Title: Topological phase transitions in line-nodal superconductors

Date: Feb 03, 2016 03:30 PM

URL: http://pirsa.org/16020101

Abstract: <p>Fathoming interplay between symmetry and topology of many-electron wave-functions has deepened understanding of quantum many body systems, especially after the discovery of topological insulators. Topology of electron wave-functions enforces and protects emergent gapless excitations, and symmetry is intrinsically tied to the topological protection in a certain class. Namely, unless the symmetry is broken, the topological nature is intact. We show novel interplay phenomena between symmetry and topology in topological phase transitions associated with line-nodal superconductors. The interplay may induce an exotic universality class in sharp contrast to that of the phenomenological Landau-Ginzburg theory. Hyper-scaling violation and emergent relativistic scaling are main characteristics, and the interplay even induces unusually large quantum critical region. We propose characteristic experimental signatures around the phase transitions in three spatial dimensions, for example, a linear phase boundary in a temperature-tuning parameter phase-diagram. </p>

Topological Phase Transitions in 3d Line-nodal Superconductors

Eun-Gook Moon

KAIST

Perimeter Institute, Feb. 3, 2016


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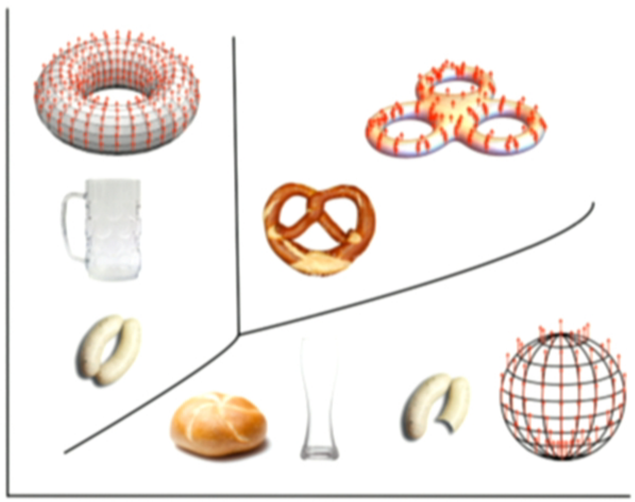
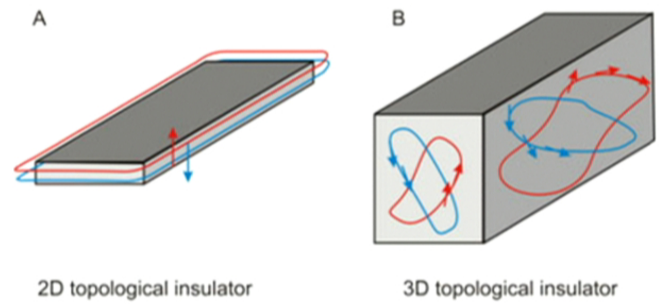
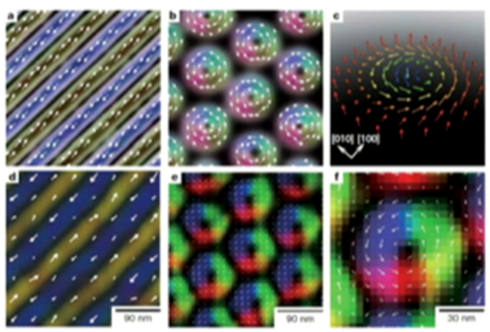
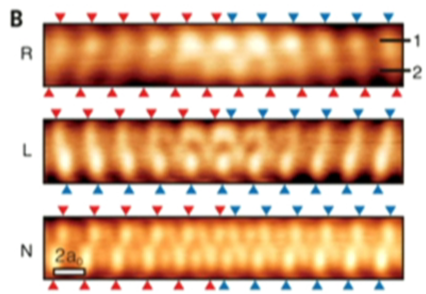
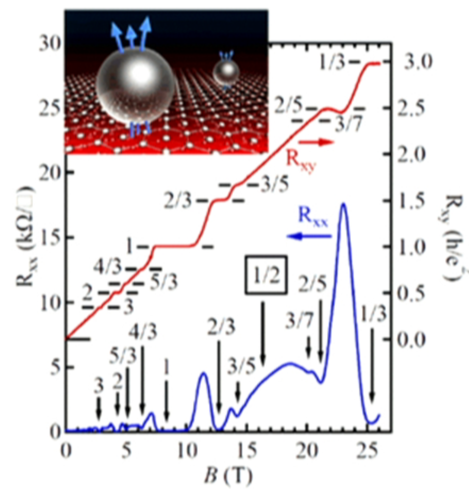
Topology, here and there





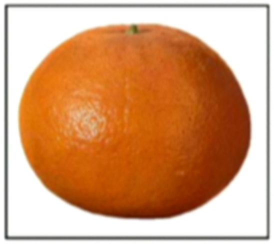






From google images with the key words "Topological matters"

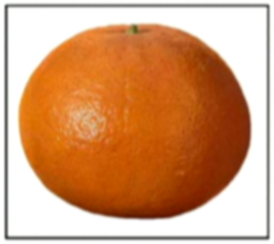
Topology

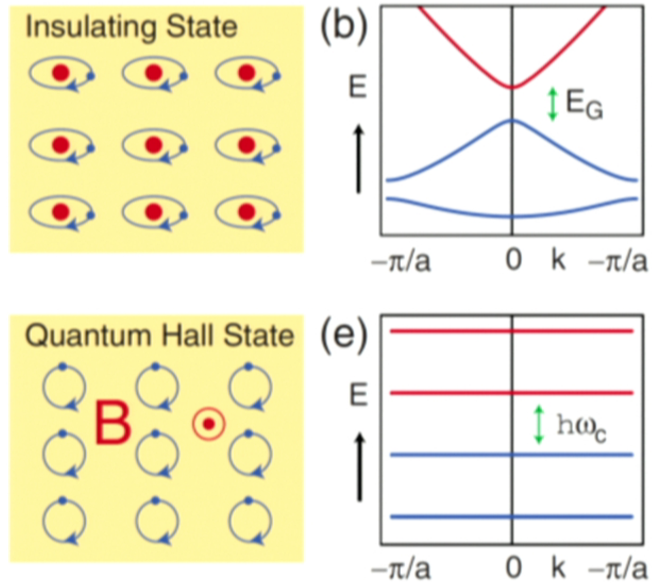


Two objects are topologically different.

: Continuous deformation cannot transform one to the other.

Topology in condensed matter systems





Hasan and Kane, *Rev. Mod. Phys. 2010*

Electron wave-function : complex number
Topological properties of electron wave-function (ex: Chern number)
Electron band structure can have different topology
Different topology ~ Different phase

Topology in condensed matter systems

Topological nature in insulators and gapped SC
: Well-understood!

Topology in condensed matter systems

Topological nature in semi-metals and gapless SC
: currently active!

PHYSICAL REVIEW X **5**, 031013 (2015)

Experimental Discovery of Weyl Semimetal TaAs

B. Q. Lv,[1] H. M. Weng,[1,2] B. B. Fu,[1] X. P. Wang,[2,3,1] H. Miao,[1] J. Ma,[1] P. Richard,[1,2] X. C. Huang,[1]
L. X. Zhao,[1] G. F. Chen,[1,2] Z. Fang,[1,2] X. Dai,[1,2] T. Qian,[1,*] and H. Ding[1,2,†]
[1]*Beijing National Laboratory for Condensed Matter Physics and Institute of Physics,*
Chinese Academy of Sciences, Beijing 100190, China
[2]*Collaborative Innovation Center of Quantum Matter, Beijing, China*
[3]*Department of Physics, Tsinghua University, Beijing 100084, China*
(Received 15 July 2015; published 31 July 2015)

LETTERS
PUBLISHED ONLINE: 17 AUGUST 2015 | DOI: 10.1038/NPHYS3426
nature physics

Observation of Weyl nodes in TaAs

B. Q. Lv[1,2†], N. Xu[2,3†], H. M. Weng[1,4†], J. Z. Ma[1,2], P. Richard[1,4], X. C. Huang[1], L. X. Zhao[1], G. F. Chen[1,4],
C. E. Matt[2], F. Bisti[2], V. N. Strocov[2], J. Mesot[2,3,5], Z. Fang[1,4], X. Dai[1,4], T. Qian[1*], M. Shi[2*] and H. Ding[1,4*]

2D MATERIALS

Observation of tunable band gap and anisotropic Dirac semimetal state in black phosphorus

Jimin Kim,[1] Seung Su Baik,[2,3] Sae Hee Ryu,[1,4] Yeongsup Sohn,[1,4] Soohyung Park,[2]
Byeong-Gyu Park,[5] Jonathan Denlinger,[6] Yeonjin Yi,[2]
Hyoung Joon Choi,[2,3] Keun Su Kim[1,4*]

Characterization of Gapless Excitation

Input: DVI - 1280x720p@60Hz
Output: SDI - 1920x1080i@60Hz

Codimension analysis

Spatial dimension : d

Codimension :
p = d − dim. of zero-energy manifold

(d, p) = (2,2) : nodal point in 2d (ex : graphene, d-wave SC)

(d, p) = (2,1) : Fermi surface in 2d (2d metal)

Characterization of Gapless Excitation

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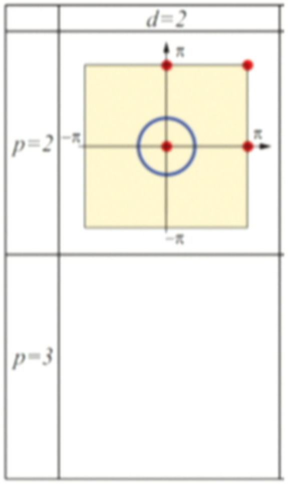


Chiu and Schnyder, *PRB 2014*

(d, p) = (2,2) : nodal point in 2d

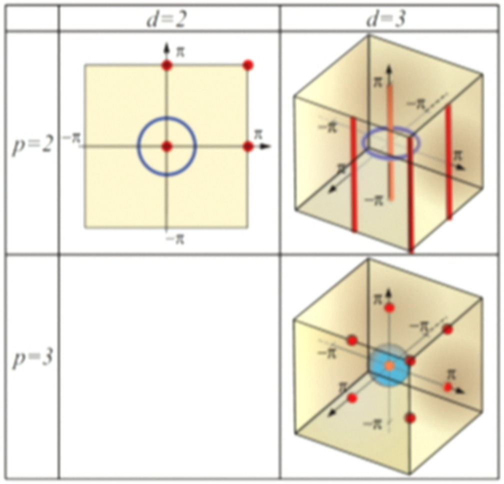
(d, p) = (2,1) : Fermi surface in 2d

(d, p) = (3,3) : nodal point in 3d

(d, p) = (3,2) : nodal line in 3d

(d, p) = (3,1) : Fermi surface in 3d

Characterization of Gapless Excitation



Chiu and Schnyder, *PRB 2014*

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(d, p) = (3,2) : nodal line in 3d

(d, p) = (3,1) : Fermi surface in 3d

Metals : p = 1 in every dimensions (not our focus)

Characterization of Topological Nature

Around a nodal point (line),
the winding number is well defined in terms of the (non-interacting) fermionic Hamiltonian.

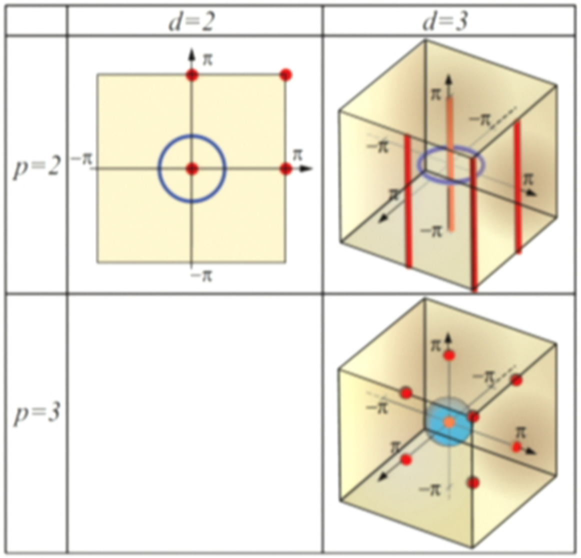
Topologically stable!

(If not stable, then nodal excitation is gapped)

Ex : (d,p) =(3,3) (ex : Weyl SM)

$$\mathcal{H}_e = d_\mu(k)\tau^\mu \quad , \quad \hat{d}_\mu = d_\mu/|d|$$

$$B_\mu(k) = \frac{1}{8\pi}\epsilon_{\mu\nu\lambda}\hat{\mathbf{d}}\cdot\partial_\nu\hat{\mathbf{d}}\times\partial_\lambda\hat{\mathbf{d}}$$



Chiu and Schnyder, *PRB 2014*

$$\partial_\mu \mathcal{B}^\mu(\vec{p}) = \delta(\vec{p})$$

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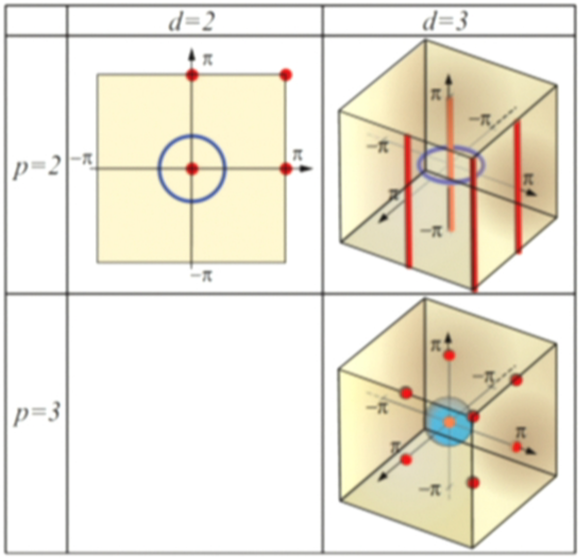


Chiu and Schnyder, *PRB 2014*

$$\partial_\mu\mathcal{B}^\mu(\vec{p}) = \delta(\vec{p})$$

Goals

1. Topology with nodal excitation in CMs

2. Topological Phase Transitions

3. TPT in 3d line-nodal SC

Goals

1. Topology with nodal excitation in CMs

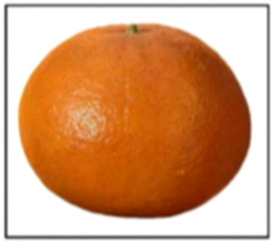
2. Topological Phase Transitions

3. TPT in 3d line-nodal SC

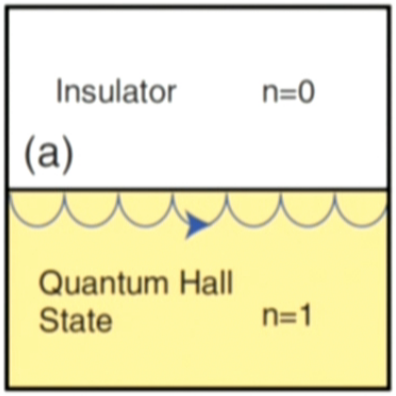
Topology....

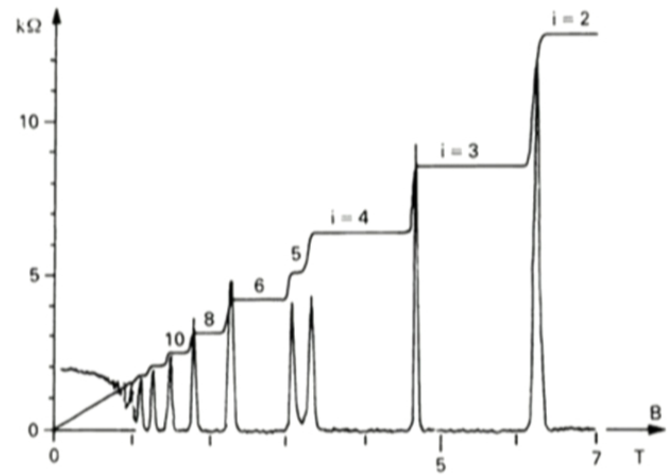
Great!

How do we observe topological signals?



Topology change in boundary





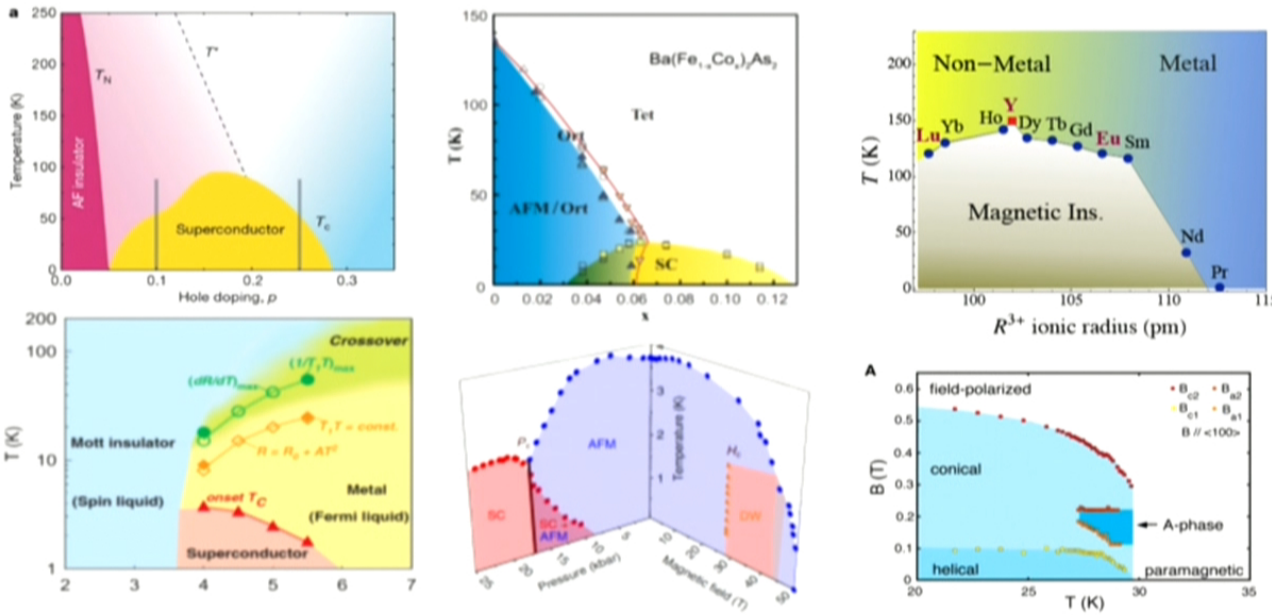
Physical boundary between topologically different states!
: proximity effects
Non-trivial boundary states (usually massless states)

Ex: Quantum Hall effects!

Useful in weakly correlated systems.

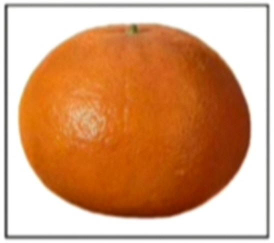
Topology change in parameter space

We have lots of tuning parameters.
(ex: doping, pressure, magnetic field, etc.)



(Especially) in strongly correlated systems.

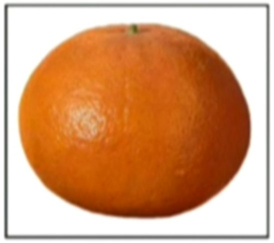
Topological phase transition



Trivial insulator

Topological insulator

Topological phase transition



Trivial insulator

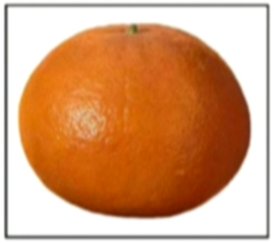
Topological
QPT

Topological insulator

Topological phase transition
: phase transition between topologically different phases

Interesting physics around topological phase transitions!
(additional massless excitation is enforced at QCP)

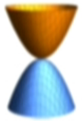
Topological phase transition



Insulator

Topological
QPT

Dirac semimetal

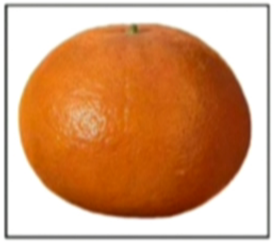


QBT.

→

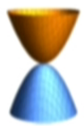
$$V_c(q) \sim \frac{e^2}{|q|}$$

Topological phase transition



Insulator

Dirac semimetal



QBT.

$Pr_2Ir_2O_7$ →

EGM, Xu, Kim, Balents, *PRL 2013*
Savory, EGM, Balents, PRX 2014
Kondo, et. al., Nat. Comm. 2015

$$V_c(q) \sim \frac{e^2}{|q|}$$

PRL **111,** 206401 (2013) PHYSICAL REVIEW LETTERS week ending 15 NOVEMBER 2013

Non-Fermi-Liquid and Topological States with Strong Spin-Orbit Coupling

Eun-Gook Moon,[1] Cenke Xu,[1] Yong Baek Kim,[2] and Leon Balents[3]

PHYSICAL REVIEW X **4,** 041027 (2014)

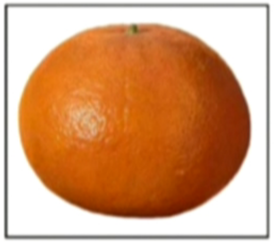
New Type of Quantum Criticality in the Pyrochlore Iridates

Lucile Savary,[1,*] Eun-Gook Moon,[1] and Leon Balents[2]

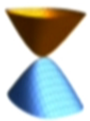
Quadratic Fermi Node in a 3D Strongly Correlated Semimetal

Takeshi Kondo,[1] M. Nakayama,[1] R. Chen,[2,3,4] J.J. Ishikawa,[1] E.-G. Moon,[2,5] T. Yamamoto,[1] Y. Ota,[1] W. Malaeb,[1,6] H. Kanai,[1] Y. Nakashima,[1] Y. Ishida,[1] R. Yoshida,[1] H. Yamamoto,[1] M. Matsunami,[7,8] S. Kimura,[7,9] N. Inami,[10] K. Ono,[10] H. Kumigashira,[10] S. Nakatsuji,[1,11] L. Balents,[12] and S. Shin[1]

Topological phase transition



Topological
QPT



Anisotropic DOS

BiTeI ?, TiO2-Vo2?

Yang, EGM, Isobe, Nagaosa, *Nat. Phys. 2014*
Cho, EGM, *Sci. Rep. 2016*

$$V_C(\mathbf{q}) \sim \frac{1}{q_\perp^{3/2} + \eta q_3^2}$$

Linear SM (Weyl, Dirac)

Many, Prof. Dai's talk

EGM, J. Lee and Y.B. Kim *in preparation*

$$V_c(q) \sim \frac{e^2}{\epsilon q^2}$$

ARTICLES
PUBLISHED ONLINE: 24 AUGUST 2014 | DOI: 10.1038/NPHYS3060
nature physics

Quantum criticality of topological phase transitions in three-dimensional interacting electronic systems

Bohm-Jung Yang[1*], Eun-Gook Moon[2], Hiroki Isobe[3] and Naoto Nagaosa[1,3*]

SCIENTIFIC REPORTS

OPEN

Novel Quantum Criticality in Two Dimensional Topological Phase transitions

Received: 02 September 2015

Gil Young Cho & Eun-Gook Moon

Goals

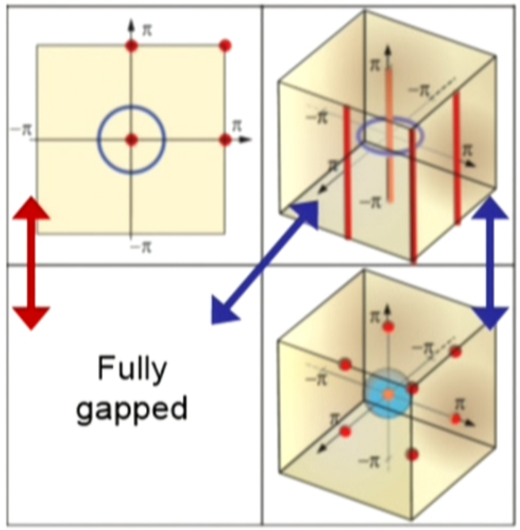
1. Topology with nodal excitation in CMs

2. Topological Phase Transitions

3. TPT in 3d line-nodal SC

Strategy

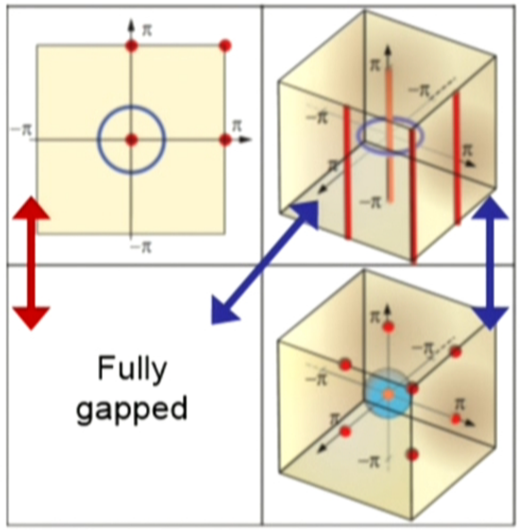
We investigate topological phase transitions.



We first consider SCs since nodal structure is more stable in SCs.

Strategy

We investigate topological phase transitions.

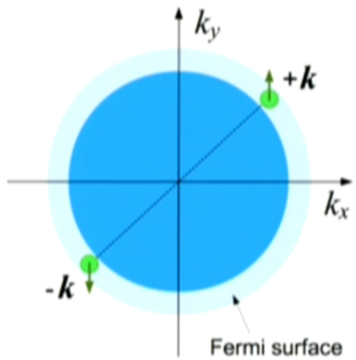


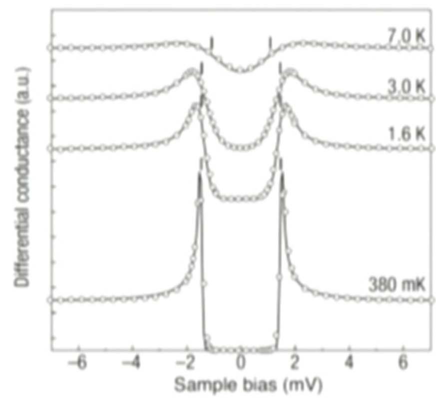
We first consider SCs since nodal structure is more stable in SCs.

In normal phases, chemical potential is (potentially) dangerous in TQP, but the same strategy applies.

Conventional Superconductors

Fermionic excitations : fully gapped (original BCS theory)





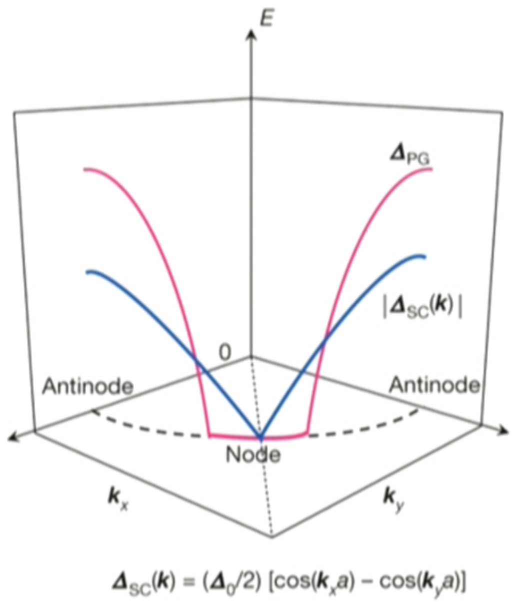
E. Hudson, *Nat. Phys. 2008*

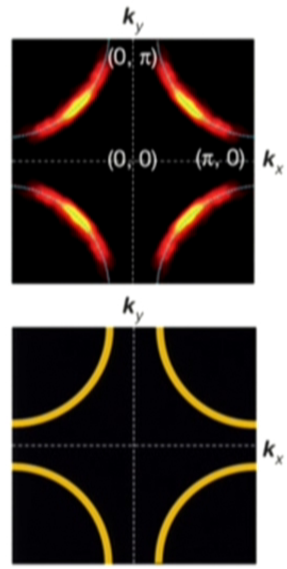
Low energy physics : Ginzburg-Landau theory of Cooper pair

$$F = F_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}\left|\left(-i\hbar\nabla - 2e\mathbf{A}\right)\psi\right|^2 + \frac{|\mathbf{B}|^2}{2\mu_0}$$

Unconventional Superconductors

Fermionic excitations : partially gapped





d-wave SC

B. Keimer et. al., *Nature. 2015*

Low energy physics : order parameter + nodal fermionic excitation

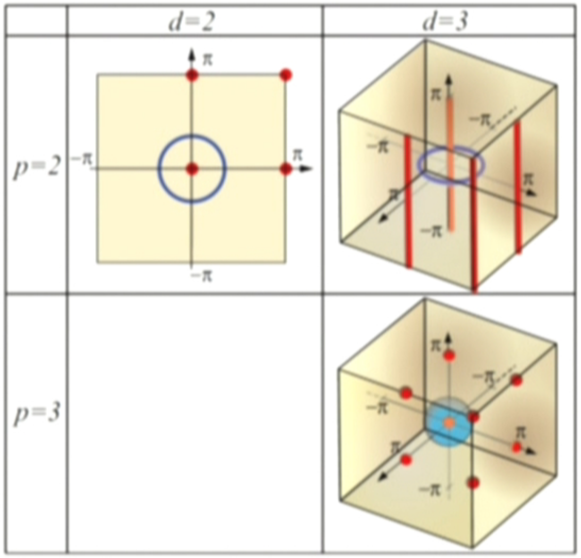
Characterization of Gapless Excitation

Spatial dimension : d

Codimension :
p = d − dim. of zero-energy manifold

(d, p) = (2,2) : nodal point in 2d
(d, p) = (3,3) : nodal point in 3d

(d, p) = (3,2) : nodal line in 3d



Chiu and Schnyder, *PRB 2014*

Line-nodal Superconductors

PRL 94, 197002 (2005) PHYSICAL REVIEW LETTERS week ending 20 MAY 2005

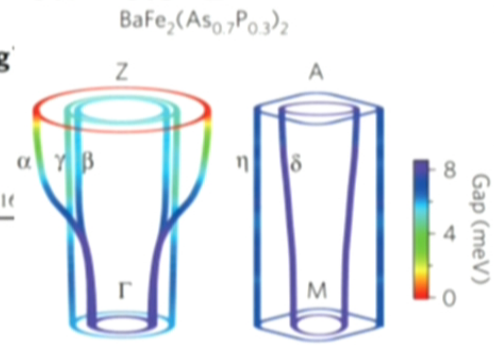
Line Nodes in the Superconducting Gap Function of Noncentrosymmetric $CePt_3Si$

K. Izawa,[1] Y. Kasahara,[1] Y. Matsuda,[1,2] K. Behnia,[1,3] T. Yasuda,[4] R. Settai,[4] and Y. Onuki[4]

nature physics LETTERS
PUBLISHED ONLINE: 4 MARCH 2012 | DOI: 10.1038/NPHYS2248

Nodal superconducting-gap structure in ferropnictide superconductor $BaFe_2(As_{0.7}P_{0.3})_2$

Y. Zhang, Z. R. Ye, Q. Q. Ge, F. Chen, Juan Jiang, M. Xu, B. P. Xie and D. L. Feng



PRL 115, 165304 (2015) PHYSICAL REVIEW LETTERS

Polar Phase of Superfluid ^3He in Anisotropic Aerogel

V. V. Dmitriev,[1,*] A. A. Senin,[1] A. A. Soldatov,[1,2] and A. N. Yudin[1]
[1]*P.L. Kapitza Institute for Physical Problems of RAS, 119334 Moscow, Russia*
[2]*Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia*
(Received 10 July 2015; published 16 October 2015)

Unconventional Superconductors

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **18** (2006) R705–R752 doi:10.1088/0953-8984/18/44/R01

TOPICAL REVIEW

Nodal structure of unconventional superconductors probed by angle resolved thermal transport measurements

Y Matsuda[1,2], K Izawa[2,3] and I Vekhter[4]

excitations. The temperature dependence of the London penetration depth $\lambda(T)$, electronic part of the specific heat $C(T)$, thermal conductivity $\kappa(T)$, and nuclear magnetic resonance (NMR) spin–lattice relaxation rate T_1^{-1} all reflect the changes in the quasiparticle occupation numbers. In the fully gapped (s wave) superconductors the quasiparticle density of states

Unconventional Superconductors

Table 1. Superconducting gap symmetry of unconventional superconductors. TRS, AFMO and FMO represent time reversal symmetry, antiferromagnetic ordering and ferromagnetic ordering, respectively.

	Node	Parity	TRS	Proposed gap function	Comments
$Sr_2Ca_{12}Cu_{24}O_{41}$	Full gap [25]	Odd [25]			Spin ladder system
κ-$(ET)_2Cu(SCN)_2$	Line (vertical) [66]	Even [148]		d_{xy} [66]	
$(TMTSF)_2PF_6$		Odd [23]			Superconductivity under pressure
$(TMTSF)_2ClO_4$	Line [173] Full gap [181]				
Sr_2RuO_4	Line (horizontal) [65] (vertical) [73]	Odd [24]	Broken [32]	$(k_x + ik_y) \times (\cos k_z c + \alpha)$ [65] $(\sin k_x + i \sin k_y)$ [73]	
$Na_xCoO_2 \cdot yH_2O$	Line [174]	Even [175, 176], Odd [26, 27]			
$(Y, Lu)Ni_2B_2C$	Point-like [67]	Even [88]		$1 - \sin^4\theta \cos(4\phi)$ [87, 177]	Very anisotropic s wave
Li_2Pt_3B	Line [35]	Even + odd			No inversion centre
$CeCu_2Si_2$	Line [178]	Even [178]			Two superconducting phases [183]
$CeIn_3$	Line [179]				Coexistence with AFMO
$CeCoIn_5$	Line (vertical)	Even [61]		$d_{x^2-y^2}$ [53, 61, 136], d_{xy} [69]	FFLO phase
$CeRhIn_5$	Line [180]	Even [180]			Coexistence with AFMO
$CePt_3Si$	Line [34]	Even + odd [182]			No inversion centre
UPd_2Al_3	Line (horizontal)	Even [109]		$\cos k_z c$ [62]	Coexistence with AFMO
UNi_2Al_3	Line [19]	Odd [19]			Coexistence with SDW
URu_2Si_2	Line [184]	Odd [20]			Coexistence with hidden order
UPt_3	Line + point [185]	Odd [18]	Broken [31]		Multiple superconducting phases
UBe_{13}	Line [189]	Odd [22]			
UGe_2	Line [186]	Odd [28]			Coexistence with FMO
URhGe		Odd [29]			Coexistence with FMO
UIr		Even + odd [30]			Coexistence with FMO and no inversion centre
$PuCoGa_5$	Line [187]	Even [125]			
$PuRhGa_5$	Line [188]				
$PrOs_4Sb_{12}$	Point [68]	Odd [22]	Broken [33]		Multiple superconducting phases

Matsuda et. al., *2006*

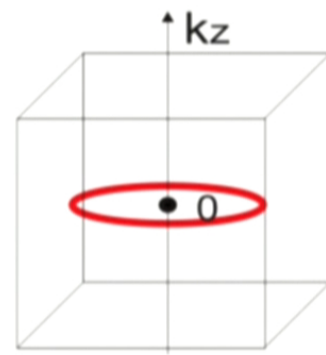
Line-nodal Superconductors

Toy model : p-wave pairing gap (~ polar phase in superfluid)

$$H_0 = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \Big(h(\boldsymbol{k})\tau^z + \Delta(\boldsymbol{k})\tau^x \Big) \Psi_{\boldsymbol{k}}$$

$$\mathcal{H}_0 = \frac{k_x^2 + k_y^2 - k_F^2}{2m}\tau^x + v_z k_z \tau^z \qquad E(\mathbf{k}) = \pm\sqrt{\frac{(k_x^2+k_y^2-k_F^2)^2}{4m^2} + v_z^2 k_z^2}$$

One line node exists in k_z=0 plane.



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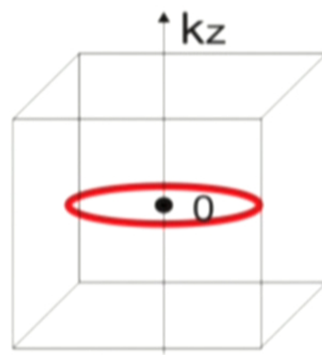
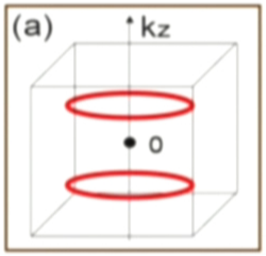
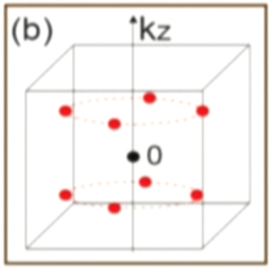


Topological phase transition



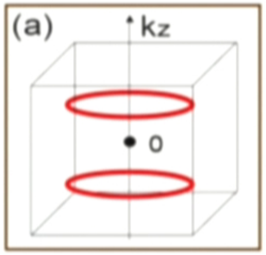
Nodal line SC



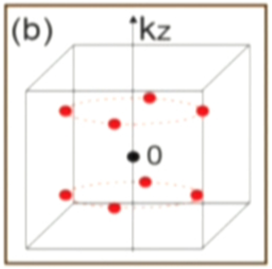
Nodal point SC

We focus on a special class
: symmetry protected topological line node.

Topological phase transition



Nodal line SC



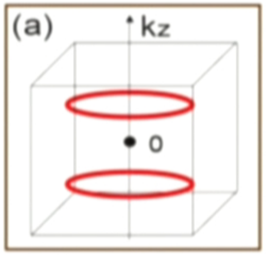
Nodal point SC

We focus on a special class
: symmetry protected topological line node.

If protecting symmetry is broken, line nodal structure is modified.

Symmetry breaking and topological change are concomitant!

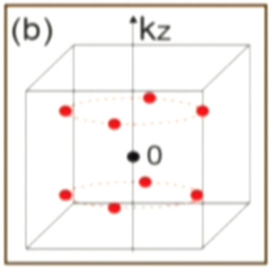
Topological phase transition



Nodal line SC

Symmetry breaking

Topological
QPT

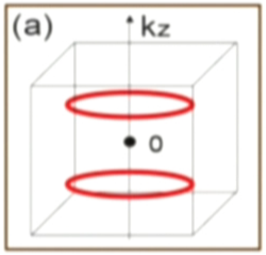


Nodal point SC

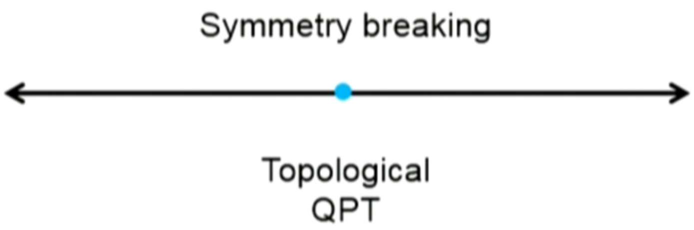
Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

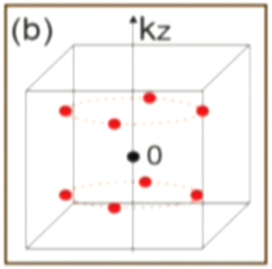
$$H_0 = \sum_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}^{\dagger} \left(h(\boldsymbol{k})\tau^z + \Delta(\boldsymbol{k})\tau^x \right) \Psi_{\boldsymbol{k}} \qquad + \qquad \phi \sum_{\boldsymbol{k}} \mathcal{F}(\boldsymbol{k}) \Psi_{\boldsymbol{k}}^{\dagger} \tau^y \Psi_{\boldsymbol{k}}$$

Topological phase transition



Nodal line SC





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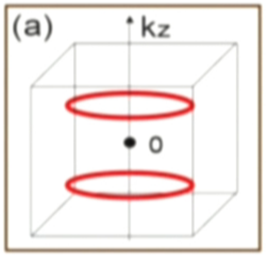
An order parameter exists.

Symmetric phase : line-node
Symmetry-broken phase : no line-node
(either point-node or fully gapped)

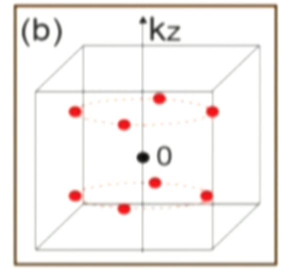
Topological phase transition



Nodal line SC

Symmetry breaking

Topological
QPT



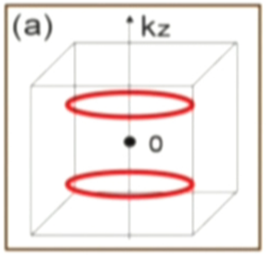
Nodal point SC

Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

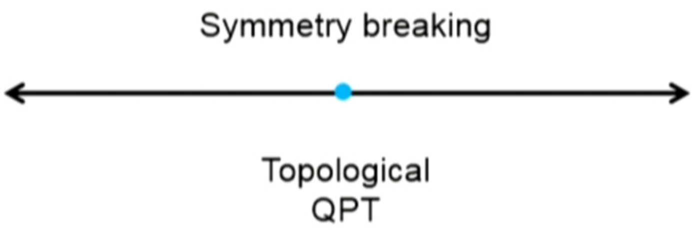
An order parameter exists.

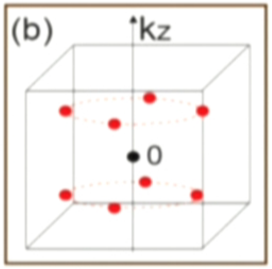
Landau-Ginzburg theory?

Topological phase transition



Nodal line SC





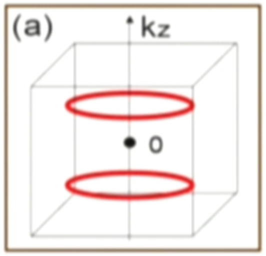
Nodal point SC

Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

Can the Landau-Ginzburg theory describe the transition?

$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

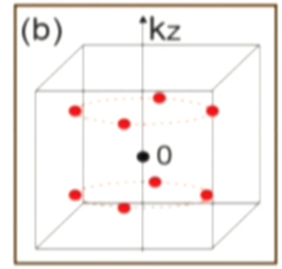
Topological phase transition



Nodal line SC

Symmetry breaking

Topological
QPT



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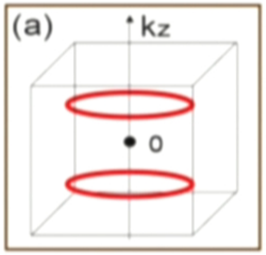
NO! (no information about topological nature in the L-G theory)

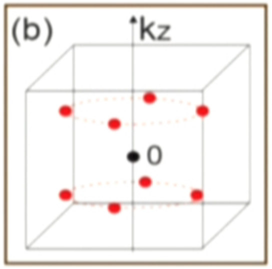
Topological phase transition



Nodal line SC

Symmetry breaking

Topological
QPT



Nodal point SC

Symmetry breaking and topological change are intrinsically tied.
(ex : time reversal symmetry(TRS))

$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

$$S_c = S_\phi + S_\psi, \quad S_\psi = \int_{x,\tau} \Psi^\dagger(\partial_\tau + \mathcal{H}_0)\Psi + g\int_\tau H_{\psi-\phi}$$

Nodal line Hamiltonian $\quad H_0 = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \Big(h(\boldsymbol{k})\tau^z + \Delta(\boldsymbol{k})\tau^x \Big) \Psi_{\boldsymbol{k}}$

Topological phase transition

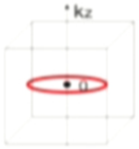
Example :

$$H_0 = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \Big(h(\boldsymbol{k})\tau^z + \Delta(\boldsymbol{k})\tau^x \Big) \Psi_{\boldsymbol{k}} \qquad H_{\psi-\phi} = \phi \sum \Psi^\dagger_k \tau^y \Psi_k$$

$$\mathcal{H}_0 = \frac{k_x^2 + k_y^2 - k_F^2}{2m}\tau^x + v_z k_z \tau^z$$

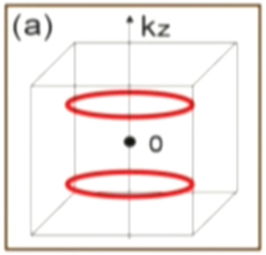
$$E(k) = \pm\sqrt{\frac{(k_x^2 + k_y^2 - k_F^2)^2}{4m^2} + v_z^2 k_z^2 + \phi^2}$$

One line node exists in the symmetric phase.

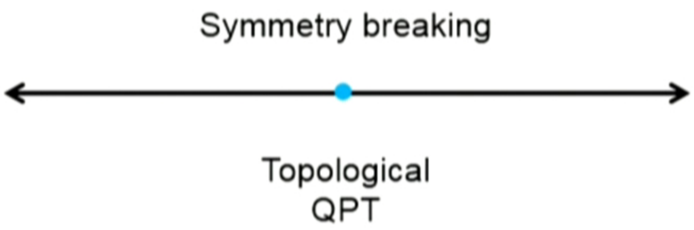


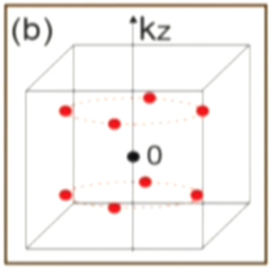
No line node exists in the symmetry-broken phase.

Topological phase transition



Nodal line SC





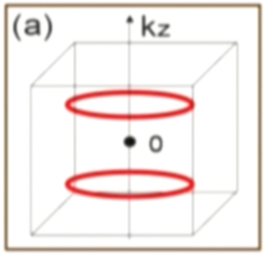
Nodal point SC

Can be generalized to general symmetry groups.

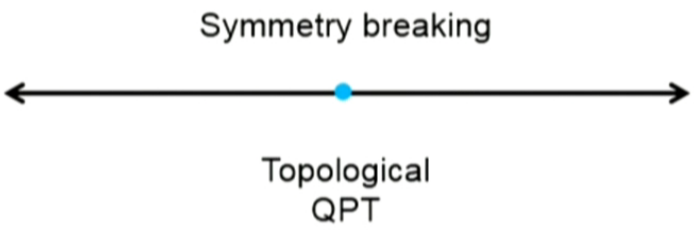
$$\mathcal{G} = C_{4v} \times \mathcal{T} \times \mathcal{P}$$

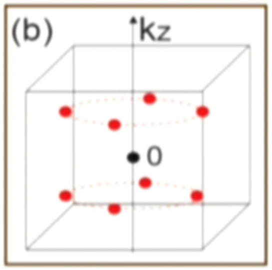
Rep.	Lattice ($\mathcal{F}_s(\boldsymbol{k})M^s$)	Continuum	#
A_1	τ^y	τ^y	0
A_2	$\sin(k_x)\sin(k_y)(\cos(k_x)-\cos(k_y))\tau^y$	$\sin(4\theta)\tau^y$	16
B_1	$(\cos(k_x)-\cos(k_y))\tau^y$	$\cos(2\theta)\tau^y$	8
B_2	$\sin(k_x)\sin(k_y)\tau^y$	$\sin(2\theta)\tau^y$	8
E	$\sin(k_x)\sin(k_z)\tau^y$, $\sin(k_y)\sin(k_z)\tau^y$	$\cos(\theta)\tau^y\mu^z$, $\sin(\theta)\tau^y\mu^z$	4

Topological phase transition



Nodal line SC





Nodal point SC

Mean field theory with $$H = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} H_{\boldsymbol{k}} \Psi_{\boldsymbol{k}} - \frac{u}{2}(\sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} \mathcal{F}(\theta_{\boldsymbol{k}})\tau_y \Psi_{\boldsymbol{k}})^2$$

$$\phi = u\langle \Psi^\dagger_{\boldsymbol{k}} \mathcal{F}(\theta_{\boldsymbol{k}})\tau_y \Psi_{\boldsymbol{k}} \rangle \qquad H_{\mathrm{MF}} = \sum_{\boldsymbol{k}} \Psi^\dagger_{\boldsymbol{k}} (H_{\boldsymbol{k}} - \phi\mathcal{F}(\theta_k)\tau_y)\Psi_{\boldsymbol{k}} + \frac{\phi^2}{2u}$$

$$\mathcal{F}_{\mathrm{MF}}(T,\phi) = -\frac{T}{V}\ln(\mathrm{tr}(e^{-\beta H_{\mathrm{MF}}}))$$

$$\mathcal{F}_{MF}(\phi) = (\frac{1}{u} - \frac{1}{u_c} + T)\phi^2 + k_f|\phi|^3 + \cdots$$

Topological phase transition

$$\mathcal{F}_{\rm MF}(T,\phi) = -\frac{T}{V}\ln(\mathrm{tr}(e^{-\beta H_{\rm MF}}))$$
$$= -\frac{T}{V}\sum_{\boldsymbol{k}}\left(\ln(1+e^{-\beta E_{\boldsymbol{k}}(\phi)}) + \ln(1+e^{\beta E_{\boldsymbol{k}}(\phi)})\right) + \frac{\phi^2}{2u}$$
$$= -\frac{T}{V}\sum_{\boldsymbol{k}}\left(\beta E_{\boldsymbol{k}}(\phi) + 2\ln(1+e^{-\beta E_{\boldsymbol{k}}(\phi)})\right) + \frac{\phi^2}{2u}$$
$$= -\frac{1}{V}\sum_{\boldsymbol{k}} E_{\boldsymbol{k}}(\phi) - \frac{2T}{V}\sum_{\boldsymbol{k}}\ln(1+e^{-\beta E_{\boldsymbol{k}}(\phi)}) + \frac{\phi^2}{2u},$$

$$\delta\mathcal{F}_{\rm MF}(\phi) = \mathcal{F}_{\rm MF}(\phi) - \mathcal{F}_{\rm MF}(0) = \frac{1}{V}\sum_{\boldsymbol{k}}(E_{\boldsymbol{k}}(0) - E_{\boldsymbol{k}}(\phi)) - \frac{2T}{V}\sum_{\boldsymbol{k}}\ln\left(\frac{1+e^{-\beta E_{\boldsymbol{k}}(\phi)}}{1+e^{-\beta E_{\boldsymbol{k}}(0)}}\right) + \frac{\phi^2}{2u}.$$

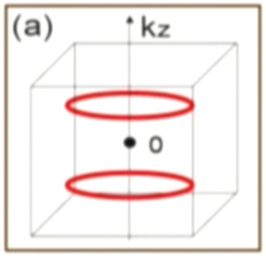
$$\frac{\delta\mathcal{F}_{MF}}{\delta\phi^2}|_{T=0} = 0 \quad \rightarrow \quad \frac{1}{2u_c} - \frac{1}{V}\sum_k \frac{\partial E_k(\phi)}{\partial\phi^2} = 0$$

u_c is not zero (cf : BCS instability)

$$\mathcal{D}(\epsilon) \sim |\epsilon|$$

$$\mathcal{F}_{MF}(\phi) = (\frac{1}{u} - \frac{1}{u_c} + T)\phi^2 + k_f|\phi|^3 + \cdots$$

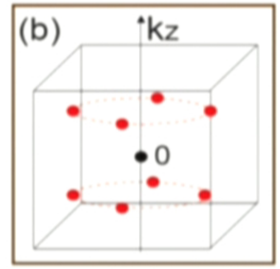
Topological phase transition



Nodal line SC

Symmetry breaking

Topological
QPT



Nodal point SC

Mean field Free energy

$$\mathcal{F}_{MF}(\phi) = (\frac{1}{u} - \frac{1}{u_c} + T)\phi^2 + k_f|\phi|^3 + \cdots$$

The cubic term appears due to nodal line fermion excitation.

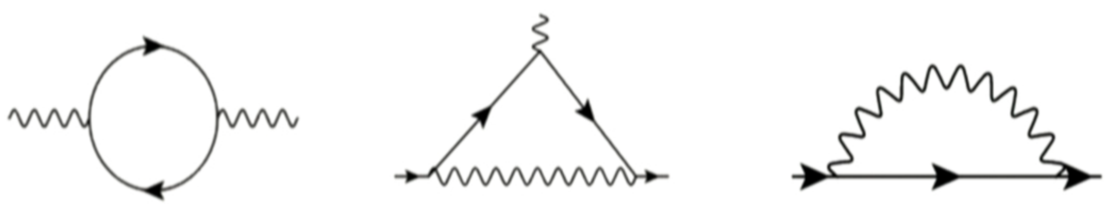
The MFT already shows the universality class is special!

Critical theory

$$S_c = S_\phi + S_\psi, \quad S_\psi = \int_{x,\tau} \Psi^\dagger(\partial_\tau + \mathcal{H}_0)\Psi + g\int_\tau H_{\psi-\phi}$$

$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau\phi)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

Quantum corrections



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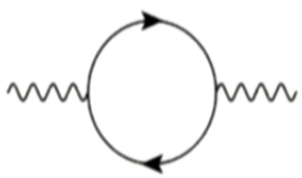

$$\rightarrow \quad N_f k_f \sqrt{\Omega^2 + v_z^2 q_z^2 + v_\perp^2 q_\perp^2} \quad \gg \Omega^2 + q^2$$

Large N theory works well!

Critical theory

$$S_c = S_\phi + S_\psi, \quad S_\psi = \int_{x,\tau} \Psi^\dagger(\partial_\tau + \mathcal{H}_0)\Psi + g\int_\tau H_{\psi-\phi}$$

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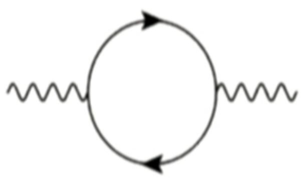

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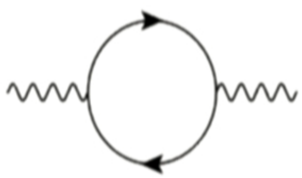


$$\rightarrow \quad N_f k_f \sqrt{\Omega^2 + v_z^2 q_z^2 + v_\perp^2 q_\perp^2} \quad \gg \Omega^2 + q^2$$

Large N theory works well!

$$\mathcal{D}(\epsilon) \sim |\epsilon|$$

$$\Sigma_b(0,0) = -\frac{g^2}{(2\pi)^3}\int_k \frac{1}{E(k)} \sim -k_f \int d\epsilon\epsilon\frac{1}{\epsilon} \sim -k_f\Lambda$$

No symmetry protection

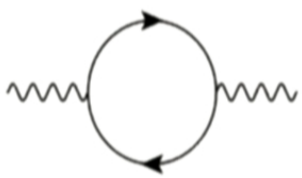
(cf. Y. Huh, EGM, Kim, PRB 2016)

$$\delta\Sigma_b(\Omega_n,0) = -\frac{g^2}{(2\pi)^3}\int d^3k\left(\frac{4E(\mathbf{k})}{\Omega_n^2 + 4E(\mathbf{k})^2} - \frac{1}{E(\mathbf{k})}\right) \sim \int_0 d\epsilon\epsilon(\frac{4\epsilon}{\Omega^2 + 4\epsilon^2} - \frac{1}{\epsilon}) \sim -|\Omega|$$

Critical theory

$$S_c = S_\phi + S_\psi, \quad S_\psi = \int_{x,\tau} \Psi^\dagger(\partial_\tau + \mathcal{H}_0)\Psi + g\int_\tau H_{\psi-\phi}$$

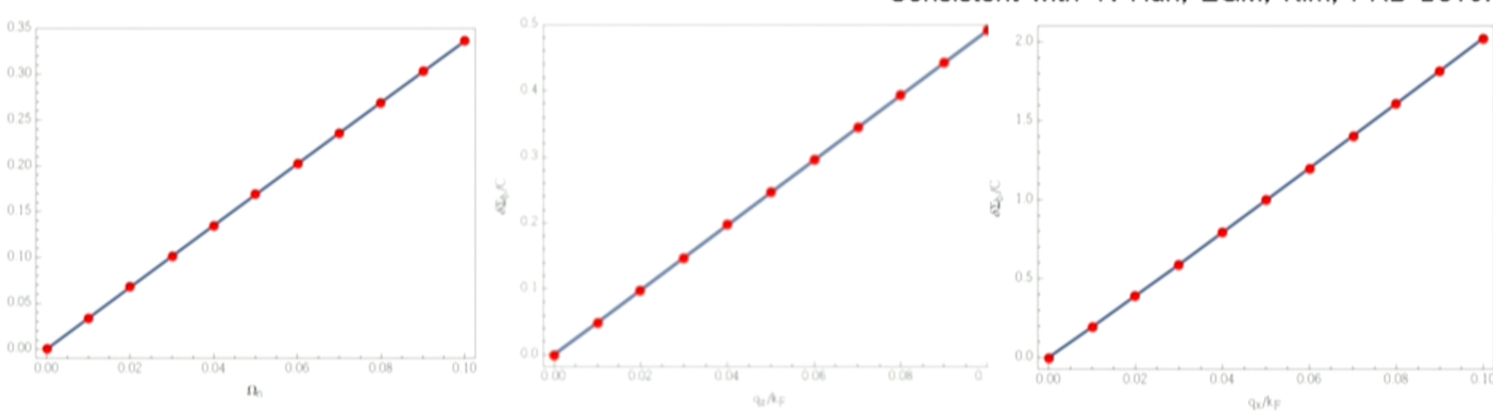
$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau\phi)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$



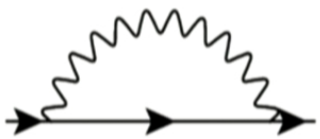
$$\rightarrow \quad N_f k_f \sqrt{\Omega^2 + v_z^2 q_z^2 + v_\perp^2 q_\perp^2} \quad \gg \Omega^2 + q^2$$

Large N theory works well!

Consistent with Y. Huh, EGM, Kim, PRB 2016!



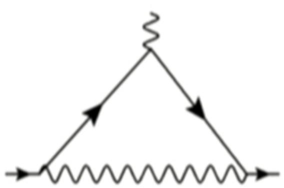
Critical theory



$$\Sigma_f(\omega, \boldsymbol{k}) = g^2 \int_{\Omega, \boldsymbol{q}} \tau^y G_f(\omega + \Omega, \boldsymbol{k} + \boldsymbol{q}) \tau^y G_b(\Omega, \boldsymbol{q})$$

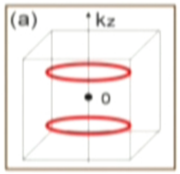
$$\frac{\delta \Sigma_f(\omega, \boldsymbol{k})}{\delta \epsilon^a} \propto \quad \frac{1}{N_f\, k_f} \times (\Lambda - \mu)$$

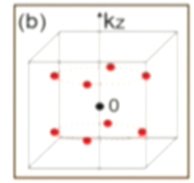
No infra-red divergence!



Similar structure as that of fermion self-energy

Critical theory





$$\mathcal{S}_\phi^c = \int_{\Omega,\boldsymbol{q}} N_f k_f \sqrt{\Omega^2 + v_z^2 q_z^2 + v_\perp^2 q_\perp^2}\, \mathcal{R}(\rho(\Omega,\boldsymbol{q}))\frac{|\phi|^2}{2}$$

$$\rho(\Omega,\mathbf{q}) = 1/\left(1 + \frac{\Omega^2 + v_z^2 q_z^2}{v_\perp^2 q_\perp^2}\right)$$

1. Large anomalous dimension.
2. Emergent Lorentz inv.
3. Hyper-scaling violation.

QCP in $3d$	z	ν	β	γ	η	HS
ϕ^4 theory[27]	1	$\frac{1}{2}$	$\frac{1}{2}$	1	0	O
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Nodal line QCP	1	1	1	1	1	X

($\Omega \sim q^z$, $\xi^{-1} \sim |r-r_c|^\nu$, $\chi_\phi \sim |r-r_c|^{-\gamma}$, and $[\phi] = \frac{d+z-2+\eta}{2}$)

Comparison

In 3d,

1. ϕ^4 theory and Higgs-Yukawa theory (upper-critical dimension)

$$S_\phi = \int_{x,\tau} \frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$$

Mean-field + logarithmic correction

2. Hertz-Millis Theory

$$\mathcal{S}_{HM} = \int_{q,\omega} (\frac{|\omega|}{\Gamma_q} + q^2 + r)|\phi(k,\omega)|^2 + \int_{x,\tau} \frac{u}{4}\phi(x,\tau)^4$$

z=2,3 + hyperscaling violation

3. Line-nodal critical theory

$$\mathcal{S}_{line} = \int_{k,\omega} k_f \left(\sqrt{\omega^2 + v_\perp^2 k_\perp^2 + v_z^2 k_z^2} + r\right)|\phi(k,\omega)|^2$$

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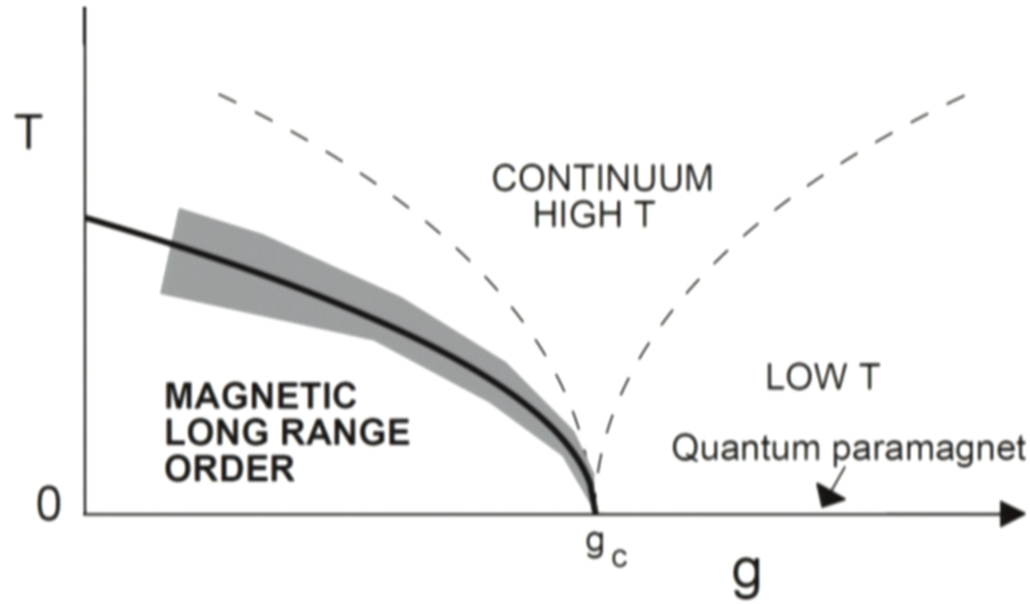
Phase diagram

Usual phase diagrams



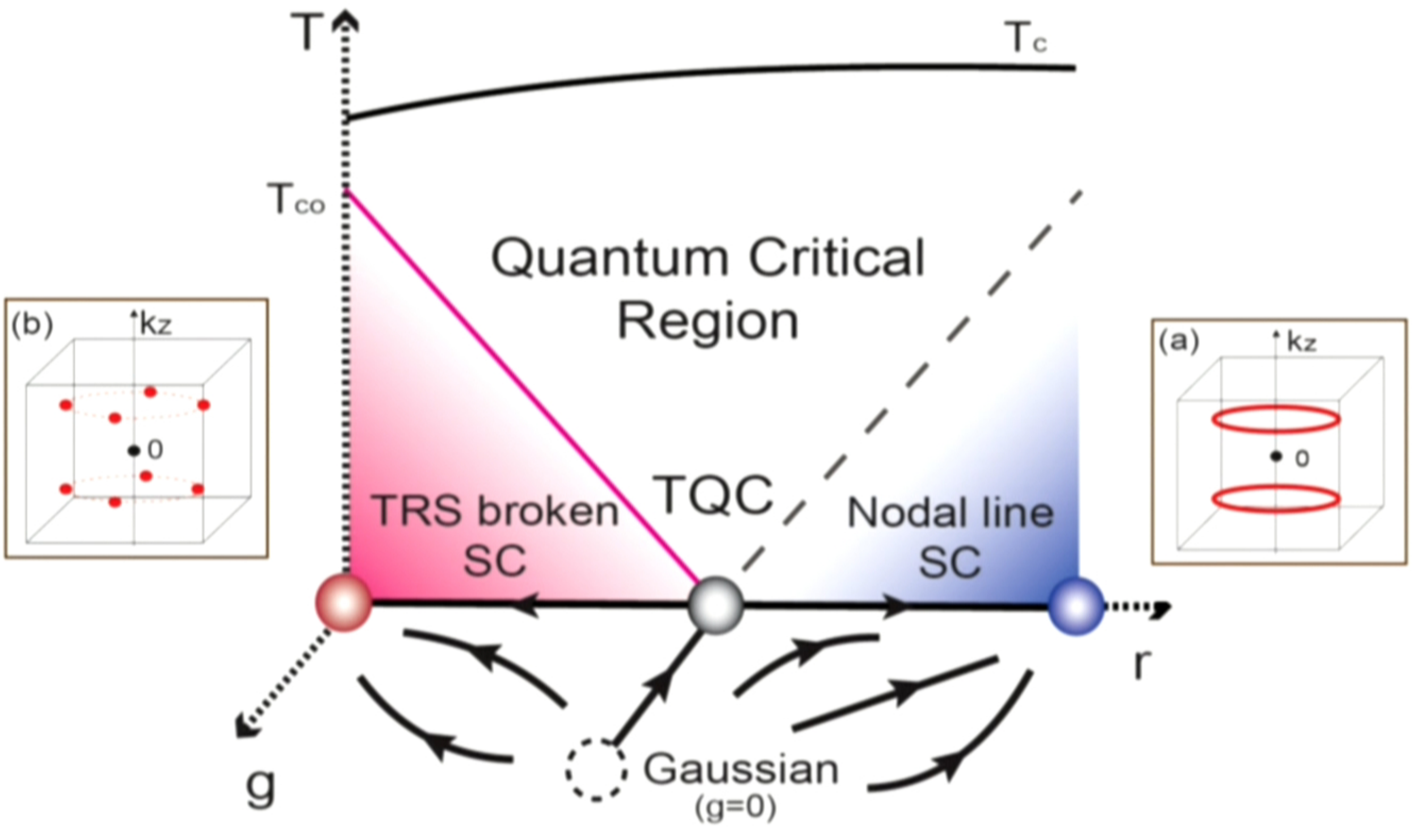
S. Sachdev, Quantum Phase Transitions

Basically, $T^2 \sim |g-g_c|$

$$T_c \sim (g_c - g)^{1/2}$$

$$\langle\phi\rangle \sim (g - g_c)^{1/2}$$

Phase diagram



Linear phase boundary

With two SCs,

If SC1 is a line-nodal SC and the transition is 2nd order, then QCP has the linear phase boundary.

If there is a 2nd order linear phase boundary between two SCs, does one of them have a line-node?

Line-nodal Superconductors

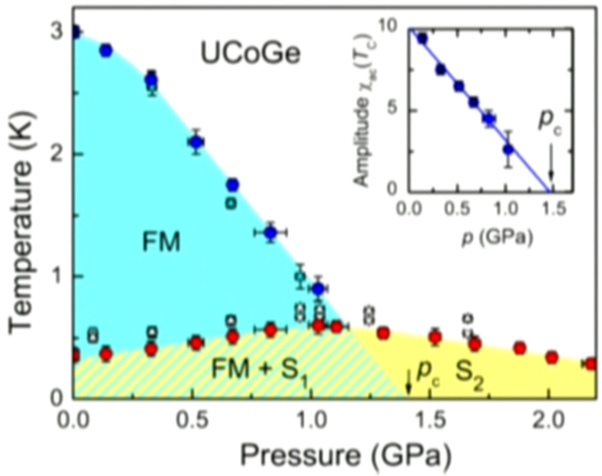


FIG. 2 (color online). Pressure-temperature phase diagram of UCoGe. Ferromagnetism (FM), blue area; superconductivity (S_1, S_2), yellow area. $T_C(p)$ extrapolates to a ferromagnetic QCP at the critical pressure $p_c = 1.40 \pm 0.05$ GPa. Superconductivity coexists with ferromagnetism below p_c, blue-yellow hatched area. Symbols: filled blue and red circles T_C and T_s^χ from $\chi_{ac}(T)$; blue and white triangles T_C and T_s^R from $\rho(T)$ (up triangles $I \parallel a$ axis, down triangles $I \parallel c$ axis); filled blue and red squares T_C and T_s^χ at $p = 0$ taken from a polycrystal [4]. Inset: Amplitude of $\chi_{ac}(T)$ at T_C as a function of pressure. The data follow a linear p dependence and extrapolate to $p_c = 1.46 \pm 0.10$ GPa.

E. Slooten et. al., *PRL 2009*

Two different symmetries, and two superconducting states.

Is it linear?

Tempting but more investigation is necessary.

Summary

Topological phase transitions appears in nodal structure change transitions.

The nodal excitation induces a novel universality class.

Hyperscaling violation, emergent Lorentz invariance, and large anomalous dimensions are characteristics.

We propose to measure the linear phase boundary for a signal of the presence of line-nodes.

Comparison

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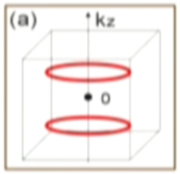
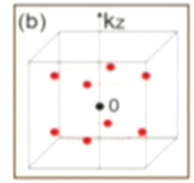
Critical theory





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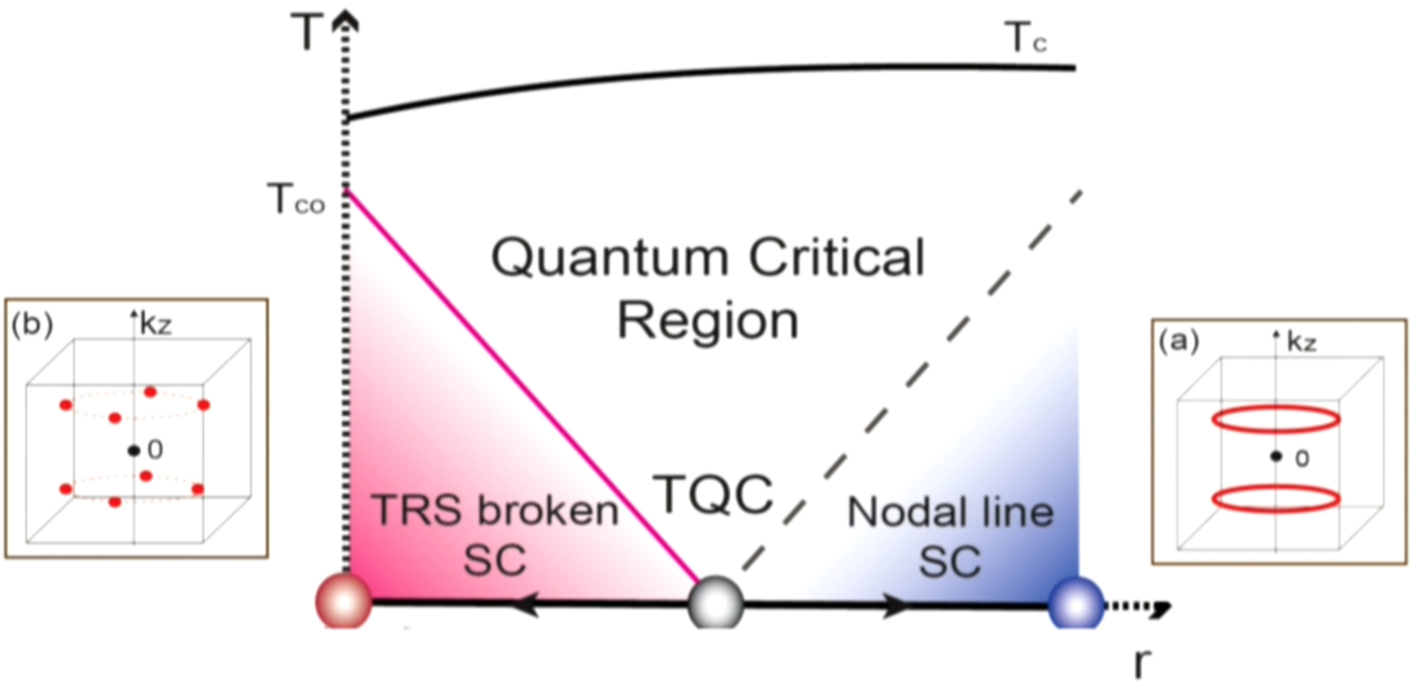
Phase diagram



Significantly larger quantum critical region due to fermion excitation

$$T_c \sim (g_c - g) \qquad \langle \phi \rangle \sim (g - g_c)$$

The linear temperature phase boundary!