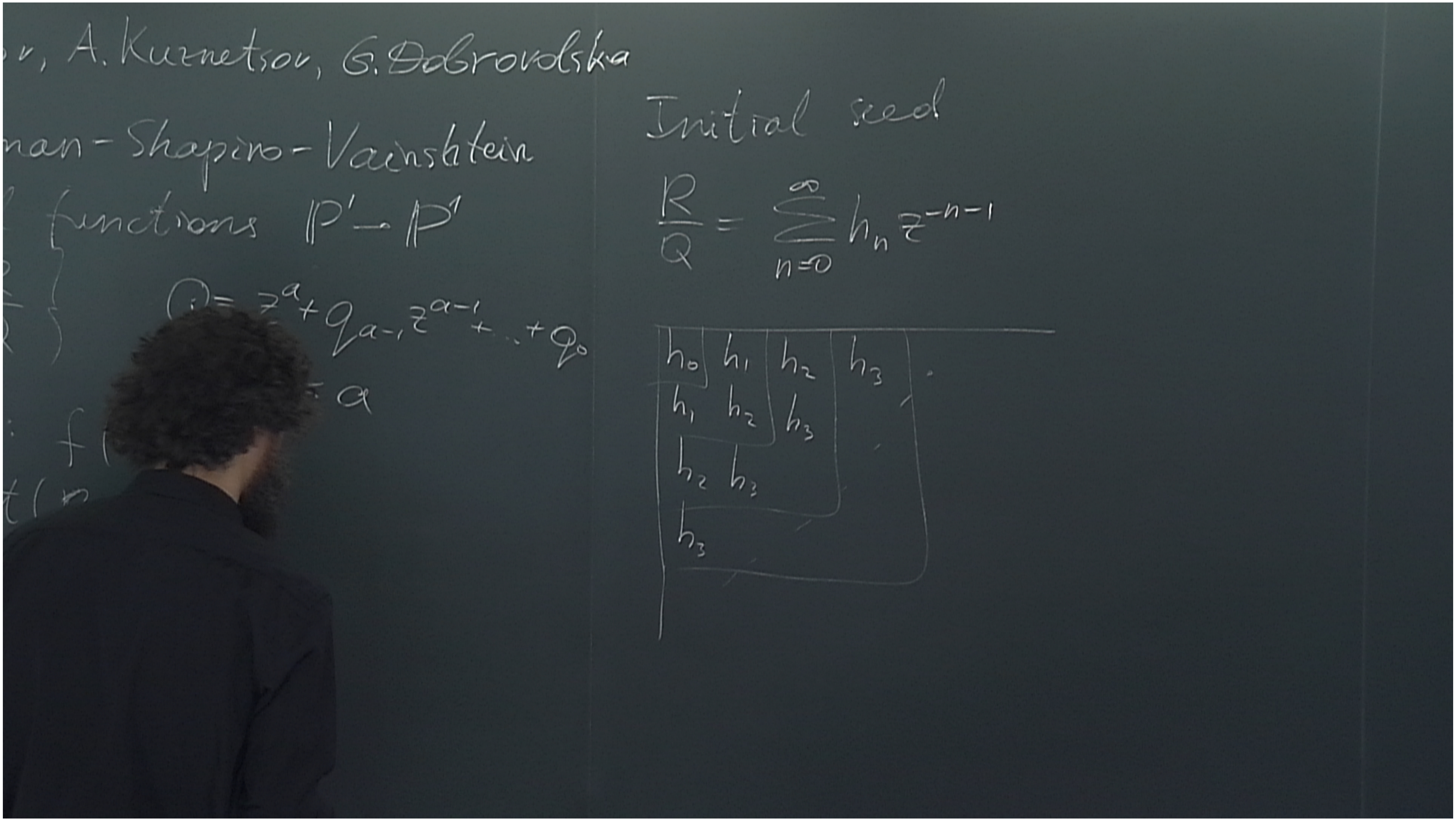
Cluster structure:

Initial seed



v, A. Kuznetsov, G. Dobrovolska

man-Shapiro-Vainshtein

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$Q = z^a + q_{a-1} z^{a-1} + \dots + q_0$

$= a$

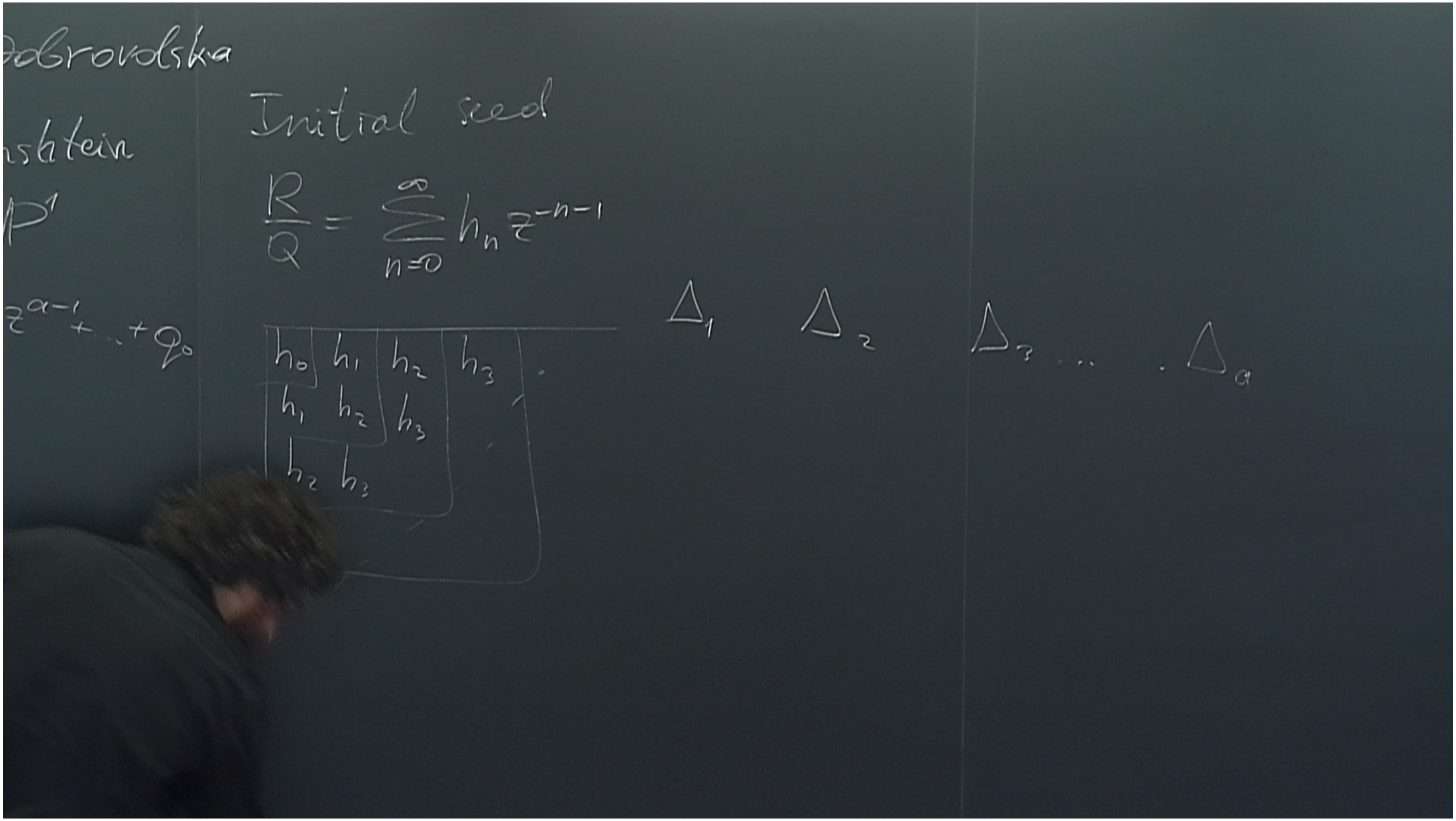
$: \quad f($

$t(n$

Initial seed

$$\frac{R}{Q} = \sum_{n=0}^{\infty} h_n z^{-n-1}$$

$$\begin{array}{|cccc}
\hline
h_0 & h_1 & h_2 & h_3 \quad . \\
h_1 & h_2 & h_3 & \\
h_2 & h_3 & & \\
h_3 & & & \\
\end{array}$$



Dobrovolska

nshtein

$\mathbb{P}^1$

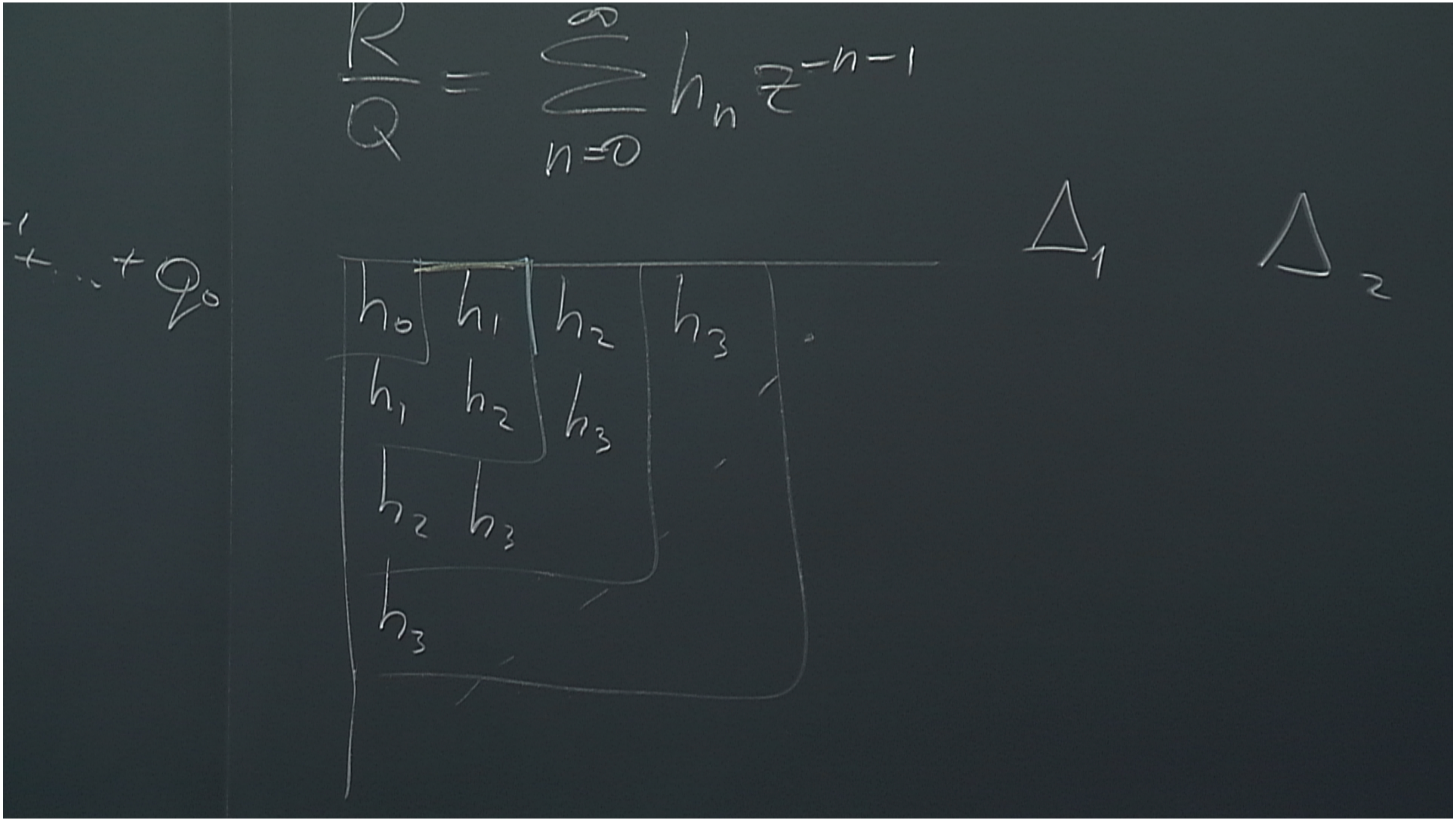
$z^{a-1}+\dots+q_0$

Initial seed

$$\frac{R}{Q}=\sum_{n=0}^{\infty}h_n z^{-n-1}$$

$$\begin{matrix} h_0 & h_1 & h_2 & h_3 & \cdot \\ h_1 & h_2 & h_3 & & \\ h_2 & h_3 & & & \end{matrix}$$

$\Delta_1 \quad \Delta_2 \quad \Delta_3 \dots \Delta_a$

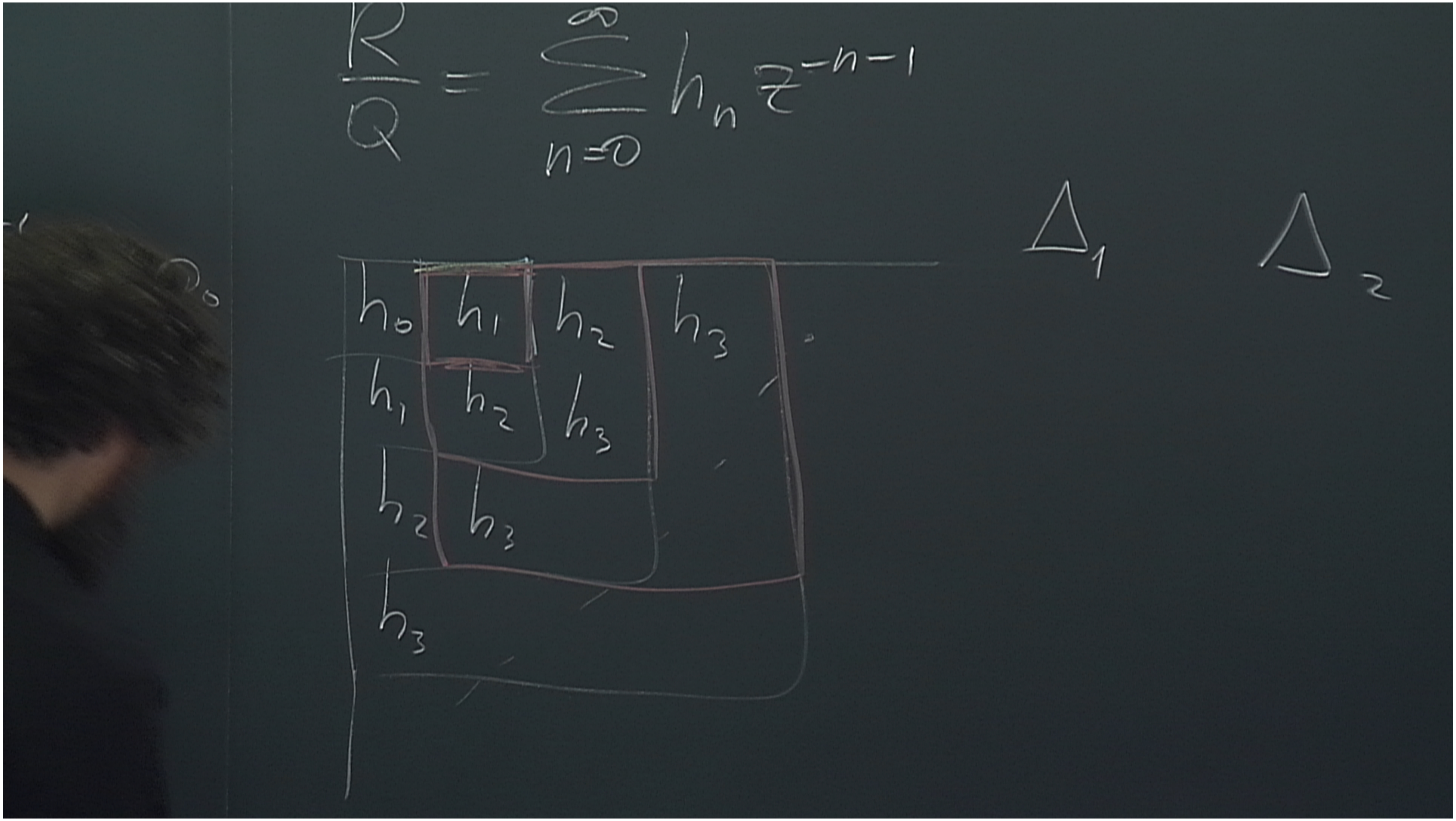


$$\frac{R}{Q} = \sum_{n=0}^{\infty} h_n z^{-n-1}$$

$^{-1}$ $+ \ldots + q_0$

$$\begin{array}{cccc} h_0 & h_1 & h_2 & h_3 \\ h_1 & h_2 & h_3 & \\ h_2 & h_3 & & \\ h_3 & & & \end{array}$$

Δ_1 Δ_2



$$\frac{R}{Q} = \sum_{n=0}^{\infty} h_n z^{-n-1}$$

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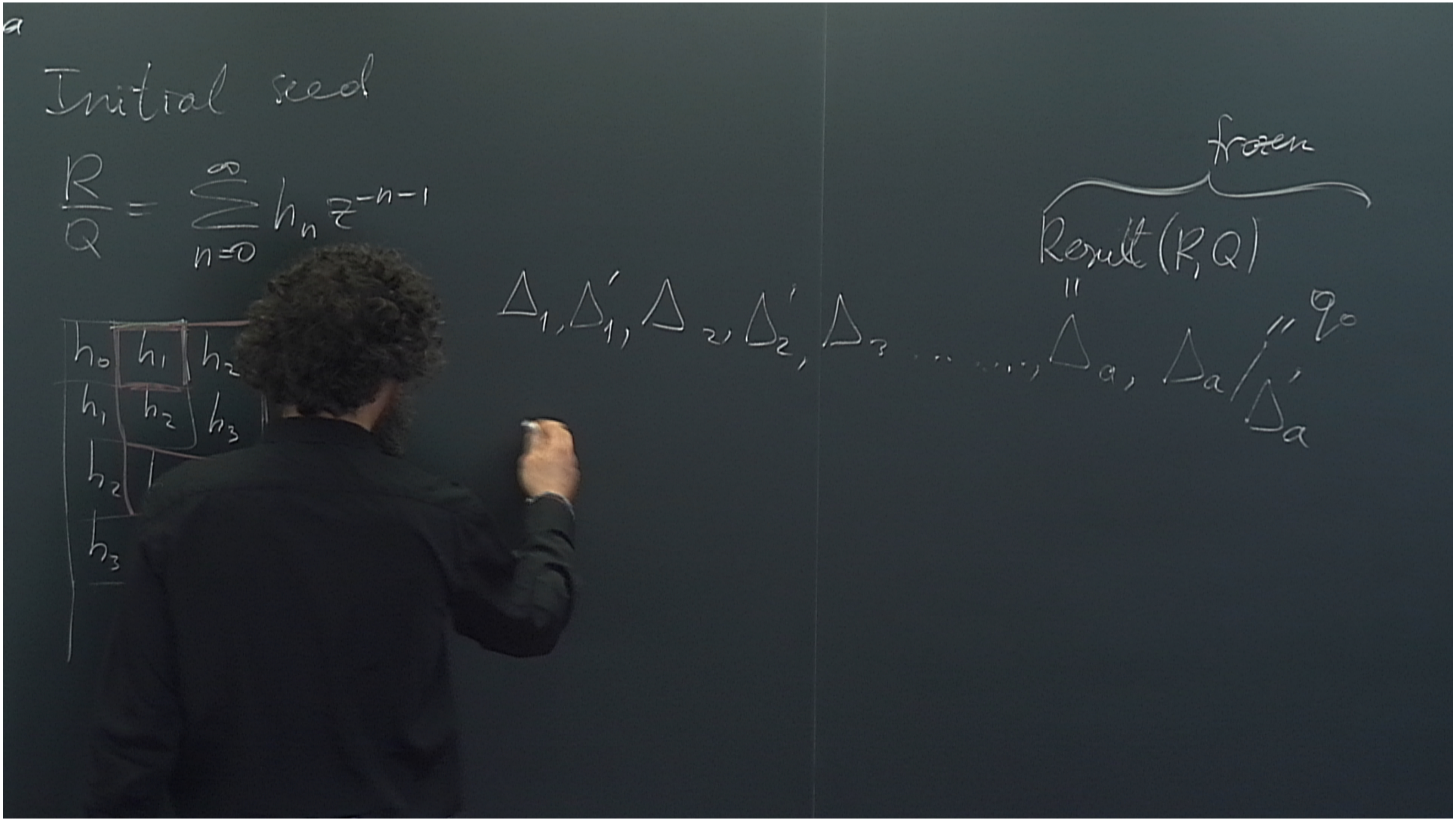


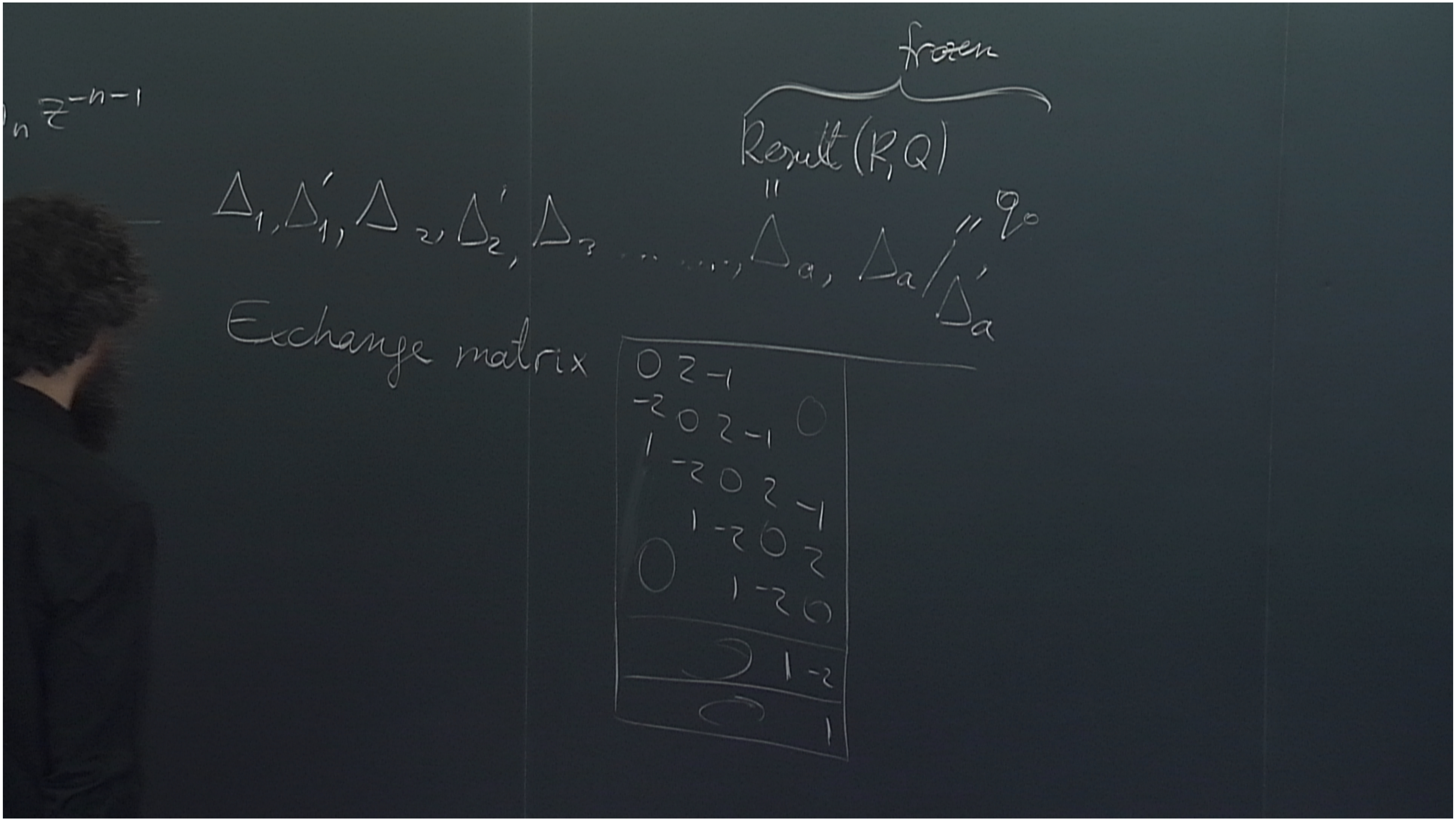
Initial seed

$$\frac{R}{Q} = \sum_{n=0}^{\infty} h_n z^{-n-1}$$

$$\begin{array}{|ccc} h_0 & \boxed{h_1} & h_2 \\ h_1 & h_2 & h_3 \\ h_2 & & \\ h_3 & & \end{array}$$

$$\overset{\overbrace{\text{Result}(R,Q)}^{\text{frozen}}}{}$$

$$\Delta_1, \Delta_1', \Delta_2, \Delta_2', \Delta_3 \ \ldots\ldots, \underset{}{\overset{\text{Result}(R,Q) \, =}{\Delta_a}}, \ \Delta_a / \Delta_a' = q_0$$

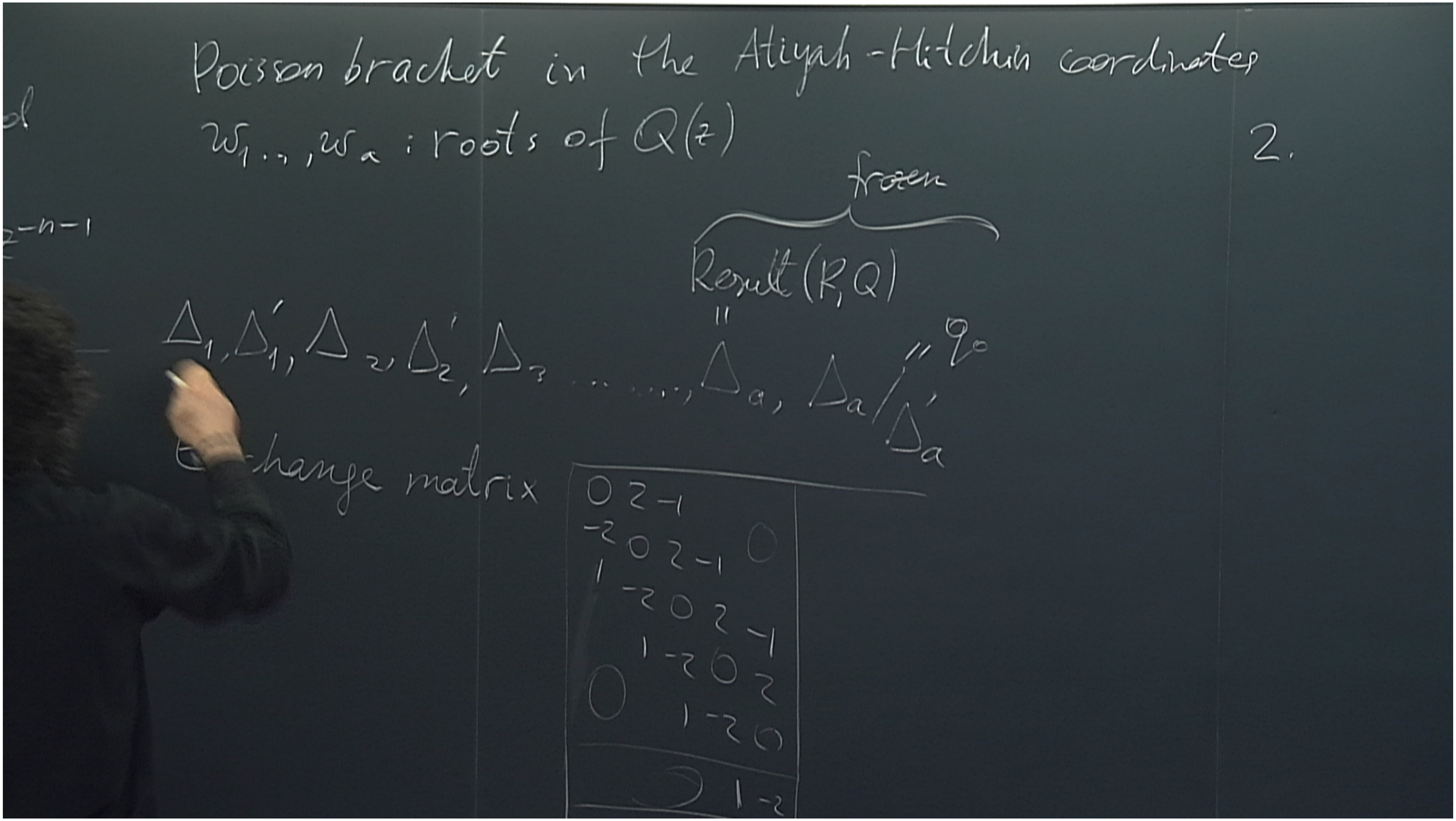


${}_n z^{-n-1}$

$$\Delta_1, \Delta_1', \Delta_2, \Delta_2', \Delta_3 \ldots\ldots, \overbrace{\overset{\mathrm{Result}(P,Q)}{\overset{\|}{\Delta_a}}, \overset{q_0}{\overset{\|}{\Delta_a / \Delta_a'}}}^{\text{frozen}}$$

Exchange matrix

$$\left[\begin{array}{ccccc} 0 & 2 & -1 & & \\ -2 & 0 & 2 & -1 & 0 \\ 1 & -2 & 0 & 2 & -1 \\ & 1 & -2 & 0 & 2 \\ 0 & & 1 & -2 & 0 \\ \hline & 0 & & 1 & -2 \\ \hline & 0 & & & 1 \end{array}\right]$$



d

z^{-n-1}

Poisson bracket in the Atiyah-Hitchin coordinates

$w_1, \dots, w_a$: roots of $Q(z)$

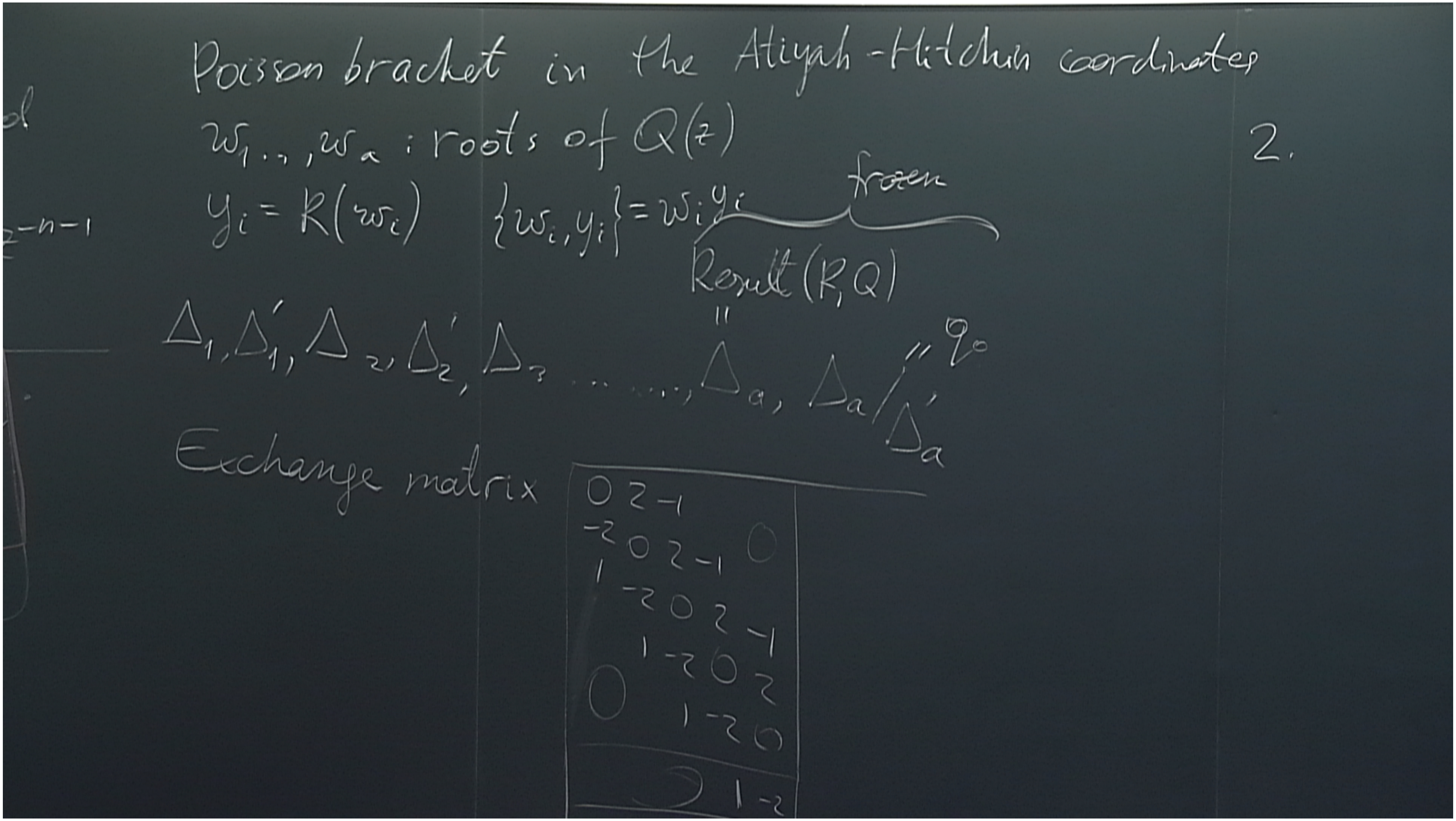
2.

$$\overbrace{\mathrm{Result}(R,Q)}^{\text{frozen}} = \Delta_a \qquad q_0 = \Delta_a/\Delta_a'$$

$$\Delta_1, \Delta_1', \Delta_2, \Delta_2', \Delta_3, \dots\dots, \Delta_a, \Delta_a/\Delta_a'$$

Exchange matrix

$$\begin{array}{|cccccc|}
0 & 2 & -1 & & & \\
-2 & 0 & 2 & -1 & & 0 \\
1 & -2 & 0 & 2 & -1 & \\
 & 1 & -2 & 0 & 2 & \\
0 & & 1 & -2 & 0 & \\
\hline
0 & & & & 1 & -2 \\
\hline
\end{array}$$



Poisson bracket in the Atiyah-Hitchin coordinates

2.

$-n-1$

$w_1, \dots, w_a$: roots of $Q(z)$

$y_i = R(w_i)$ $\quad \{w_i, y_i\} = w_i y_i$

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Exchange matrix

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0 & 2 & -1 & & & 0 & \\
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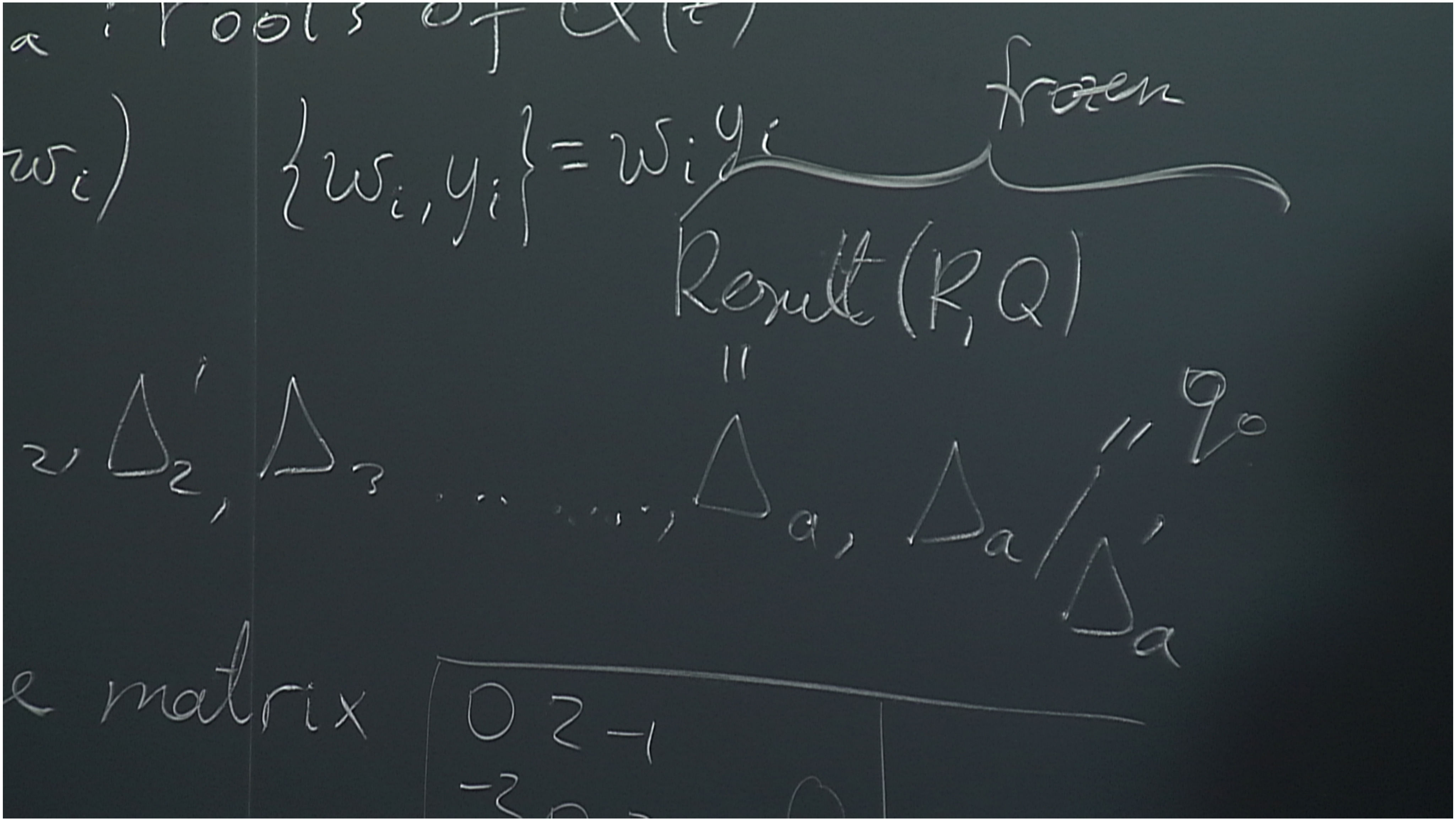
0 & & 1 & -2 & 0 & & \\
\hline
 & 0 & & 1 & -2 & & \\
\end{array}\right.$$



$_a$: roots of $Q(z)$

$w_i)$ $\{w_i, y_i\} = w_i y_i$

frozen

Result(R, Q)

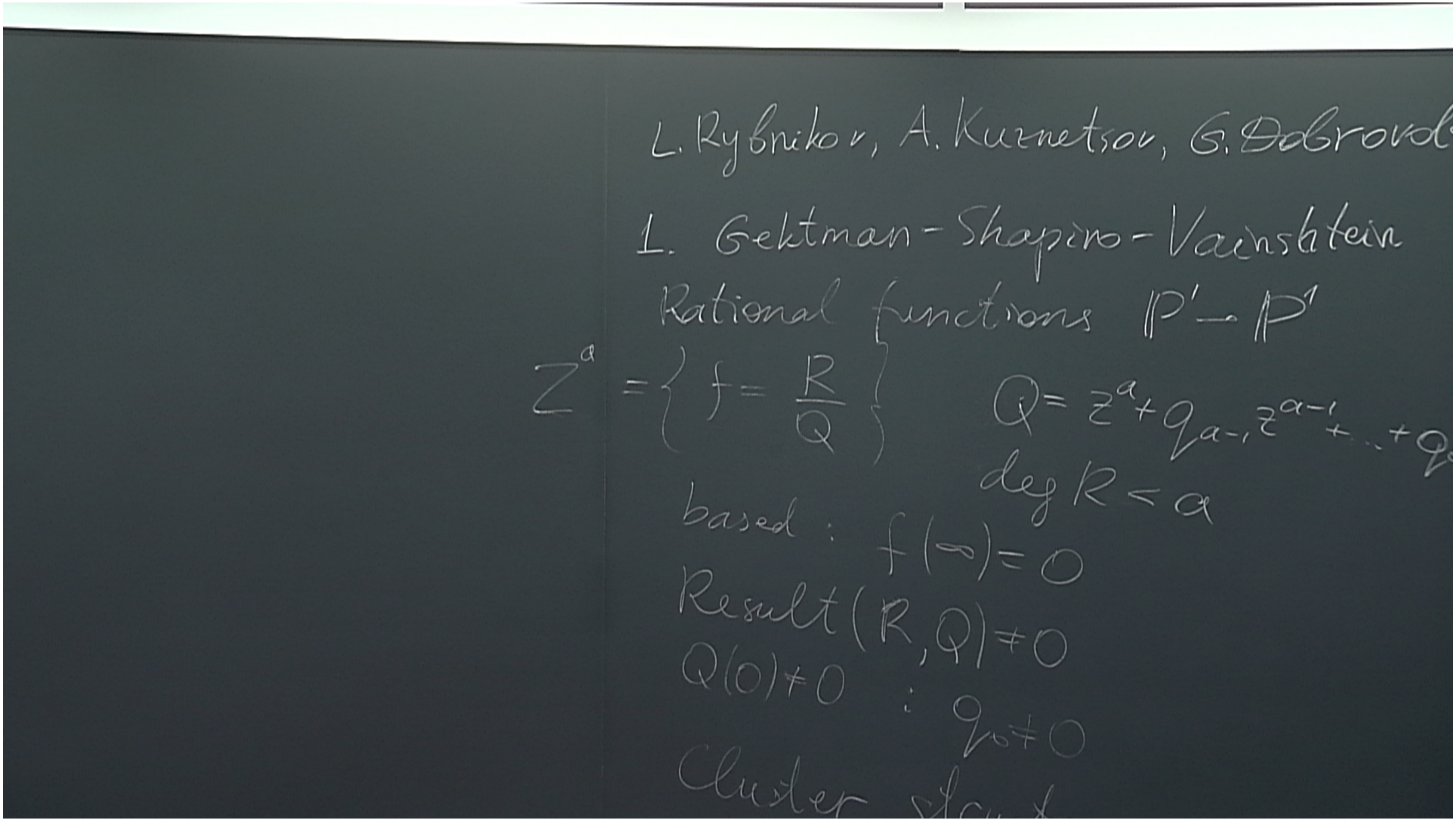
$=$

$_2, \Delta_2', \Delta_3, \ldots\ldots, \Delta_a, \Delta_a / \Delta_a' = q_0$

e matrix

0 2 −1

−3 0



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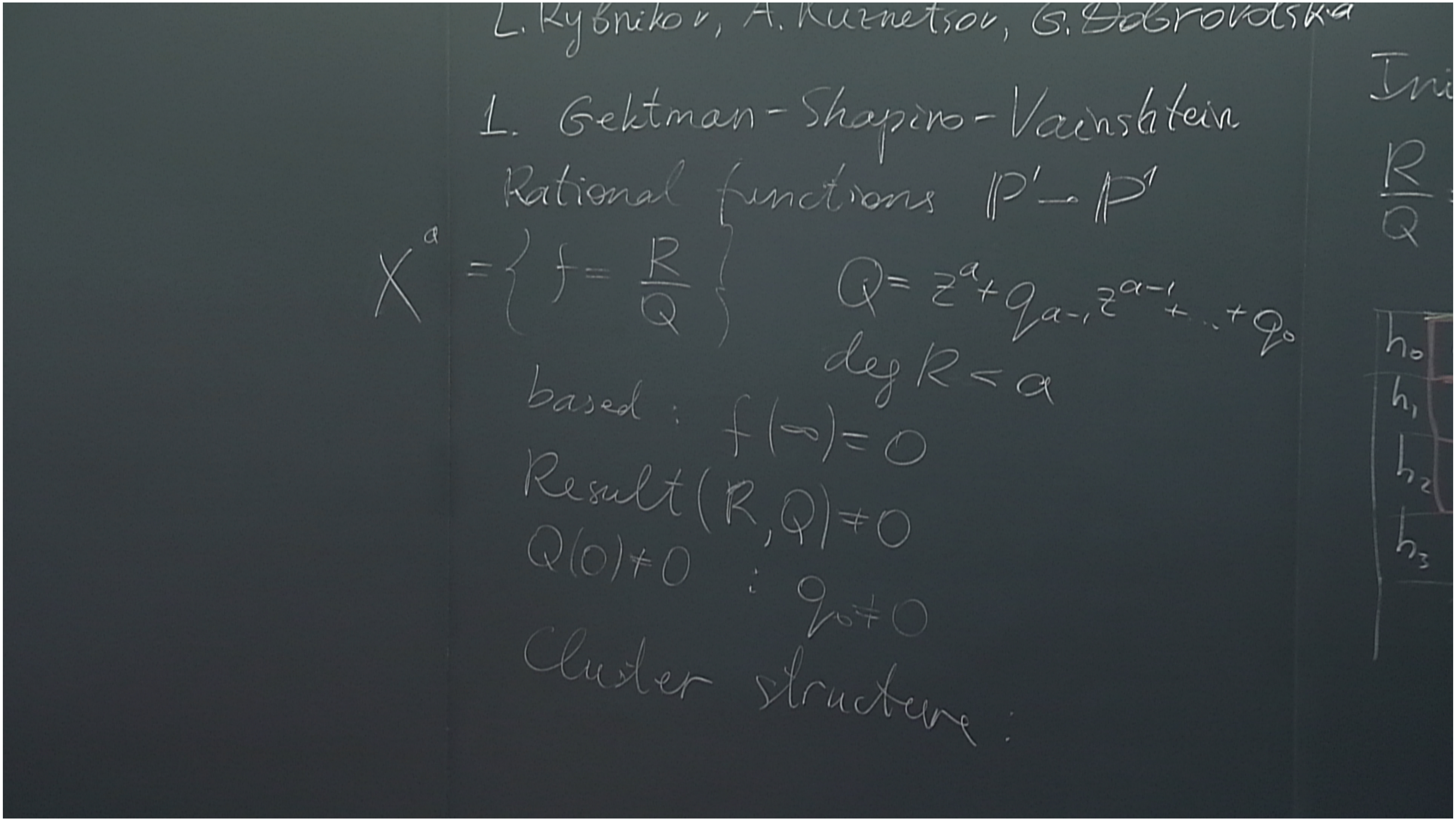
Cluster



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Cluster structure:

Ini

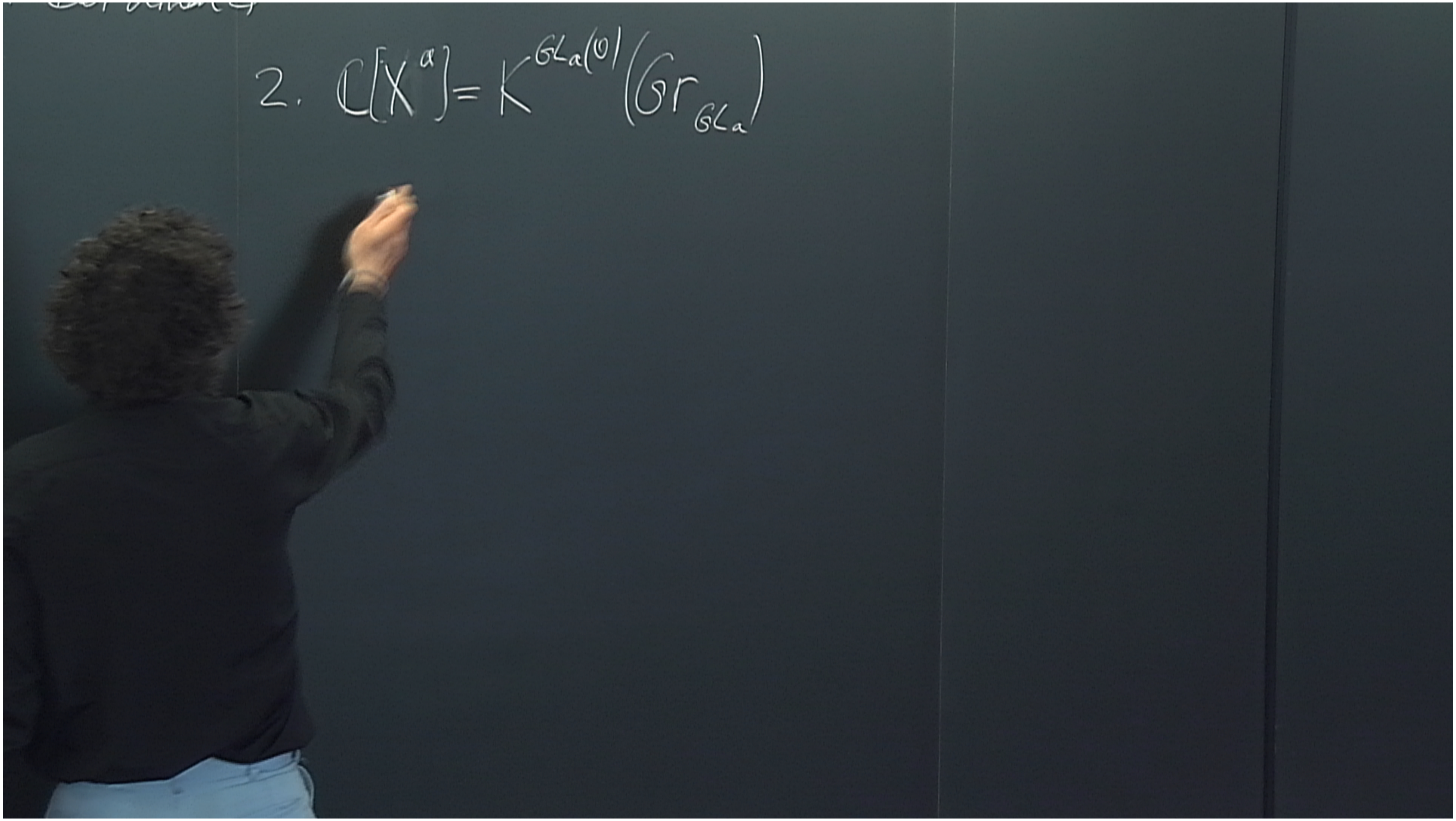
$\frac{R}{Q}$

h_0

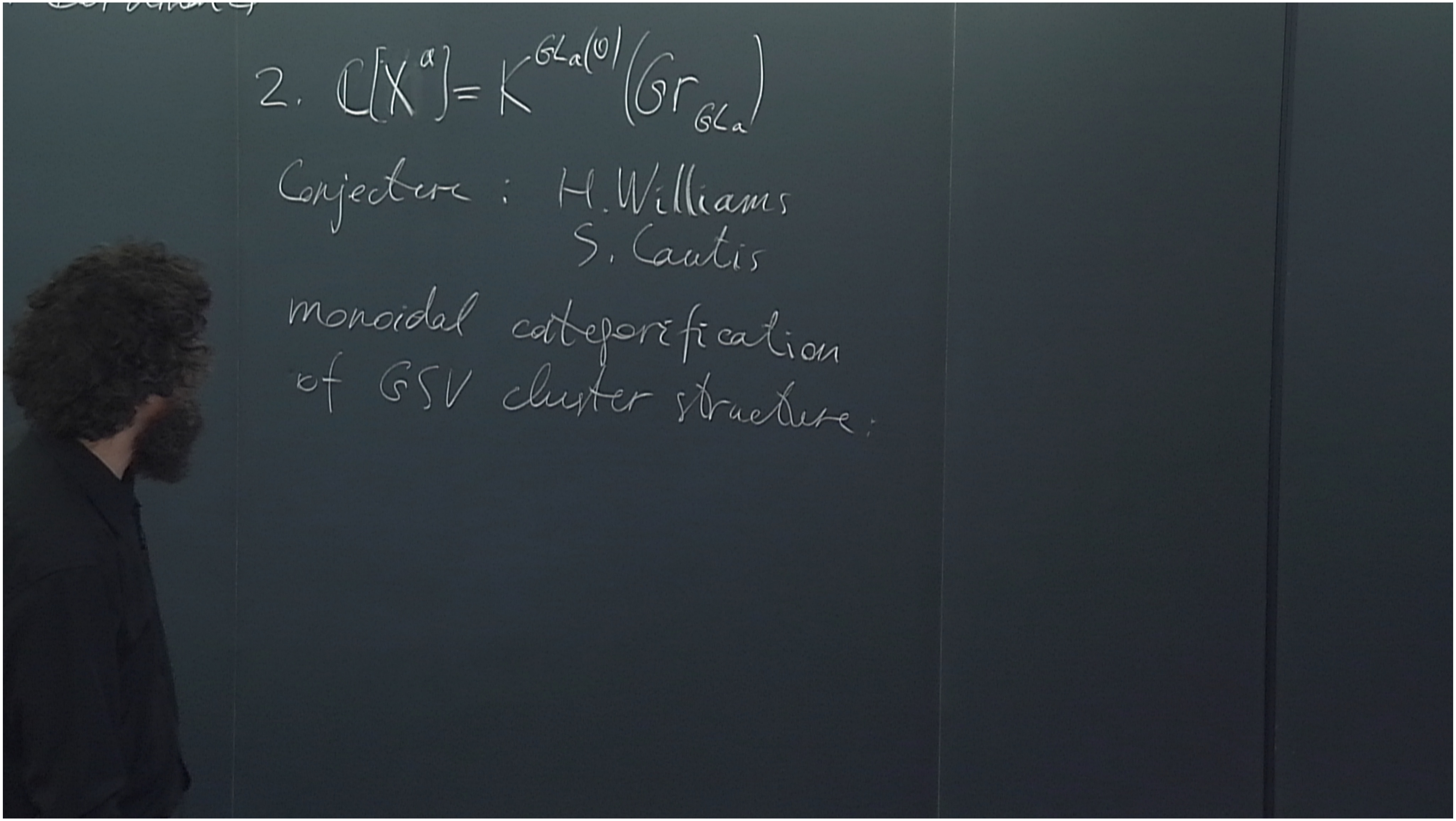
h_1

h_2

h_3



2. $\mathbb{C}[X^a] = K^{GL_a(O)}(Gr_{GL_a})$



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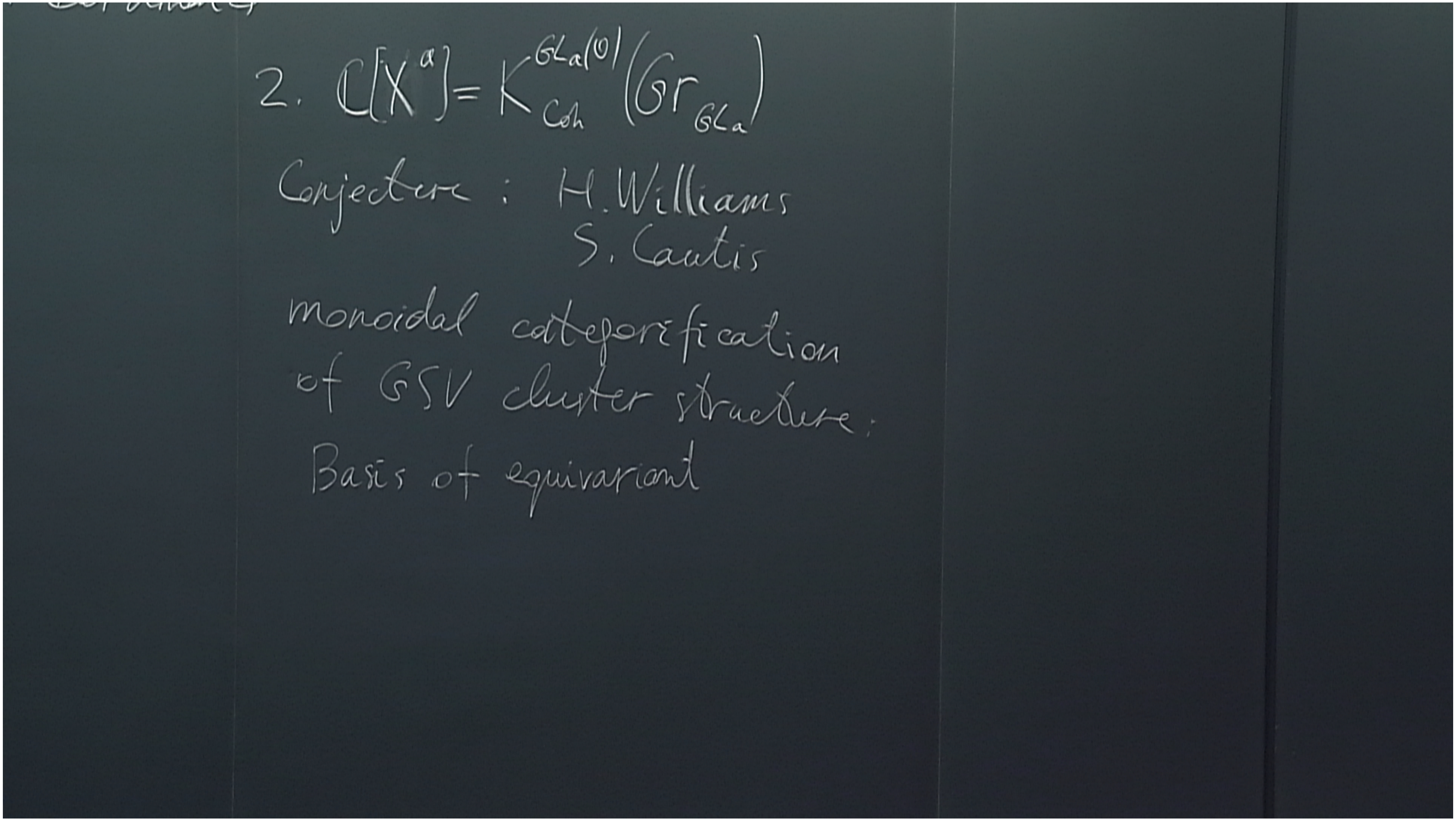
Conjecture: H. Williams
S. Cautis

monoidal categorification of GSV cluster structure:



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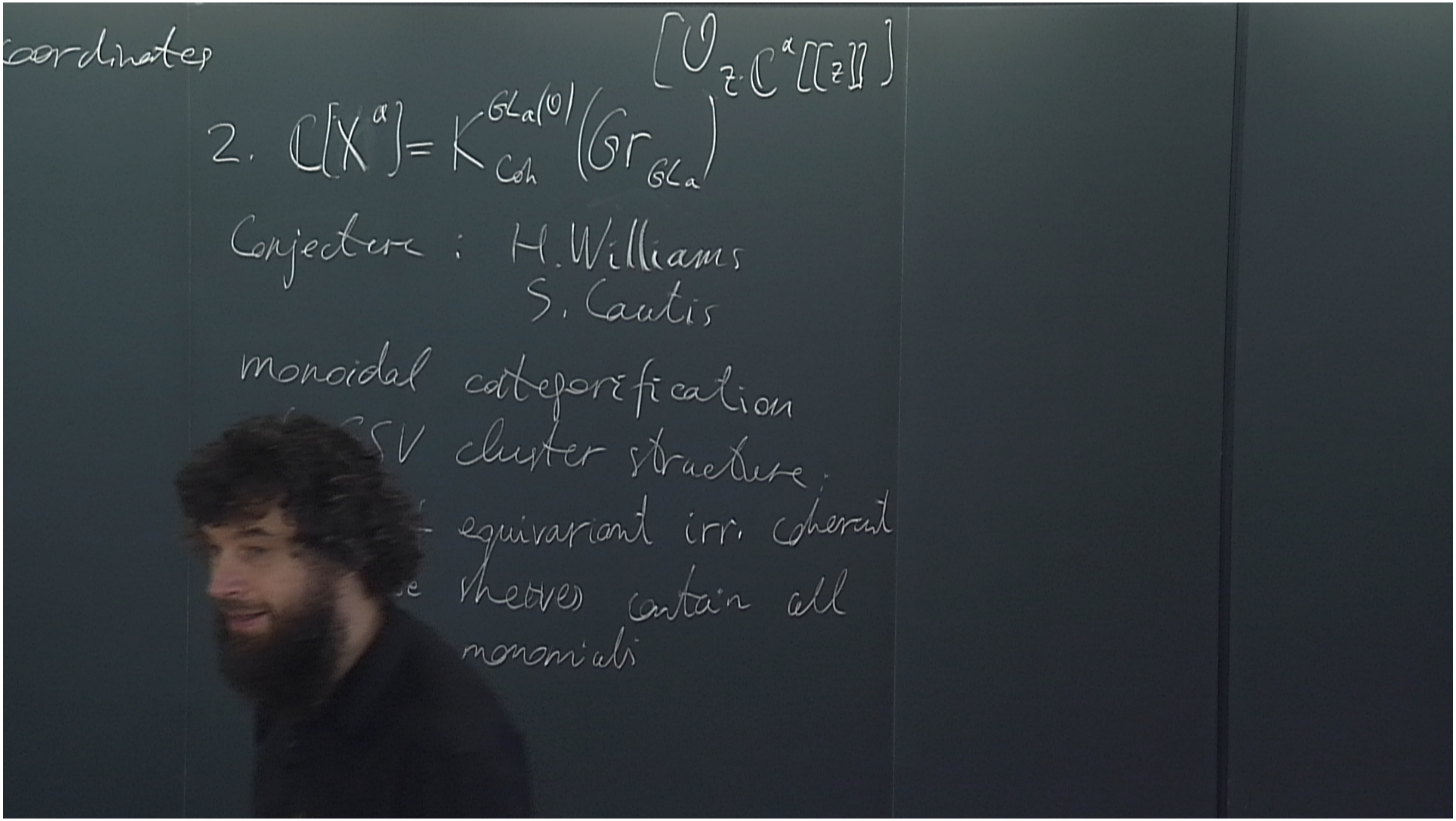
Basis of equivariant irr. coherent
perverse sheaves contain all
cluster monomials



coordinates

$[\mathcal{O}_z \cdot \mathbb{C}^a[[z]]]$

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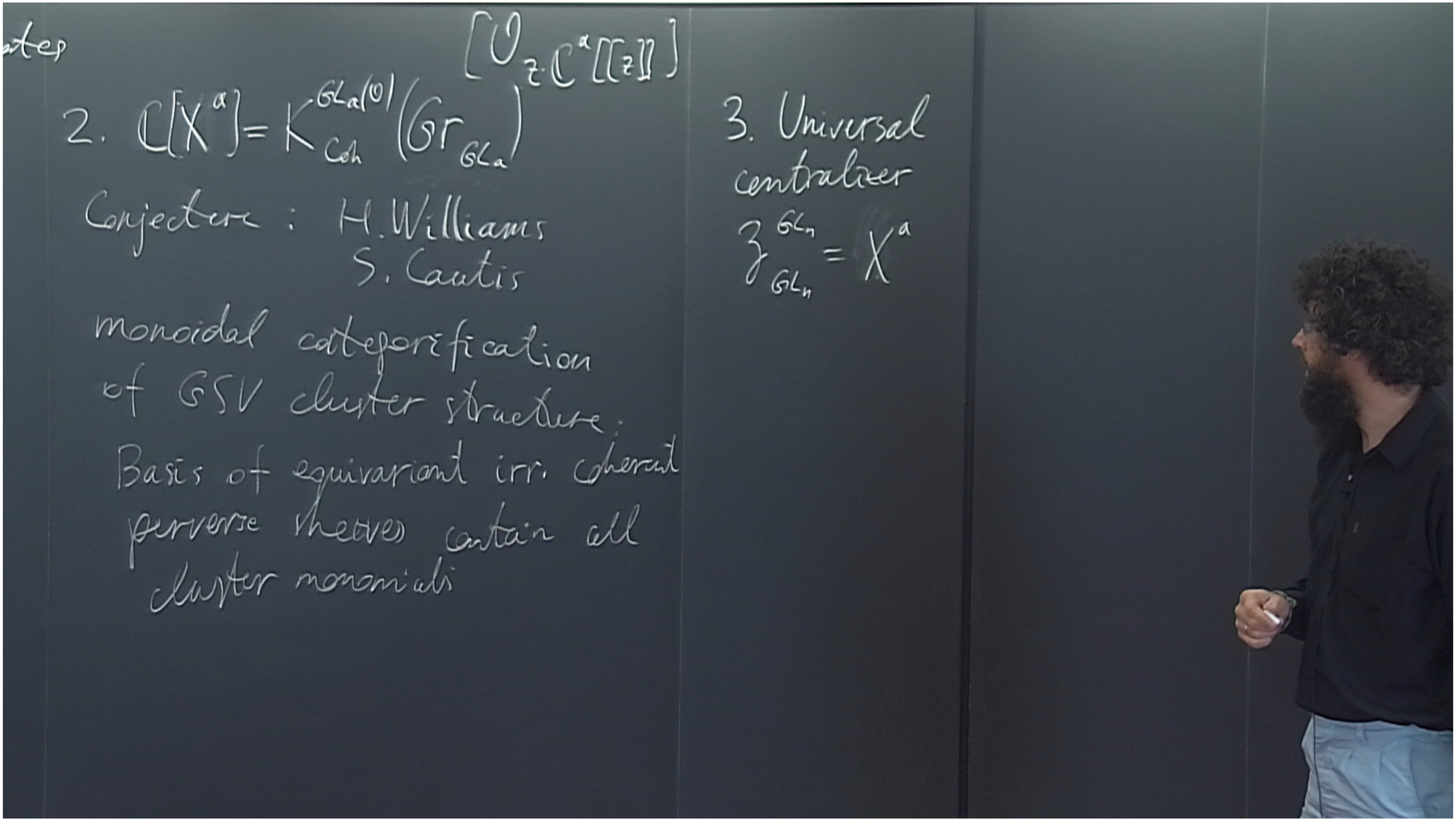
monoidal categorification
GSV cluster structure;
equivariant irr. coherent
sheaves contain all
monomials



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monoidal categorification of GSV cluster structure:

Basis of equivariant irr. coherent perverse sheaves contain all cluster monomials

3. Universal centralizer

$Z^{GL_n}_{GL_n} = X^a$

rent

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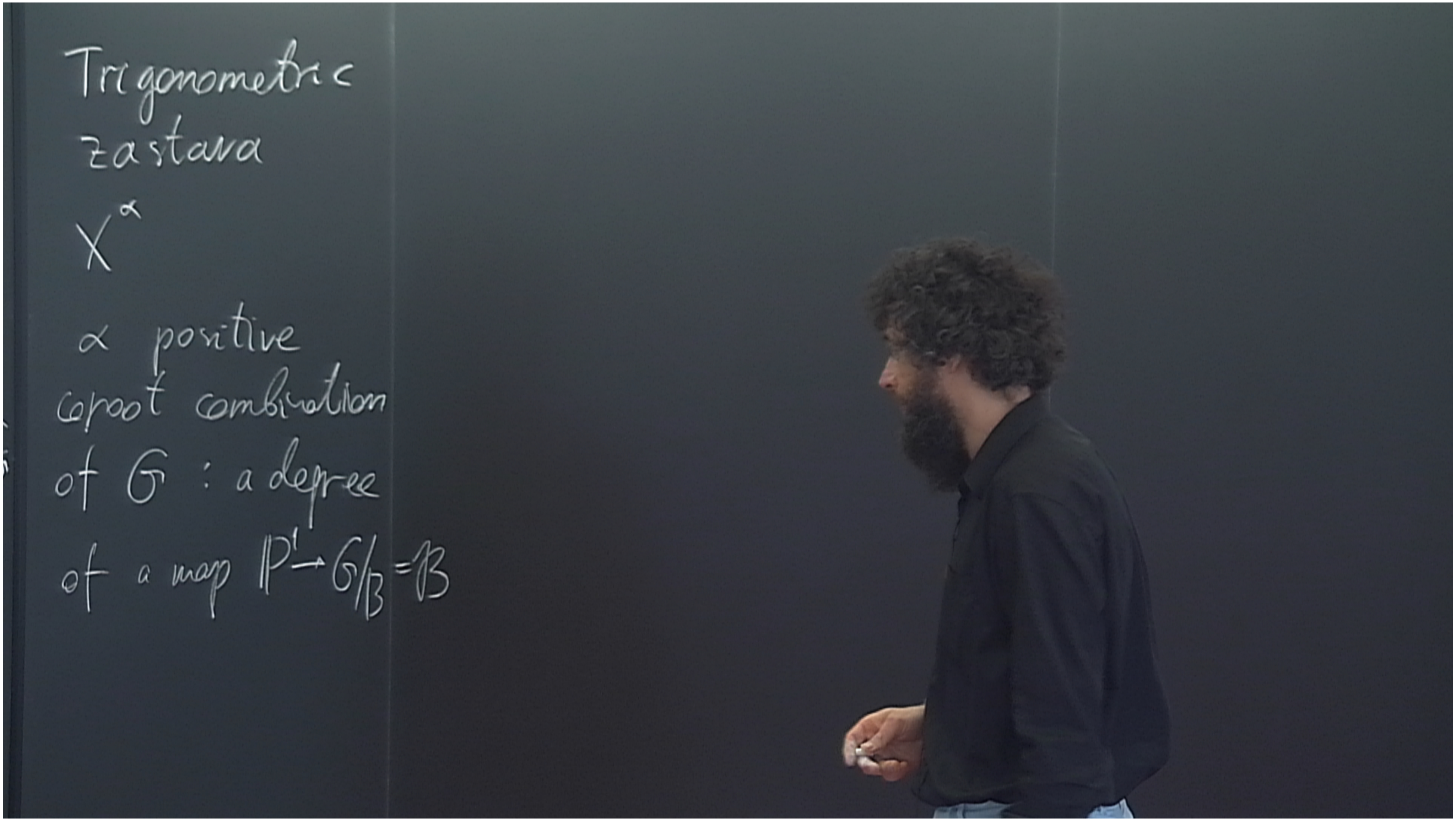
$$\mathfrak{Z}^{GL_n}_{GL_n} = X^a$$

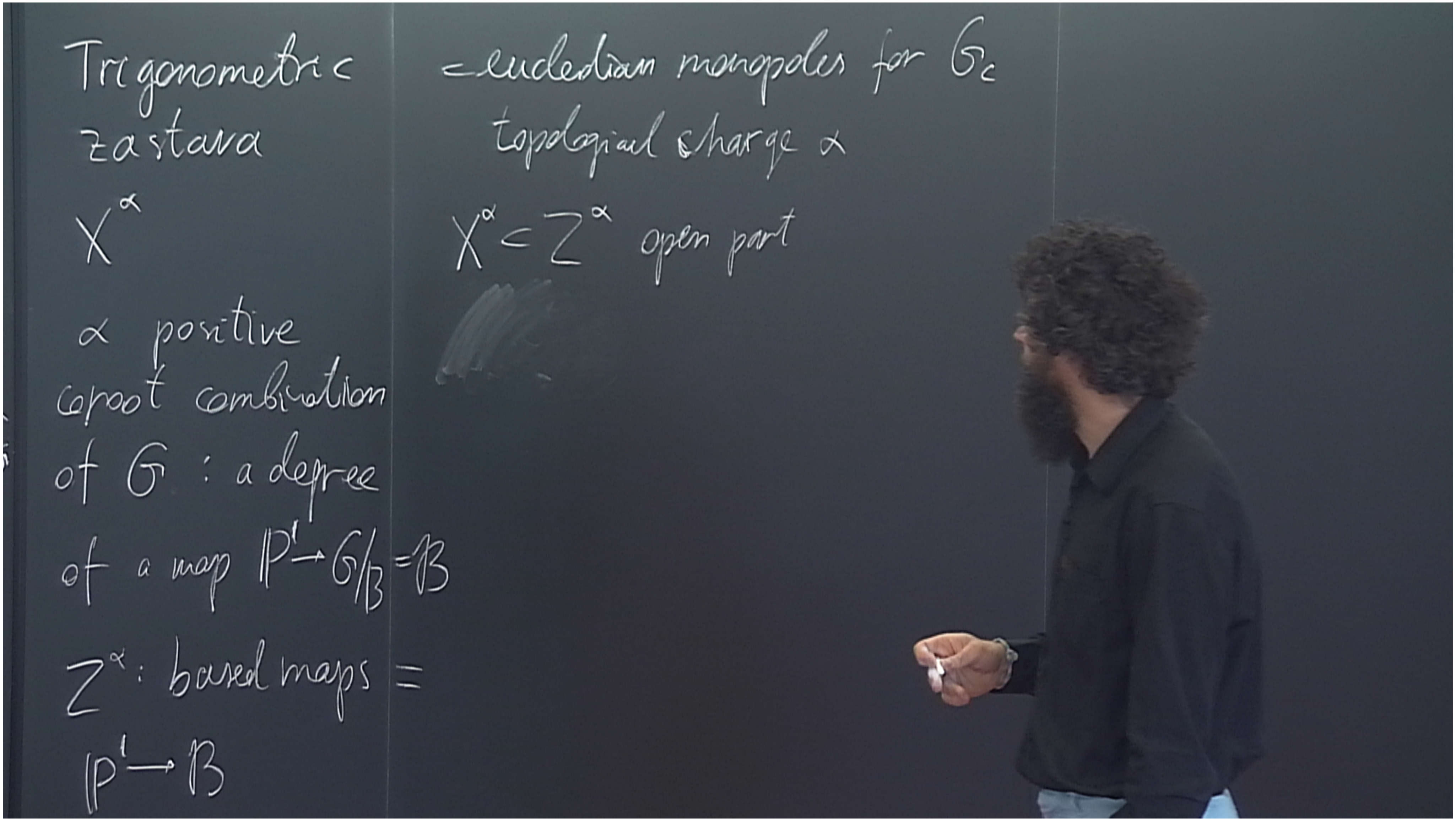
$$\mathfrak{Z}^{G}_{G} \simeq \mathrm{Spe}\, K^{G(O)} \mathrm{Gr}_G$$



Trigonometric zastava

X^{α}

α positive coroot combination of G : a degree of a map $\mathbb{P}^1 \to G/B = \mathcal{B}$



Trigonometric zastava

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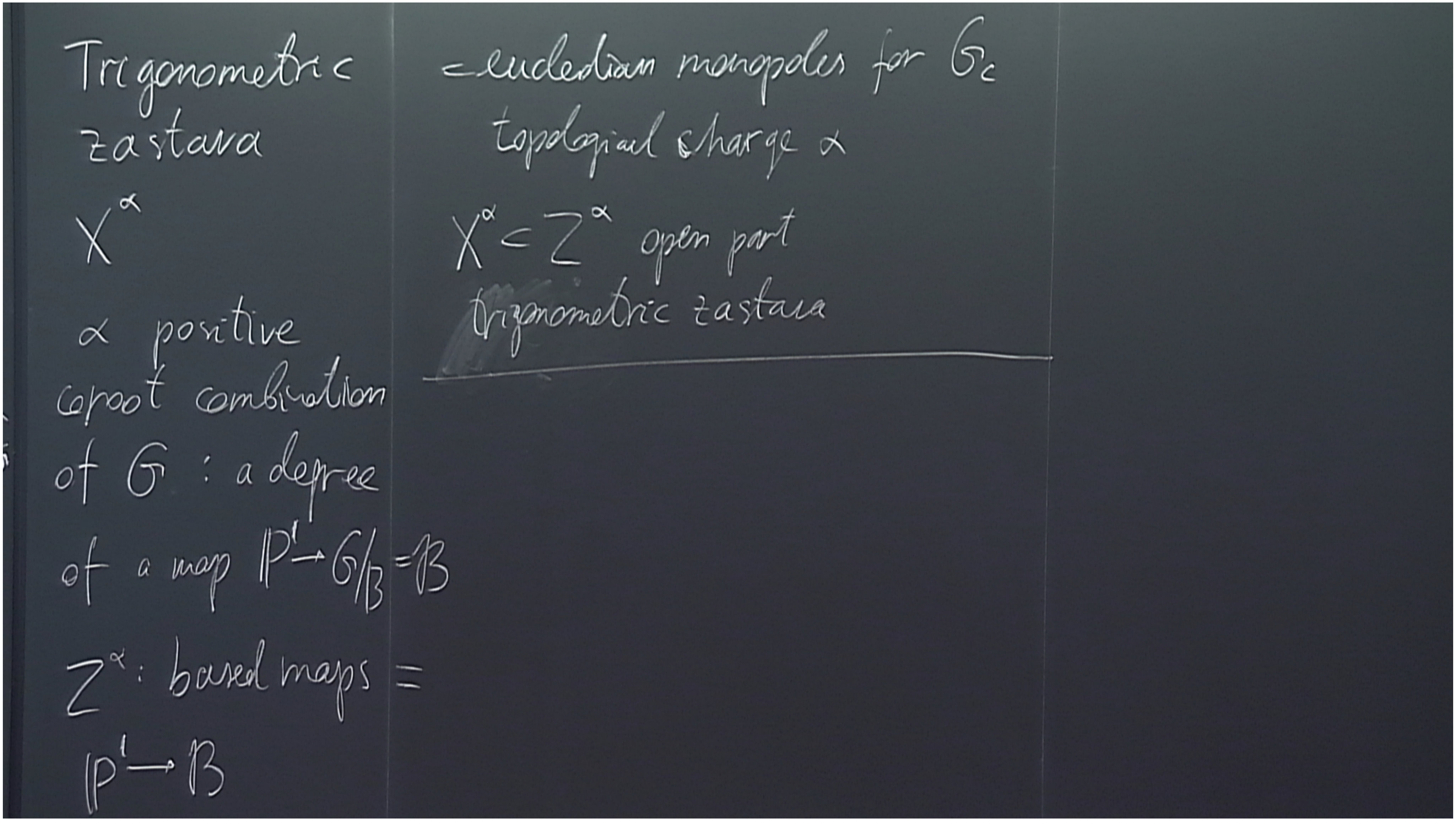
Z^{α}: based maps $=$ $\mathbb{P}^1 \to \mathcal{B}$

$=$ euclidean monopoles for G_c topological charge α

$X^{\alpha} \subset Z^{\alpha}$ open part



Trigonometric zastava

X^α

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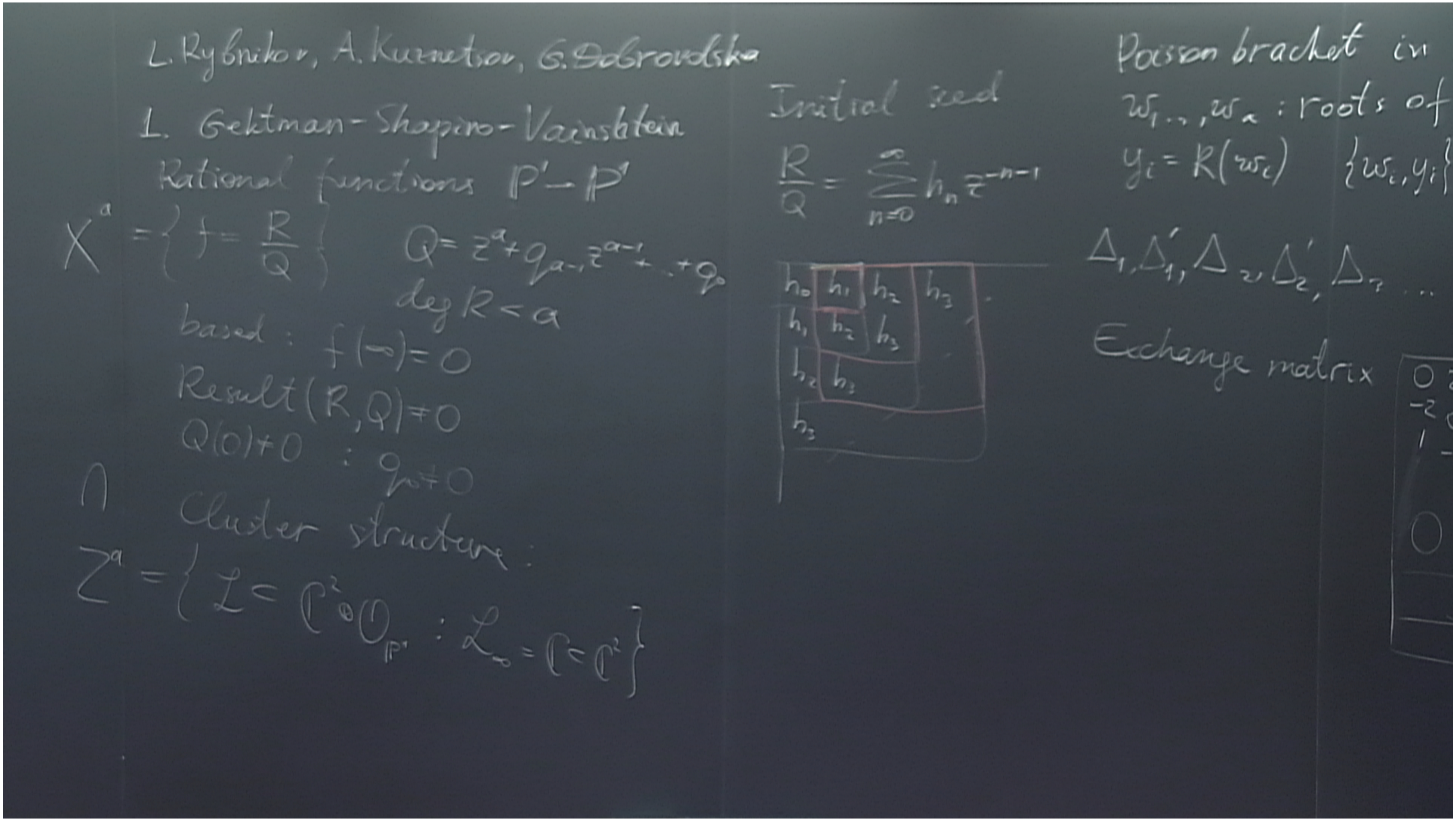
Z^α : based maps = $\mathbb{P}^1 \to \mathcal{B}$

= eucledian monopoles for G_c topological charge α

$X^\alpha \subset Z^\alpha$ open part trigonometric zastava



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$\deg R < a$

based: $f(\infty) = 0$

$\mathrm{Result}(R, Q) \neq 0$

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$\cap$ Cluster structure:

$Z^a = \left\{ \mathcal{L} \subset \mathbb{C}^2 \otimes \mathcal{O}_{\mathbb{P}^1} : \mathcal{L}_\infty = \mathbb{C} \subset \mathbb{C}^2 \right\}$

Initial seed

$$\frac{R}{Q} = \sum_{n=0}^{\infty} h_n z^{-n-1}$$

$$\begin{matrix} h_0 & h_1 & h_2 & h_3 \\ h_1 & h_2 & h_3 & \\ h_2 & h_3 & & \\ h_3 & & & \end{matrix}$$

Poisson bracket in

$w_1, \dots, w_a$: roots of

$y_i = R(w_i)$ $\quad \{w_i, y_i\}$

$\Delta_1, \Delta_1', \Delta_2, \Delta_2', \Delta_3 \dots$

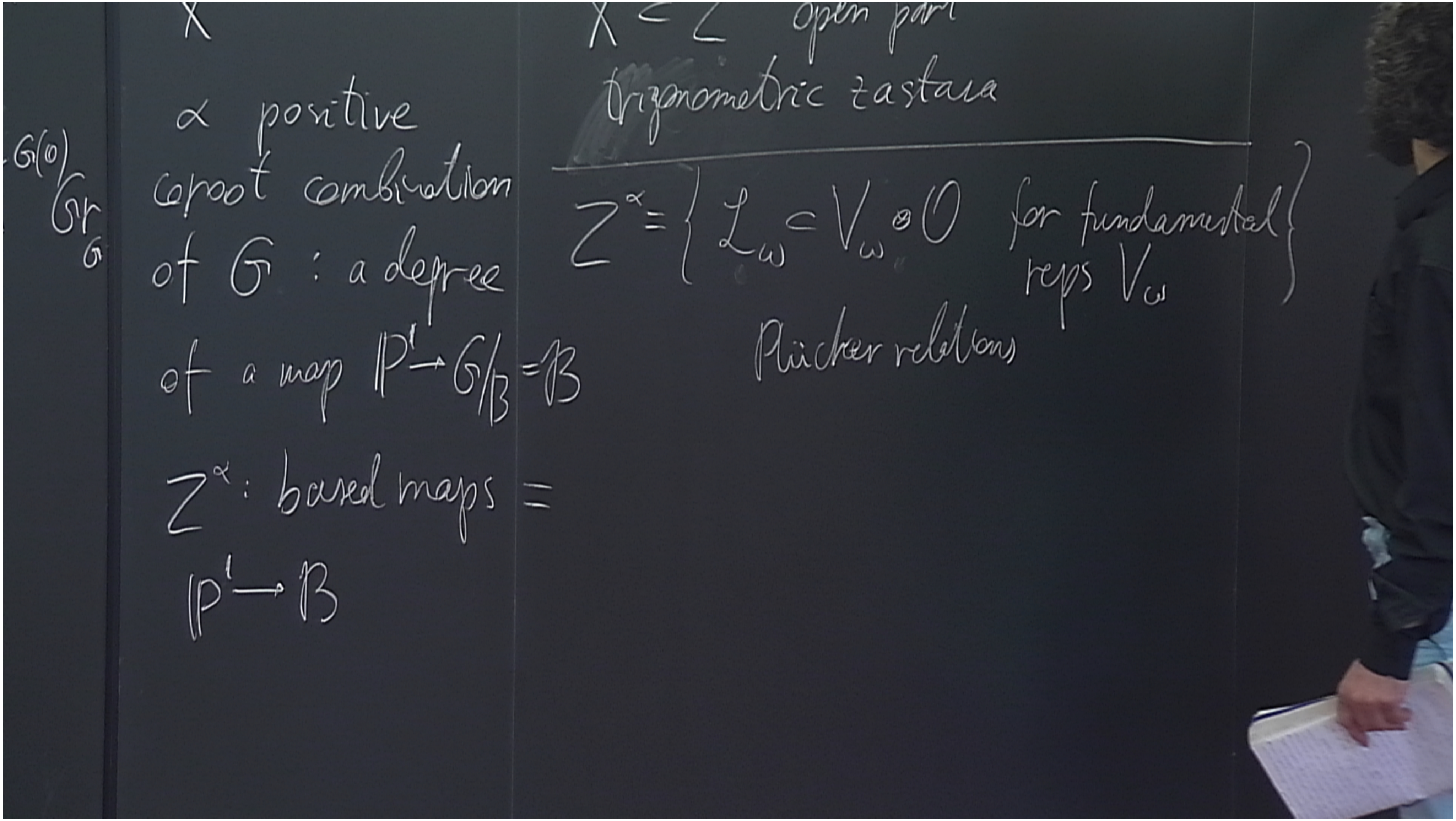
Exchange matrix

0

-2

1

0



$G(\mathcal{O})/Gr_G$

X

α positive coroot combination of G : a degree of a map $\mathbb{P}^1 \to G/B = \mathcal{B}$

Z^α : based maps $=$ $\mathbb{P}^1 \to \mathcal{B}$

$X \subset Z$ open part

trigonometric zastava

$Z^\alpha = \{ \mathcal{L}_\omega \subset V_\omega \otimes \mathcal{O}$ for fundamental reps $V_\omega \}$

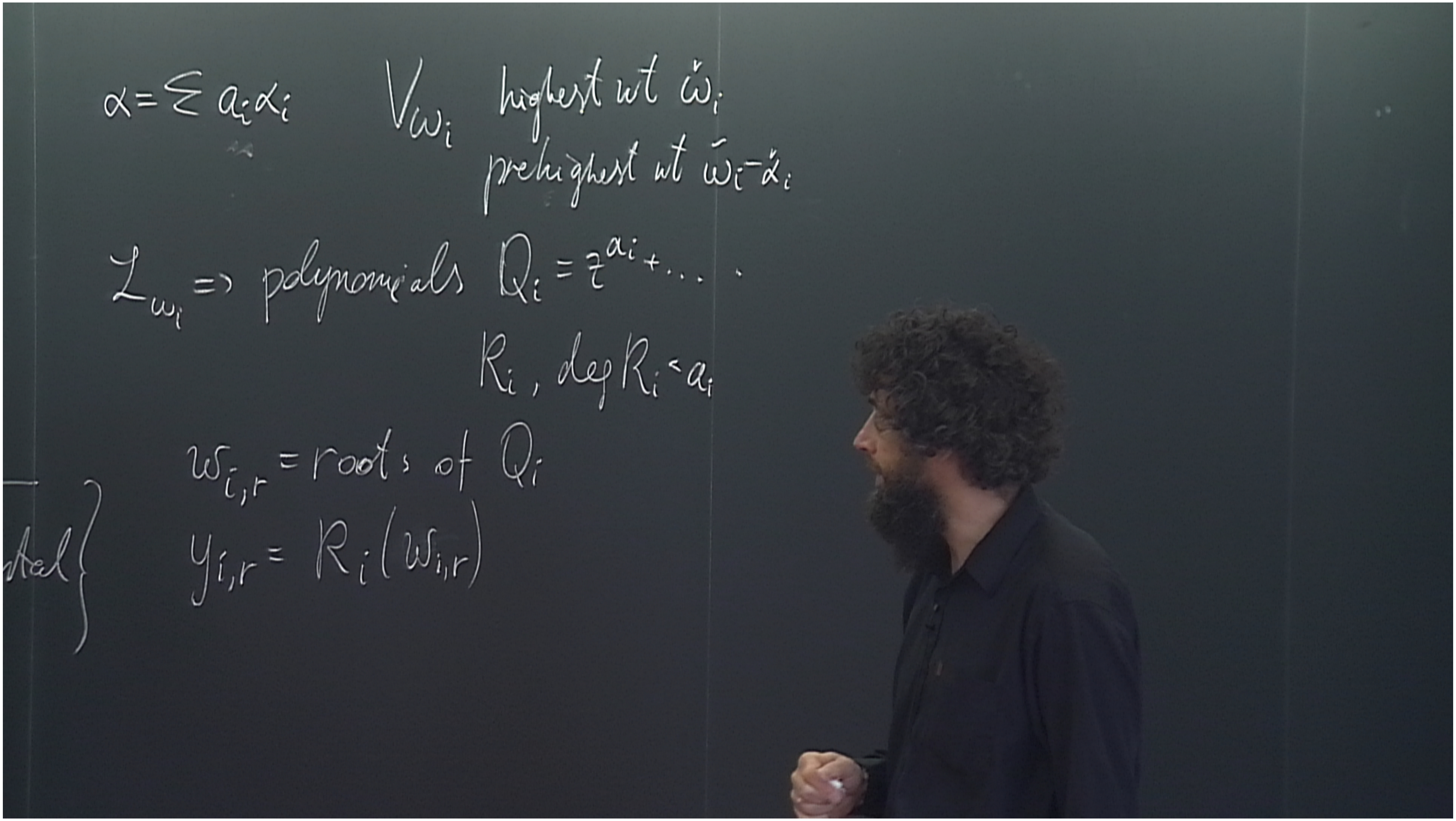
Plücker relations

$\alpha = \sum a_i \alpha_i$ $\quad V_{\omega_i}$ highest wt $\check{\omega}_i$

prehighest wt $\check{\omega}_i - \check{\alpha}_i$

$\mathcal{L}_{\omega_i} \Rightarrow$ polynomials $Q_i = z^{a_i} + \ldots$.

R_i, $\deg R_i < a_i$



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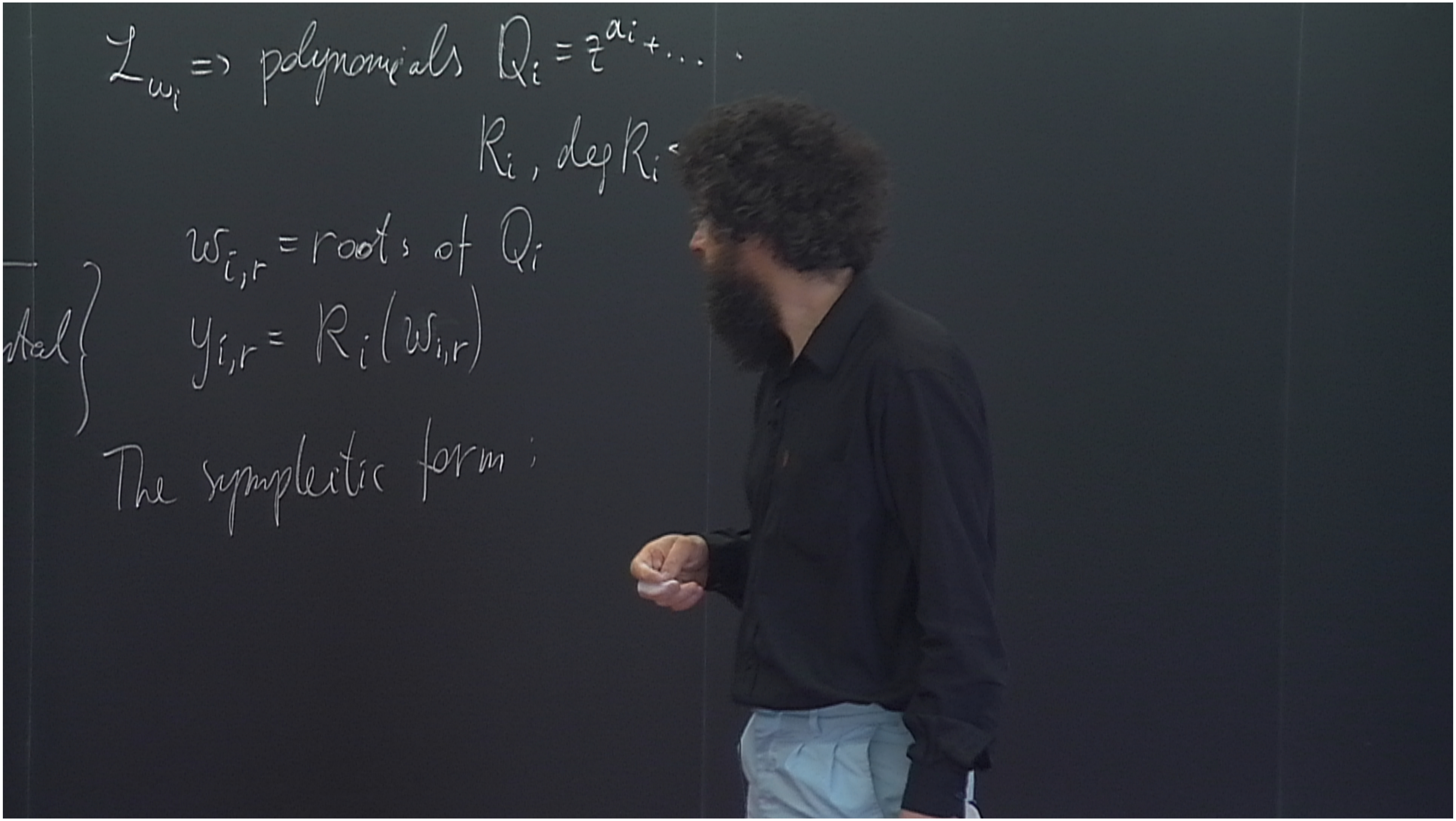
R_i, $\deg R_i < a_i$

$w_{i,r}$ = roots of Q_i

tal $\Big\{$ $y_{i,r} = R_i(w_{i,r})$



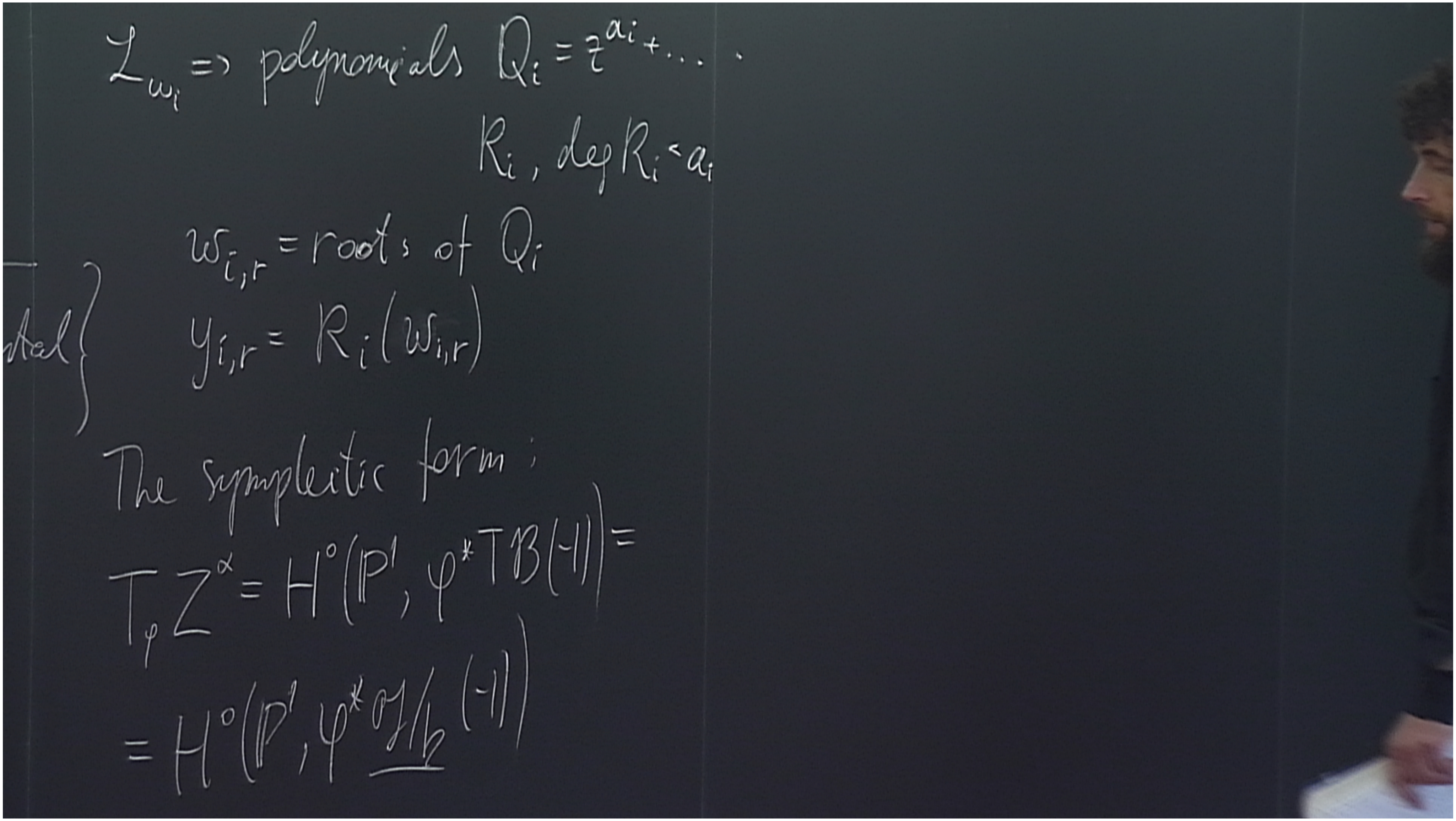
$\mathcal{L}_{w_i} \Rightarrow$ polynomials $Q_i = z^{a_i} + \dots$.

R_i, $\deg R_i <$

$w_{i,r}$ = roots of Q_i

tal $\Big\{$ $y_{i,r} = R_i(w_{i,r})$

The symplectic form :



$\mathcal{L}_{w_i} \Rightarrow$ polynomials $Q_i = z^{a_i} + \dots$.

R_i, $\deg R_i < a_i$

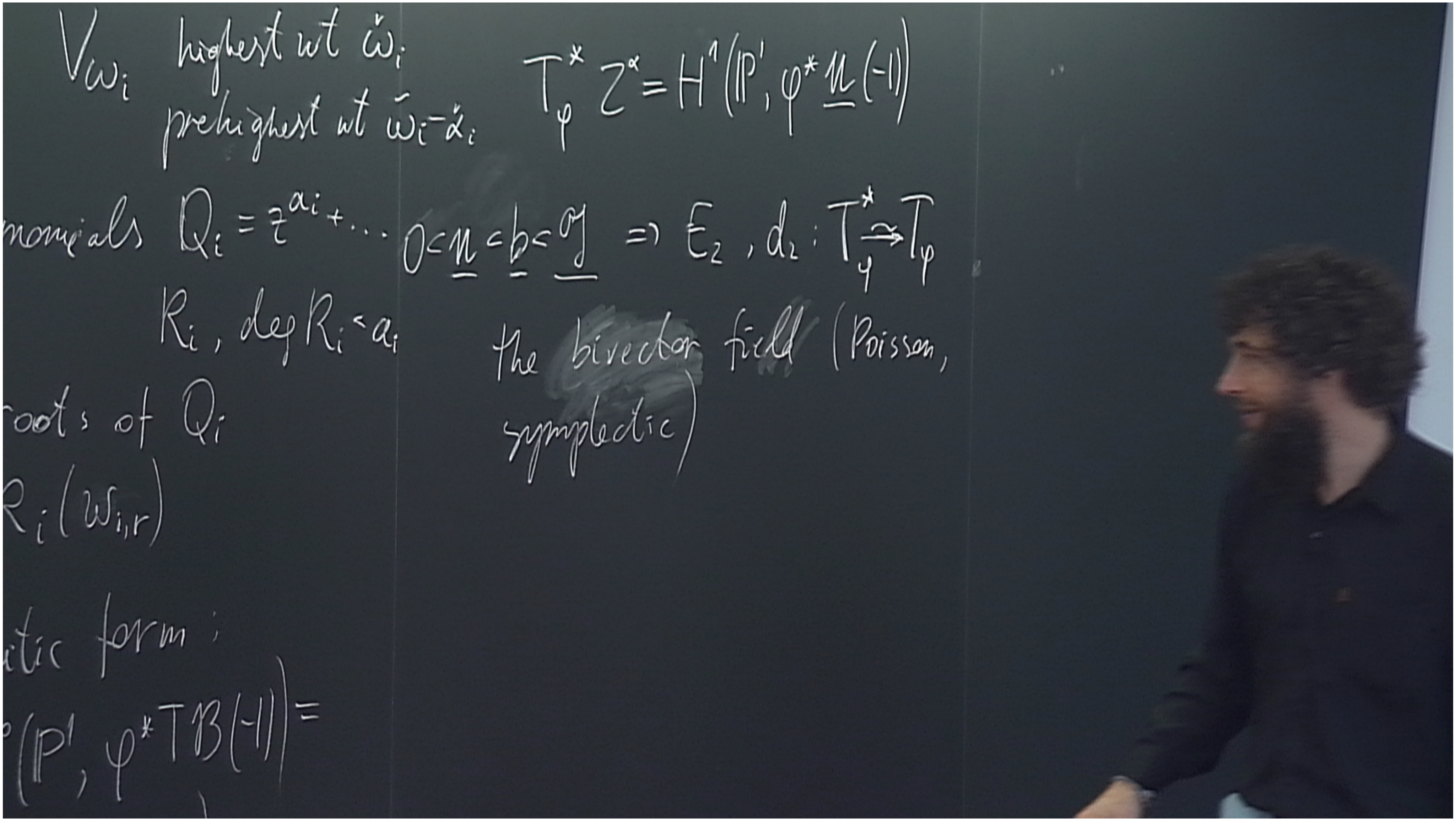
$w_{i,r}$ = roots of Q_i

tal $\left\{ y_{i,r} = R_i(w_{i,r}) \right.$

The symplectic form:

$$T_\varphi Z^\alpha = H^0(\mathbb{P}^1, \varphi^* T\mathcal{B}(-1)) =$$

$$= H^0(\mathbb{P}^1, \varphi^* \underline{\mathfrak{g}/\mathfrak{b}}(-1))$$



V_{ω_i} highest wt $\check{\omega}_i$

prehighest wt $\check{\omega}_i - \check{\alpha}_i$

nomials $Q_i = z^{a_i} + \dots$

R_i, $\deg R_i < a_i$

roots of Q_i

$R_i(w_{i,r})$

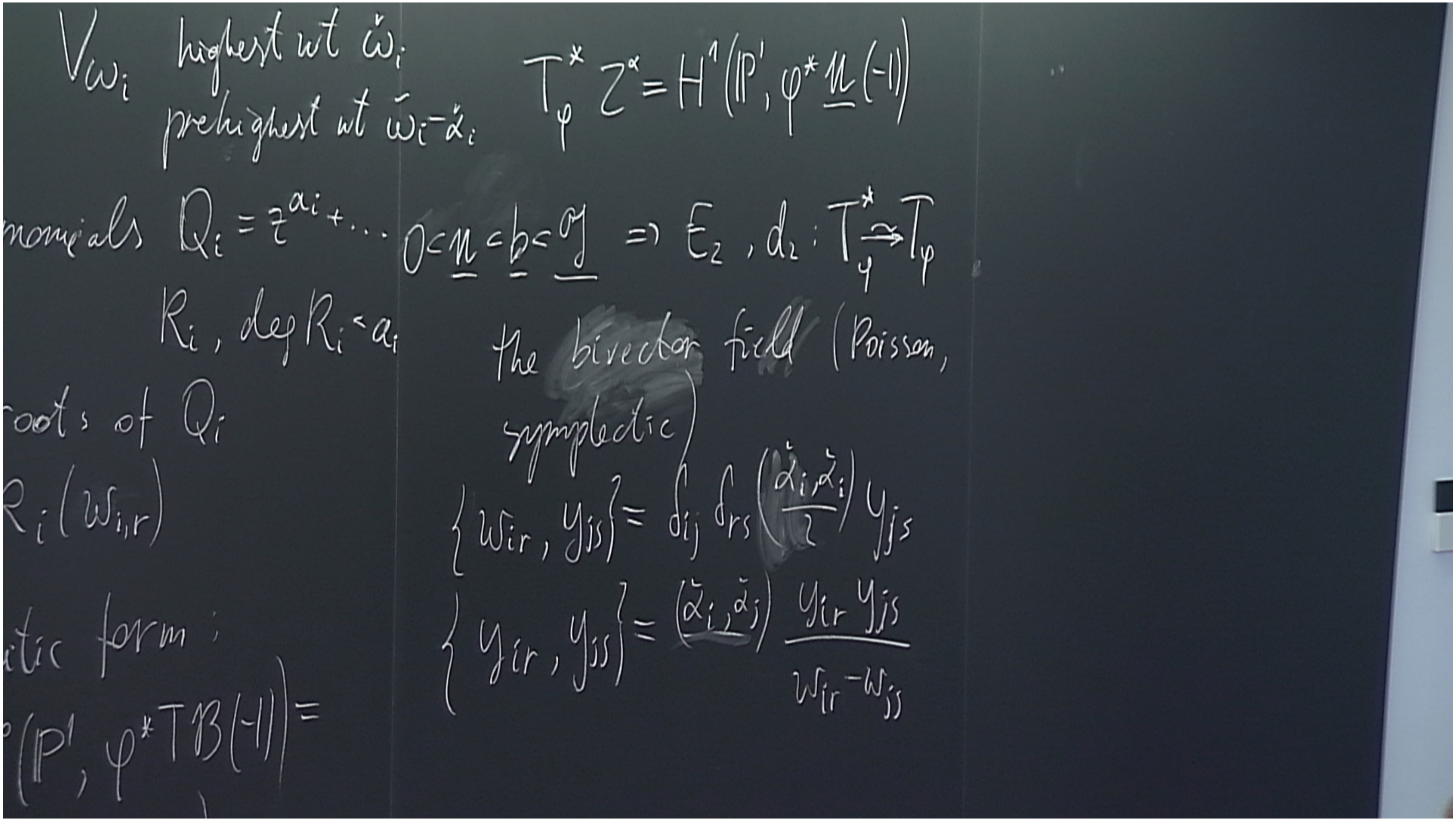
itic form:

$(\mathbb{P}^1, \varphi^* T\mathcal{B}(-1)) =$

$$T^*_\varphi Z^\alpha = H^1(\mathbb{P}^1, \varphi^* \underline{\mathfrak{n}}(-1))$$

$$0 < \underline{\mathfrak{n}} < \underline{\mathfrak{b}} < \underline{\mathfrak{g}} \Rightarrow E_2\,,\ d_2 : T^*_\varphi \xrightarrow{\sim} T_\varphi$$

the bivector field (Poisson, symplectic)



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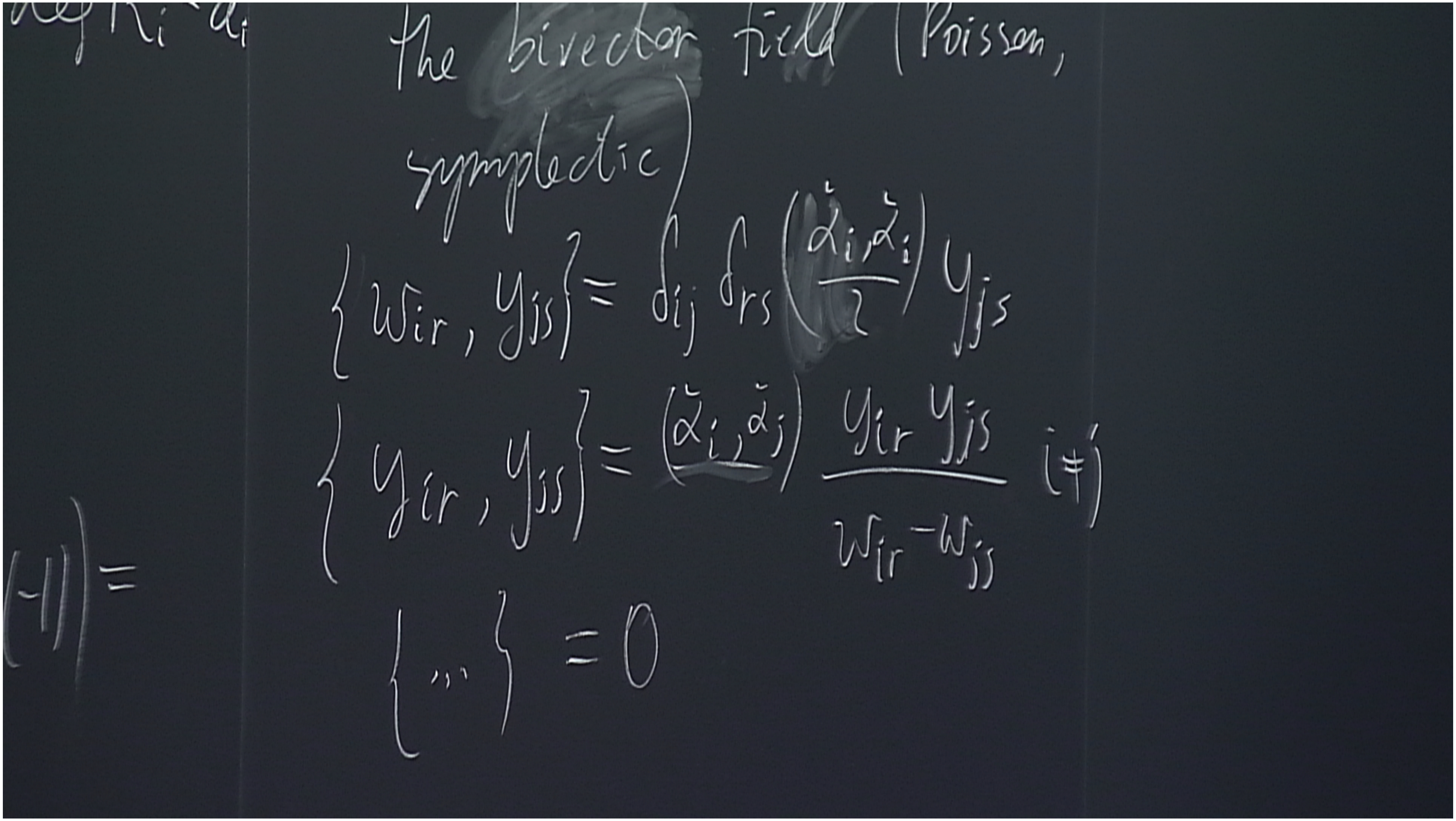
the bivector field (Poisson, symplectic)

$$\{w_{ir}, y_{js}\} = \delta_{ij}\,\delta_{rs}\left(\frac{\check{\alpha}_i, \check{\alpha}_i}{2}\right) y_{js}$$

$$\{y_{ir}, y_{js}\} = (\check{\alpha}_i, \check{\alpha}_j)\,\frac{y_{ir}\,y_{js}}{w_{ir} - w_{js}}$$



$\deg R_i \sim a_i$

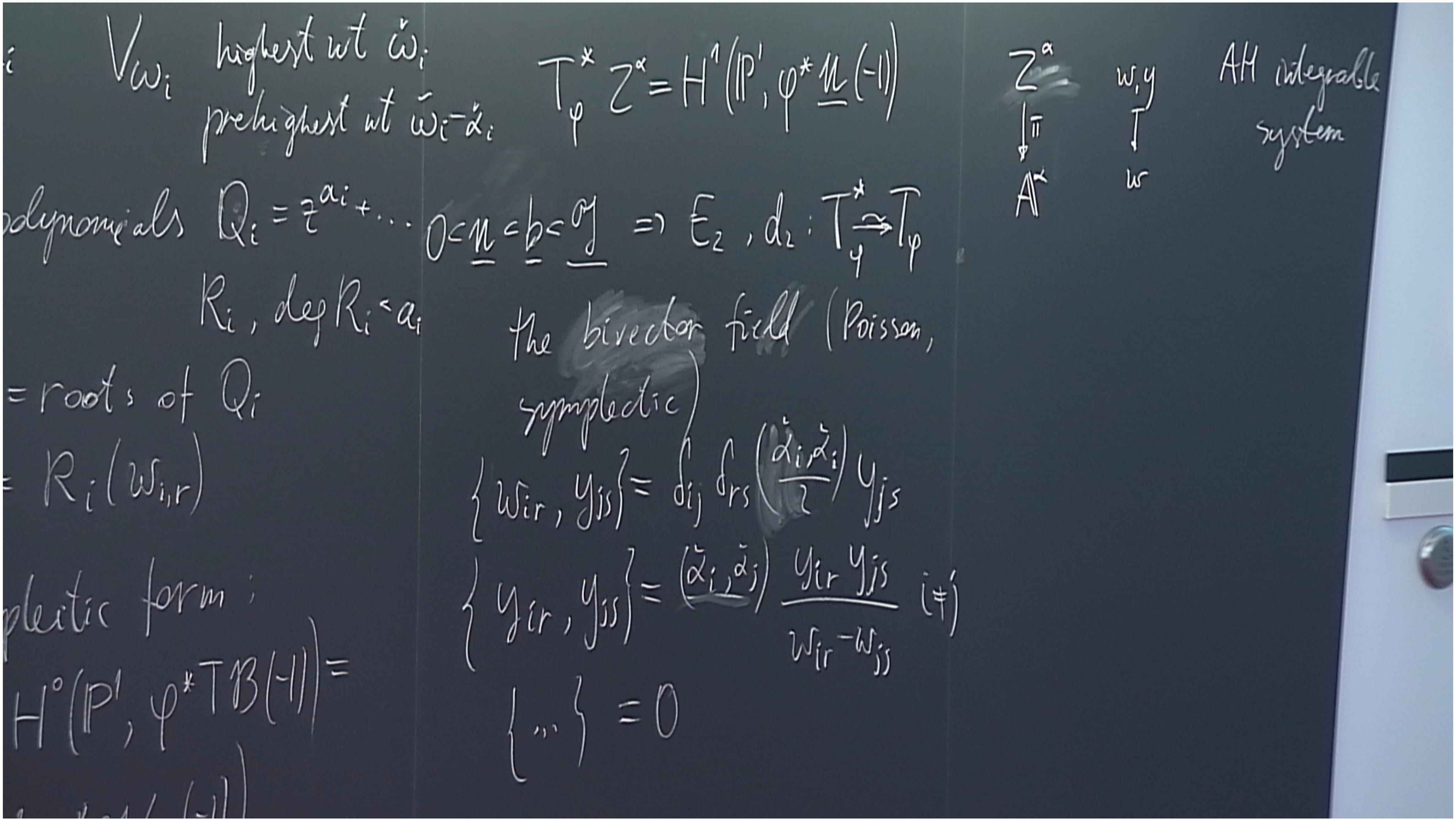
$(-1)) =$

the bivector field (Poisson, symplectic)

$$\{w_{ir}, y_{js}\} = \delta_{ij}\,\delta_{rs}\left(\frac{\check{\alpha}_i, \check{\alpha}_i}{2}\right) y_{js}$$

$$\{y_{ir}, y_{js}\} = (\check{\alpha}_i, \check{\alpha}_j)\,\frac{y_{ir}\,y_{js}}{w_{ir} - w_{js}} \quad i \neq j$$

$$\{\ ,\ \} = 0$$



i V_{ω_i} highest wt $\check{\omega}_i$

prehighest wt $\check{\omega}_i - \check{\alpha}_i$

olynomials $Q_i = z^{a_i} + \dots$

R_i, $\deg R_i < a_i$

$=$ roots of Q_i

$= R_i(w_{i,r})$

plectic form :

$H^0(\mathbb{P}^1, \varphi^* T\mathcal{B}(-1)) =$

$$T^*_\varphi Z^\alpha = H^1(\mathbb{P}^1, \varphi^* \underline{\mathfrak{n}}(-1))$$

$$0 \subset \underline{\mathfrak{n}} \subset \underline{\mathfrak{b}} \subset \underline{\mathfrak{g}} \Rightarrow \mathcal{E}_2 , d_2 : T^*_\varphi \xrightarrow{\sim} T_\varphi$$

the bivector field (Poisson, symplectic)

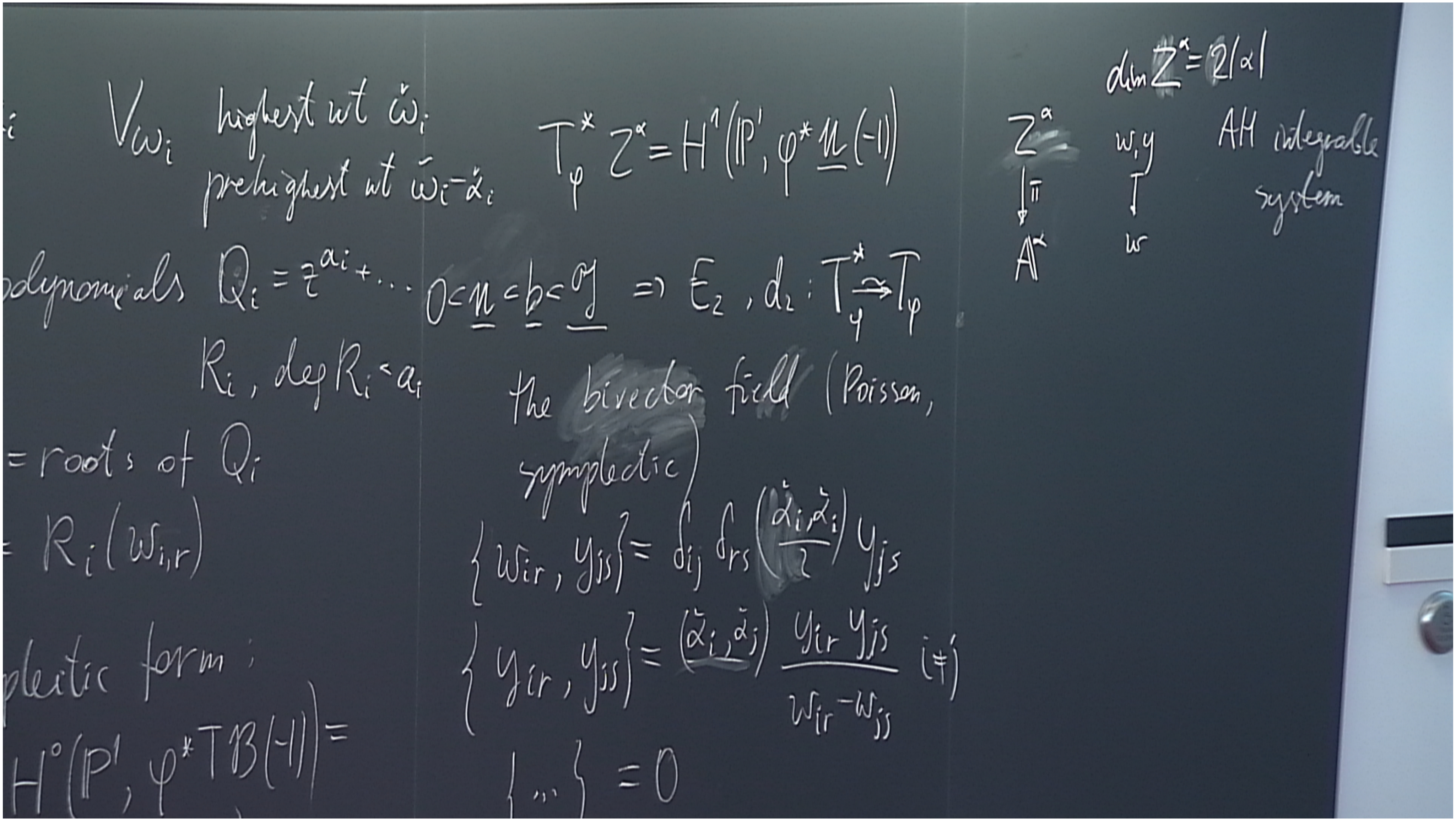
$$\{w_{ir}, y_{js}\} = \delta_{ij} \delta_{rs} \left(\frac{\check{\alpha}_i, \check{\alpha}_i}{2}\right) y_{js}$$

$$\{y_{ir}, y_{js}\} = (\check{\alpha}_i, \check{\alpha}_j) \frac{y_{ir} y_{js}}{w_{ir} - w_{js}} \quad (i \neq j)$$

$$\{\dots\} = 0$$

$$\begin{tikzcd} Z^\alpha \arrow[d, "\pi"] & w, y \arrow[d] \\ \mathbb{A}^\alpha & w \end{tikzcd}$$

AH integrable system



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$$\dim Z^\alpha = 2|\alpha|$$

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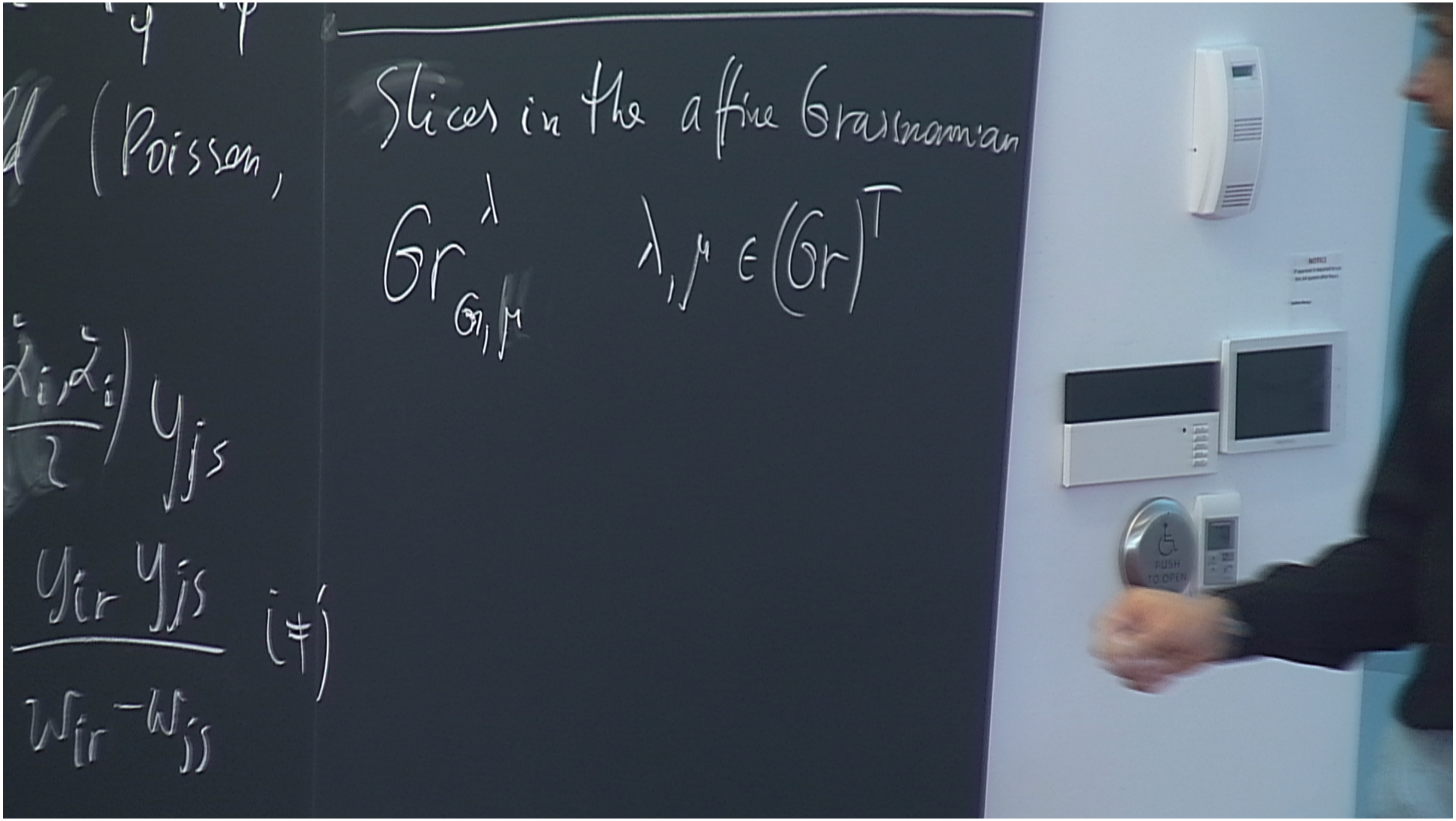
AH integrable system



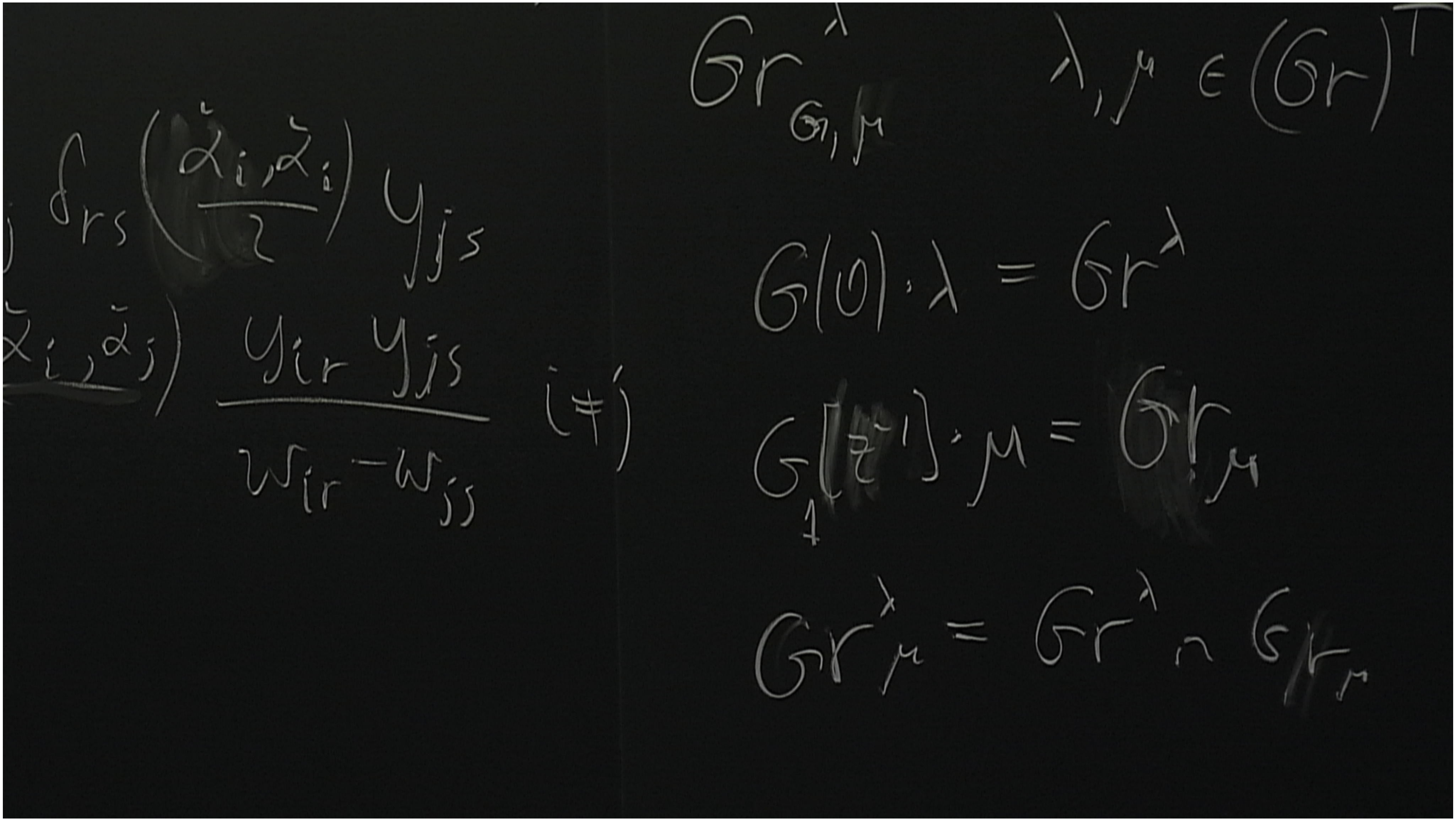
d (Poisson,

$\frac{\check{\alpha}_i, \check{\alpha}_i}{2}) y_{js}$

$\frac{y_{ir} y_{js}}{w_{ir} - w_{js}}$ $(i \neq j)$

Slices in the affine Grassmannian

$Gr^{\lambda}_{G, \mu}$ $\lambda, \mu \in (Gr)^T$



$$j \ \delta_{rs}\left(\frac{\check{\alpha}_i,\check{\alpha}_i}{2}\right) y_{js}$$

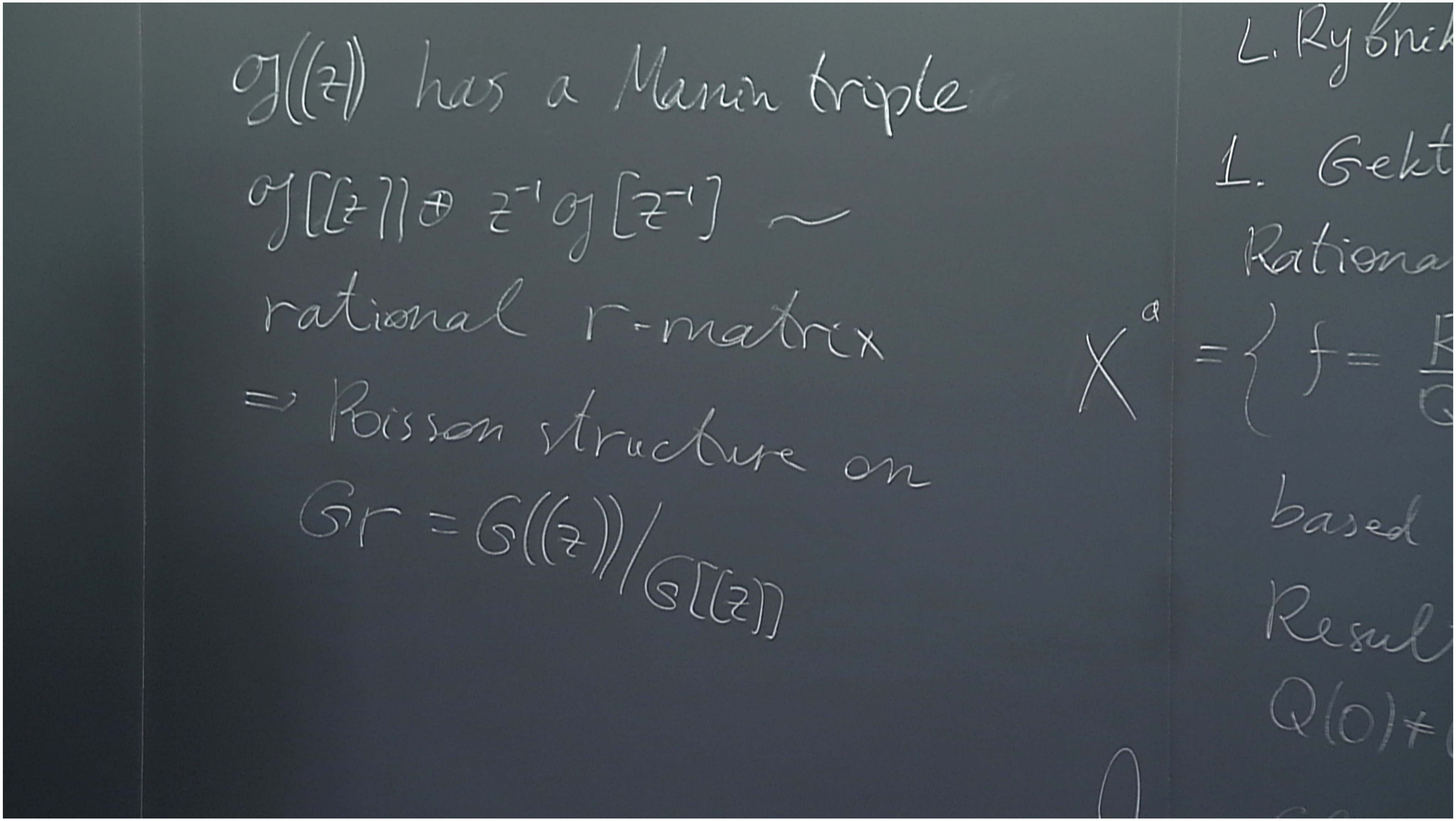
$$\check{\alpha}_i,\check{\alpha}_j) \ \frac{y_{ir}\,y_{js}}{w_{ir}-w_{js}} \quad (i \neq j)$$

$$Gr^{\lambda}_{G,\mu} \qquad \lambda,\mu \in (Gr)^T$$

$$G(\mathcal{O})\cdot\lambda = Gr^{\lambda}$$

$$G_1[z^{-1}]\cdot\mu = Gr_{\mu}$$

$$Gr^{\lambda}_{\mu} = Gr^{\lambda} \cap Gr_{\mu}$$



$\mathfrak{g}((z))$ has a Manin triple

$\mathfrak{g}[[z]] \oplus z^{-1}\mathfrak{g}[z^{-1}]$ ~

rational r-matrix

⇒ Poisson structure on

$Gr = G((z)) / G[[z]]$

L. Rybnik

1. Gekt

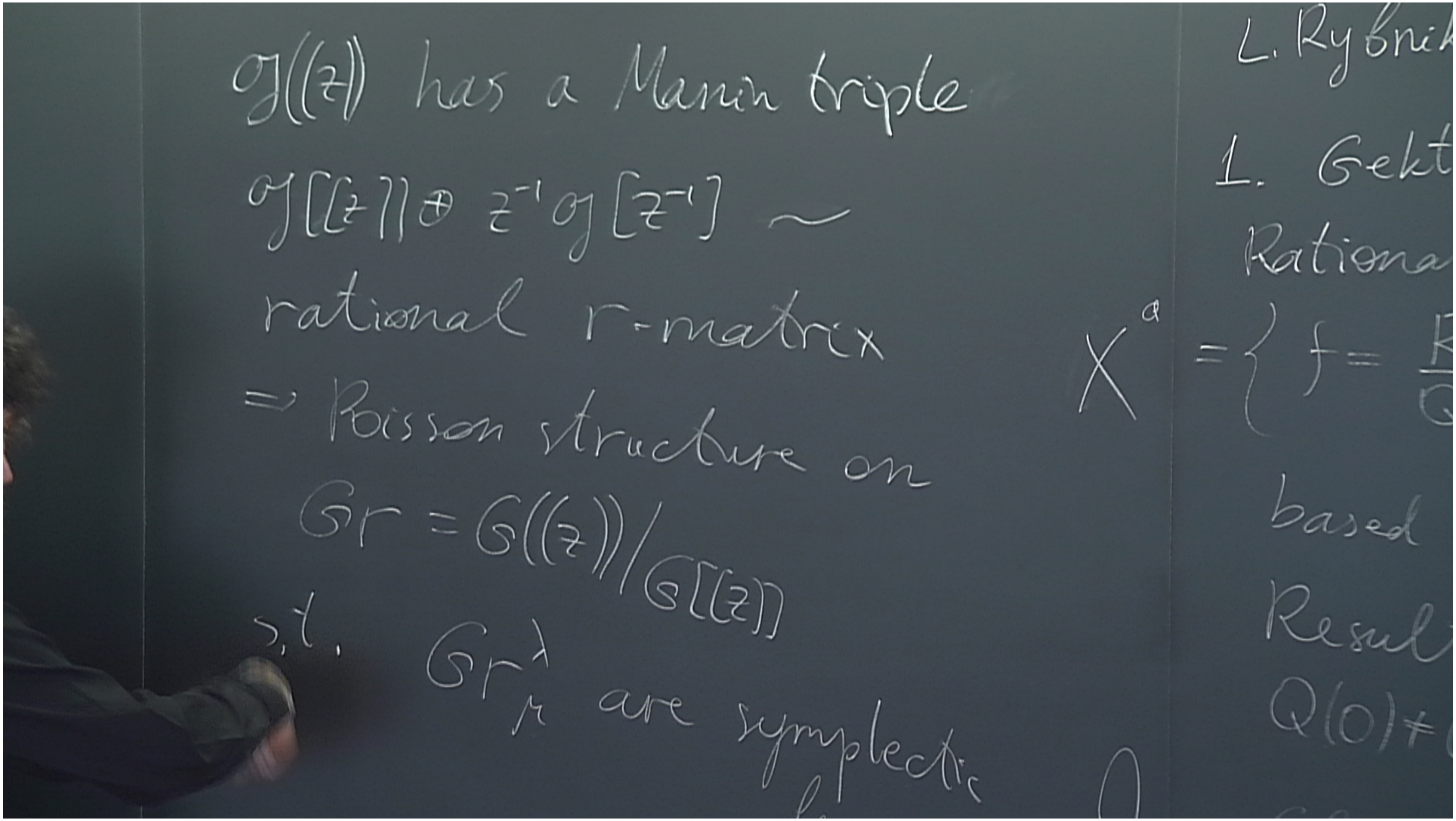
Rationa

$X^a = \{ f = \frac{R}{Q}$

based

Resul

$Q(0) \neq$



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$Gr = G((z))/G[[z]]$

s.t. Gr^{λ}_{μ} are symplectic

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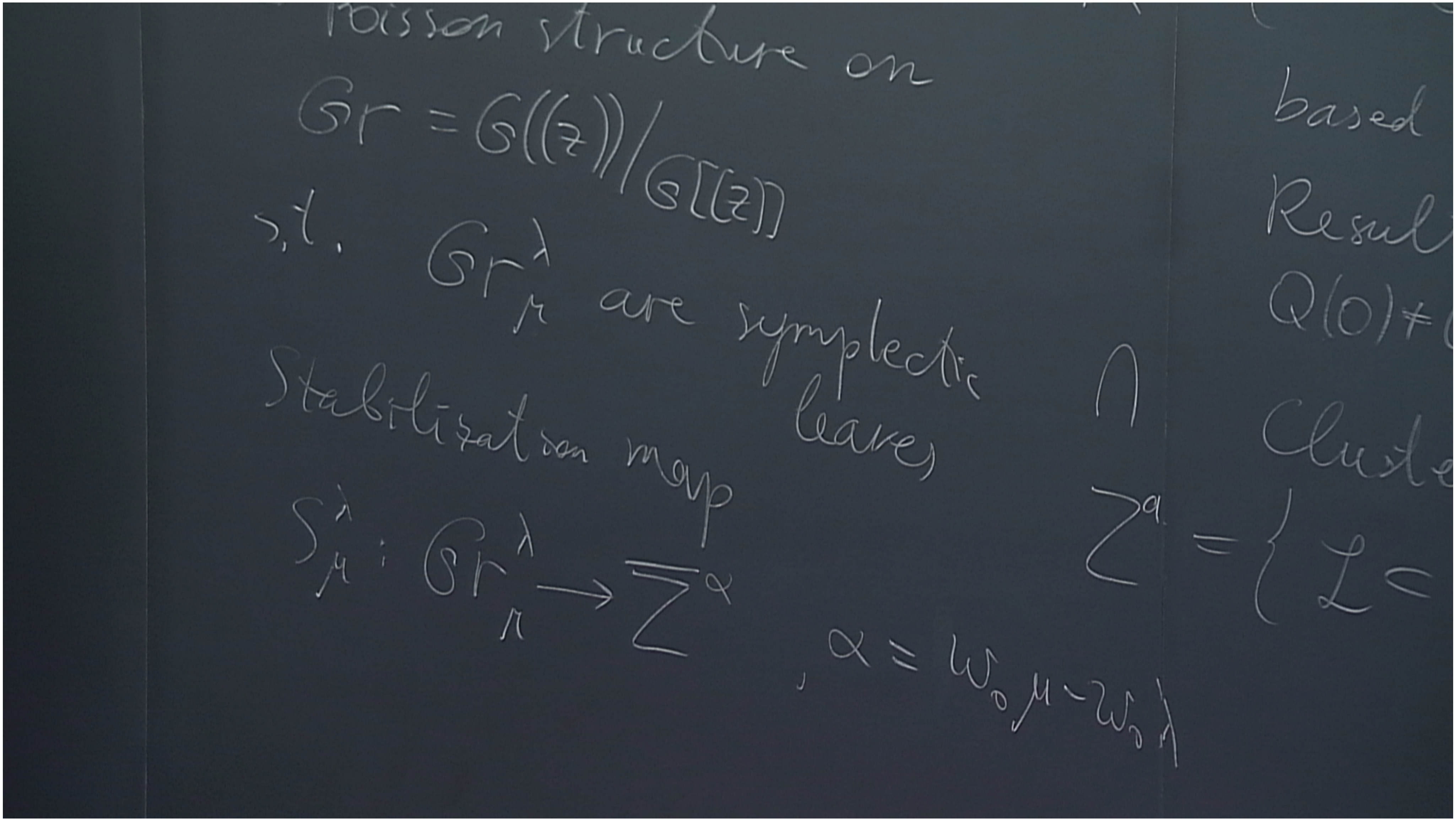


Poisson structure on

$\mathrm{Gr} = G((z))/G[[z]]$

s.t. $\mathrm{Gr}^{\lambda}_{\mu}$ are symplectic leaves

Stabilization map

$s^{\lambda}_{\mu} : \mathrm{Gr}^{\lambda}_{\mu} \longrightarrow \Sigma^{\alpha}$, $\alpha = w_0\mu - w_0\lambda$

$\cap$

$\Sigma^{a} = \{ \mathcal{L} \subset$

based

Resul

$Q(0) \neq ($

Cluste

symplectic leaves

Stabilization map

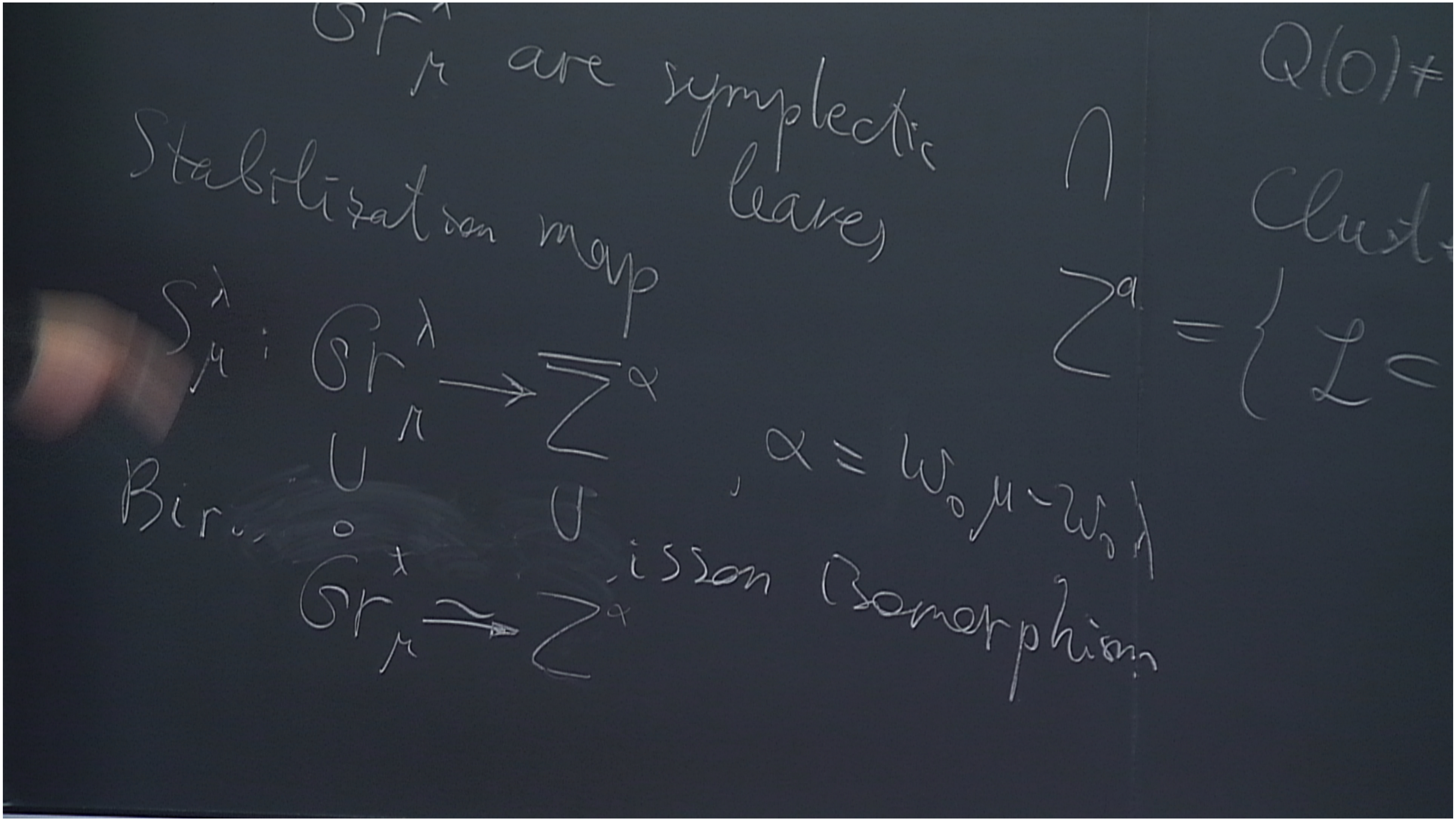
$$S^{\lambda}_{\mu} : Gr^{\lambda}_{\mu} \longrightarrow \overline{\Sigma}^{\alpha} \quad , \alpha = w_0\mu - w_0\lambda$$

Birational Poisson isomorphism

$\cap$

Cluste

$\Sigma^{a} = \{ \mathcal{L} \subset$



Gr^λ_μ are symplectic leaves

Stabilization map

$$S^\lambda_\mu : Gr^\lambda_\mu \longrightarrow \overline{\Sigma}^\alpha$$

$$\begin{tikzcd} Gr^\lambda_\mu \arrow[r] & \overline{\Sigma}^\alpha \\ \overset{\circ}{Gr}{}^\lambda_\mu \arrow[u, phantom, "\cup"] \arrow[r, "\sim"] & \Sigma^\alpha \arrow[u, phantom, "\cup"] \end{tikzcd}$$

, $\alpha = w_0\mu - w_0\lambda$

Bir… …isson isomorphism

$\cap$

$\Sigma^a = \{ \mathcal{L} \subset$

Q(0)+

Clust

$\mathrm{Gr}^{\lambda}_{\mu}$ are symplectic leaves

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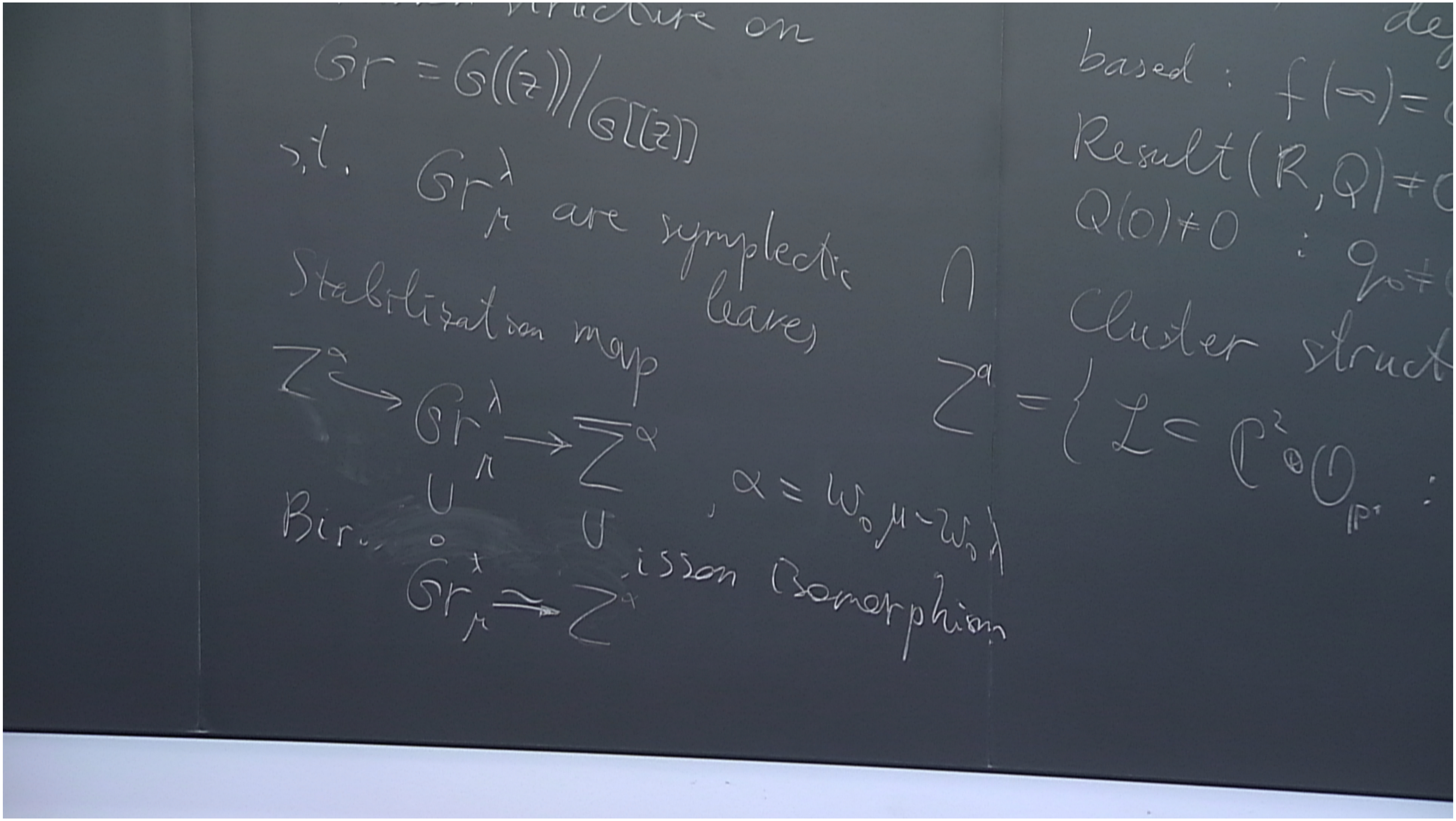
Bir... ...isson isomorphism

$Q(0) \neq$

$\cap$

Clust

$\mathcal{Z}^{a} = \{ \mathcal{L} \subset$



structure on

$Gr = G((z))/G[[z]]$

s.t. Gr^λ_μ are symplectic leaves

Stabilization map

$$\begin{tikzcd} Z^a \arrow[r, hook] & Gr^\lambda_\mu \arrow[r] & \overline{Z}^\alpha \\ & \overset{\circ}{Gr}{}^\lambda_\mu \arrow[u, phantom, "\cup"] \arrow[r, "\simeq"] & \overset{\circ}{Z}{}^\alpha \arrow[u, phantom, "\cup"] \end{tikzcd}$$

, $\alpha = w_0\mu - w_0\lambda$

Bir... isson Isomorphism

$\cap$

de

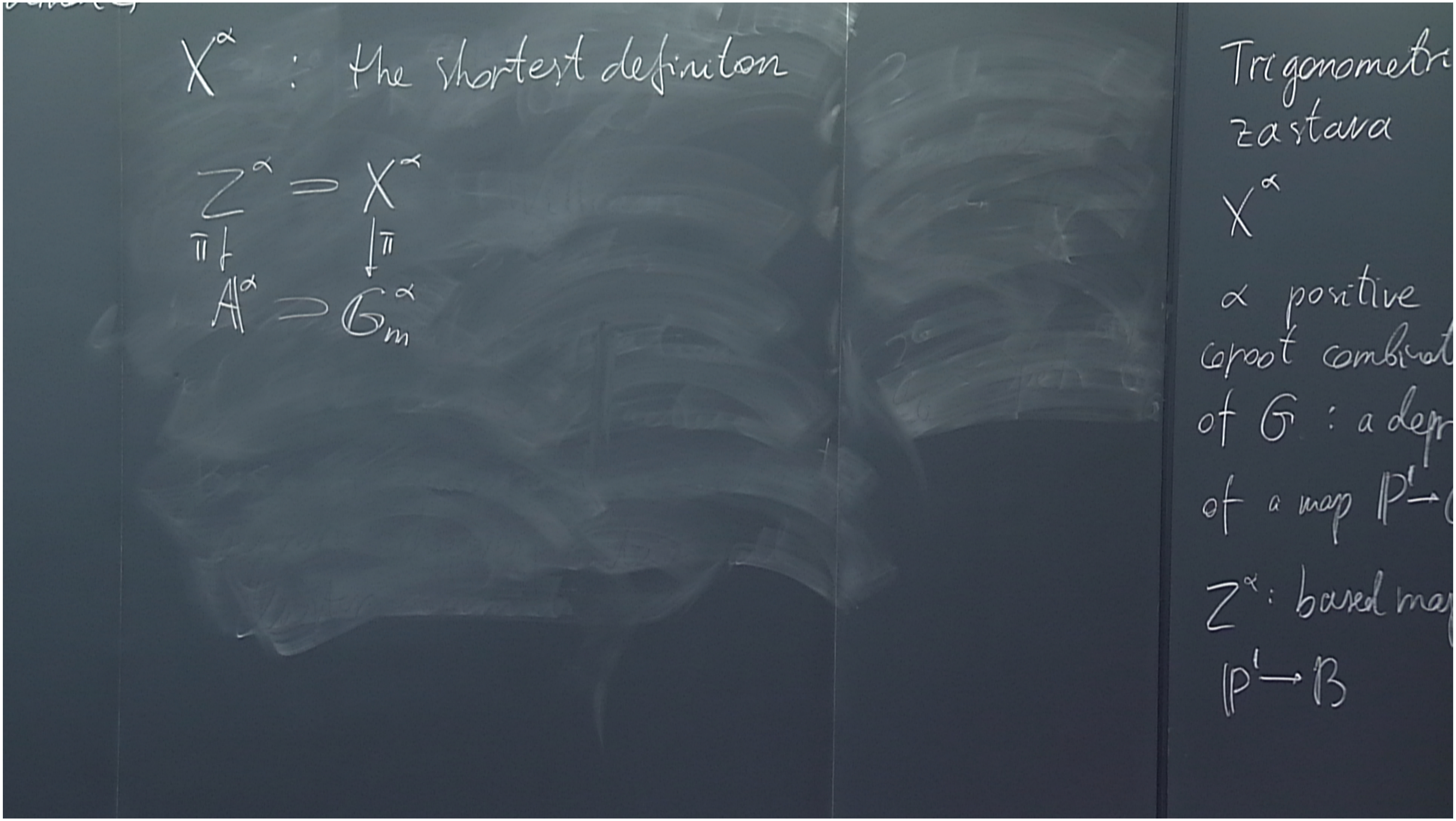
based : $f(\infty) =$

$\mathrm{Result}(R,Q) \neq$

$Q(0) \neq 0$: $q_0 \neq$

Cluster struct

$Z^a = \{ \mathcal{L} \subset \mathbb{C}^2 \otimes \mathcal{O}_{\mathbb{P}^1} :$



X^α : the shortest definiton

$$\begin{tikzcd} Z^\alpha \arrow[d, "\pi"'] & \supset & X^\alpha \arrow[d, "\pi"] \\ \mathbb{A}^\alpha & \supset & \mathbb{G}_m^\alpha \end{tikzcd}$$

Trigonometri

zastava

X^α

α positive

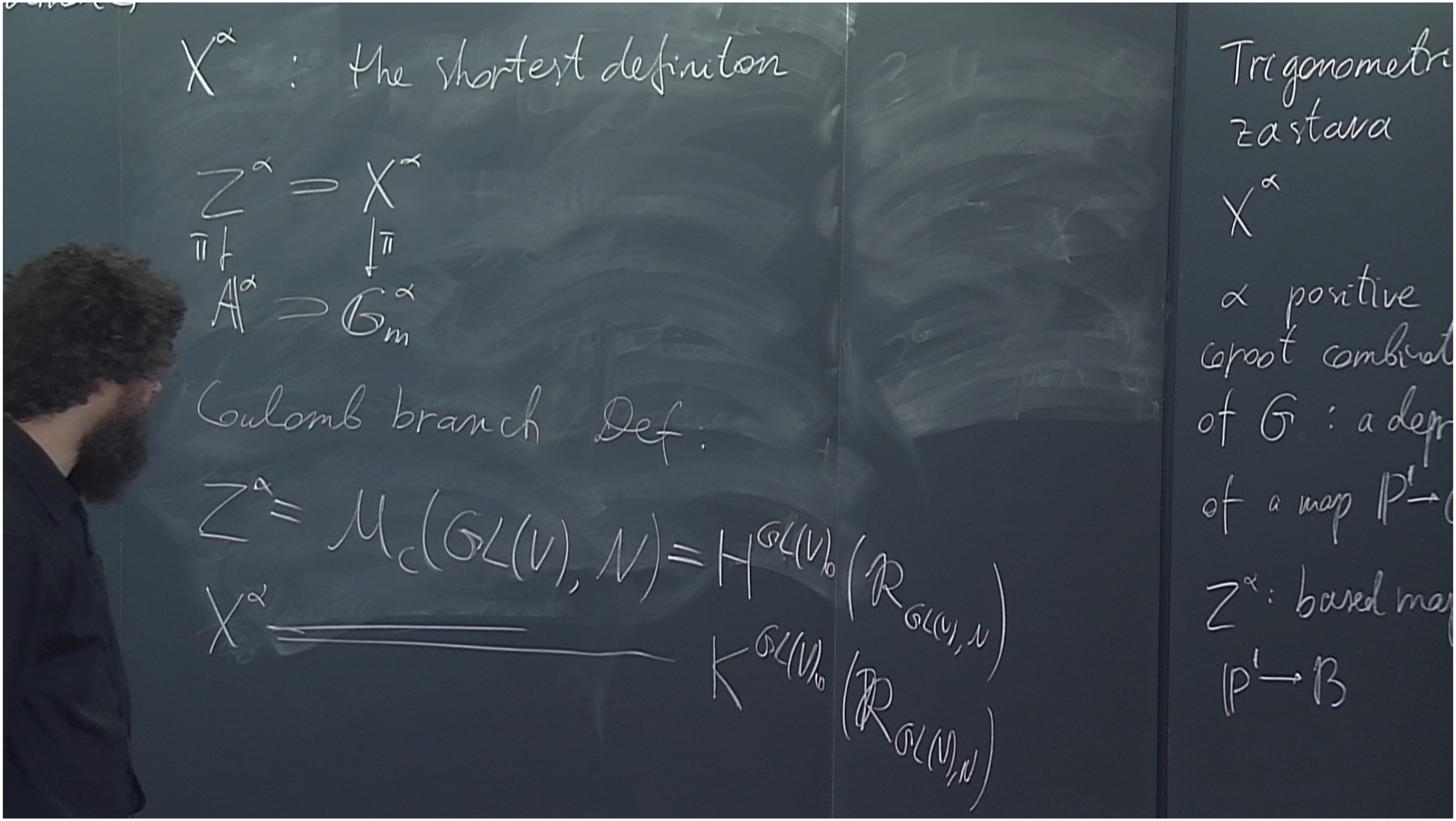
coroot combinat

of G : a degr

of a map $\mathbb{P}^1 \to$

Z^α : based ma

$\mathbb{P}^1 \to \mathcal{B}$



X^α : the shortest definiton

$$\begin{tikzcd} Z^\alpha \arrow[r, phantom, "\supset"] \arrow[d, "\pi"'] & X^\alpha \arrow[d, "\pi"] \\ \mathbb{A}^\alpha \arrow[r, phantom, "\supset"] & \mathbb{G}_m^\alpha \end{tikzcd}$$

Coulomb branch Def:

$$Z^\alpha = \mathcal{M}_C(GL(V), N) = H^{GL(V)_\mathcal{O}}(\mathcal{R}_{GL(V),N})$$

$$X^\alpha = K^{GL(V)_\mathcal{O}}(\mathcal{R}_{GL(V),N})$$

Trigonometri
zastava

X^α

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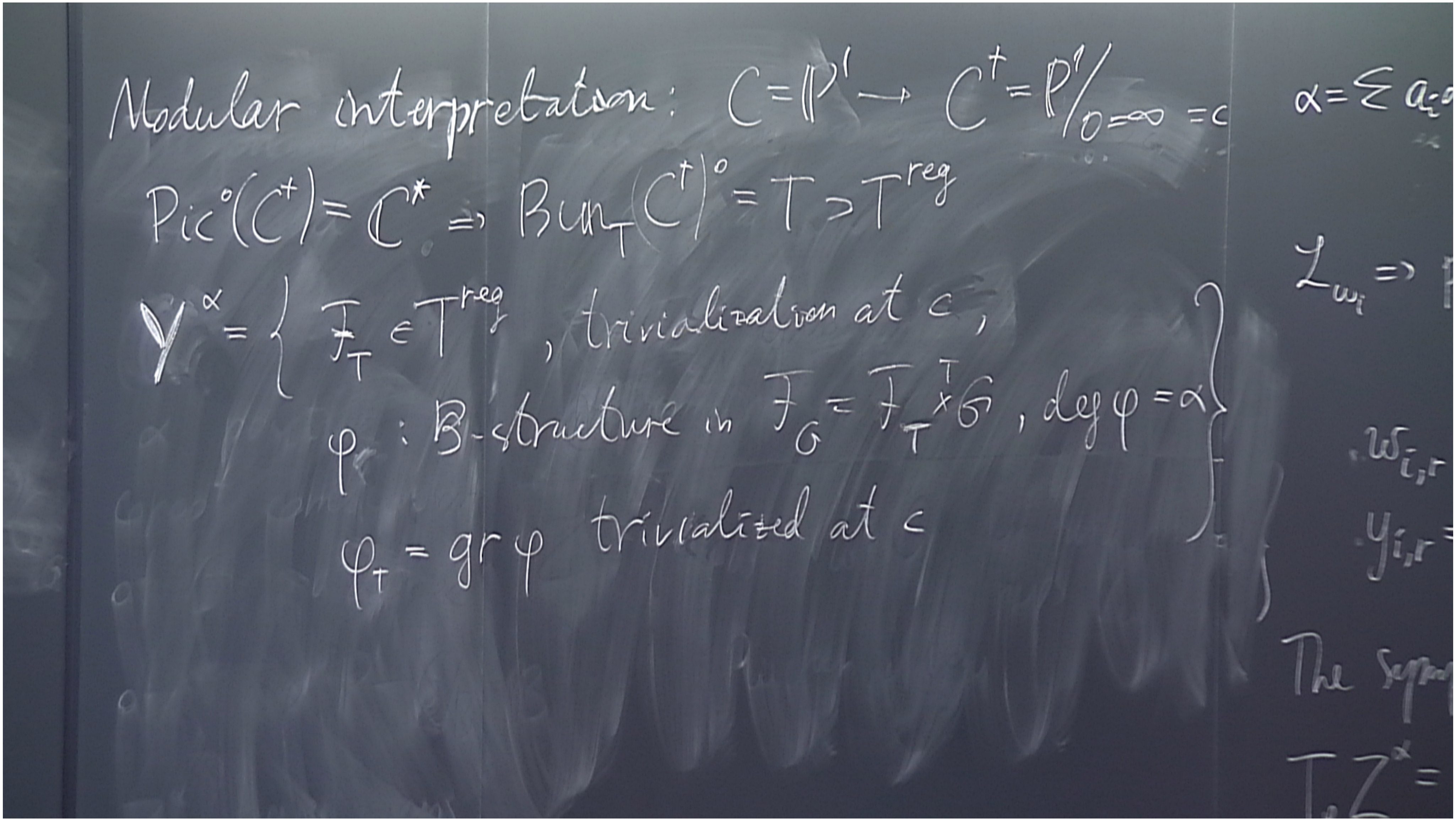
$\mathbb{P}^1 \to \mathcal{B}$

$\mathbb{A} \supset \mathbb{G}_m$

oulomb branch Def:

$$Z^\alpha = \mathcal{M}_C(GL(V), N) = H^{GL(V)_0}(\mathcal{R}_{GL(V),N})$$

$$K^\alpha = K^{GL(V)_0}(\mathcal{R}_{GL(V),N})$$



Modular interpretation: $C=\mathbb{P}^1 \to C^+=\mathbb{P}^1/_{0=\infty}=c$

$\mathrm{Pic}^0(C^+)=\mathbb{C}^* \Rightarrow \mathrm{Bun}_T(C^+)^0 = T \supset T^{reg}$

$$Y^\alpha = \left\{ \begin{array}{l} \mathcal{F}_T \in T^{reg}, \text{ trivialization at } c, \\ \varphi : B\text{-structure in } \mathcal{F}_G = \mathcal{F}_T \overset{T}{\times} G, \ \deg\varphi=\alpha \\ \varphi_T = gr\,\varphi \text{ trivialized at } c \end{array} \right\}$$

$\alpha = \Sigma\, a_i$

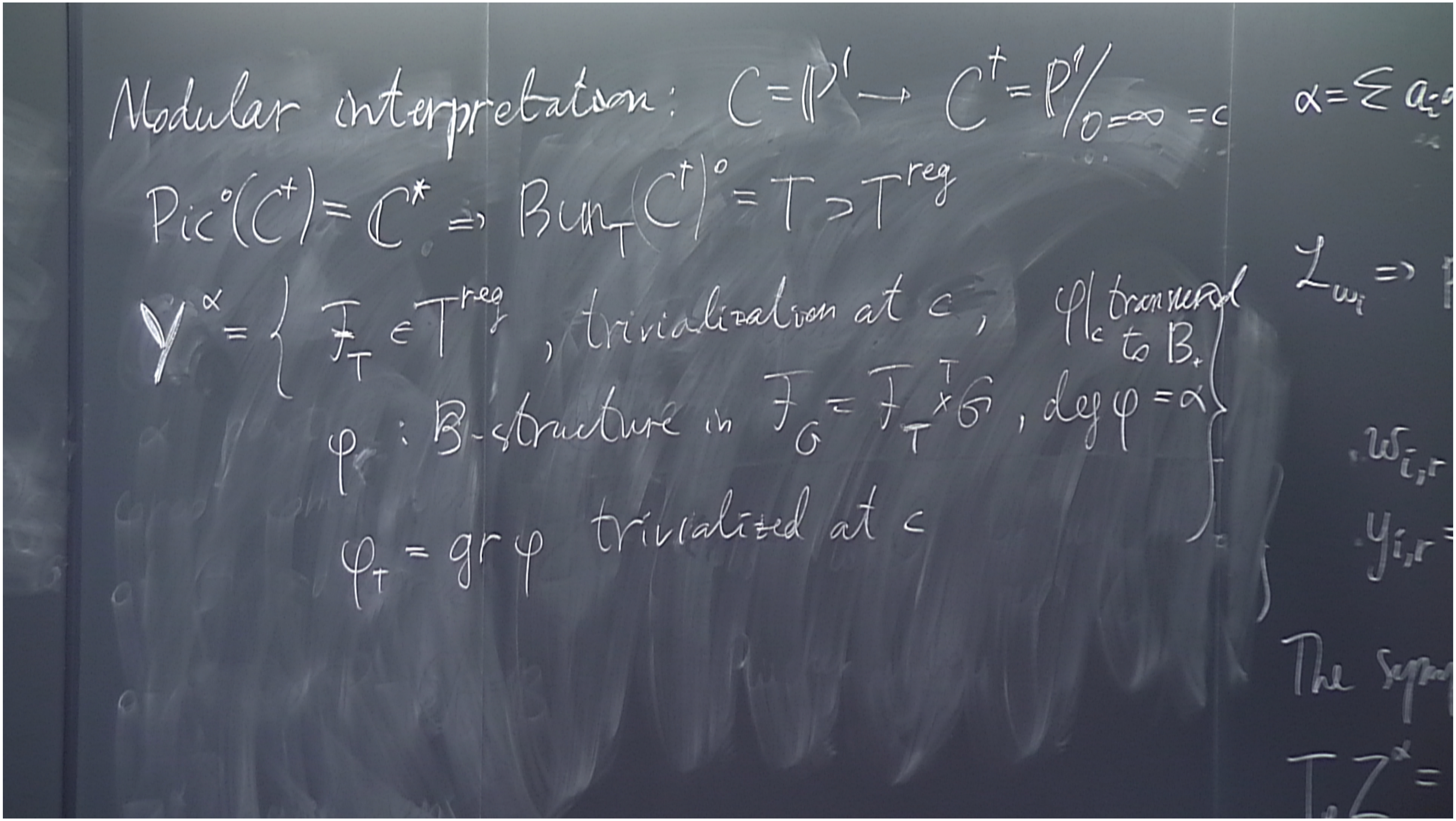
$\mathcal{L}_{\omega_i} \Rightarrow$

$w_{i,r}$

$y_{i,r} =$

The

$T_c Z^\alpha =$



Modular interpretation: $C=\mathbb{P}^1 \longrightarrow C^\dagger = \mathbb{P}^1/_{0=\infty} = c$

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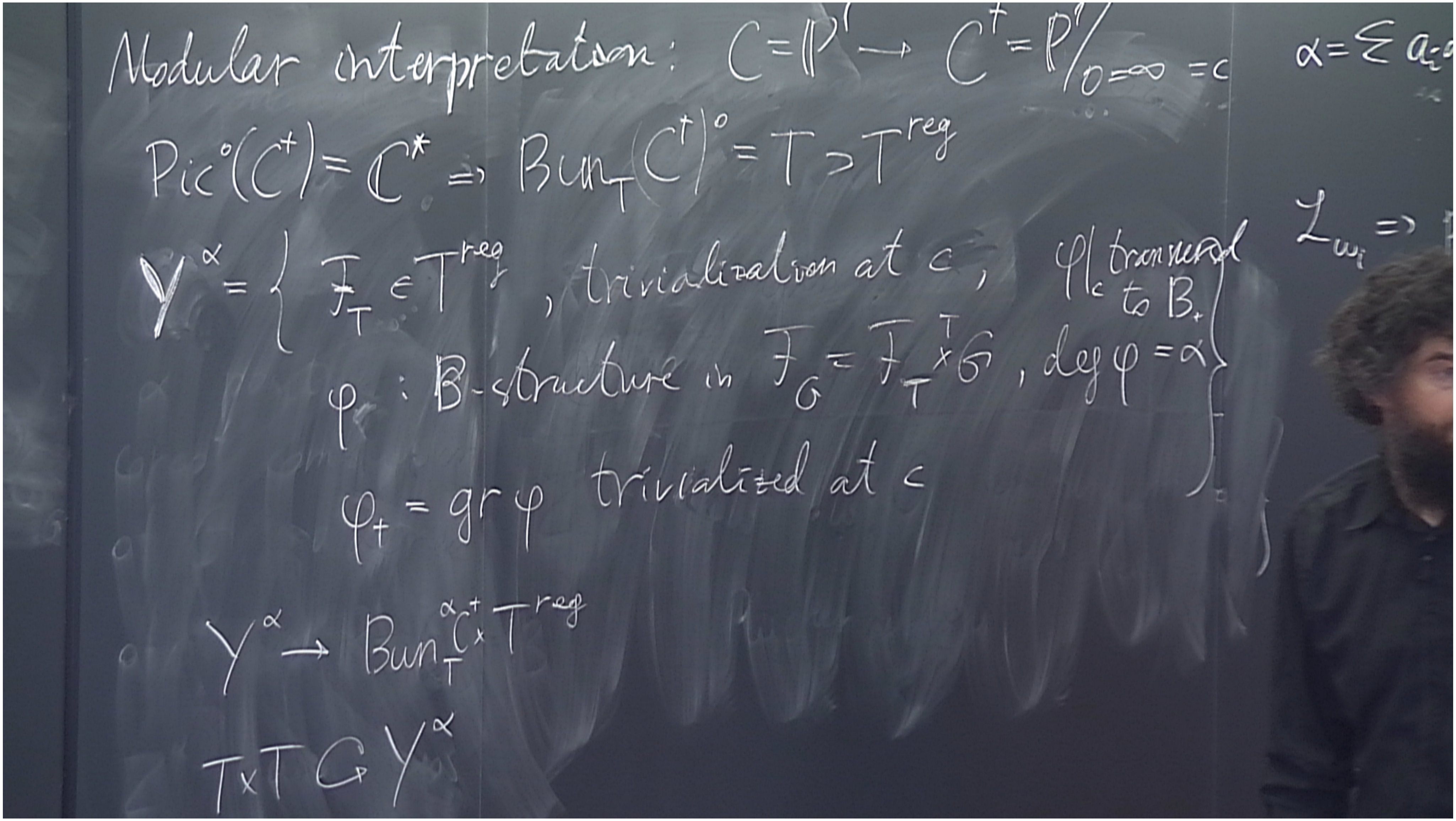


Modular interpretation: $C = \mathbb{P}^1 \longrightarrow C^+ = \mathbb{P}^1/_{0=\infty} = c$

$\mathrm{Pic}^\circ(C^+) = \mathbb{C}^* \Rightarrow \mathrm{Bun}_T(C^+)^\circ = T \supset T^{reg}$

$Y^\alpha = \{ \mathcal{F}_T \in T^{reg}$, trivialization at c, $\varphi|_c$ transversal to B_+

φ : B-structure in $\mathcal{F}_G = \mathcal{F}_T \overset{T}{\times} G$, $\deg \varphi = \alpha \}$

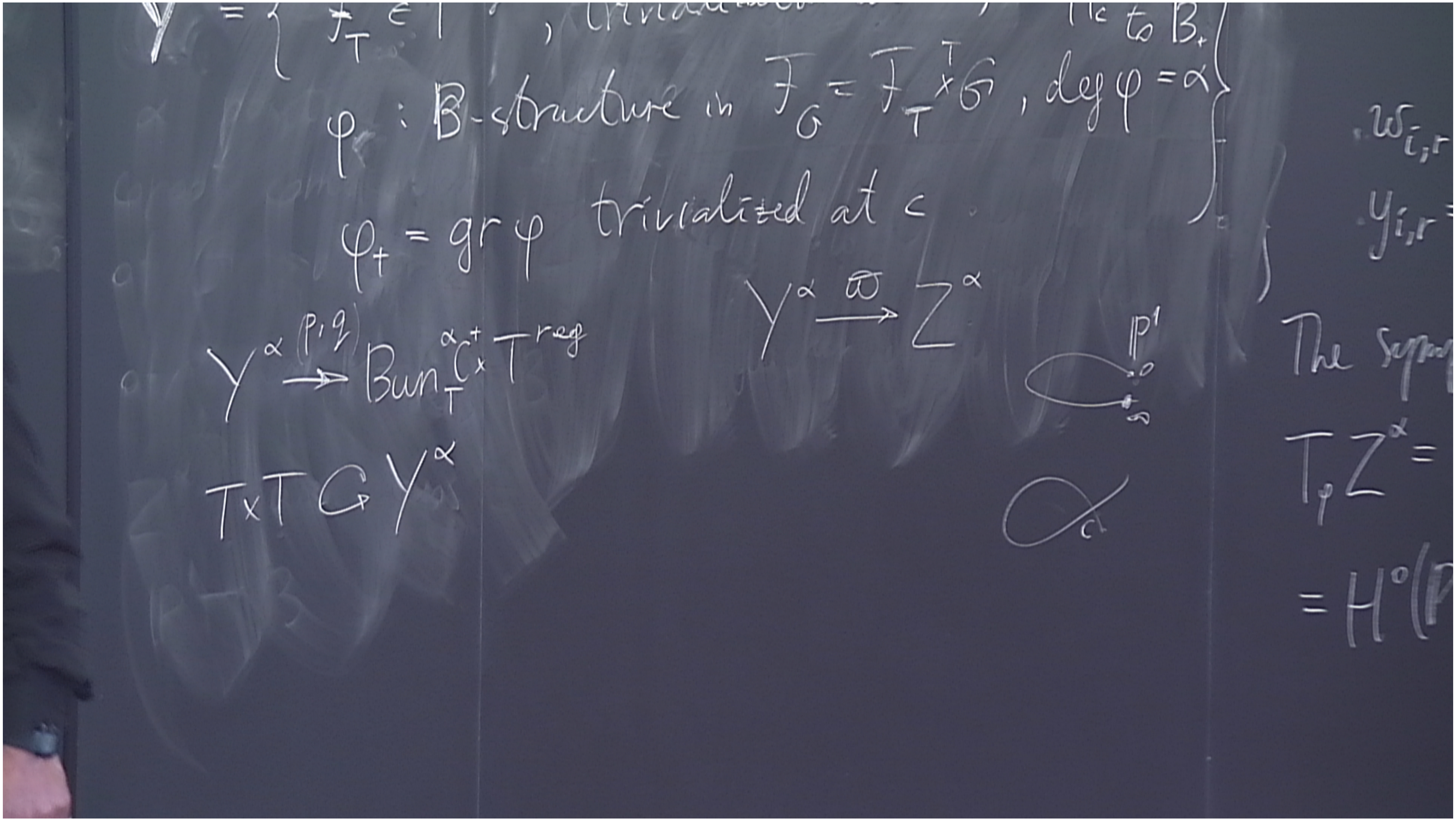
$\varphi_+ = gr\, \varphi$ trivialized at c

$Y^\alpha \longrightarrow \mathrm{Bun}_T^\alpha C^+ \times T^{reg}$

$T \times T \circlearrowright Y^\alpha$

$\alpha = \Sigma\, a_i$

$\mathcal{L}_{\omega_i} \Rightarrow$



$Y = \{ \mathcal{F}_T \in$, [illegible] to B_+

φ : B-structure in $\mathcal{F}_G = \mathcal{F}_T \overset{T}{\times} G$, $\deg \varphi = \alpha$

$\varphi_+ = gr\,\varphi$ trivialized at c $\}$

$Y^\alpha \xrightarrow{\varpi} Z^\alpha$

$Y^\alpha \xrightarrow{(p,q)} \mathrm{Bun}_T^{\alpha} C^{+}_{\times} T^{reg}$

$T \times T \curvearrowright Y^\alpha$

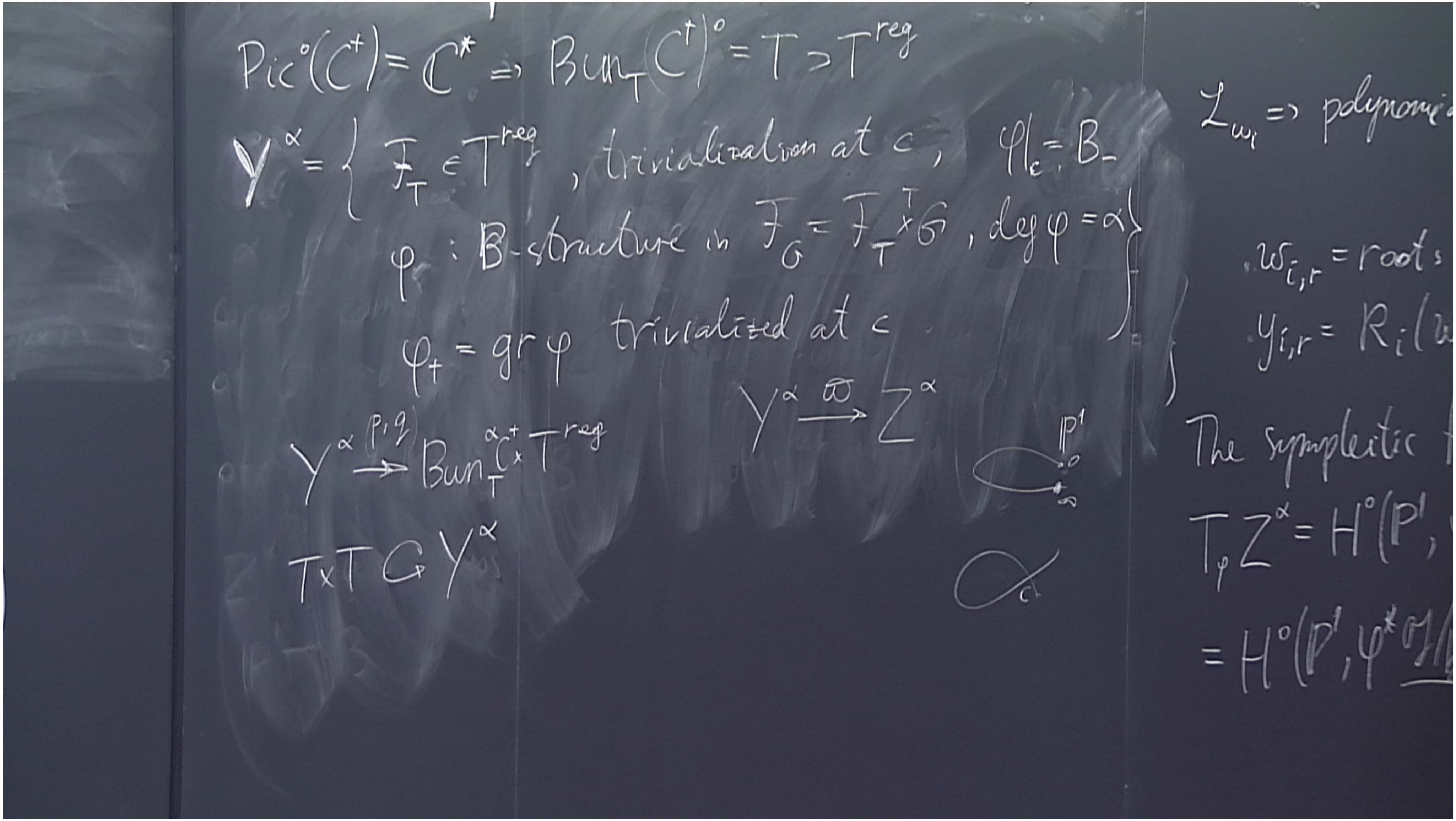
$w_{i,r}$

$y_{i,r} =$

The sym

$T_\varphi Z^\alpha =$

$= H^0(\mathbb{P}$



$\mathrm{Pic}^0(C^+) = \mathbb{C}^* \Rightarrow \mathrm{Bun}_T(C^+)^0 = T \supset T^{reg}$

$Y^\alpha = \{ \mathcal{F}_T \in T^{reg}$, trivialization at c, $\varphi|_c = B_-$

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$\varphi_+ = gr\,\varphi$ trivialized at c

$Y^\alpha \xrightarrow{\varpi} Z^\alpha$

$Y^\alpha \xrightarrow{(p,q)} \mathrm{Bun}_T^{\alpha} C^+ \times T^{reg}$

$T \times T \curvearrowright Y^\alpha$

$\mathcal{L}_{\omega_i} \Rightarrow$ polynomi

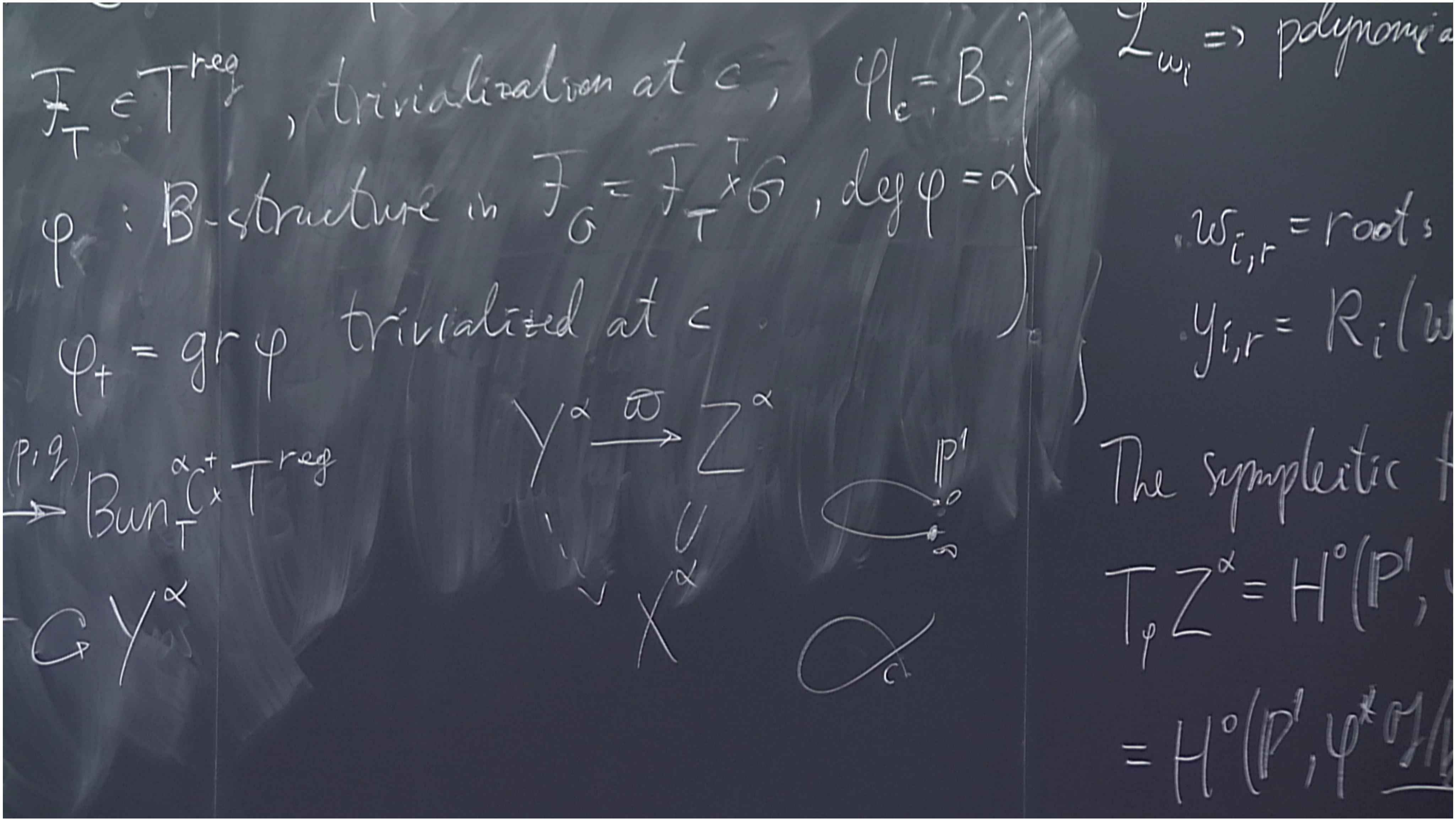
$w_{i,r}$ = roots

$y_{i,r} = R_i(w$

The symplectic f

$T_\varphi Z^\alpha = H^0(\mathbb{P}^1,$

$= H^0(\mathbb{P}^1, \varphi^* \mathfrak{g}/$



$\mathcal{F}_T \in T^{reg}$, trivialization at c, $\varphi|_c = B_-$

φ : B-structure in $\mathcal{F}_G = \mathcal{F}_T \overset{T}{\times} G$, $\deg \varphi = \alpha$

$\varphi_+ = gr\, \varphi$ trivialized at c

(p,q)

$\to \mathrm{Bun}_T^{\alpha^+} C \times T^{reg}$

$T \circlearrowright Y^\alpha$

$$\begin{tikzcd} Y^\alpha \arrow[r, "\varpi"] \arrow[rd] & Z^\alpha \arrow[d, phantom, "\cup"] \\ & X^\alpha \end{tikzcd}$$

$\mathcal{L}_{w_i} \Rightarrow$ polynomia

$w_{i,r}$ = roots

$y_{i,r} = R_i(w$

The symplectic f

$T_\varphi Z^\alpha = H^0(\mathbb{P}^1,$

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X^α : the shortest definition

$$\begin{tikzcd} Z^\alpha \arrow[r, phantom, "\supset"] \arrow[d, "\pi"'] & X^\alpha \arrow[d, "\pi"] \\ \mathbb{A}^\alpha \arrow[r, phantom, "\supset"] & \mathbb{G}_m^\alpha \end{tikzcd}$$

Prop: for any $\xi \in T^{reg}$

$$\varpi^{-1}(\xi) / 1\times T \simeq X^\alpha$$

Similar construction equips Y^α with a bivector field

Modular interpretation: $C = \mathbb{P}^1 \to C^+ = \mathbb{P}^1/_{0=}$

$\mathrm{Pic}^\circ(C^+) = \mathbb{C}^* \Rightarrow \mathrm{Bun}_T(C^+)^\circ = T \supset T^{reg}$

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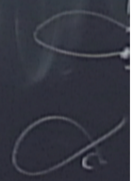
φ : B-structure in $\mathcal{F}_G = \mathcal{F}_T \overset{T}{\times} G$, deg

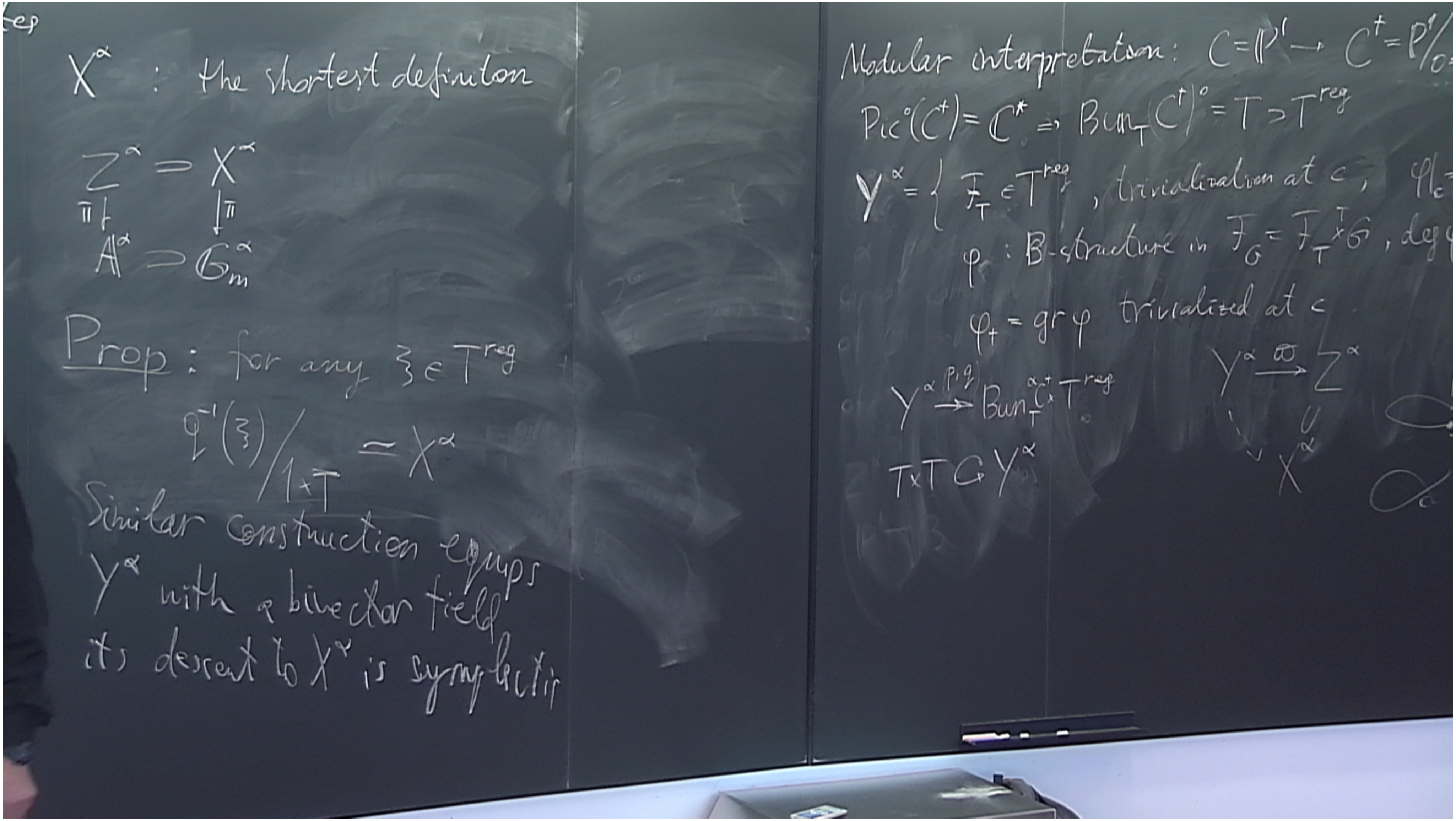
$\varphi_+ = \mathrm{gr}\,\varphi$ trivialized at c

$$Y^\alpha \xrightarrow{(p,q)} \mathrm{Bun}_T^{\alpha}C^+ \times T^{reg}$$

$T \times T \circlearrowright Y^\alpha$

$$\begin{tikzcd} Y^\alpha \arrow[r, "\varpi"] \arrow[rd, dashed] & Z^\alpha \arrow[d, phantom, "\cup"] \\ & X^\alpha \end{tikzcd}$$





X^α : the shortest definition

$$\begin{tikzcd} Z^\alpha \arrow[r, phantom, "\supset"] \arrow[d, "\pi"'] & X^\alpha \arrow[d, "\pi"] \\ \mathbb{A}^\alpha \arrow[r, phantom, "\supset"] & \mathbb{G}_m^\alpha \end{tikzcd}$$

<u>Prop</u>: for any $\xi \in T^{reg}$

$$q^{-1}(\xi) \big/ 1\times T \simeq X^\alpha$$

Similar construction equips Y^α with a bivector field its descent to X^α is symplectic

Modular interpretation: $C = \mathbb{P}^1 \to C^+ = \mathbb{P}^1/_{0=}$

$\mathrm{Pic}^\circ(C^+) = \mathbb{C}^* \Rightarrow \mathrm{Bun}_T(C^+)^\circ = T \supset T^{reg}$

$Y^\alpha = \{ \mathcal{F}_T \in T^{reg}$, trivialization at c, $\varphi|_c =$

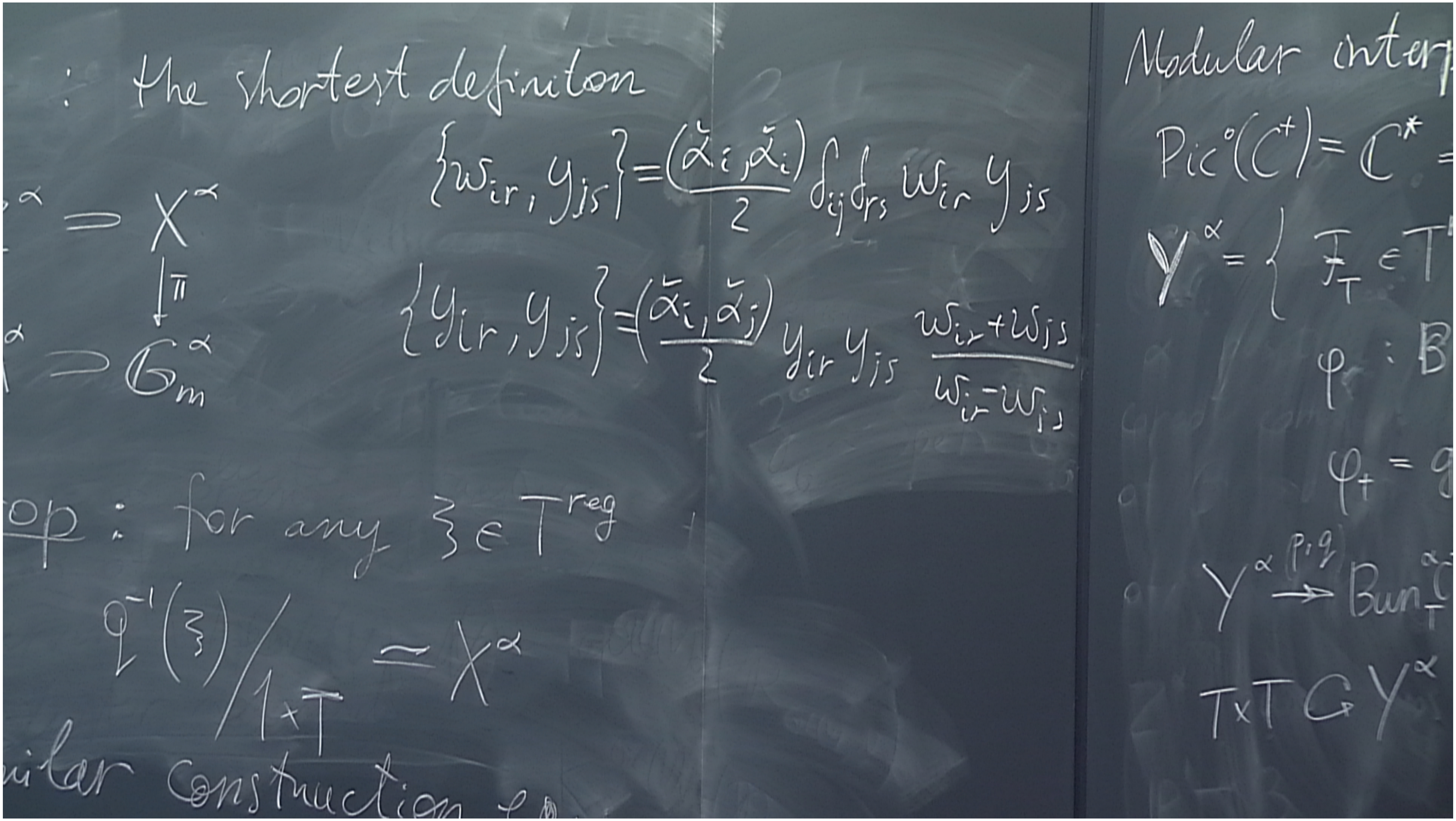
φ : B-structure in $\mathcal{F}_G = \mathcal{F}_T \overset{T}{\times} G$, deg

$\varphi_+ = \mathrm{gr}\,\varphi$ trivialized at c

$$Y^\alpha \xrightarrow{(p,q)} \mathrm{Bun}_T^\alpha C^+ \times T^{reg}$$

$$T\times T \circlearrowright Y^\alpha$$

$$\begin{tikzcd} Y^\alpha \arrow[r, "\varpi"] \arrow[dr, dashed] & Z^\alpha \\ & X^\alpha \arrow[u, phantom, "\cup"] \end{tikzcd}$$



: the shortest definiton

$$\{w_{ir}, y_{js}\} = \frac{(\check{\alpha}_i, \check{\alpha}_i)}{2} \delta_{ij} \delta_{rs} w_{ir} y_{js}$$

$$^\alpha \supset X^\alpha$$

$$\downarrow \pi$$

$$^\alpha \supset \mathbb{G}_m^\alpha$$

$$\{y_{ir}, y_{js}\} = \frac{(\check{\alpha}_i, \check{\alpha}_j)}{2} y_{ir} y_{js} \frac{w_{ir} + w_{js}}{w_{ir} - w_{js}}$$

op : for any $\xi \in T^{reg}$

$$q^{-1}(\xi) / 1 \times T \simeq X^\alpha$$

ilar construction

Modular interp

$$\mathrm{Pic}^\circ(C^+) = \mathbb{C}^* =$$

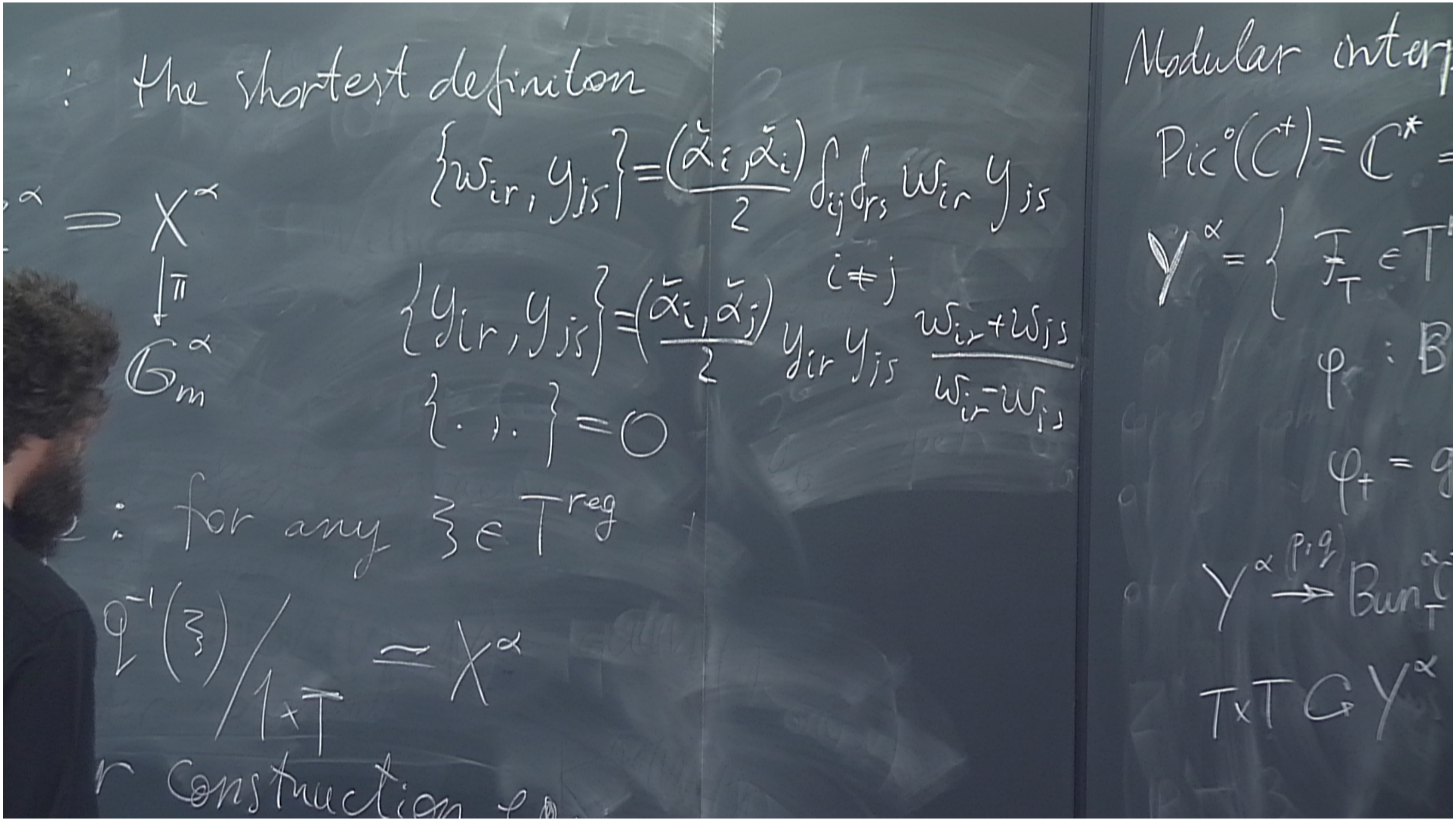
$$Y^\alpha = \{ \mathcal{F}_T \in T$$

$$\varphi : \mathcal{B}$$

$$\varphi_+ = q$$

$$Y^\alpha \xrightarrow{p, q} \mathrm{Bun}_T^\alpha(C$$

$$T \times T \curvearrowright Y^\alpha$$



: the shortest definiton

$^\alpha \simeq X^\alpha$

$\downarrow \pi$

$\mathbb{G}_m^\alpha$

$$\{w_{ir}, y_{js}\} = \frac{(\check{\alpha}_i, \check{\alpha}_i)}{2} \delta_{ij} \delta_{rs} w_{ir} y_{js}$$

$i \neq j$

$$\{y_{ir}, y_{js}\} = \frac{(\check{\alpha}_i, \check{\alpha}_j)}{2} y_{ir} y_{js} \frac{w_{ir} + w_{js}}{w_{ir} - w_{js}}$$

$$\{\cdot, \cdot\} = 0$$

: for any $\xi \in T^{reg}$

$$q^{-1}(\xi) / _{1 \times T} \simeq X^\alpha$$

r construction

Modular interp

$\mathrm{Pic}^\circ(C^+) = \mathbb{C}^* =$

$Y^\alpha = \{ \mathcal{F}_T \in T$

$\varphi : \mathcal{B}$

$\varphi_+ = q$

$Y^\alpha \xrightarrow{p,q} \mathrm{Bun}_T^\alpha($

$T \times T \circlearrowright Y^\alpha$

$w_{ir}\, y_{js}$

$$\frac{w_{ir}+w_{js}}{w_{ir}-w_{js}}$$

$Fl = G((z))/Iw$

Bl

$Gr = G((z))/G[[z]]$

$\mathfrak{g}((z))$ has trigonometric r-matrix $\Leftrightarrow$

Manin triple in $\mathfrak{g}((z))\oplus\mathfrak{h}$

$\Rightarrow$ Poisson structure on Fl

$\alpha = \sum a_i$

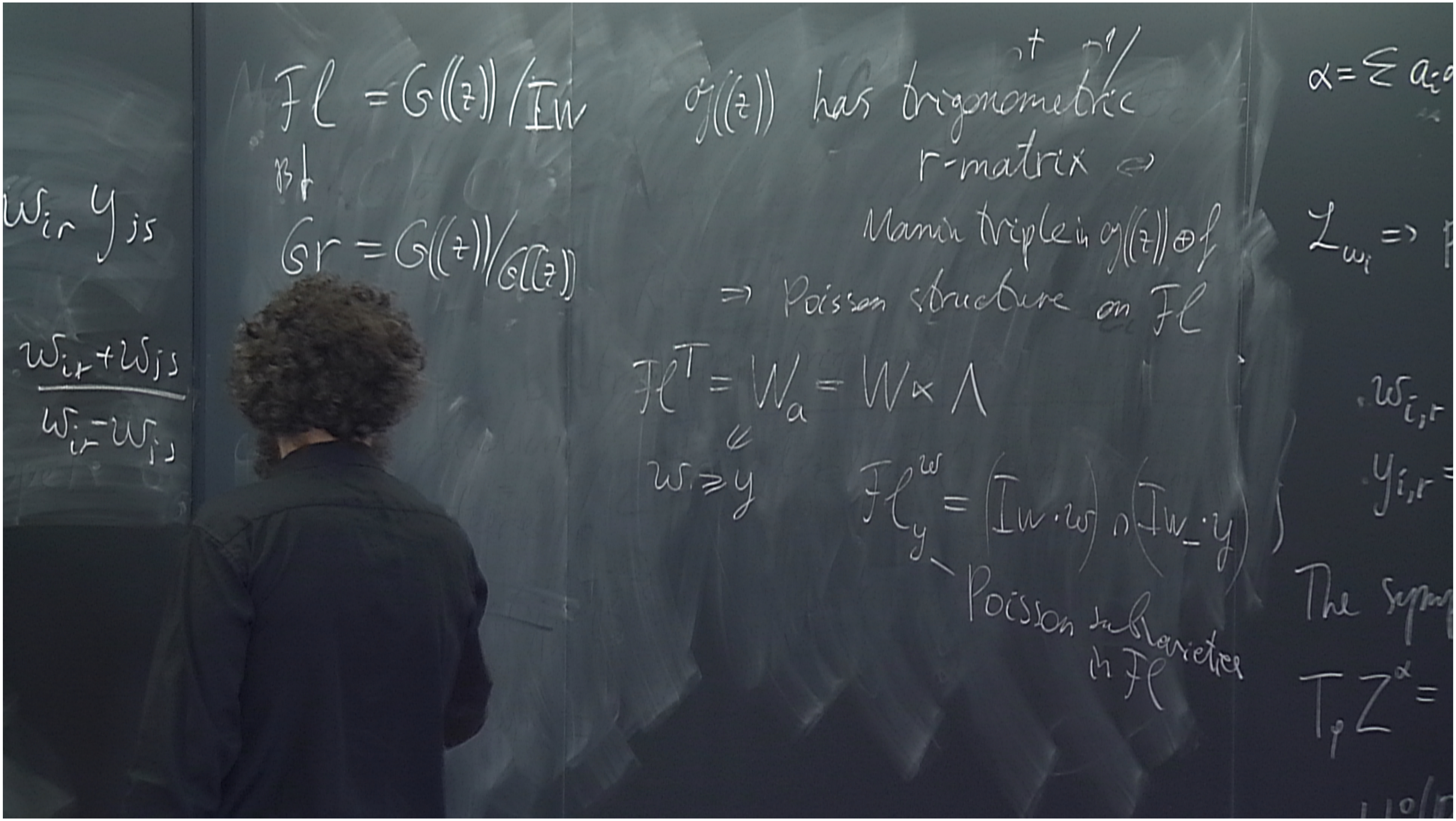
$L_{w_i} \Rightarrow$

$w_{i,r}$

$y_{i,r} =$

The symp

$T_\varphi Z^\alpha =$



$w_{ir}\, y_{js}$

$$\frac{w_{ir}+w_{js}}{w_{ir}-w_{js}}$$

$Fl = G((z))/Iw$

$Gr = G((z))/G[[z]]$

$\mathfrak{g}((z))$ has trigonometric r-matrix ⇔ Manin triple in $\mathfrak{g}((z)) \oplus \mathfrak{h}$

⇒ Poisson structure on Fl

$Fl^T = W_a = W \ltimes \Lambda$

$w \geq y$

$Fl^w_y = (Iw \cdot w) \cap (Iw_- \cdot y)$ — Poisson subvarieties in Fl

$\alpha = \sum a_i$

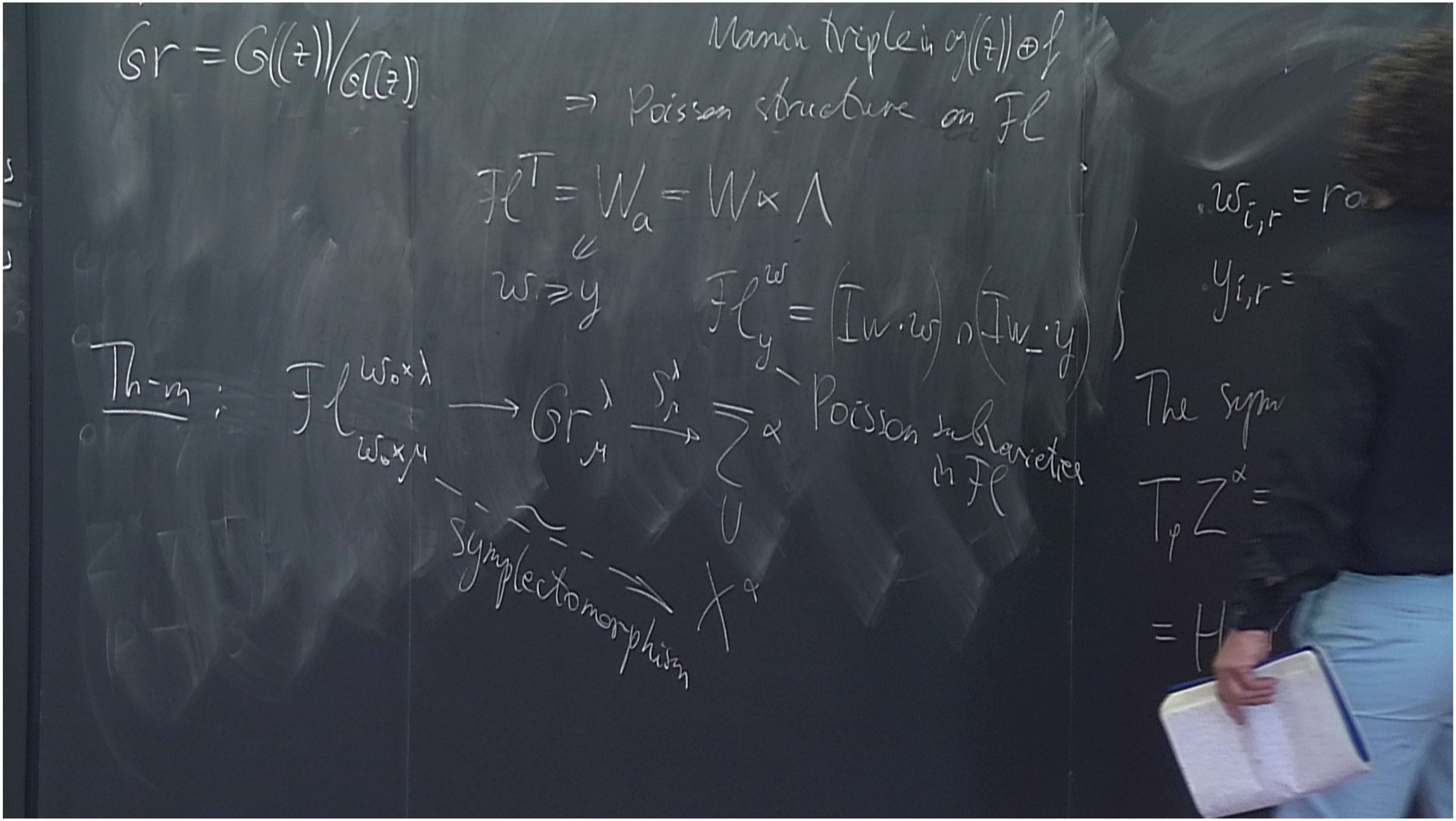
$L_{w_i} \Rightarrow$

$w_{i,r}$

$y_{i,r} =$

The sym

$T_\varphi Z^\alpha =$



$Gr = G((z))/G[[z]]$

Manin triple in $\mathfrak{g}((z)) \oplus \mathfrak{h}$

$\Rightarrow$ Poisson structure on $\mathcal{F}\ell$

$\mathcal{F}\ell^T = W_a = W \ltimes \Lambda$

$\overset{\in}{w} \geq y$ $\quad \mathcal{F}\ell^w_y = (\mathrm{Iw} \cdot w) \cap (\mathrm{Iw}_- \cdot y)$ — Poisson subvarieties in $\mathcal{F}\ell$

Th-m:

$$\begin{tikzcd}
\mathcal{F}\ell^{w_0 \times \lambda}_{w_0 \times \mu} \arrow[r] \arrow[rrd, dashed, "\sim", "\text{Symplectomorphism}"'] & Gr^\lambda_\mu \arrow[r, "s^\lambda_\mu"] & \Sigma^\alpha \arrow[d, phantom, sloped, "\subset"] \\
 & & X^\alpha
\end{tikzcd}$$

$w_{i,r} = ro$

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The symp

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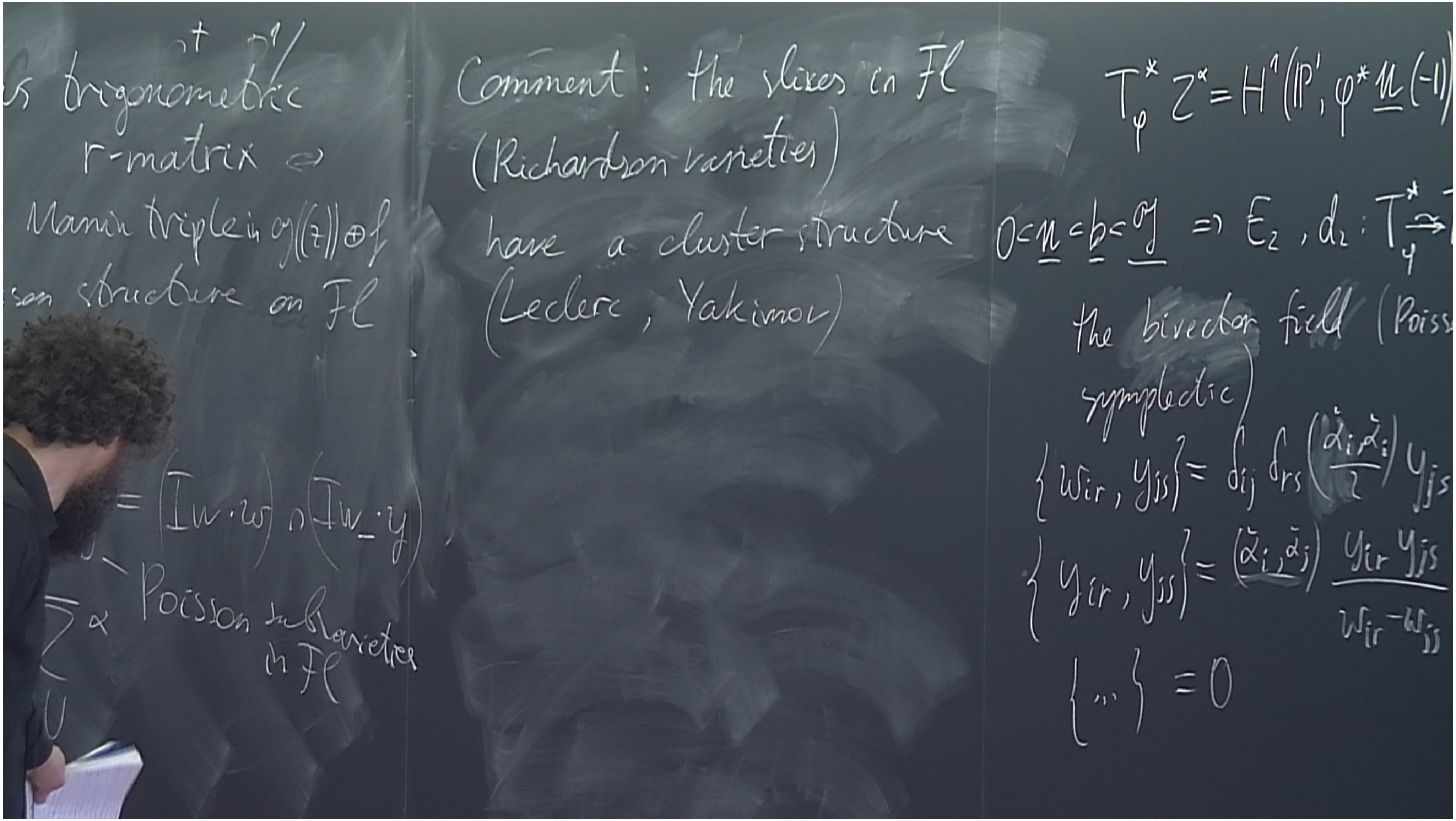
$= H$



t 1/

…s trigonometric

r-matrix ⇔

Manin triple in $\mathfrak{g}((z))\oplus\mathfrak{f}$

…son structure on $\mathcal{F}l$

$= (I w_+ \cdot w) \cap (I w_- \cdot y)$

Poisson subvarieties in $\mathcal{F}l$

$\sum \alpha$

$\cup$

Comment: the slices in $\mathcal{F}l$

(Richardson varieties)

have a cluster structure

(Leclerc, Yakimov)

$$T^*_\varphi Z^\alpha = H^1(\mathbb{P}^1, \varphi^* \underline{\mathfrak{n}}(-1))$$

$$0 \subset \underline{\mathfrak{n}} \subset \underline{\mathfrak{b}} \subset \underline{\mathfrak{g}} \Rightarrow E_2 , d_2 : T^*_\varphi \xrightarrow{\sim}$$

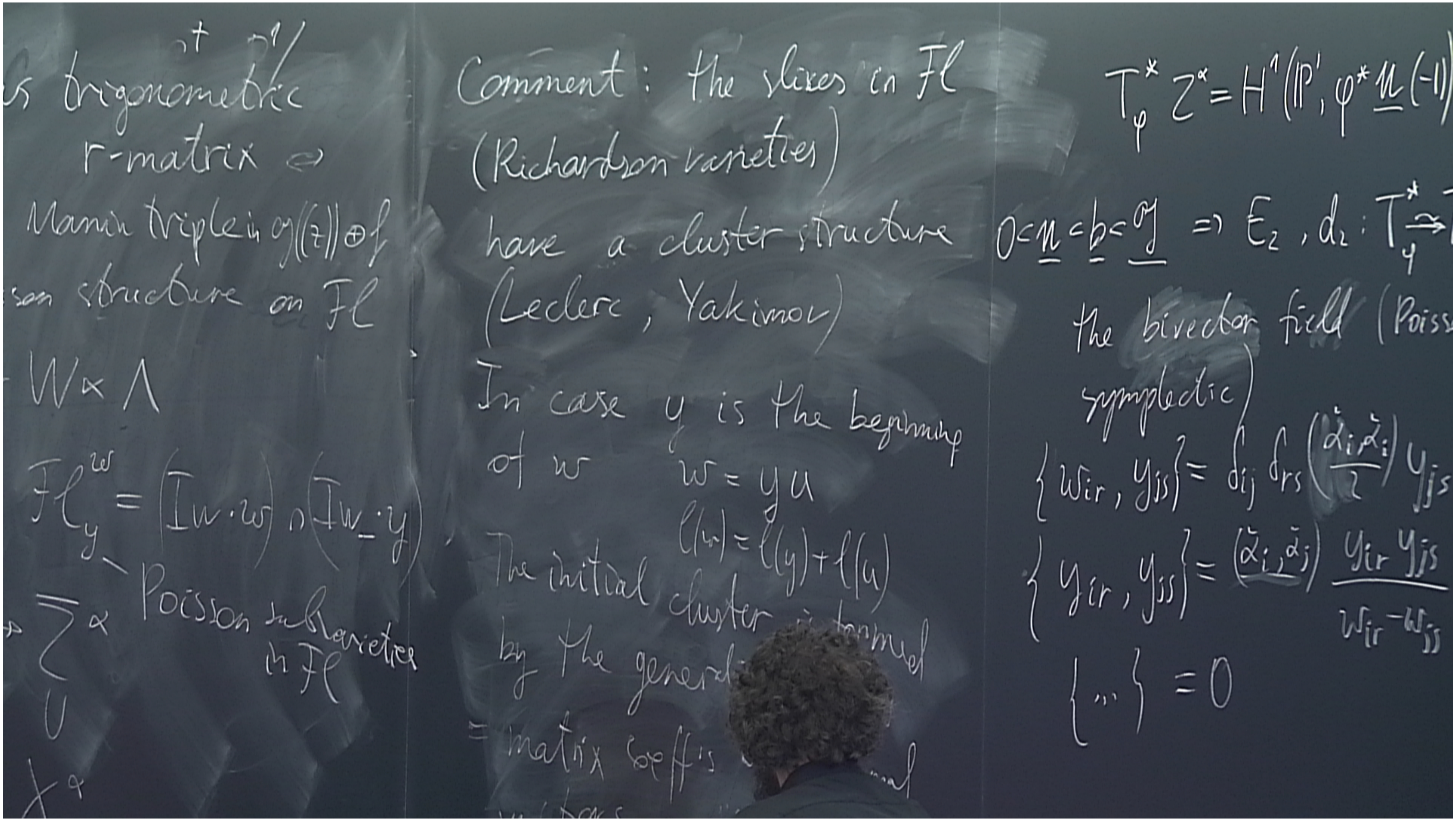
the bivector field (Poiss…

symplectic)

$$\{w_{ir}, y_{js}\} = \delta_{ij}\,\delta_{rs}\left(\frac{\check{\alpha}_i, \check{\alpha}_i}{2}\right) y_{js}$$

$$\{y_{ir}, y_{js}\} = (\check{\alpha}_i, \check{\alpha}_j)\,\frac{y_{ir}\,y_{js}}{w_{ir} - w_{js}}$$

$$\{\ldots\} = 0$$



$\mathfrak{n}^{+}$ $\mathfrak{n}^{1}$/

...s trigonometric
r-matrix ⇔
Manin triple in $\mathfrak{g}((z)) \oplus \mathfrak{f}$

...son structure on $\mathcal{F}\ell$

$W \ltimes \Lambda$

$\mathcal{F}\ell^{w}_{y} = (\overline{Iw \cdot w}) \cap (\overline{Iw_{-} \cdot y})$ ↖ Poisson subvarieties in $\mathcal{F}\ell$

$\Rightarrow \sum^{\alpha}$

$\cup$

$\mathcal{X}^{\alpha}$

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In case y is the beginning of w $\quad w = yu$
$\ell(w) = \ell(y) + \ell(u)$

The initial cluster is formed
by the general...
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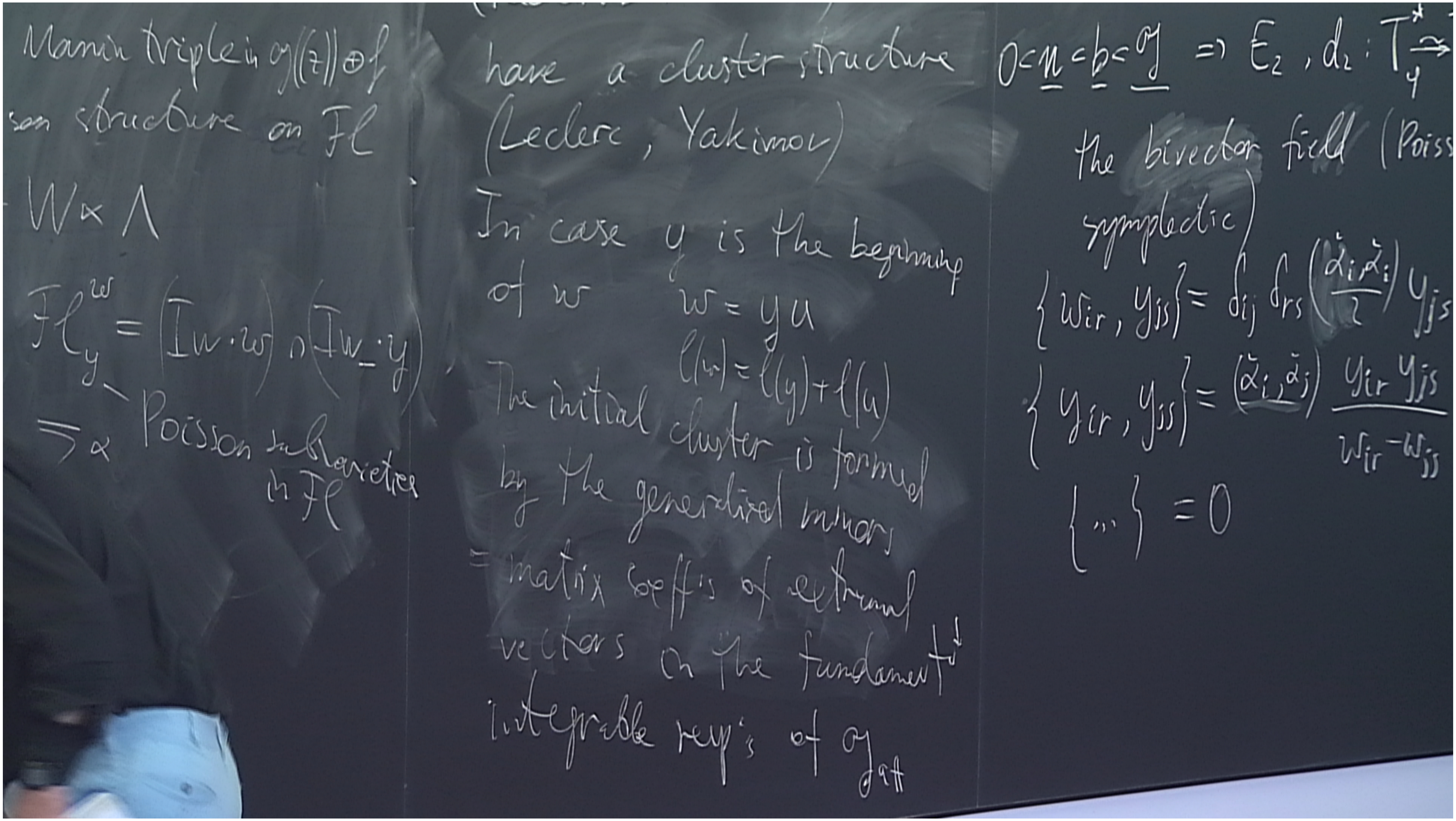

Manin triple in $\mathfrak{g}((z)) \oplus f$

son structure on $\mathcal{F}\ell$

$W \ltimes \Lambda$

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In case y is the beginning of w $w = yu$

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The initial cluster is formed by the generalized minors

= matrix coeff's of extremal vectors in the fundament'l integrable rep's of $\mathfrak{g}_{aff}$

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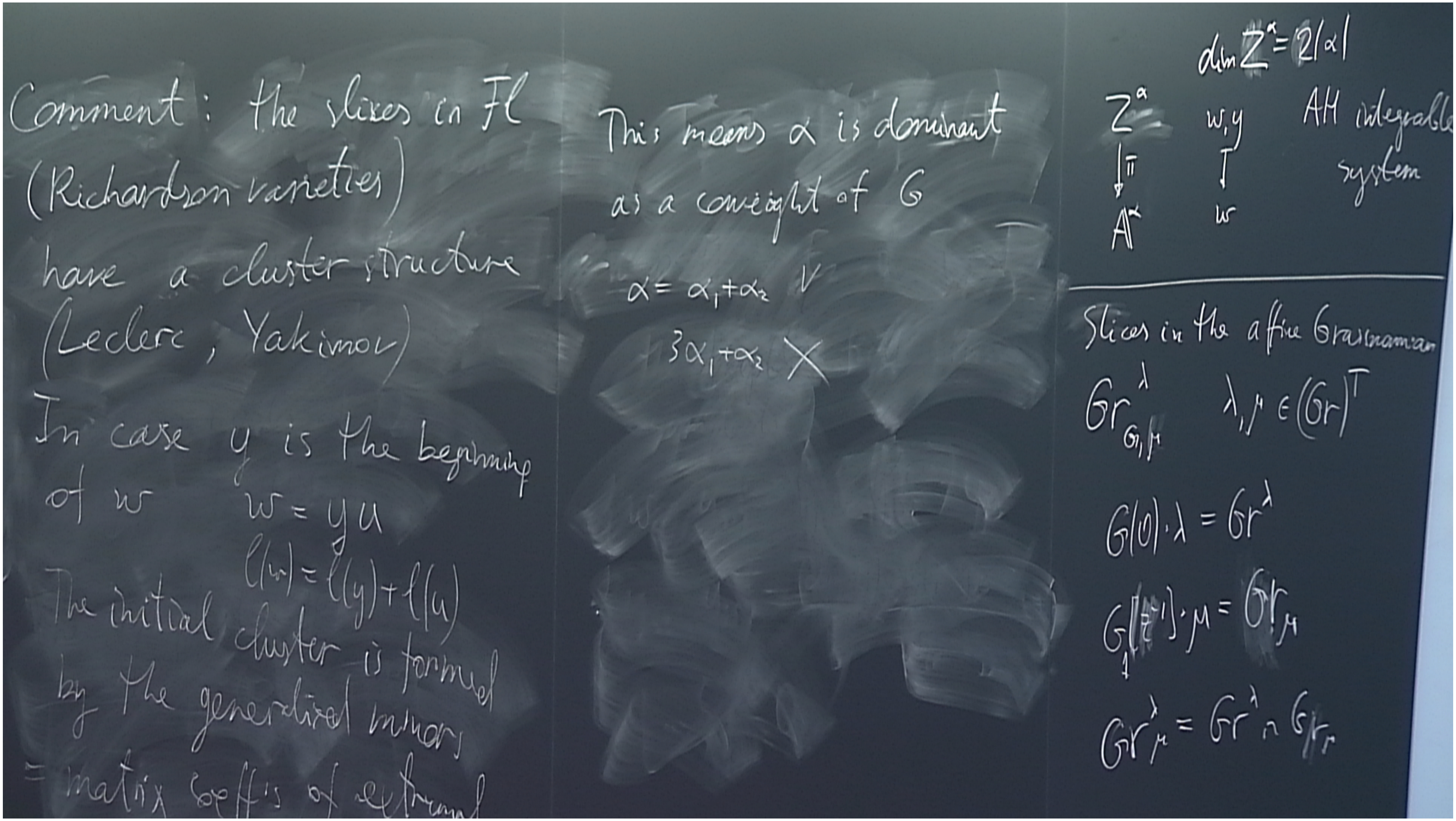

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This means α is dominant
as a coweight of G

$\alpha = \alpha_1 + \alpha_2$ ✓

$3\alpha_1 + \alpha_2$ ✗

$\dim Z^\alpha = 2|\alpha|$

$$\begin{tikzcd} Z^\alpha \arrow[d, "\pi"] & w, y \arrow[d] \\ \mathbb{A}^\alpha & w \end{tikzcd}$$

AH integrable
system

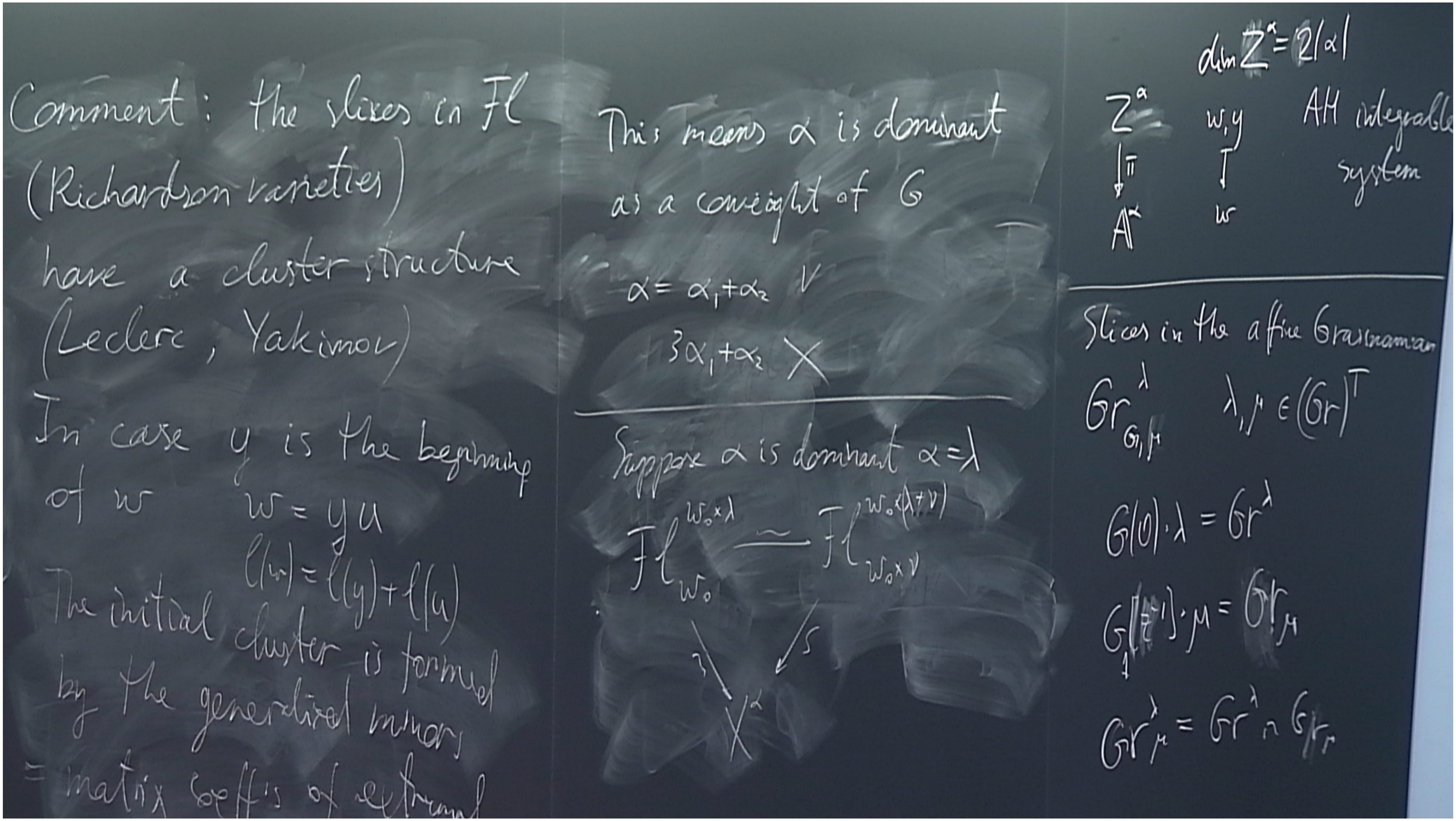
Slices in the affine Grassmannian

$Gr^\lambda_{G,\mu}$ $\quad \lambda, \mu \in (Gr)^T$

$G(\mathcal{O}) \cdot \lambda = Gr^\lambda$

$G_1[t^{-1}] \cdot \mu = Gr_\mu$

$Gr^\lambda_\mu = Gr^\lambda \cap Gr_\mu$



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Suppose α is dominant $\alpha = \lambda$

$$\begin{tikzcd} \mathcal{F}\ell^{w_0 \times \lambda}_{w_0} \arrow[rr, "\sim"] \arrow[dr, "\sim"'] & & \mathcal{F}\ell^{w_0 \times (\lambda+\nu)}_{w_0 \times \nu} \arrow[dl, "s"] \\ & X^{\alpha} & \end{tikzcd}$$

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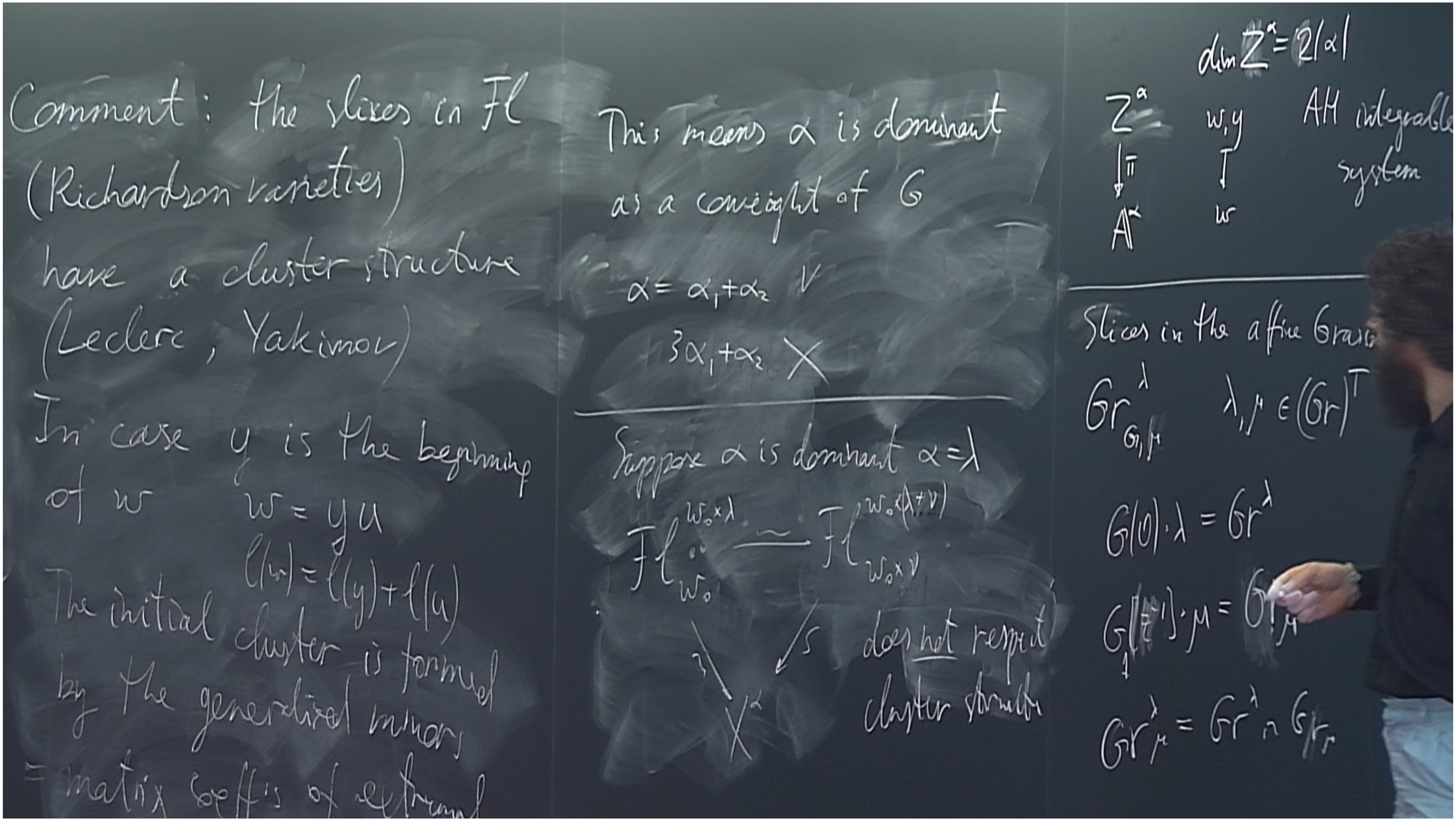

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\end{tikzcd}$$

does not respect cluster structure

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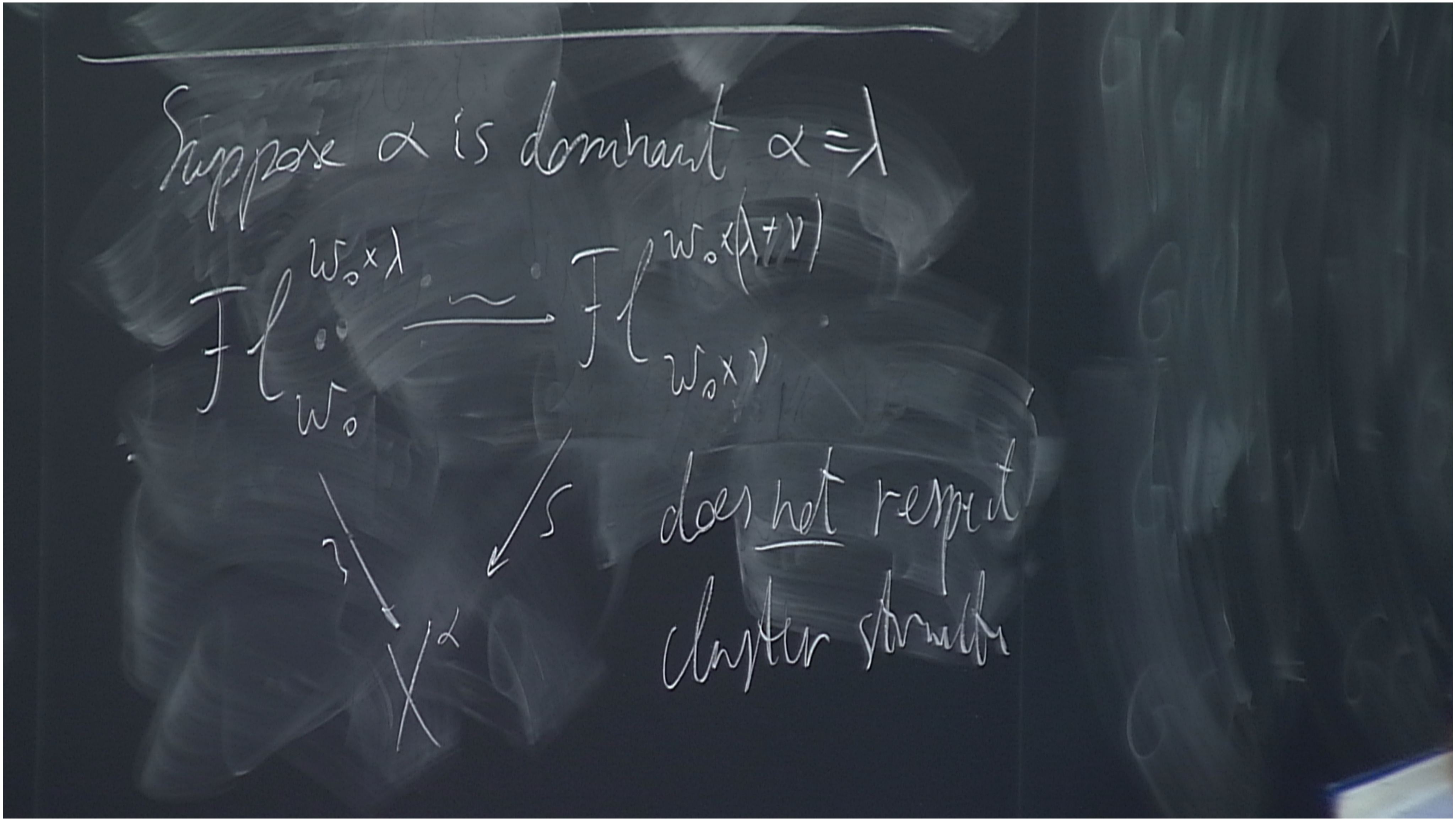

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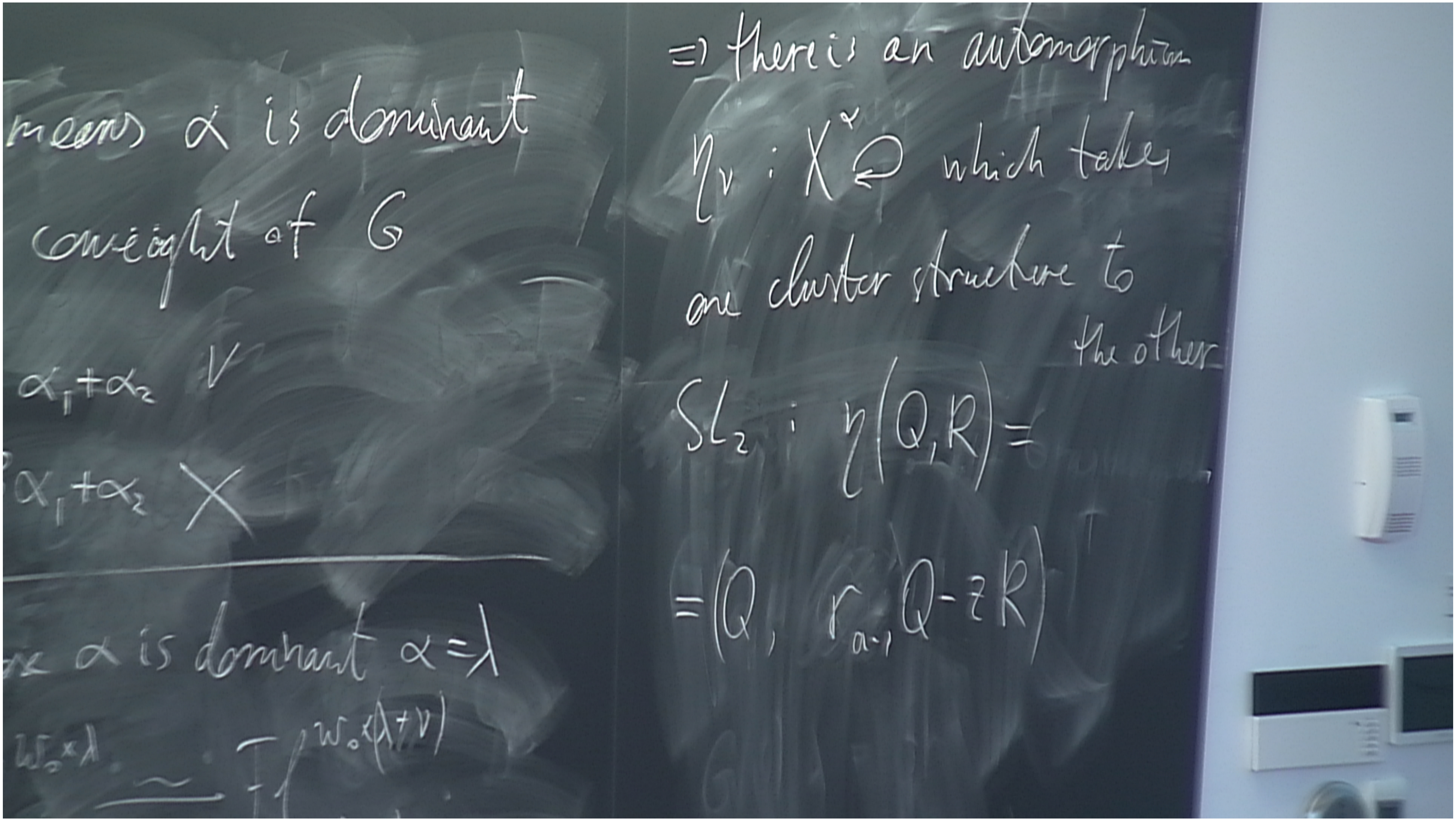


means α is dominant coweight of G

$\alpha_1+\alpha_2$ ✓

$\alpha_1+\alpha_2$ ✗

---

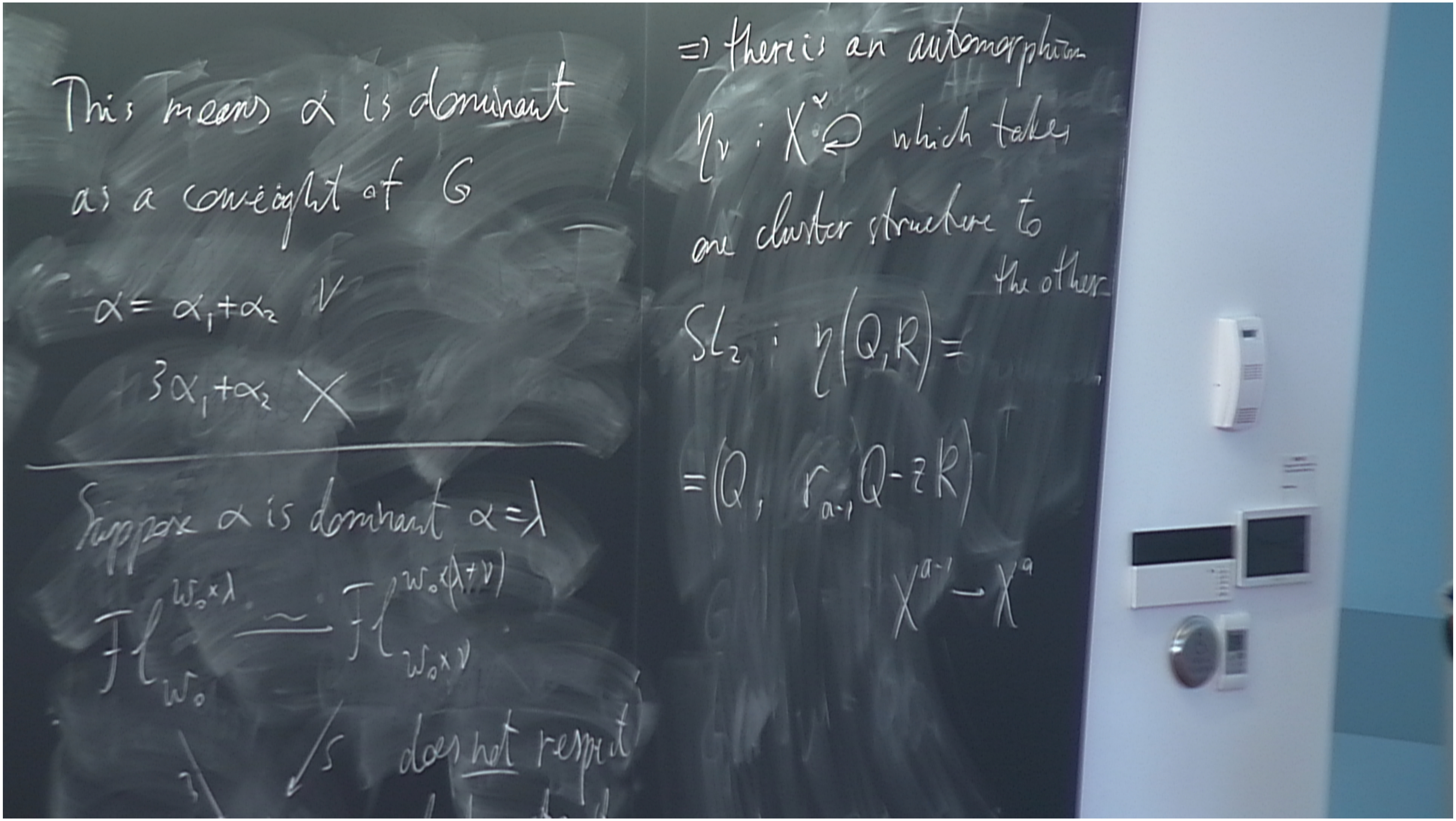
x α is dominant $\alpha=\lambda$

$w_0 x \lambda \xrightarrow{\sim} I^{w_0 x(\lambda+\nu)}$

$\Rightarrow$ there is an automorphism $\eta_\nu : X^\alpha \circlearrowleft$ which takes one cluster structure to the other

SL_2 : $\eta(Q,R) =$

$=(Q,\ r_{a-1}Q - zR)$



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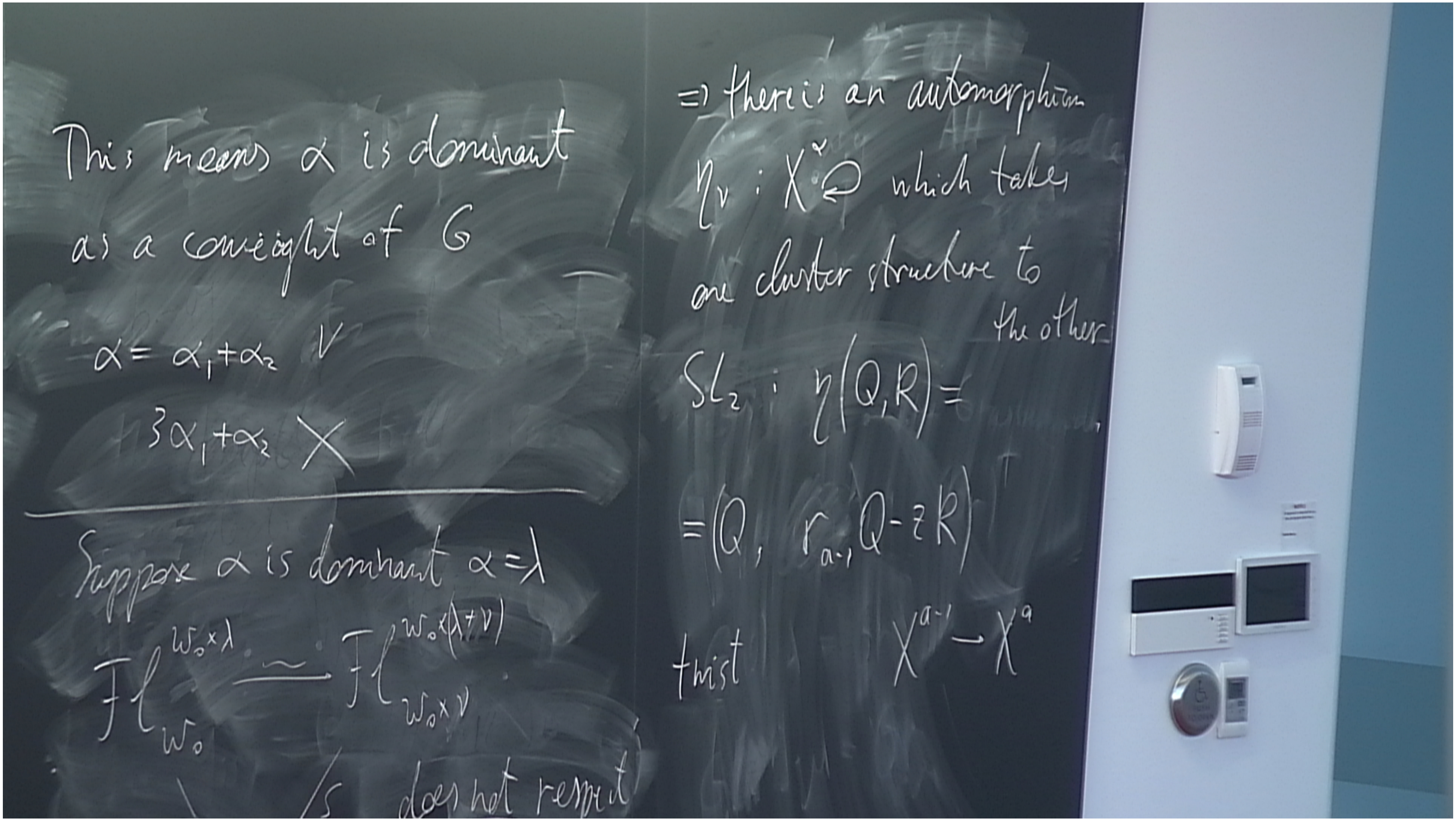

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$K^{GL_a} Gr_{GL_a} = \mathbb{C}[X^a]$

$\mathrm{PervCoh}^{GL_a}(Gr_{GL_a})$ is not commutative

there are left and right rigidities

$$\mathrm{Hom}(A \otimes B, C) = \mathrm{Hom}(A, C \otimes B^*)$$

$\mathcal{F}l = G((z))/Iw$

$p \downarrow$

$Gr = G((z))/G[[z]]$

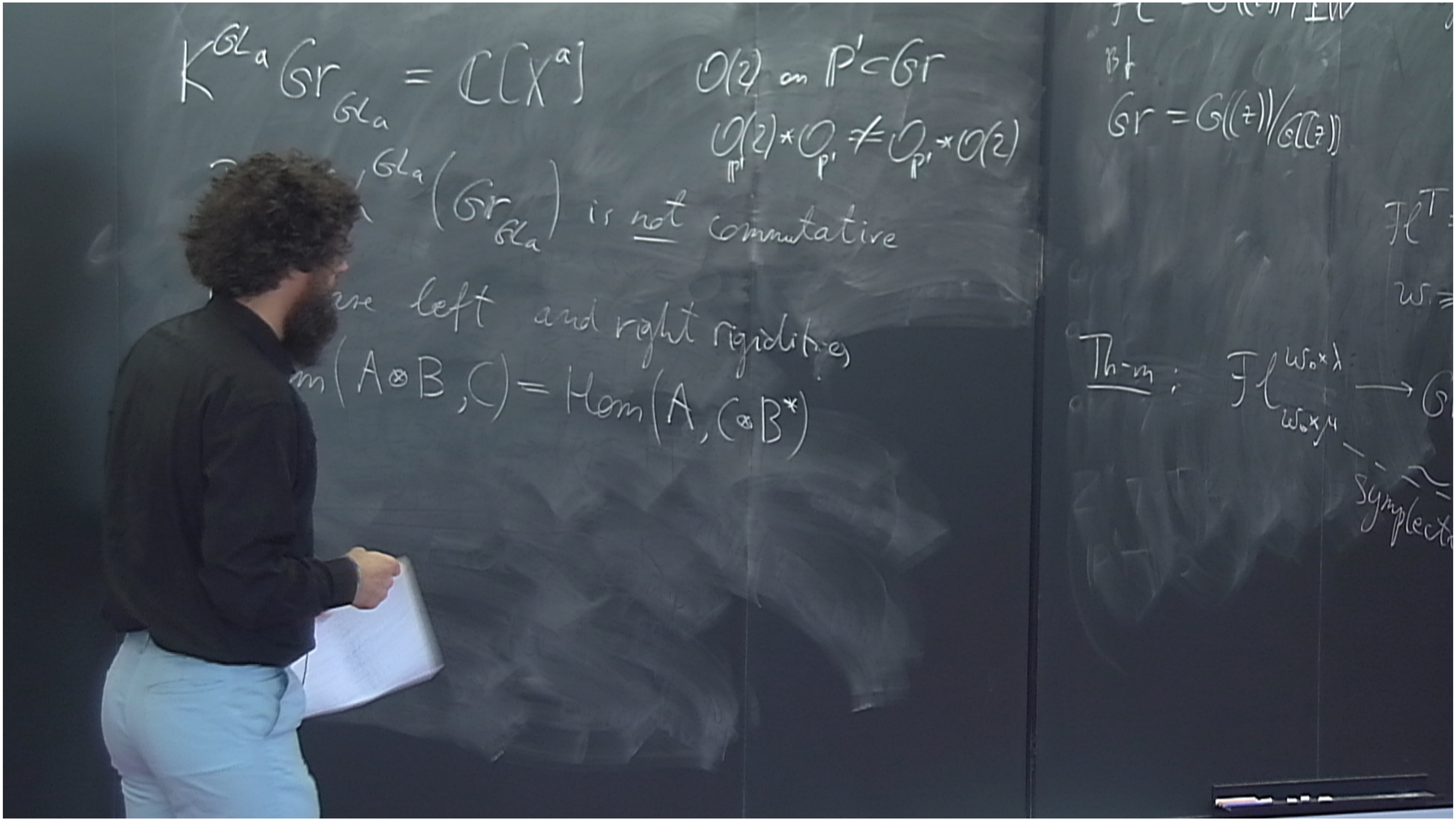
$\mathcal{F}l^T =$

$w_0 =$

Th-m : $\mathcal{F}l^{w_0 * \lambda}_{w_0 * \mu} \longrightarrow G$

$\sim$

Symplect



$K^{GL_a} Gr_{GL_a} = \mathbb{C}[X^a]$

$\mathcal{O}(2)$ on $\mathbb{P}^1 \subset Gr$

$\mathcal{O}_{\mathbb{P}^1}(2) * \mathcal{O}_{\mathbb{P}^1} \neq \mathcal{O}_{\mathbb{P}^1} * \mathcal{O}(2)$

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$m(A \otimes B, C) = Hom(A, C \otimes B^*)$

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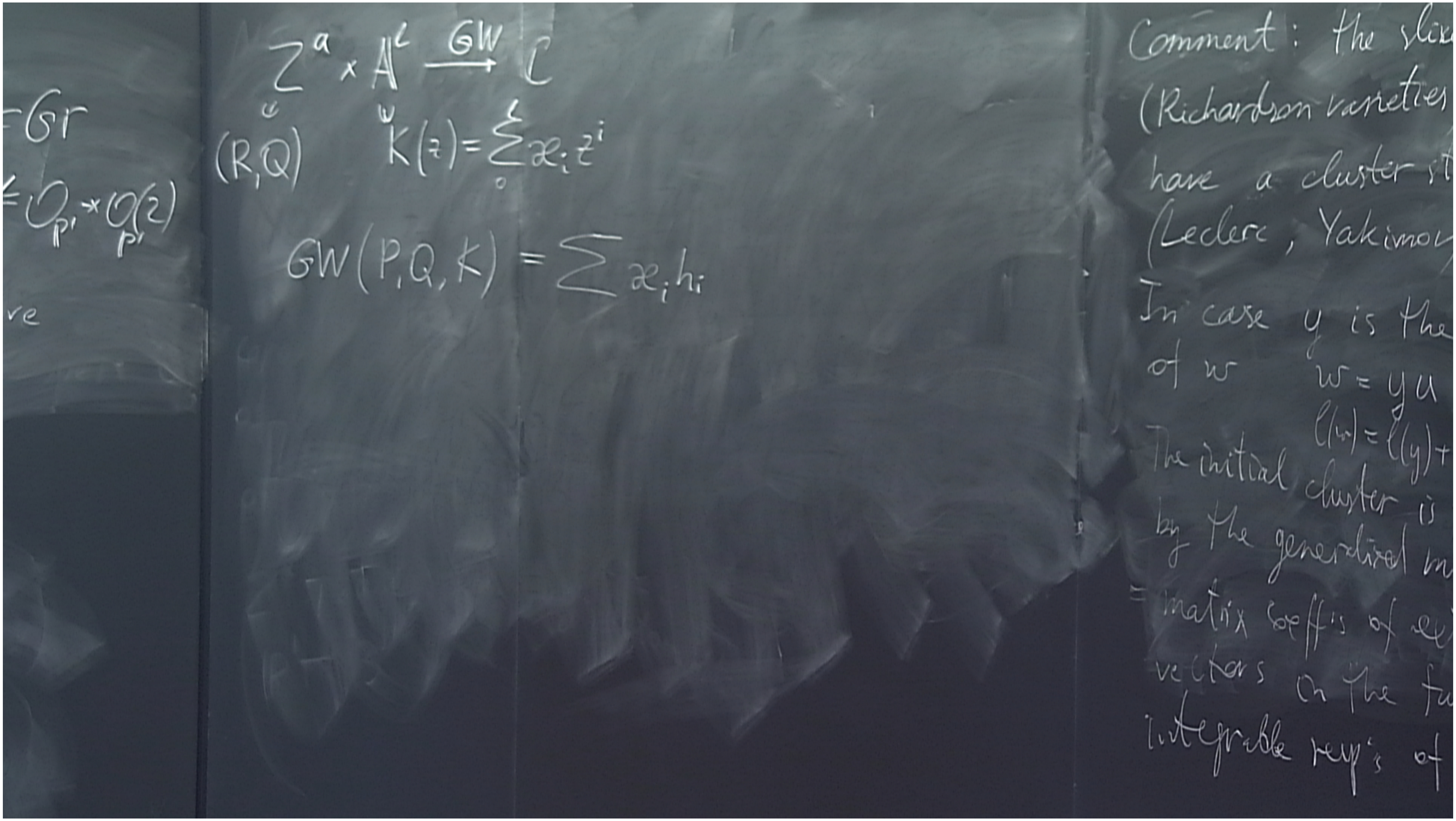
$w_0 =$

<u>Th-m</u>: $Fl^{w_0 * \lambda}_{w_0 * \mu} \longrightarrow G$

symplecto



$= Gr$

$\leq \mathcal{O}_{\mathbb{P}^1} * \mathcal{O}_{\mathbb{P}^1}(2)$

re

$$\begin{array}{ccccc} Z^a & \times & \mathbb{A}^\ell & \xrightarrow{GW} & \mathbb{C} \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} & & \\ (R,Q) & & K(z) = \sum_0^\ell x_i z^i & & \end{array}$$

$$GW(P,Q,K) = \sum x_i h_i$$

Comment: the slic

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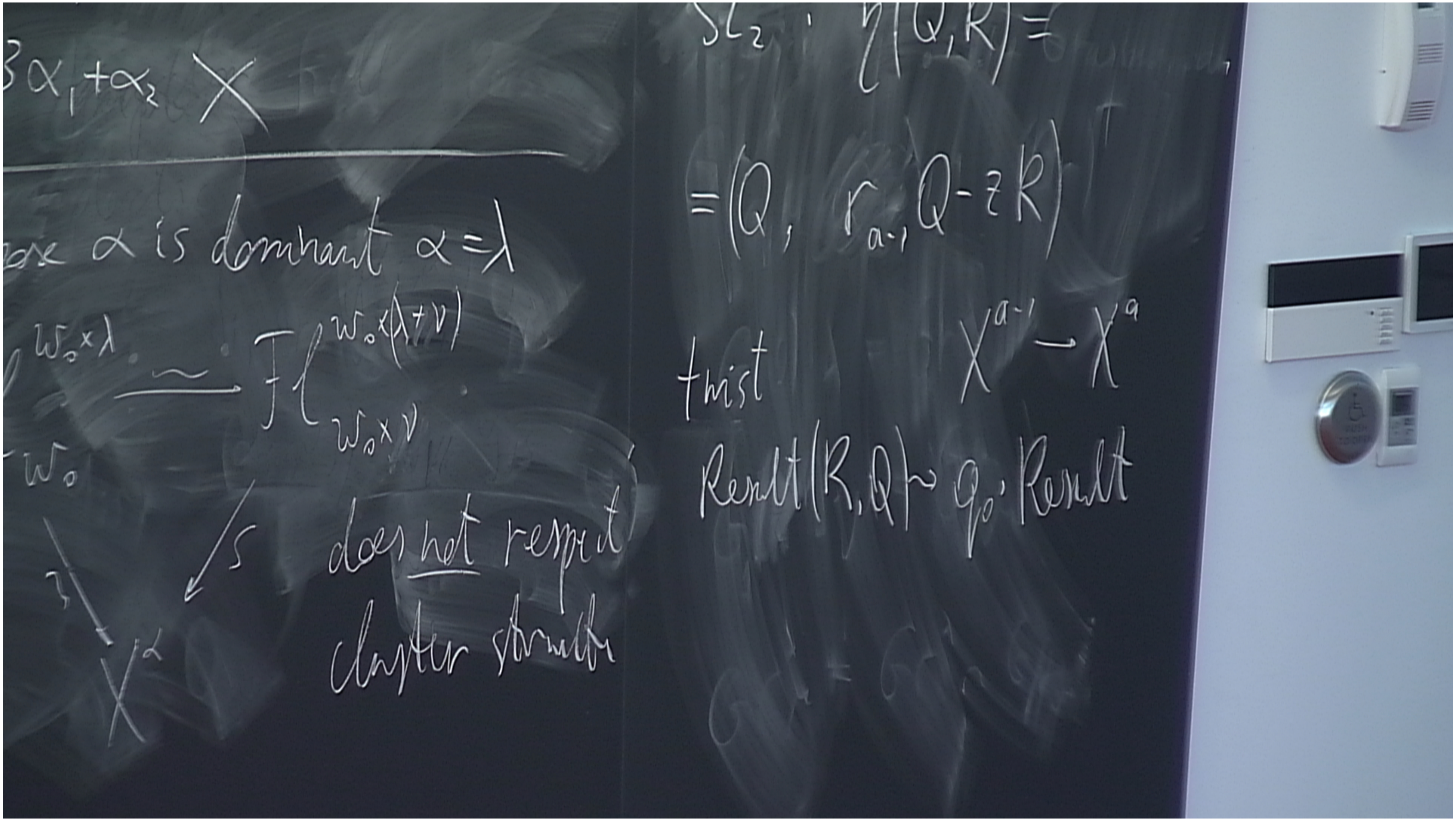

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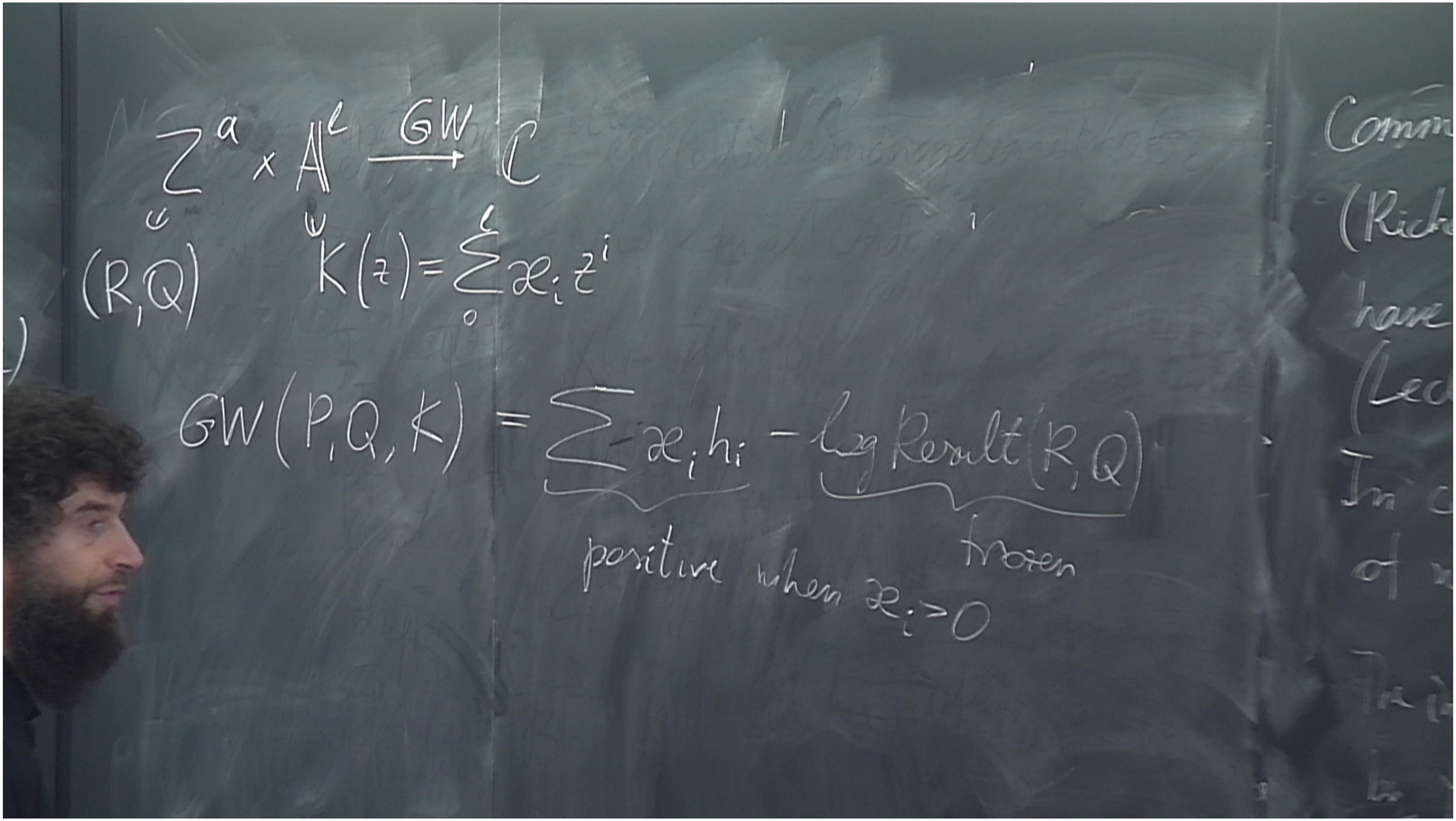
Result$(R,Q) \rightsquigarrow q_0\cdot$ Result



$$\begin{array}{ccccc} \Sigma^a & \times & \mathbb{A}^\ell & \xrightarrow{GW} & \mathbb{C} \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} & & \\ (P,Q) & & K(z)=\sum_0^\ell x_i z^i & & \end{array}$$

$$GW(P,Q,K) = \underbrace{\sum x_i h_i}_{\text{positive when } x_i>0} - \underbrace{\log \mathrm{Result}(P,Q)}_{\text{frozen}}$$

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