Title: Large Instantons in Heterotic Supergravity

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Abstract: <p>I will discuss a class of non-compact solutions to the Strominger-Hull system, the first order system of equations for preserving N=1 supersymmetry in heterotic compactifications to four dimensions. The solutions consists of the conifold and its Z2 orbifold with Abelian gauge fields and non-zero three-form flux. The heterotic Bianchi Identity is solved in a large charge limit of the gauge fields, where it is shown that the topological term p1(TX) can be consistently neglected. At large distances, these solutions are locally Ricci-flat. For a given flux, the family of solutions has three real parameters, the size of the pair of two spheres in the IR and the dilaton zero mode. There exists an explicit analytic solution for the decoupled near horizon region where for a given flux, the size of the cycles is frozen and the only parameter is the dilaton zero mode. This near horizon region also has an exactly solvable worldsheet CFT. When one of the two cycles has vanishing size the near horizon region disappears, but a solution on the unorbifolded resolved conifold still exists.</p>

Large Charge Instantons in Heterotic Supergravity

Eirik Eik Svanes (ILP, LPTHE)
based on work in collaboration with Nick Halmagyi and Dan Israel,
arXiv:1601.07561, (arXiv:0910.3190)

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This talk is concerned with heterotic supergravity at $\mathcal{O}(\alpha')$, and its four-dimensional effective supergravity. In particular, we will discuss on-shell solutions on the conifold. We cover:

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This talk is concerned with heterotic supergravity at $\mathcal{O}(\alpha')$, and its four-dimensional effective supergravity. In particular, we will discuss on-shell solutions on the conifold. We cover:

- Brief introduction to heterotic supergravity.
- BPS equations and Bianchi Identity.
- Analytic near horizon (NH) solution on $\mathbb{Z}_2$-orbifolded conifold.
- Numerical asymptotically Ricci flat solution.
- Solution on unorbifolded resolved conifold.
- Discussion of gauge charges, and five-brane charge.
- Conclusions and outlook..

General String Compactifications

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Heterotic supergravity is ten-dimensional. It is standard to postulate a "compactification":

$$\mathcal{M}_{10} = \mathcal{M}_4 \times X_6 \, ,$$

where $\mathcal{M}_4$ is assumed Minkowski, and X_6 is the "internal space". For compact geometries at $\mathcal{O}(\alpha'^0)$ one requires X to be Kähler and hence Calabi-Yau [Strominger 86]. Special case of Maldacena-Nunez no go theorem.

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Deformations $\delta X \Rightarrow$ give rise to low-energy moduli fields. Not observed and so must be lifted (moduli problem).

Type II: RR-fluxes available. Used to stabilize moduli.

Heterotic: Only NS-flux H. Worse still: Supersymmetry $\Rightarrow H = i(\partial - \bar{\partial})\omega \Rightarrow$ torsional geometries!

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Non-compact "local" geometries $\Rightarrow$ flux and torsion even at $\mathcal{O}(\alpha'^0)$. This talk: Local geometry (the conifold) + addition of α'-effects through the non-trivial heterotic Bianchi identity.

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The low energy theory of the heterotic string is a 10d $N = 1$ supergravity coupled to $E_8 \times E_8$ or $SO(32)$ Yang-Mills.

Good for phenomenology, but hard to stabilize moduli. Need to leave CY-locus and consider α'-effects (anomaly, etc).

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Complications:

- torsional geometries not well understood, but some progress [Strominger 86, Becker et al 2003, Ivanov 2009, ..].
- Complicated expressions to deal with, e.g. non-trivial Bianchi Identity.
- Very few non-trivial examples. Compact examples: [Dasgupta et al 99, Yau et al 06, ..]. Non-compact: [Israel et al 09, Dasgupta et al 13, Fei 15, etc, This talk!].

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A better understanding of compact and non-compact torsional heterotic compactifications is required.

Action, BPS conditions and Bianchi Identity

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The bosonic heterotic action reads [Regshhoeff-de Roo 89]

$$S = \int_{M_{10}} e^{-2\phi} \left[*\mathcal{R} + 4|\mathrm{d}\phi|^2 - \frac{1}{2}|H|^2 + \alpha' \left(\mathrm{tr}\,|F|^2 - \mathrm{tr}\,|R|^2 \right) \right] ,$$

where $\mathcal{R}$ is the Einstein-Hilbert term, ϕ is the dilaton, H is the NS-flux and F is the curvature of the gauge-connection. R is the curvature of some tangent bundle connection ∇.

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where $\mathcal{R}$ is the Einstein-Hilbert term, ϕ is the dilaton, H is the NS-flux and F is the curvature of the gauge-connection. R is the curvature of some tangent bundle connection ∇.

BPS conditions:

$$\nabla^-_m \epsilon = \nabla_m \epsilon - \frac{1}{8} H_{m\,ab}\gamma^{ab}\epsilon = 0$$

$$\nabla\!\!\!/ \phi\,\epsilon - \frac{1}{2} H\!\!\!/\,\epsilon = 0$$

$$F_{mn}\gamma^{mn}\epsilon = R_{mn}\gamma^{mn}\epsilon = 0\,.$$

Bianchi identity:

$$\mathrm{d}H = \alpha'\left(\mathrm{tr}\,F^2 - \mathrm{tr}\,R^2\right) + \delta(\mathit{sources})\,.$$

We work in a large charge limit, which means that we can consistently drop the topological tr R^2 term.

Compactification and $SU(3)$-structures

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A non-vanishing globally defined spinor ϵ on the internal space X_6 gives rise to an $SU(3)$-structure:

$$J \wedge \Omega = 0\,, \quad \frac{1}{3!} J \wedge J \wedge J = *1 = -\frac{i}{8} \Omega \wedge \overline{\Omega}\,,$$

where Ω is a complex locally decomposable three-form, and J is the hermitian two-form.

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where Ω is a complex locally decomposable three-form, and J is the hermitian two-form.

On this space, the BPS equations take the form [Strominger, Hull 86]

$$\mathrm{d}\left(e^{-2\phi}\Omega\right) = 0 \,, \ \mathrm{d}\left(e^{-2\phi} J \wedge J\right) = 0 \,, \ H = *e^{2\phi}\mathrm{d}\left(e^{-2\phi} J\right) \,.$$

The first condition implies that X_6 is a *complex manifold*. The second condition is known as the *conformally balanced* condition.

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The first condition implies that X_6 is a *complex manifold*. The second condition is known as the *conformally balanced* condition.The BPS conditions for the bundles are:

$$F \wedge \Omega = R \wedge \Omega = 0\,, \quad J \lrcorner F = J \lrcorner R = 0\,.$$

The first condition says that the bundles are *holomorphic*, while the second condition is referred to as the *Yang-Mills condition*, often referred to as the slope stability condition even in the non-compact case.

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Solution on $\mathbb{Z}_2$-orbifolded Conifold

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We consider the six-dimensional non-compact "internal" geometries as cones over the coset space $T^{(1,1)}$. The five-dimensional space $T^{(p,q)}$ is given by the coset

$$T^{(p,q)} = SU(2) \times SU(2)/U(1)\,,$$

where the coprime integers $\{p, q\}$ describe the embedding of $U(1)$ in $Spin(4)$.

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To be more precise, recall that $SU(2) \times SU(2)$ can locally be expressed as

$$SU(2) \times SU(2) = \left(\mathbb{C}P^1 \times \mathbb{C}P^1\right) \times [U(1) \times U(1)]\ ,$$

where $U(1) \times U(1)$ is the diagonal of $SU(2) \times SU(2)$. To get $T^{(p,q)}$ one factors out by the $U(1)$ which sends $z \in U(1)$ to diag$(z^p, z^{-p}, z^{-q}, z^q)$ inside $U(1) \times U(1)$.

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We will focus on cones over $T^{(1,1)}$. In fact, the BPS equations will only allow this choice of integers for our frame choice.

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The metric ansatz as a cone over $T^{(1,1)}$ reads

$$\mathrm{d}s_6^2 = \frac{3H}{2}\frac{\mathrm{d}r^2}{f^2} + r^2\left[\frac{H_1+H_2}{4}\left(\sigma_1^2+\sigma_2^2\right) + \frac{H_1-H_2}{4}\left(\hat{\sigma}_1^2+\hat{\sigma}_2^2\right) + \frac{f^2 H}{6}\eta^2\right],$$

where all functions are functions of the radial coordiante r. The function H will later be fixed in terms of $H_{1,2}$, fixing reparametrization of r.

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where all functions are functions of the radial coordiante r. The function H will later be fixed in terms of $H_{1,2}$, fixing reparametrization of r.

Here $\{\sigma_i, \hat{\sigma}_i\}$ are the left-invariant one-forms on $SU(2) \times SU(2)$ satisfying

$$d\sigma_i = \frac{1}{2}\epsilon_{ijk}\sigma_j \wedge \sigma_k\ , \quad d\hat{\sigma}_i = \frac{1}{2}\epsilon_{ijk}\hat{\sigma}_j \wedge \hat{\sigma}_k\ ,$$

and $\eta = \sigma^3 + \hat{\sigma}_3$, giving the Lie algebra or tangent space of $SU(2) \times SU(2)$. The coset space $T^{(1,1)}$ is constructed by $SU(2) \times SU(2)$ reduced by the diagonal $U(1)$ generated by $\theta = \sigma_3 - \hat{\sigma}_3$.

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The complex frame ansatz is:

$$E_1 = \sqrt{\frac{3H}{2}}\left(\frac{\mathrm{d}r}{f}\right)\,,\quad E_2 = -ir\sqrt{\frac{H_1+H_2}{4}}\left(\sigma_1+i\sigma_2\right)\,,\quad E_3 = -ir\sqrt{\frac{H_1-H_2}{4}}\left(\hat{\sigma}_1+i\hat{\sigma}_2\right)$$

$$\Omega = E_1\wedge E_2\wedge E_3\,,\quad J = \frac{1}{2i}\sum_{i=1}^{3}E_i\wedge\overline{E}_i\,.$$

Flux and Bundle Ansatz

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We further have the ansatze for the flux and field-strengths

$$H = \alpha' \left[h_1 \, \eta \wedge (\Omega_1 - \Omega_2) + h_2 \, \eta \wedge (\Omega_1 + \Omega_2) \right]$$

$$F = -\frac{1}{4} \left[(\Omega_1 - \Omega_2 - \mathrm{d}[g_1 \, \eta]) \, \mathbf{p} - \mathrm{d}[g_2 \, \eta] \mathbf{q} \right] \cdot \mathbf{H} \, ,$$

where

$$\Omega_1 = -\mathrm{d}\sigma_3 = -\sigma_1 \wedge \sigma_2 \, , \;\; \Omega_2 = -\mathrm{d}\hat{\sigma}_3 = -\hat{\sigma}_1 \wedge \hat{\sigma}_2 \, .$$

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$$\Omega_1 = -\mathrm{d}\sigma_3 = -\sigma_1 \wedge \sigma_2 \, , \;\; \Omega_2 = -\mathrm{d}\hat{\sigma}_3 = -\hat{\sigma}_1 \wedge \hat{\sigma}_2 \, .$$

We focus on Abelian bundles, where $\mathbf{p}$ and $\mathbf{q}$ are vectors embedded in the Cartan $\{\mathbf{H}\}$ of either $\mathbf{so}(32)$ or $\mathbf{e}_8 \times \mathbf{e}_8$. Crutially, we require

$$\mathbf{p} \cdot \mathbf{q} = 0 \, .$$

The charges $\mathbf{p}$ and $\mathbf{q}$ will be assumed large, that is

$$Tr(\mathbf{p} \cdot \mathbf{H})^2 = 2p^2 >> 1 \, , \quad Tr(\mathbf{q} \cdot \mathbf{H})^2 = 2q^2 >> 1 \, .$$

The large charges also provide a natural small dimensionless expansion parameter.

It can be checked that the topological charge $p_1(TX)$ in the Bianchi identity can be consistently dropped to leading order in the $(1/q)$-expansion. Intuitively, we get metric parameters of $\mathcal{O}(p^2, q^2)$, while $R = \mathrm{d}\omega$ for some connection one-form $\omega \sim g^{-1}\mathrm{d}g = \mathcal{O}(1)$.

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Plugging the ansatz into the BPS equations with radial parametrization

$$H = (H_1^2 - H_2^2)/H_1 \,, \quad \rho = \frac{r}{a}$$

gives three equations for four unsolved functions $\{\phi, f, H_1, H_2\}$:

$$\log\left[\frac{f^2}{a^2\rho^2 H_1}\right]' = \frac{6}{\rho f^2} - \frac{8}{\rho}$$

$$(\rho^4 g_1)' = \frac{4\rho^3 H_2}{H_1}$$

$$-2a^2\,\rho f^2\left(\frac{2\rho H_2^2}{H_1} + \rho^2 H_1'\right) = 3\left[p^2(-1+g_1^2) + q^2 g_2^2\right] \,.$$

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The functions $\{g_1, g_2, h_1, h_2, \phi\}$ are given by

$$e^{2(\phi-\phi_0)} = a^2\left(H_1^2 - H_2^2\right) \, , \quad g_1 = \frac{a^2\rho f^2}{3p^2}\left(\rho^2 H_2\right)' + g_c \, , \quad g_2 = \frac{1}{\rho^4}$$

$$\alpha' h_1 = -\frac{a^2\rho f^2}{12}\left(\rho^2 H_2\right)' \, , \quad \alpha' h_2 = -\frac{a^2\rho f^2}{12}\left(\frac{2\rho H_2^2}{H_1} + \rho^2 H_1'\right) \, ,$$

where $\{a, g_c, \phi_0\}$ are constants.

Exact Near-Horizon Solution

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In the chosen radial coordinate, we find an analytic non-Ricci-flat solution to the BPS equations

$$H_1 = \frac{p^2}{a^2\rho^2}\,, \quad H_2 = \frac{g_c p^2}{a^2\rho^2}\,, \quad f^2 = \frac{3}{4}\left(1 - \frac{q^2}{(1-g_c^2)p^2}\,\frac{1}{\rho^8}\right)$$

$$g_1 = g_c\,, \quad g_2 = \frac{1}{\rho^4}\,, \quad e^{2(\phi-\phi_0)} = \frac{p^4(1-g_c^2)}{a^4\rho^4}\,.$$

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$$H_1 = \frac{p^2}{a^2\rho^2}\,, \quad H_2 = \frac{g_c p^2}{a^2\rho^2}\,, \quad f^2 = \frac{3}{4}\left(1 - \frac{q^2}{(1-g_c^2)p^2}\frac{1}{\rho^8}\right)$$

$$g_1 = g_c\,, \quad g_2 = \frac{1}{\rho^4}\,, \quad e^{2(\phi-\phi_0)} = \frac{p^4(1-g_c^2)}{a^4\rho^4}\,.$$

The parameter a can be viewed as a blow-up parameter for a $S^2 \times S^2$ in the conifold base. By an appropriate rescaling of r and ϕ, we can absorb $\{a, q^2\}$:

$$ds_6^2 = \frac{2\alpha' p^2(1-g_c^2)}{R^2}\left\{\frac{dR^2}{1-\frac{1}{R^8}} + \frac{R^2}{8}\left(\frac{\sigma_1^2+\sigma_2^2}{1-g_c} + \frac{\hat\sigma_1^2+\hat\sigma_2^2}{1+g_c} + \frac{1}{2}\left(1-\frac{1}{R^8}\right)\eta^2\right)\right\}\,,$$

leaving three independent parameters p^2, ϕ_0, g_c. At large R, $f^2 \to 3/4 \neq 1$, and the solution is never Ricci-flat.

Exact Near-Horizon Solution

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leaving three independent parameters p^2, ϕ_0, g_c. At large R, $f^2 \to 3/4 \neq 1$, and the solution is never Ricci-flat.

At constant R, the transverse space is $T^{(1,1)}/\mathbb{Z}_2$, where the orbifolding is needed to have a regular solution at the bolt $R = 1$, where

$$M_6 \sim \mathbb{R}^2 \times S^2 \times S^2\,.$$

Moreover, the dilaton is *finite* at the bolt.

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Sending the blow-up parameter $a \to 0$ we obtain a *singular* solution:

$$\mathrm{d}s_6^2 = 2\alpha' p^2 (1 - g_c^2) \left\{ \mathrm{d}\log(r)^2 + \frac{1}{8} \left[\frac{1}{1-g_c}(\sigma_1^2 + \sigma_2^2) + \frac{1}{1+g_c}(\hat{\sigma}_1^2 + \hat{\sigma}_2^2) + \frac{1}{2}\eta^2 \right] \right\}$$

$$F = -\frac{1}{4}\left((1+g_c)\Omega_1 - (1-g_c)\Omega_2\right) \mathbf{p} \cdot \mathbf{H}$$

$$H = \frac{\alpha' p^2}{8}(1 - g_c^2)\, \eta \wedge (\Omega_1 + \Omega_2)$$

$$e^{\phi - \tilde{\phi}_0} = \frac{\alpha' p^2 \sqrt{1 - g_c^2}}{r^2}\,.$$

This can also be viewed as the UV limit of the NH-solution. The metric is still regular, but factorises to a "linear dilaton" radial direction times a non-Einstein compact $T^{(1,1)}$. Backgrounds with such asymptotics have been studied by [Aharony et al 98] in the context of holography. The dilaton is divergent for $r \to 0$. As there is no bolt, the orbifolding is no longer required for regularity.

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We will later discuss a branch of solutions on the resolved conifold which exhibit the same feature. These solutions have no NH region.

It should also be mentioned that the analytic NH-solution, incuding this singular limit, has a world-sheet description in terms of a solvable $(0,2)$-CFT.

Interpolating Nomerical Solution

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We have found numerical solutions which interpolate between the NH solution for small r and the Ricci-flat conifold [Candelas et al 90]:

$$H_1 \to \text{constant}\,, \quad H_2 \to 0\,, \quad f^2 \to 1\,.$$

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$$H_1 \to \text{constant}\,, \quad H_2 \to 0\,, \quad f^2 \to 1\,.$$

For a given g_c, the solution space is two-dimensional, parameterized by the values of H_i (IR values) at the bolt $h_{i,0}$. We plot $f(\rho)$ for some choices of parameters:

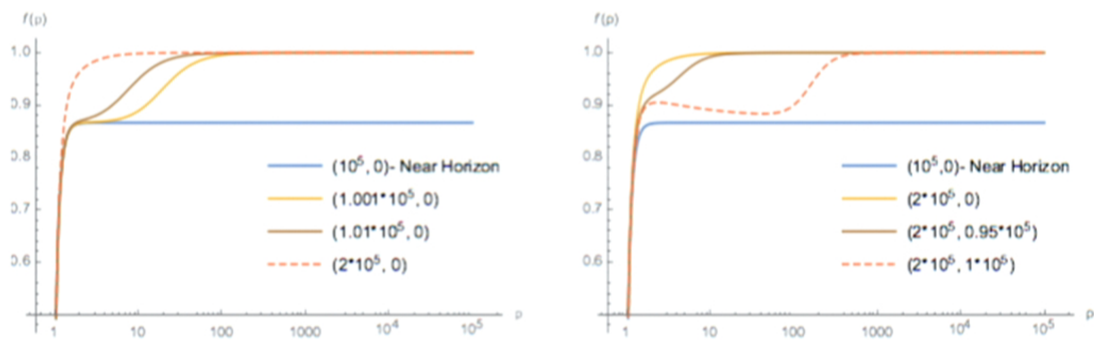

Figure 1: $f(\rho)$ with $g_c = 0$, vanishing H_2 at bolt (left) and non-vanishing H_2 (right). Note that introducing a non-zero $h_{2,0}$ washes away NH-solution.

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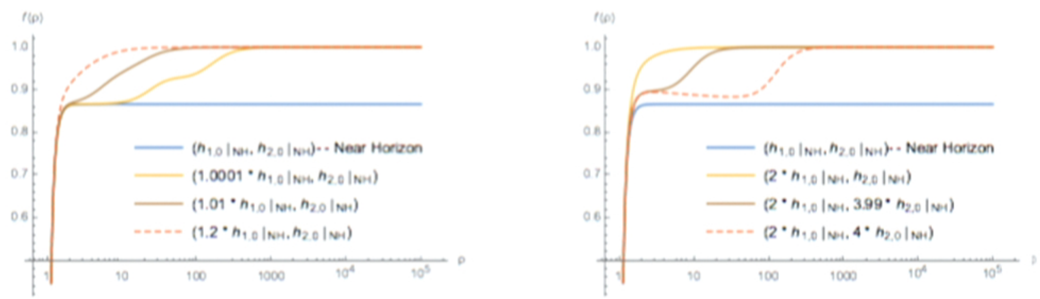

Figure 2: The metric function $f(\rho)$ with $g_c = \frac{1}{3}$ with varying $h_{1,0}$ (left) and varying $h_{2,0}$ (right).

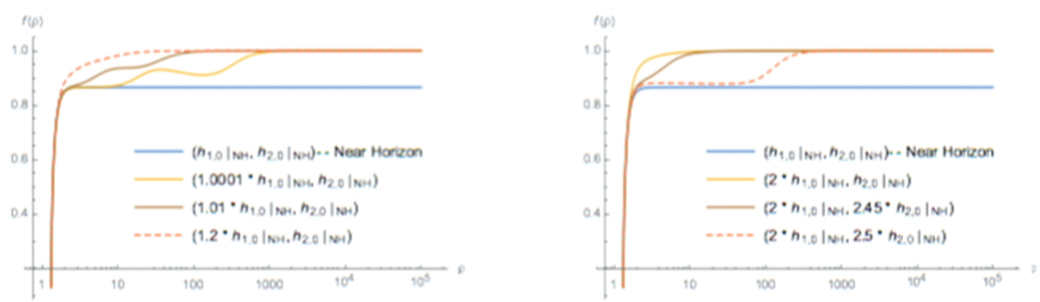

Figure 3: The metric function $f(\rho)$ with $g_c = \frac{2}{3}$ with varying $h_{1,0}$ (left) and varying $h_{2,0}$ (right). Not the appearence of the dip for larger g_c. As g_c approaches 1 this dip moves to the left and ultimately pinches off the NH region.

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We have also found a set of solutions at the boundary of the parameter space $g_c \to 1^-, a \to 0^+$.

A full three-sphere of $T^{(1,1)}$ shrinks at $r = 0$, which is the familiar resolved conifold, but with a non-Kähler metric due to the non-trivial flux. These geometries need not be orbifolded for regularity.

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The IR expansion reads

$$H_1 = \frac{p^2}{r^2} + h_{2,1}\, r^2 + \dots\,, \quad H_2 = \frac{p^2}{r^2} - h_{2,1}\, r^2 + \dots\,,$$
$$e^{2(\phi-\phi_0)} = 4p^2\, h_{2,1} - \frac{5h_{2,1}^2}{6}\, r^4 + \dots$$
$$f^2 = \frac{3}{4} + \frac{h_{2,1}}{4p^2}\, r^4 + \dots\,, \quad g_1 = 1 - \frac{h_{2,1} r^4}{p^2} + \dots\,, \quad g_2 = 0\,,$$

giving a one-parameter set of solutions, parameterized by $h_{2,1}$, which formally connects to the above singular solution.

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giving a one-parameter set of solutions, parameterized by $h_{2,1}$, which formally connects to the above singular solution. The IR metric reads ($\hat{r} = r^2,\ R_{S^2}^2 = 1/4h_{2,1}$):

$$\mathrm{d}s_6^2 = 2h_{2,1}p^2 \left[\mathrm{d}\hat{r}^{\,2} + \frac{\hat{r}^{\,2}}{4}(\hat{\sigma}_1^2 + \hat{\sigma}_2^2 + \eta^2) + R_{S^2}^2(\sigma_1^2 + \sigma_2^2) + \dots \right]\,,$$

where a three-sphere shrinks.

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We have found a set of numerical solution inerpolating to a Ricci-flat solution in the UV. There is no analytic near-horizon region in IR that can be decoupled, i.e. the *solution has different IR dynamics*.

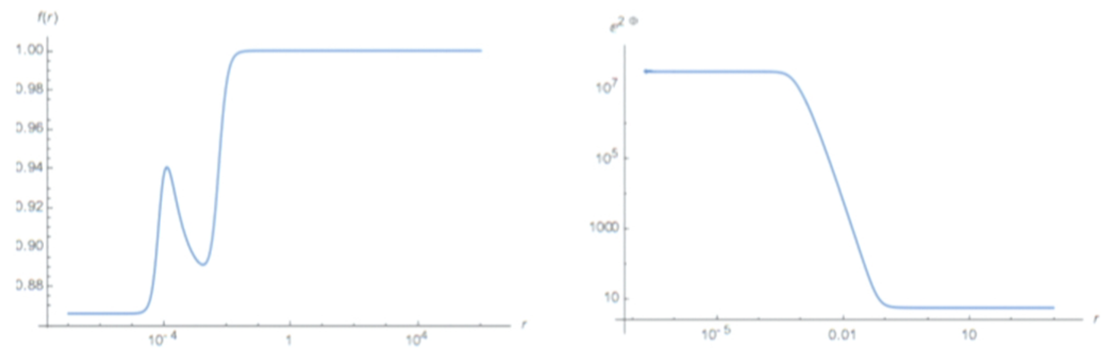

Figure 4: Numerical results for the metric function $f(r)$ and the dilaton $e^{2\Phi}$. We have set $\{p^2 = 10^6,\ h_{2,1} = 10\}$.

Note that the overall scale of $e^{2\Phi}$ can be absorbed into Φ_0; only the ratio $e^{2\Phi|_{IR}}/e^{2\Phi|_{UV}} \sim p^2$ is physical. Note also that in the limit $g_c \to 1^-$, the dilaton is no longer singular.

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Gauge Charges

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The solutions obtained from the metric and gauge ansats presented here have an asymptotic magnetic monopole charge

$$q_{\text{UV}} = \frac{1}{2\pi}\int_{S^2_\infty} F = \frac{1}{2}\mathbf{p}\cdot\mathbf{H}$$

signaling the presence of a non-normalizable mode, which does not fall off fast enough to vanish at infinity.

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Further, there are IR charges associated with the compact two-cycles of the geometry at the bolt

$$q_{1,\text{IR}} = \frac{1}{2\pi}\int_{S^2_1} F = \frac{1}{2}\Big[-(1+g_c)\mathbf{p} - \frac{p}{q}\sqrt{1-g_c^2}\,\mathbf{q}\Big]\cdot\mathbf{H} \equiv -(s_1\,\mathbf{p}+\hat{\mathbf{q}})\cdot\mathbf{H}$$

$$q_{2,\text{IR}} = \frac{1}{2\pi}\int_{S^2_2} F = \frac{1}{2}\Big[(1-g_c)\mathbf{p} - \frac{p}{q}\sqrt{1-g_c^2}\,\mathbf{q}\Big]\cdot\mathbf{H} \equiv -(s_2\,\mathbf{p}+\hat{\mathbf{q}})\cdot\mathbf{H}\,.$$

Either integer or half-integer Dirac quantization conditions can be imposed:

$$\forall\ell\in\{0,1,..,16\}\,,\ \begin{cases} s_1\,p_\ell+\hat{q}_\ell\in\mathbb{Z} \\ s_2\,p_\ell-\hat{q}_\ell\in\mathbb{Z}\end{cases} \quad \text{or} \quad \forall\ell\in\{0,1,..,16\}\,,\ \begin{cases} s_1\,p_\ell+\hat{q}_\ell\in\mathbb{Z}+1/2 \\ s_2\,p_\ell-\hat{q}_\ell\in\mathbb{Z}+1/2\end{cases}$$

This agrees with quantization conditions obtained from the worldsheet [israel et al 09, Israel Halmagyi EES 16].

Five-Brane Charge

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There is a subtlety regarding the five-brane charge as we generically have $\mathrm{d}H \neq 0$. We define two charges, the Maxwell charge and the Page charge

$$\mathcal{Q}_M = \frac{1}{2\pi^2\alpha'}\int_{M_3} H\,, \quad \mathcal{Q}_P = \frac{1}{2\pi^2\alpha'}\int_{M_3} \left(H - \alpha' CS(A)\right)\,,$$

where $CS(A)$ is the Chern-Simons form

$$CS(A) = \mathrm{tr}\, F \wedge A\,.$$

The Page charge is required to be integer quantized [Rohm Witten 86].

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As $b_3(T^{(1,1)}) = 1$, there is a single three-cycle in the geometry, in this case $S^3/\mathbb{Z}_2$. Note however that the Maxwell charge will depend in general on the choice of three-cycle representative, as

$$\mathcal{Q}_{M,1} - \mathcal{Q}_{M,2} = \frac{1}{2\pi^2}\int_{M_4} \mathrm{tr}\, F \wedge F\,,$$

where M_4 is the four-chain connection the cycles.

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where M_4 is the four-chain connection the cycles.

There are two canonical representatives in the geometry, $S^3_{1,2}/\mathbb{Z}_2$. We get

$$\mathcal{Q}_{M,1} = \frac{1}{2\pi^2\alpha'}\int_{S^3_1/\mathbb{Z}_2} H = 4(h_1 + h_2)\,, \quad \mathcal{Q}_{M,1} = \frac{1}{2\pi^2\alpha'}\int_{S^3_1/\mathbb{Z}_2} H = 4(h_1 - h_2)\,.$$

The Maxwell Charge

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The Maxwell charge is also radially dependent:

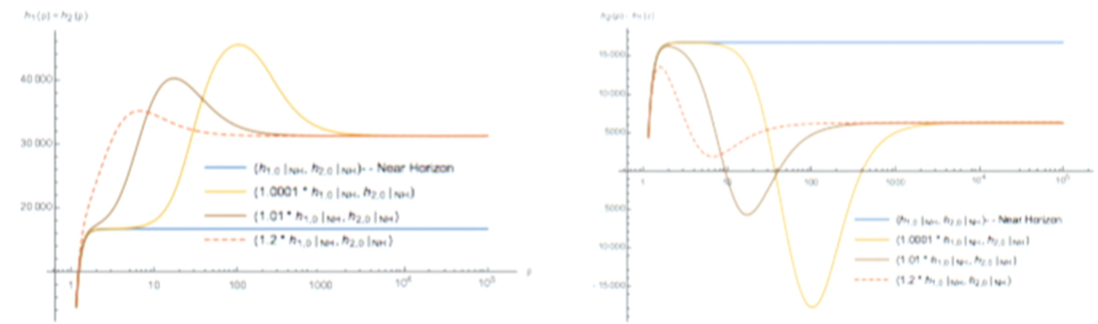

Figure 5: The flux functions $h_2(\rho) \pm h_1(\rho)$ with $g_c = \frac{1}{3}$. Note that the near-horizon solution has $h_1 = 0$.

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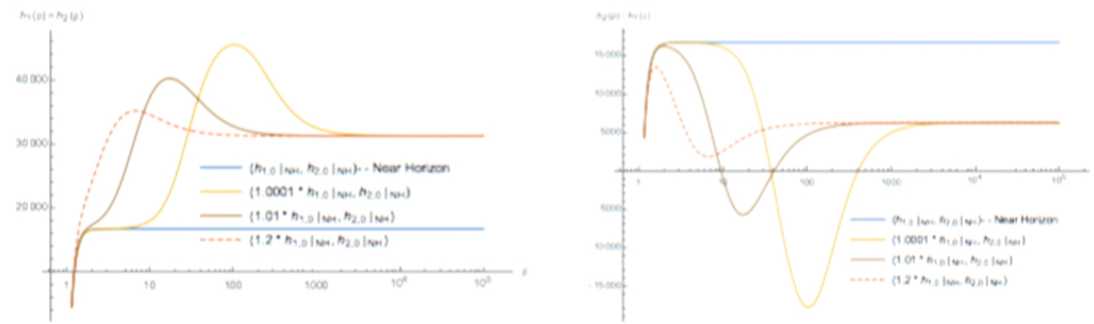
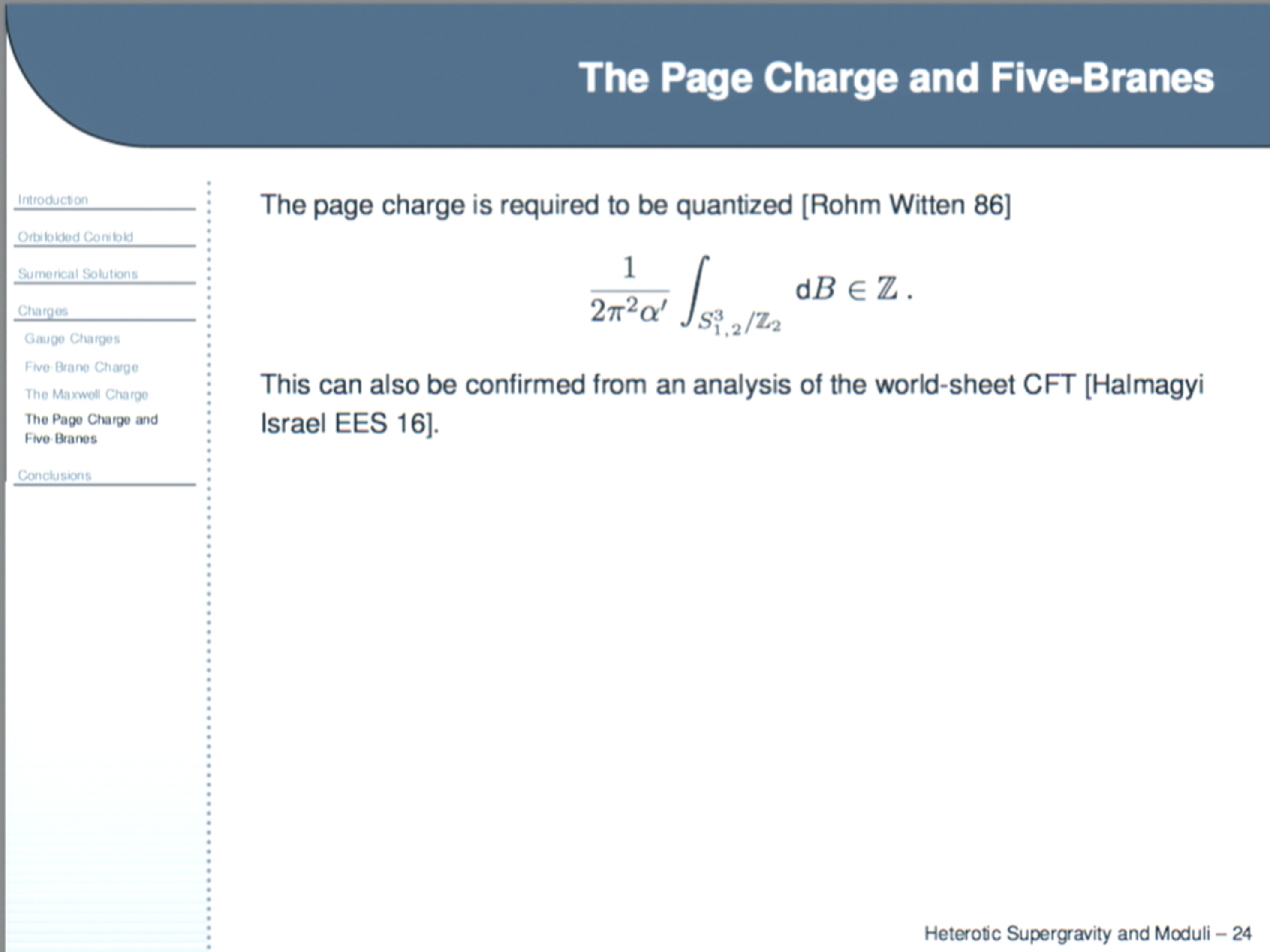


Figure 5: The flux functions $h_2(\rho) \pm h_1(\rho)$ with $g_c = \frac{1}{3}$. Note that the near-horizon solution has $h_1 = 0$.

Note that as $r \to \infty$ the charges in the UV for the locally Ricci-flat solutions differ from that of the near horizon solution, signaling a discrete change in the UV dynamics.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The page charge is required to be quantized [Rohm Witten 86]
$$\frac{1}{2\pi^2\alpha'}\int_{S^3_{1,2}/\mathbb{Z}_2} \mathrm{d}B \in \mathbb{Z}\,.$$
This can also be confirmed from an analysis of the world-sheet CFT [Halmagyi Israel EES 16].
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Recall the Bianchi Identity

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where j_P denotes the page current. A non-zero j_P signals the presence of a five-brane, usually by the appearance of non-regularities and δ-functions in the solution.

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Our regular solutions have $j_P = 0$ where $h_2 = 0$ at the bolt. However the solution in the blow-down limit $a \to 0$ has a singular metric in the Einstein frame, where h_2 is constant along r. We interpret this as implying that the contribution to the Maxwell charge in the blow-down limit comes from explicit brane source in the IR, i.e. the RHS of the Bianchi identity has an explicit δ-function source.

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- On the boundary of parameter space, in the blow-down limit, we have also found regular numerical solutions on the resolved conifold. These have no near-horizon region and therefore different IR dynamics.

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- Inclusion of nonabelian gauge fields. Any subgroup of either $SO(32)$ or $E_8 \times E_8$ is allowed. Useful for a better understanding of phenomenology from such heterotic local models.

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- Move away from the large charge limit and consider α'-corrections. In particular, the NH-solution corresponds to a exactly solvable world-sheet CFT. Including higher corrections should hence be possible.

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- The conifold provides concrete examples of holography, particularly the NH-solution which has an asymptotically "linear dilaton backgrounds" [Aharony et al 98]. In the heterotic case, the dual field theory should be a special kind of little string theory [Seidberg 97]. Holography has been studied in this context before [Israel et al 09], but it would be interesting to expand upon this analysis.

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- It would also be interesting to look for more exotic versions of the internal space, where e.g. one or both S^2 have been replaced by Riemann surfaces. This probably requires a generalisation of the fram ansatz, etc. Work in this direction is underway.

Thank you!

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Thank you for your attention!