Title: Superradiant instabilities of asymptotically AdS black holes

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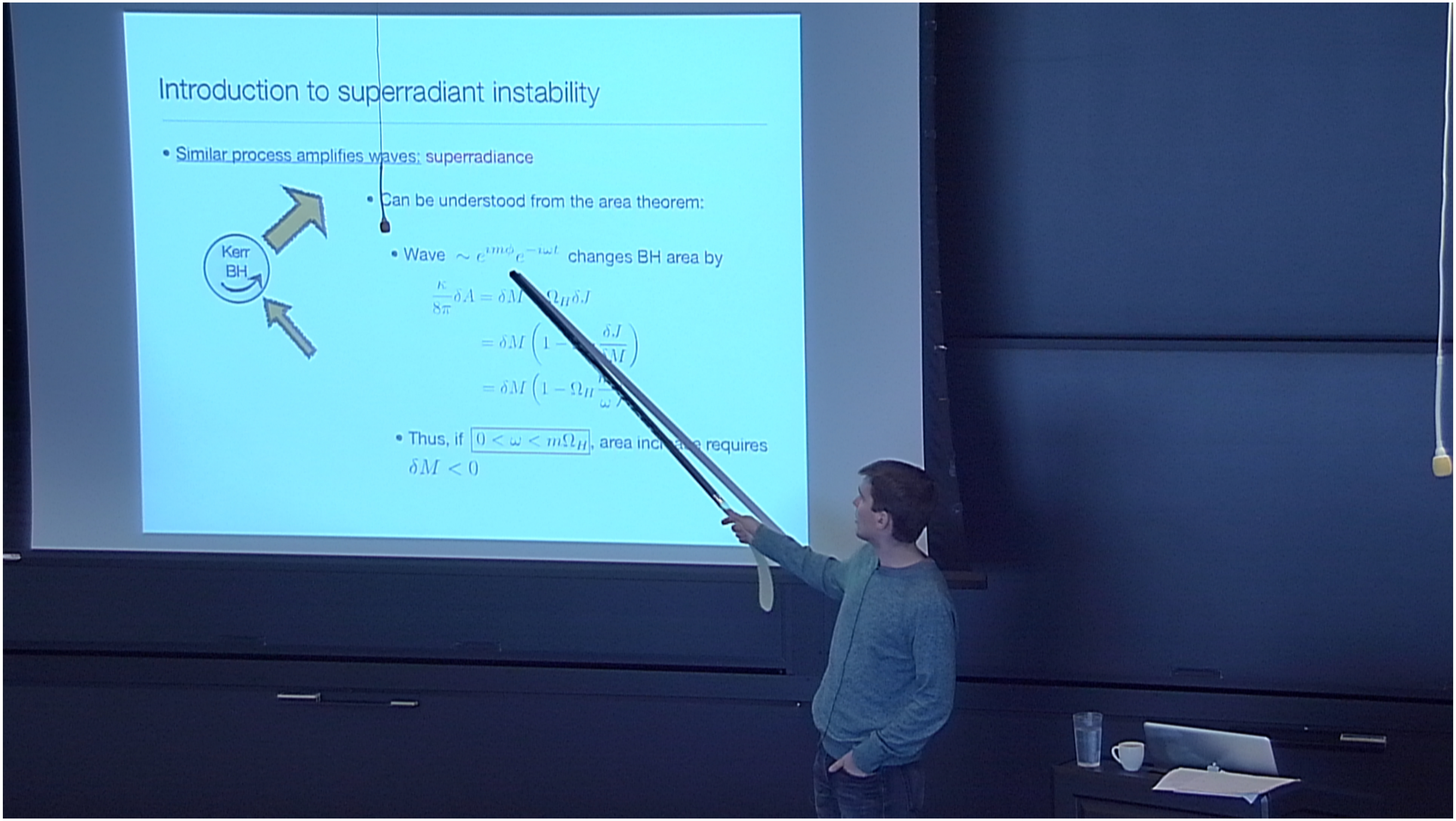
Abstract: <p>Asymptotically AdS spacetimes with reflecting boundary conditions represent a natural setting for studying superradiant instabilities of rotating or charged black holes. In the first part of this talk, I prove that all asymptotically AdS black holes with ergoregions in dimension d â‰¥ 4 are linearly unstable to gravitational perturbations. This proof uses the canonical energy method of Hollands and Wald in a WKB limit. In the second part of the talk, I consider a charged Reissner-Nordstrom-AdS black hole---which is superradiantly unstable to charged scalar field perturbations at the linear level---and study the full *nonlinear* evolution of the instability. In this special case, the instability occurs even for spherically symmetric perturbations, which simplifies the analysis and allows for the use of numerical general relativity simulations. Our results show that nonlinear backreaction causes the black hole to lose charge and mass to the scalar field as the instability proceeds. Eventually, higher scalar field harmonics become nonsuperradiant, and they are reabsorbed into the black hole. The final state is described by a â€œhairyâ€• black hole, surrounded by a scalar condensate in the fundamental (lowest) mode. I discuss implications of this work on the original problem of the rotating black hole superradiant instability.</p>



Introduction to superradiant instability

- Similar process amplifies waves: superradiance

- Can be understood from the area theorem:
  - Wave $\sim e^{im\phi}e^{-i\omega t}$ changes BH area by

$$\frac{\kappa}{8\pi}\delta A = \delta M \quad \Omega_H \delta J$$
$$= \delta M \left(1 - \quad \frac{\delta J}{\delta M}\right)$$
$$= \delta M \left(1 - \Omega_H \frac{\quad}{\omega}\right)$$

  - Thus, if $\boxed{0 < \omega < m\Omega_H}$, area inc e requires $\delta M < 0$

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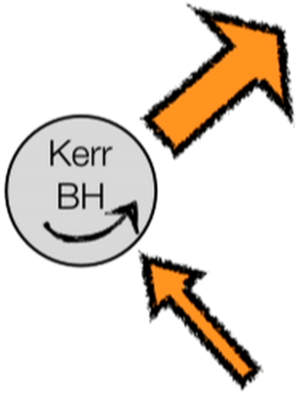

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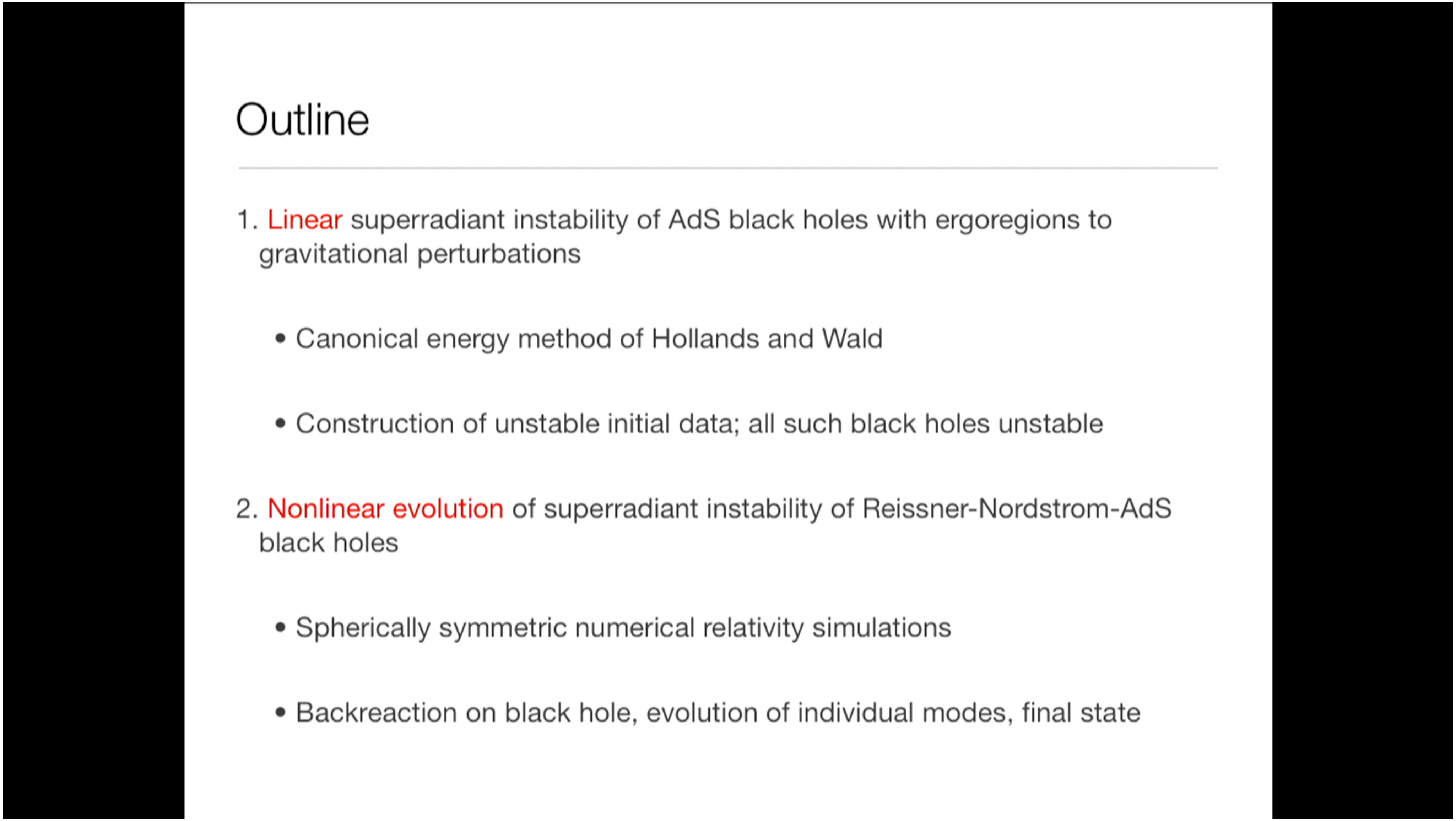
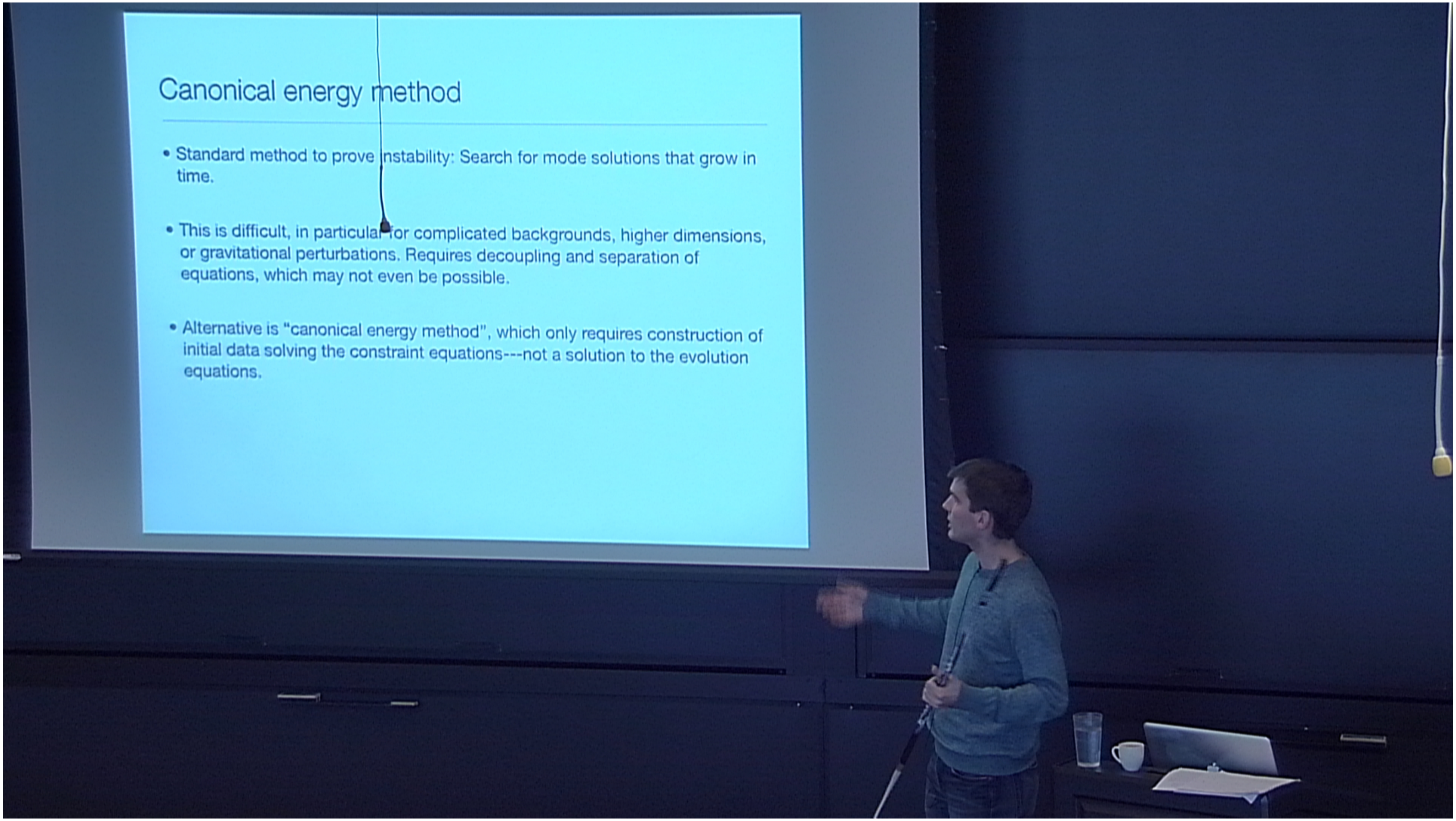
$$\begin{aligned}\frac{\kappa}{8\pi}\delta A &= \delta M - \Omega_H \delta J \\ &= \delta M\left(1-\Omega_H\frac{\delta J}{\delta M}\right) \\ &= \delta M\left(1-\Omega_H\frac{m}{\omega}\right)\end{aligned}$$

  - Thus, if $\boxed{0<\omega<m\Omega_H}$, area increase requires $\delta M < 0$



Outline

1. Linear superradiant instability of AdS black holes with ergoregions to gravitational perturbations
    - Canonical energy method of Hollands and Wald
    - Construction of unstable initial data; all such black holes unstable
2. Nonlinear evolution of superradiant instability of Reissner-Nordstrom-AdS black holes
    - Spherically symmetric numerical relativity simulations
    - Backreaction on black hole, evolution of individual modes, final state

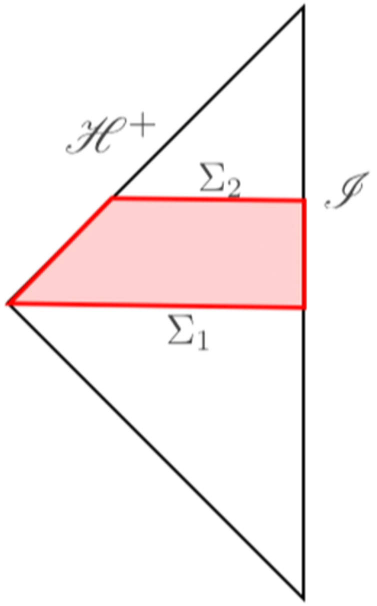


Canonical energy method

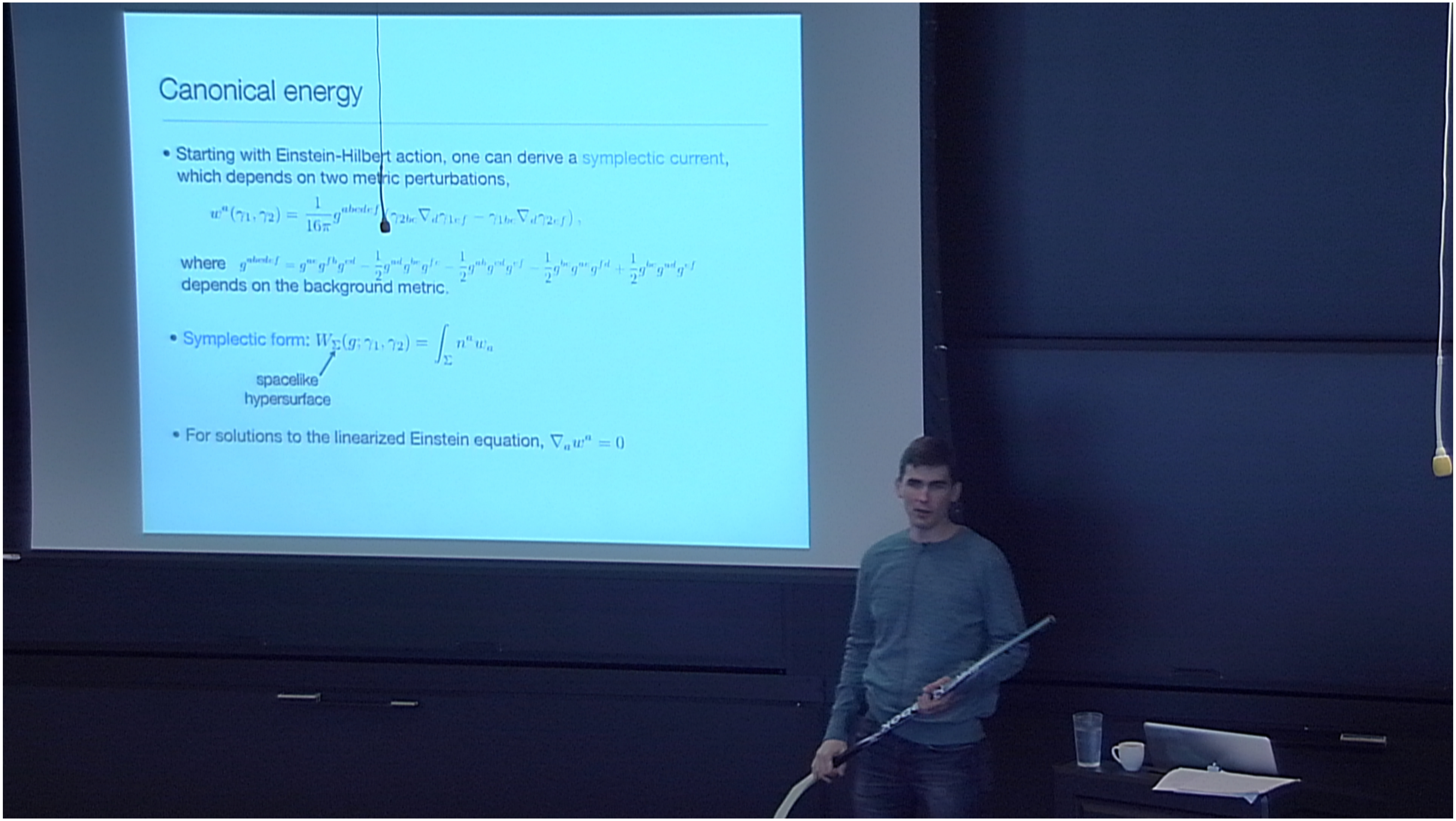
- Standard method to prove instability: Search for mode solutions that grow in time.
- This is difficult, in particular for complicated backgrounds, higher dimensions, or gravitational perturbations. Requires decoupling and separation of equations, which may not even be possible.
- Alternative is "canonical energy method", which only requires construction of initial data solving the constraint equations---not a solution to the evolution equations.

Canonical energy method

- Canonical energy $\mathcal{E}$ is an integral over a Cauchy hypersurface Σ, quadratic in the perturbation γ_{ab}, satisfying
    - Gauge invariance
    - Degeneracy precisely on perturbations to other stationary black holes
    - Conservation
    - Positive flux at horizon and infinity



- Then $\mathcal{E}_{\Sigma_2} < \mathcal{E}_{\Sigma_1}$, and if a solution to the constraints γ_{ab} exists such that $\mathcal{E}_{\Sigma_1}(\gamma) < 0$, instability follows.



Canonical energy

- Starting with Einstein-Hilbert action, one can derive a symplectic current, which depends on two metric perturbations,

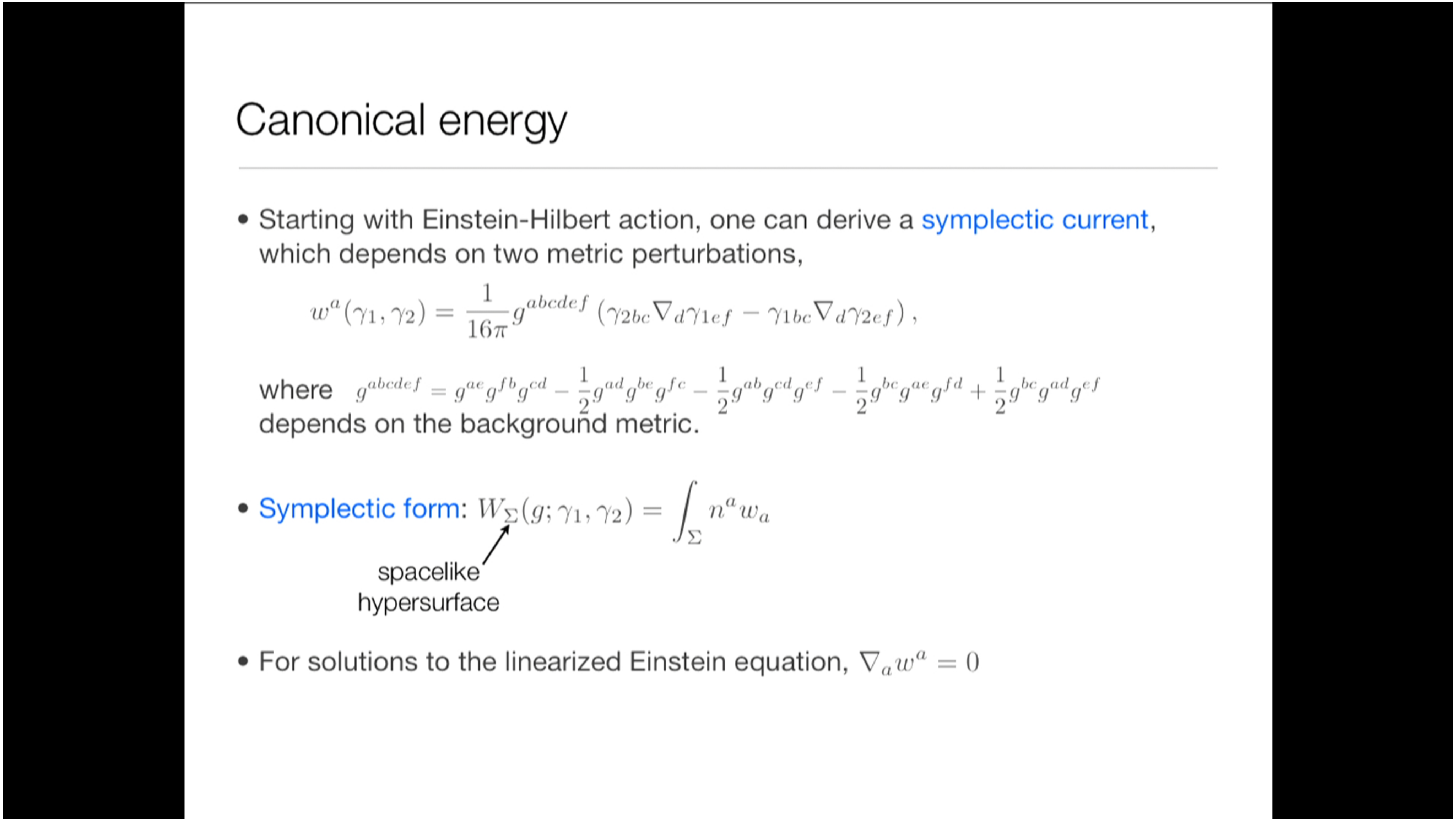
$$w^a(\gamma_1, \gamma_2) = \frac{1}{16\pi} g^{abcdef} (\gamma_{2bc} \nabla_d \gamma_{1ef} - \gamma_{1bc} \nabla_d \gamma_{2ef}),$$

where $g^{abcdef} = g^{ae} g^{fb} g^{cd} - \frac{1}{2} g^{ad} g^{be} g^{fc} - \frac{1}{2} g^{ab} g^{cd} g^{ef} - \frac{1}{2} g^{bc} g^{ae} g^{fd} + \frac{1}{2} g^{bc} g^{ad} g^{ef}$ depends on the background metric.

- Symplectic form: $W_\Sigma(g; \gamma_1, \gamma_2) = \int_\Sigma n^a w_a$

spacelike hypersurface

- For solutions to the linearized Einstein equation, $\nabla_a w^a = 0$



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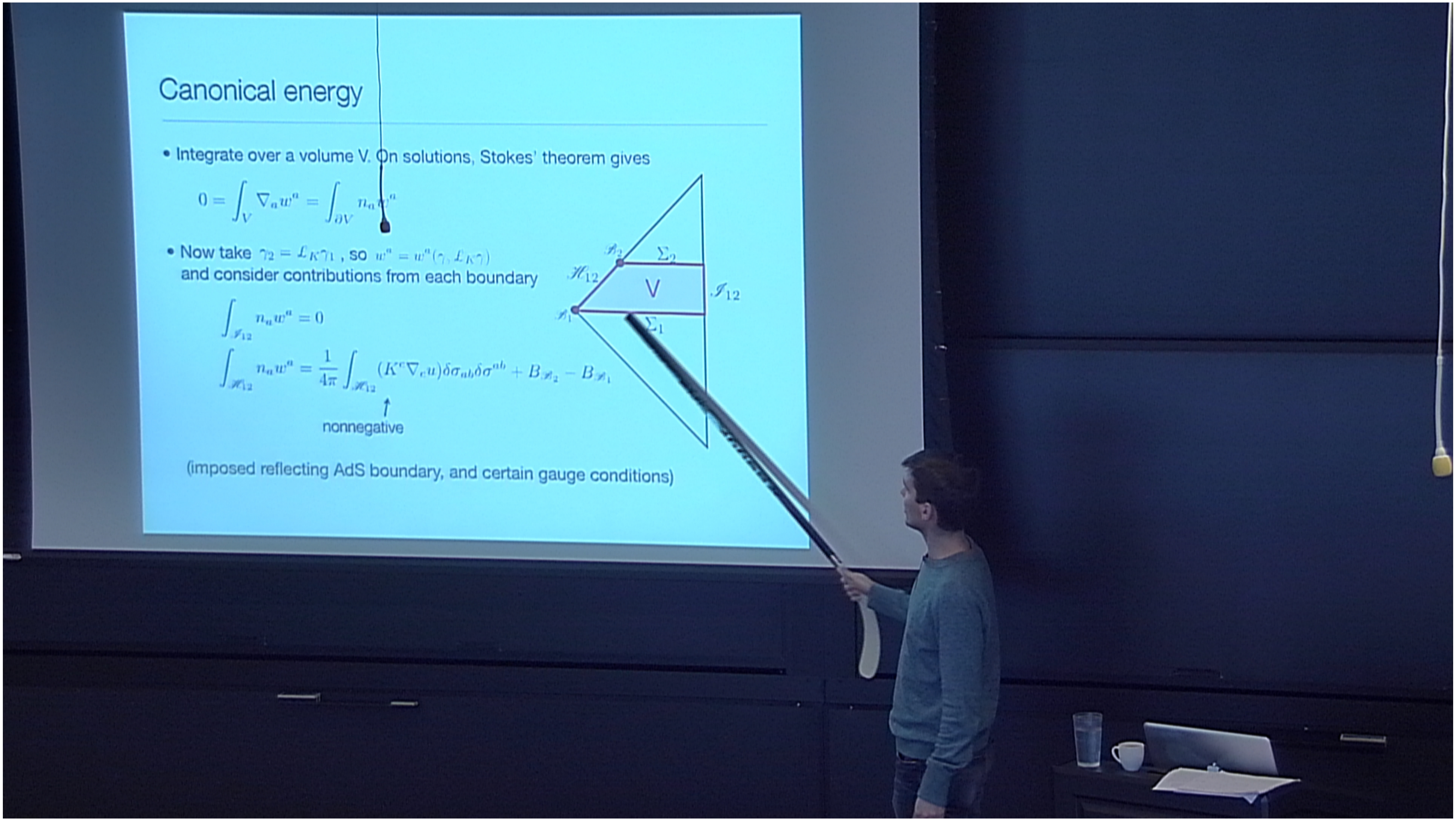

Canonical energy

- Integrate over a volume V. On solutions, Stokes' theorem gives

$$0 = \int_V \nabla_a w^a = \int_{\partial V} n_a w^a$$

- Now take $\gamma_2 = \pounds_K \gamma_1$, so $w^a = w^a(\gamma, \pounds_K \gamma)$ and consider contributions from each boundary

$$\int_{\mathscr{I}_{12}} n_a w^a = 0$$

$$\int_{\mathscr{H}_{12}} n_a w^a = \frac{1}{4\pi} \int_{\mathscr{H}_{12}} (K^c \nabla_c u) \delta\sigma_{ab} \delta\sigma^{ab} + B_{\mathscr{B}_2} - B_{\mathscr{B}_1}$$

↑
nonnegative

(imposed reflecting AdS boundary, and certain gauge conditions)

Canonical energy

- So define the canonical energy

$$\mathcal{E}_K(\gamma, \Sigma) = W_\Sigma(g; \gamma, \pounds_K \gamma) - B_{\mathscr{B}}(g; \gamma)$$

- Above implies $\mathcal{E}_K(\gamma, \Sigma_2) \leq \mathcal{E}_K(\gamma, \Sigma_1)$ (decreases in time)

- Under restriction to certain gauge conditions at $\mathscr{H}^+$ and $\mathscr{I}$, together with $\delta A = 0$ and $\delta H_X = 0$ for all asymptotic symmetries X^a, it can be shown that $\mathcal{E}_K(\gamma, \Sigma)$ is gauge-invariant and degenerate precisely on perturbations to other stationary black holes.

Stability criterion

Canonical energy non-negative
for all such perturbations
STABLE

Canonical energy negative for
some perturbation
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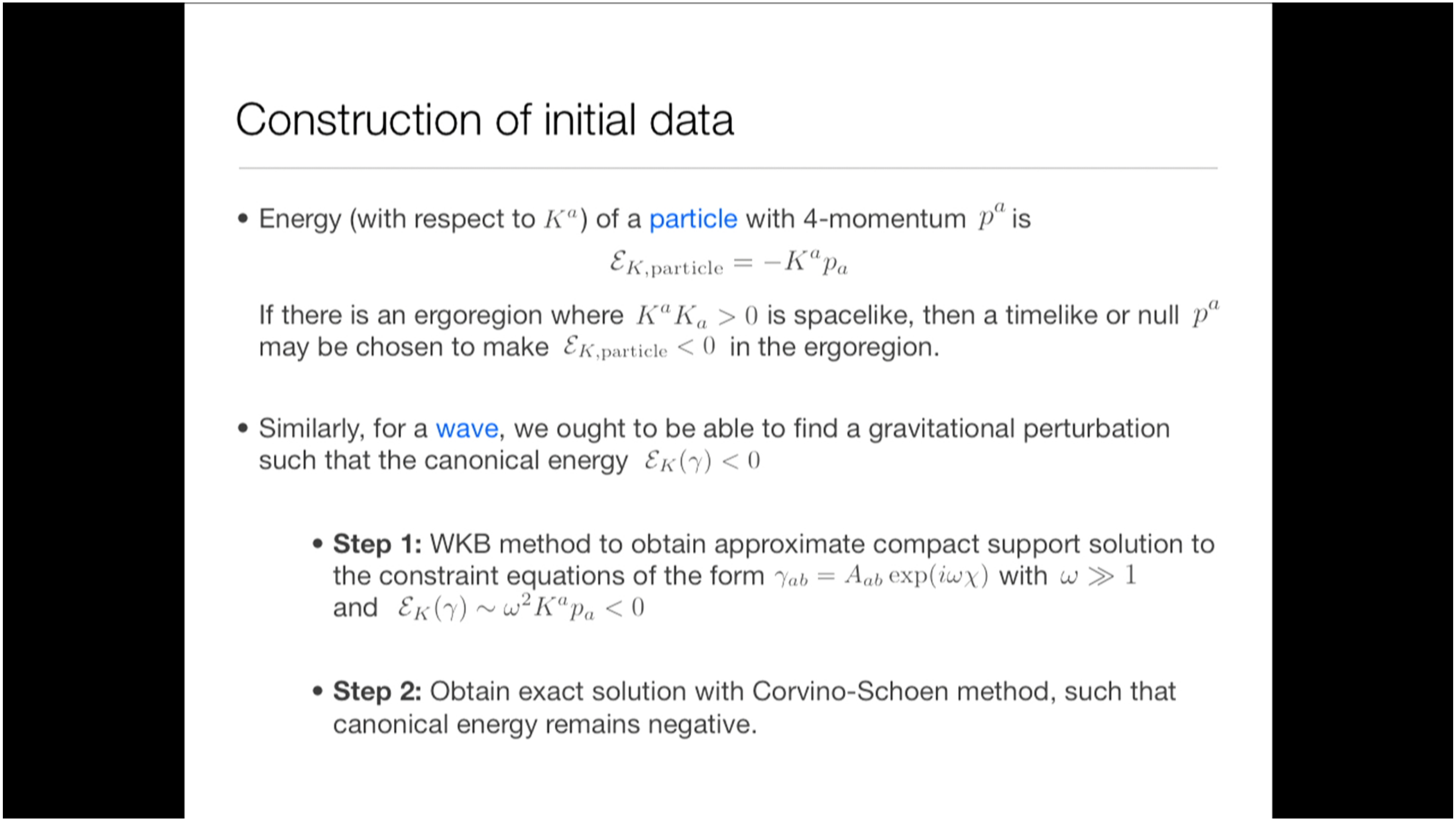


Construction of initial data

- Energy (with respect to K^a) of a particle with 4-momentum p^a is

$$\mathcal{E}_{K,\text{particle}} = -K^a p_a$$

If there is an ergoregion where $K^a K_a > 0$ is spacelike, then a timelike or null p^a may be chosen to make $\mathcal{E}_{K,\text{particle}} < 0$ in the ergoregion.

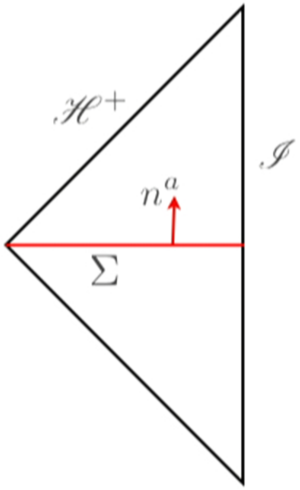
- Similarly, for a wave, we ought to be able to find a gravitational perturbation such that the canonical energy $\mathcal{E}_K(\gamma) < 0$
    - **Step 1:** WKB method to obtain approximate compact support solution to the constraint equations of the form $\gamma_{ab} = A_{ab} \exp(i\omega\chi)$ with $\omega \gg 1$ and $\mathcal{E}_K(\gamma) \sim \omega^2 K^a p_a < 0$
    - **Step 2:** Obtain exact solution with Corvino-Schoen method, such that canonical energy remains negative.

Construction of initial data

- Convenient to trade spacetime quantities g_{ab} and γ_{ab} for initial data quantities defined on Σ

$$q_{ab} = g_{ab} + n_a n_b$$
$$p^{ab} = \sqrt{q}(k^{ab} - q^{ab} k^c{}_c)$$

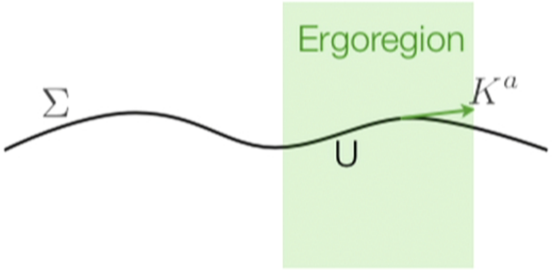
$$\delta q_{ab} = q_a{}^c q_b{}^d \gamma_{cd}$$
$$\delta p^{ab} = \sqrt{q}(q^{ac} q^{bd} - q^{ab} q^{cd}) \frac{1}{2} \pounds_n \gamma_{cd}$$



Construction of initial data

- Assume there is a region where K^a is spacelike. Construct approximate initial data of compact support in this region.

- Trick: In this region, choose Σ such that it is tangent to K^a (possible since spacelike). This leads to the expression



$$\mathcal{E}_K(\delta q_{ab}, \delta p^{ab}) = -\frac{1}{16\pi}\int_\Sigma K^a \left(-2\delta p^{bc} D_a \delta q_{bc} + 4\delta p^{cb} D_b \delta q_{ac} + 2\delta q_{ac} D_b \delta p^{cb} \right.$$
$$\left. -2p^{cb}\delta q_{ad} D_b \delta q_c{}^d + p^{cb}\delta q_{ad} D^d \delta q_{cb}\right)$$

- Constraints

$$\boldsymbol{C}(\delta q_{ab}, \delta p^{ab}) \equiv \begin{pmatrix} q^{\frac{1}{2}}\left(D^a D_a \delta q_c{}^c - D^a D^b \delta q_{ab} + Ric(q)^{ab}\delta q_{ab}\right) + \\ q^{-\frac{1}{2}}\left(-\delta q_c{}^c p^{ab} p_{ab} + 2p_{ab}\delta p^{ab} + 2p^{ac} p^b{}_a \delta q_{bc} + \right. \\ \left. \frac{1}{d-2} p^c{}_c p^d{}_d \delta q^a{}_a - \frac{2}{d-2} p^a{}_a \delta p^b{}_b - \frac{2}{d-2}\delta q_{ab} p^{ab} p_c{}^c\right) \\ \\ -2q^{\frac{1}{2}} D^b(q^{-\frac{1}{2}}\delta p_{ab}) + D_a \delta q_{cb} p^{cb} - 2D_c \delta q_{ab} p^{bc} \end{pmatrix} = 0$$

Construction of initial data

- WKB expansion of initial data $\delta q_{ab} = \left(\sum_{n \geq 0} Q_{ab}^{(n)} (i\omega)^{-n} \right) \exp(i\omega\chi),$

$$\delta p_{ab} = \left(\sum_{n \geq 0} P_{ab}^{(n)} (i\omega)^{-n+1} \right) \exp(i\omega\chi)$$

WKB parameter

phase function

- Constraints become

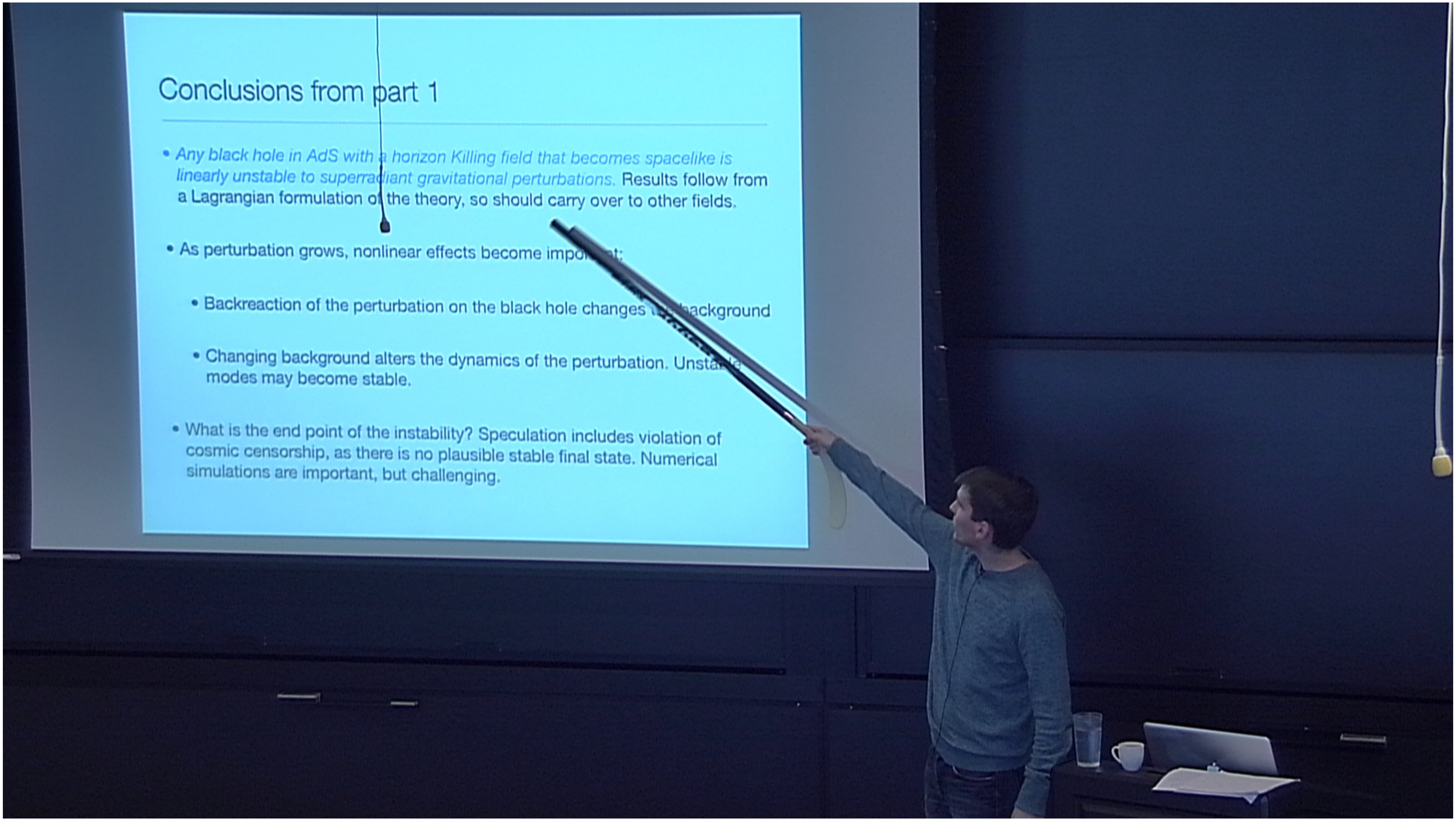
$$\left(\begin{array}{c} -D^a\chi(D_a\chi)Q_c^{(n)c} + D^a\chi(D^b\chi)Q_{ab}^{(n)} \\ P_{ab}^{(n)} D^b\chi \end{array} \right) = \boldsymbol{C}^{(n)}$$

Depends on lower order (m<n)
WKB approximations

  - 0th order, choose

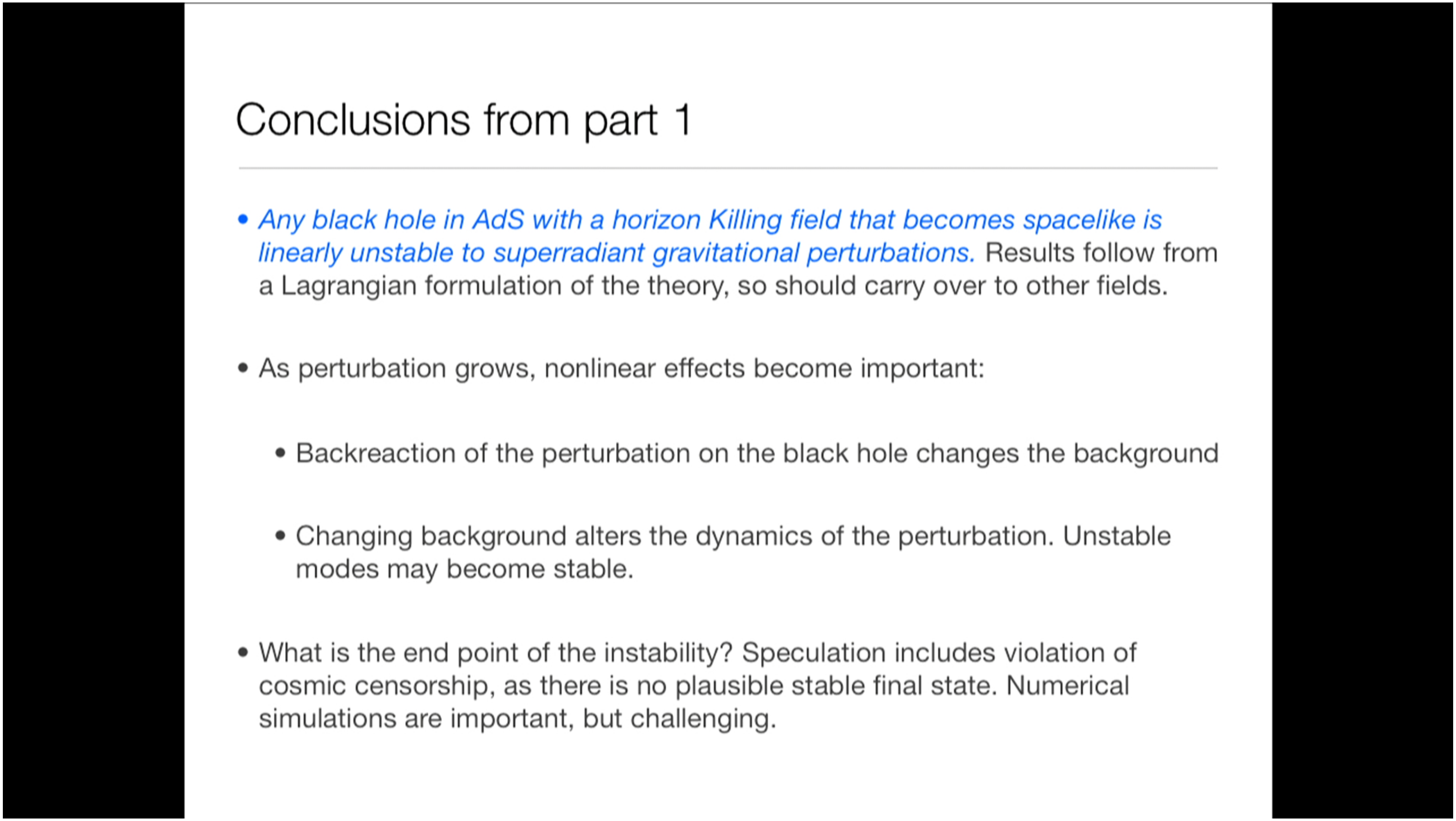
$$P_{ab}^{(0)} = -Q_{ab}^{(0)}, \qquad Q_a^{(0)a} = 0, \qquad Q_{ab}^{(0)} D^b\chi = 0$$

  - Higher orders algebraic



Conclusions from part 1

- *Any black hole in AdS with a horizon Killing field that becomes spacelike is linearly unstable to superradiant gravitational perturbations.* Results follow from a Lagrangian formulation of the theory, so should carry over to other fields.
- As perturbation grows, nonlinear effects become important:
  - Backreaction of the perturbation on the black hole changes the background
  - Changing background alters the dynamics of the perturbation. Unstable modes may become stable.
- What is the end point of the instability? Speculation includes violation of cosmic censorship, as there is no plausible stable final state. Numerical simulations are important, but challenging.



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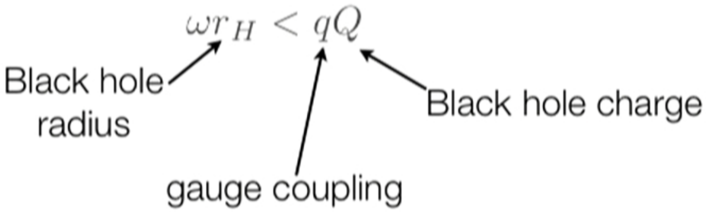
Part 2: RN-AdS superradiant instability

arXiv:1601.01384 [gr-qc], with P. Bosch and L. Lehner

- Reissner-Nordstrom-AdS black holes are also subject to the superradiant instability, with charge playing the role of angular momentum.

- Charged scalar field mode $\psi \sim e^{-i\omega t}$ superradiantly amplified if



$$\omega r_H < qQ$$

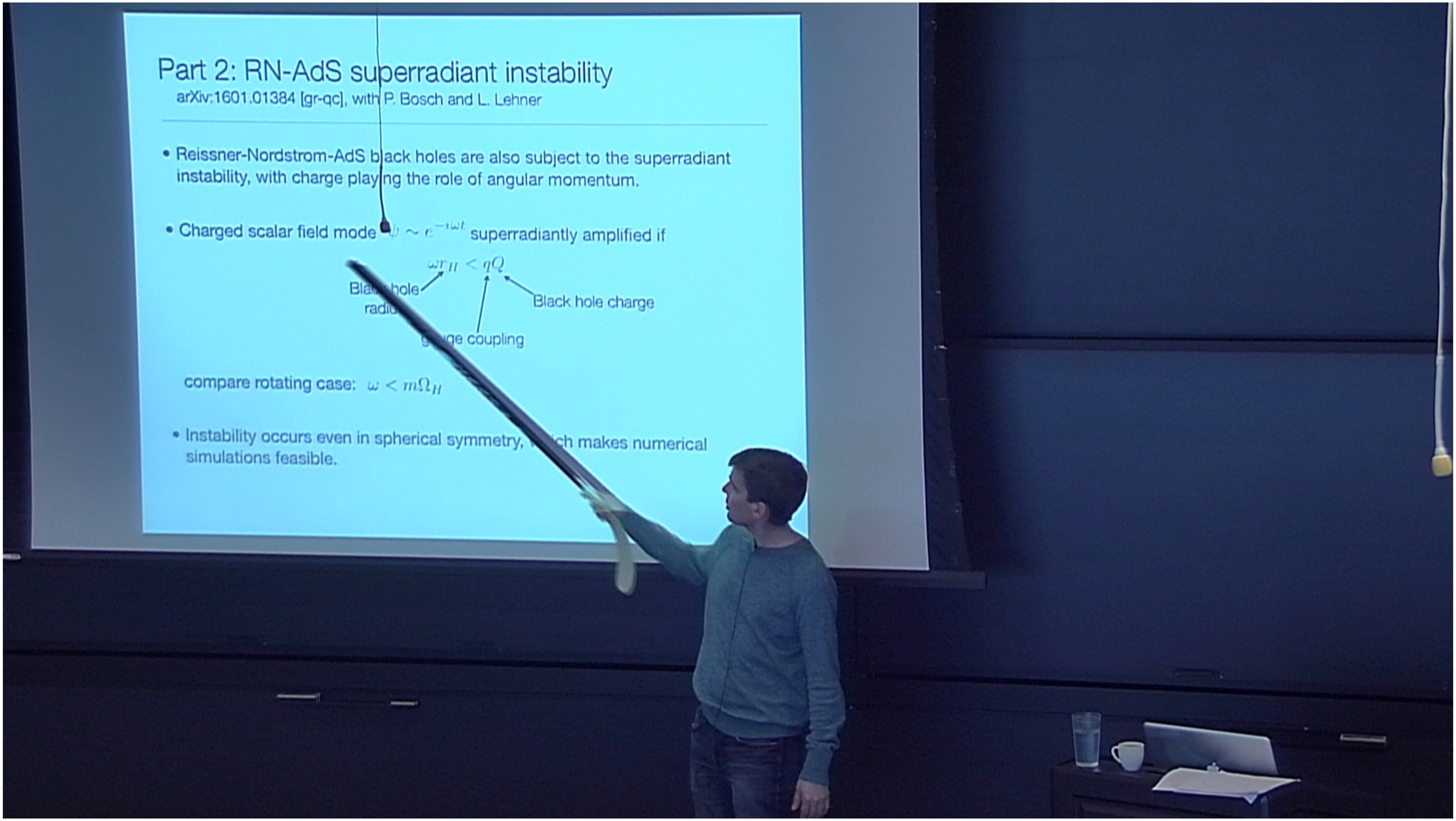
Black hole radius

gauge coupling

Black hole charge

compare rotating case: $\omega < m\Omega_H$

- Instability occurs even in spherical symmetry, which makes numerical simulations feasible.



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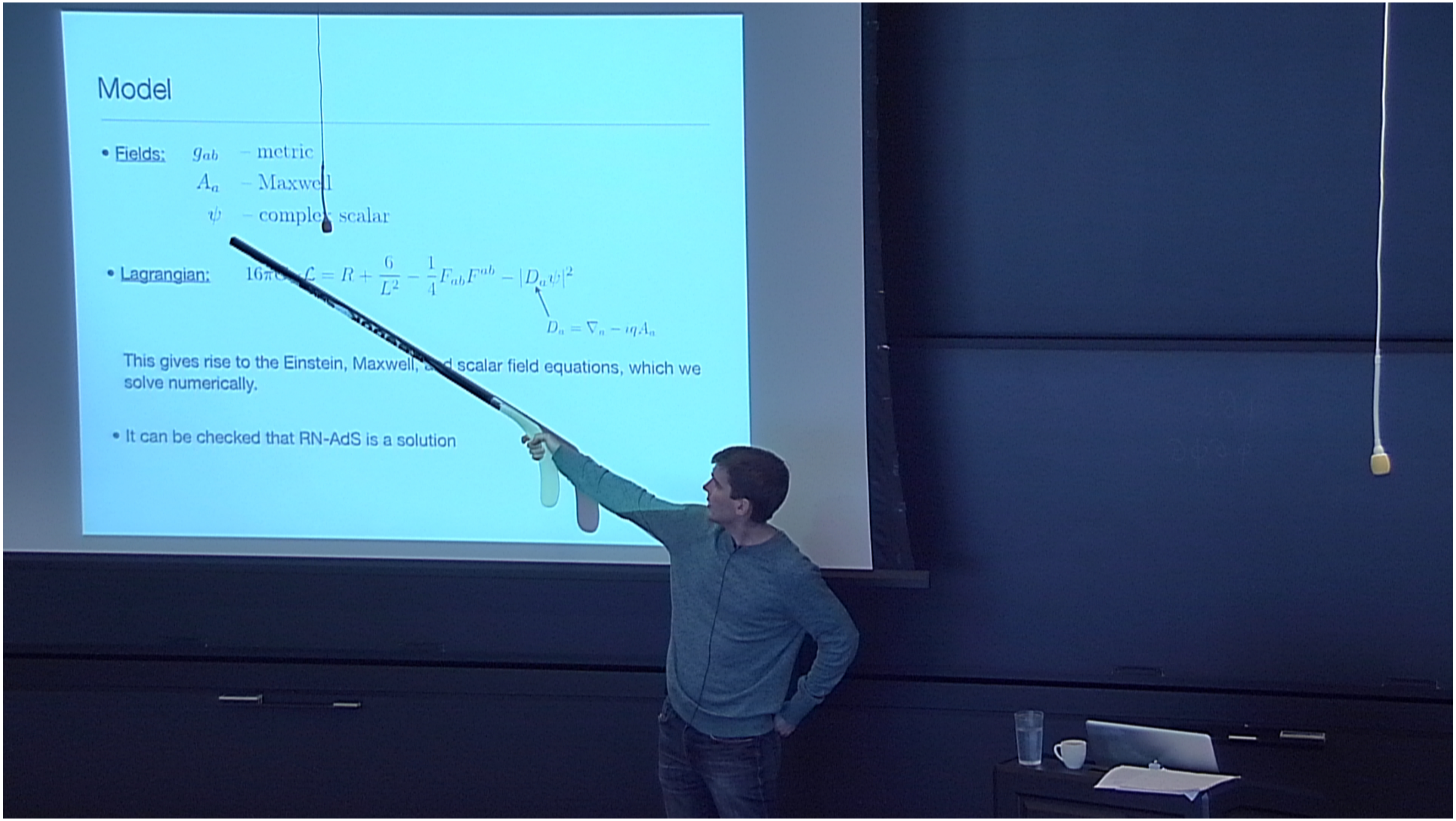


Model

- **Fields:** g_{ab} – metric
  A_a – Maxwell
  ψ – complex scalar

- **Lagrangian:** $16\pi G_N \mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F_{ab}F^{ab} - |D_a\psi|^2$

  $D_a = \nabla_a - iqA_a$

  This gives rise to the Einstein, Maxwell, and scalar field equations, which we solve numerically.

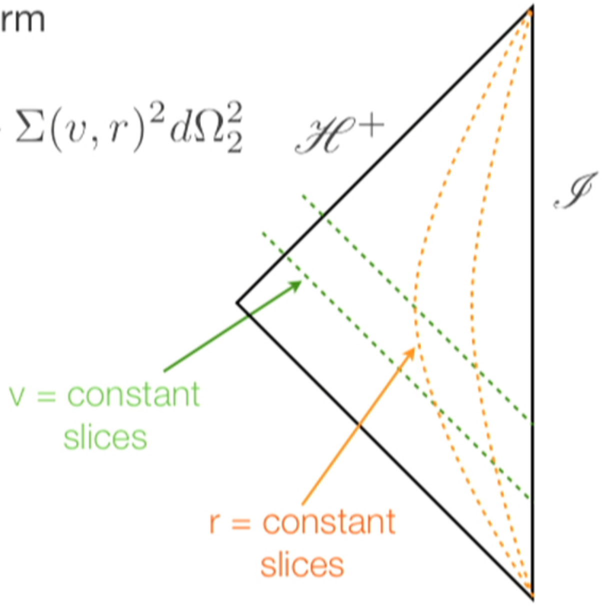
- It can be checked that RN-AdS is a solution

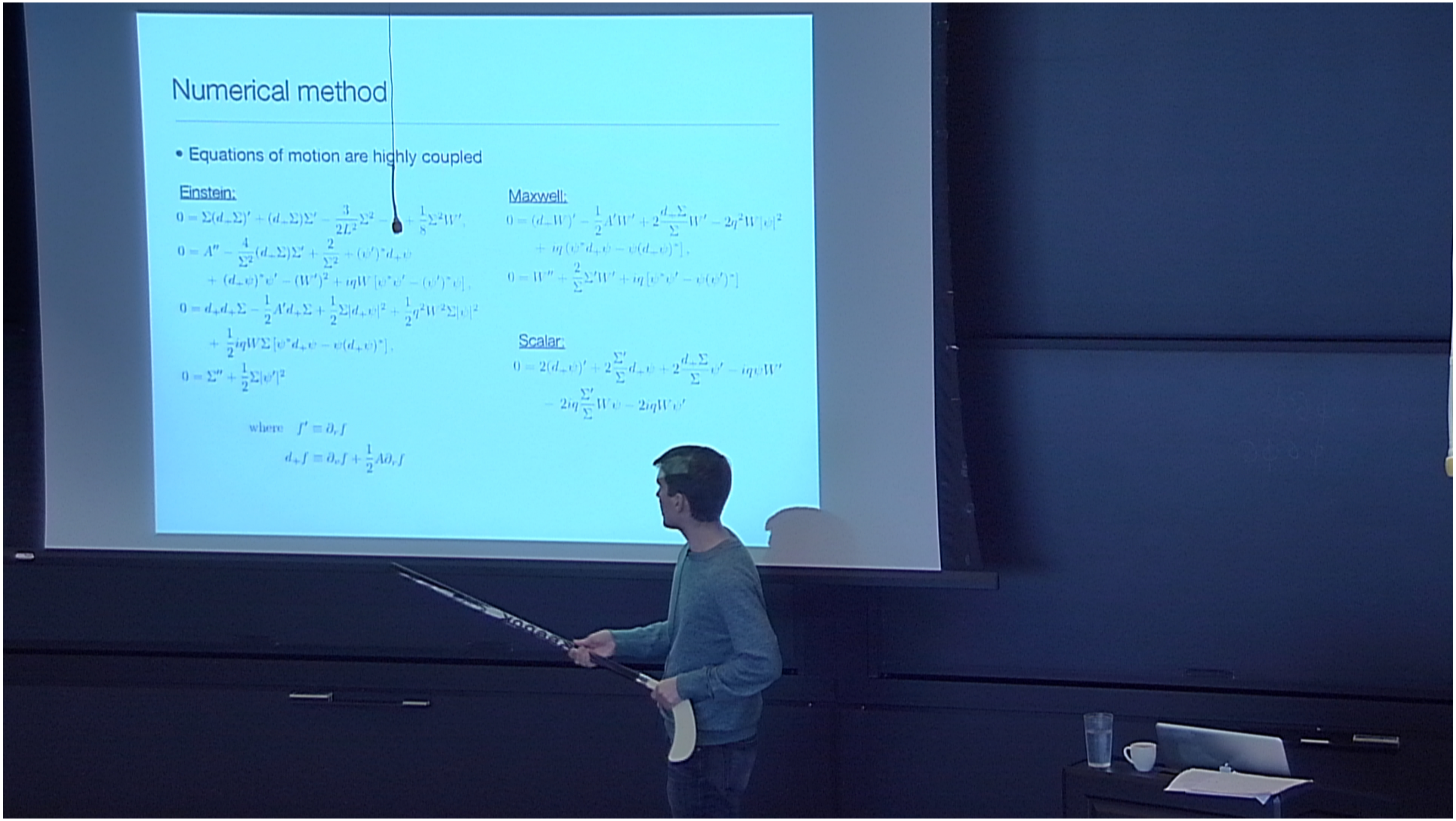
Numerical method

- We work in Eddington-Finkelstein coordinates and spherical symmetry. Metric and Maxwell fields can be put in the form

$$ds^2 = -A(v,r)dv^2 + 2dvdr + \Sigma(v,r)^2 d\Omega_2^2$$

$$A_\mu dx^\mu = W(v,r)dv$$





Numerical method

- Equations of motion are highly coupled

Einstein:

$$0 = \Sigma(d_+\Sigma)' + (d_+\Sigma)\Sigma' - \frac{3}{2L^2}\Sigma^2 - \text{[illegible]} + \frac{1}{8}\Sigma^2 W'^2,$$

$$0 = A'' - \frac{4}{\Sigma^2}(d_+\Sigma)\Sigma' + \frac{2}{\Sigma^2} + (\psi')^* d_+\psi$$
$$+ (d_+\psi)^*\psi' - (W')^2 + iqW\left[\psi^*\psi' - (\psi')^*\psi\right],$$

$$0 = d_+d_+\Sigma - \frac{1}{2}A'd_+\Sigma + \frac{1}{2}\Sigma|d_+\psi|^2 + \frac{1}{2}q^2W^2\Sigma|\psi|^2$$
$$+ \frac{1}{2}iqW\Sigma\left[\psi^* d_+\psi - \psi(d_+\psi)^*\right],$$

$$0 = \Sigma'' + \frac{1}{2}\Sigma|\psi'|^2$$

$$\text{where} \quad f' = \partial_r f$$
$$d_+ f = \partial_v f + \frac{1}{2}A\partial_r f$$

Maxwell:

$$0 = (d_+W)' - \frac{1}{2}A'W' + 2\frac{d_+\Sigma}{\Sigma}W' - 2q^2W|\psi|^2$$
$$+ iq\,(\psi^* d_+\psi - \psi(d_+\psi)^*],$$

$$0 = W'' + \frac{2}{\Sigma}\Sigma'W' + iq\left[\psi^*\psi' - \psi(\psi')^*\right]$$

Scalar:

$$0 = 2(d_+\psi)' + 2\frac{\Sigma'}{\Sigma}d_+\psi + 2\frac{d_+\Sigma}{\Sigma}\psi' - iq\psi W'$$
$$- 2iq\frac{\Sigma'}{\Sigma}W\psi - 2iqW\psi'$$

Numerical method

- Integration procedure

$$\text{initial data: } \psi(v = v_0, r)$$

Integrate equations, radially inward in r
Impose M and Q as boundary conditions

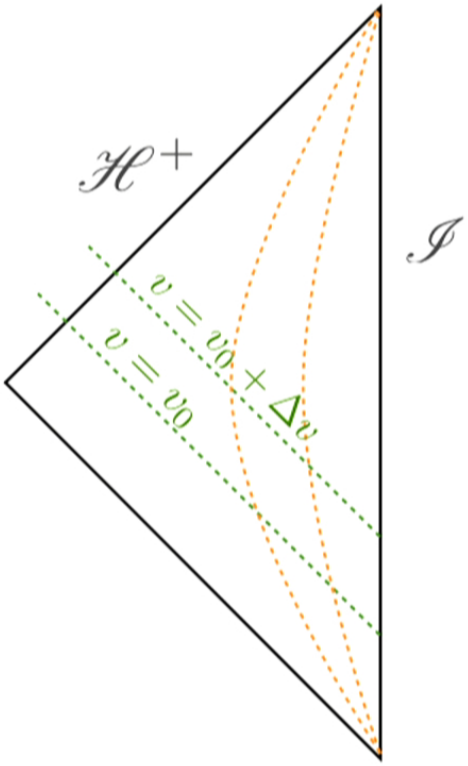
$$d_+\Sigma, A, \Sigma, W, \text{and } d_+\psi \text{ at } v = v_0$$

$$\partial_v \psi = d_+\psi - \frac{1}{2} A \partial_r \psi$$

$$\partial_v \psi(v = v_0, r)$$

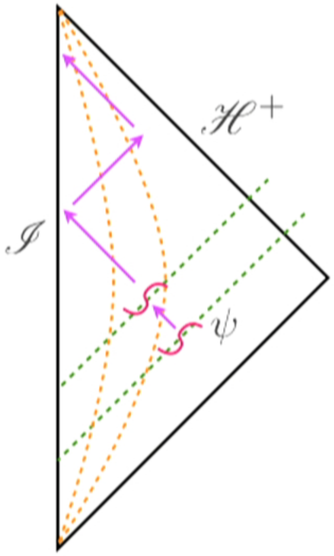
Integrate one step in v

$$\psi(v = v_0 + \Delta v, r)$$

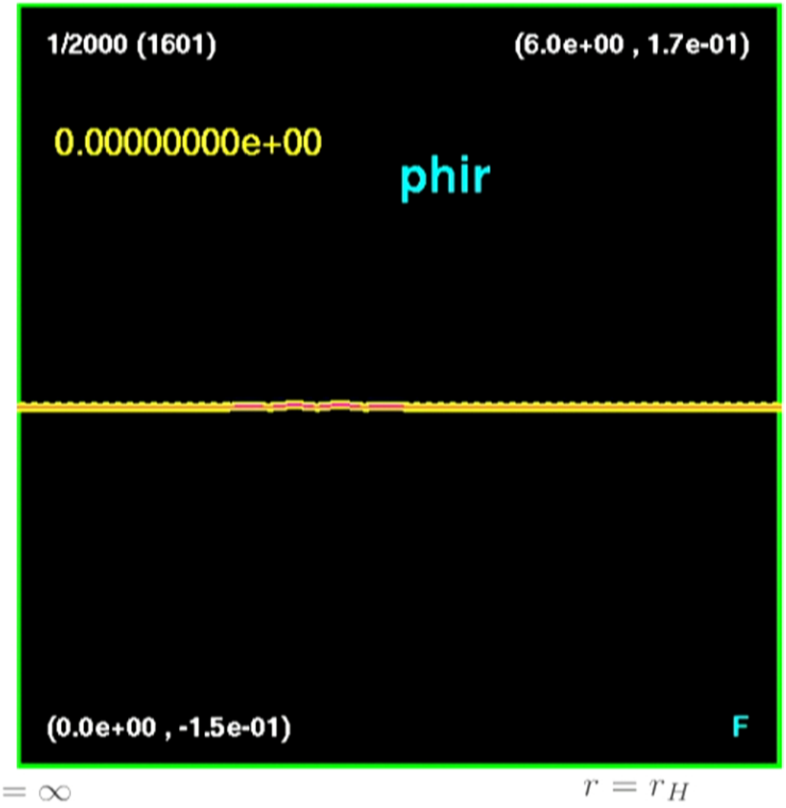


Sample evolution

- We consider small black holes in AdS, so $r_H \ll L$
- Compactly supported initial data for ψ, small amplitude.



rH = 0.2
L = 1
Q/Qmax = 0.8
q = 12

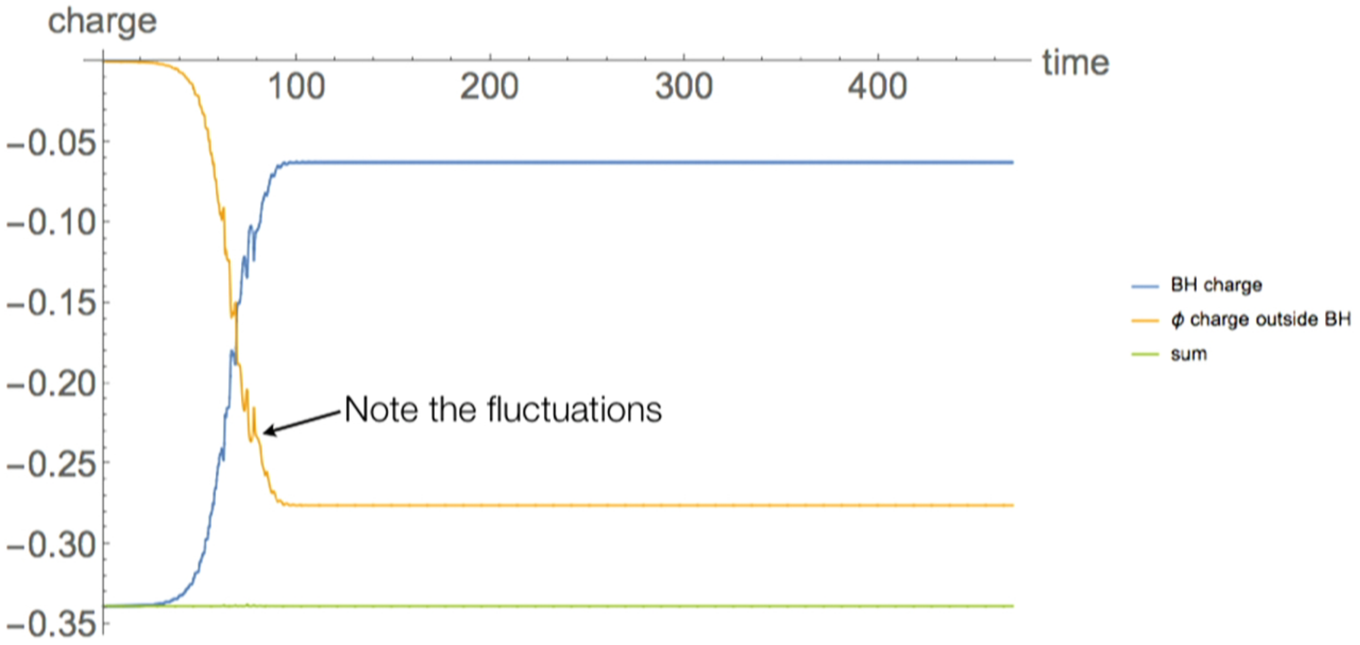


Apparent horizon area vs time

- Area always increases

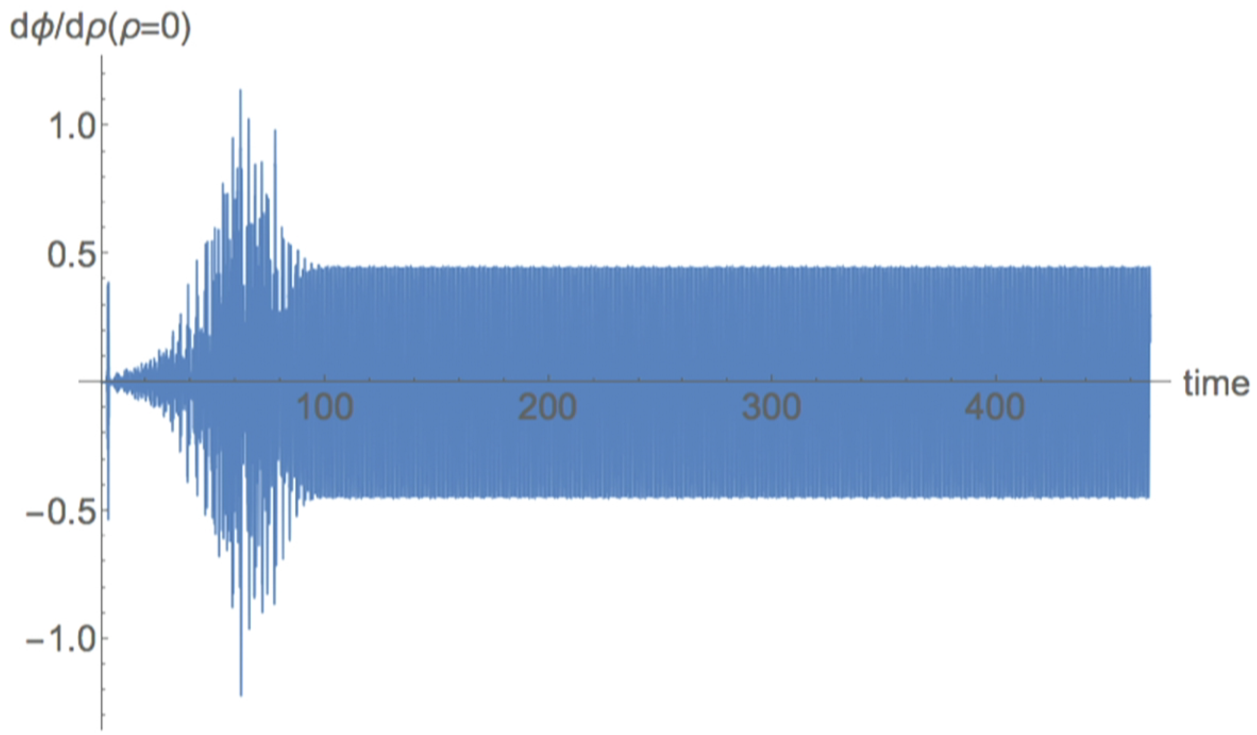
Charge vs time

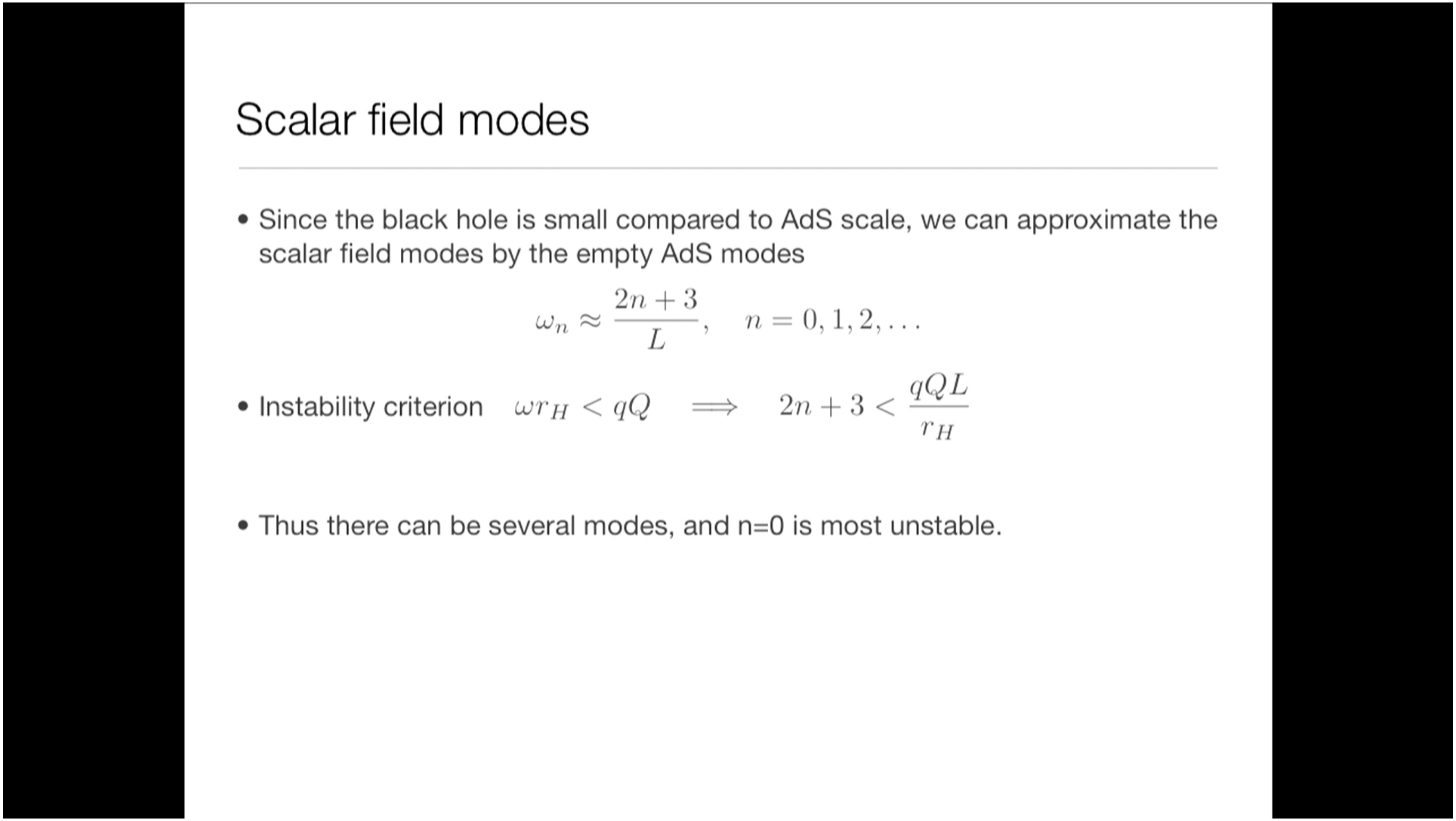
- Most of the charge is extracted by the scalar field



Note the fluctuations

Boundary field $\varphi_3(v)$





Scalar field modes

- Since the black hole is small compared to AdS scale, we can approximate the scalar field modes by the empty AdS modes

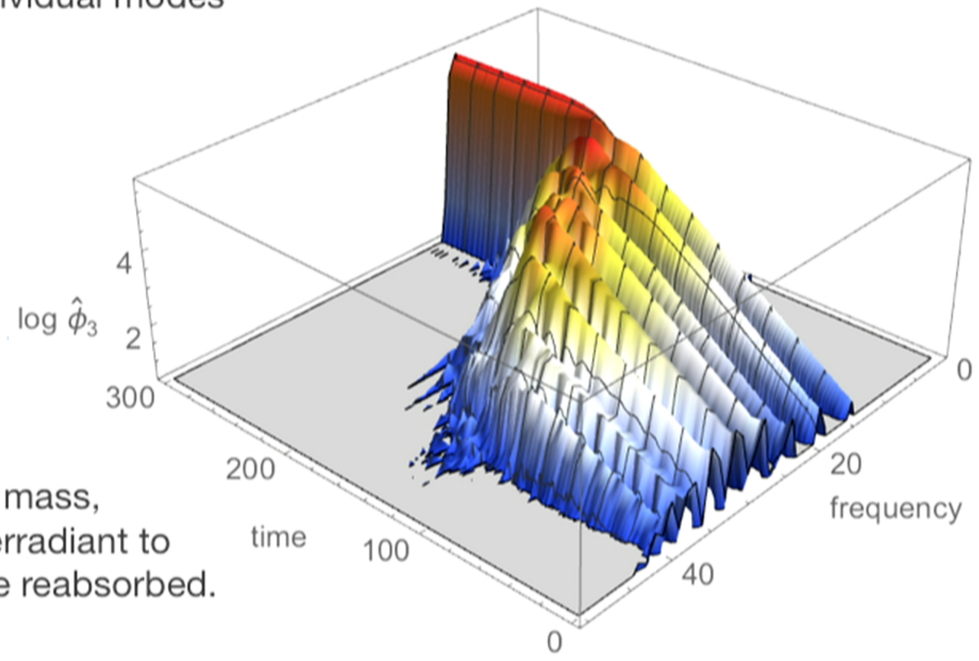
$$\omega_n \approx \frac{2n+3}{L}, \quad n = 0, 1, 2, \dots$$

- Instability criterion $\omega r_H < qQ \quad \Longrightarrow \quad 2n+3 < \frac{qQL}{r_H}$

- Thus there can be several modes, and n=0 is most unstable.

Scalar field modes

- Spectrogram reveals individual modes



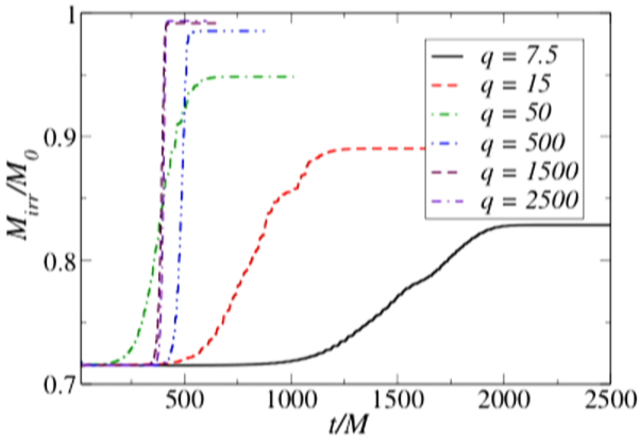
- As BH loses charge and mass, modes switch from superradiant to nonsuperradiant, and are reabsorbed.

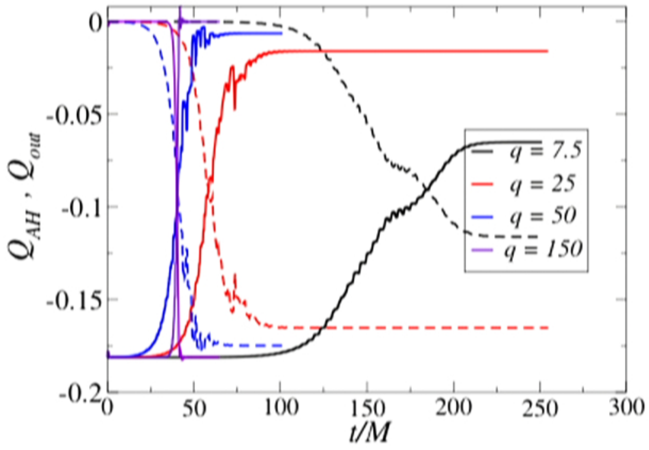
- Final state is BH + lowest mode, with zero growth rate.

What happens when q is increased?

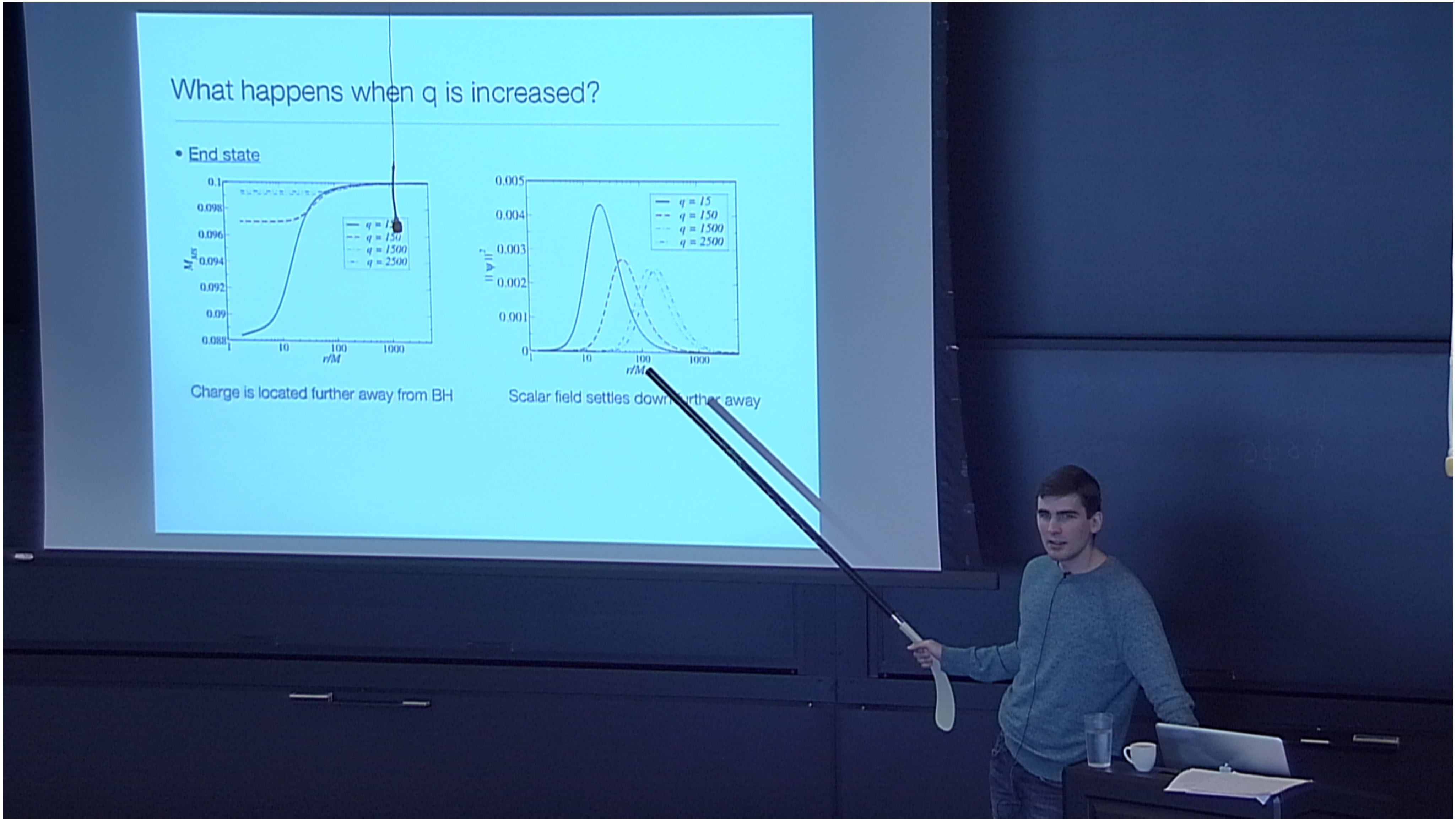
- Larger q excites more modes -> faster process.

Recall $2n+3 < \frac{qQL}{r_H}$





- Larger q extracts more charge



What happens when q is increased?

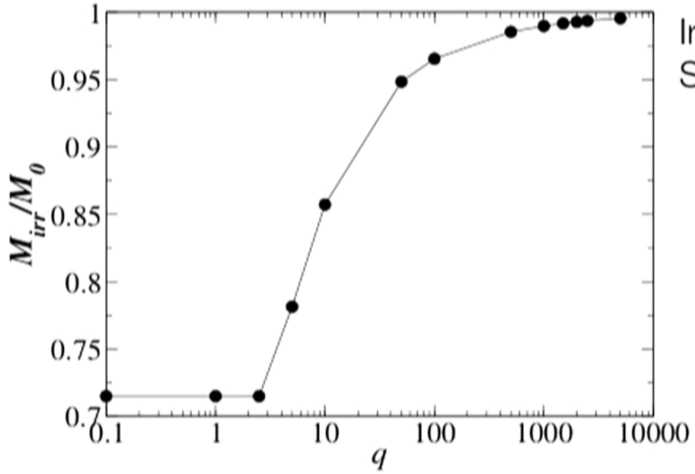
- End state

Charge is located further away from BH

Scalar field settles down further away

What happens when q is increased?

- End state

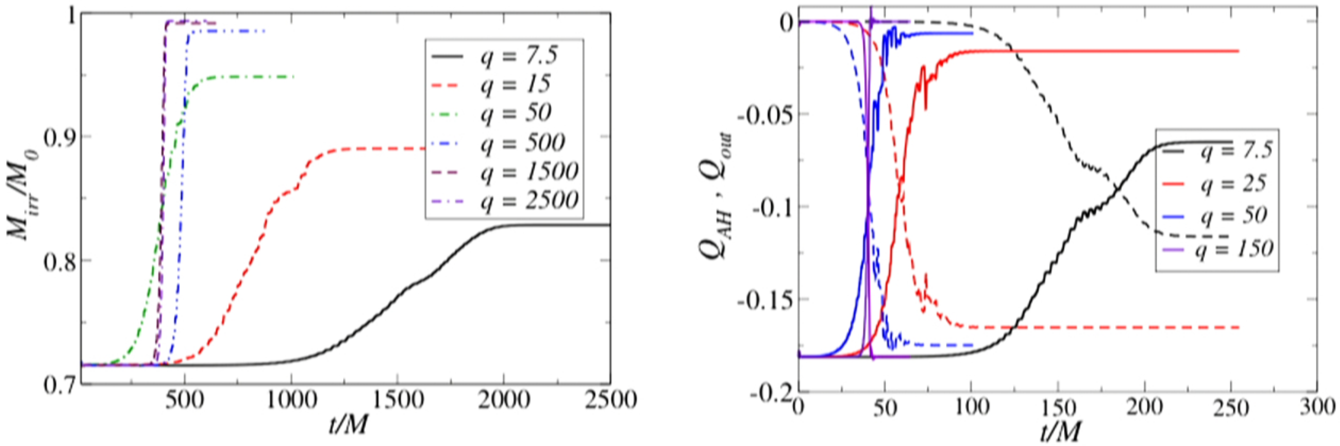


Irreducible mass (area) approaches that of a Schwarzschild-AdS BH of mass M

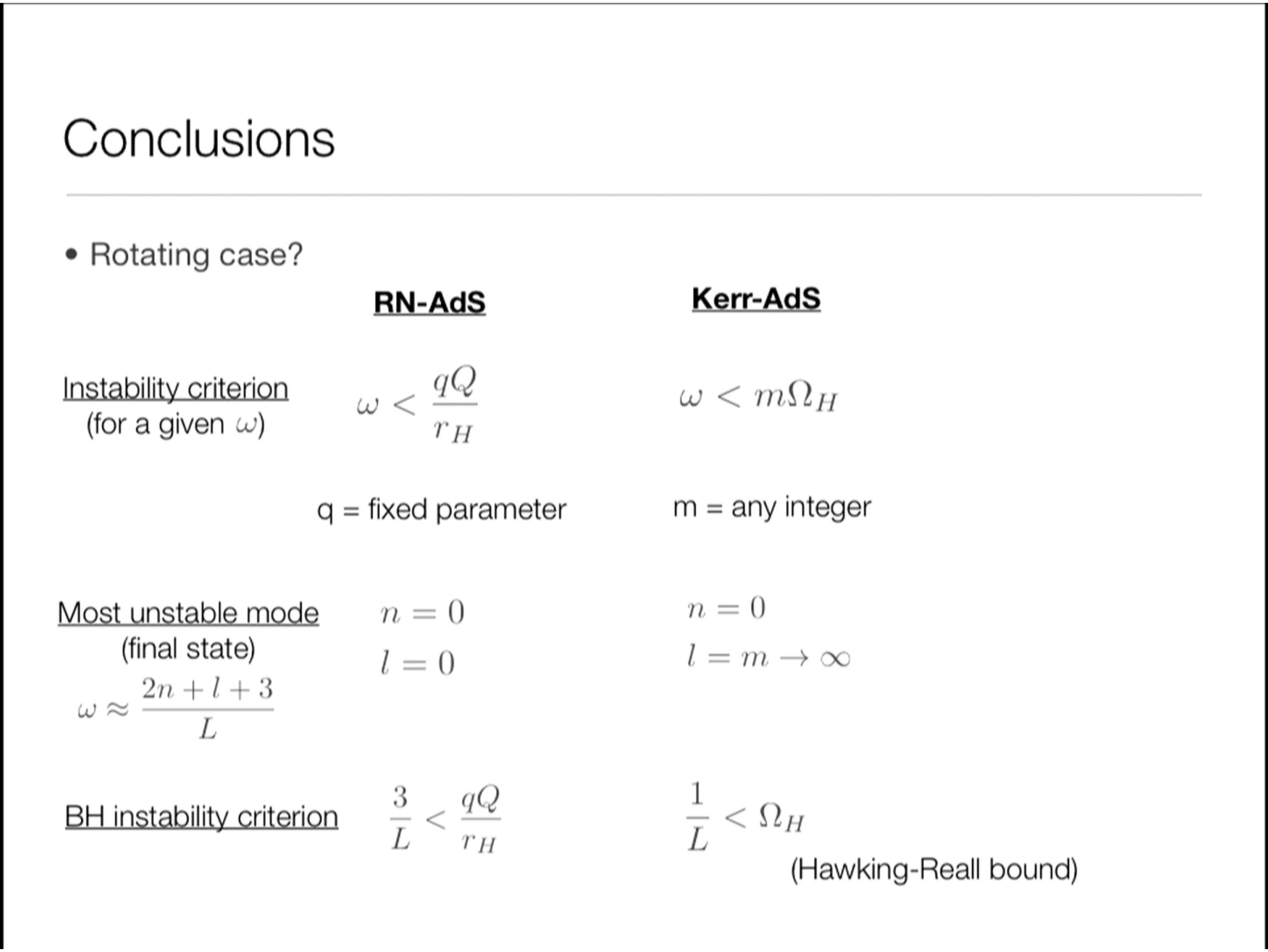
- *For larger q, the scalar field charge/mass ratio is increased. As the scalar field extracts nearly the full ADM charge Q, it extracts very little mass. Final state approaches Schwarzschild-AdS, surrounded by a distant low-mass/high-charge condensate.*

What happens when q is increased?

- Larger q excites more modes -> faster process. Recall $2n + 3 < \frac{qQL}{r_H}$



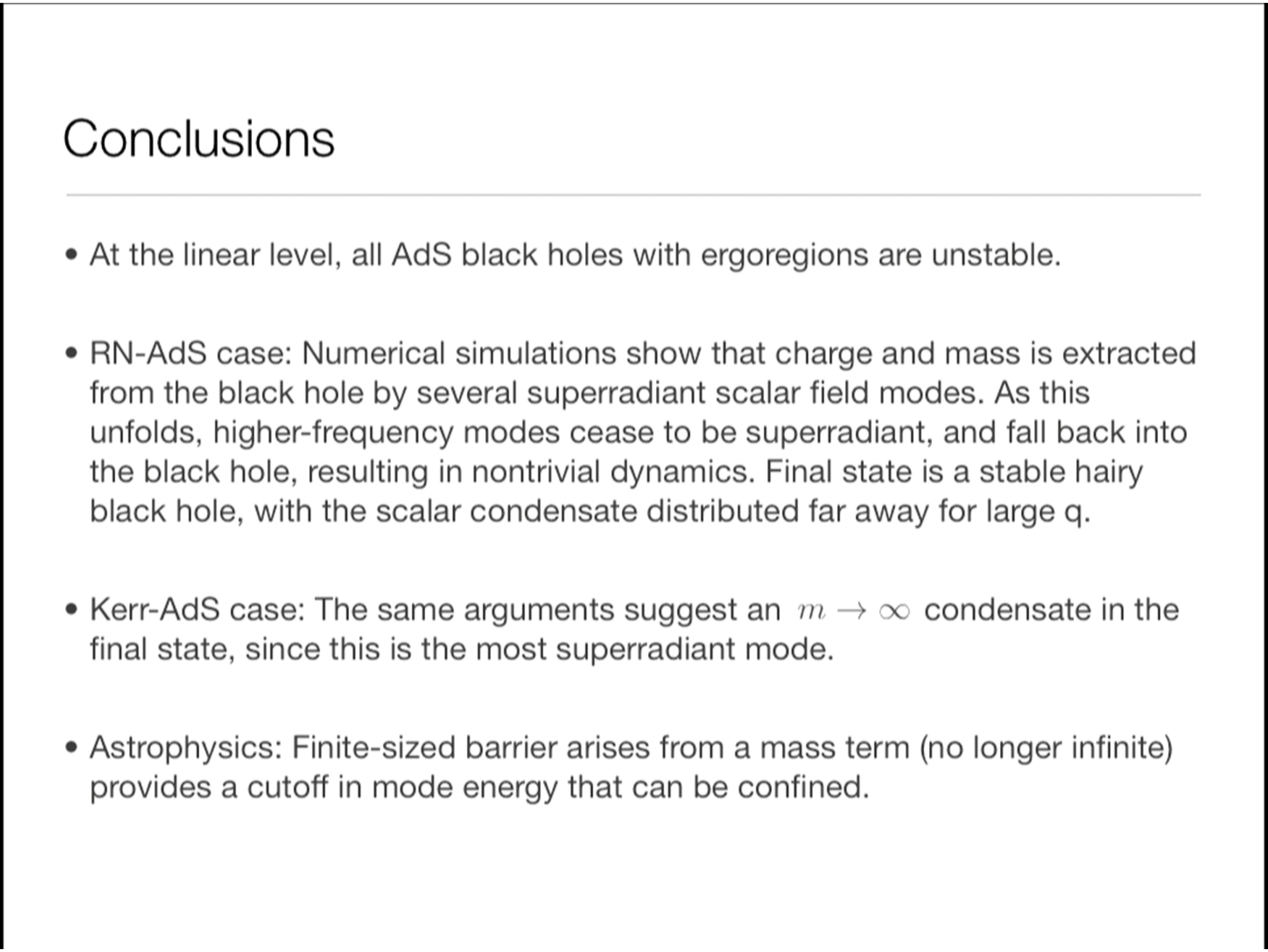
- Larger q extracts more charge



Conclusions

- Rotating case?

	RN-AdS	**Kerr-AdS**
Instability criterion (for a given ω)	$\omega < \frac{qQ}{r_H}$	$\omega < m\Omega_H$
	q = fixed parameter	m = any integer
Most unstable mode (final state) $\omega \approx \frac{2n+l+3}{L}$	$n = 0$ $l = 0$	$n = 0$ $l = m \to \infty$
BH instability criterion	$\frac{3}{L} < \frac{qQ}{r_H}$	$\frac{1}{L} < \Omega_H$ (Hawking-Reall bound)



Conclusions

- At the linear level, all AdS black holes with ergoregions are unstable.

- RN-AdS case: Numerical simulations show that charge and mass is extracted from the black hole by several superradiant scalar field modes. As this unfolds, higher-frequency modes cease to be superradiant, and fall back into the black hole, resulting in nontrivial dynamics. Final state is a stable hairy black hole, with the scalar condensate distributed far away for large q.

- Kerr-AdS case: The same arguments suggest an $m \to \infty$ condensate in the final state, since this is the most superradiant mode.

- Astrophysics: Finite-sized barrier arises from a mass term (no longer infinite) provides a cutoff in mode energy that can be confined.