Title: Quantum theory with indefinite causal structure

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Abstract: <p>Quantum theory can be understood as a theory of information processing in the circuit framework for operational probabilistic theories. This approach presupposes a definite casual structure as well as a preferred time direction. But in general relativity, the causal structure of space-time is dynamical and not predefined, which indicates that a quantum theory that could incorporate gravity requires a more general operational paradigm. In this talk, I will describe recent progress in this direction. First, I will show how relaxing the assumption that local operations take place in a global causal structure leads to a generalized framework that unifies all signaling and non-signaling quantum correlations in space-time via an extension of the density matrix called the process matrix. This framework also contains a new kind of correlations incompatible with any definite causal structure, which violate causal inequalities, the general theory of which I am going to present. I will then present an extension of the process matrix framework, in which no predefined causal structure is assumed even locally. This is based on a more general, time-neutral notion of operation, which leads to new insights into the problem of time-reversal symmetry in quantum mechanics, the meaning of causality, and the fact that we remember the past but not the future. In the resultant generalized formulation of quantum theory, operations are associated with regions that can be connected in networks with no directionality assumed for the connections. The theory is compatible with timelike loops and other acausal structures.</p>

Quantum theory with indefinite causal structure

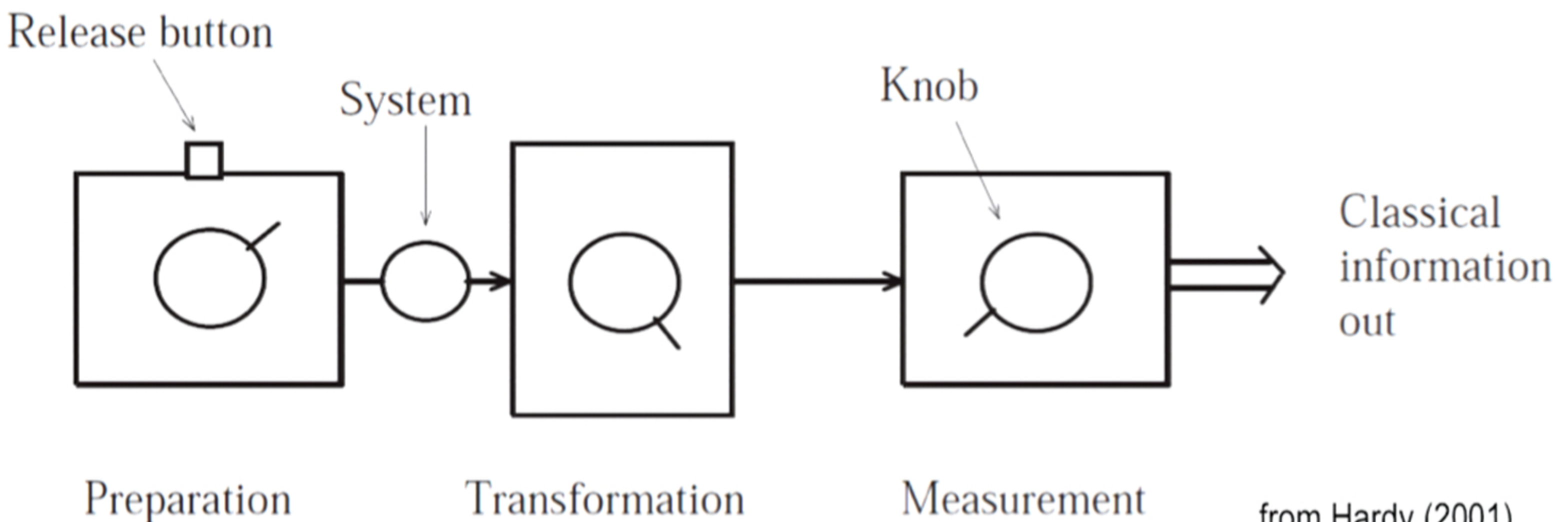
Ognyan Oreshkov

Centre for Quantum Information and Communication, Université Libre de Bruxelles

Based on work with:

Caslav Brukner, Nicolas Cerf, Fabio Costa, Christina Giarmatzi

Operational Approach

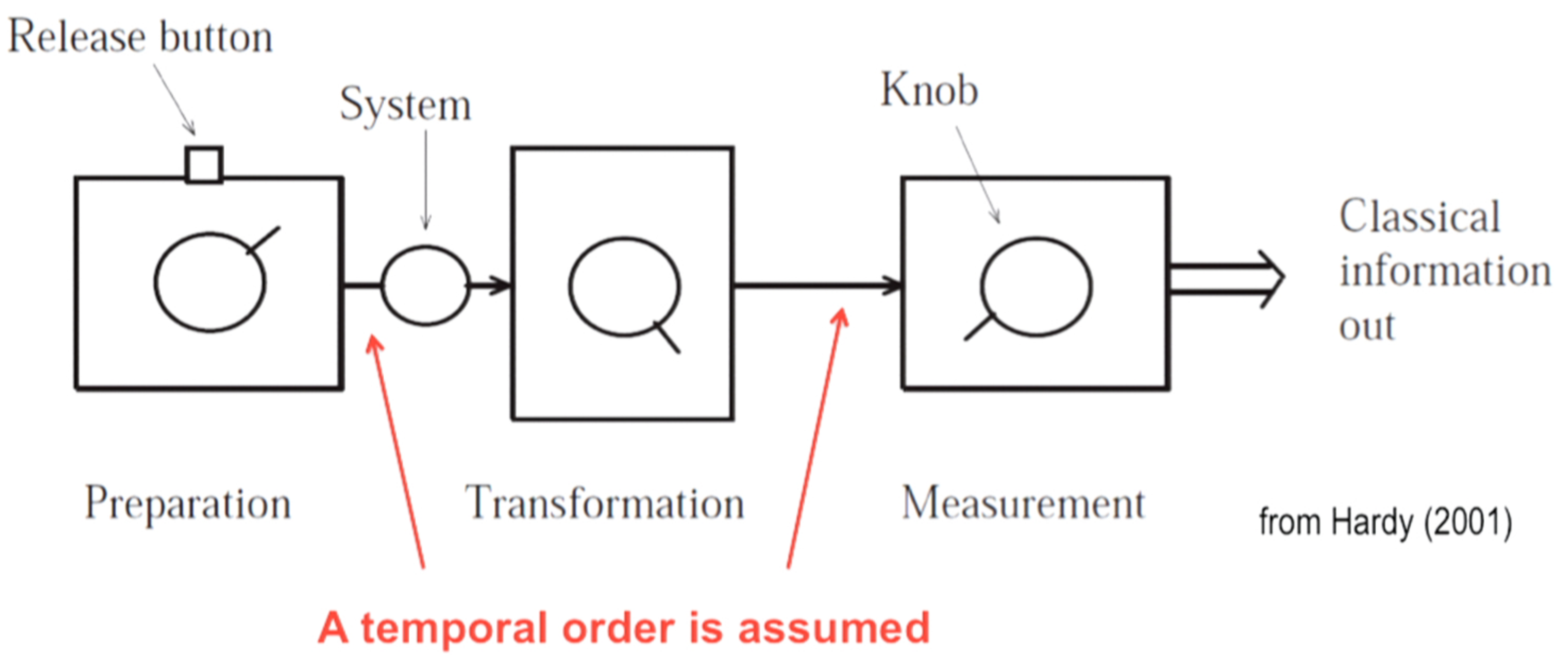


from Hardy (2001)

Significant progress in understanding QM from an **operational** perspective.

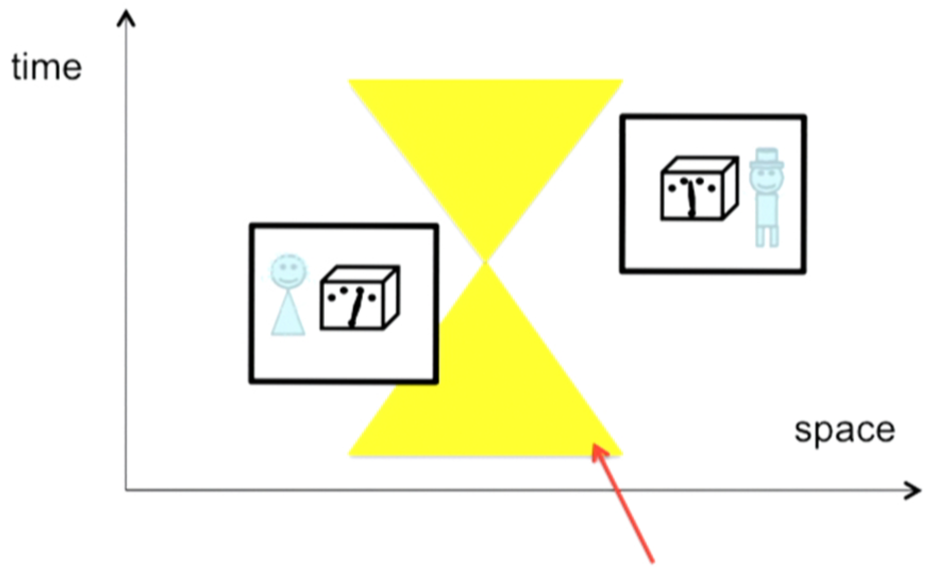
Hardy (2001), Barrett (2005), Dakic and Brukner (2009), Massanes and Mülelr (2010), Chiribella, D'Ariano, and Perinotti (2010), Hardy (2011) ….

Operational Approach



from Hardy (2001)

Correlations between experiments in space-time



A causal structure is assumed

Questions

1. Could we understand causal structure from more primitive concepts (e.g., signaling from A to B → A is in the past of B)?

2. Why does signalling always go forward in time?

3. Can we generalize quantum theory so that a causal structure is not presumed? (Motivation: quantum gravity)

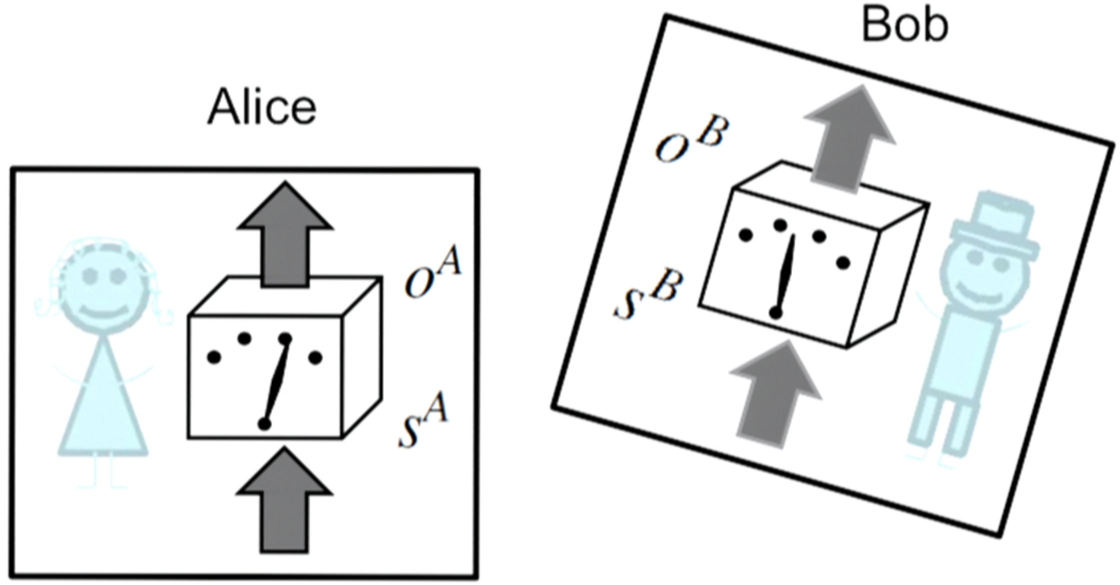
   Hardy, arXiv:0509120

4. What new physical possibilities would this imply?

Outline

- The process framework for operations with no causal order
- A time-symmetric operational approach to quantum theory
- Quantum theory without any prior notion of time

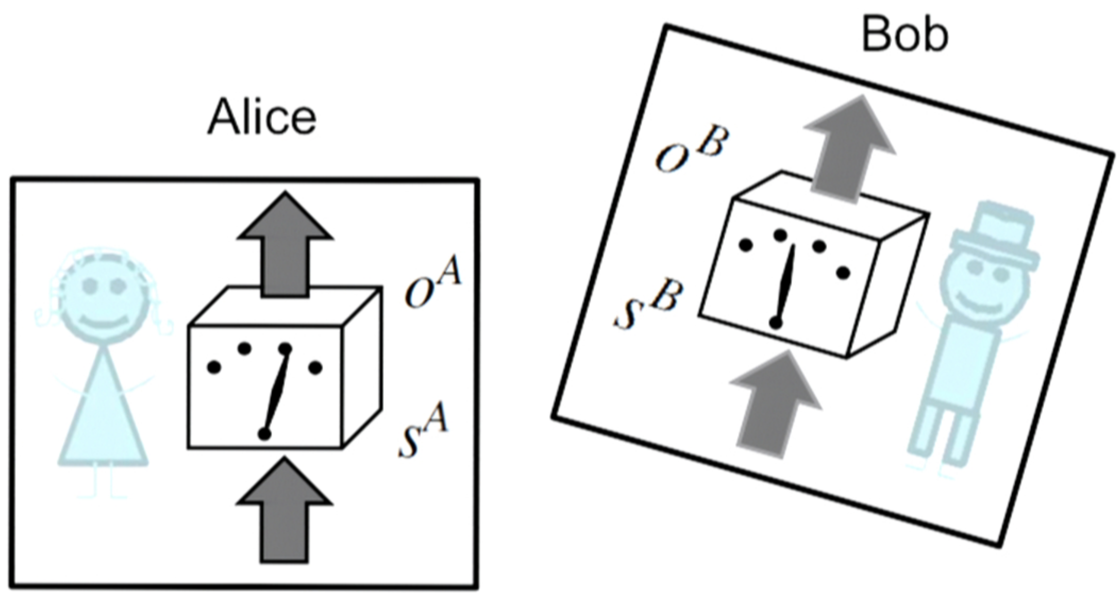
The process framework



No assumption of pre-existing causal order.

O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

The process framework



'Process' →
(catalogue of probabilities)

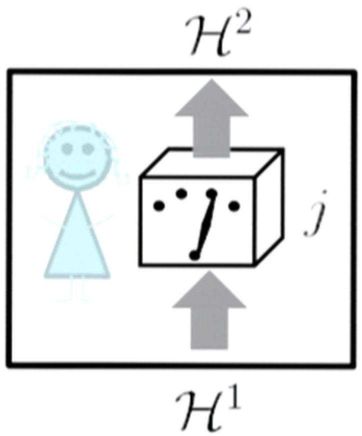
…

$$P(o^A, o^B | s^A, s^B)$$

…

Quantum processes

Local descriptions agree with quantum mechanics



Transformations = completely positive (CP) maps

$$\mathcal{M}_j : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2)$$

Kraus representation: $$\mathcal{M}_j(\rho) = \sum_k E_{jk}\rho E_{jk}^\dagger$$

Completeness relation: $$\sum_j \sum_k E_{jk}^\dagger E_{jk} = I$$

Quantum processes

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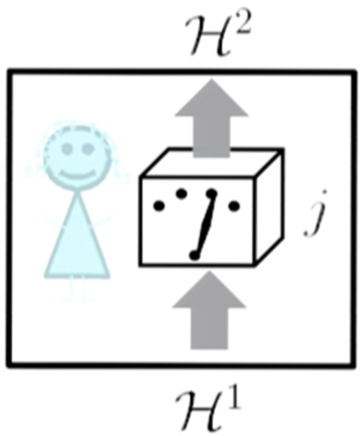

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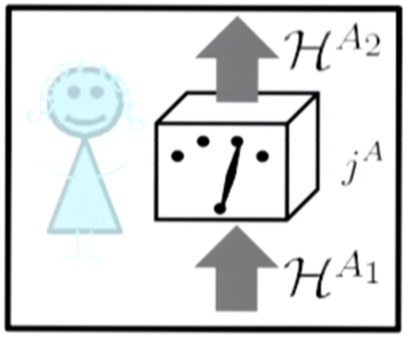
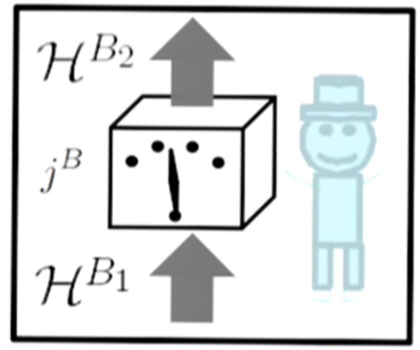


$$\mathcal{M}^A_{j^A} : \mathcal{L}(\mathcal{H}^{A_1}) \rightarrow \mathcal{L}(\mathcal{H}^{A_2})$$



$$\mathcal{M}^B_{j^B} : \mathcal{L}(\mathcal{H}^{B_1}) \rightarrow \mathcal{L}(\mathcal{H}^{B_2})$$

Assumption 1: The probabilities are functions of the local CP maps,

$$P(\mathcal{M}^A_{j^A}, \mathcal{M}^B_{j^B}, \cdots)$$

Local validity of QM ➡ $P(\mathcal{M}^A, \mathcal{M}^B, \cdots)$ is **linear** in $\mathcal{M}^A$, $\mathcal{M}^B$, ...

Choi-Jamiołkowski isomorphism

CP maps

Positive semidefinite matrices

$$\mathcal{M}^{A} : \mathcal{L}(\mathcal{H}^{A_1}) \to \mathcal{L}(\mathcal{H}^{A_2}) \quad \longleftrightarrow \quad M^{A_1 A_2} \in \mathcal{L}(\mathcal{H}^{A_1}) \otimes \mathcal{L}(\mathcal{H}^{A_2})$$

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CP maps		Positive semidefinite matrices
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$$M^{12} := [\mathcal{I} \otimes \mathcal{M}(|\Phi^+\rangle\langle\Phi^+|)]^T$$

$$|\Phi^+\rangle = \sum_i |i\rangle|i\rangle$$

The process matrix

Representation

$$P(\mathcal{M}^A_{j^A}, \mathcal{M}^B_{j^B}, \cdots) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2 \cdots}\left(M^{A_1 A_2}_{j^A} \otimes M^{B_1 B_2}_{j^B} \otimes \cdots\right)\right]$$

Process matrix

The process matrix

Representation

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Process matrix

Similar to Born's rule but can describe signalling!

The process matrix

Conditions on W (assuming the parties can share entanglement):

1. Non-negative probabilities: $W^{A_1A_2B_1B_2\cdots} \geq 0$

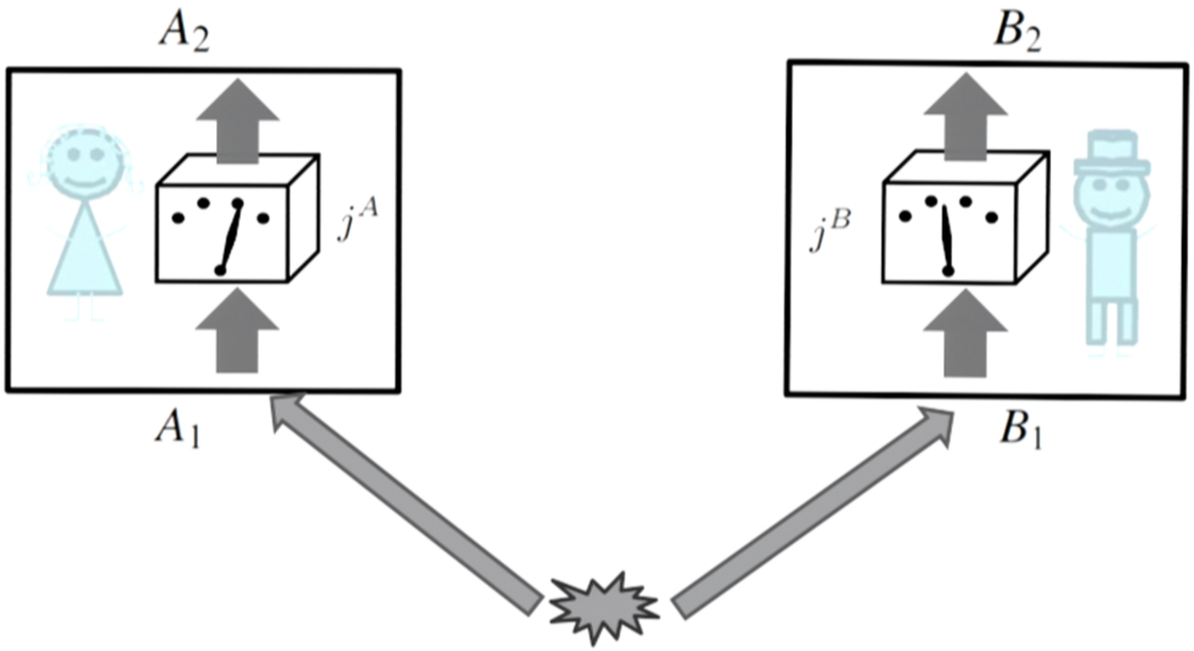
2. Probabilities sum up to 1:

$$\mathrm{Tr}\left[W^{A_1A_2B_1B_2\cdots}\left(M^{A_1A_2} \otimes M^{B_1B_2} \otimes \cdots\right)\right] = 1$$

on all CPTP maps $M^{A_1A_2}$, $M^{B_1B_2}$, ...

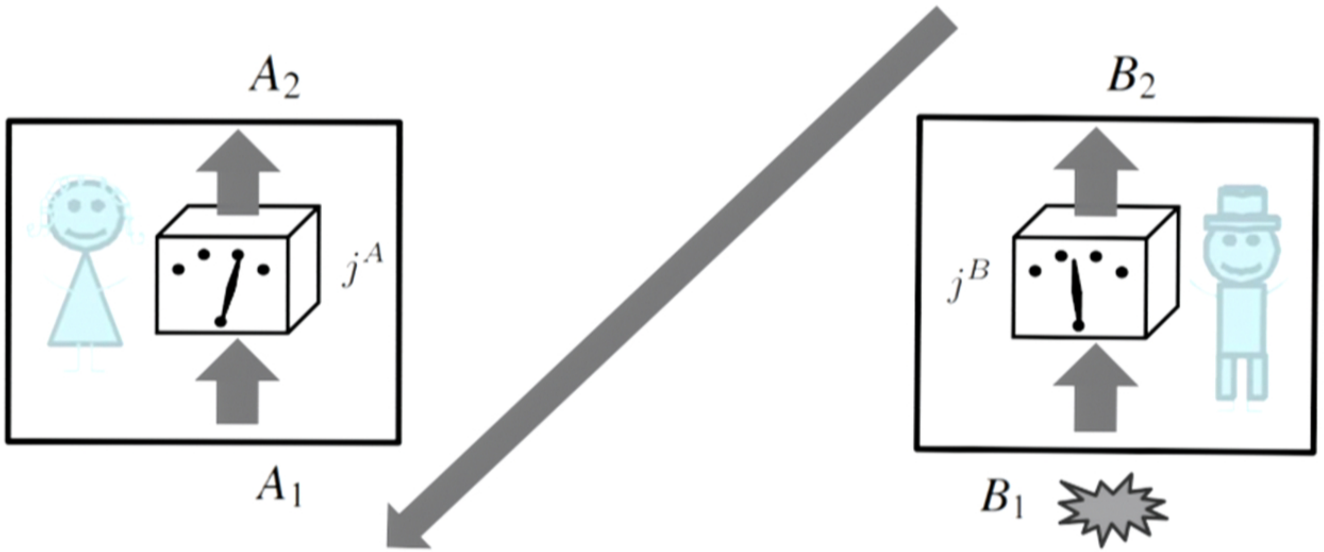
⟶ Simple characterization via the allowed terms in a Hilbert-Schmidt basis

Example: bipartite state



$$W^{A_1 A_2 B_1 B_2} = \rho^{A_1 B_1} \otimes \mathbb{1}^{A_2 B_2}$$

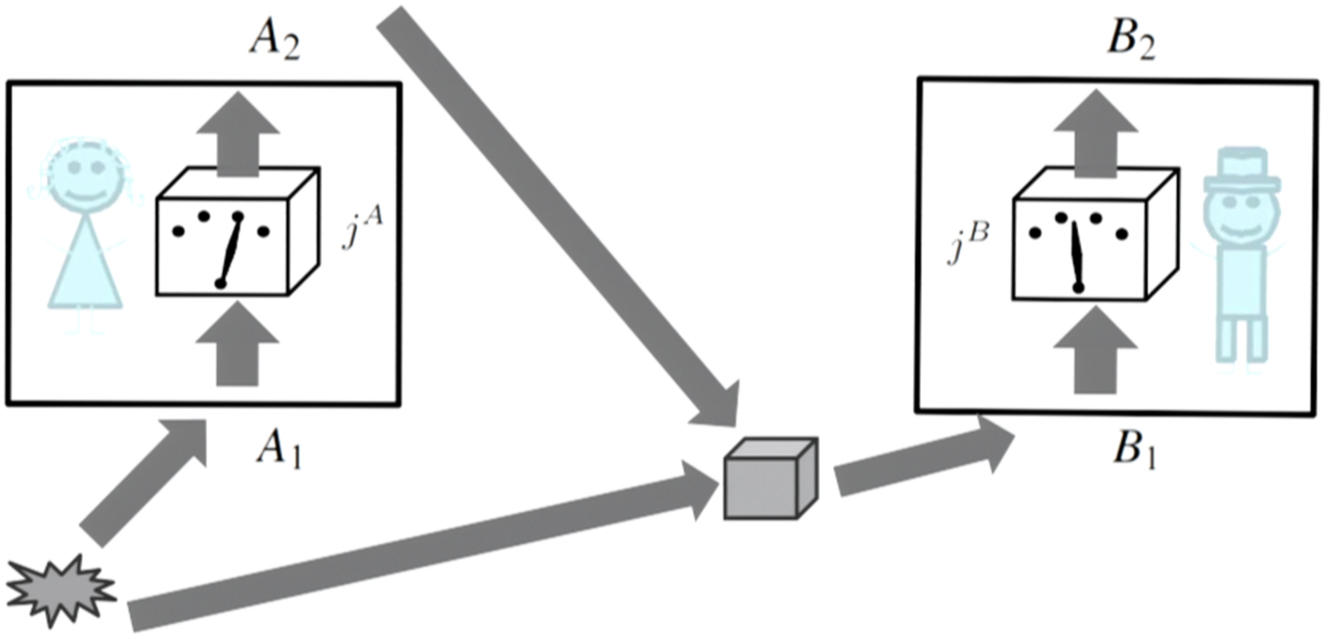
Example: channel B→A



$$W^{A_1 A_2 B_1 B_2} = \mathbb{1}^{A_2} \otimes (C^{A_1 B_2})^T \otimes \rho^{B_1}$$

Example: channel with memory A→ B

(The most general possibility compatible with no signalling from B to A)



$$W^{A_1 A_2 B_1 B_2} = W^{A_1 A_2 B_1} \otimes \mathbb{1}^{B_2}$$

Bipartite processes with causal realization

$W^{A \npreceq B}$ – no signalling from A to B (ch. with memory from B to A)

$W^{B \npreceq A}$ – no signalling from B to A (ch. with memory from A to B)

Bipartite processes with causal realization

$W^{A \not\leq B}$ – no signalling from A to B (ch. with memory from B to A)

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More generally, we may conceive **causally separable** processes (probabilistic mixtures of fixed-order processes):

$$W^{A_1 A_2 B_1 B_2} = q W^{B \not\leq A} + (1-q) W^{A \not\leq B}$$

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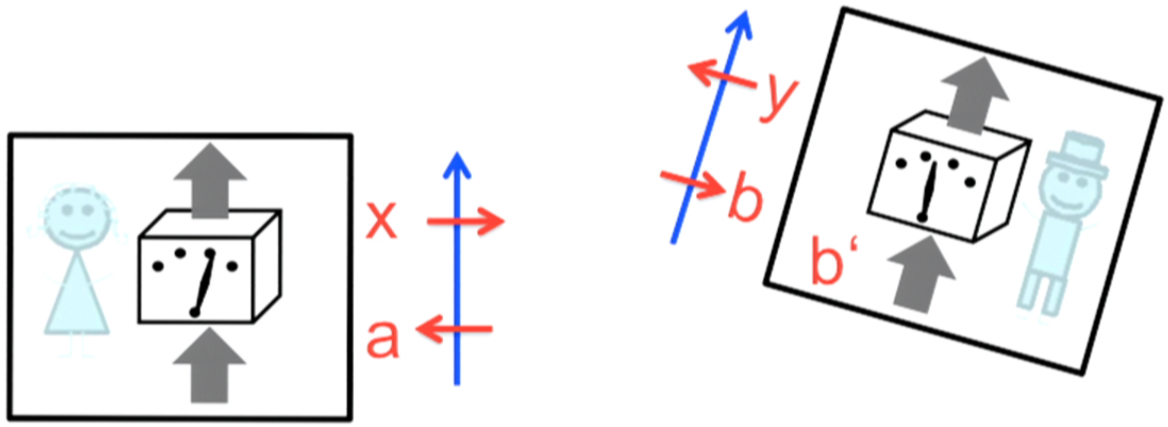
Are all possible *W* causally separable?

Causal game



- Alice is given bit **a** and Bob bit **b**.
- Bob is given an additional bit **b'** that tells him whether he should guess her bit (**b'=1**) or she should guess his bit (**b'=0**).
- Alice produces **x** and Bob **y**, which are their best guesses for the value of the bit given to the other.
- The goal is to maximize the probability for correct guess:

$$p_{succ} = \frac{1}{2}[P(x = b|b' = 0) + P(y = a|b' = 1)]$$

A non-causal process

Can achieve probability of success $p_{succ} = \frac{2+\sqrt{2}}{4} > \frac{3}{4}$

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$

two-level systems

The operations of Alice and Bob do not occur in a definite order!

More info: O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

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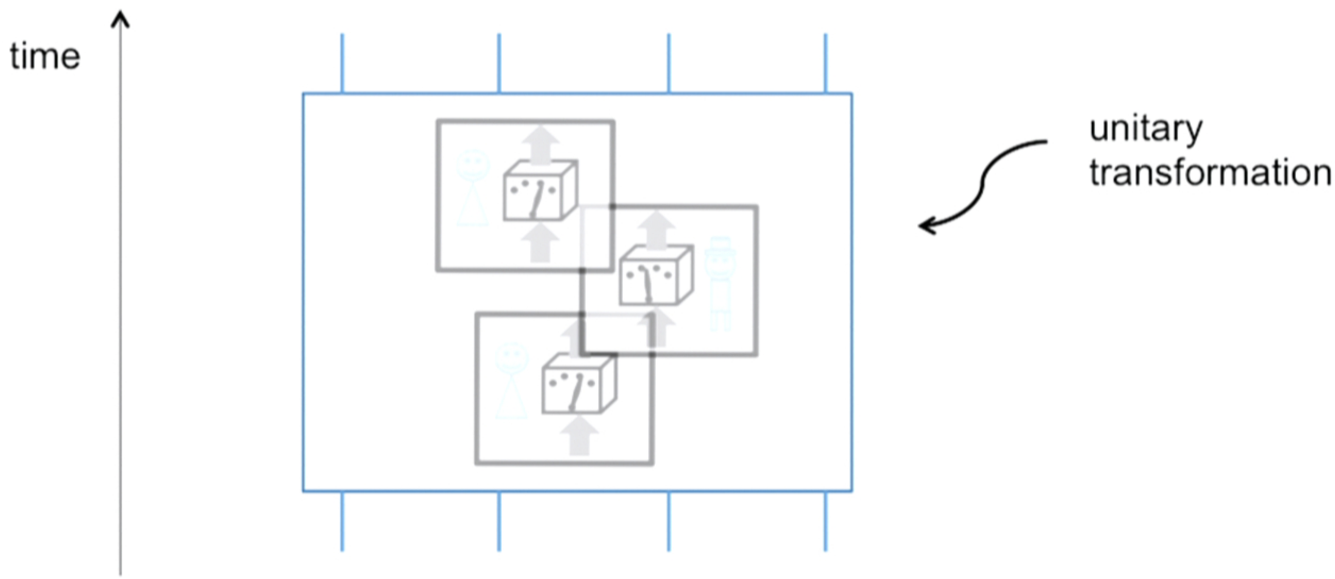
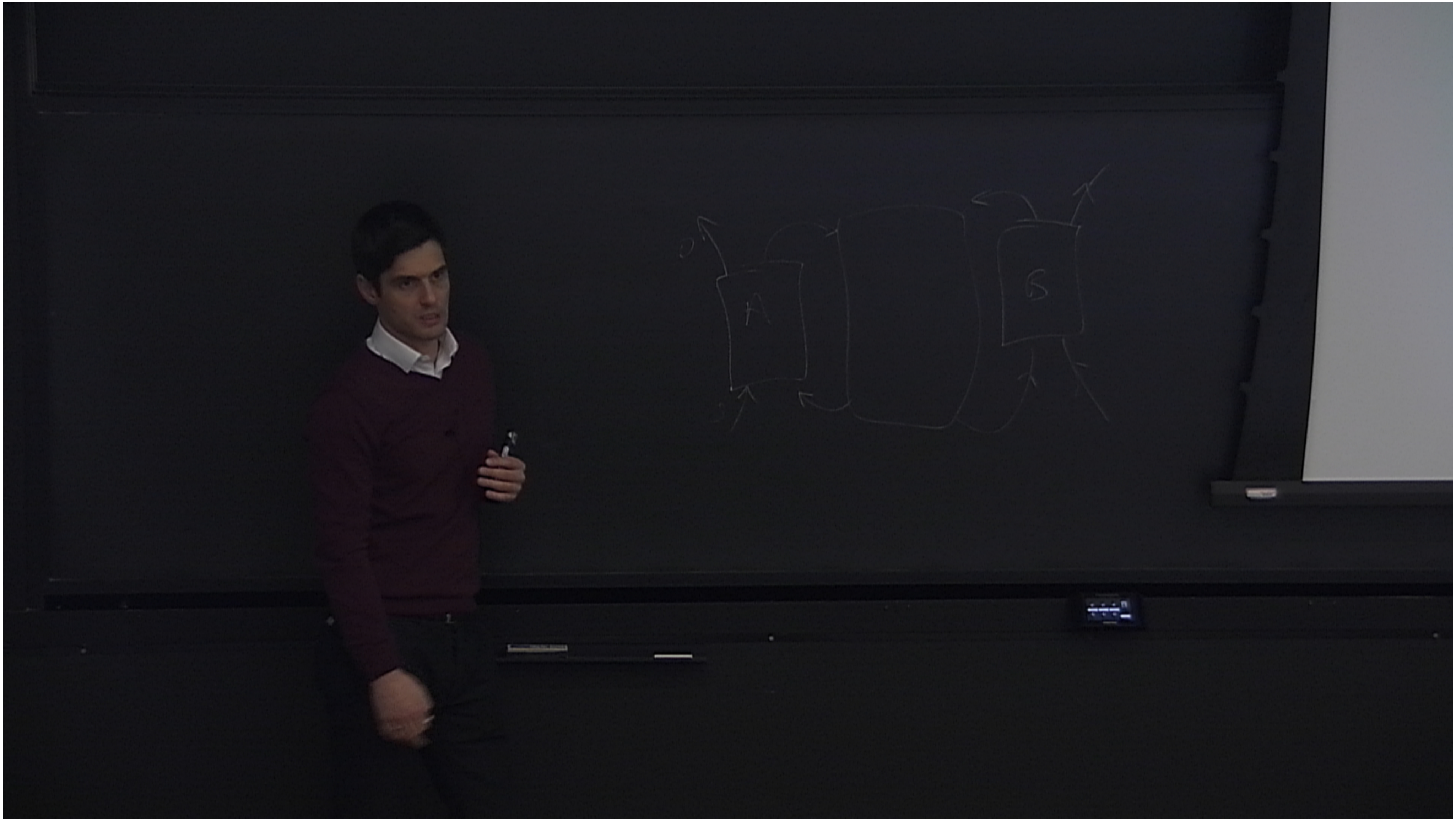
Can such process be realized in practice?

We don't know.

But it is not a priori impossible

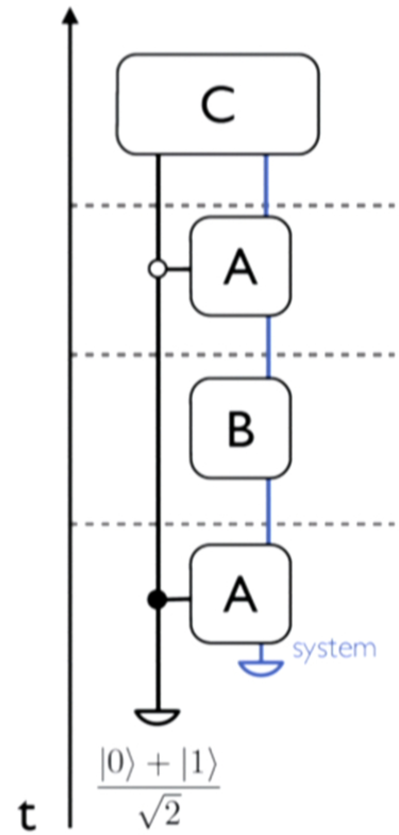
From the outside the experiment may still agree with standard unitary evolution in time.




A
B

The quantum switch

Chiribella, D'Ariano, Perinotti and Valiron,
arXiv:0912.0195, PRA 2013



The *tripartite* process is not causally separable!

O. Oreshkov and C. Giarmatzi, arXiv:1506.05449

(also Araujo et al., NJP 17, 102001 (2015))

Other causal inequalities and violations

Simplest bipartite inequalities:

Branciard, Araujo, Feix, Costa, Brukner, NJP 18, 013008 (2016)

Multiparite inequalities:

Baumeler and Wolf

- violation with perfect signaling: Proc. ISIT 2014, 526-530 (2014)

- violation by classical local operations: PRA 90, 042106 (2014)
NJP 18, 013036, 2016

Biased version of the original inequality:

Bhattacharya and Banik, arXiv:1509.02721 (2015)

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

A notion of causality should:

- **have a universal expression** (implies the multipartite case)
- **allow of *dynamical* causal order** (a given event can influence the order of other events in its future)
- **capture our intuition of causality**

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

General process: $\mathcal{W}^{A,B,\cdots} \equiv \{P(o^A, o^B, ...|s^A, s^B, ...)\}$

Intuition: The probability for a set of events to occur outside of the causal future of Alice and for these events to have a particular causal configuration with Alice is independent of the choice of setting of Alice.

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O. O. and C. Giarmatzi, arXiv:1506.05449

General process: $\mathcal{W}^{A,B,\cdots} \equiv \{P(o^A, o^B, ...|s^A, s^B, ...)\}$

A process is **causal** iff there exists a random partial order $\kappa(A, B, \cdots)$ and and a probability distribution $P(\kappa(A, B, \cdots), o^A, o^B, \cdots | s^A, s^B, \cdots)$ such that for every party, e.g., *A*, and every subset *X*, *Y*, … of the other parties,

$$P(\kappa(A, X, Y, \cdots), A \npreceq X, A \npreceq Y, \cdots, o^X, o^Y, \cdots | s^A, s^B, \cdots)$$
$$= P(\kappa(A, X, Y, \cdots), A \npreceq X, A \npreceq Y, \cdots, o^X, o^Y, \cdots | s^B, \cdots).$$

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$$= P(\kappa(A, X, Y, \cdots), A \npreceq X, A \npreceq Y, \cdots, o^X, o^Y, \cdots |s^B, \cdots).$$

(background-independent understanding of causal order)

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

Consider $\mathcal{W}^{X^1,\cdots X^n} \equiv \mathcal{W}^{\mathcal{A},\mathcal{B}}$

$$\mathcal{A} = \{X^1, \cdots, X^k\}$$
$$\mathcal{B} = \{X^{k+1}, \cdots, X^n\}$$

If no signaling from $\mathcal{B}$ to $\mathcal{A}$ ➡ exists **reduced process** $\mathcal{W}^{\mathcal{A}}$

$$\mathcal{W}^{\mathcal{A},\mathcal{B}} \equiv \mathcal{W}^{\mathcal{B}|\mathcal{A}} \circ \mathcal{W}^{\mathcal{A}}$$

↑ **conditional process**

Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449

Theorem (canonical causal decomposition)**:**

$$\mathcal{W}_c^{X^1,\cdots,X^n} = \sum_{i=1}^{n} q_i \mathcal{W}^{(X^1,\cdots,X^{i-1},X^{i+1},\cdots,X^n)\not\preceq X^i}, \quad q_i \geq 0$$

where

$$\mathcal{W}^{(X^1,\cdots,X^{i-1},X^{i+1},\cdots,X^n)\not\preceq X^i} = \mathcal{W}_c^{X^1,\cdots,X^{i-1},X^{i+1},\cdots,X^n|X^i} \circ \mathcal{W}^{X^i}$$

(iterative formulation)

Describes causal 'unraveling' of the events in the process.

Formal theory of causality for processes

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(iterative formulation)

Causal correlations form polytopes! [For the bipartite case, see Branciard *et al.*, NJP 18, 013008 (2016)]

Causal separability

O. O. and C. Giarmatzi, arXiv:1506.05449

A *quantum* process is called **causally separable** iff it can be written in a canonical causal form with every reduced and conditional process being a valid quantum process.

(analogy with Bell local and separable quantum states)

→ Agrees with the bipartite definition $W^{A_1 A_2 B_1 B_2} = q W^{B \not\preceq A} + (1-q) W^{A \not\preceq B}$

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In the multipartite case, causality and causal separability are not equivalent!

Extensive causality and causal separability

O. O. and C. Giarmatzi, arXiv:1506.05449

Non-causality can be *activated* by shared entanglement!

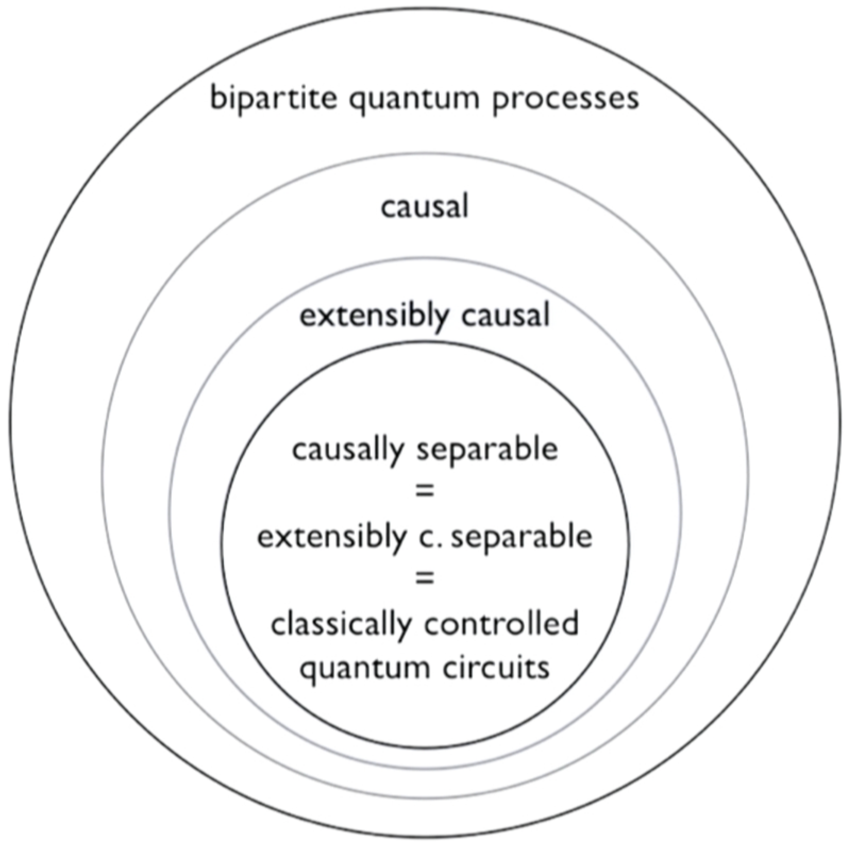
→ Define **extensively causal** / **extensively causally separable** processes

(remain causal / causally separable under extension with arbitrary input ancilla)

Simple characterization of multipartite *extensively causally separable* processes!
(see paper)

What we know about the classes of quantum processes

O. O. and C. Giarmatzi, arXiv:1506.05449



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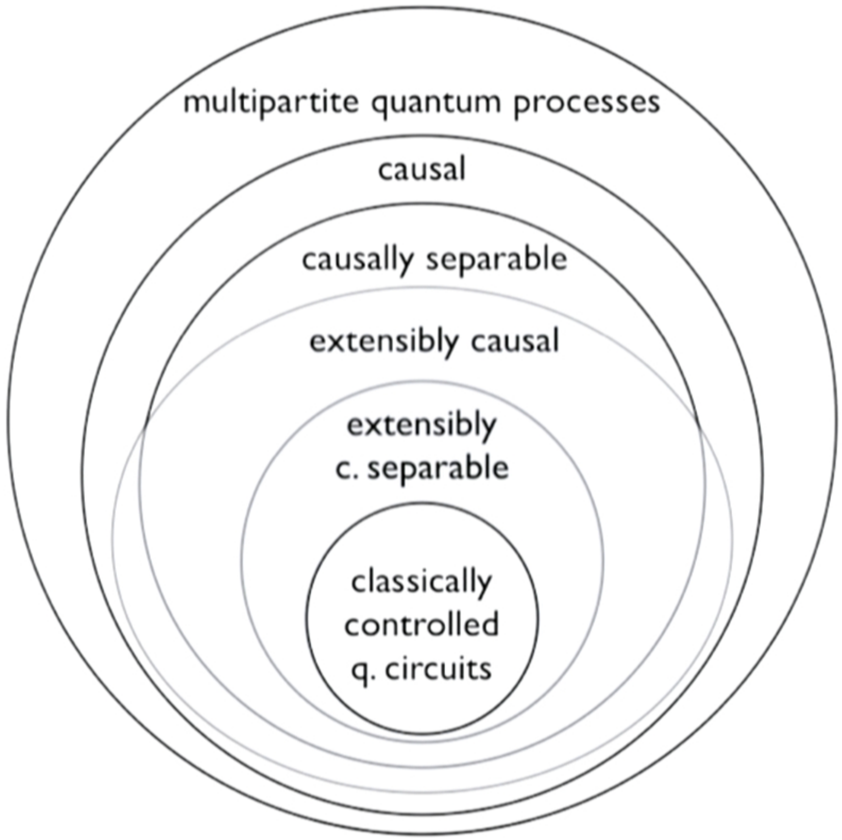


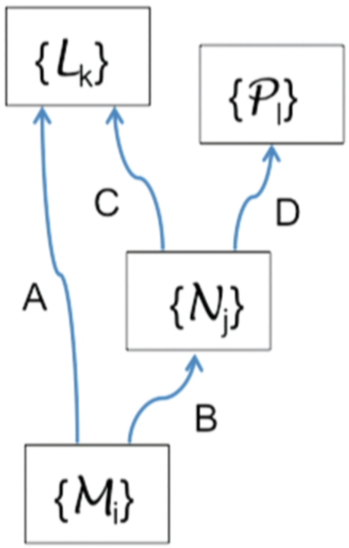
This framework still assumes time locally, and it is time-asymmetric.

What is the origin of this time asymmetry?

Could we relax the assumption of time also locally?

The circuit framework for operational probabilistic theories

Circuit (an acyclic composition of operations with no open wires):



Probabilistic structure

Joint probabilities

p(i, j, k, l)

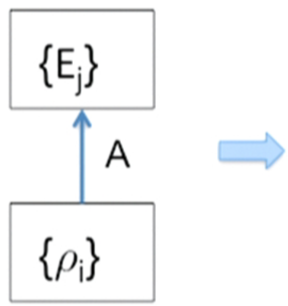
p(i, j, k, l) $\geq$ 0, $\sum_{ijkl}$ p (i, j, k, l)=1

Hardy, PIRSA:09060015; Chiribella, D'Ariano, Perinotti, PRA 81, 062348 (2010) [arXiv 2009]

Time-asymmetry of standard quantum theory

Causality axiom [Chiribella, D'Ariano, Perinotti, PRA 81, 062348 (2010), PRA 84, 012311 (2011)]:

Also Pegg, PLA 349, 411 (2006), ('**weak causality**').



$p(i, j) \equiv p(\rho_i, E_j)$

In quantum theory, $p(\rho_i, E_j) = Tr(\hat{\rho}_i \hat{E}_j)$.

The marginal probabilities of the preparation events are independent of the measurement:

$$p(\rho_i \,|\, \{E_j\}) = p(\rho_i)$$

'No signalling from the future'

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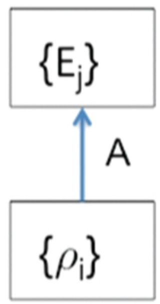

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O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Two ideas:

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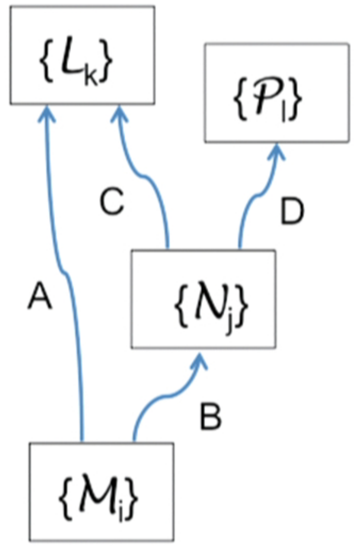
O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Two ideas:

Idea 1. The closed-box assumption

The events in a box are correlated with other events only as a result of information exchange through the wires

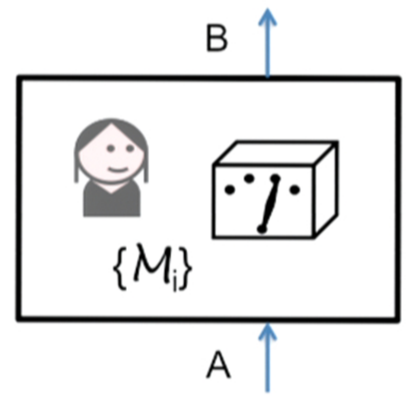


What do we call ‘operation’?

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Two ideas:

Idea 1. The closed-box assumption



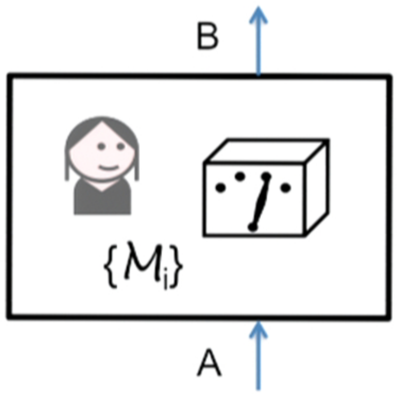
→ An operation can be realized inside an isolated box.

What do we call 'operation'?

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Two ideas:

Idea 2. No post-selection



The 'choice' of operation can be known *before* the operation is applied

(Underlies the interpretation that an operation can be 'chosen'.)

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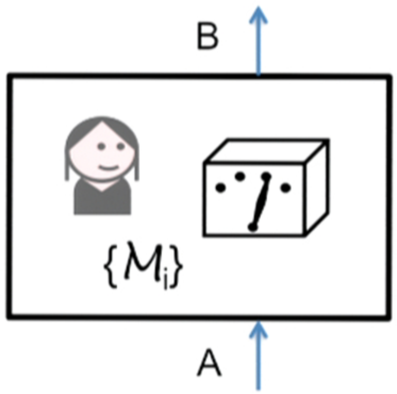


The ‘choice’ of operation can be known *before* the operation is applied

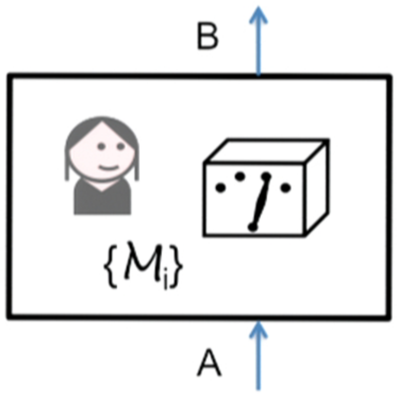
(Underlies the interpretation that an operation can be ‘chosen’.)

→ The causality axiom describes a constraint on pre-selected operations.

What do we call 'operation'?

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Proposal: **drop the 'no post-selection' criterion**



Operation =

description of the possible events in a box conditional on local information

What do we call ‘operation’?

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

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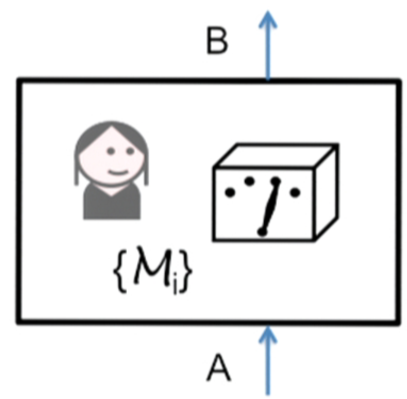

Idea 2. No post-selection

The ‘choice’ of operation can be known *before* the operation is applied

(Underlies the interpretation that an operation can be ‘chosen’.)

The very concept of operations is time-asymmetric!

What do we call ‘operation’?

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

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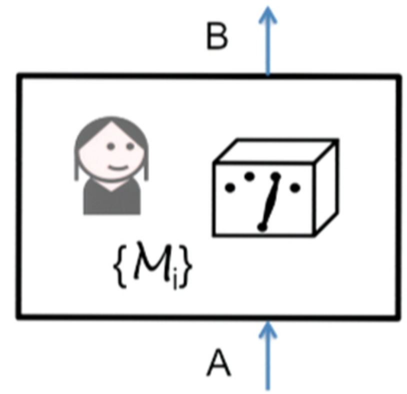

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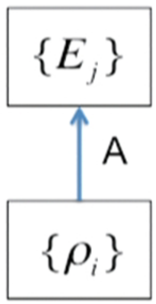
description of the possible events in a box conditional on local information

Time-symmetric quantum theory

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Joint probabilities:

The basic probability rule.



$$p(i,j) = \frac{Tr(\rho_i E_j)}{Tr(\bar{\rho}\bar{E})}$$

where

$$\bar{\rho} = \sum_i \rho_i, \quad Tr(\bar{\rho}) = 1$$

$$\bar{E} = \sum_j E_j, \quad Tr(\bar{E}) = d$$

[Also Pegg, Barnett, Jeffers, J. Mod. Opt. 49, 913 (2002).]

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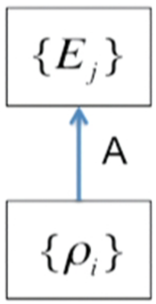

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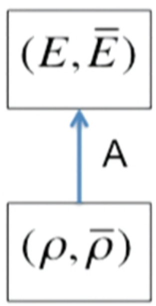
New states and effects

States (equivalent preparation events): $(\rho, \bar{\rho})$, where $0 \le \rho \le \bar{\rho},\ Tr(\bar{\rho}) = 1$.

Effects (equivalent measurement events): $(E, \bar{E})$, where $0 \le E \le \bar{E},\ Tr(\bar{E}) = d$.

Joint probabilities:



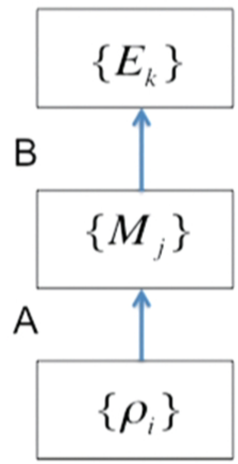
$$p[(\rho,\bar{\rho}),(E,\bar{E})] = \frac{Tr(\rho E)}{Tr(\bar{\rho}\bar{E})}, \qquad Tr(\bar{\rho}\bar{E}) \neq 0$$

$$= 0, \qquad Tr(\bar{\rho}\bar{E}) = 0$$

States can be thought of as functions on effects and vice versa.

Time reversal symmetry

Example:



$$p(i,j,k) = \frac{Tr(E_k^B M_j^{A\to B}(\rho_i^A))}{Tr(\bar{E}^B \bar{M}^{A\to B}(\bar{\rho}^A))}$$

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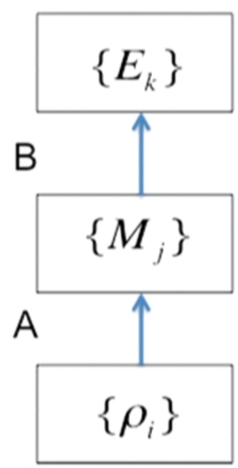

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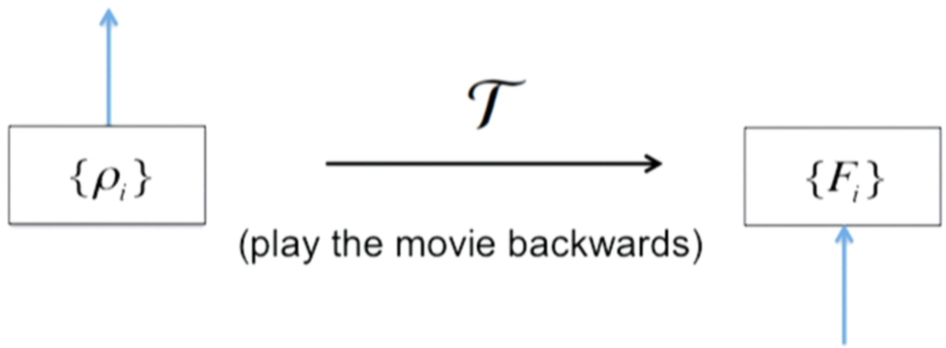
Time reversal symmetry

The exact form of time-reversal is not implicit in the formalism!



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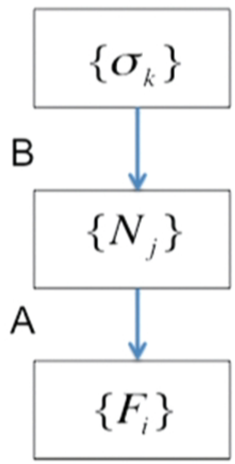

$$p(i,j,k) = \frac{Tr(F_i^A N_j^{B\to A}(\sigma_k^B))}{Tr(\overline{F}^A \overline{N}^{B\to A}(\overline{\sigma}^B))}$$

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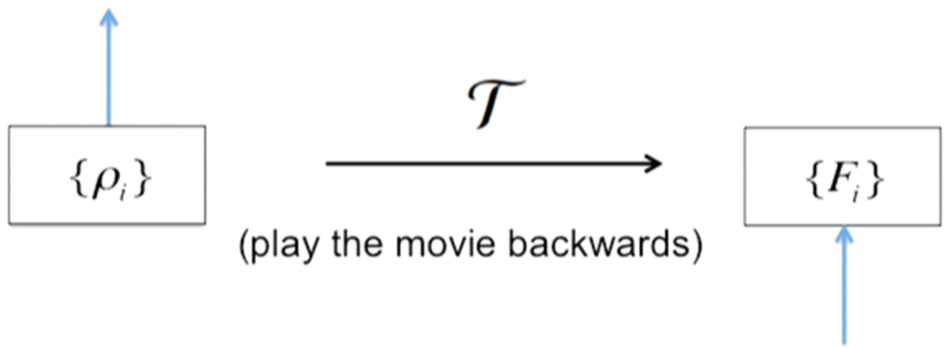

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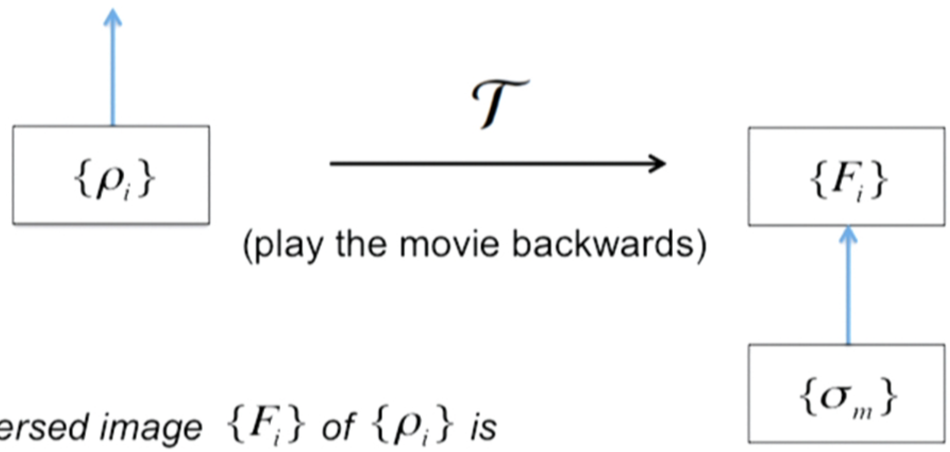

The *time-reversed image* $\{F_i\}$ *of* $\{\rho_i\}$ *is* determined relative to preparations $\{\sigma_m\}$ that have not been time-reversed.

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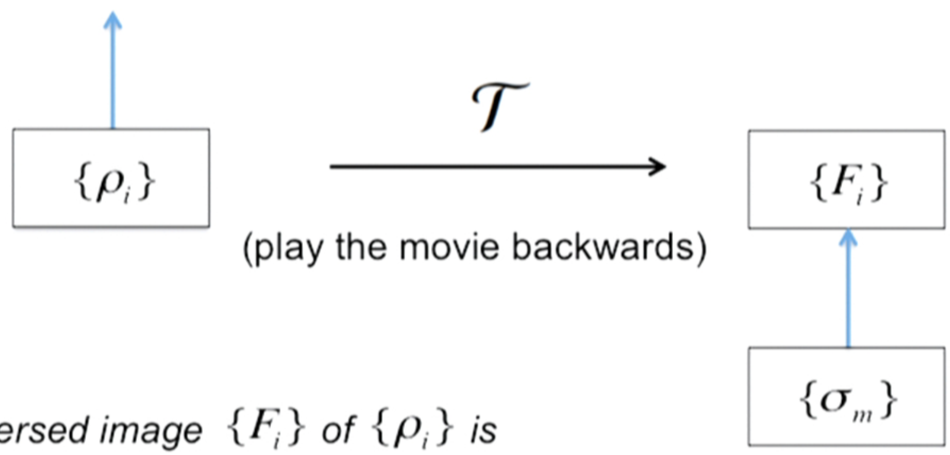


The *time-reversed image* $\{F_i\}$ *of* $\{\rho_i\}$ *is* determined relative to preparations $\{\sigma_m\}$ that have not been time-reversed.

Generalized Wigner's theorem

Important: states and effects are objects that live in *different* spaces.

There is *no natural isomorphism* between the two spaces!

We represent them by operators in the same space based on the bilinear form

$$(E^{A^*}, \rho^A) \;=\; \langle \rho^A, E^A \rangle \;=\; \mathrm{Tr}[\rho^A E^A] \;,$$

which defines an isomorphism $E^{A^*} \;\leftrightarrow\; E^A$.

This isomorphism has no physical meaning! It is simply based on the choice of bilinear form, and should not be confused with time reversal!

Generalized Wigner's theorem

Two types of symmetry transformation:

Type I - States go to states, and effects go to effects: $(\hat{S}^{A}_{s\to s}, \hat{S}^{A}_{e\to e})$

Type II - States go to effects, and effects go to states: $(\hat{S}^{A}_{s\to e}, \hat{S}^{A}_{e\to s})$

Generalized Wigner's theorem

- Symmetries of type I are described by:

$$\hat{S}_{s\to s}(\rho;\overline{\rho}) = (\sigma;\overline{\sigma}) = (\frac{S\rho S^{\dagger}}{\mathrm{Tr}(S\overline{\rho}S^{\dagger})};\frac{S\overline{\rho}S^{\dagger}}{\mathrm{Tr}(S\overline{\rho}S^{\dagger})}),$$

$$\hat{S}_{e\to e}(E;\overline{E}) = (F;\overline{F}) = (d\frac{S^{-1\dagger}ES^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}S^{-1})};d\frac{S^{-1\dagger}\overline{E}S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}S^{-1})}),$$

or

$$\hat{S}_{s\to s}(\rho;\overline{\rho}) = (\sigma;\overline{\sigma}) = (\frac{S\rho^{T}S^{\dagger}}{\mathrm{Tr}(S\overline{\rho}^{T}S^{\dagger})};\frac{S\overline{\rho}^{T}S^{\dagger}}{\mathrm{Tr}(S\overline{\rho}^{T}S^{\dagger})}),$$

$$\hat{S}_{e\to e}(E;\overline{E}) = (F;\overline{F}) = (d\frac{S^{-1\dagger}E^{T}S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^{T}S^{-1})};d\frac{S^{-1\dagger}\overline{E}^{T}S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^{T}S^{-1})}),$$

where S is an invertible operator, and T is a transposition is some basis.

Generalized Wigner's theorem

If the evolution under time reversal is described by Schrödinger's equation, positivity of energy → time reversal is an *anti-unitary* operation.

Thus**, time reversal is in the class**:

$$\hat{S}_{s\to e}(\rho;\overline{\rho}) = (F;\overline{F}) = (d\frac{S\rho^T S^\dagger}{\mathrm{Tr}(S\overline{\rho}^T S^\dagger)};d\frac{S\overline{\rho}^T S^\dagger}{\mathrm{Tr}(S\overline{\rho}^T S^\dagger)}),$$

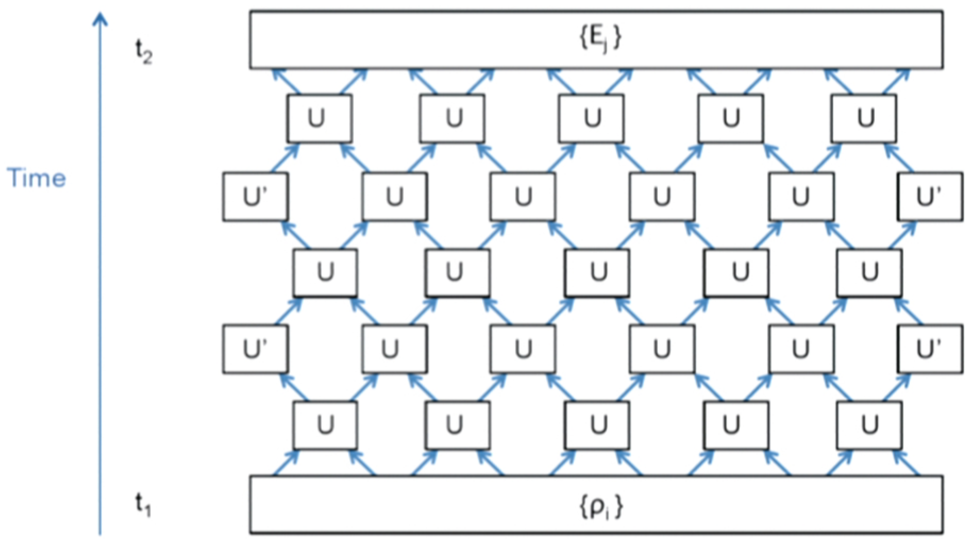
$$\hat{S}_{e\to s}(E;\overline{E}) = (\sigma;\overline{\sigma}) = (\frac{S^{-1\dagger}E^T S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^T S^{-1})};\frac{S^{-1\dagger}\overline{E}^T S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^T S^{-1})}).$$

Since we are generalizing quantum theory, we leave open the possibility for non-unitary S.

Understanding the observed asymmetry

A toy model of the universe:

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)



For an observer at t_1, all future circuits contain standard operations iff $\sum_{j \in Q} E_j = \mathbb{1}$.

(linked to the fact that we can remember the past and not the future)

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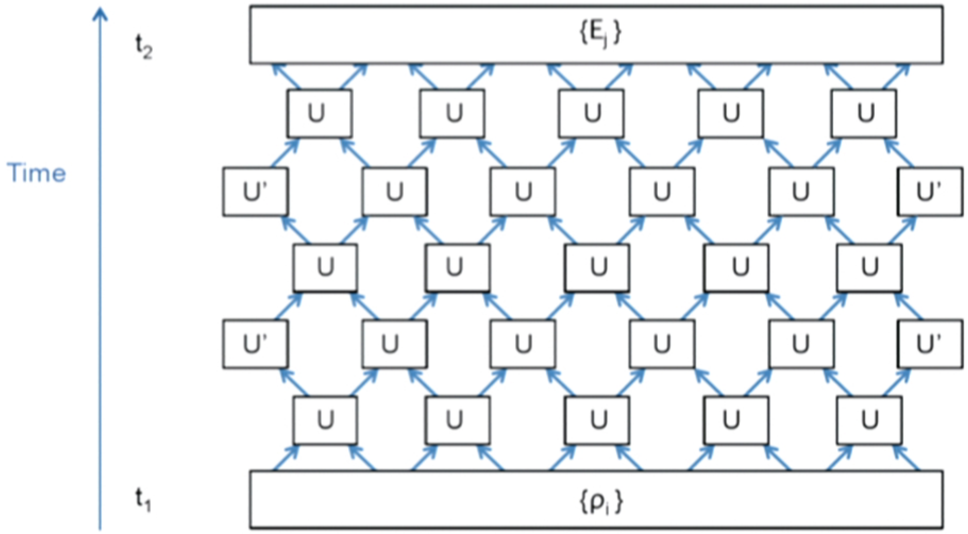
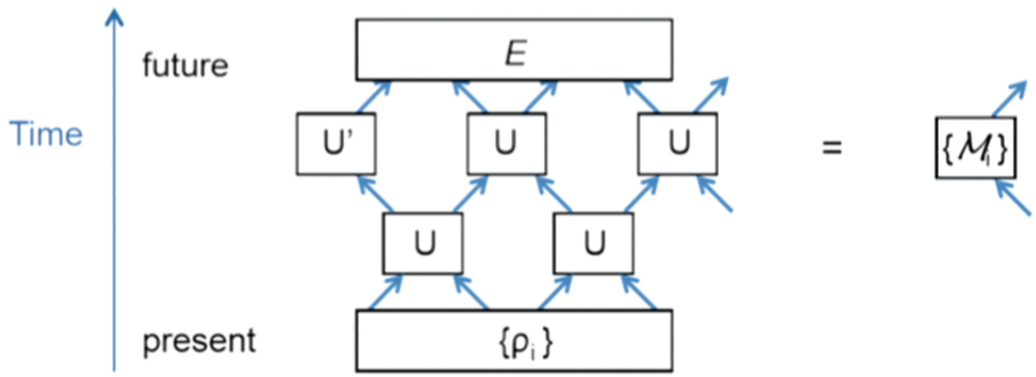


For an observer at t_1, all future circuits contain standard operations iff $\sum_{j \in Q} E_j = \mathbb{1}$.

(linked to the fact that we can remember the past and not the future)

Note: it is logically possible that non-standard operations were obtainable without post-selection



A time-neutral formalism

An isomorphism dependent on time reversal

TRANSFORMATIONS EFFECTS ON PAIRS OF SYSTEMS

$$(\mathcal{M}^{A_1 \to B_1}; \overline{\mathcal{M}}^{A_1 \to B_1}) \leftrightarrow (M^{A_1 B_2}; \overline{M}^{A_1 B_2})$$

O.O. and N. Cerf, arXiv: 1406.3829

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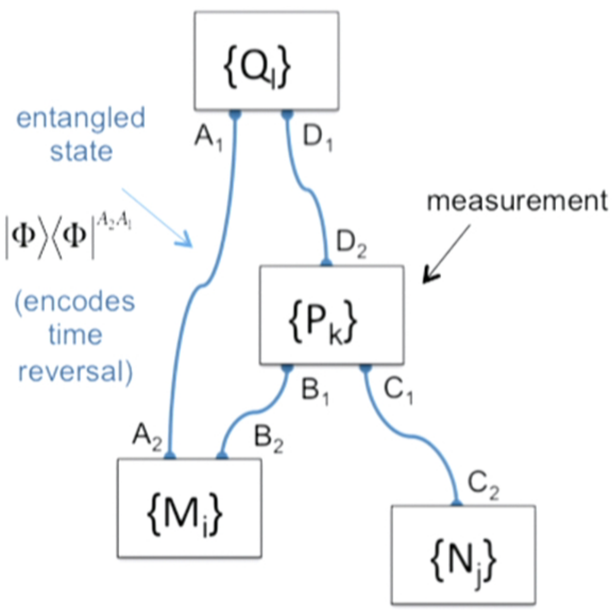

Joint probabilities:

$$p(i,j,k,l\,|\,\{M_i^{A_2B_2}\},\{N_j^{C_2}\},\cdots,W)=$$

$$\frac{Tr[W^{A_1A_2B_1B_2C_1C_2D_1D_2}(M_i^{A_2B_2}\otimes N_j^{C_2}\otimes P_k^{B_1C_1D_2}\otimes Q_l^{A_1D_1})]}{\sum_{i,j,k,l} Tr[W^{A_1A_2B_1B_2C_1C_2D_1D_2}(M_i^{A_2B_2}\otimes N_j^{C_2}\otimes P_k^{B_1C_1D_2}\otimes Q_l^{A_1D_1})]}$$

'process matrix' (encodes the connections)

$$=|\Phi\rangle\langle\Phi|^{A_1A_2}\otimes|\Phi\rangle\langle\Phi|^{B_1B_2}\otimes|\Phi\rangle\langle\Phi|^{C_1C_2}\otimes|\Phi\rangle\langle\Phi|^{D_1D_2}$$

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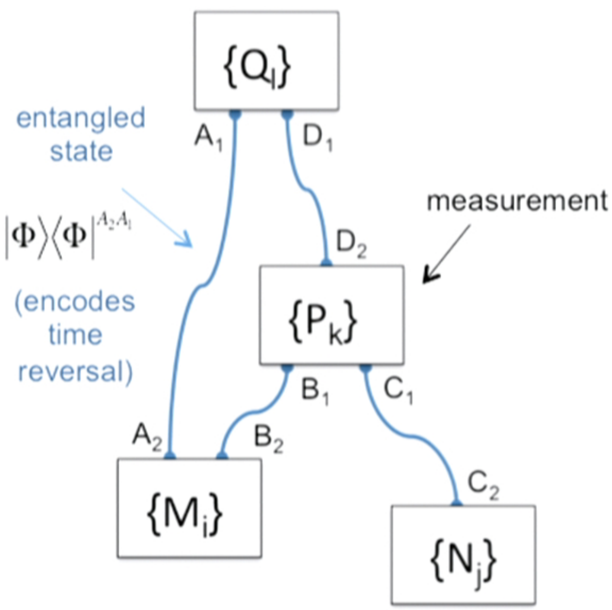

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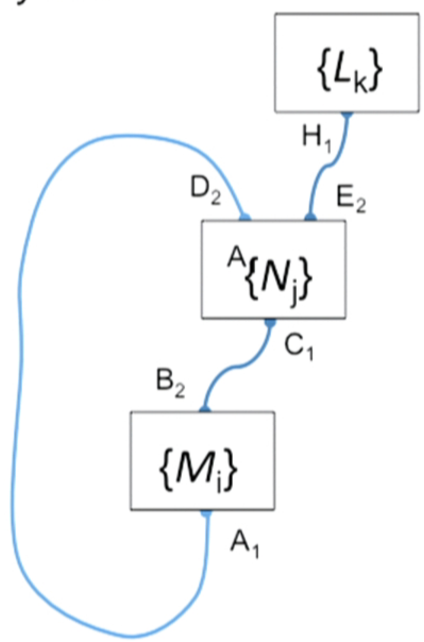
O.O. and N. Cerf, arXiv: 1406.3829

A time-neutral formalism

Can describe circuits with *cycles*:



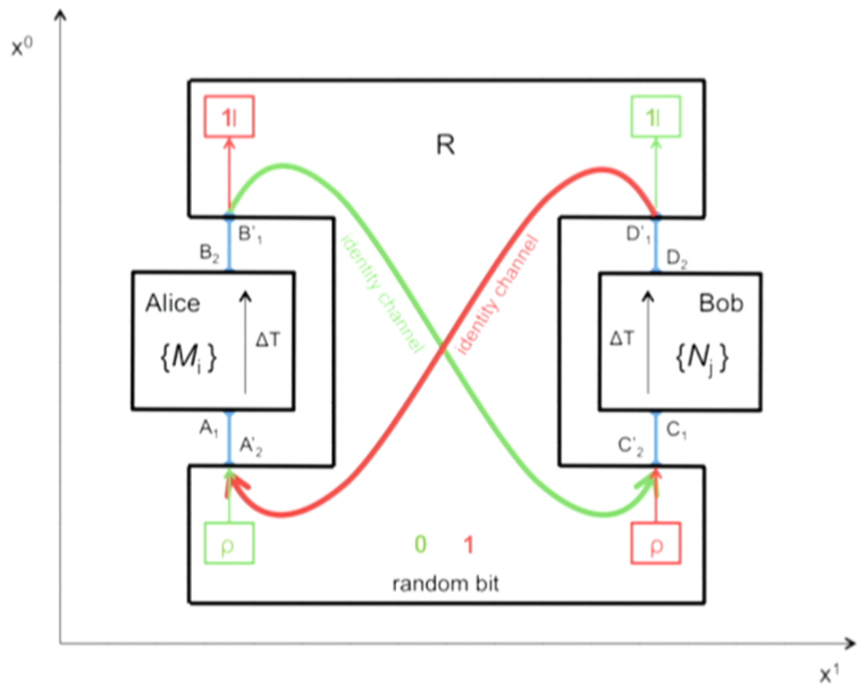
All such circuits can be realized using post-selection.

(Compatible with closed timelike curves)

O.O. and N. Cerf, arXiv: 1406.3829

A time-neutral formalism

There exist circuits with cycles that can be obtained without post-selection!

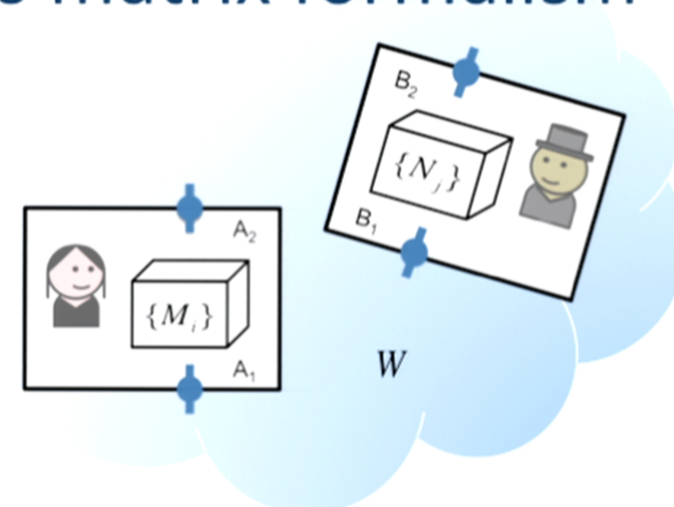


the idea of *background independence* extended to random events

(provides a framework for understanding experiments realizing the quantum switch)

Time-symmetric process matrix formalism

Equivalently:



external variables

$$p(i,j,\cdots|\{M_i^{A_1A_2}\},\{M_j^{B_1B_2}\},\cdots,W)=\frac{Tr[W^{A_1A_2B_1B_2\cdots}(M_i^{A_1A_2}\otimes M_j^{B_1B_2}\otimes\cdots)]}{Tr[W^{A_1A_2B_1B_2\cdots}(\bar{M}^{A_1A_2}\otimes\bar{M}^{B_1B_2}\otimes\cdots)]}$$

The 'process matrix':

$$W^{A_1A_2B_1B_2\cdots}\geq 0,\quad Tr(W^{A_1A_2B_1B_2\cdots})=1$$

Note: Any process matrix is allowed.

O.O. and N. Cerf, arXiv: 1406.3829

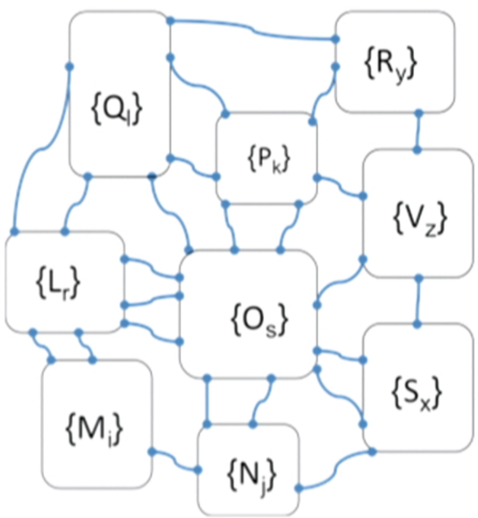
Dropping the assumption of local time

Observation: The predictions are the same whether the systems are of type 1 or type 2.

Proposal: There is no a priori distinction between systems of type 1 and 2.

The concept of time should come out from properties of the dynamics!

The general picture:



Main probability rule

$$p(i,j,\cdots|\{M_i^{\cdots}\},\{N_j^{\cdots}\},\cdots) = \frac{Tr[W_{wires}^{\cdots}(M_i^{\cdots}\otimes N_j^{\cdots}\otimes\cdots)]}{Tr[W_{wires}^{\cdots}(\bar{M}^{\cdots}\otimes\bar{N}^{\cdots}\otimes\cdots)]}$$

O.O. and N. Cerf, arXiv: 1406.3829

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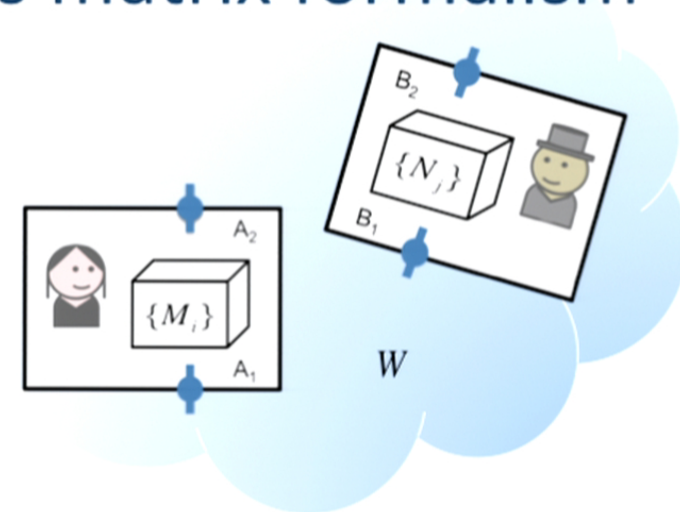

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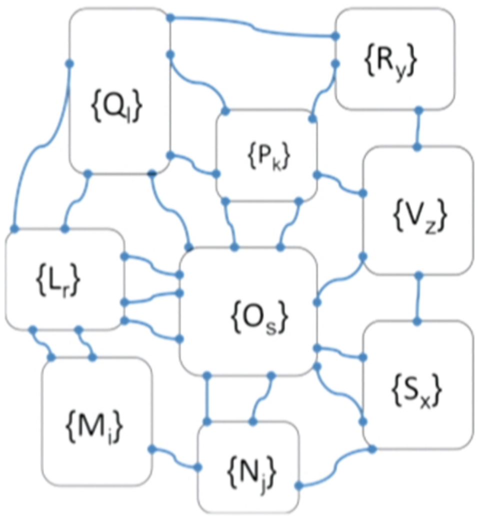

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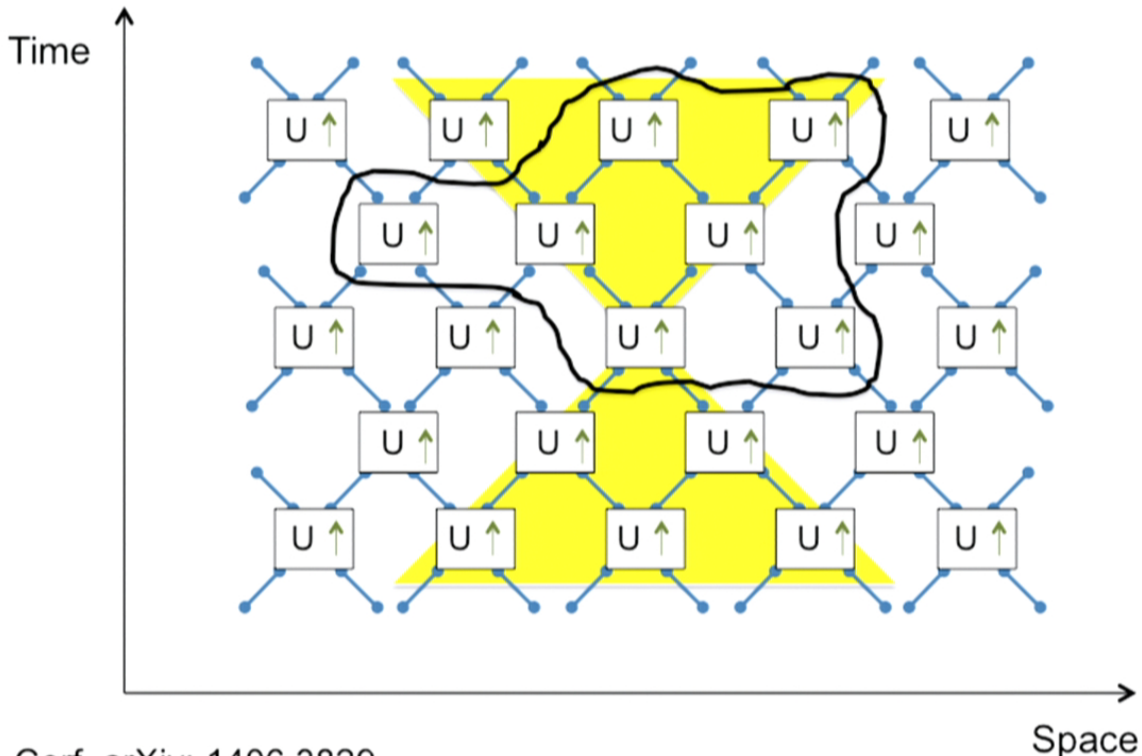
O.O. and N. Cerf, arXiv: 1406.3829

Limit of quantum field theory

R. Oeckl, Phys. Lett. B 575, 318 (2003), ... , Found. Phys. 43, 1206 (2013)

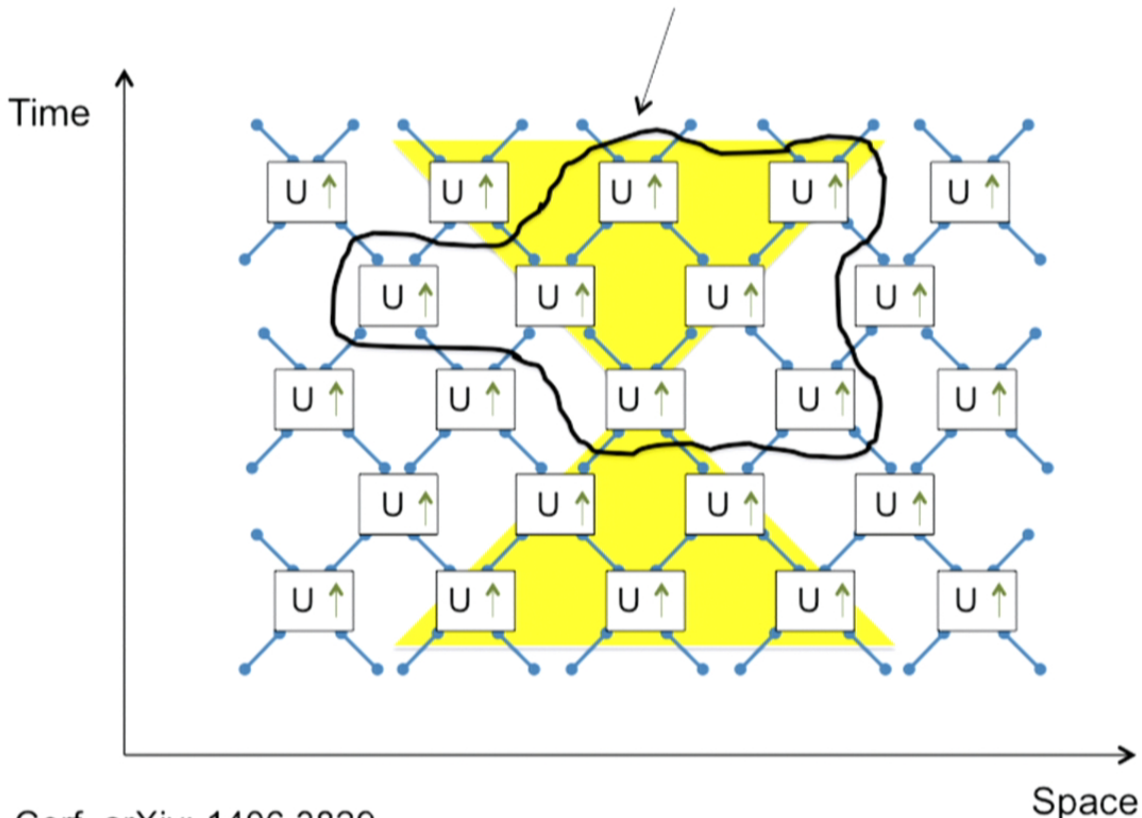
(the 'general boundary' approach with a few generalization)



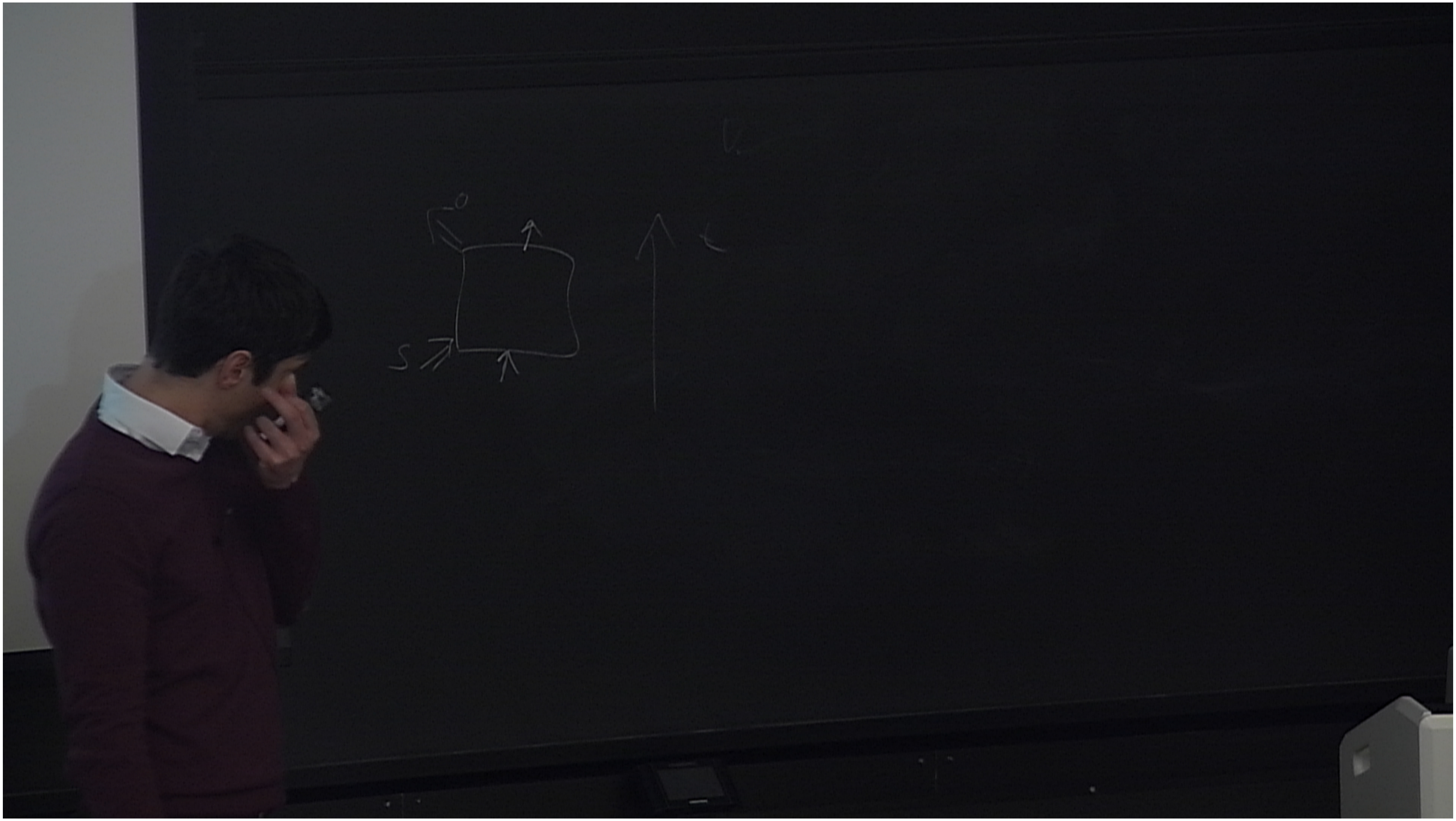
O.O. and N. Cerf, arXiv: 1406.3829

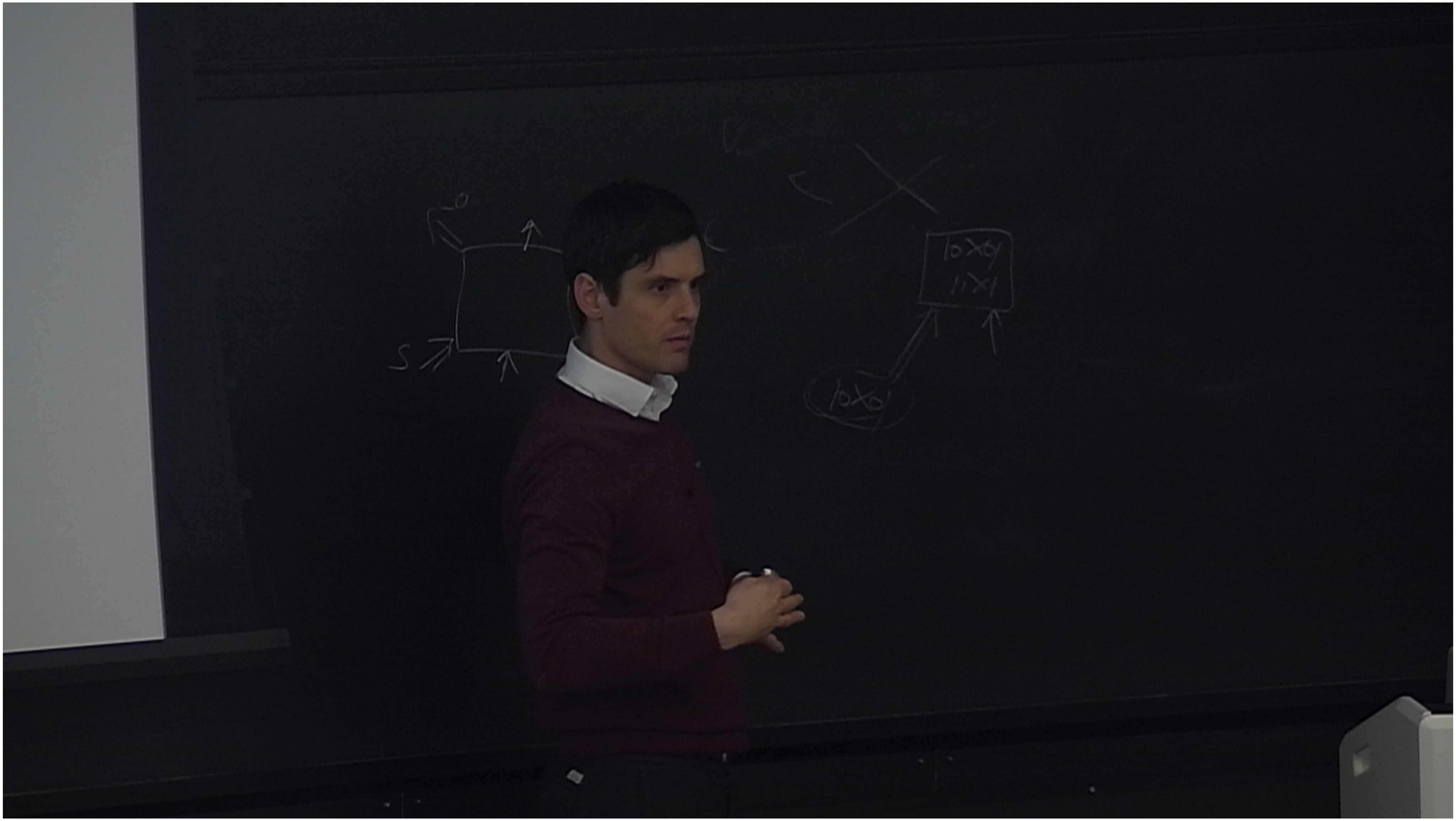
Proposal: causal structure from correlations

The causal structure underlying the dynamics in the region is reflected in correlation properties of the state on the boundary.



O.O. and N. Cerf, arXiv: 1406.3829

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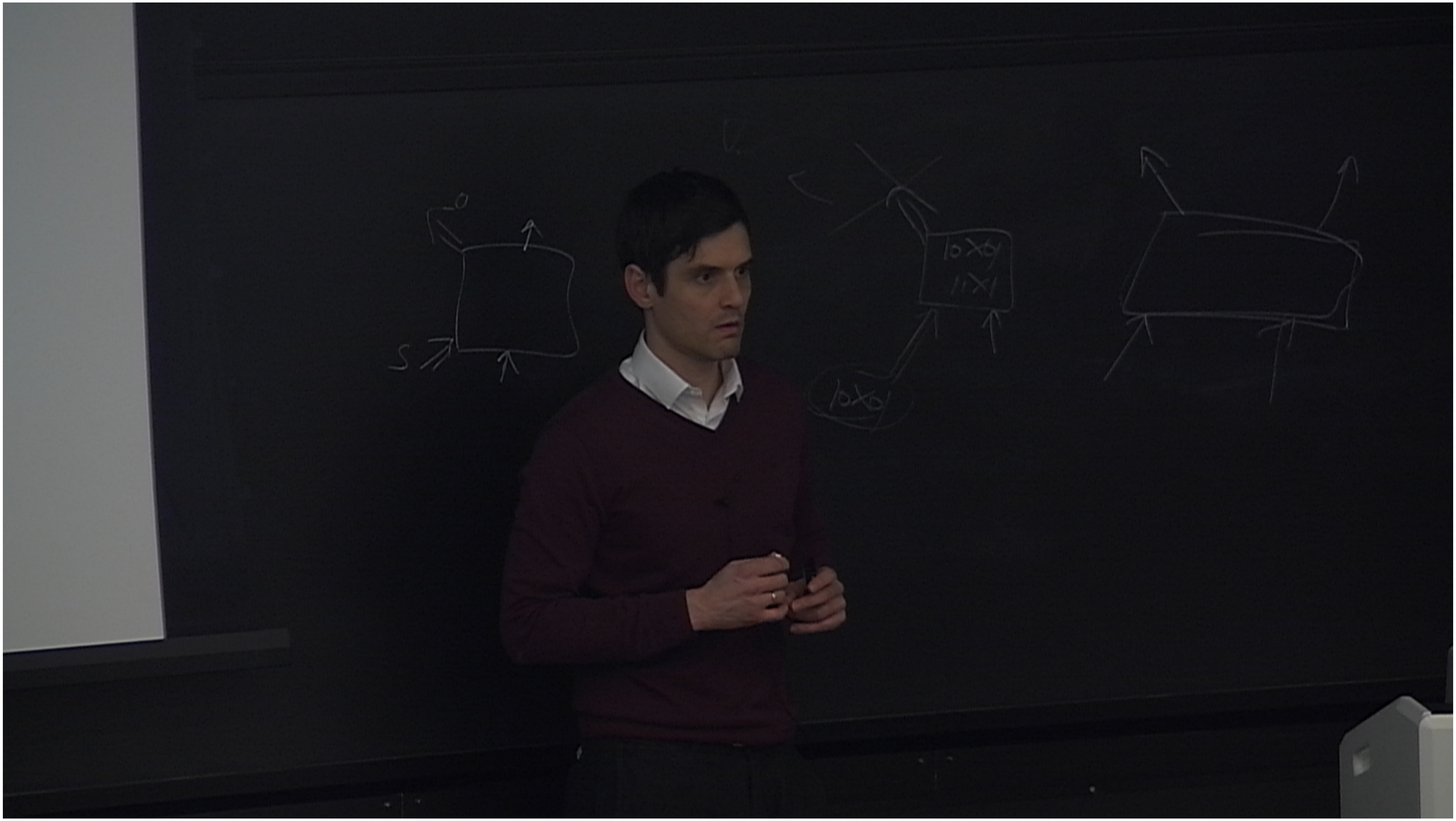
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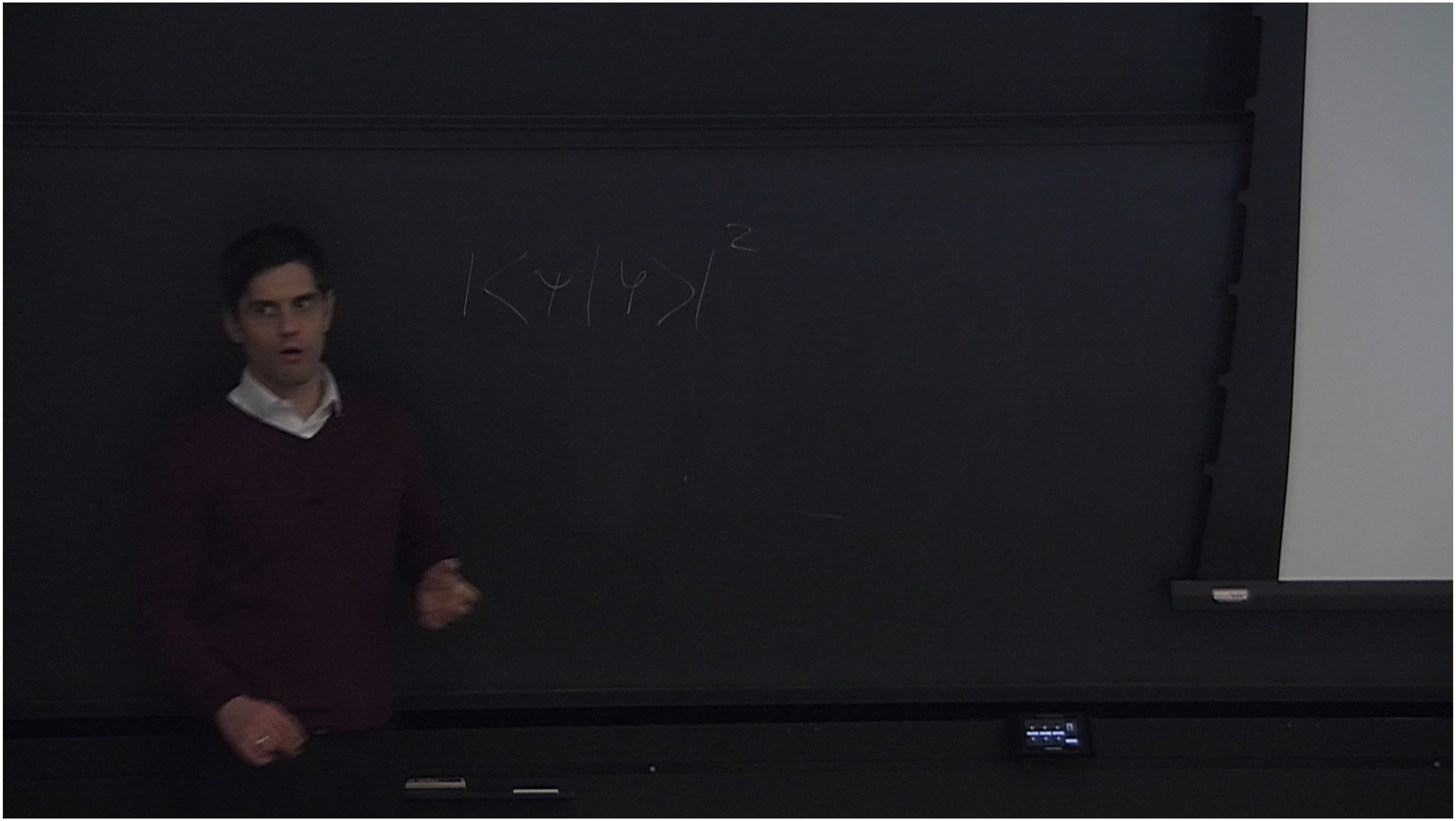
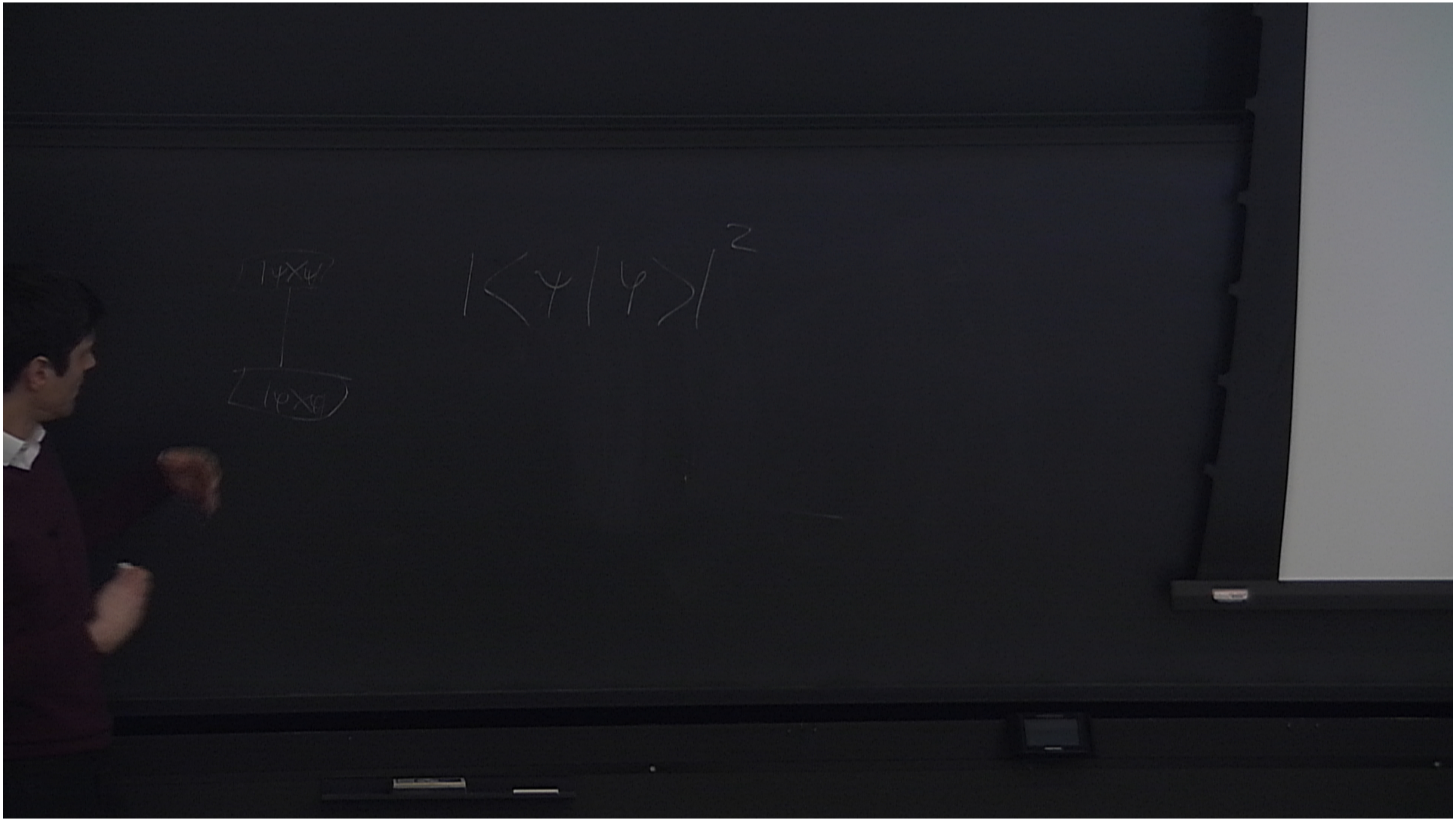
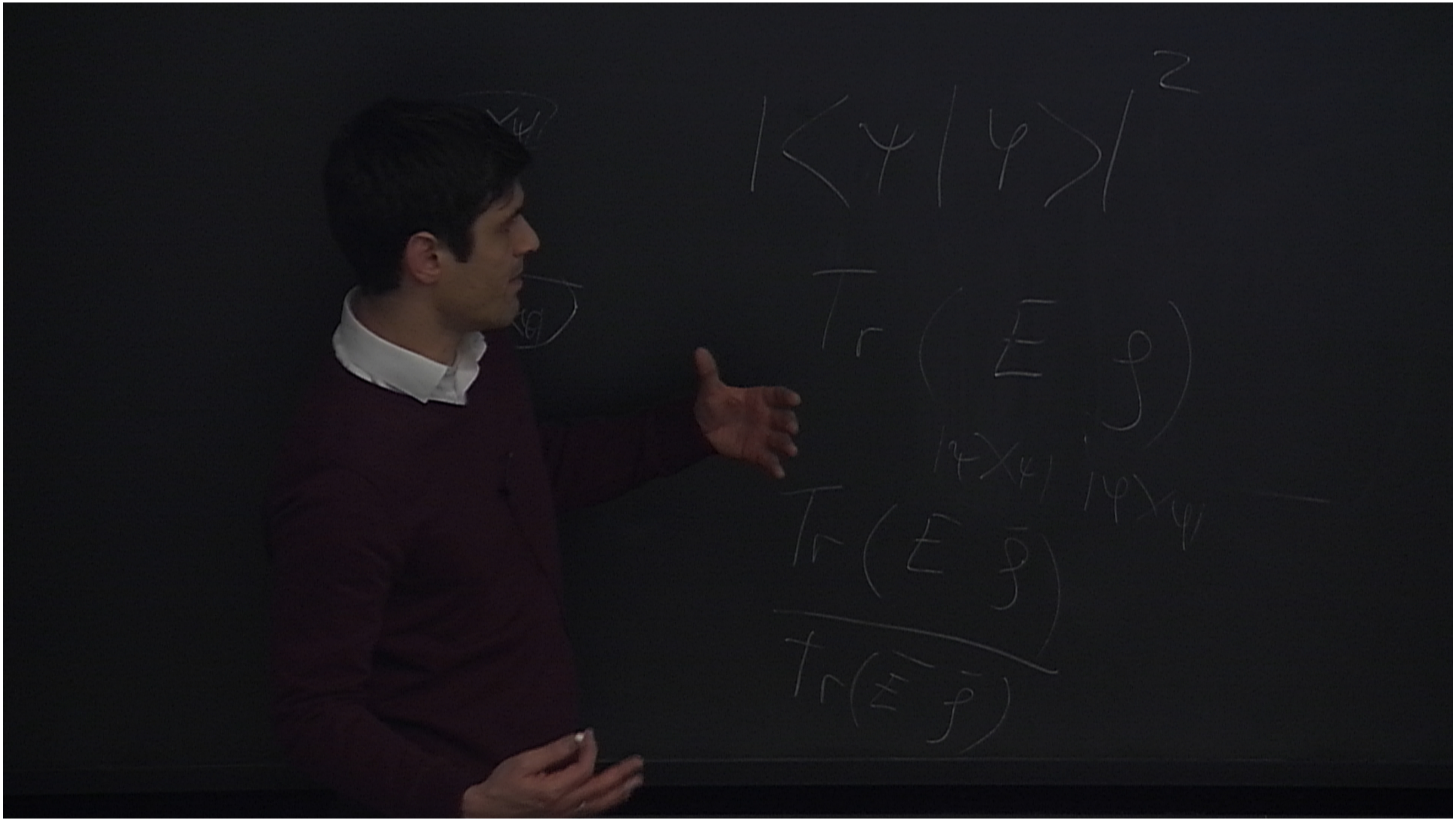

$|\langle \psi | \varphi \rangle|^2$


|⟨ψ|φ⟩|²


|<ψ|φ>|²
Tr(E ρ)
|ψ><ψ| |φ><φ|