Title: Interferometry and the Global 21-cm Signal

Date: Feb 09, 2016 11:00 AM

URL: http://pirsa.org/16020093

Abstract: <p>The global redshifted 21-cm radiation background is expected to be a powerful probe of the re-heating and re-ionization of the intergalactic medium. However, its measurement is technically challenging: one must extract the small, frequency-dependent signal from under much brighter and spectrally smooth foregrounds. Traditional approaches to study the global signal have used single-antenna systems, where one must calibrate out frequency-dependent structure in the overall system gain, as well as remove the noise bias from auto-correlating a single amplifier output. I will review these approaches, and critically examine several recent proposals to measure the global background using interferometric setups. In particular, using very general principles, I will show that the latters' sensitivity is directly related to two characteristics: the cross-talk between the readout channels (i.e. the signal picked up at one antenna when the other one is driven) and the correlated noise due to thermal fluctuations of lossy elements (e.g. absorbers or the ground) radiating into both channels. I will also briefly discuss the implications and future prospects for interferometric methods.</p>

Interferometry and the global 21-cm signal

Tejaswi Venumadhav

Institute for Advanced Study

Interferometry and the global 21-cm signal

Tejaswi Venumadhav

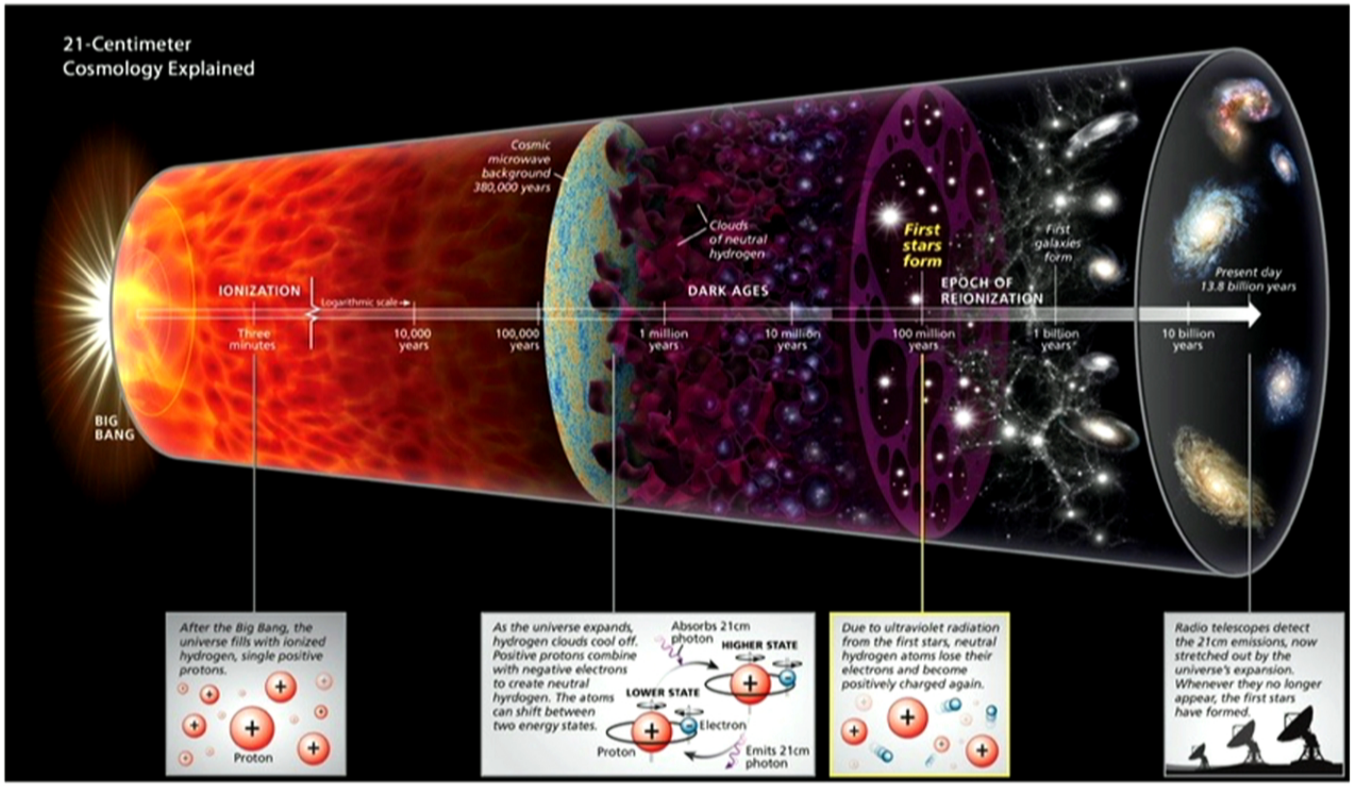
Institute for Advanced Study

Interferometry and the global 21-cm signal

Tejaswi Venumadhav

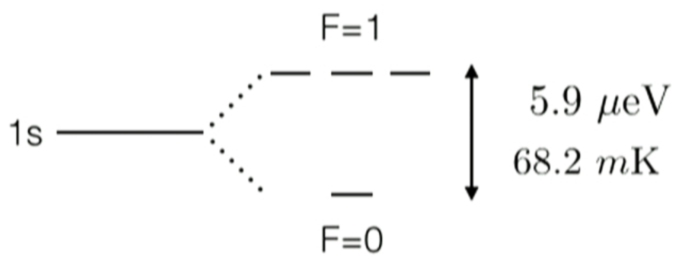
Institute for Advanced Study

Collaborators

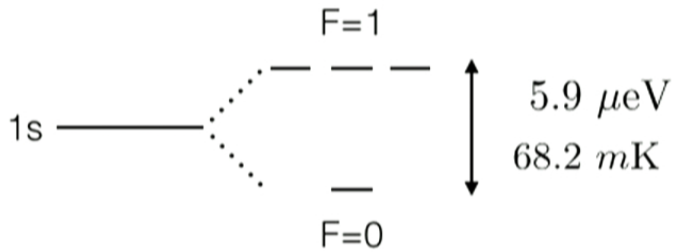


Discover Magazine, (2014)

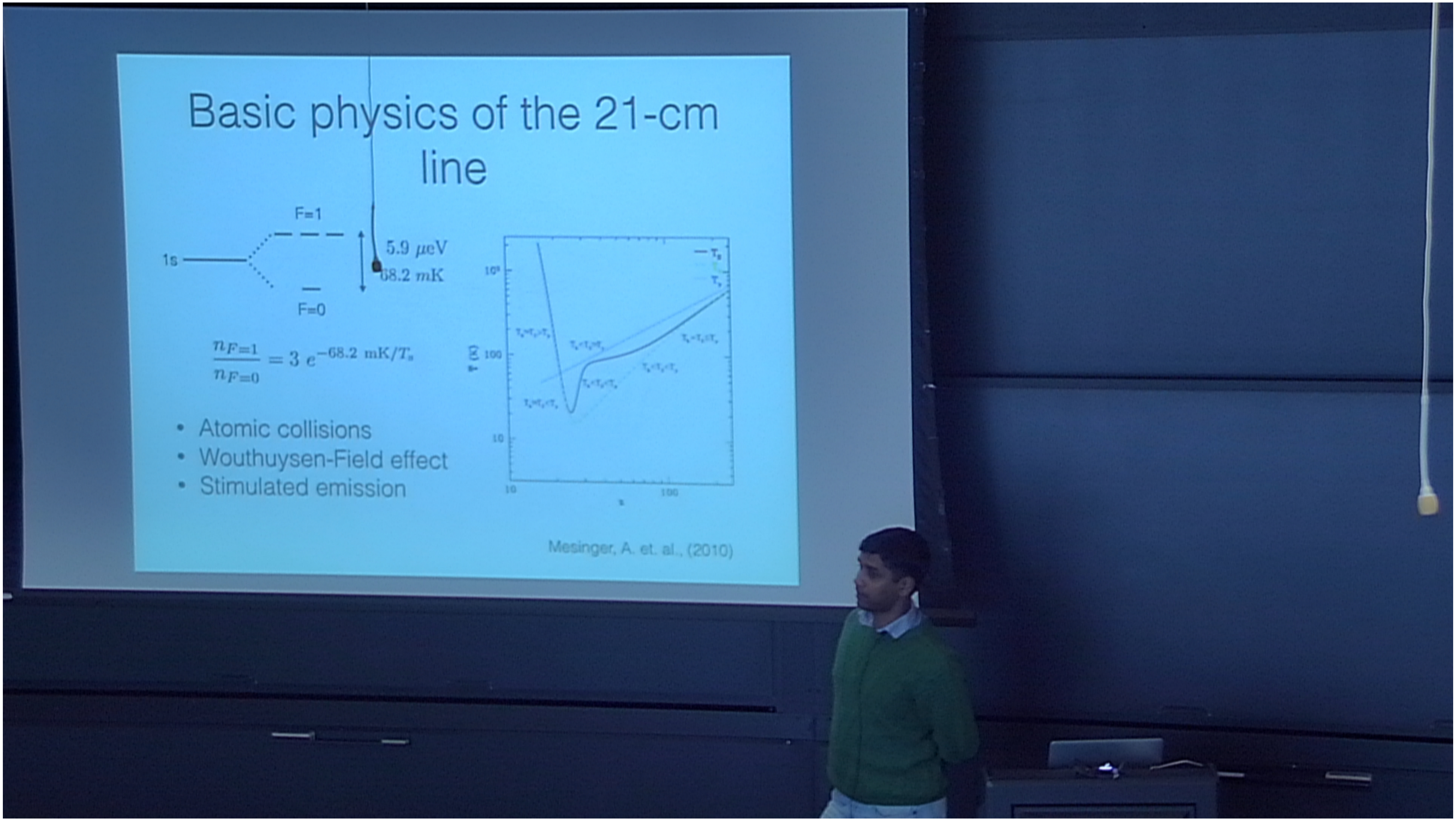
Basic physics of the 21-cm line



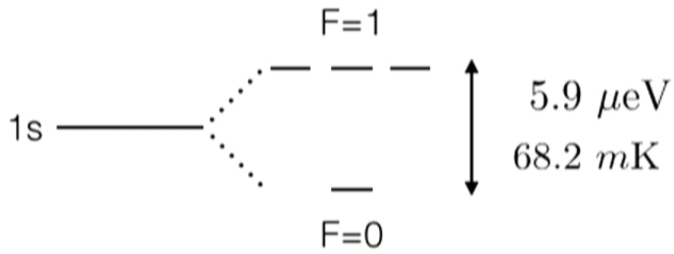
Basic physics of the 21-cm line



$$\frac{n_{F=1}}{n_{F=0}} = 3\ e^{-68.2\ \mathrm{mK}/T_\mathrm{s}}$$

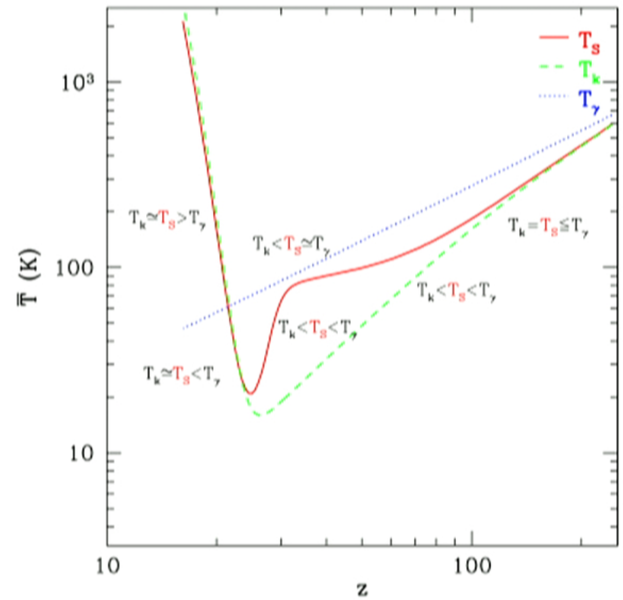

Basic physics of the 21-cm line
F=1
1s
F=0
5.9 μeV
68.2 mK
$\frac{n_{F=1}}{n_{F=0}} = 3\, e^{-68.2 \text{ mK}/T_s}$
• Atomic collisions
• Wouthuysen-Field effect
• Stimulated emission
Mesinger, A. et. al., (2010)

Basic physics of the 21-cm line



$$\frac{n_{F=1}}{n_{F=0}} = 3\ e^{-68.2\ \mathrm{mK}/T_s}$$

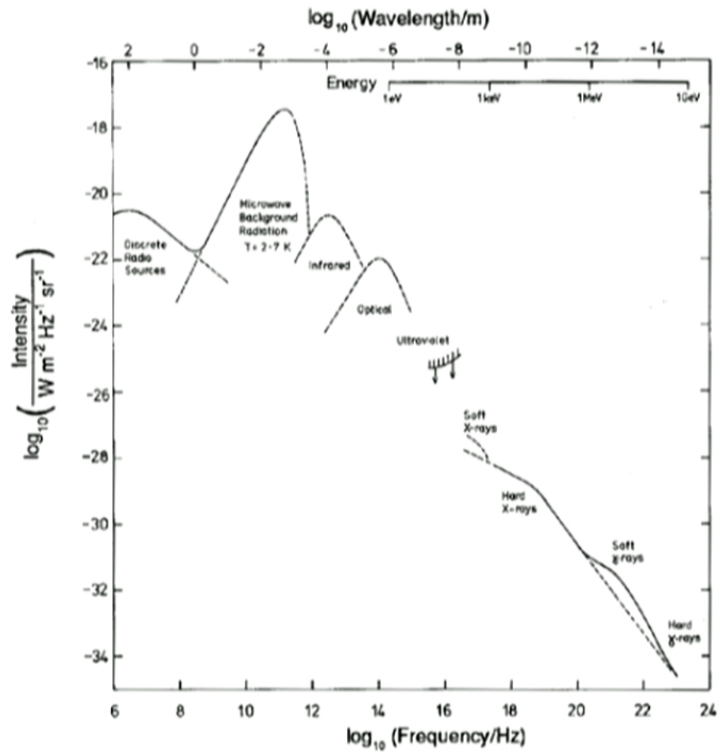
- Atomic collisions
- Wouthuysen-Field effect
- Stimulated emission



Mesinger, A. et. al., (2010)

Brightness temperature measures absorption or emission against the CMB

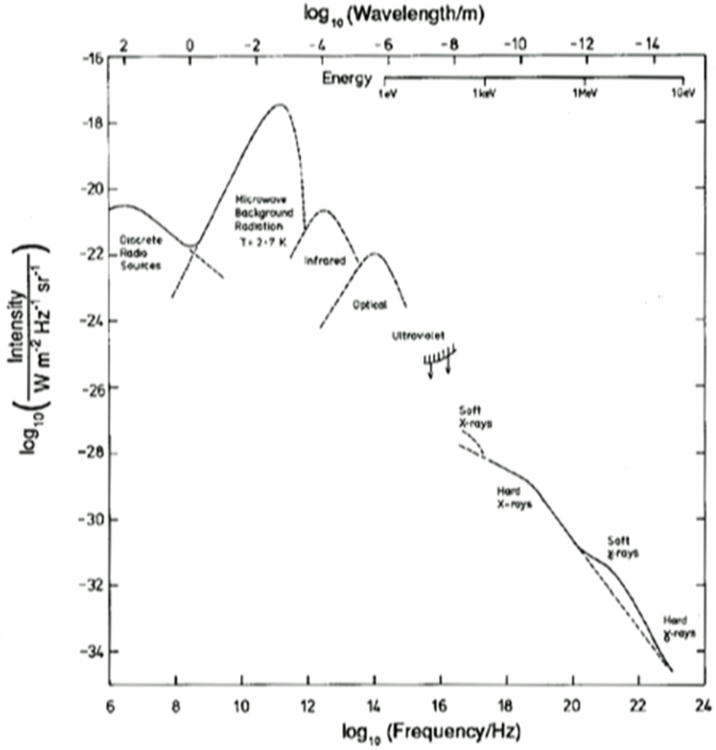
$$\delta T_{\rm b} = \frac{1}{1+z}(T|_{\rm out} - T_\gamma)$$
$$= \boxed{26.4\ {\rm mK}}\, x_{1s}\left(1 - \frac{T_\gamma}{T_{\rm s}}\right) \times$$
$$(1+\delta_{\rm b})\,\frac{H(z)}{\partial_\parallel v_\parallel}\left(\frac{1+z}{10}\right)^{1/2}$$

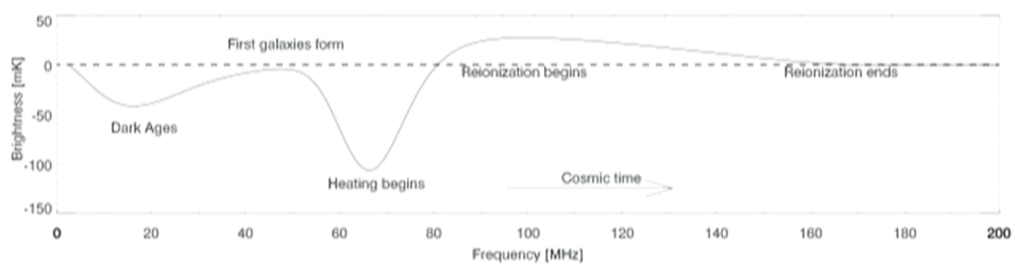


Longair, M. S., & Sunyaev, R., (1971)

Brightness temperature measures absorption or emission against the CMB

$$\delta T_{\rm b} = \frac{1}{1+z}(T|_{\rm out} - T_\gamma)$$
$$= \boxed{26.4\ {\rm mK}}\, x_{1s}\left(1 - \frac{T_\gamma}{T_{\rm s}}\right) \times$$
$$(1+\delta_{\rm b})\,\frac{H(z)}{\partial_{\|} v_{\|}}\left(\frac{1+z}{10}\right)^{1/2}$$

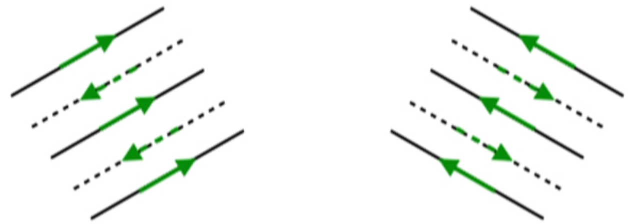




Pritchard, J., & Loeb, A., (2010) Longair, M. S., & Sunyaev, R., (1971)

Single antenna setup

$$\langle |E(\hat{\mathbf{n}}, \nu, \alpha)|^2 \rangle \sim (1/2) I(\hat{\mathbf{n}}, \nu)$$
$$= (1/\lambda^2) T_{\text{sky}}(\hat{\mathbf{n}}, \nu)$$

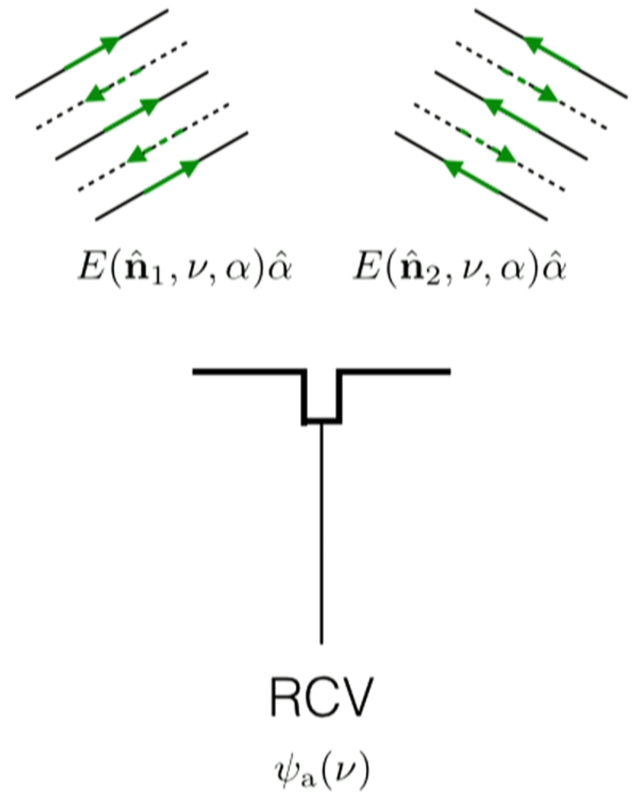


$E(\hat{\mathbf{n}}_1, \nu, \alpha)\hat{\alpha}$ $E(\hat{\mathbf{n}}_2, \nu, \alpha)\hat{\alpha}$

Single antenna setup

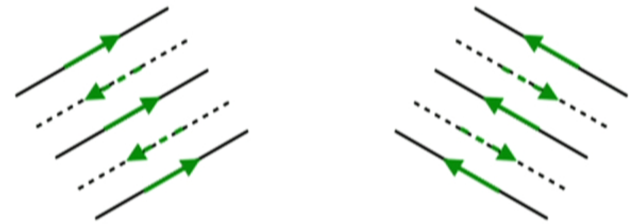
$$\langle |E(\hat{\mathbf{n}}, \nu, \alpha)|^2 \rangle \sim (1/2) I(\hat{\mathbf{n}}, \nu)$$
$$= (1/\lambda^2) T_{\rm sky}(\hat{\mathbf{n}}, \nu)$$

$$\psi_{\rm a}(\nu) = \sum_{i,\alpha} F(\hat{\mathbf{n}}_i, \nu) E(\hat{\mathbf{n}}_i, \nu, \alpha) \ (\hat{\alpha} \cdot \hat{\mathbf{a}})$$



Single antenna setup

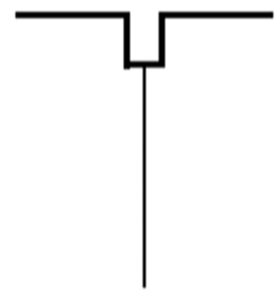
$$\langle |E(\hat{\mathbf{n}}, \nu, \alpha)|^2 \rangle \sim (1/2) I(\hat{\mathbf{n}}, \nu)$$
$$= (1/\lambda^2) T_{\rm sky}(\hat{\mathbf{n}}, \nu)$$



$$\psi_{\rm a}(\nu) = \sum_{i,\alpha} F(\hat{\mathbf{n}}_i, \nu) E(\hat{\mathbf{n}}_i, \nu, \alpha) \ (\hat{\alpha} \cdot \hat{\mathbf{a}})$$

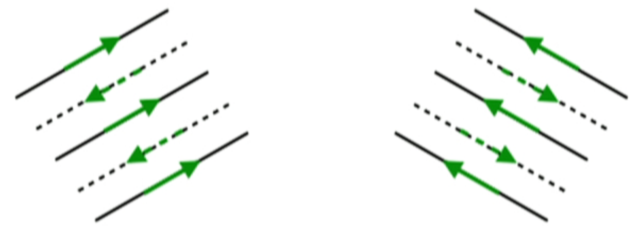
$$\langle |\psi_{\rm a}(\nu)|^2 \rangle = T_{\rm a}(\nu) = \int d\hat{\mathbf{n}} (1/\lambda^2) A(\hat{\mathbf{n}}, \nu) T_{\rm sky}(\hat{\mathbf{n}}, \nu)$$

$$\boxed{A(\hat{\mathbf{n}}, \nu) \sim |F(\hat{\mathbf{n}}, \nu)|^2}$$



Single antenna setup

$$\langle |E(\hat{\mathbf{n}}, \nu, \alpha)|^2 \rangle \sim (1/2) I(\hat{\mathbf{n}}, \nu) = (1/\lambda^2) T_{\rm sky}(\hat{\mathbf{n}}, \nu)$$



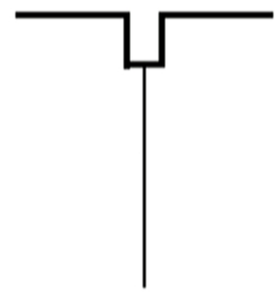
$E(\hat{\mathbf{n}}_1, \nu, \alpha)\hat{\alpha}$ $E(\hat{\mathbf{n}}_2, \nu, \alpha)\hat{\alpha}$

$$\psi_{\rm a}(\nu) = \sum_{i,\alpha} F(\hat{\mathbf{n}}_i, \nu) E(\hat{\mathbf{n}}_i, \nu, \alpha)\ (\hat{\alpha} \cdot \hat{\mathbf{a}})$$

$$\langle |\psi_{\rm a}(\nu)|^2 \rangle = T_{\rm a}(\nu) = \int d\hat{\mathbf{n}} (1/\lambda^2) A(\hat{\mathbf{n}}, \nu) T_{\rm sky}(\hat{\mathbf{n}}, \nu)$$

$$A(\hat{\mathbf{n}}, \nu) \sim |F(\hat{\mathbf{n}}, \nu)|^2$$

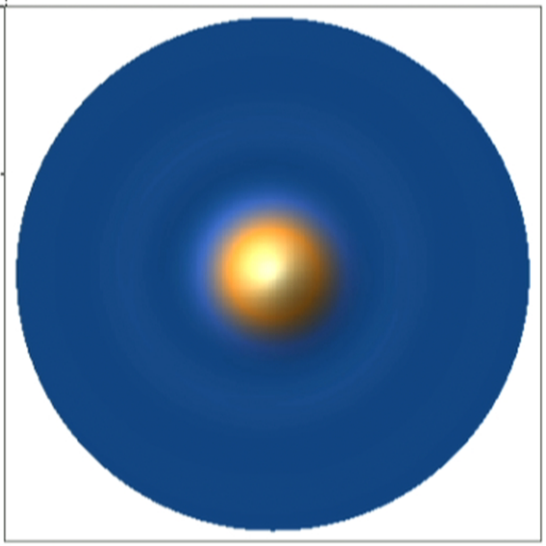
$$\int d\hat{\mathbf{n}}\ A(\hat{\mathbf{n}}, \nu) = \lambda^2$$



RCV

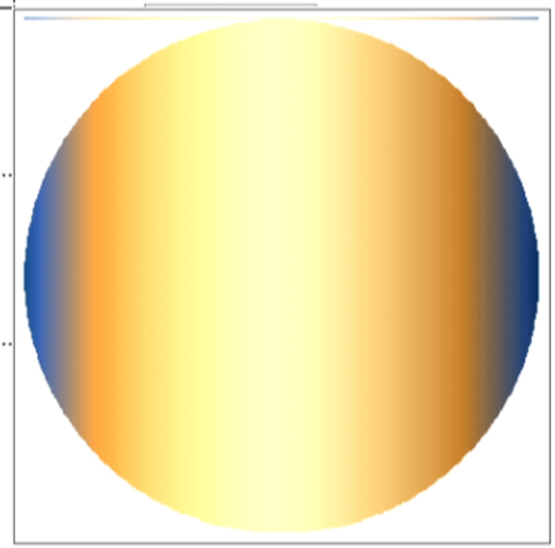
$\psi_{\rm a}(\nu)$

Aside: Beam profiles

Antenna	Parameters	$A(\hat{\mathbf{n}})$
Circular slit	Diameter d	

Aside: Beam profiles

Antenna	Parameters	$A(\hat{\mathbf{n}})$
Circular slit	Diameter d	
Short dipole	$d \ll \lambda$	



Pitfall: Foregrounds

Galactic synchrotron radiation

$$T_{\rm sky}(\nu) \sim 400 \text{ K} \left(\frac{1+z}{9}\right)^{2.55}$$

For a bandwidth $\delta\nu = 5 \text{ MHz}$
and temperature step $\delta T = 25 \text{ mK}$
integration time needed for
1-sigma detection satisfies

$$\delta T = \frac{T_{\rm sky}}{\sqrt{\delta\nu\delta t}}$$

Pitfall: Foregrounds

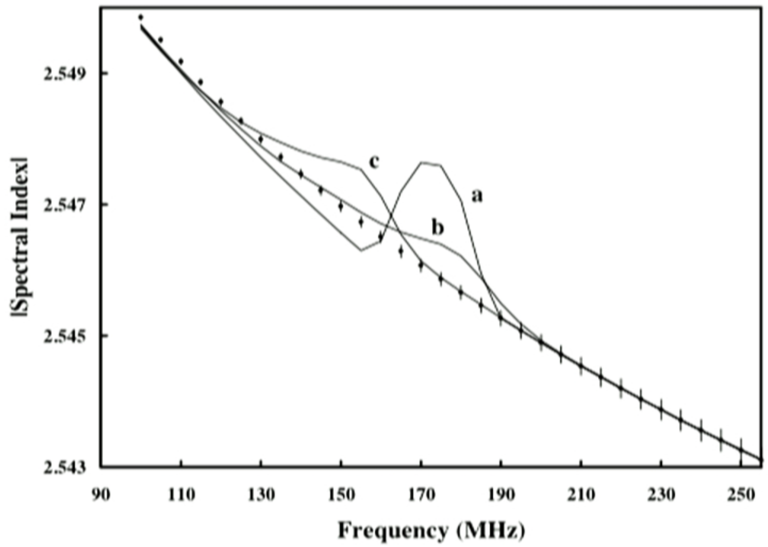
Galactic synchrotron radiation

$$T_{\mathrm{sky}}(\nu) \sim 400\ \mathrm{K} \left(\frac{1+z}{9}\right)^{2.55}$$

For a bandwidth $\delta\nu = 5\ \mathrm{MHz}$
and temperature step $\delta T = 25\ \mathrm{mK}$
integration time needed for
1-sigma detection satisfies

$$\delta T = \frac{T_{\mathrm{sky}}}{\sqrt{\delta\nu \delta t}}$$

$$\delta t = 51.2\ \mathrm{s}\ !$$



Shaver, P.A. et. al., (1999)

Pitfalls

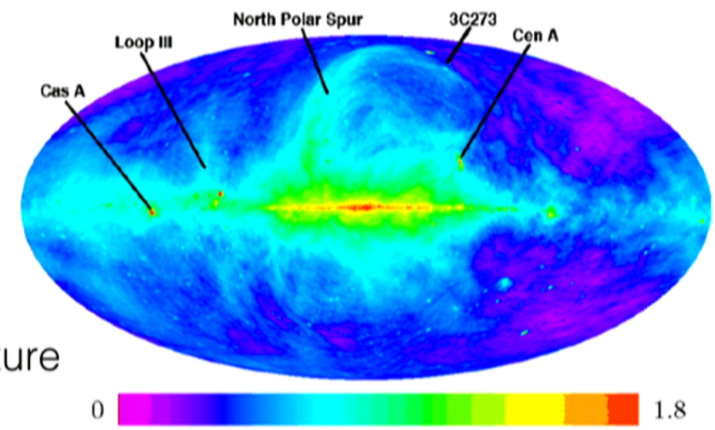
- Foregrounds
- Varying foregrounds
- Johnson Noise
- Internal reflections

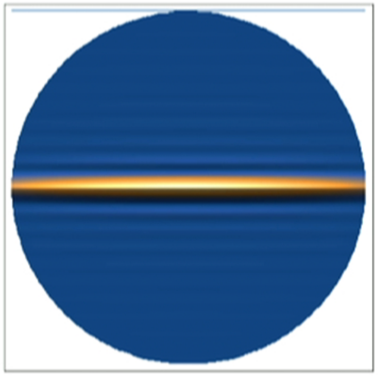
Pitfall: Varying Foregrounds

$$T_{\rm a}(\nu) = \int d\hat{\mathbf{n}}(1/\lambda^2)A(\hat{\mathbf{n}},\nu)\boxed{T_{\rm sky}(\hat{\mathbf{n}},\nu)}$$

$$T_{\rm sky}(\nu) \sim F(\hat{\mathbf{n}})\ \mathrm{K}\left(\frac{1+z}{9}\right)^{G(\hat{\mathbf{n}})}$$

Spatial structure -> frequency structure





de Oliveira-Costa et. al., (2008)

Pitfall: Varying Foregrounds

$$T_{\rm a}(\nu) = \int d\hat{\mathbf{n}}(1/\lambda^2)A(\hat{\mathbf{n}},\nu)\boxed{T_{\rm sky}(\hat{\mathbf{n}},\nu)}$$

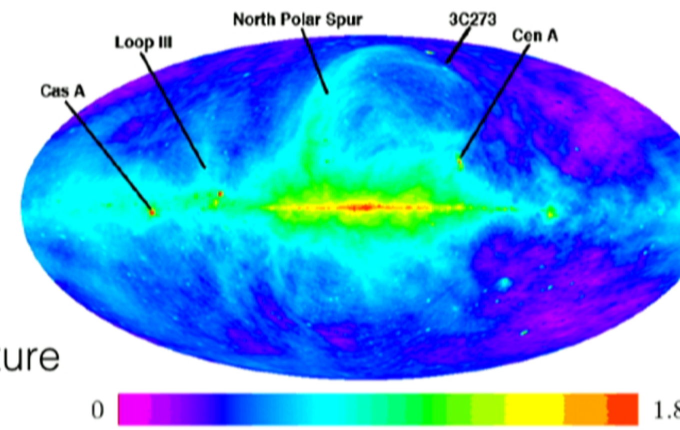
$$T_{\rm sky}(\nu) \sim F(\hat{\mathbf{n}})\ \mathrm{K}\left(\frac{1+z}{9}\right)^{G(\hat{\mathbf{n}})}$$

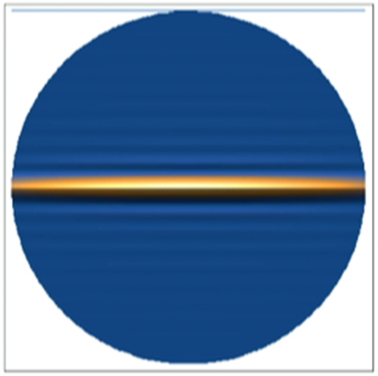
Spatial structure -> frequency structure

Typically modeled by

$$T_{\rm model}(\nu) = \sum_{i=0}^{N-1} a_i \left(\frac{\nu}{\nu_0}\right)^{-2.5+i}$$

Is this a good model?
Look at fit residuals*





de Oliveira-Costa et. al., (2008)

Pitfall: Varying Foregrounds

$$T_{\mathrm{a}}(\nu) = \int d\hat{\mathbf{n}}(1/\lambda^2)A(\hat{\mathbf{n}},\nu)\boxed{T_{\mathrm{sky}}(\hat{\mathbf{n}},\nu)}$$

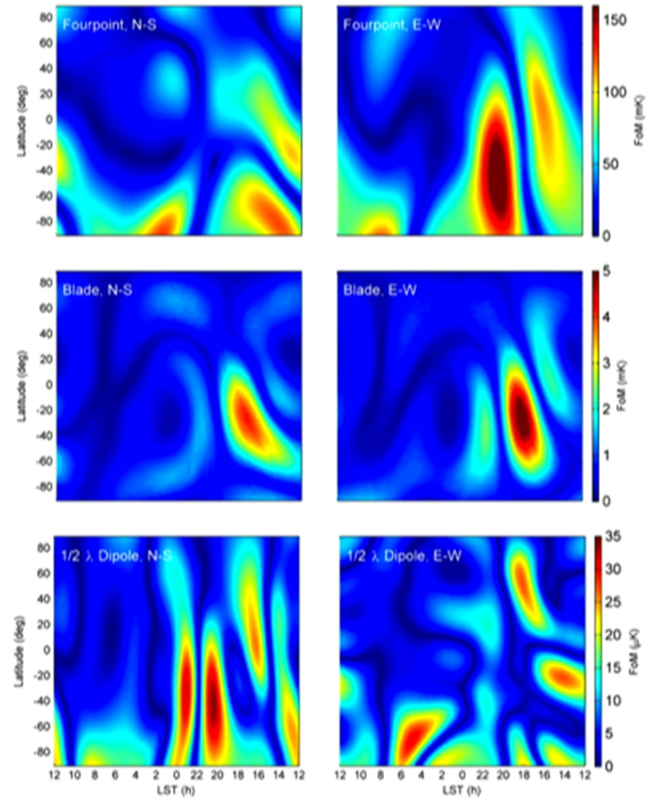
$$T_{\mathrm{sky}}(\nu) \sim F(\hat{\mathbf{n}})\ \mathrm{K}\left(\frac{1+z}{9}\right)^{G(\hat{\mathbf{n}})}$$

Spatial structure -> frequency structure

Typically modeled by

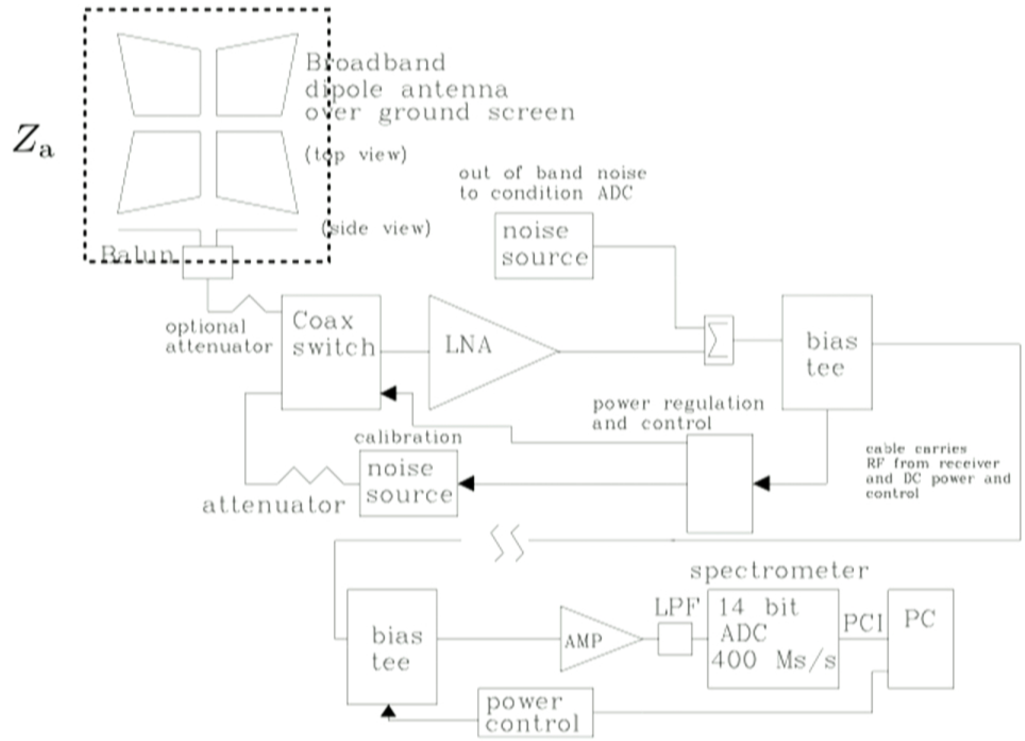
$$T_{\mathrm{model}}(\nu) = \sum_{i=0}^{N-1} a_i \left(\frac{\nu}{\nu_0}\right)^{-2.5+i}$$

Is this a good model?
Look at fit residuals*



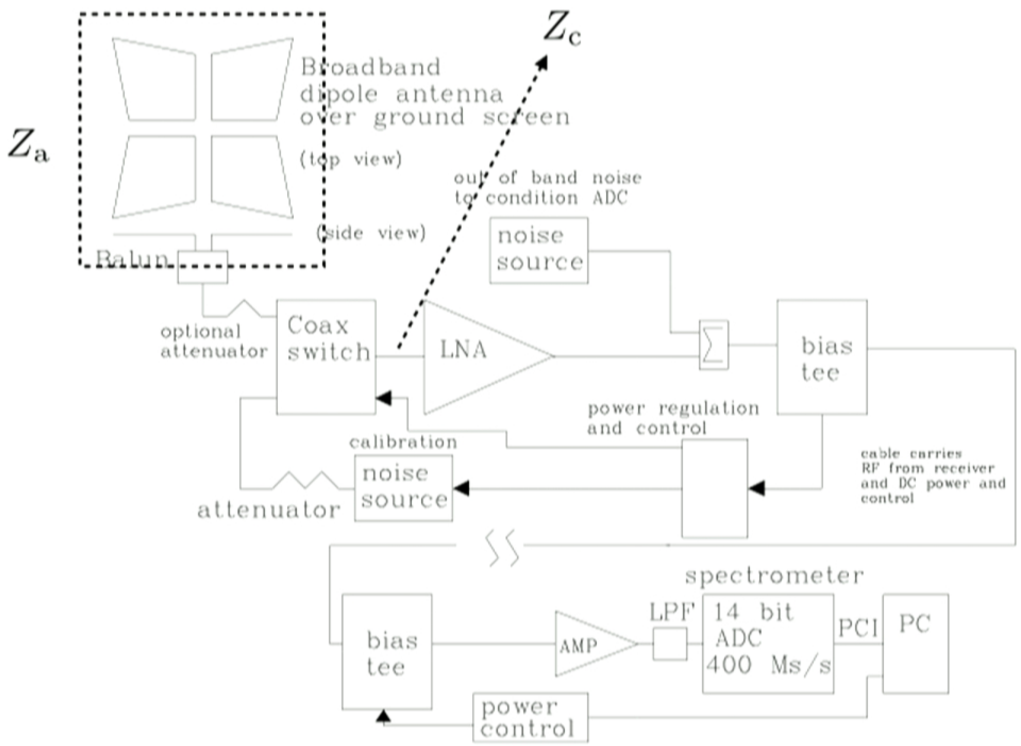
Mozdzen, T.J. et. al., (2015)

Pitfall: Internal Reflections



Rogers, A. E. E. & Bowman, J. D., (2012)

Pitfall: Internal Reflections

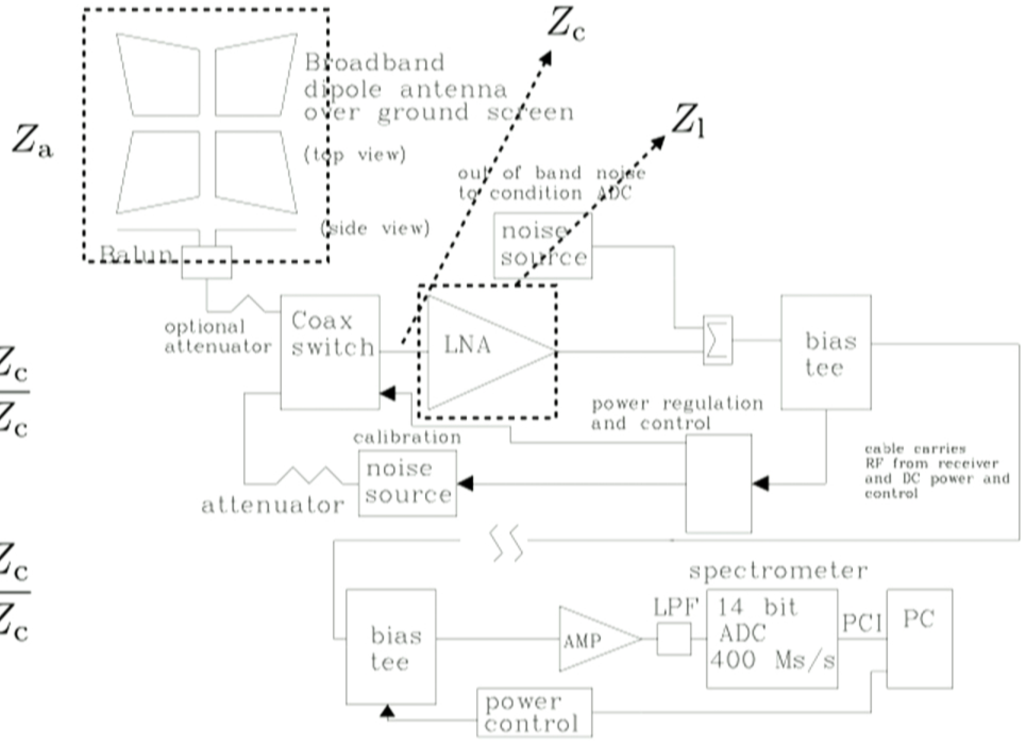


Rogers, A. E. E. & Bowman, J. D., (2012)

Pitfall: Internal Reflections

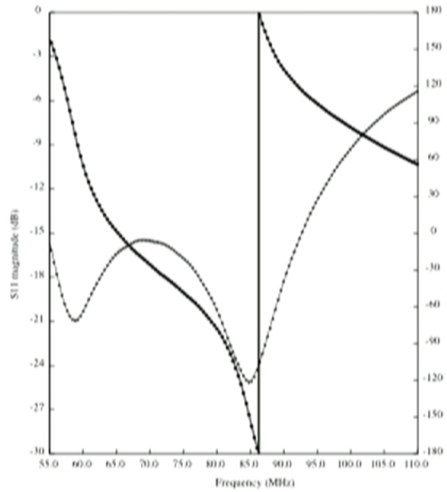
$$\Gamma_a = \frac{Z_a - Z_c}{Z_a + Z_c}$$

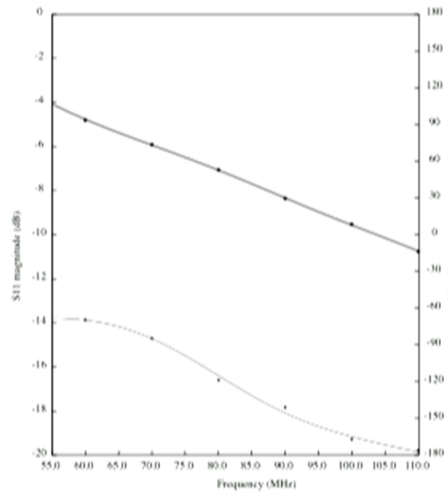
$$\Gamma_l = \frac{Z_l - Z_c}{Z_l + Z_c}$$

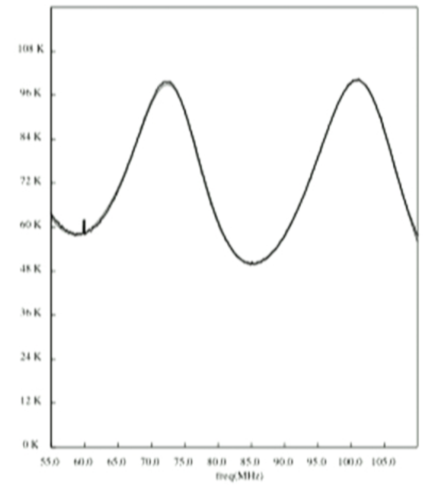


Rogers, A. E. E. & Bowman, J. D., (2012)

Pitfall: Internal Reflections





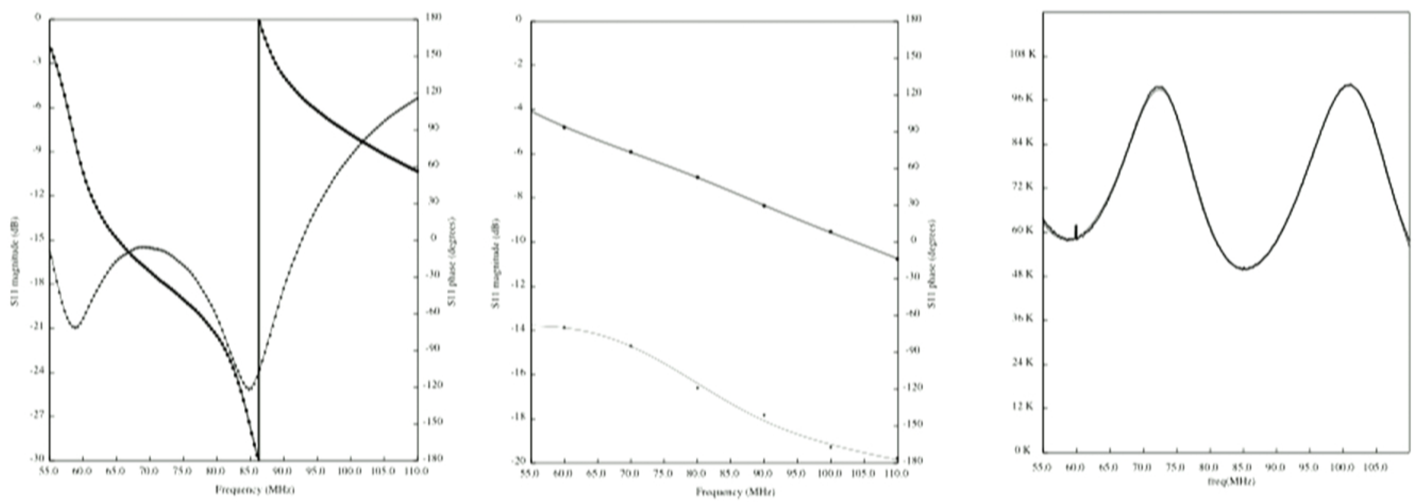


Parameter changed	rms (K)	rms2 (K)
Γ_a amplitude	1.8	0.09
Γ_a phase	0.25	0.06
Γ_l amplitude	0.13	0.06
Γ_l phase	0.19	0.04
Z_b amplitude	0.38	0.09
Z_b phase	0.004	0.001
T_{amb} temperature	1.2	0.8

[a] rms2 is for antenna with -20 dB reflection coefficient

Rogers, A. E. E. & Bowman, J. D., (2012)

Pitfall: Internal Reflections

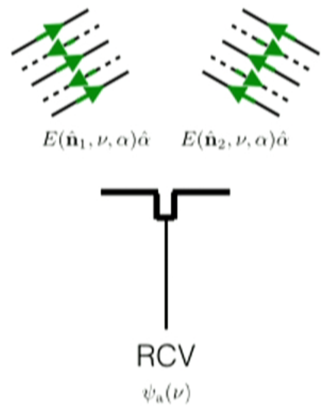


Parameter changed	rms (K)	rms2 (K)
Γ_a amplitude	1.8	0.09
Γ_a phase	0.25	0.06
Γ_l amplitude	0.13	0.06
Γ_l phase	0.19	0.04
Z_b amplitude	0.38	0.09
Z_b phase	0.004	0.001
T_{amb} temperature	1.2	0.8

[a] rms2 is for antenna with -20 dB reflection coefficient

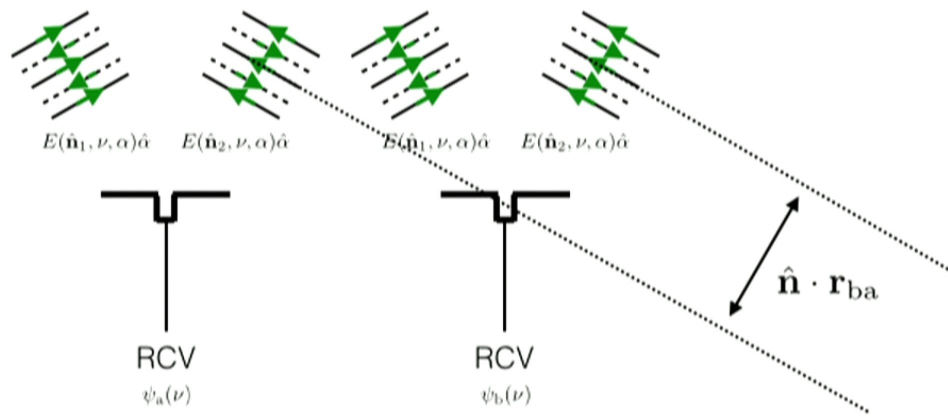
Rogers, A. E. E. & Bowman, J. D., (2012)

Interferometric setup

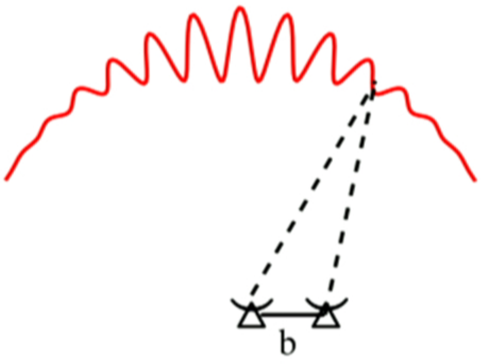


$$\psi_{\mathbf{a}}(\nu) = \sum_{i,\alpha} F(\hat{\mathbf{n}}_i, \nu) E(\hat{\mathbf{n}}_i, \nu, \alpha)\ (\hat{\alpha} \cdot \hat{\mathbf{a}})$$

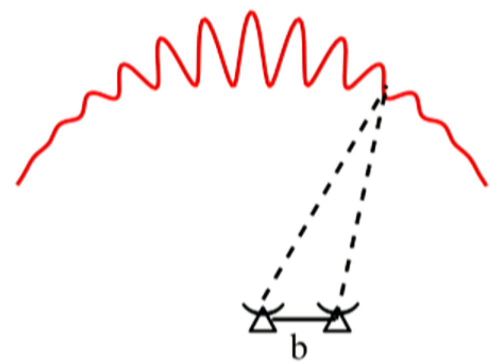
Interferometric setup



$$\psi_{\rm a}(\nu) = \sum_{i,\alpha} F({\rm a}, \hat{\mathbf{n}}_i, \nu) E(\hat{\mathbf{n}}_i, \nu, \alpha)(\hat{\alpha} \cdot \hat{\mathbf{a}}) \boxed{e^{-ik\hat{\mathbf{n}}\cdot\mathbf{r}_a}}$$


b

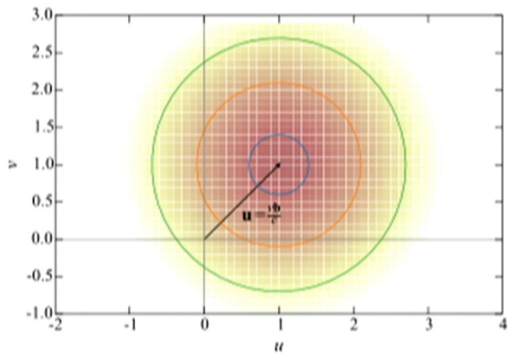
Packed interferometers

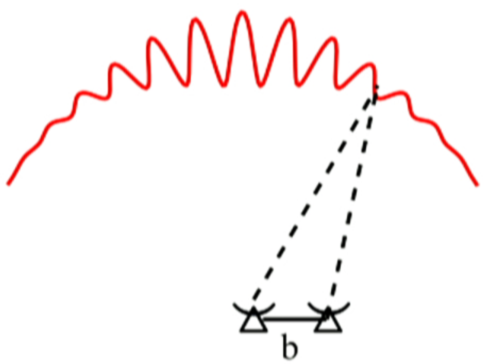


$$V(\nu) = \int d\hat{\mathbf{n}}\ (\nu^2/c^2) A(\hat{\mathbf{n}}, \nu) T_{\rm sky}(\hat{\mathbf{n}}, \nu) e^{ik\hat{\mathbf{n}}\cdot\mathbf{b}}$$

Presley, M., Parsons, A., Liu, A. (2015)

Packed interferometers





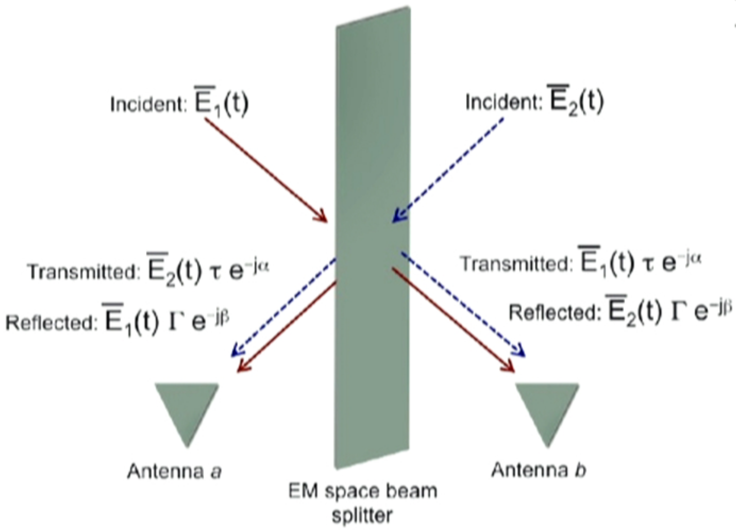
$$V(\nu) = \int d\hat{\mathbf{n}}\ (\nu^2/c^2) A(\hat{\mathbf{n}}, \nu) T_{\rm sky}(\hat{\mathbf{n}}, \nu) e^{ik\hat{\mathbf{n}}\cdot\mathbf{b}}$$

Simple to understand in the flat-sky approximation

$$V(\nu) = (\nu^2/c^2)\left[\tilde{A} \otimes \tilde{T}_{\rm sky}\right](\nu\mathbf{b}/c)$$

Presley, M., Parsons, A., Liu, A. (2015)

Beam splitters



Beam splitter with reflectivity $\Gamma e^{-i\beta}$ and transmittivity $\tau e^{-i\alpha}$

$$\psi_{\rm a} = \Gamma e^{-i\beta} E_1 + \tau e^{-i\alpha} E_2$$

$$\psi_{\rm b} = \tau e^{-i\alpha} E_1 + \Gamma e^{-i\beta} E_2$$

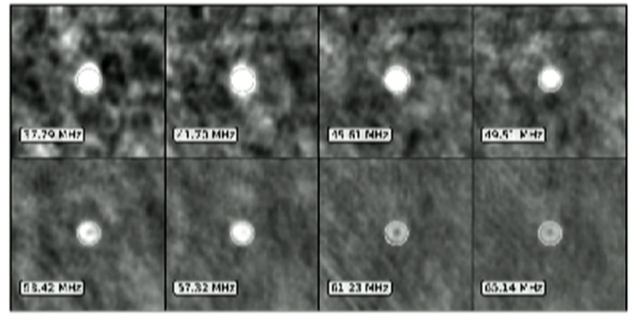
Mahesh, N., et. al., (2014)

Lunar occultation

- At radio frequencies, the moon is approximately a black-body with T ~ 230 K.

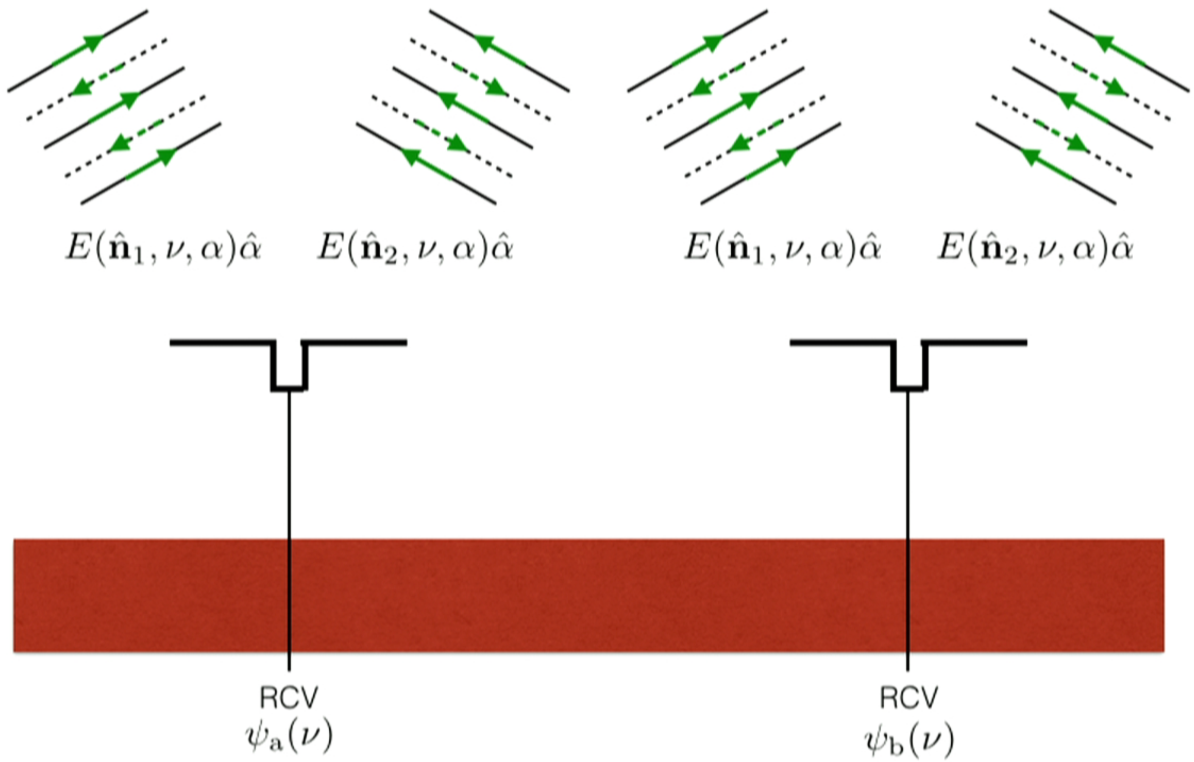
- If M is a masking function, the sky-brightness is

$$T_{\rm sky} = T_{\rm B}\,(1 - M) + T_{\rm M}\,M$$
$$= \underbrace{(T_{\rm M} - T_{\rm B})M}_{\rm occulted} + \underbrace{T_{\rm B}}_{\rm non-occulted},$$

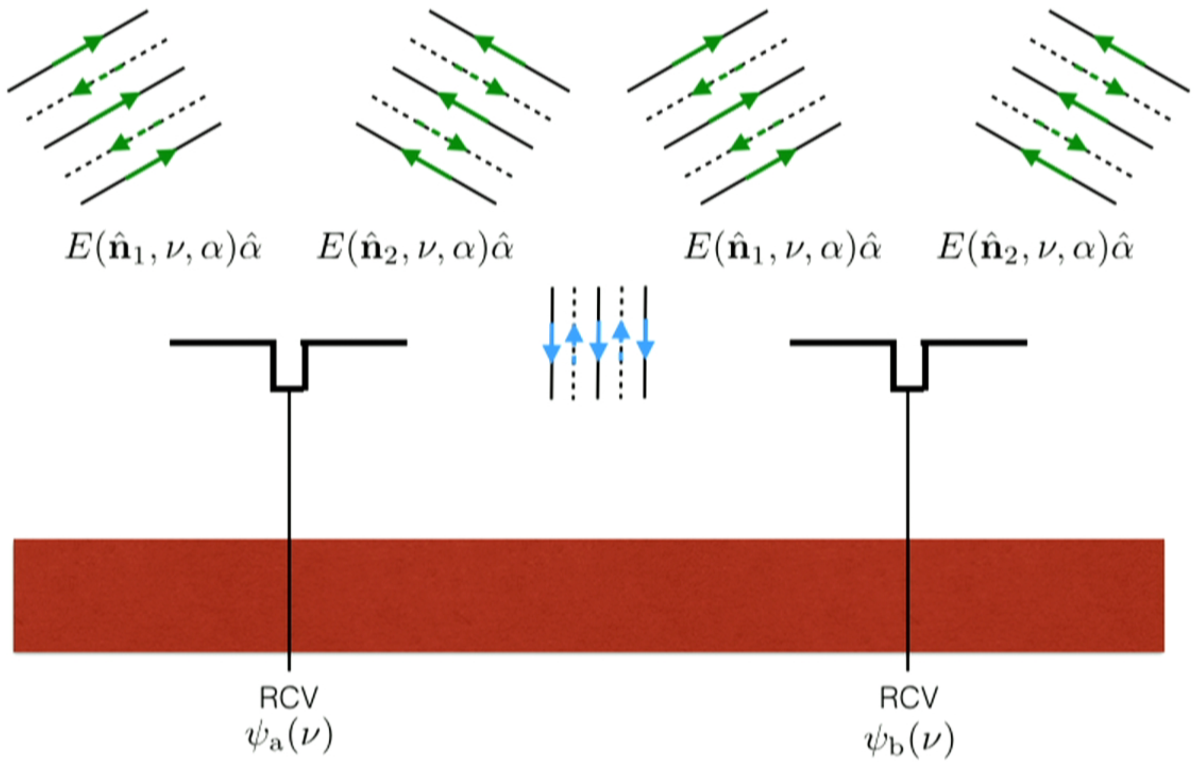


Vedantham, H. K., et. al. (2015)

Pitfalls: spurious correlations



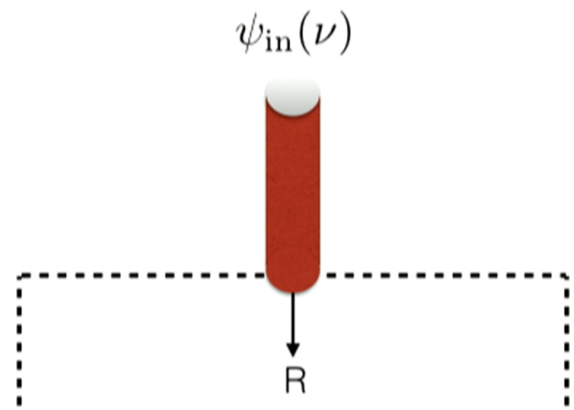
Pitfalls: spurious correlations



Antenna Crosstalk

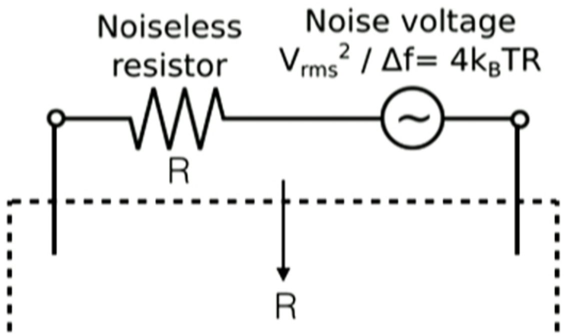
Input-output channels

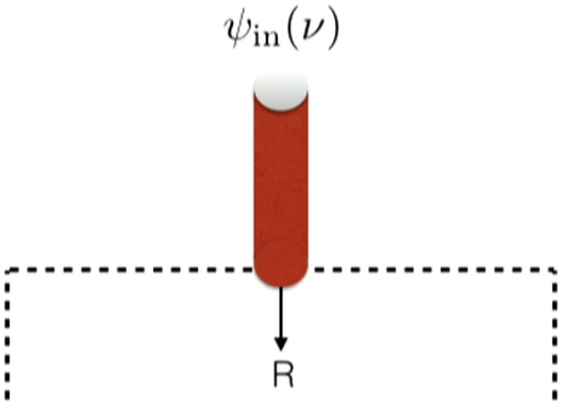
Lossy materials



Input-output channels

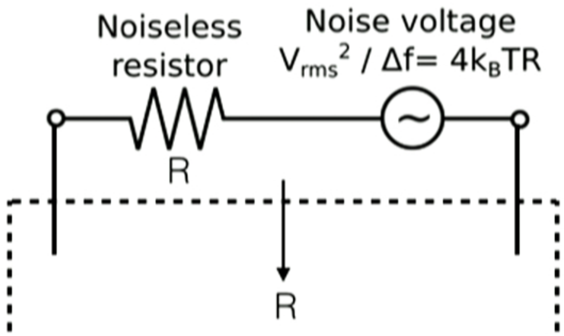
Lossy materials



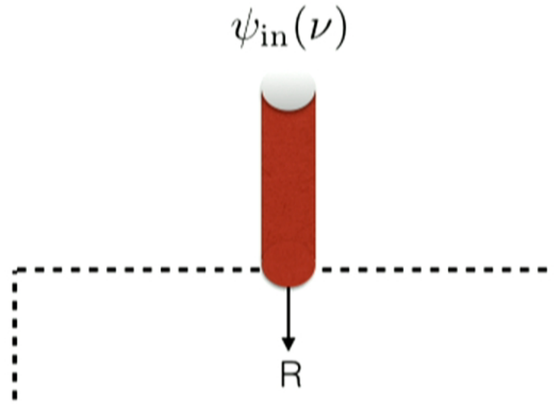


Input-output channels

Lossy materials



$$\frac{dP_{\rm in}}{d\nu} = \frac{1}{\Delta\nu}\frac{V_{\rm rms}^2}{4R} = k_{\rm B}T$$

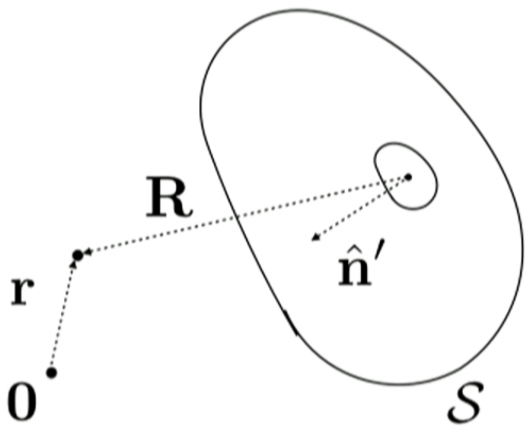


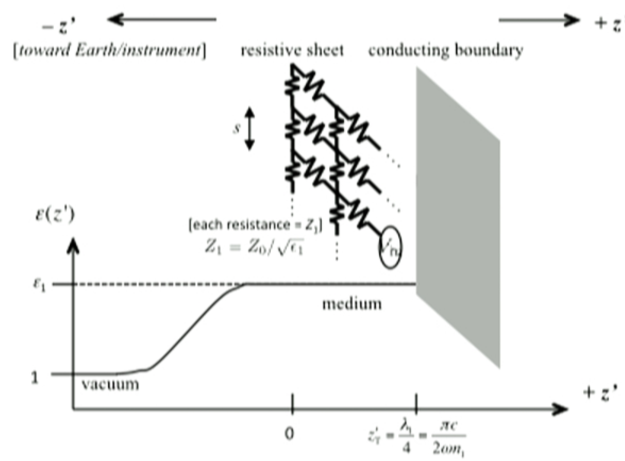
$$\frac{dP_{\rm in}}{d\nu} = |\psi_{\rm in}(\nu)|^2 = k_{\rm B}T$$

Input-output channels

Input-output channels

Sky sources





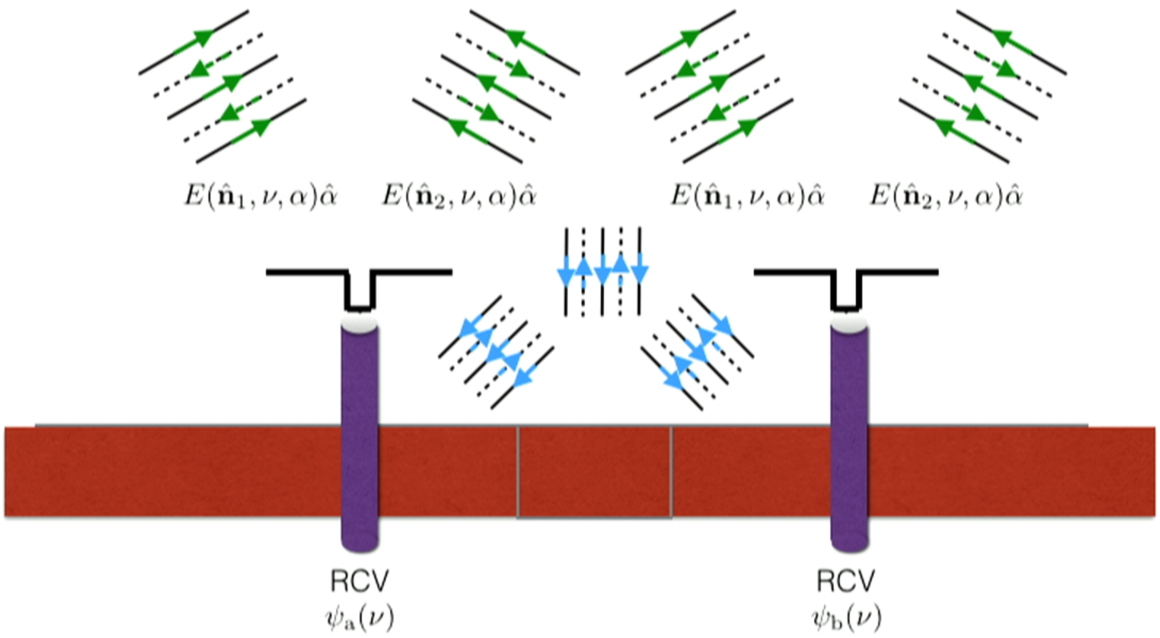
Diffraction integral

$$\mathbf{E}(\mathbf{r}) = \frac{k}{2\pi i}\int_{\mathcal{S}} da\ \frac{e^{ikR}}{R}\boxed{\mathbf{E}(a)}\hat{\mathbf{n}}'\cdot\hat{\mathbf{R}}$$

$$\int da\ E_x(a) = sV_{\rm in}$$

$$\langle V_{\rm in}\rangle^2 = 4Z_1\Delta\nu|\psi_{\rm in}(\nu)|^2$$

Linear Blob



$E(\hat{\mathbf{n}}_1, \nu, \alpha)\hat{\alpha}$
$E(\hat{\mathbf{n}}_2, \nu, \alpha)\hat{\alpha}$
$E(\hat{\mathbf{n}}_1, \nu, \alpha)\hat{\alpha}$
$E(\hat{\mathbf{n}}_2, \nu, \alpha)\hat{\alpha}$
RCV
$\psi_{\rm a}(\nu)$
RCV
$\psi_{\rm b}(\nu)$

Usual Identities

	$(\mathrm{H}, \hat{\mathbf{n}}_1)$ $\cdots$	d_1 $\cdots$	c_1 $\cdots$	
$(\mathrm{H}, \hat{\mathbf{n}}_1)$ $\vdots$	U_{ss}	U_{sd}	U_{sc}	$\sqrt{T_{\mathrm{s}}}$
d_1 $\vdots$	U_{ds}	U_{dd}	U_{dc}	$\sqrt{T_{\mathrm{d}}}$
c_1 $\vdots$	U_{cs}	U_{cd}	U_{cc}	$\sqrt{T_{\mathrm{c}}}$

Usual Identities

- Single matched antenna, no dissipation:

$$T_{\rm a} = U_{\rm as} T_{\rm s} \mathbb{1} U_{\rm as}^{\dagger}$$

$$\equiv$$

$$T_{\rm a} = (\nu^2/c^2) \int d\hat{\mathbf{n}} A(\hat{\mathbf{n}}, \nu) T_{\rm sky}(\hat{\mathbf{n}}, \nu)$$

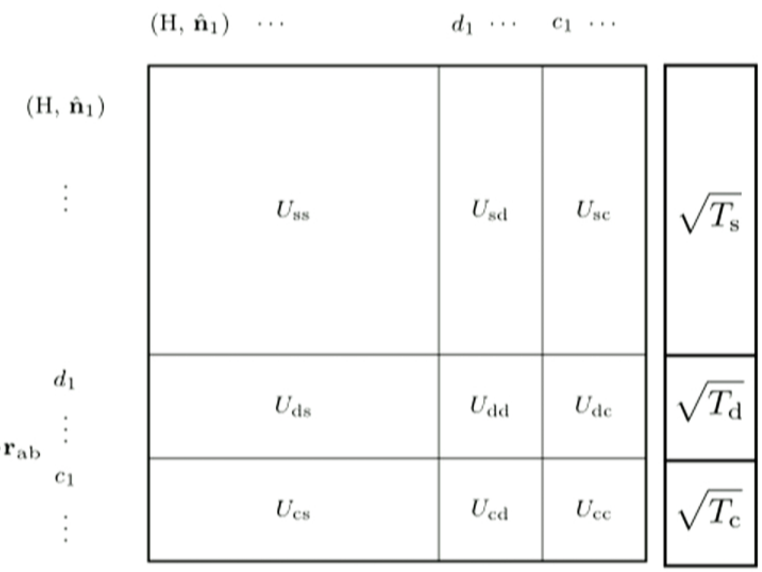
- Pair of matched antennas, no dissipation or crosstalk:

$$V_{\rm ab} = U_{\rm as} T_{\rm s} \mathbb{1} U_{\rm bs}^{\dagger}$$

$$\equiv$$

$$V_{\rm ab} = (\nu^2/c^2) \int d\hat{\mathbf{n}} A(\hat{\mathbf{n}}, \nu) T_{\rm sky}(\hat{\mathbf{n}}, \nu) e^{ik\hat{\mathbf{n}}\cdot\mathbf{r}_{\rm ab}}$$

- We get identities on A for free from unitarity



Usual Identities

- Single matched antenna, no dissipation:

$$T_{\rm a} = U_{\rm as} T_{\rm s} \mathbb{1} U_{\rm as}^{\dagger}$$

$$\equiv$$

$$T_{\rm a} = (\nu^2/c^2) \int d\hat{\mathbf{n}} A(\hat{\mathbf{n}}, \nu) T_{\rm sky}(\hat{\mathbf{n}}, \nu)$$

- Pair of matched antennas, no dissipation or crosstalk:

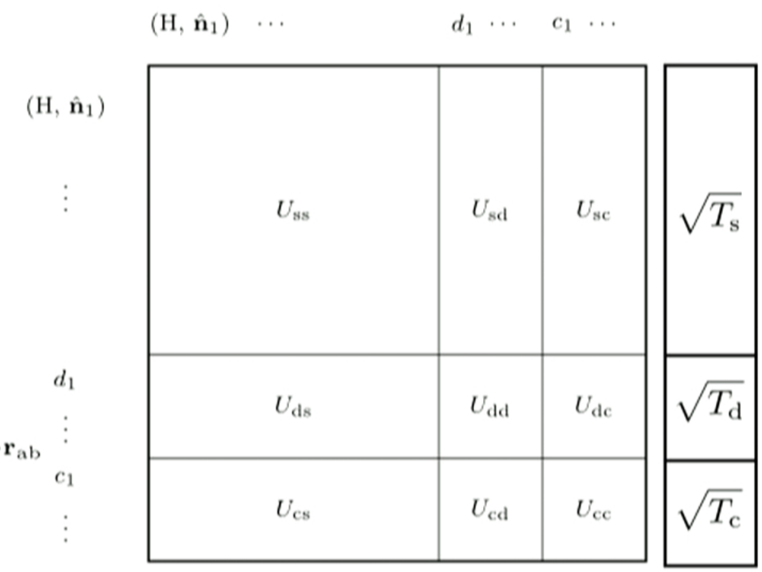
$$V_{\rm ab} = U_{\rm as} T_{\rm s} \mathbb{1} U_{\rm bs}^{\dagger}$$

$$\equiv$$

$$V_{\rm ab} = (\nu^2/c^2) \int d\hat{\mathbf{n}} A(\hat{\mathbf{n}}, \nu) T_{\rm sky}(\hat{\mathbf{n}}, \nu) e^{ik\hat{\mathbf{n}}\cdot\mathbf{r}_{\rm ab}}$$

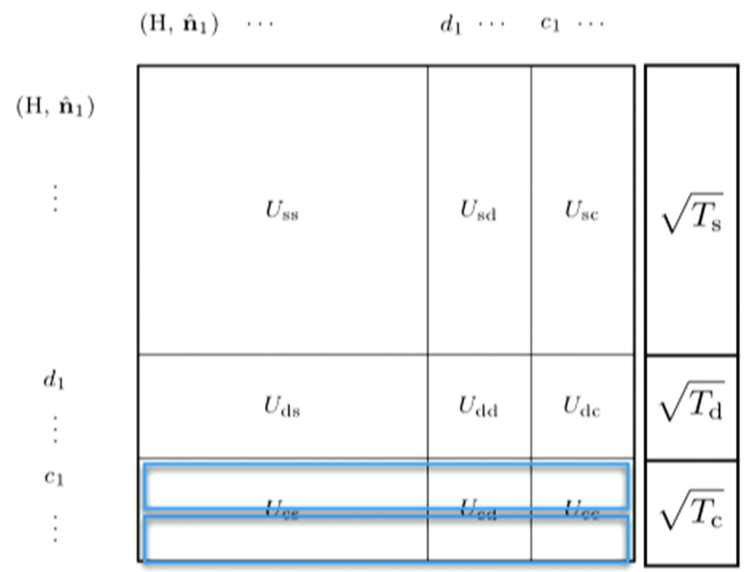
- We get identities on A for free from unitarity

- We also observe two consequences of crosstalk.



Global signal measurement

- Consider two channels being interfered.
  $\langle \psi^*_{c_i} \psi_{c_j} \rangle = \langle (\psi^*_{c_i,\mathrm{in}} + \psi^*_{c_i,\mathrm{out}})(\psi_{c_j,\mathrm{in}} + \psi_{c_j,\mathrm{out}}) \rangle$
- From the relation between the input and outputs, and the channels
  $$\langle \psi^*_{c_i} \psi_{c_j} \rangle = U(c_j; c_i) k_B T_{c_i} + U(c_i; c_j)^* k_B T_{c_j} + \sum_I U(c_i; I)^* U(c_j; I) k_B T_I.$$
- From unitarity, and a uniform sky
  $$\begin{aligned} \langle \psi^*_{c_i} \psi_{c_j} \rangle &= U(c_j; c_i) k_B T_{c_i} + U(c_i; c_j)^* k_B T_{c_j} + \sum_I U(c_i; I)^* U(c_j; I) k_B (T_I - T_s) \\ &= U(c_j; c_i) k_B T_{c_i} + U(c_i; c_j)^* k_B T_{c_j} + \sum_k U(c_i; c_k)^* U(c_j; c_k) k_B (T_{c_k} - T_s) \\ &\quad \sum_k U(c_i; d_k)^* U(c_j; d_k) k_B (T_{d_k} - T_s) \end{aligned}$$



Global signal measurement

- Consider two channels being interfered.
  $\langle \psi^*_{c_i} \psi_{c_j} \rangle = \langle (\psi^*_{c_i,\mathrm{in}} + \psi^*_{c_i,\mathrm{out}})(\psi_{c_j,\mathrm{in}} + \psi_{c_j,\mathrm{out}}) \rangle$
- From the relation between the input and outputs, and the channels

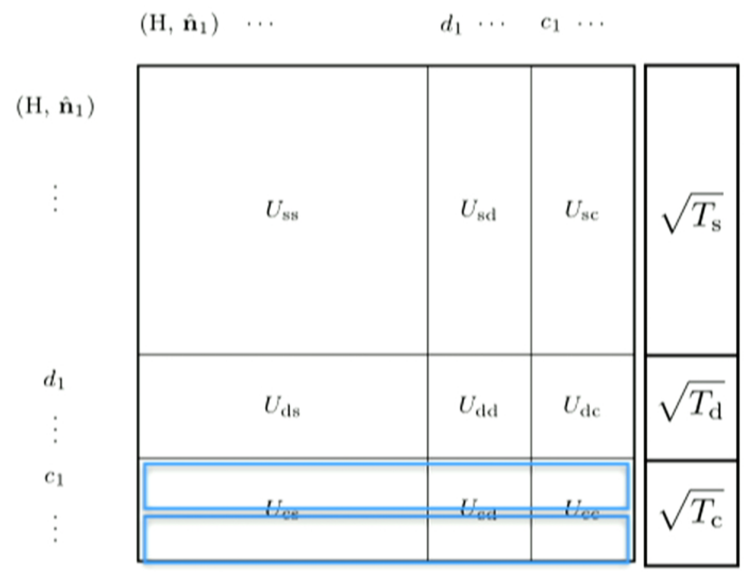
$$\langle \psi^*_{c_i} \psi_{c_j} \rangle = U(c_j; c_i) k_B T_{c_i} + U(c_i; c_j)^* k_B T_{c_j} + \sum_I U(c_i; I)^* U(c_j; I) k_B T_I .$$

- From unitarity, and a uniform sky

$$\begin{aligned} \langle \psi^*_{c_i} \psi_{c_j} \rangle &= U(c_j; c_i) k_B T_{c_i} + U(c_i; c_j)^* k_B T_{c_j} + \sum_I U(c_i; I)^* U(c_j; I) k_B (T_I - T_s) \\ &= U(c_j; c_i) k_B T_{c_i} + U(c_i; c_j)^* k_B T_{c_j} + \sum_k U(c_i; c_k)^* U(c_j; c_k) k_B (T_{c_k} - T_s) \\ &\quad \sum_k U(c_i; d_k)^* U(c_j; d_k) k_B (T_{d_k} - T_s) \end{aligned}$$

- Rephrasing in terms of crosstalk and correlated noise

$$\begin{aligned} \frac{1}{k_B} \frac{\partial}{\partial T_{\mathrm{sky}}} \langle \psi^*_{c_i} \psi_{c_j} \rangle &= -\left[U(c_i; c_i)^* U(c_j; c_i) + U(c_i; c_j)^* U(c_j; c_j) \right] - \\ &\quad \sum_{k \neq i,j} U(c_i; c_k)^* U(c_j; c_k) - \sum_{d_k} \frac{1}{k_B} \frac{\partial}{\partial T_{d_k}} \langle \psi^*_{c_i} \psi_{c_j} \rangle \end{aligned}$$



Conclusions and outlook

- Any interferometric global signal experiment must operate in a regime close to a single antenna experiment.
- A possibility is that this offers additional calibration handles, but in general this requires careful understanding of the systematics of the setup.