Title: Studying Strongly-Coupled Dynamics with Conformal Truncation

Date: Feb 23, 2016 02:00 PM

URL: http://pirsa.org/16020096

Abstract: <p>I will discuss ongoing work developing Hamiltonian truncation methods for studying strongly-coupled IR physics originating from a perturbed UV conformal field theory. This method uses a UV basis of conformal Casimir eigenstates, which is truncated at some maximum Casimir eigenvalue, to approximate the low energy spectrum of the IR theory. So far, such methods have been limited to theories in 2D, and I will present a new framework for generalizing this approach to higher dimensions. Focusing specifically on the case of scalar fields in 3D, I will then show tests of this framework by comparing with known analytic results at weak coupling and in the O(N) model at large-N, before discussing the possible application to strongly-coupled systems like the 3D Ising model.</p>

Studying Strongly-Coupled Dynamics with Conformal Truncation

Matthew T. Walters
Boston University

with E. Katz, Z. Khandker; N. Anand, V. Genest

arXiv: 1603.XXXXX

Perimeter Institute Theory Seminar, 2/23/16

Goal

Compute / approximate low-energy spectrum, correlation functions in strongly-coupled systems

Big Picture

UV CFT

$+ g^{d-\Delta}\mathcal{O}$

Strongly-Coupled IR

- ✦ Many calculational tools
  - Bootstrap, AdS/CFT, etc
- ✦ Potentially **non-interacting** (Fock space)

Can we use **UV tools** to study **IR dynamics**?

Punchline: Conformal Truncation

- Use **UV basis** of **conformal Casimir** eigenstates up to maximum eigenvalue $\mathcal{C}_{\max}$
- Diagonalize **truncated** Hamiltonian to obtain approximate **IR spectrum**, study correlators
- **Today:** Scalar field theory in $d = 2 + 1$
  - Develop framework for $d > 2$

Outline

- Conceptual Example: Rayleigh-Ritz Method
- Why Casimir Eigenstates?
- Difficulties Moving Beyond 2D
- Proposed Basis for Scalar Field Theory in 3D
- Testing the Method (Pert. Theory, Large N)
- Future Directions / Applications

Rayleigh-Ritz Method

Simple method for approximating energy eigenstates

- Lord Rayleigh (1877), Walter Ritz (1908)

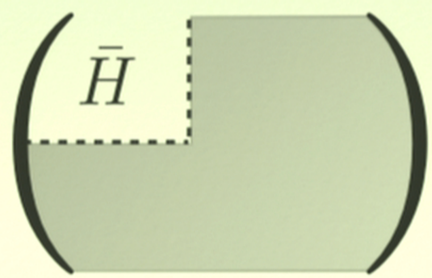
1) Start with complete basis of states $|n\rangle$

2) Truncate basis at some level $|n_{\max}\rangle$

3) Construct truncated matrix $\bar{H} \equiv \langle n|H|n'\rangle \quad (n, n' \leq n_{\max})$

4) Diagonalize $\bar{H}$

$$H =$$



Rayleigh-Ritz Method

QM Example: Anharmonic Oscillator

$$H = \frac{1}{2}p^2 + \frac{1}{2}x^2 + \lambda x^4$$

Perturbation series in λ **asymptotic**, doesn't converge

Use original harmonic oscillator states $|n\rangle$ to build:

$$\bar{H} = (n + \tfrac{1}{2})\delta_{nn'} + \lambda\langle n|(a^\dagger + a)^4|n'\rangle \quad (n, n' \leq n_{\max})$$

Unperturbed Hamiltonian

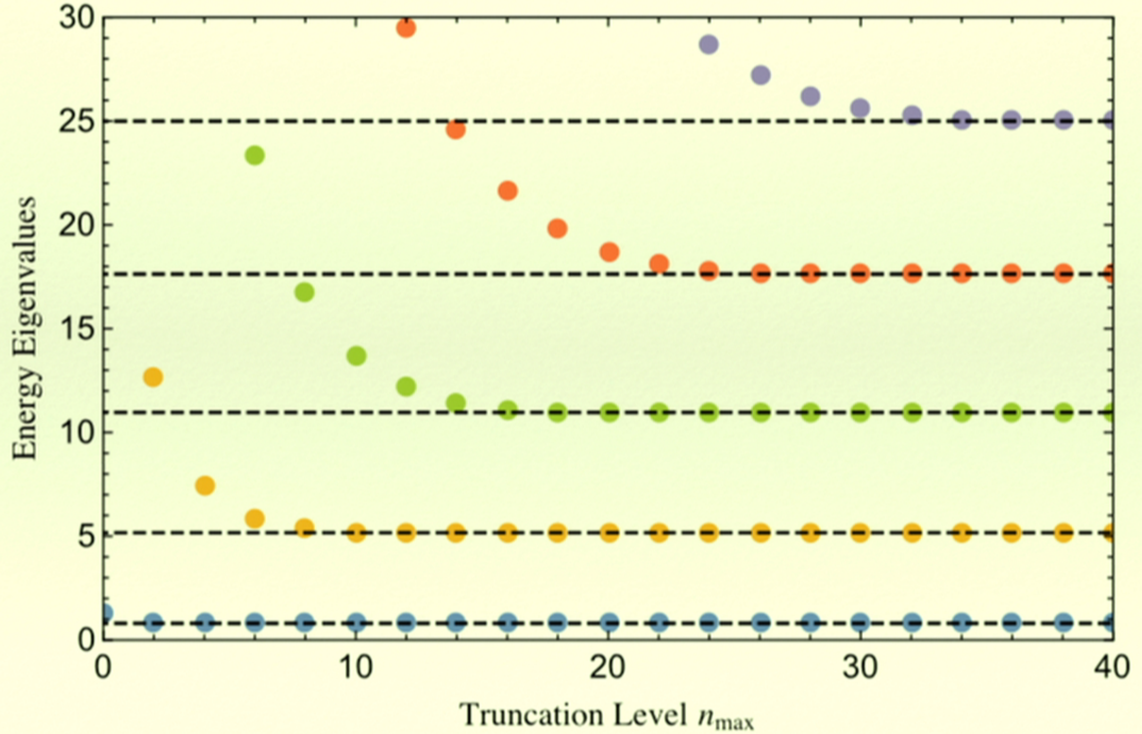
"Tree Level" Matrix Elements

Finite Matrix

7

Rayleigh-Ritz Method

Easy to diagonalize truncated matrix $\bar{H}$ to obtain: $(\lambda = 1)$



What's a Good Basis?

Need truncated basis with **large overlap** with IR states

Simpler Question: What's natural basis for UV CFT?

- Hilbert space built from local operators $\mathcal{O}(x)|0\rangle$
- Organized into eigenstates of quadratic Casimir:

$$\mathcal{C} = -D^2 - \frac{1}{2}(P_\mu K^\mu + K_\mu P^\mu) + \frac{1}{2}L_{\mu\nu}L^{\mu\nu}$$

- Eigenvalues set by dimension and spin of operator:

$$\mathcal{C} \to \Delta(\Delta - d) + \ell(\ell + d - 2) \sim \Delta^2$$

9

AdS/CFT Intuition

- Operator $\mathcal{O}(x)$
- Conformal Casimir $\mathcal{C}$

$\longleftrightarrow$

- "Object" $\Phi(x, z)$
- Invariant Mass M^2_{AdS}

Operators with large Casimir correspond to **heavy** objects in AdS

EFT in AdS: Operators with large scaling dimension **decouple** from low dimension ones

Heemskerk et al., '09; Fitzpatrick et al., '10

AdS/CFT Intuition

- Operator $\mathcal{O}(x)$
- Conformal Casimir $\mathcal{C}$

$\longleftrightarrow$

- "Object" $\Phi(x, z)$
- Invariant Mass M^2_{AdS}

Operators with large Casimir correspond to **heavy** objects in AdS

EFT in AdS: Operators with large scaling dimension **decouple** from low dimension ones

Heemskerk et al., '09; Fitzpatrick et al., '10

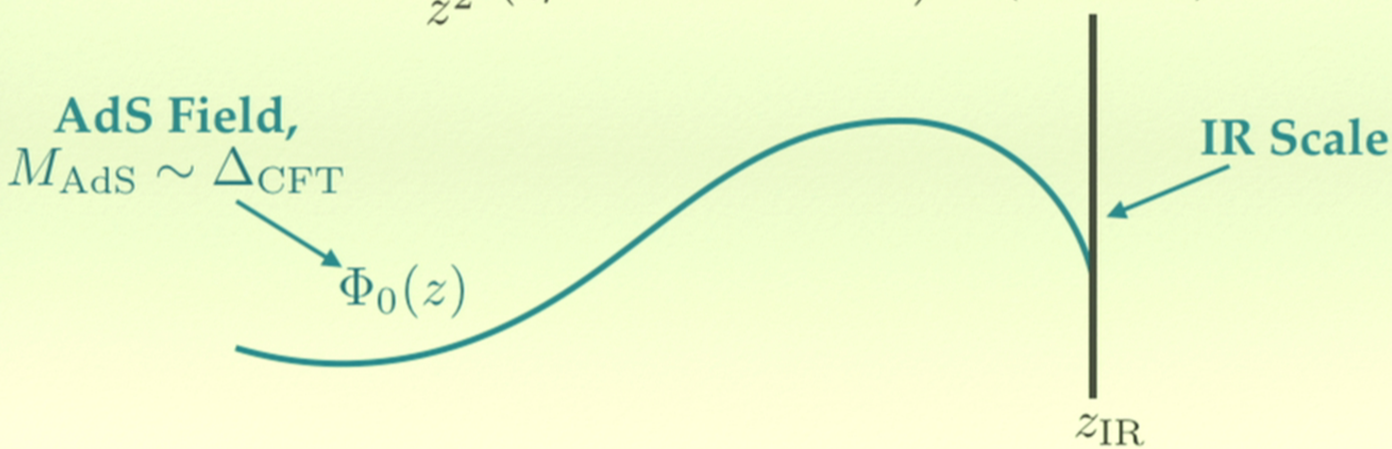
Decoupling of Heavy Operators

Fitzpatrick et al., '13

What about after breaking conformal symmetry?

Simple RS example: Scalar field with IR brane $(\mathrm{AdS}_5/\mathrm{CFT}_4)$

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right) \quad (z \leq z_{\mathrm{IR}})$$



What's the typical 4D mass scale from single AdS field?

Decoupling of Heavy Operators

Fitzpatrick et al., '13

Adding the IR boundary term $S_{\rm bdy} = -2b \int d^4x \sqrt{-g}\, \Phi^2$

leads to lowest KK mode 4D mass

$$m_0^2 \approx \frac{2}{z_{\rm IR}^2}(\Delta - b)(\Delta - 1) \sim \boxed{\frac{\Delta^2}{z_{\rm IR}^2}}$$

Unless tuned, high dimension operators typically **decouple** from IR states

Proof is in the Pudding

Holographic **intuition**: Low-dimension operators in UV CFT have large overlap with low-energy states in IR QFT

- Clearly theory-dependent

Field theory **demonstration**: Let's just try it!

Conformal Truncation

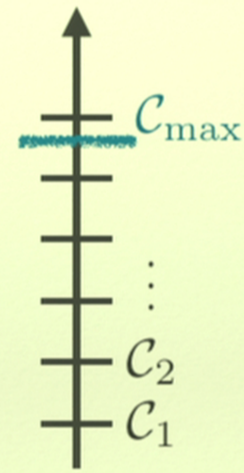
Strategy:

- UV basis of Casimir eigenstates $|\mathcal{C}\rangle \equiv \mathcal{O}(x)|0\rangle$
- Truncate basis at maximum eigenvalue $\mathcal{C}_{\max}$
- Diagonalize truncated Hamiltonian:

$$\bar{H} \equiv \langle \mathcal{C}_i|(H_{\mathrm{CFT}} + \delta H)|\mathcal{C}_j\rangle$$

- Obtain approximate energy eigenstates $|\bar{E}\rangle \approx |E\rangle$

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \to \langle \mathcal{O}(x) \sum_{\bar{E}} |\bar{E}\rangle\langle \bar{E}|\mathcal{O}(0)\rangle$$



Truncated Conformal Space Approach

Yurov, Zamolodchikov, '90

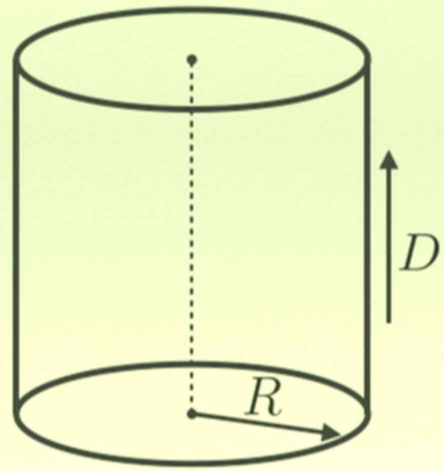
"Radial quantization" framework: $\mathbb{R} \times S^{d-1}$ ← **Finite Volume**

Scaling Dimension → Energy $E = \Delta/R$

Set UV cutoff $\Lambda_{\mathrm{UV}} \to \Delta_{\mathrm{max}}$

Successful in many 2D theories

- Minimal models, sine-Gordon, etc



15

TCSA in Higher Dimensions

Hogervorst et al., '14

Generalized TCSA to arbitrary d

Vacuum energy shifted by perturbation $\mathcal{L} = \mathcal{L}_{\mathrm{CFT}} + g^{d-\Delta}\mathcal{O}$

$(\sim g^2)$

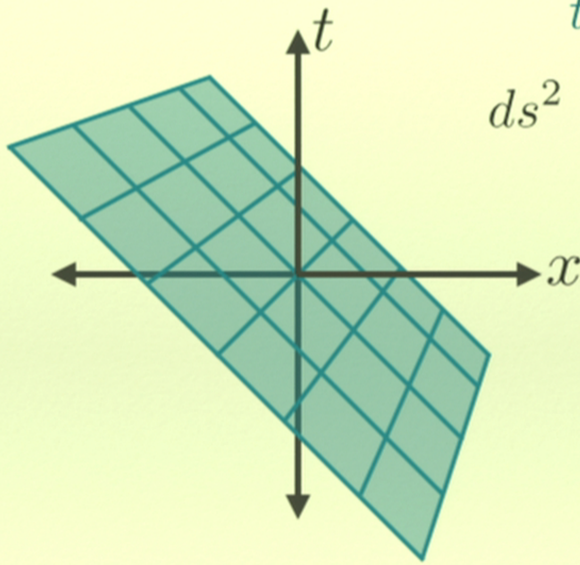
Diverges when $\Delta > \dfrac{d}{2}$

Example: Scalar field theory with $\mathcal{O} = \phi^4$

$$\Delta = 2(d-2) \to \text{diverges for } \boxed{d > \frac{8}{3}}$$

Alternative Method

Conformal truncation on the **lightcone**:



$$ds^2 = 2dx^+dx^- - dx^{\perp 2}$$

(dx^+: $t+x$; dx^-: $t-x$; $dx^{\perp}$: $y, z, \cdots$)

Quantize on surface of constant $x^+ \equiv \frac{1}{\sqrt{2}}(t+x)$

Lightcone Vacuum

Translationally-invariant interactions mix vacuum with excited states with $\vec{P} = 0$

Lightcone momentum: $P^2 = 2P_+ P_- - P_\perp^2$

Lightcone Energy (→ P_+)

Lightcone Momentum (→ P_-)

For all states **except** vacuum: $P_- > 0$

Interactions preserve lightcone vacuum!

$$\boxed{|0\rangle = |\Omega\rangle}$$

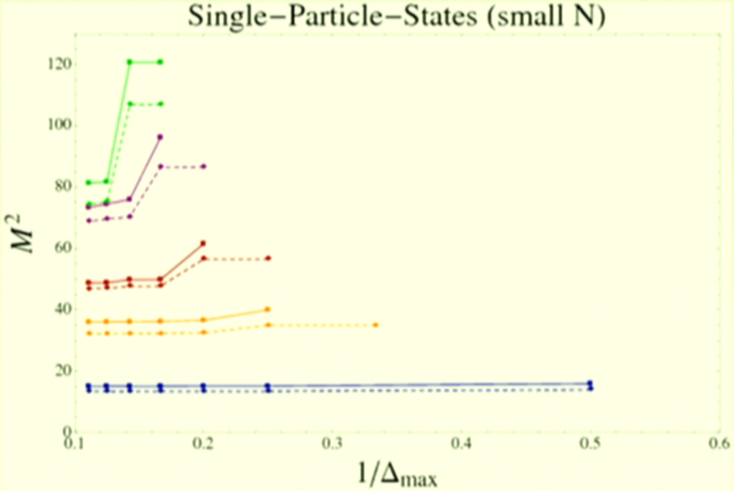
Yamakawi, '98 **DLCQ, etc**

Lightcone Truncation

Proposal:

Katz et al., '14

- UV basis of Casimir eigenstates on single lightcone slice $x^+ = 0$ with $\mathcal{C} \leq \mathcal{C}_{\max}$
- Successfully applied to 2D QCD (Hadron spectrum)



Going to Higher Dimensions

Lightcone momentum has **automatic** cutoff:

$$p_{i-} < P_-$$

Transverse momentum is **unbounded**, leading to divergences when $|p_{i\perp}| \to \infty$

Strategy: Hard UV cutoff on **invariant mass** of basis states

$$\mu^2 \leq \Lambda^2$$

Introduction of regulator **does not** spoil conformal truncation

Going to Higher Dimensions

Lightcone momentum has **automatic** cutoff:

$$p_{i-} < P_-$$

Transverse momentum is **unbounded**, leading to divergences when $|p_{i\perp}| \to \infty$

Strategy: Hard UV cutoff on **invariant mass** of basis states

$$\mu^2 \leq \Lambda^2$$

Introduction of regulator **does not** spoil conformal truncation

Scalar Field Theory in $d = 2 + 1$

UV CFT: Free field theory $\mathcal{L}_{\text{CFT}} = \frac{1}{2} {:}\partial_\mu \phi \partial^\mu \phi{:}$

Normal-Ordered

Relevant perturbations

$$\delta\mathcal{L} = -\frac{1}{2} m^2 {:}\phi^2{:} - \frac{1}{3!} g {:}\phi^3{:} - \frac{1}{4!} \lambda {:}\phi^4{:}$$

Free Scalar Field

$+ \, \delta\mathcal{L}(m, \lambda)$

Future Application

Ising Model $\langle \phi\phi \rangle \to \langle \sigma\sigma \rangle, \quad \langle \phi^2\phi^2 \rangle \to \langle \epsilon\epsilon \rangle$

21

Scalar Field Theory in $d = 2 + 1$

UV CFT: Free field theory $\mathcal{L}_{\text{CFT}} = \frac{1}{2}{:}\partial_\mu\phi\partial^\mu\phi{:}$ ← **Normal-Ordered**

Relevant perturbations

$$\delta\mathcal{L} = -\frac{1}{2}m^2{:}\phi^2{:} - \frac{1}{3!}g{:}\phi^3{:} - \frac{1}{4!}\lambda{:}\phi^4{:}$$

- **Free Scalar Field**

  $+\,\delta\mathcal{L}(m, \lambda)$

  Future Application

- **Ising Model** $\langle\phi\phi\rangle \to \langle\sigma\sigma\rangle, \quad \langle\phi^2\phi^2\rangle \to \langle\epsilon\epsilon\rangle$

21

Constructing Basis

Work in momentum space at fixed lightcone time:

$$\mathcal{O}(x) \to |\mathcal{O}(P_-, P_\perp)\rangle$$

Fix total momentum: $\vec{P} \equiv (P_-, P_\perp) = \boxed{(P_-, 0)}$

$$\boxed{|\mathcal{O}(P)\rangle = |\mathcal{C}, P_-, P_\perp\rangle}$$

Operators of the form

$$\mathcal{O}(x) = \sum_{\{m_n\}} c_{\{m_n\}} :\partial^{m_1}\phi(x)\partial^{m_2}\phi(x)\cdots\partial^{m_n}\phi(x):$$

Write in terms of Fock space modes:

$$\phi(x) \to a_p^\dagger, a_p$$

Basis Functions

Momentum space form:

$$|\mathcal{O}(P)\rangle = \int \frac{d^2p_1 \cdots d^2p_n}{(2\pi)^{2n} 2p_{1-} \cdots 2p_{n-}} (2\pi)^2 \delta^2(P - p_i)\, F(p) |p_1, \cdots, p_n\rangle$$

Lorentz-Invariant Measure

Total Momentum Fixed

***n*-Particle State**

$$F(p) \equiv \sum_{\{m_n\}} c_{\{m_n\}}\, p_1^{m_1} \cdots p_n^{m_n}$$

Need to find basis of **Casimir eigenfunctions**: $\mathcal{C}F(p) = \lambda F(p)$

Conformal Casimir

Example: **Two-particle** states $\boxed{\mathcal{O} \sim \phi \partial_{\mu_1} \cdots \partial_{\mu_\ell} \phi}$ $\quad (\partial^2 \phi = 0)$

Expect $\Delta = 2\Delta_\phi + \ell \quad (\Delta_\phi = \frac{1}{2}) \to \mathcal{C} = \boxed{2(\Delta - 1)^2 - 2}$

Total momentum **fixed**: $p_1, p_2 \to p$

Can write Casimir as differential operator:

$$\mathcal{C} = -2 - 2p_-(P_- - p_-)\frac{\partial^2}{\partial p_-^2} + (2p_- - P_-)\frac{\partial}{\partial p_-} + 2p_\perp \frac{\partial}{\partial p_\perp} + 2p_\perp(2p_- - P_-)\frac{\partial^2}{\partial p_- \partial p_\perp} - \frac{p_\perp^2 (2p_- - P_-)^2}{2p_-(P_- - p_-)} \frac{\partial^2}{\partial p_\perp^2}$$

Casimir Eigenstates

Invariant mass for two-particle states:

$$M^2 = 2P_+P_- - P_\perp^2 = \frac{p_\perp^2 P_-^2}{p_-(P_- - p_-)}$$

Define new variable:

$$\tilde{p} \equiv p_\perp \frac{P_-}{\sqrt{p_-(P_- - p_-)}}$$

Compute Casimir in new coordinates p_-, $\tilde{p}$

$$\mathcal{C} = -2 - 2p_-(P_- - p_-)\frac{\partial^2}{\partial p_-^2} + (2p_- - P_-)\frac{\partial}{\partial p_-}$$

Independent of new variable $\tilde{p}$! $\left([\mathcal{C}, M^2] = 0\right)$

Two-Particle Basis

$$\boxed{F(p_-, p_\perp) \to F(p_-, \tilde{p})}$$

Imposing hard cutoff on invariant mass $\tilde{p}^2 \leq \Lambda^2$:

$$F(p) = P_{\Delta_{\rm UV}-1}^{(-\frac{1}{2},-\frac{1}{2})}\left(\frac{2p_-}{P_-} - 1\right) P_k\left(\frac{\tilde{p}}{\Lambda}\right)$$

Specifies Casimir Multiplet

Specifies State in Multiplet

Casimir Eigenvalues: $\boxed{\mathcal{C} = 2(\Delta_{\rm UV} - 1)^2 - 2}$

Conformal multiplets of primary + lightcone descendants

$$\tilde{p} \sim P_+ \to \boxed{\mathcal{O},\ P_+\mathcal{O},\ P_+^2\mathcal{O},\ \cdots}$$

26

Casimir Eigenstates

Invariant mass for two-particle states:

$$M^2 = 2P_+P_- - P_\perp^2 = \frac{p_\perp^2 P_-^2}{p_-(P_- - p_-)}$$

Define new variable:

$$\tilde{p} \equiv p_\perp \frac{P_-}{\sqrt{p_-(P_- - p_-)}}$$

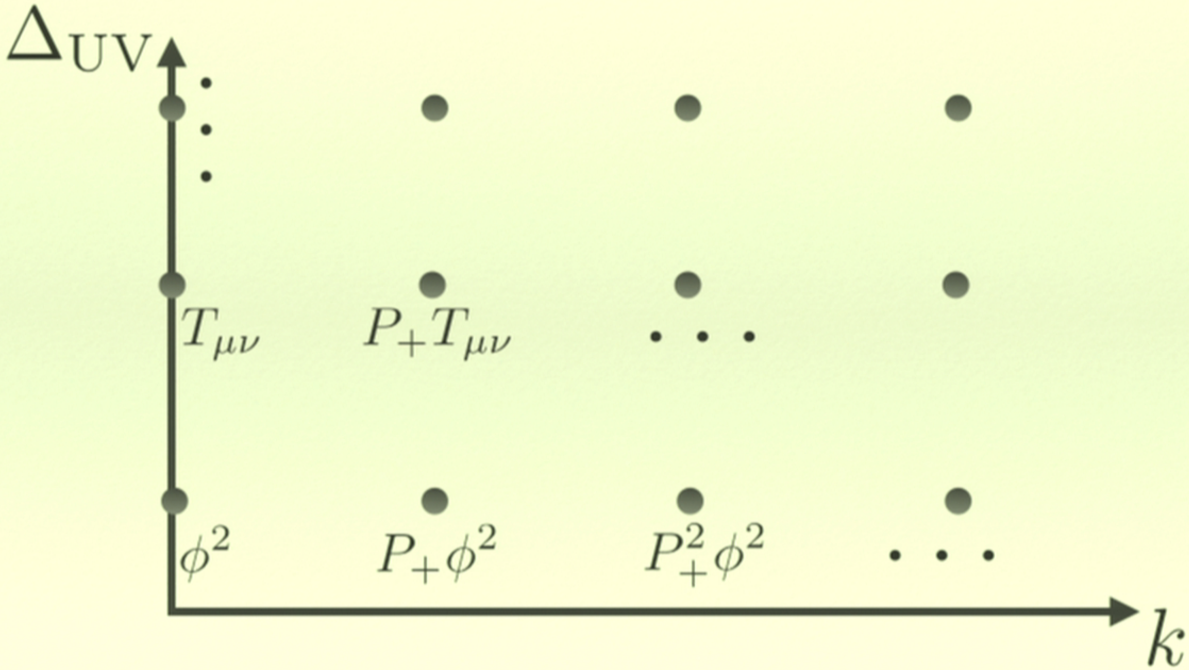
Compute Casimir in new coordinates p_-, $\tilde{p}$

$$\mathcal{C} = -2 - 2p_-(P_- - p_-)\frac{\partial^2}{\partial p_-^2} + (2p_- - P_-)\frac{\partial}{\partial p_-}$$

Independent of new variable $\tilde{p}$! $\left([\mathcal{C}, M^2] = 0\right)$

Two-Particle Basis

$$F(p) = P^{(-\frac{1}{2},-\frac{1}{2})}_{\Delta_{\mathrm{UV}}-1}\left(\frac{2p_-}{P_-}-1\right) P_k\left(\frac{\tilde{p}}{\Lambda}\right)$$



Symmetrization of Basis

Basis states built from **indistinguishable** particles

Need to restrict to functions invariant under exchange:

$$p_1 \leftrightarrow p_2$$

Reduces basis to: odd $\Delta_{\mathrm{UV}} \times$ even k OR even $\Delta_{\mathrm{UV}} \times$ odd k

More generally, need functions invariant under symmetric group S_n:

$$p_i \leftrightarrow p_j \quad (\forall\, i, j)$$

Lightcone Hamiltonian

For **Lorentz-invariant** spectrum, truncate and diagonalize:

$$M^2 = 2P_+P_- - P_\perp^2$$

"Tree Level"

Calculate matrix elements: $\mathcal{M}_{ij} \equiv 2P_- \langle \mathcal{C}_i | P_+ | \mathcal{C}_j \rangle$

Have **two** parameters to truncate: $\Delta_{\max},\ k_{\max}$

Number of Multiplets **Size of Multiplets**

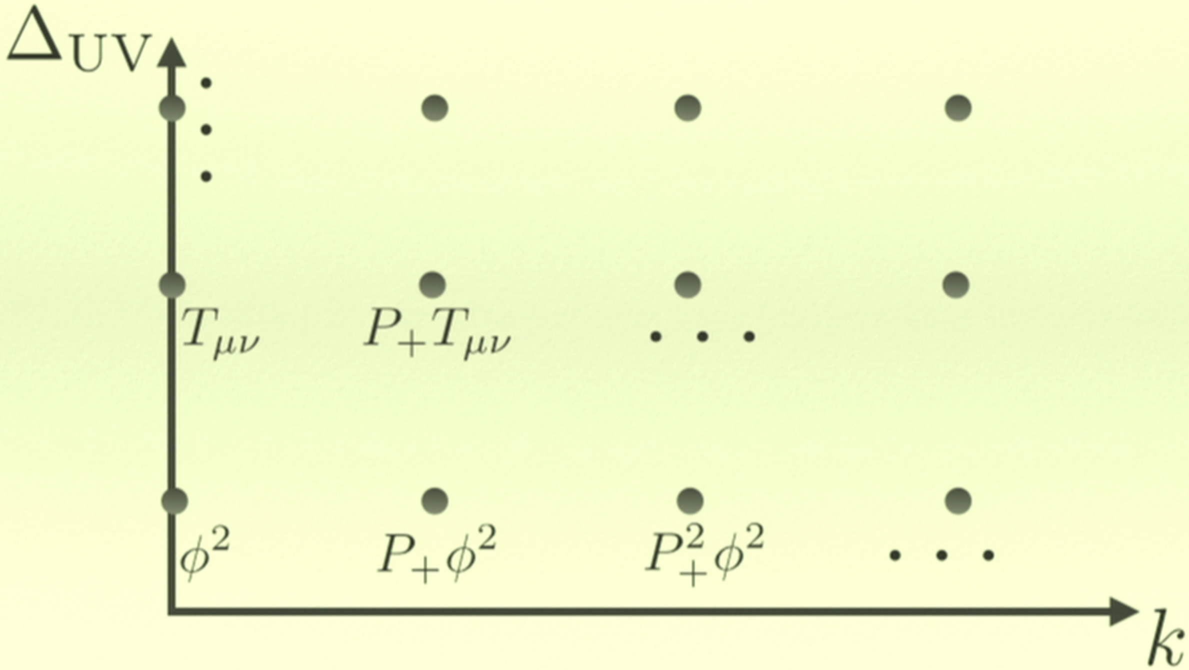
Simple test: CFT Hamiltonian

$$P_+^{(\mathrm{CFT})} = \int \frac{d^2p}{(2\pi)^2} a_p^\dagger a_p \boxed{\frac{p_\perp^2}{2p_-}} \sim \tilde{p}^2$$

Only mixes states within **same** multiplet! $\Big([\mathcal{C}, P_+] = 0\Big)$

Two-Particle Basis

$$F(p) = P^{(-\frac{1}{2},-\frac{1}{2})}_{\Delta_{\mathrm{UV}}-1}\left(\frac{2p_-}{P_-}-1\right)P_k\left(\frac{\tilde{p}}{\Lambda}\right)$$



27

Lightcone Hamiltonian

For **Lorentz-invariant** spectrum, truncate and diagonalize:

$$M^2 = 2P_+P_- - P_\perp^2$$

"Tree Level"

Calculate matrix elements: $\mathcal{M}_{ij} \equiv 2P_- \langle \mathcal{C}_i | P_+ | \mathcal{C}_j \rangle$

Have **two** parameters to truncate: $\Delta_{\max}$, $k_{\max}$

Number of Multiplets **Size of Multiplets**

Simple test: CFT Hamiltonian

$$P_+^{(\mathrm{CFT})} = \int \frac{d^2p}{(2\pi)^2} a_p^\dagger a_p \boxed{\frac{p_\perp^2}{2p_-}} \sim \tilde{p}^2$$

Only mixes states within **same** multiplet! $\left([\mathcal{C}, P_+] = 0\right)$

29

Observable: Spectral Density

Result: Approximate mass eigenstates $|\mu_i\rangle$

Compute Källén-Lehmann spectral densities:

$$\rho_{\mathcal{O}}(\mu^2) \equiv \sum_i |\langle \mathcal{O}(0)|\mu_i\rangle|^2 \delta(\mu^2 - \mu_i^2)$$

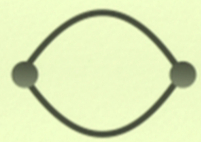
Encodes **dynamical** information from 2-pt functions:

$$\langle T\{\mathcal{O}(x)\mathcal{O}(0)\}\rangle = \int \frac{d^3P}{(2\pi)^3} e^{iP\cdot x} \int d\mu^2 \frac{\rho_{\mathcal{O}}(\mu^2)}{P^2 - \mu^2 + i\epsilon}$$

Example: ϕ^2

In practice, simpler to calculate **integrated density**:

$$I_{\mathcal{O}}(\mu^2) \equiv \int_0^{\mu^2} d\mu'^2 \rho_{\mathcal{O}}(\mu'^2) = \sum_{\mu_i \leq \mu} |\langle \mathcal{O}(0)|\mu_i\rangle|^2$$

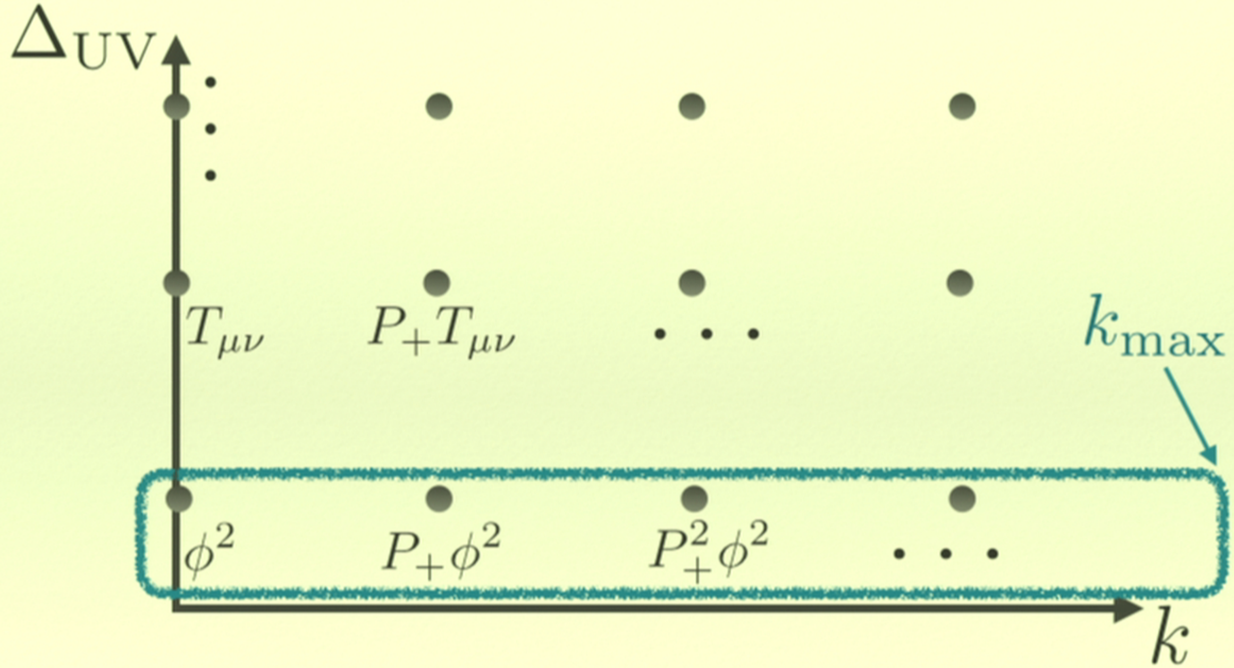
$\langle \phi^2(x)\phi^2(0)\rangle =$  $\rightarrow I_{\phi^2}(\mu^2) = \dfrac{\mu}{2\pi}$ $\left(\sim \mu^{2\Delta_{\phi^2}-1}\right)$

Only need ϕ^2 Casimir multiplet $\rightarrow \Delta_{\rm UV} = 1$

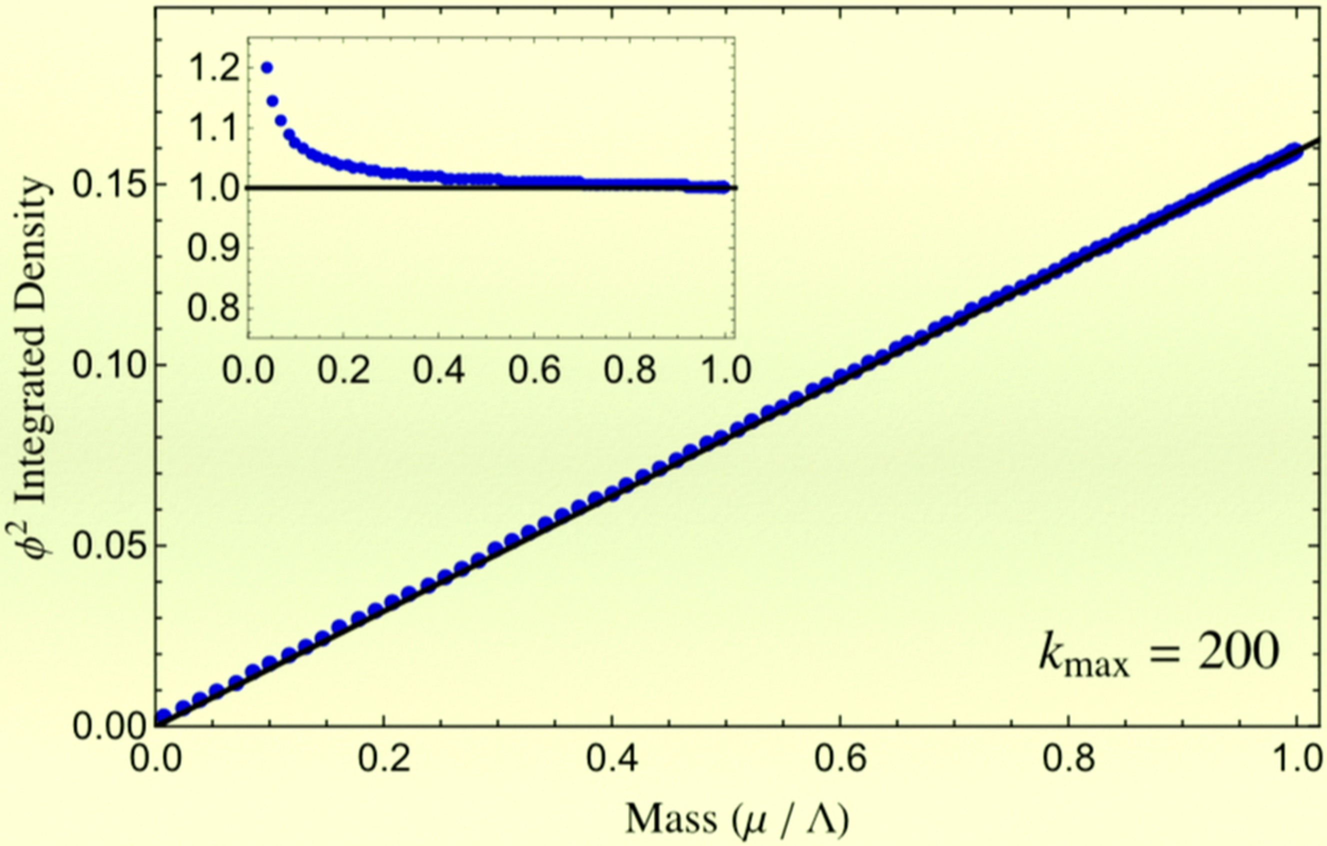
Truncate multiplet at $k_{\rm max}$

31

ϕ^2 Spectral Density



ϕ^2 Spectral Density



CFT Mass Eigenstates

Primary + descendants combine to form approximate mass eigenstates:

$$F_{\mu_i}(p_-, \tilde{p}) \sim \sum_{k=0}^{k_{\max}} (k + \tfrac{1}{2}) P_k(\mu_i) P_k(\tilde{p}) \approx \delta(\tilde{p}^2 - \mu_i^2)$$

Reconstructing continuum of **KK modes** associated with each Casimir multiplet (AdS field)

Generalizing to $O(N)$

Can extend basis to N free fields ϕ_I

$$\mathcal{L}_{\rm CFT} = \sum_{I=1}^{N} \frac{1}{2} :\partial_\mu \phi_I \partial^\mu \phi_I:$$

Basis of two-particle singlets **identical** to single-field case

Perturb CFT with relevant interaction:

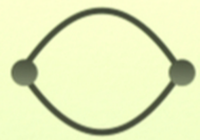
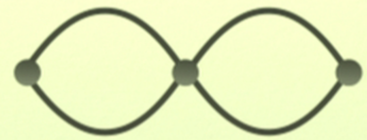
$$\delta \mathcal{L} = -\sum_{I,J} \frac{1}{4} \lambda :\phi_I^2 \phi_J^2:$$

$N \to \infty$ limit with $\kappa \equiv \lambda N$ **fixed**: Particle number mixing **suppressed** by $1/N$

Large N Limit

Can resum leading contributions to spectral density for

$$\vec{\phi}^2 \equiv \frac{1}{\sqrt{N}} \sum_I :\phi_I^2:$$

$$\langle \vec{\phi}^2(x) \vec{\phi}^2(0) \rangle =$$  $$+$$  $$+ \cdots$$

$$\rightarrow \boxed{I_{\vec{\phi}^2}(\mu^2) = \frac{\mu}{2\pi} - \frac{\kappa}{16\pi} \tan^{-1}\left(\frac{8\mu}{\kappa}\right)}$$

$$\text{UV: } I_{\vec{\phi}^2}(\mu^2) \rightarrow \frac{\mu}{2\pi} \quad (\Delta_{\vec{\phi}^2} \rightarrow 1)$$

$$\text{IR: } I_{\vec{\phi}^2}(\mu^2) \rightarrow \frac{32\mu^3}{3\pi\kappa^2} \quad (\Delta_{\vec{\phi}^2} \rightarrow 2)$$

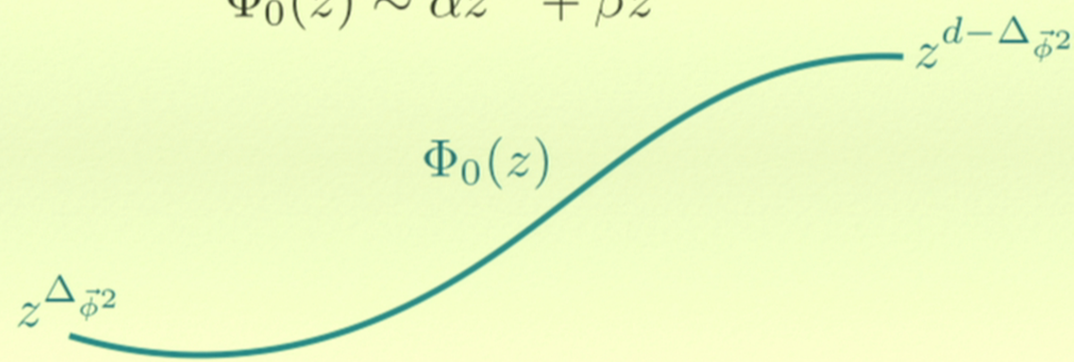
36

Matching AdS Intuition

"Single-trace" operator $\vec{\phi}^2(x) \leftrightarrow$ AdS field $\Phi(x, z)$

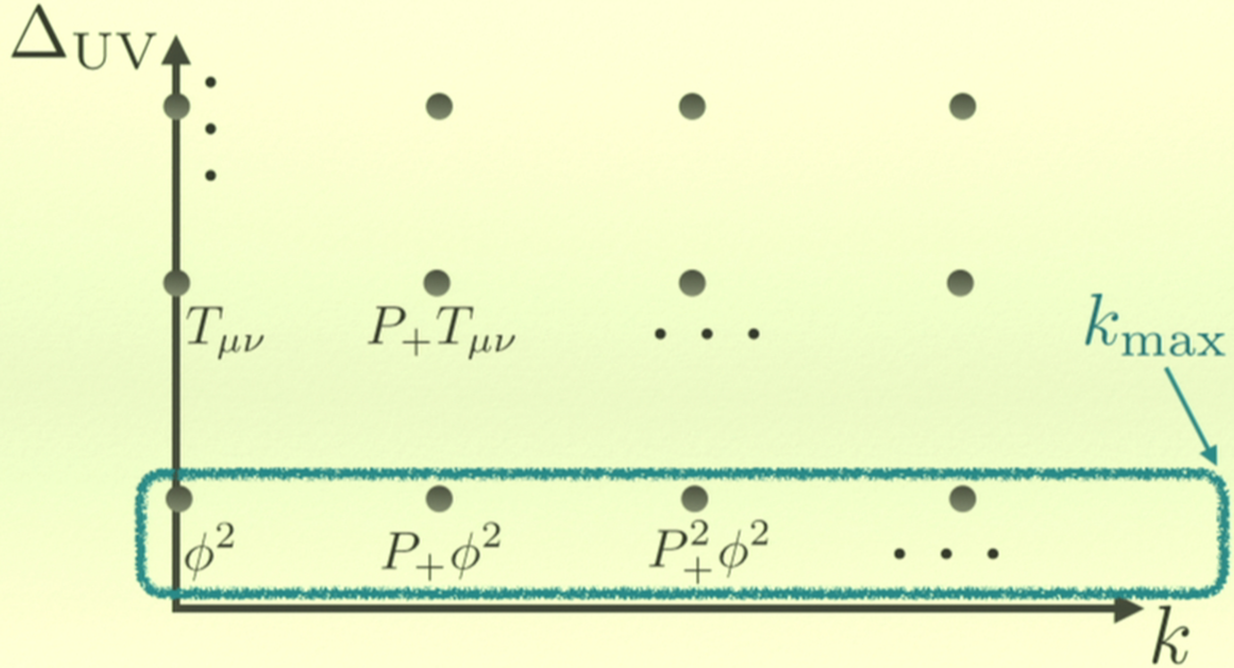
Adding "double-trace" perturbation $\left(\vec{\phi}^2\right)^2$ equivalent to modifying boundary conditions for Φ Witten, '01

$$\Phi_0(z) \sim \alpha z^{\Delta} + \beta z^{d-\Delta}$$

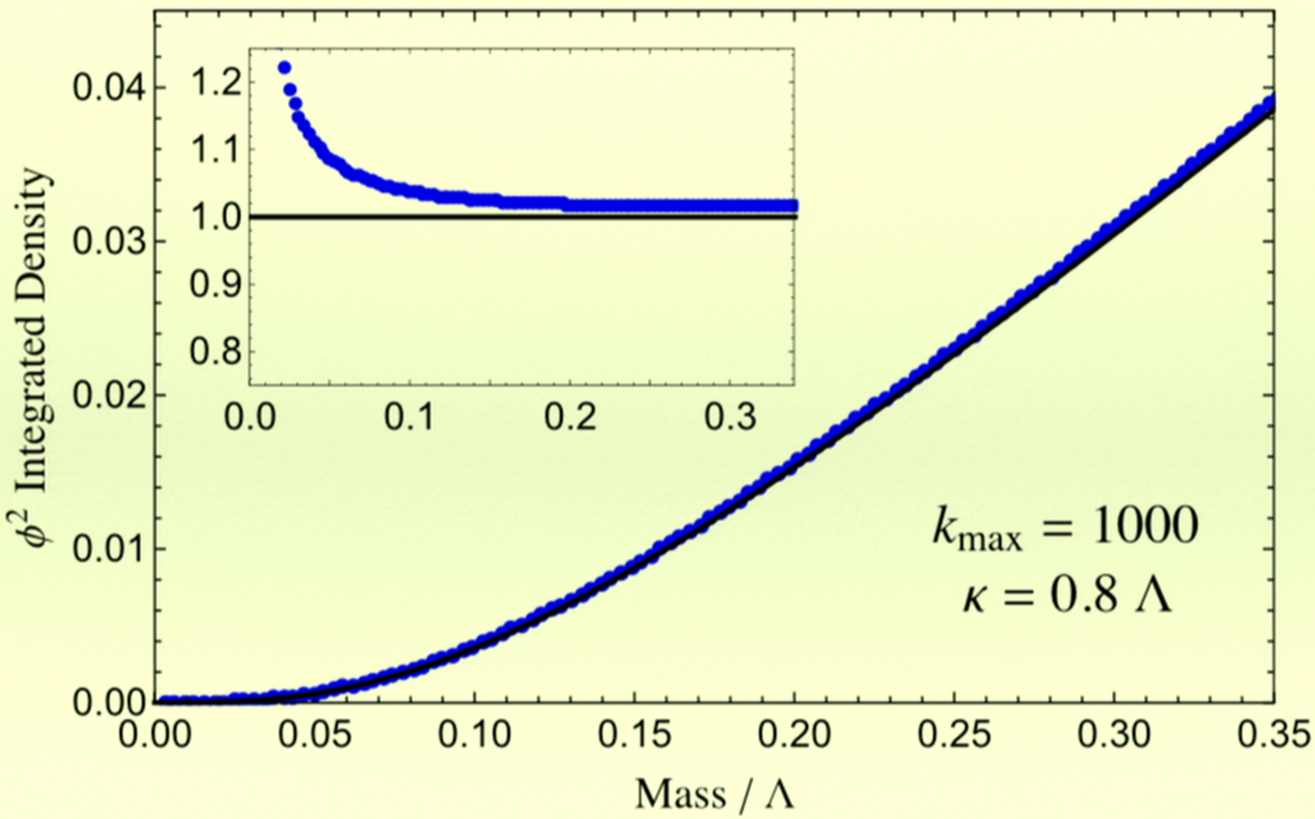


Interaction only affects $\vec{\phi}^2$ multiplet ($\Delta_{\mathrm{UV}} = 1$), consistent with AdS expectations

Large N $\vec{\phi}^2$ Spectral Density



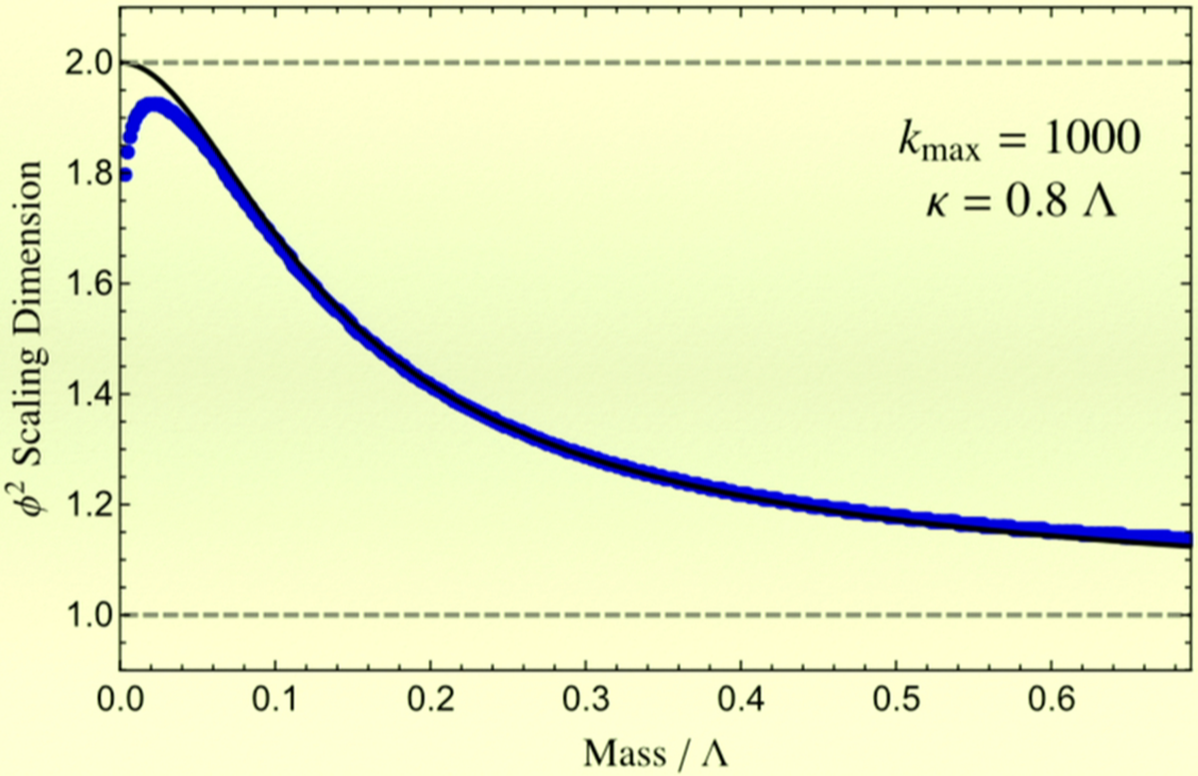
Large N $\vec{\phi}^2$ Spectral Density



39

Large N RG Flow

$$I_{\vec{\phi}^2}(\mu^2) \sim \mu^{2\Delta_{\vec{\phi}^2}-1} \rightarrow \Delta_{\vec{\phi}^2} \sim \frac{1}{2}\left(\mu \frac{I'(\mu^2)}{I(\mu^2)} + 1\right)$$



Adding Mass

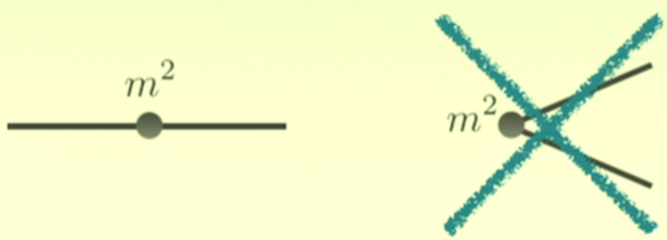
Mass term shifts lightcone Hamiltonian:

$$\delta P_+^{(m)} = \int \frac{d^2p}{(2\pi)^2} a_p^\dagger a_p \boxed{\frac{m^2}{2p_-}}$$

Independent of $\tilde{p}$

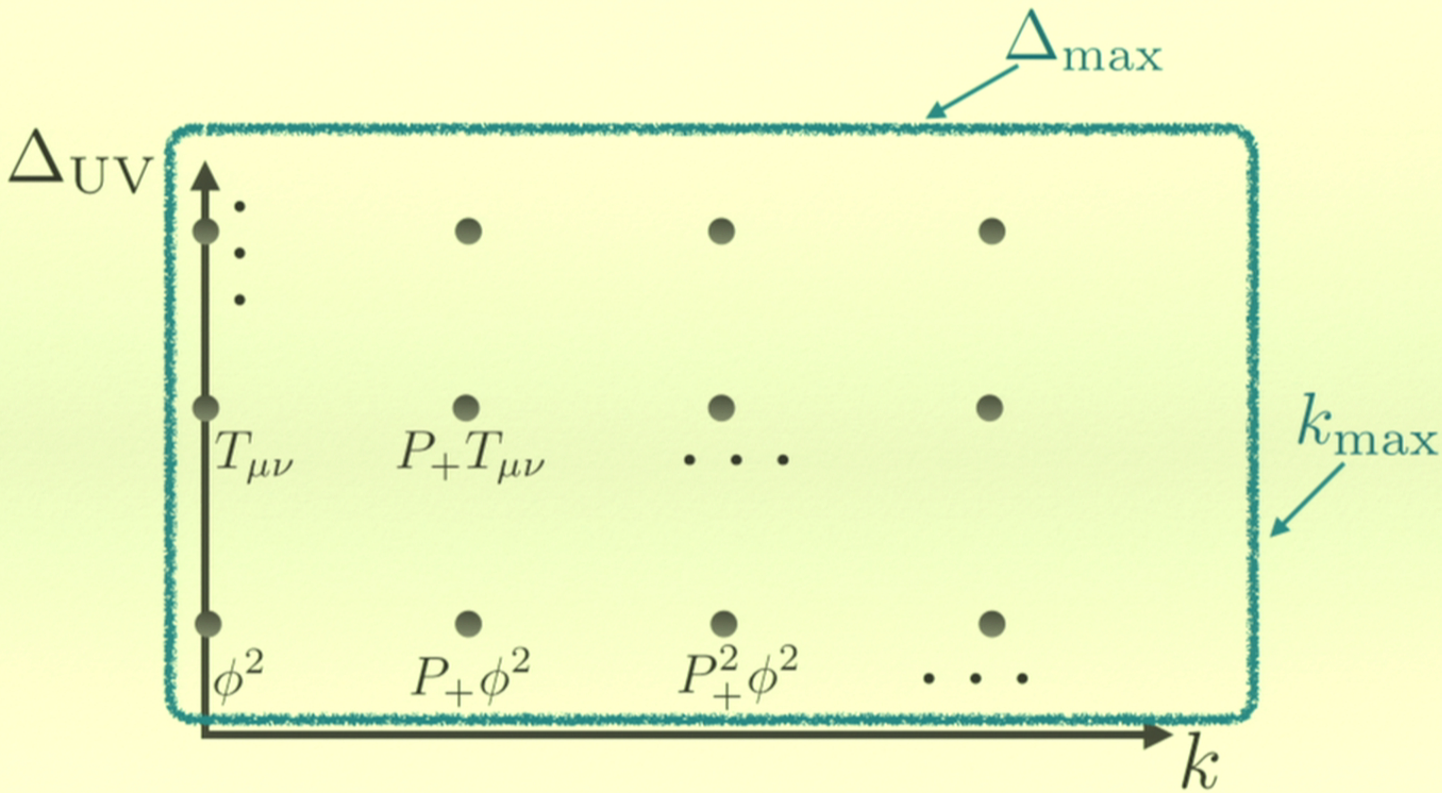
Mixes Casimir multiplets, **preserves** combination of primary + descendants

Also preserves **particle number** $\rightarrow$ advantage of lightcone quantization



41

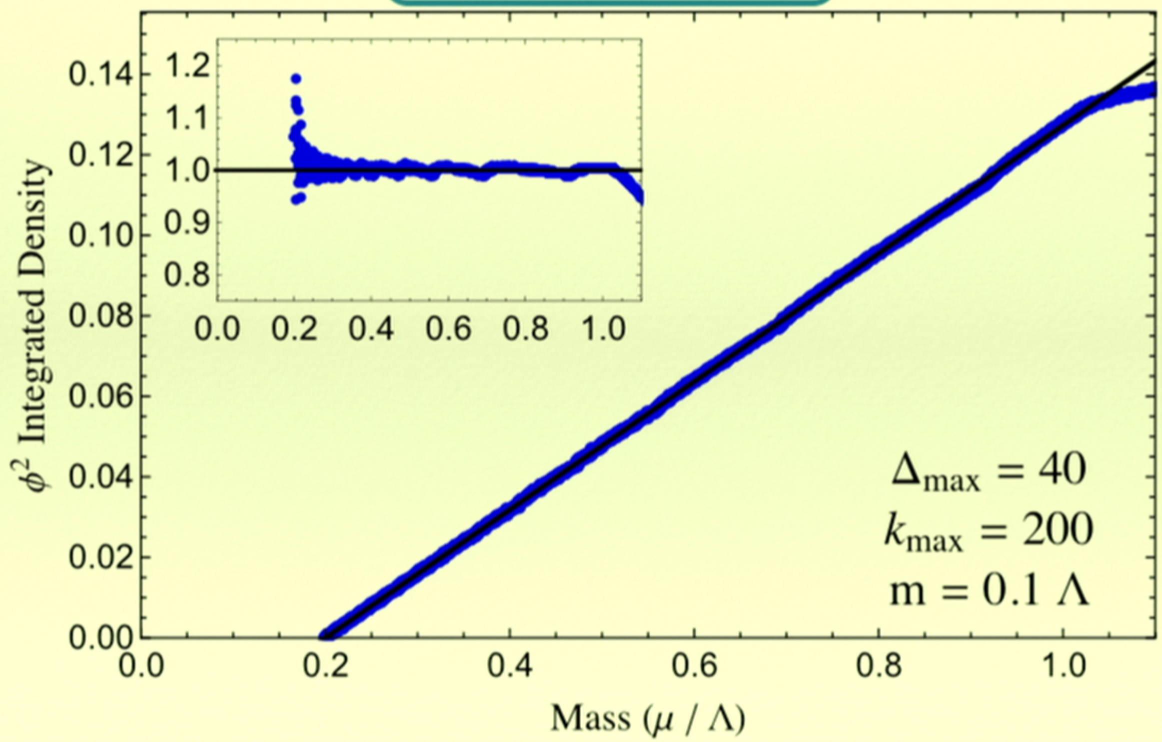
Adding Mass



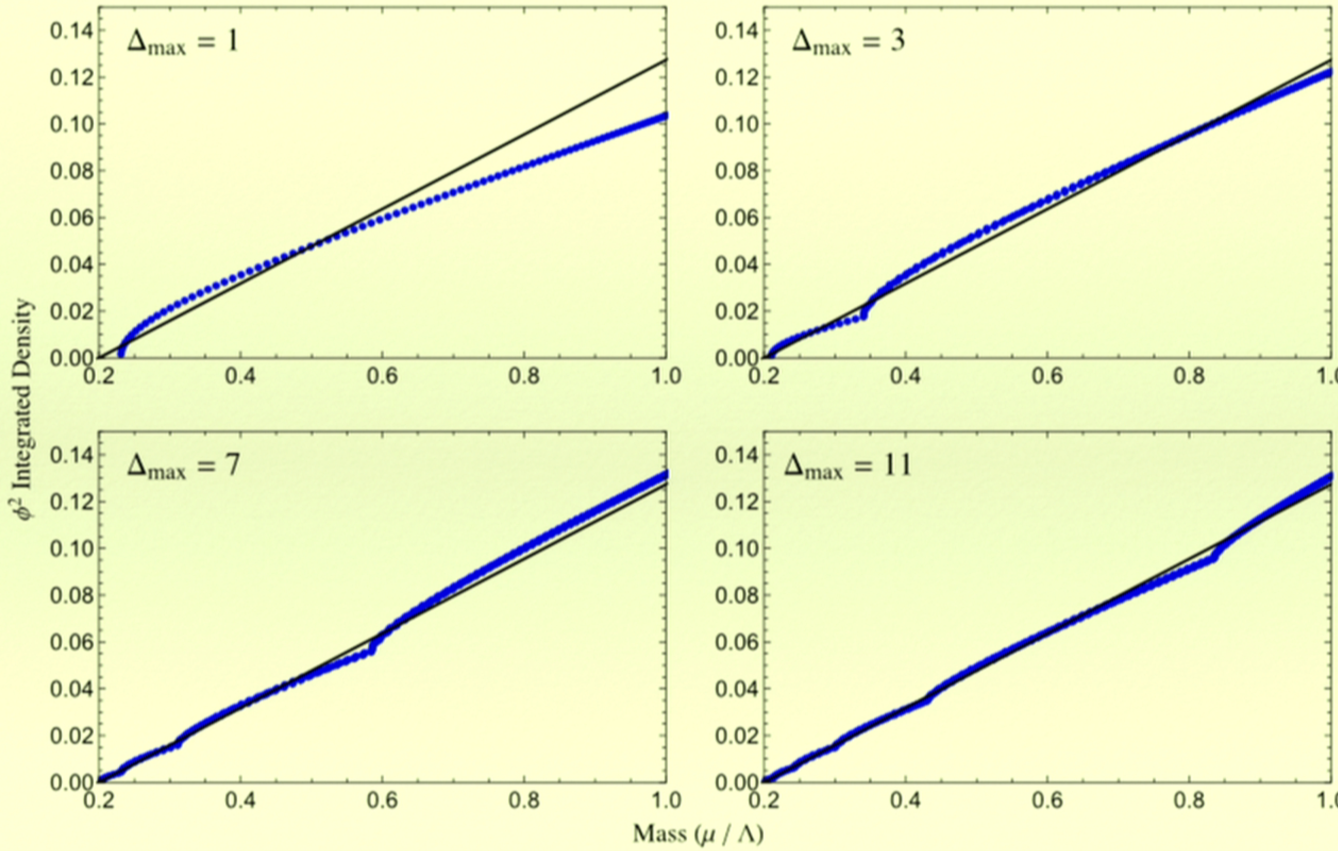
42

ϕ^2 Spectral Density

$$I_{\phi^2}(\mu^2) = \frac{\mu - 2m}{2\pi}$$



Varying Δ_{max}



Large N $\vec{\phi}^2$ Spectral Density

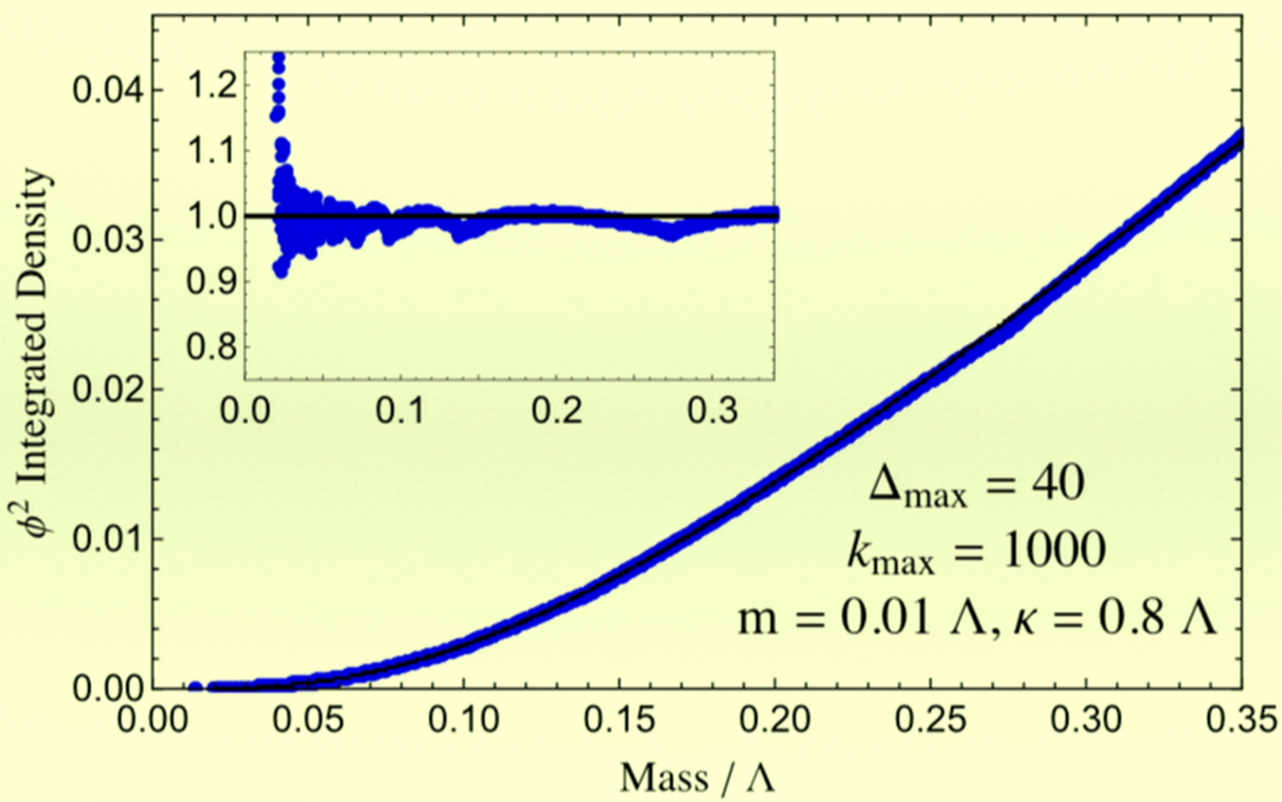
Free Field Theory in UV

$$\rho_{\vec{\phi}^2}(\mu^2) = \frac{\frac{1}{4\pi\mu}}{\left(1+\frac{\kappa}{8\pi\mu}\log\left(\frac{\mu+2m}{\mu-2m}\right)\right)^2+\left(\frac{\kappa}{8\mu}\right)^2}$$

Mass Gap in Far IR

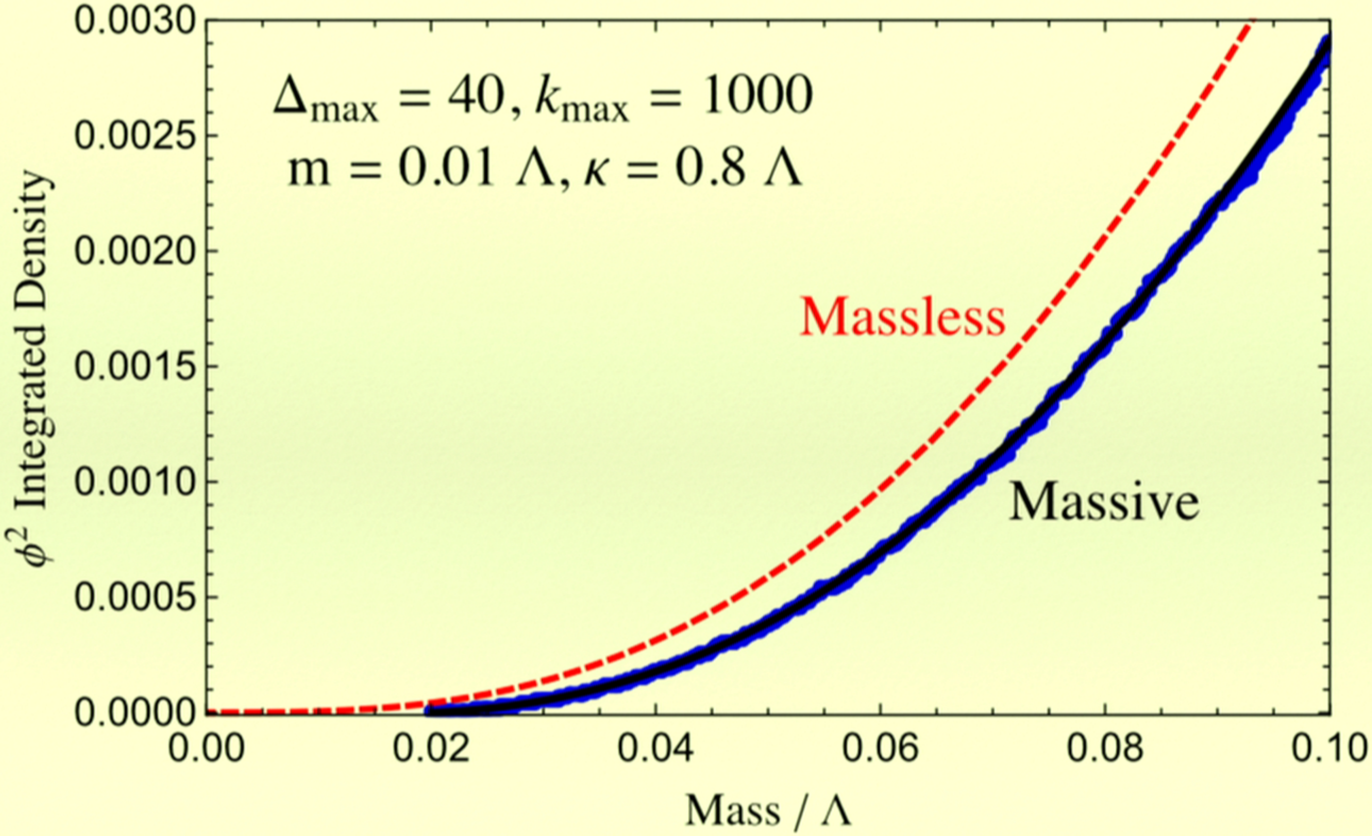
New Scaling in IR

Large N $\vec{\phi}^2$ Spectral Density



46

Large N $\vec{\phi}^2$ Spectral Density



47

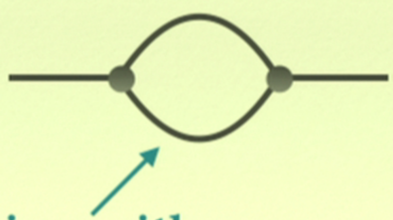
Mixing Particle Number

Return to single-field case; add perturbative interaction

$$\delta\mathcal{L} = -\frac{1}{3!} g {:}\phi^3{:}$$

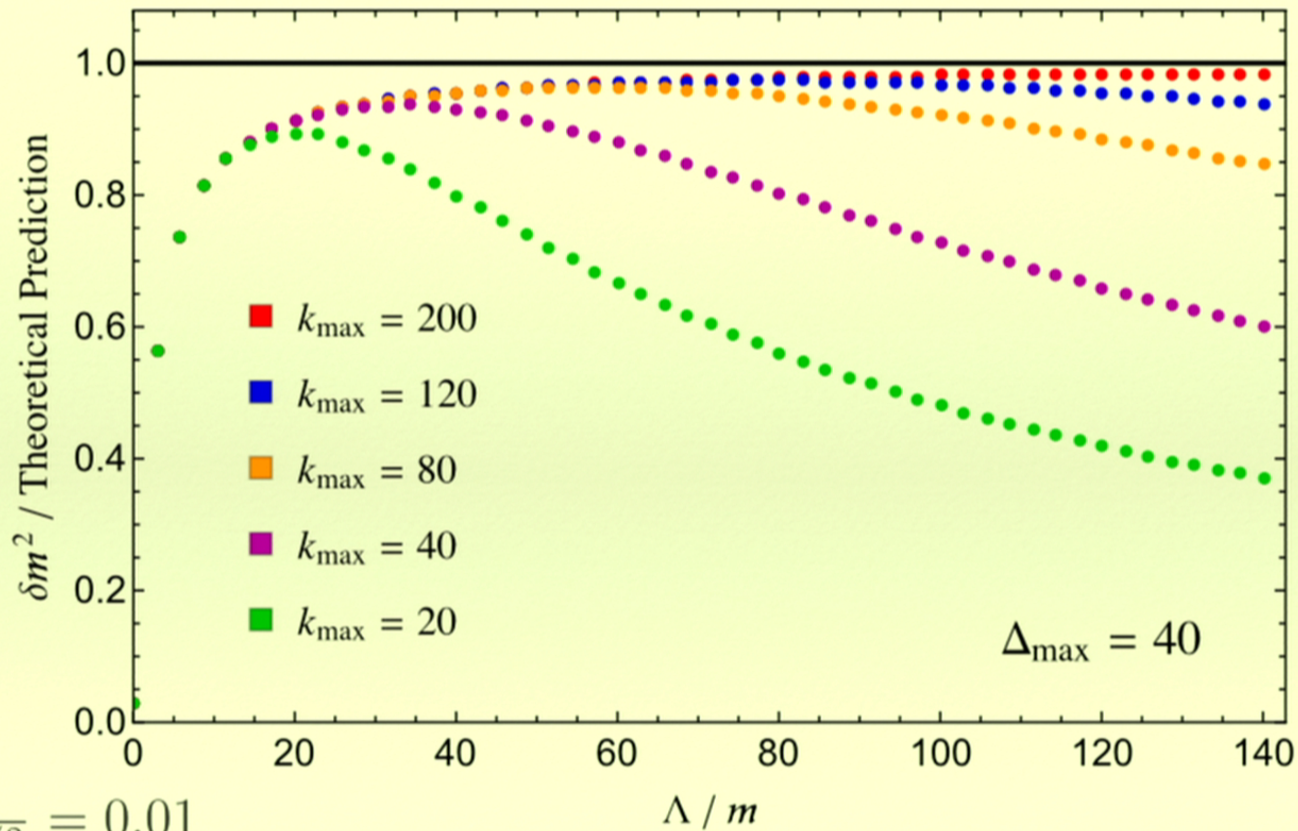
Mixes states with different particle number

Example: One-particle mass shift

$\delta m^2 =$  $= \boxed{-\frac{g^2 \log 3}{16\pi m}}$

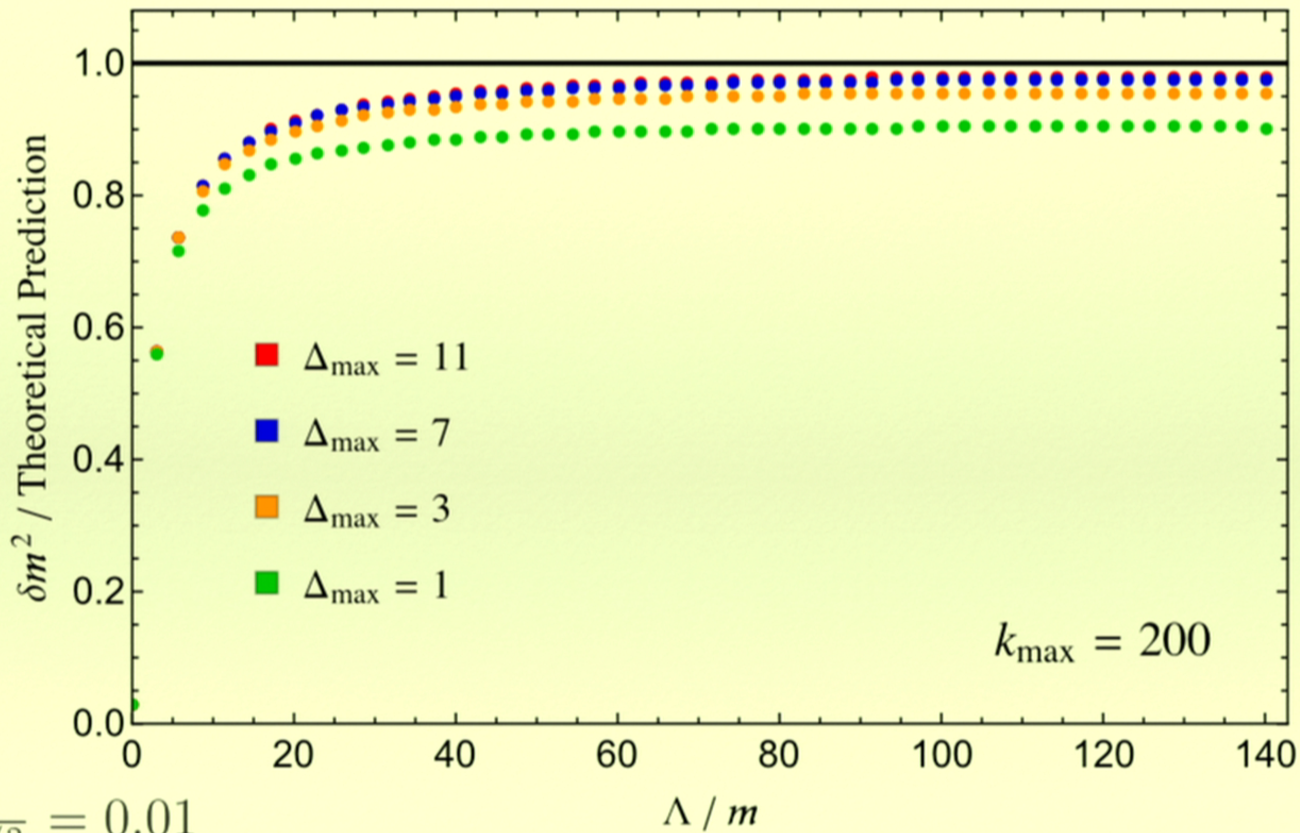
Add one-particle state to our basis

One-Particle Mass Shift



$$\frac{g}{m^{3/2}} = 0.01$$

One-Particle Mass Shift



$$\frac{g}{m^{3/2}} = 0.01$$

Future Work

Now that we have a framework for higher dimensions:

- Improve symmetrization procedure for higher particle number
- 2D and 3D Ising model near critical coupling $\frac{\lambda_*}{m}$
- Symmetry breaking phase $(Q|0\rangle = 0)$
- Generalize to gauge theories, SUSY (3D Scalar QED)
- Correlation functions at finite temperature or chemical potential
- Dimensional transmutation (Gross-Neveu)
- Continue to $d = 4$ (QCD??)

Summary

- UV basis of conformal Casimir eigenstates can reproduce IR spectrum
- Conformal truncation on the lightcone can be used in higher dimensions ($d > 2$)
- General lesson: Hamiltonian truncation are a useful tool for strongly-coupled systems

52