Title: Bottom-up approach to logarithmic CFTs

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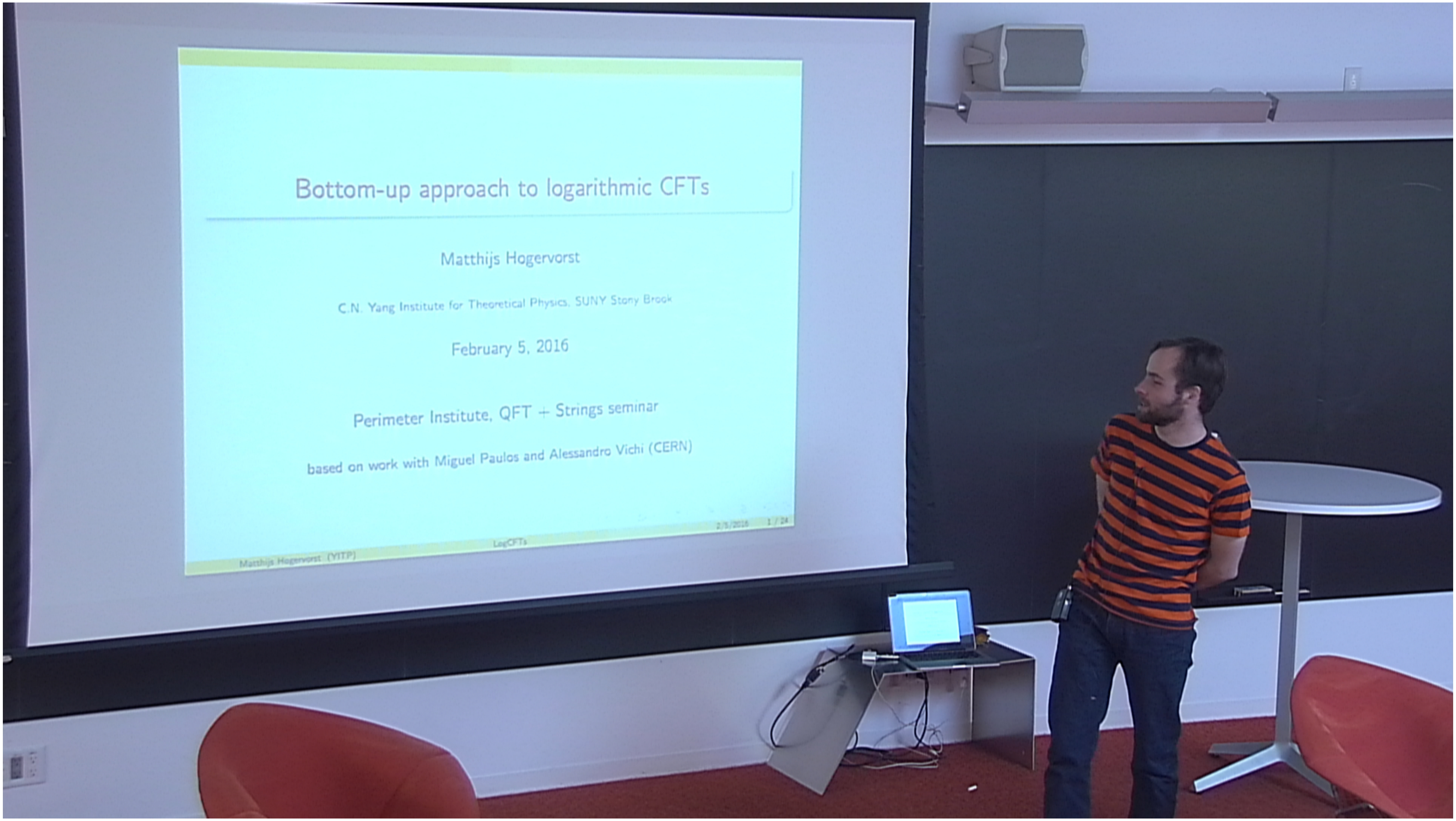
Abstract:


Bottom-up approach to logarithmic CFTs
Matthijs Hogervorst
C.N. Yang Institute for Theoretical Physics, SUNY Stony Brook
February 5, 2016
Perimeter Institute, QFT + Strings seminar
based on work with Miguel Paulos and Alessandro Vichi (CERN)
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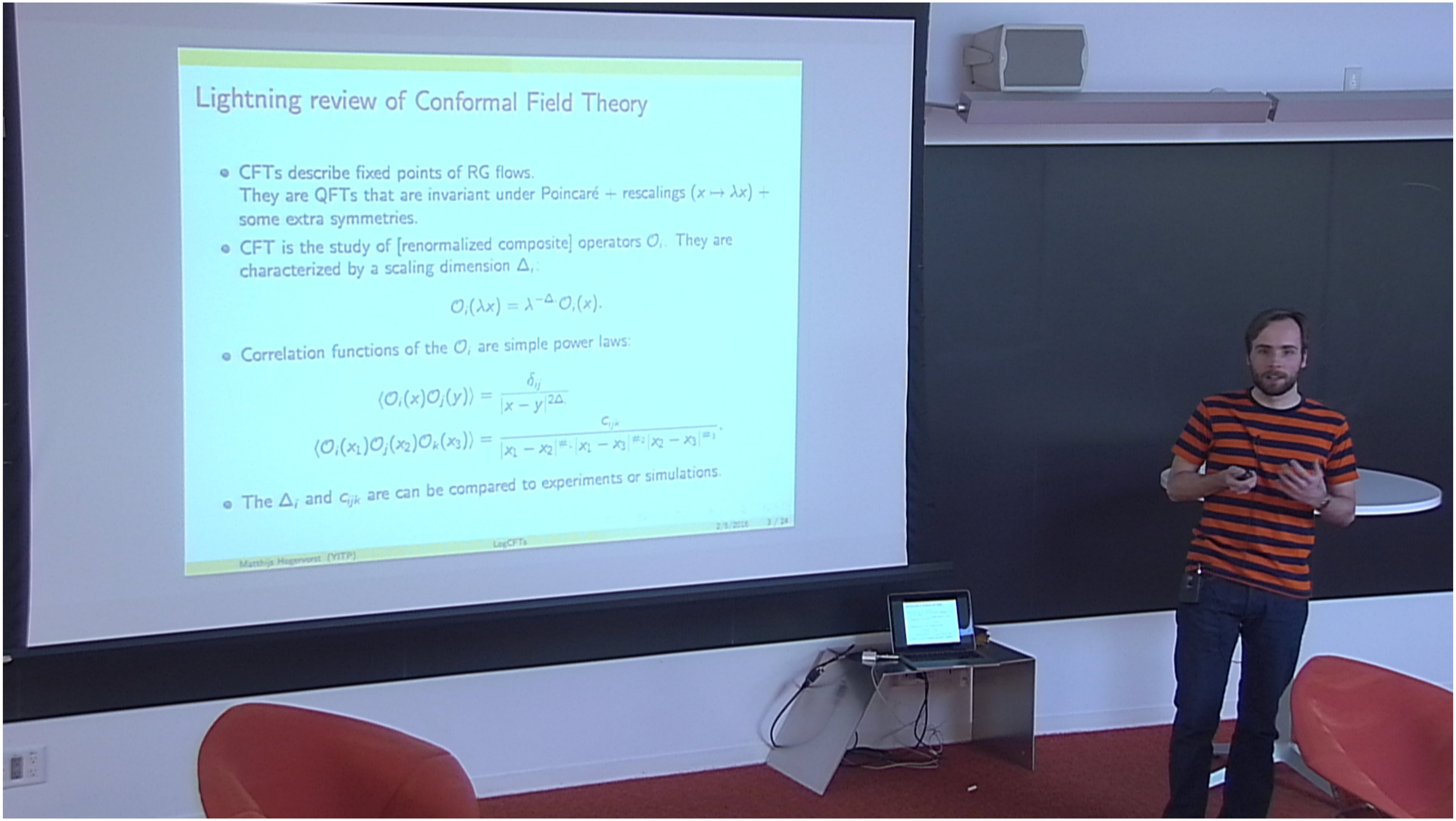
Lightning review of Conformal Field Theory
CFTs describe fixed points of RG flows.
They are QFTs that are invariant under Poincaré + rescalings $(x \mapsto \lambda x)$ + some extra symmetries.
CFT is the study of [renormalized composite] operators $\mathcal{O}_i$. They are characterized by a scaling dimension Δ_i:
$\mathcal{O}_i(\lambda x) = \lambda^{-\Delta_i} \mathcal{O}_i(x).$
Correlation functions of the $\mathcal{O}_i$ are simple power laws:
$\langle \mathcal{O}_i(x) \mathcal{O}_j(y) \rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}}$
$\langle \mathcal{O}_i(x_1) \mathcal{O}_j(x_2) \mathcal{O}_k(x_3) \rangle = \frac{c_{ijk}}{|x_1-x_2|^{\#_1} |x_1-x_3|^{\#_2} |x_2-x_3|^{\#_3}}.$
The Δ_i and c_{ijk} are can be compared to experiments or simulations.
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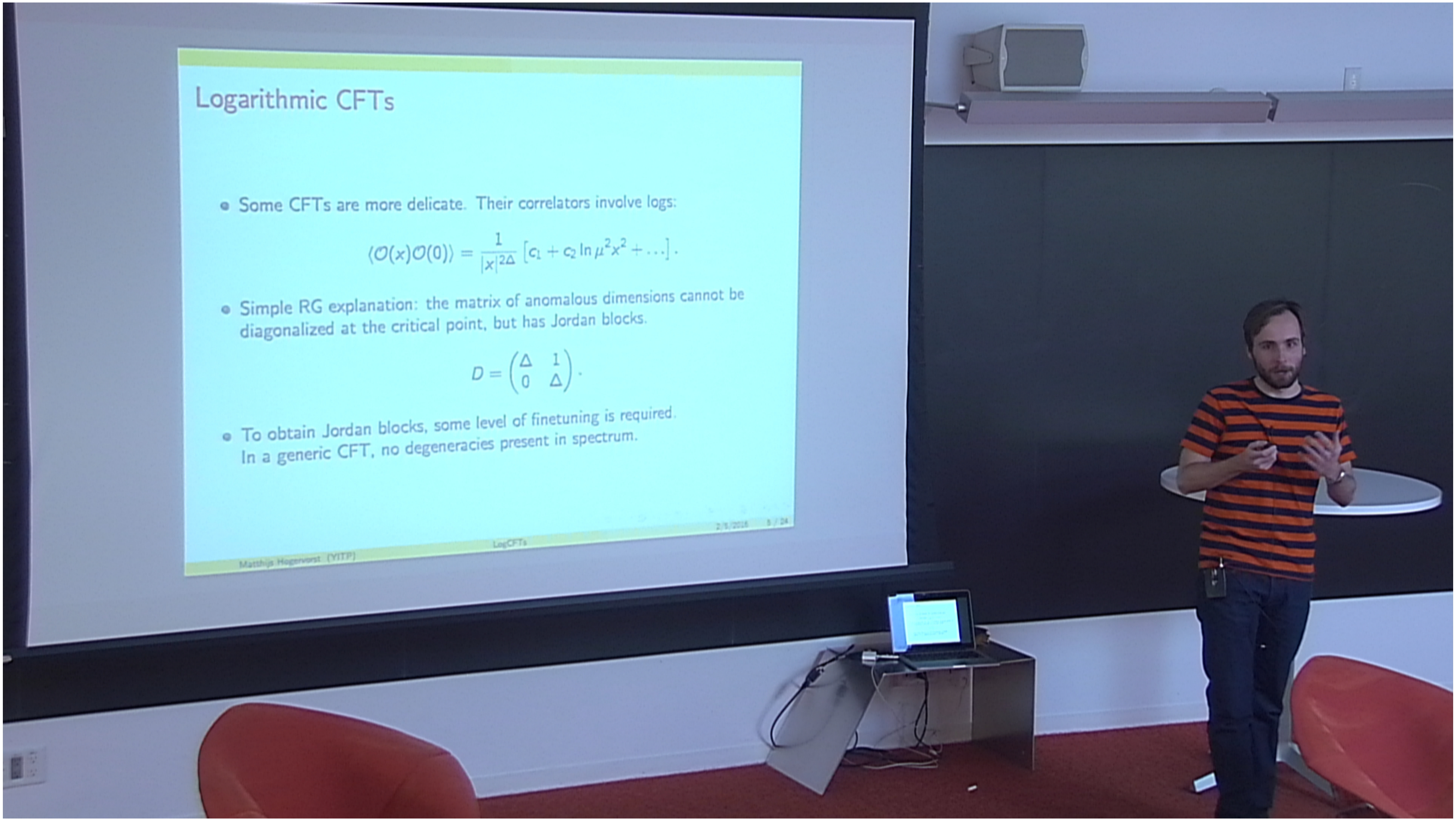
Logarithmic CFTs
Some CFTs are more delicate. Their correlators involve logs:
$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \frac{1}{|x|^{2\Delta}}\left[c_1 + c_2 \ln \mu^2 x^2 + \ldots\right].$
Simple RG explanation: the matrix of anomalous dimensions cannot be diagonalized at the critical point, but has Jordan blocks.
$D = \begin{pmatrix} \Delta & 1 \\ 0 & \Delta \end{pmatrix}.$
To obtain Jordan blocks, some level of finetuning is required.
In a generic CFT, no degeneracies present in spectrum.
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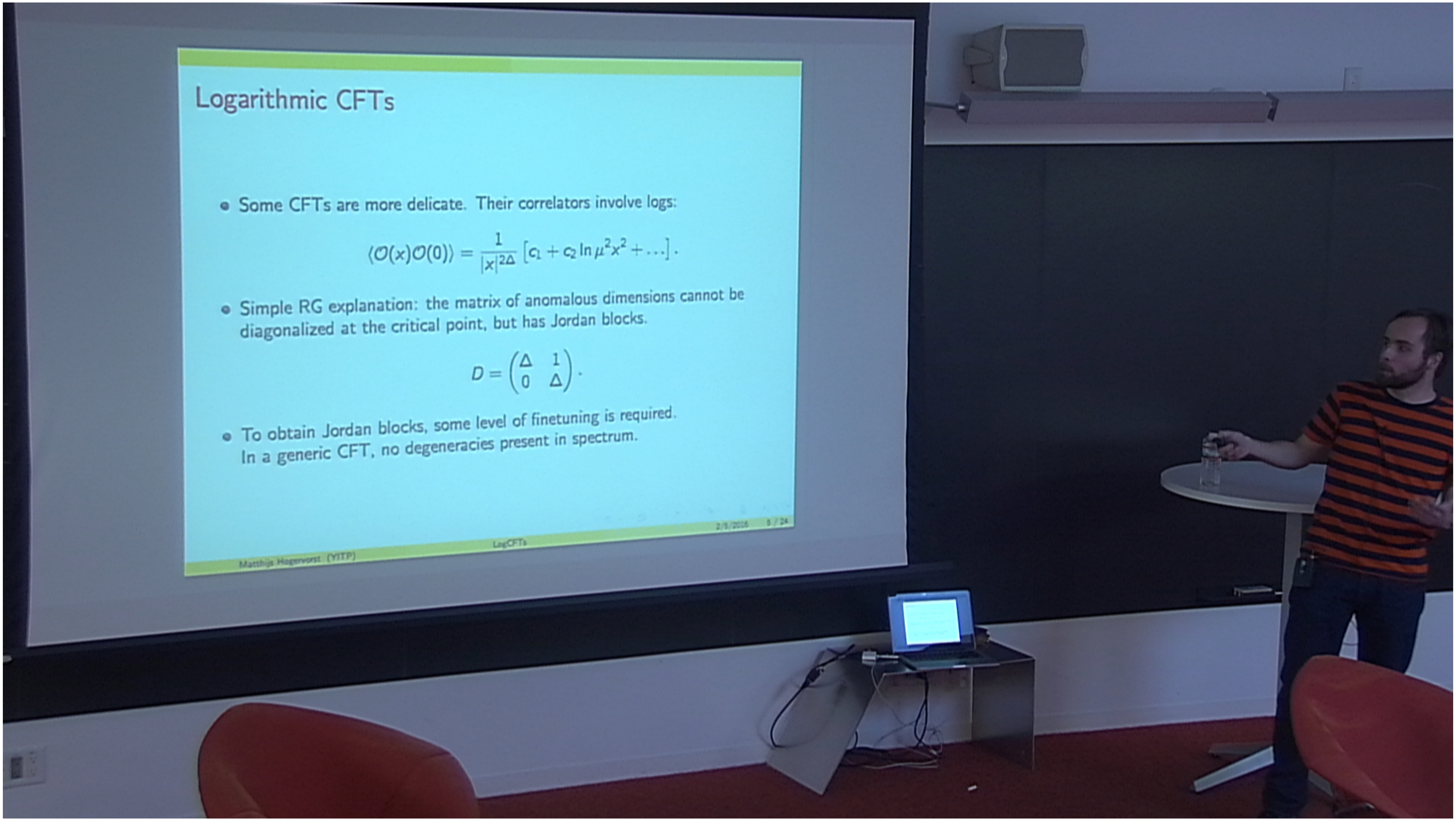
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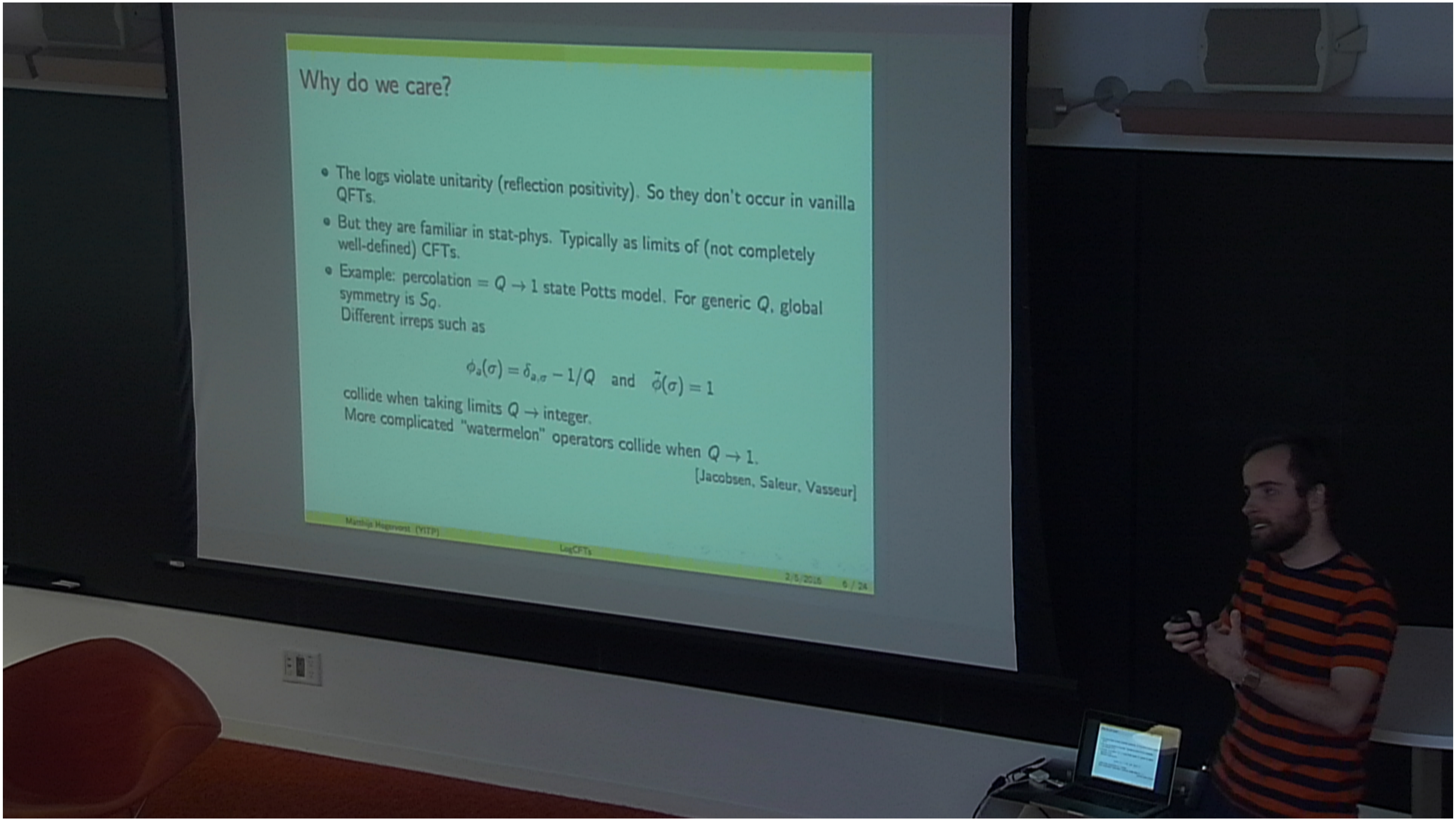
Why do we care?
The logs violate unitarity (reflection positivity). So they don't occur in vanilla QFTs.
But they are familiar in stat-phys. Typically as limits of (not completely well-defined) CFTs.
Example: percolation = $Q \to 1$ state Potts model. For generic Q, global symmetry is S_Q.
Different irreps such as
$\phi_a(\sigma) = \delta_{a,\sigma} - 1/Q$ and $\tilde{\phi}(\sigma) = 1$
collide when taking limits $Q \to$ integer.
More complicated "watermelon" operators collide when $Q \to 1$.
[Jacobsen, Saleur, Vasseur]
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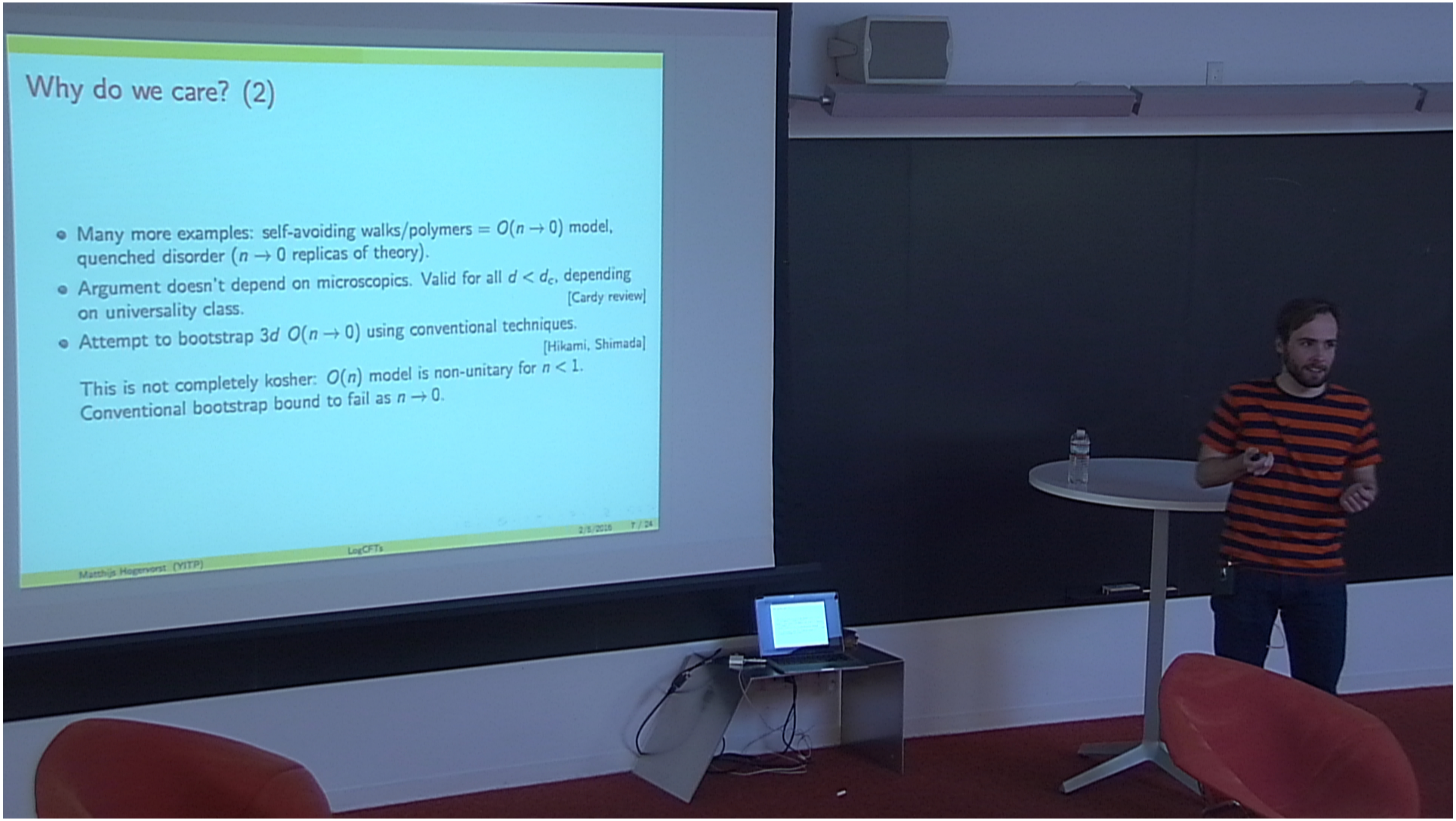
Why do we care? (2)
Many more examples: self-avoiding walks/polymers = $O(n \to 0)$ model, quenched disorder ($n \to 0$ replicas of theory).
Argument doesn't depend on microscopics. Valid for all $d < d_c$, depending on universality class.
[Cardy review]
Attempt to bootstrap 3d $O(n \to 0)$ using conventional techniques.
[Hikami, Shimada]
This is not completely kosher: $O(n)$ model is non-unitary for $n < 1$.
Conventional bootstrap bound to fail as $n \to 0$.
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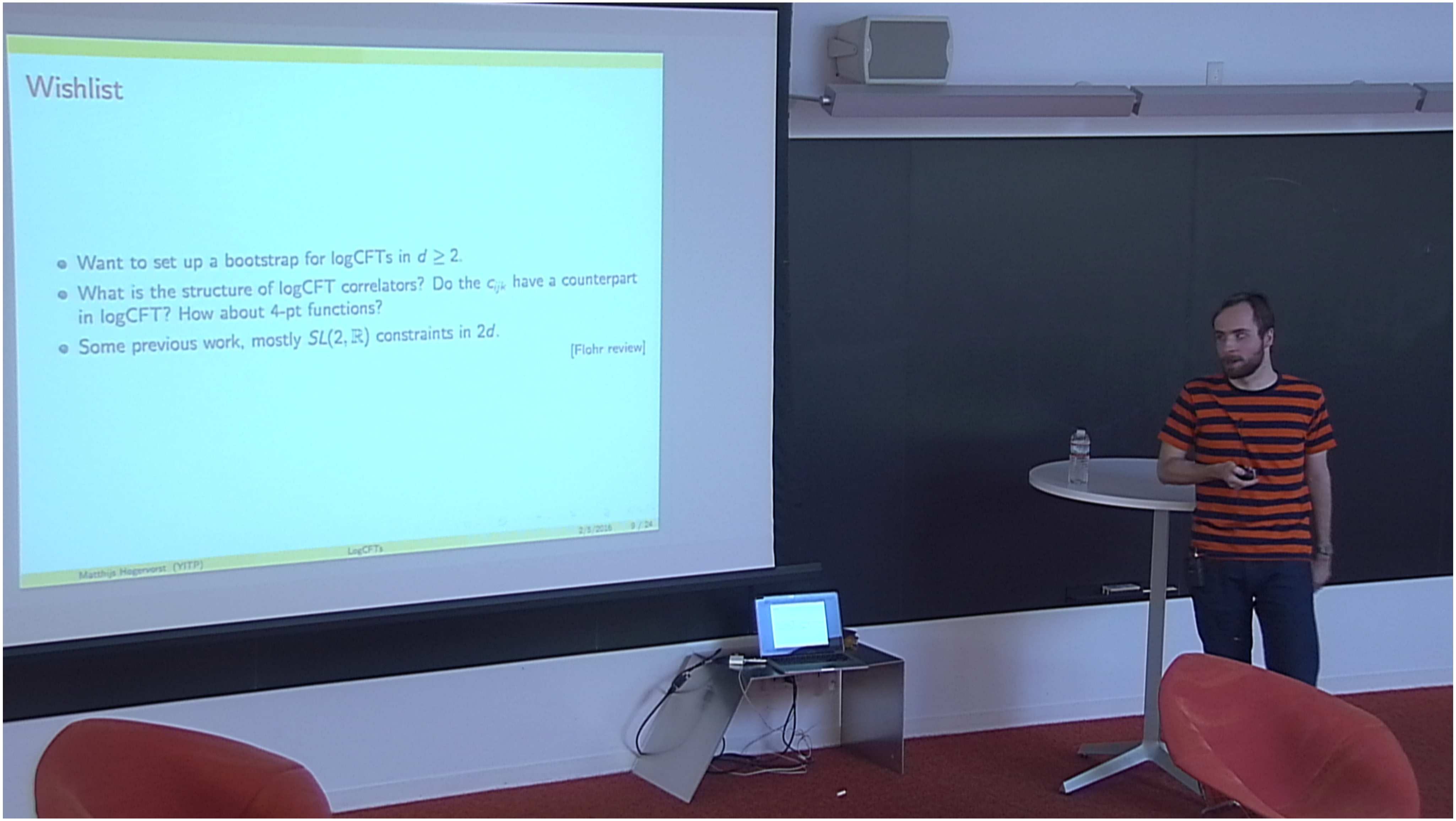
Wishlist
Want to set up a bootstrap for logCFTs in $d \geq 2$.
What is the structure of logCFT correlators? Do the c_{ijk} have a counterpart in logCFT? How about 4-pt functions?
Some previous work, mostly $SL(2,\mathbb{R})$ constraints in $2d$.
[Flohr review]
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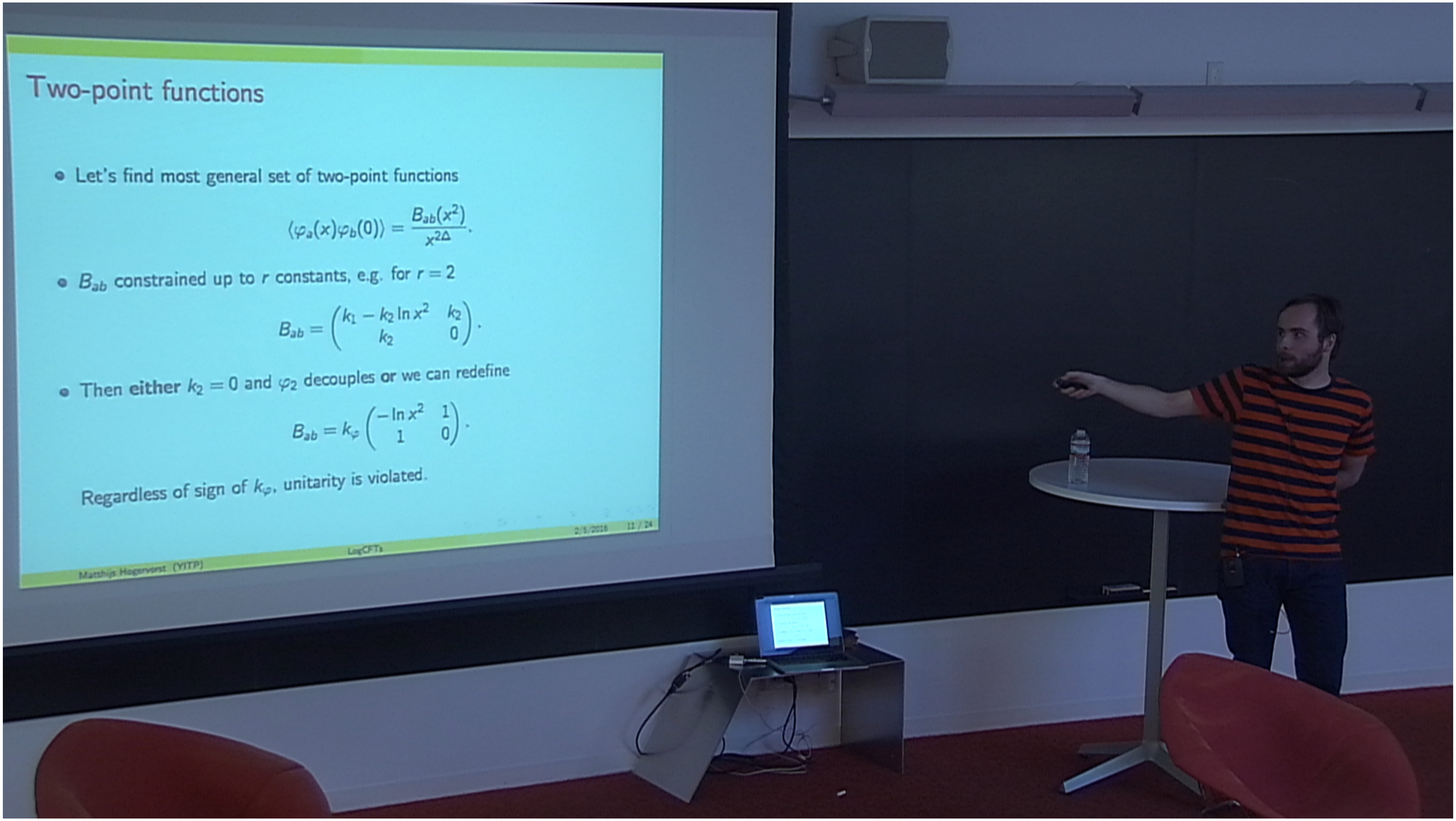
Two-point functions
Let's find most general set of two-point functions
$\langle \varphi_a(x)\varphi_b(0)\rangle = \frac{B_{ab}(x^2)}{x^{2\Delta}}$.
B_{ab} constrained up to r constants, e.g. for $r = 2$
$B_{ab} = \begin{pmatrix} k_1 - k_2 \ln x^2 & k_2 \\ k_2 & 0 \end{pmatrix}$.
Then either $k_2 = 0$ and φ_2 decouples or we can redefine
$B_{ab} = k_\varphi \begin{pmatrix} -\ln x^2 & 1 \\ 1 & 0 \end{pmatrix}$.
Regardless of sign of k_φ, unitarity is violated.
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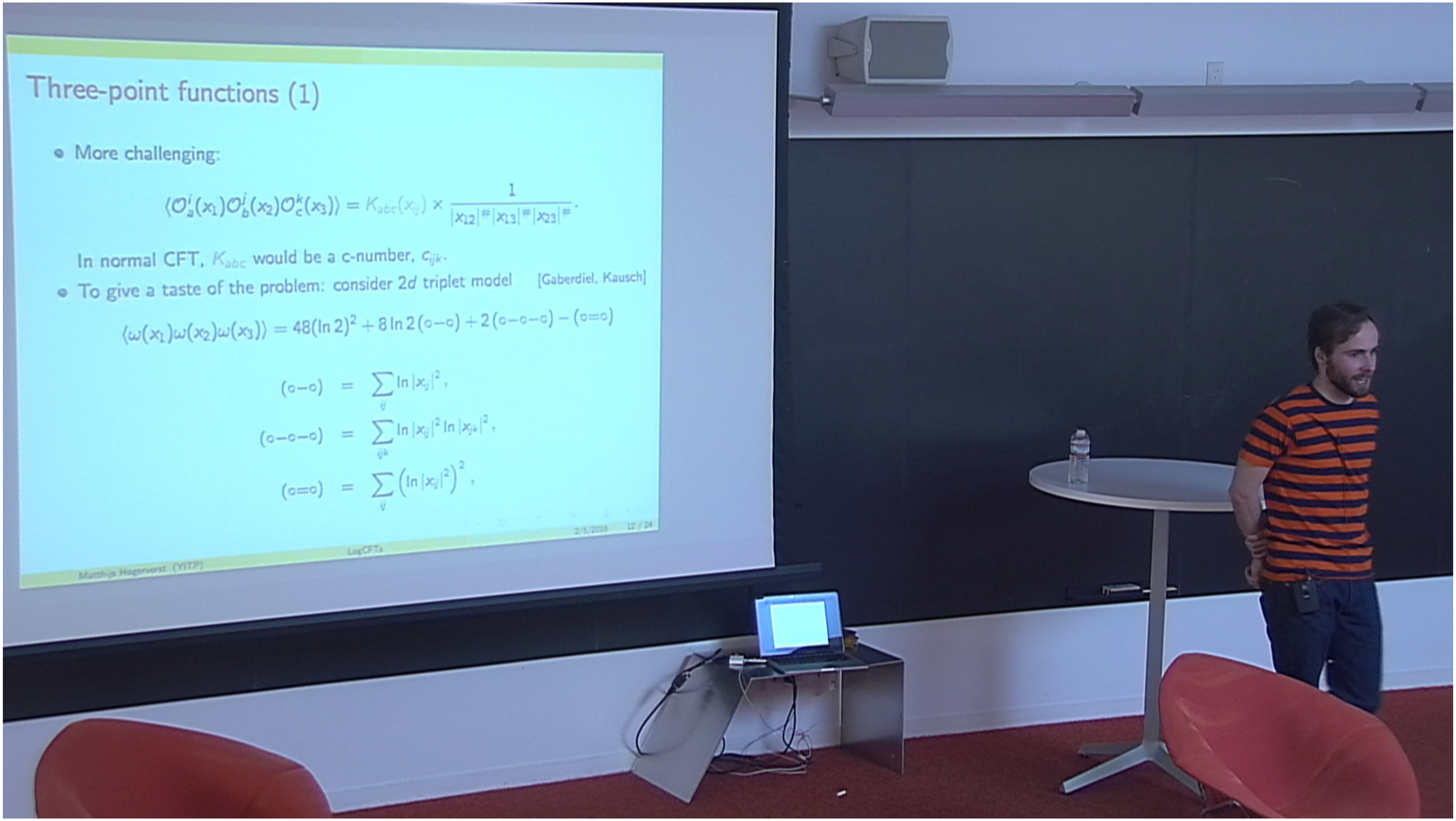
Three-point functions (1)
More challenging:
$\langle \mathcal{O}_a^i(x_1)\mathcal{O}_b^j(x_2)\mathcal{O}_c^k(x_3)\rangle = K_{abc}(x_{ij}) \times \frac{1}{|x_{12}|^{\#}|x_{13}|^{\#}|x_{23}|^{\#}}.$
In normal CFT, K_{abc} would be a c-number, c_{ijk}.
To give a taste of the problem: consider 2d triplet model
[Gaberdiel, Kausch]
$\langle \omega(x_1)\omega(x_2)\omega(x_3)\rangle = 48(\ln 2)^2 + 8\ln 2(\circ\!-\!\circ) + 2(\circ\!-\!\circ\!-\!\circ) - (\circ\!=\!\circ)$
$(\circ\!-\!\circ) = \sum_{ij} \ln|x_{ij}|^2,$
$(\circ\!-\!\circ\!-\!\circ) = \sum_{ijk} \ln|x_{ij}|^2 \ln|x_{jk}|^2,$
$(\circ\!=\!\circ) = \sum_{ij} \left(\ln|x_{ij}|^2\right)^2,$
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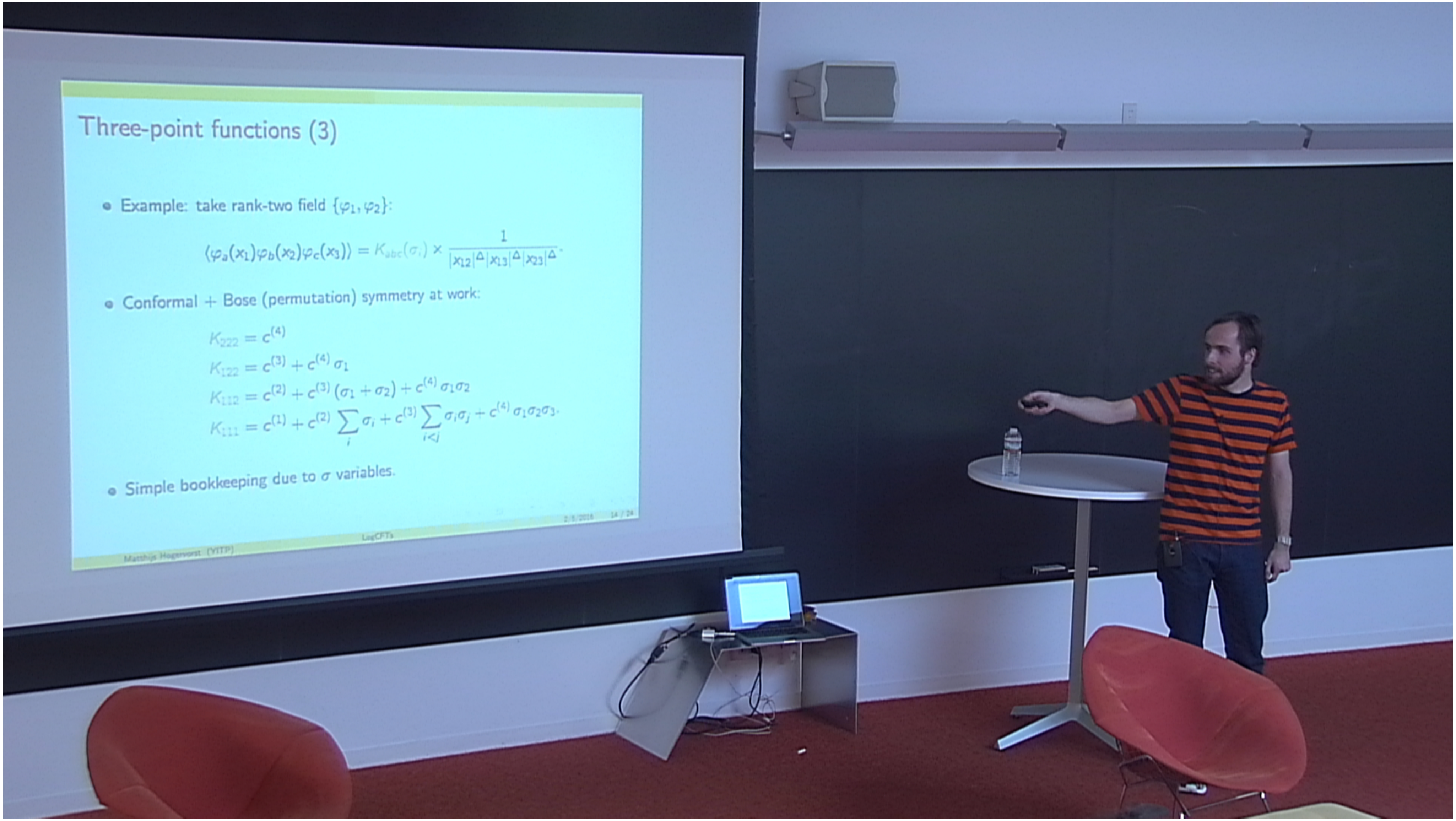
Three-point functions (3)
Example: take rank-two field $\{\varphi_1, \varphi_2\}$:
$\langle \varphi_a(x_1)\varphi_b(x_2)\varphi_c(x_3)\rangle = K_{abc}(\sigma_i) \times \frac{1}{|x_{12}|^\Delta |x_{13}|^\Delta |x_{23}|^\Delta}.$
Conformal + Bose (permutation) symmetry at work:
$K_{222} = c^{(4)}$
$K_{122} = c^{(3)} + c^{(4)}\sigma_1$
$K_{112} = c^{(2)} + c^{(3)}(\sigma_1 + \sigma_2) + c^{(4)}\sigma_1\sigma_2$
$K_{111} = c^{(1)} + c^{(2)}\sum_i \sigma_i + c^{(3)}\sum_{i<j}\sigma_i\sigma_j + c^{(4)}\sigma_1\sigma_2\sigma_3.$
Simple bookkeeping due to σ variables.
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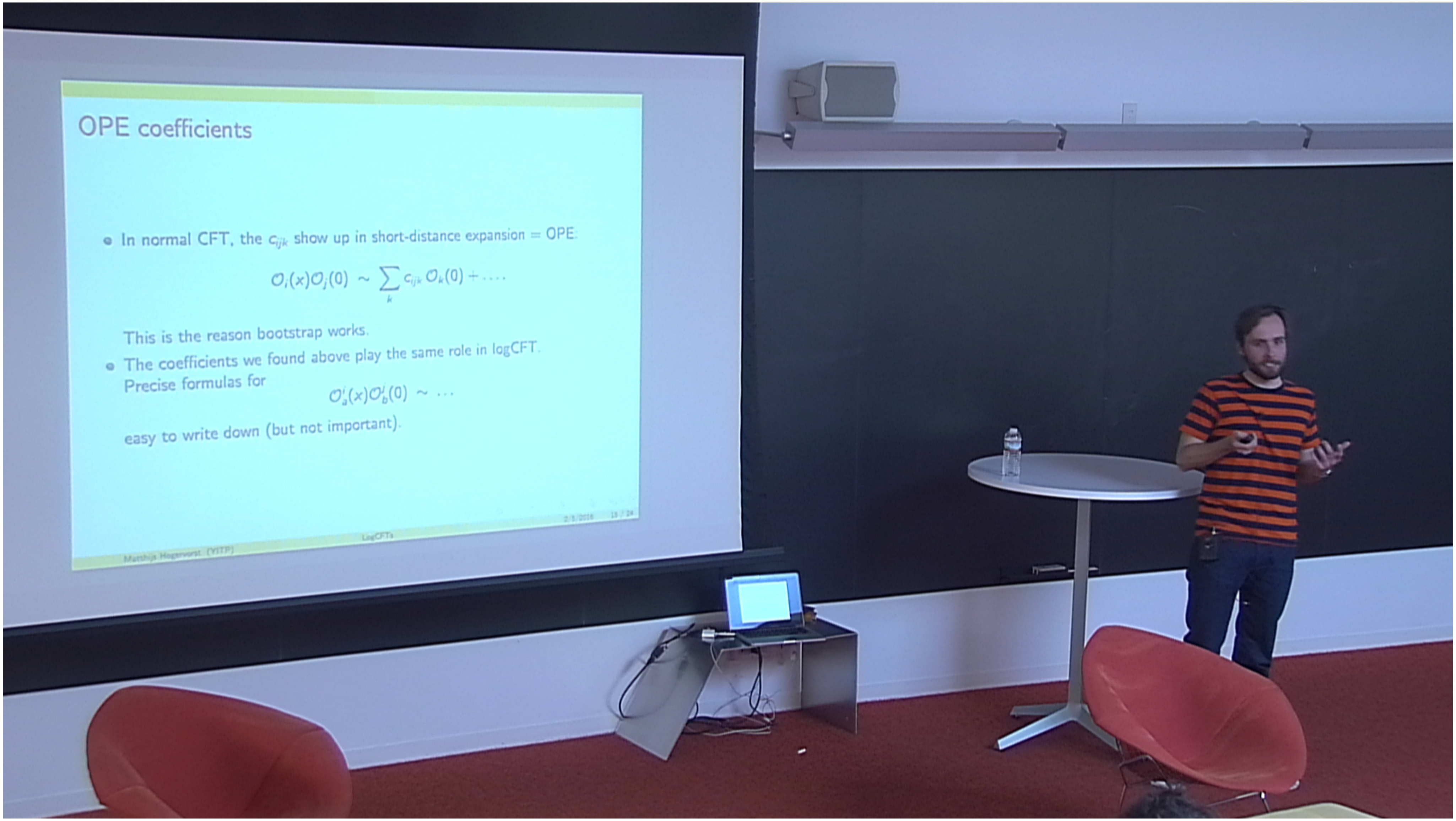
OPE coefficients
In normal CFT, the c_{ijk} show up in short-distance expansion = OPE:
$\mathcal{O}_i(x)\mathcal{O}_j(0) \sim \sum_k c_{ijk}\,\mathcal{O}_k(0) + \dots$
This is the reason bootstrap works.
The coefficients we found above play the same role in logCFT. Precise formulas for
$\mathcal{O}^i_a(x)\mathcal{O}^j_b(0) \sim \dots$
easy to write down (but not important).
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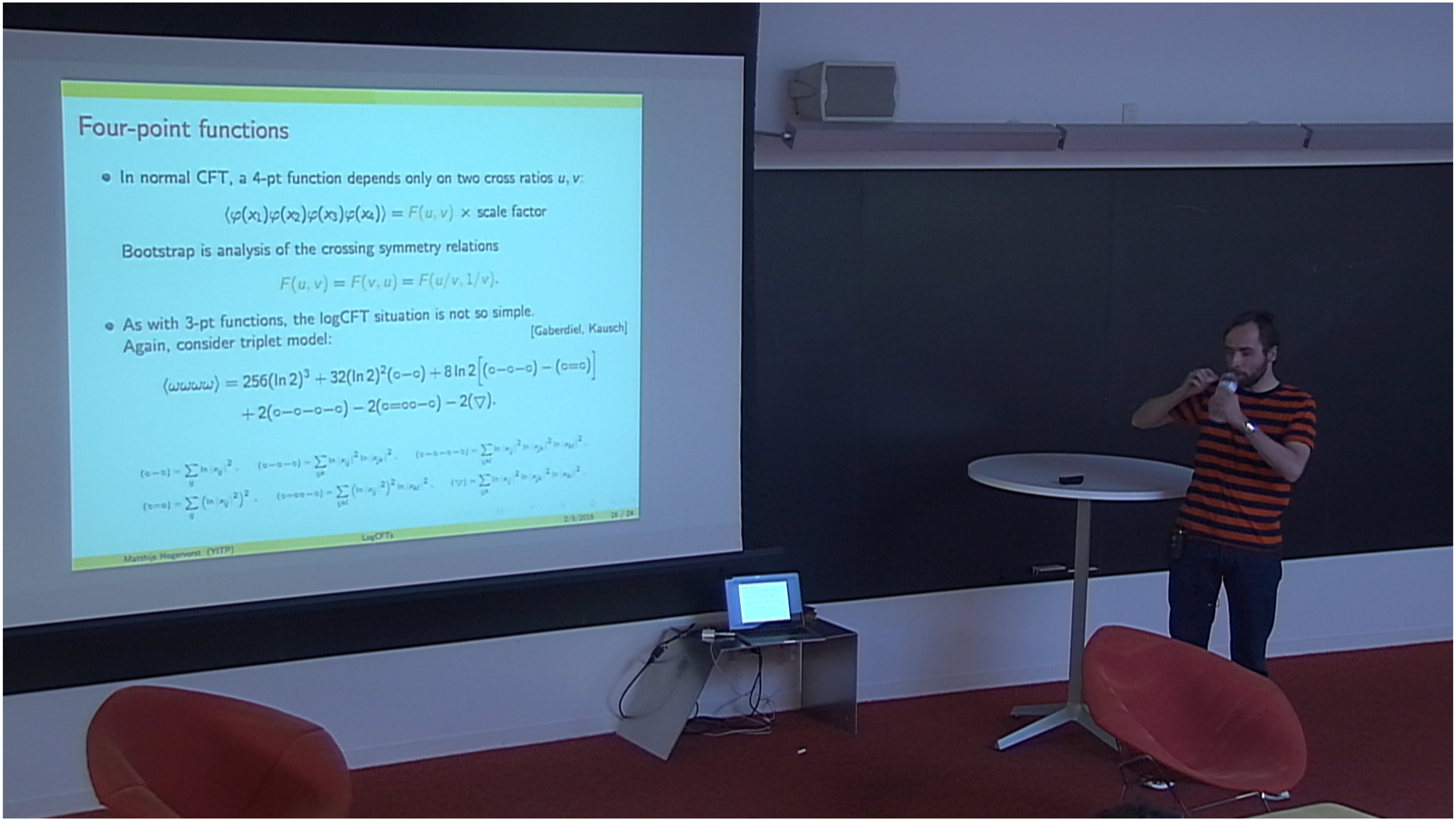
Four-point functions
In normal CFT, a 4-pt function depends only on two cross ratios u, v:
$\langle\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\rangle = F(u, v) \times$ scale factor
Bootstrap is analysis of the crossing symmetry relations
$F(u, v) = F(v, u) = F(u/v, 1/v)$.
As with 3-pt functions, the logCFT situation is not so simple.
[Gaberdiel, Kausch]
Again, consider triplet model:
$\langle\omega\omega\omega\omega\rangle = 256(\ln 2)^3 + 32(\ln 2)^2(\circ\!-\!\circ) + 8\ln 2\left[(\circ\!-\!\circ\!-\!\circ) - (\circ\!=\!\circ)\right] + 2(\circ\!-\!\circ\!-\!\circ\!-\!\circ) - 2(\circ\!=\!\circ\circ\!-\!\circ) - 2(\triangledown)$.
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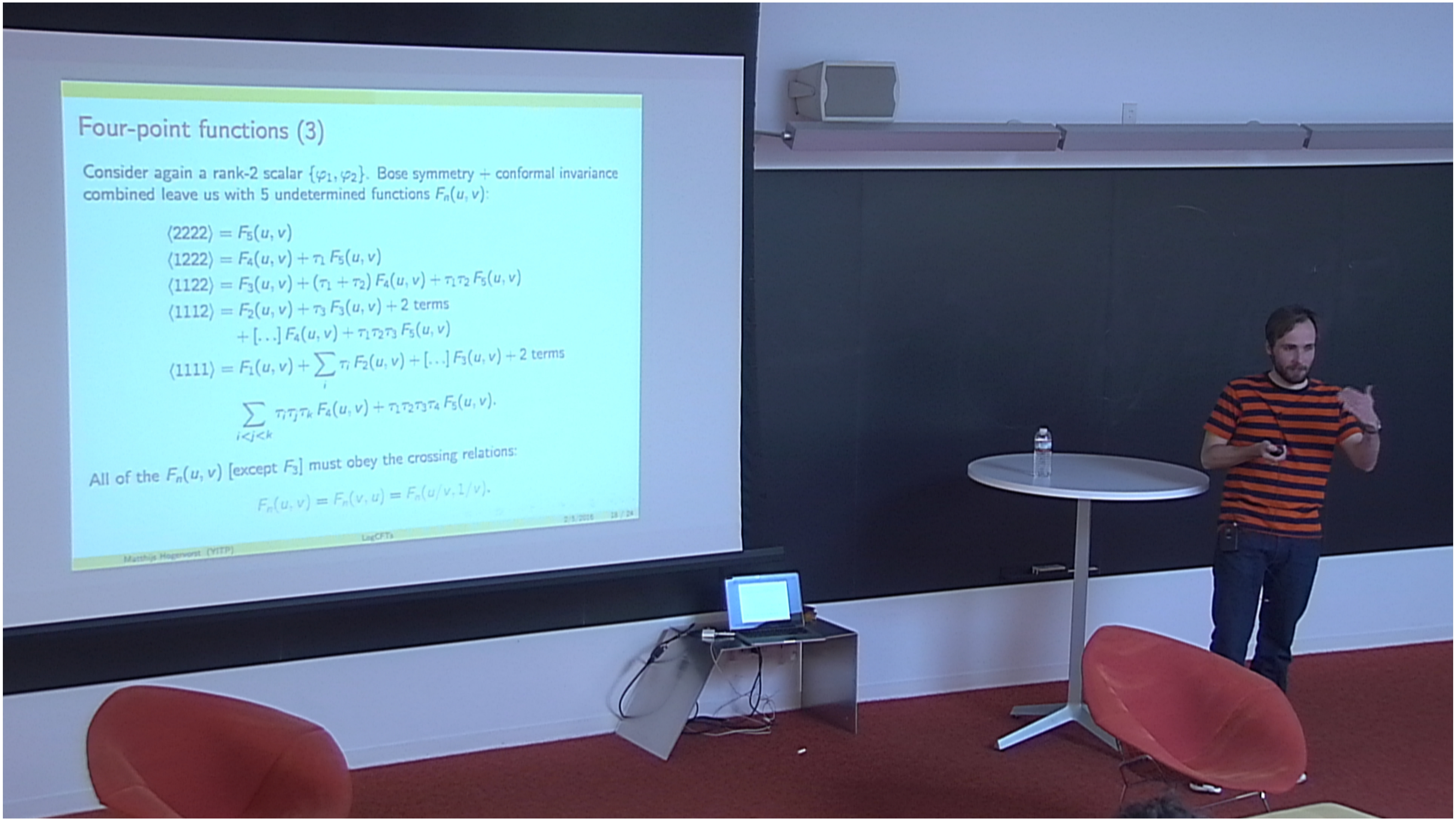
Four-point functions (3)
Consider again a rank-2 scalar $\{\varphi_1, \varphi_2\}$. Bose symmetry + conformal invariance combined leave us with 5 undetermined functions $F_n(u, v)$:
$\langle 2222 \rangle = F_5(u,v)$
$\langle 1222 \rangle = F_4(u,v) + \tau_1 F_5(u,v)$
$\langle 1122 \rangle = F_3(u,v) + (\tau_1 + \tau_2) F_4(u,v) + \tau_1\tau_2 F_5(u,v)$
$\langle 1112 \rangle = F_2(u,v) + \tau_3 F_3(u,v) + 2$ terms
$+ [\ldots] F_4(u,v) + \tau_1\tau_2\tau_3 F_5(u,v)$
$\langle 1111 \rangle = F_1(u,v) + \sum_i \tau_i F_2(u,v) + [\ldots] F_3(u,v) + 2$ terms
$\sum_{i<j<k} \tau_i\tau_j\tau_k F_4(u,v) + \tau_1\tau_2\tau_3\tau_4 F_5(u,v).$
All of the $F_n(u,v)$ [except F_3] must obey the crossing relations:
$F_n(u,v) = F_n(v,u) = F_n(u/v, 1/v).$
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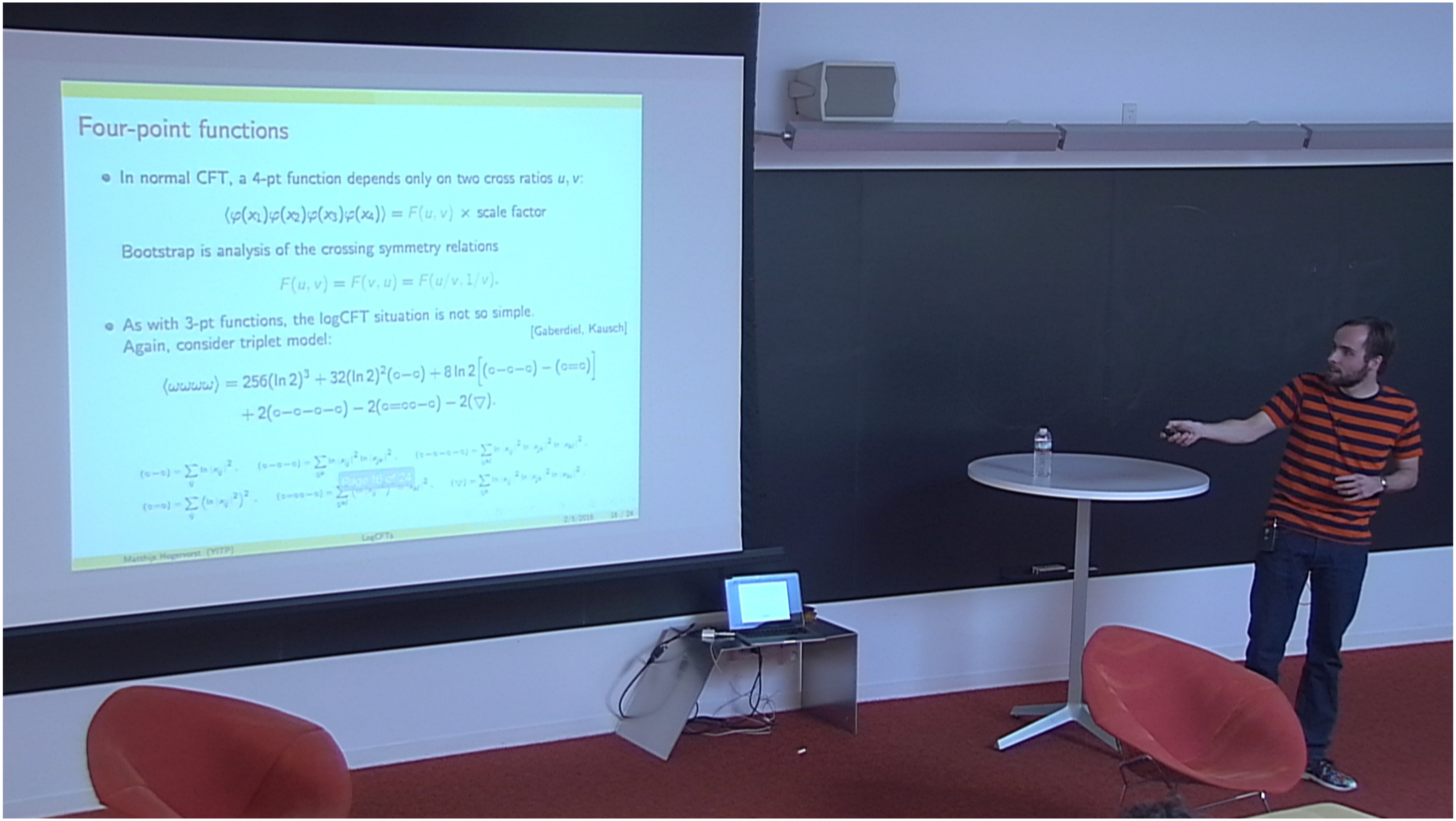
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Again, consider triplet model:
⟨ωωωω⟩ = 256(ln 2)^3 + 32(ln 2)^2(∘−∘) + 8 ln 2[(∘−∘−∘) − (∘=∘)]
+ 2(∘−∘−∘−∘) − 2(∘=∘∘−∘) − 2(▽).
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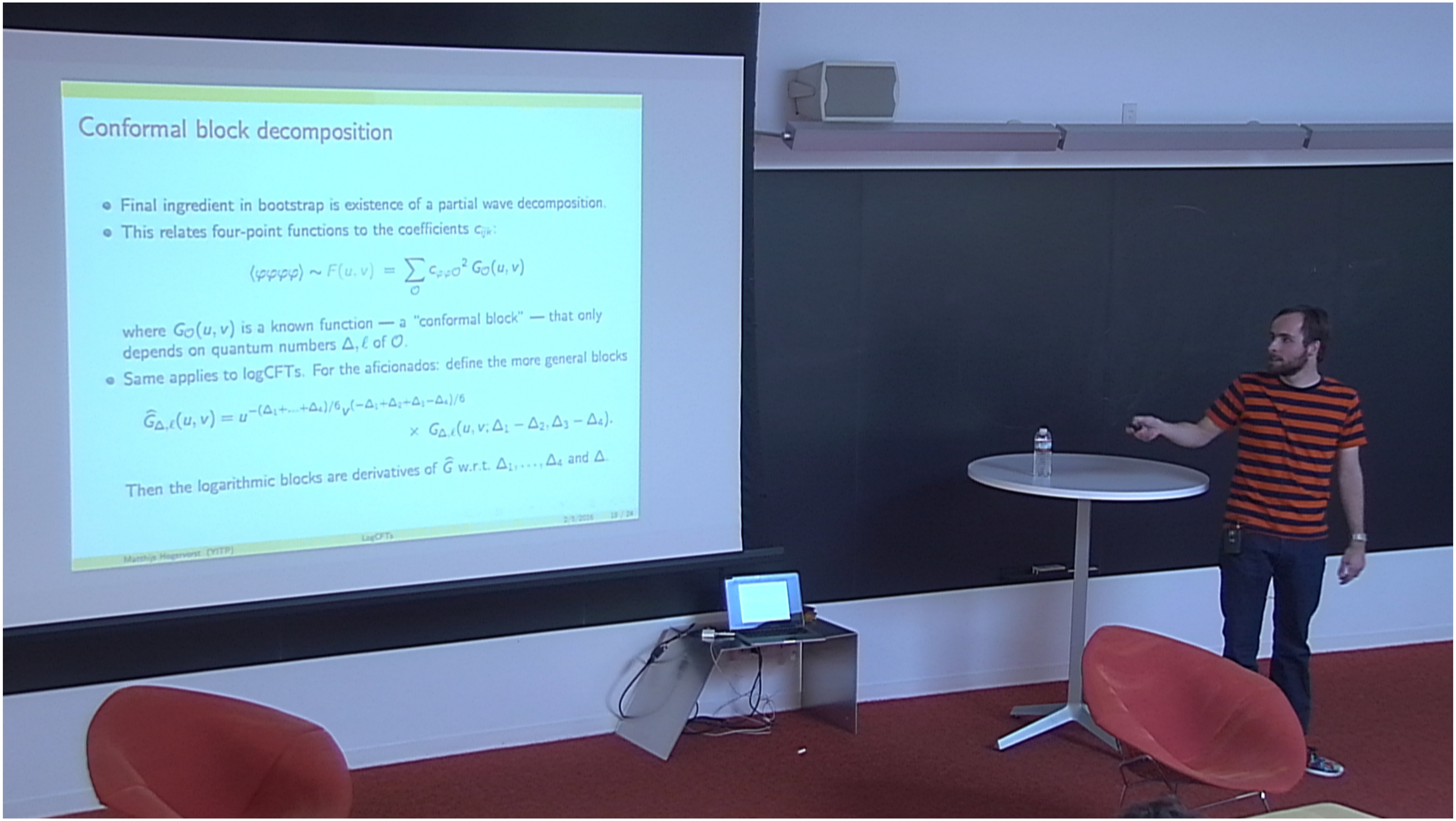
Conformal block decomposition
Final ingredient in bootstrap is existence of a partial wave decomposition.
This relates four-point functions to the coefficients c_{ijk}:
$\langle\varphi\varphi\varphi\varphi\rangle \sim F(u,v) = \sum_{\mathcal{O}} c_{\varphi\varphi\mathcal{O}}{}^2\, G_{\mathcal{O}}(u,v)$
where $G_{\mathcal{O}}(u,v)$ is a known function — a "conformal block" — that only depends on quantum numbers Δ, ℓ of $\mathcal{O}$.
Same applies to logCFTs. For the aficionados: define the more general blocks
$\widehat{G}_{\Delta,\ell}(u,v) = u^{-(\Delta_1+\ldots+\Delta_4)/6} v^{(-\Delta_1+\Delta_2+\Delta_3-\Delta_4)/6} \times G_{\Delta,\ell}(u,v;\Delta_1-\Delta_2,\Delta_3-\Delta_4).$
Then the logarithmic blocks are derivatives of $\widehat{G}$ w.r.t. $\Delta_1,\ldots,\Delta_4$ and Δ.
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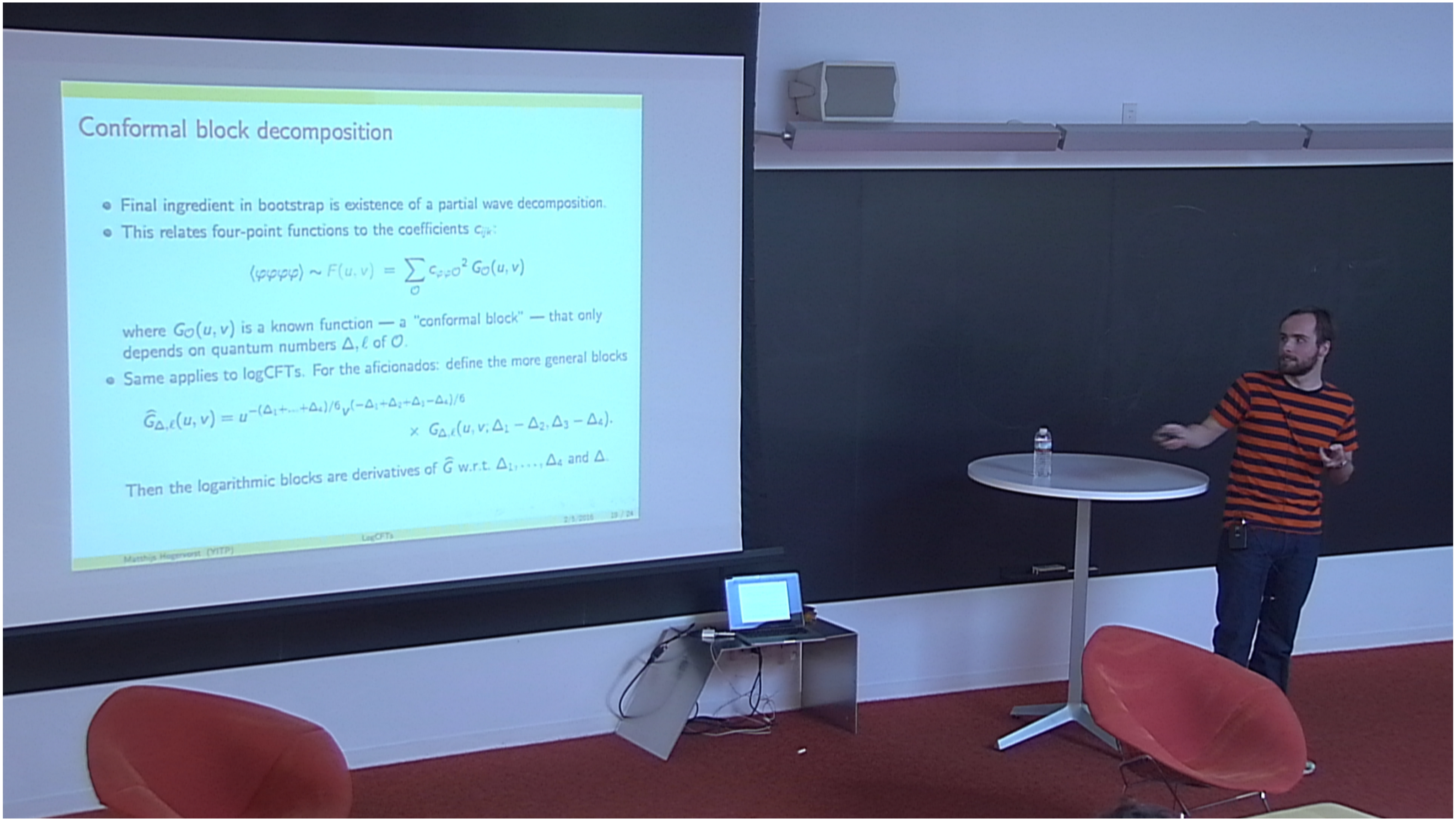
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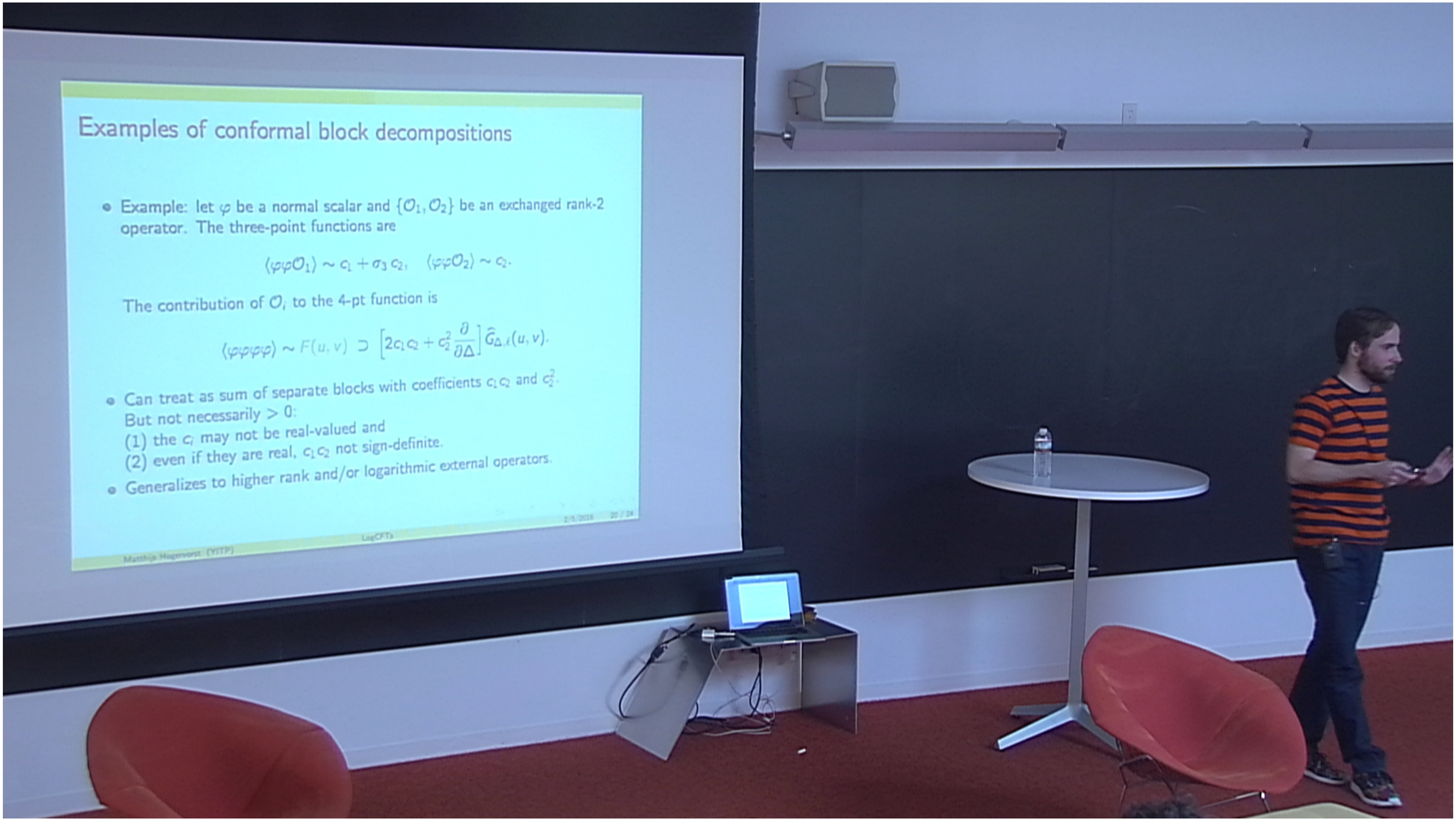
Examples of conformal block decompositions
Example: let φ be a normal scalar and $\{\mathcal{O}_1, \mathcal{O}_2\}$ be an exchanged rank-2 operator. The three-point functions are
$\langle \varphi\varphi\mathcal{O}_1 \rangle \sim c_1 + \sigma_3 c_2, \quad \langle \varphi\varphi\mathcal{O}_2 \rangle \sim c_2.$
The contribution of $\mathcal{O}_i$ to the 4-pt function is
$\langle \varphi\varphi\varphi\varphi \rangle \sim F(u,v) \supset \left[2c_1c_2 + c_2^2 \frac{\partial}{\partial\Delta} \right] \widehat{G}_{\Delta,\ell}(u,v).$
Can treat as sum of separate blocks with coefficients c_1c_2 and c_2^2.
But not necessarily > 0:
(1) the c_i may not be real-valued and
(2) even if they are real, c_1c_2 not sign-definite.
Generalizes to higher rank and/or logarithmic external operators.
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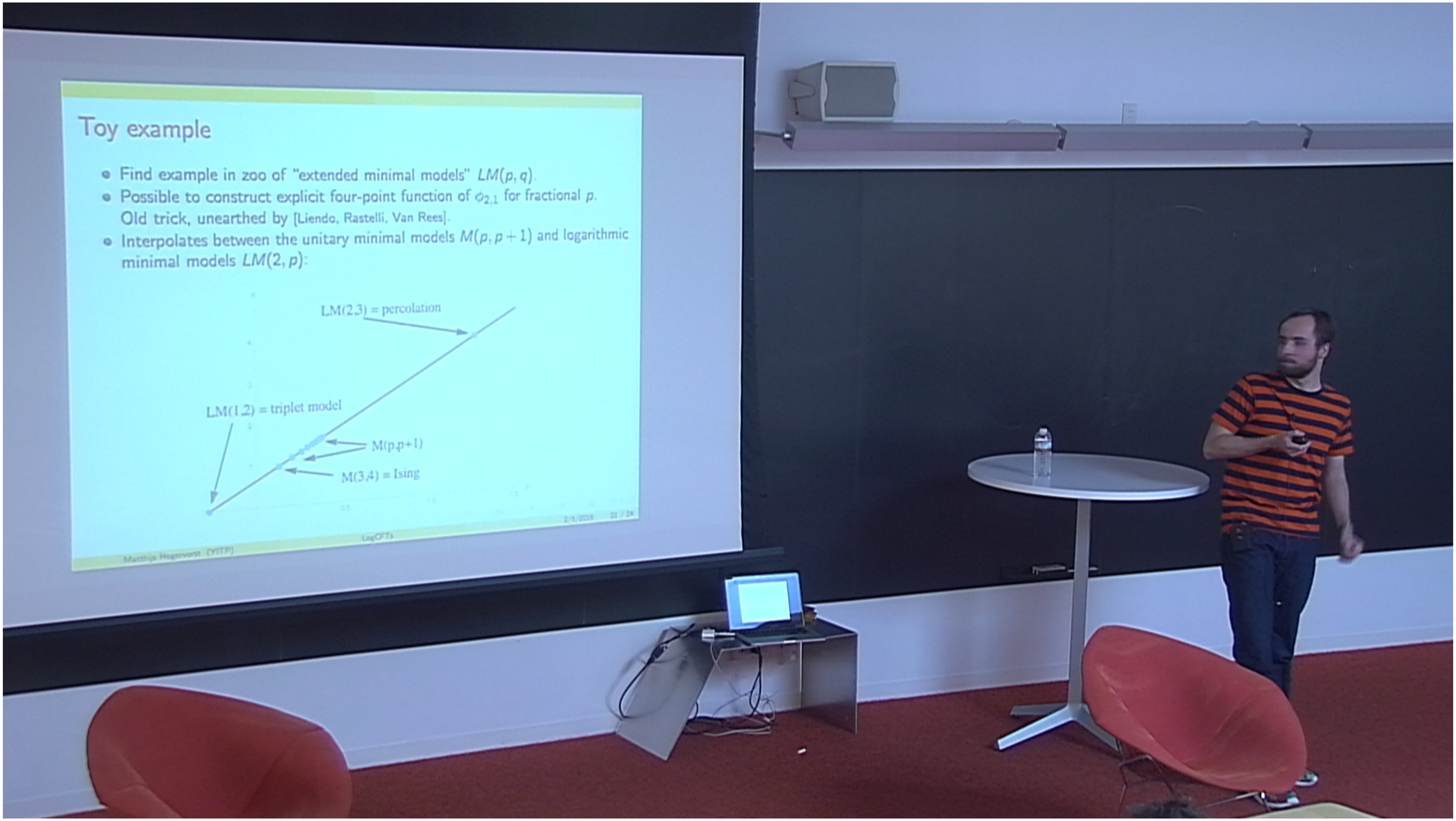

Toy example
Find example in zoo of "extended minimal models" LM(p, q).
Possible to construct explicit four-point function of φ2,1 for fractional p.
Old trick, unearthed by [Liendo, Rastelli, Van Rees].
Interpolates between the unitary minimal models M(p, p+1) and logarithmic minimal models LM(2, p):
LM(2,3) = percolation
LM(1,2) = triplet model
M(p,p+1)
M(3,4) = Ising
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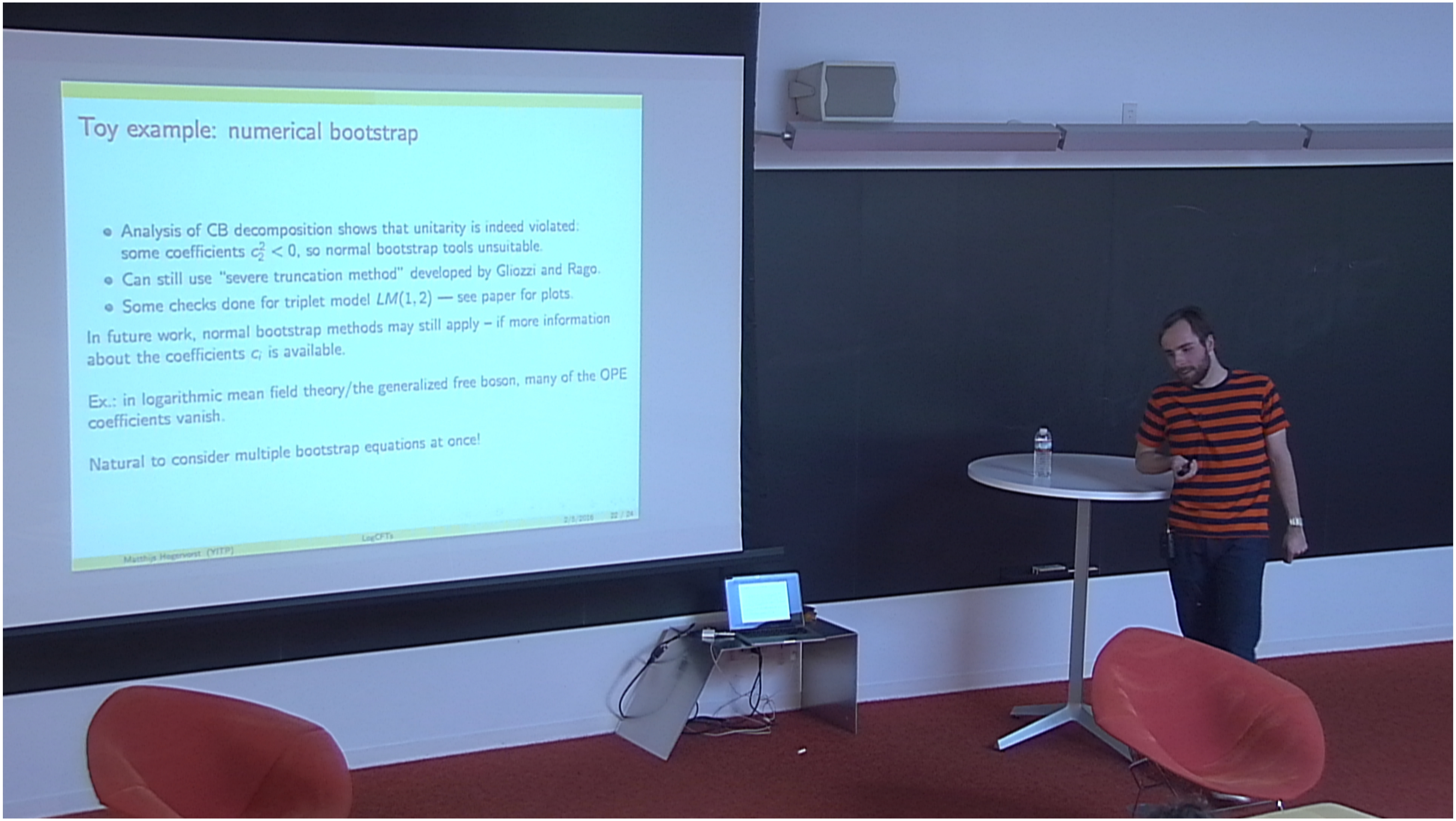
Toy example: numerical bootstrap
Analysis of CB decomposition shows that unitarity is indeed violated: some coefficients $c_2^2 < 0$, so normal bootstrap tools unsuitable.
Can still use "severe truncation method" developed by Gliozzi and Rago.
Some checks done for triplet model $LM(1,2)$ — see paper for plots.
In future work, normal bootstrap methods may still apply – if more information about the coefficients c_i is available.
Ex.: in logarithmic mean field theory/the generalized free boson, many of the OPE coefficients vanish.
Natural to consider multiple bootstrap equations at once!
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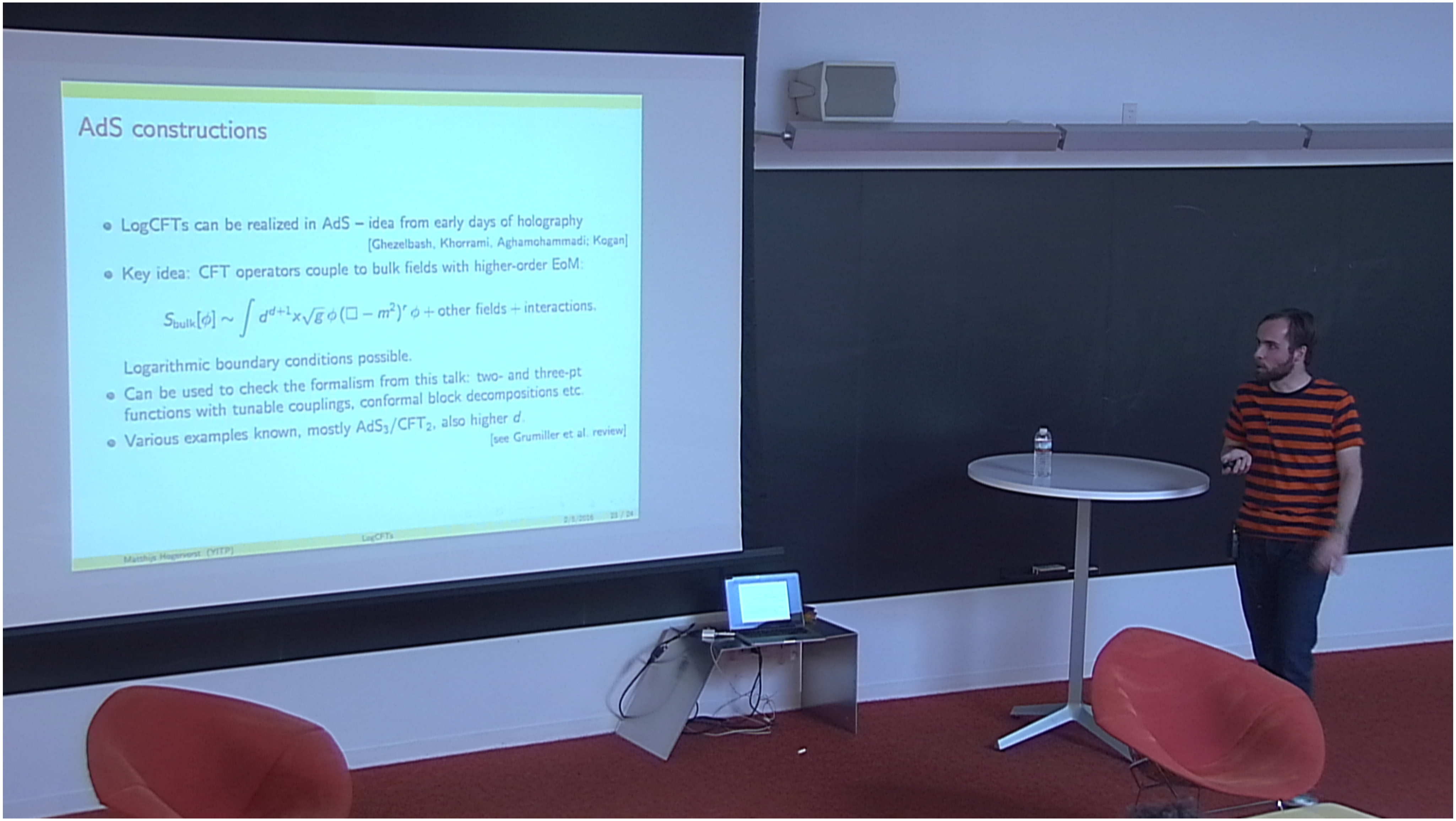
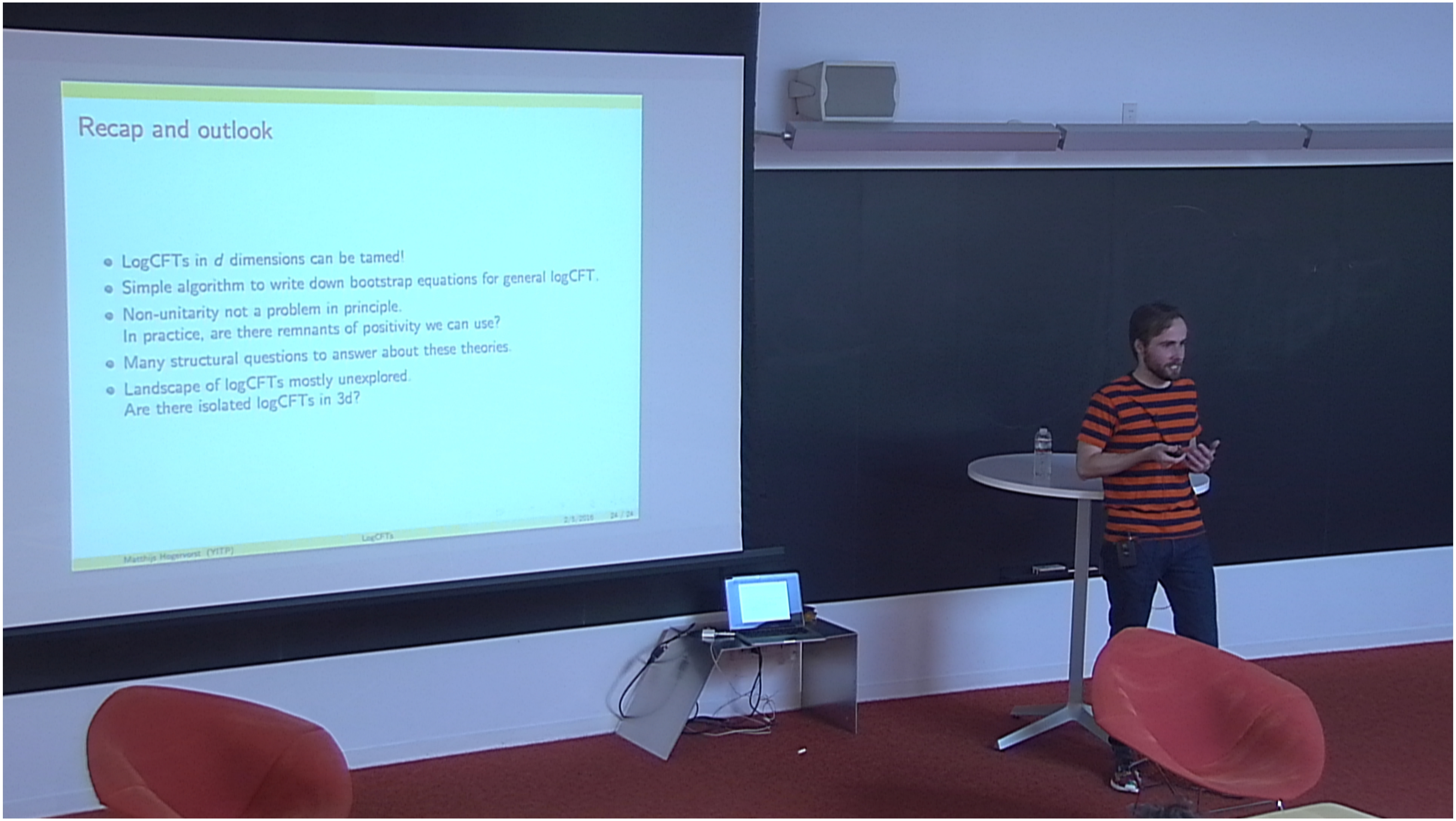

AdS constructions
LogCFTs can be realized in AdS – idea from early days of holography
[Ghezelbash, Khorrami, Aghamohammadi; Kogan]
Key idea: CFT operators couple to bulk fields with higher-order EoM:
$S_{\text{bulk}}[\phi] \sim \int d^{d+1}x\sqrt{g}\,\phi\,(\Box - m^2)^r\,\phi + \text{other fields} + \text{interactions.}$
Logarithmic boundary conditions possible.
Can be used to check the formalism from this talk: two- and three-pt functions with tunable couplings, conformal block decompositions etc.
Various examples known, mostly AdS_3/CFT_2, also higher d.
[see Grumiller et al. review]



Recap and outlook
LogCFTs in d dimensions can be tamed!
Simple algorithm to write down bootstrap equations for general logCFT.
Non-unitarity not a problem in principle.
In practice, are there remnants of positivity we can use?
Many structural questions to answer about these theories.
Landscape of logCFTs mostly unexplored.
Are there isolated logCFTs in 3d?
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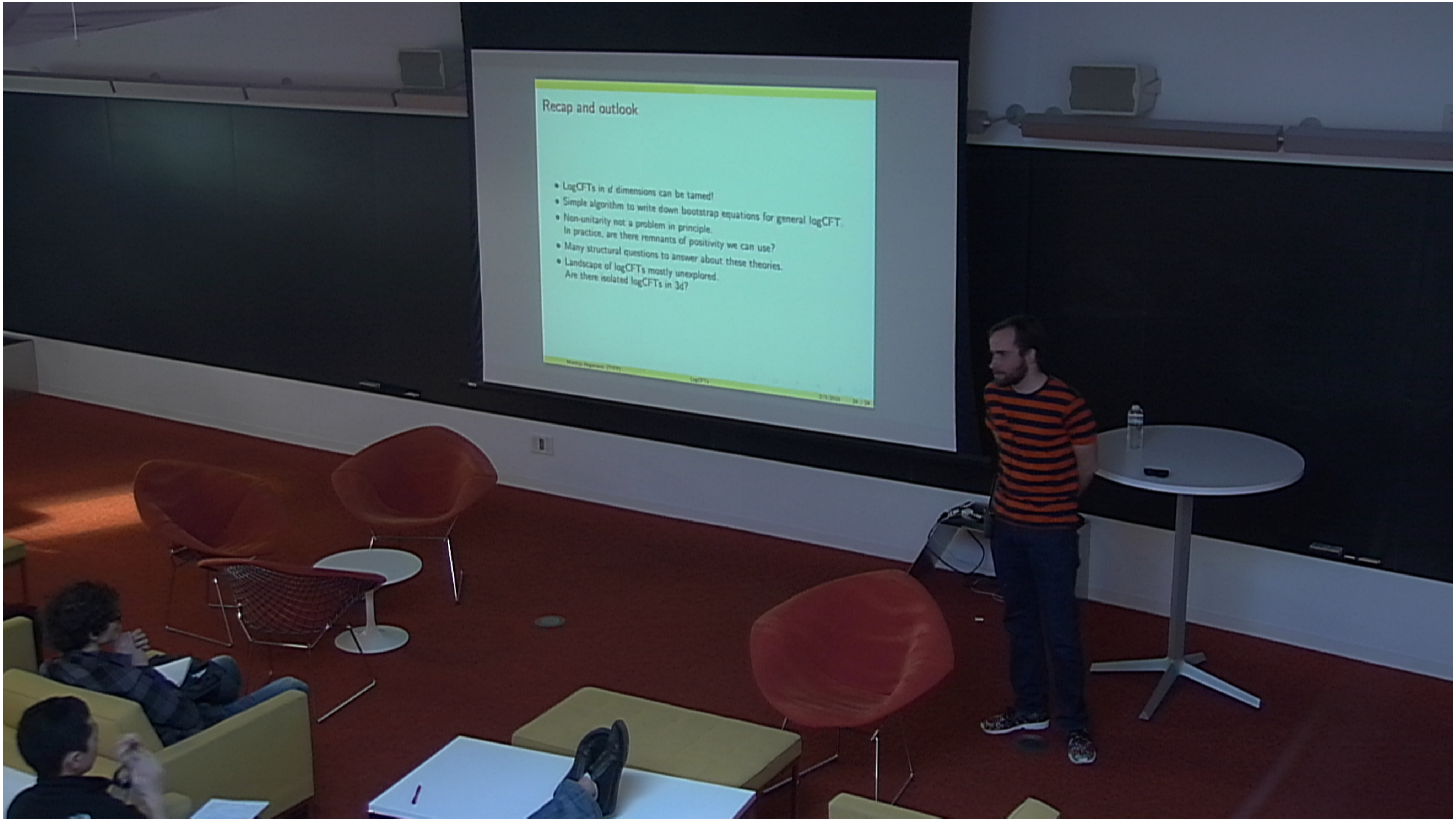
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