

Title: Postquantum steering

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URL: <http://pirsa.org/16020092>

Abstract: <p>The discovery of postquantum nonlocality, i.e. the existence of nonlocal correlations stronger than any quantum correlations but nevertheless consistent with the no-signaling principle, has deepened our understanding of the foundations quantum theory. In this work, we investigate whether the phenomenon of Einstein-Podolsky-Rosen steering, a different form of quantum nonlocality, can also be generalized beyond quantum theory. While postquantum steering does not exist in the bipartite case, we prove its existence in the case of three observers. Importantly, we show that post-quantum steering is a genuinely new phenomenon, fundamentally different from postquantum nonlocality. Our results provide new insight into the nonlocal correlations of multipartite quantum systems.

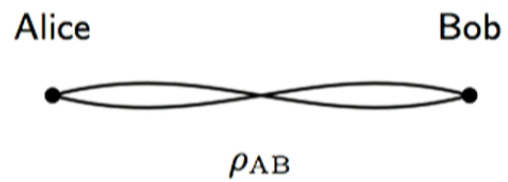
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Postquantum steering

Ana Belén Sainz, Nicolas Brunner, Daniel Cavalcanti
Paul Skrzypczyk and Tamás Vértesi

Phys. Rev. Lett. 115, 190403 (2015)

Entanglement



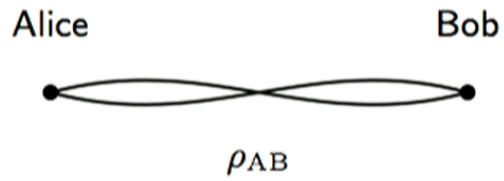
- Quantum teleportation
- Quantum Key Distribution

Nonlocality



- Device Independent QKD
- Randomness

Entanglement



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Nonlocality

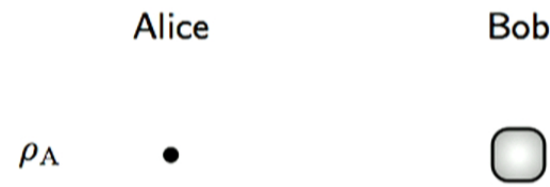


- Device Independent QKD
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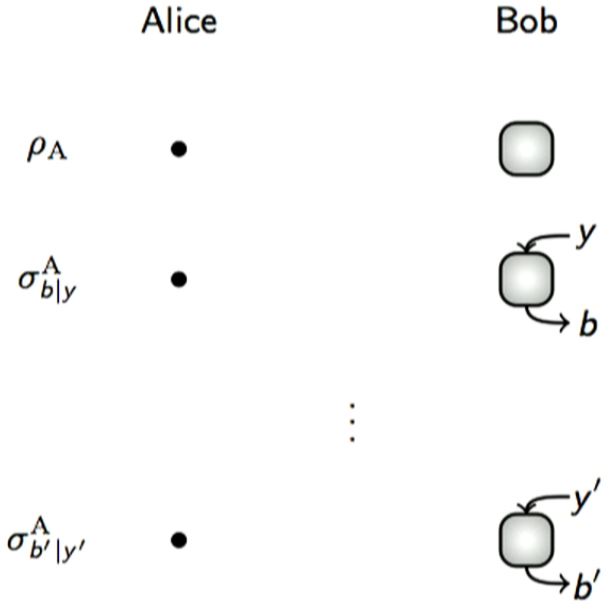
Steering



Steering



Steering



Steering

Fix $y \longrightarrow$ ensemble: $\{\sigma_{b|y}^\Lambda\}_b, \longrightarrow \rho_\Lambda = \sum_b \sigma_{b|y}^\Lambda$

Assemblage: $\{\sigma_{b|y}^\Lambda\}_{b,y}. \quad p(b|y) = \text{tr}(\sigma_{b|y}^\Lambda).$

Steering

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Quantum: $\sigma_{b|y}^A = \text{tr}_B(\mathbb{1}_A \otimes M_{b|y} \rho_{AB})$

Given an assemblage, **could it have a classical explanation?**

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Steering Inequality:

$$\text{tr} \sum_{by} F_{by} \sigma_{b|y}^A \leq \beta_{\text{US}}$$

Set of "classical" assemblages

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Given an assemblage, **could it have a quantum explanation?**

Bipartite steering

$$\text{Given } \{\sigma_{b|y}^A\}_{b,y}, \quad \rho_A = \sum_b \sigma_{b|y}^A, \quad \text{tr}(\rho_A) = 1$$

$$\exists \rho_{AB}, \quad \{M_{b|y}\}_{b,y} \quad \text{st} \quad \sigma_{b|y}^A = \text{tr}_B(\mathbb{1}_A \otimes M_{b|y} \rho_{AB})$$

¹N. Gisin, Helvetica Physica Acta 62, 363 (1989).
L. P. Hughston, R. Jozsa and W. K. Wootters, Phys. Lett. A 183, 14 (1993).

Bipartite steering

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- Alice and Bob: **Yes !** GHJW theorem¹
- **Multipartite scenarios?**

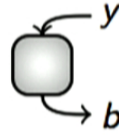
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Steering: multipartite scenarios

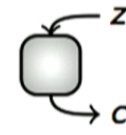
Alice

•
 $\sigma_{bc|yz}^A$

Bob

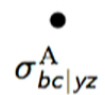


Charlie

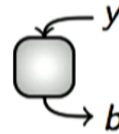


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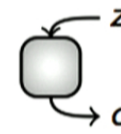
Alice



Bob



Charlie

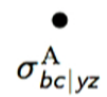


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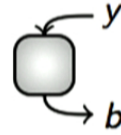
Steering: multipartite scenarios

Alice

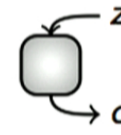


$\sigma_{bc|yz}^A$

Bob



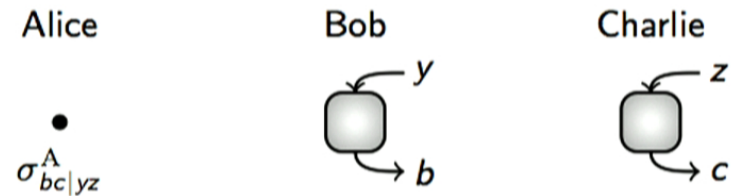
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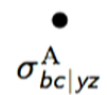
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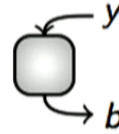
No Signalling: $\sum_b \sigma_{bc|yz}^A = \sum_b \sigma_{bc|y'z}^A, \quad \sum_c \sigma_{bc|yz}^A = \sum_c \sigma_{bc|yz'}^A$

Steering: multipartite scenarios

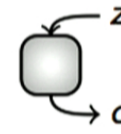
Alice



Bob



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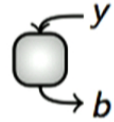
$\exists \rho_{ABC}, \{M_{b|y}\}_{b,y}, \{M_{c|z}\}_{c,z} \text{ st } \sigma_{bc|yz}^A = \text{tr}_B(\mathbb{1}_A \otimes M_{b|y} \otimes M_{c|z} \rho_{ABC})$

Postquantum steering: example

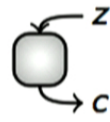
Alice

$$\bullet$$
$$\sigma_{bc|yz}^A$$

Bob



Charlie



$$b, c, y, z \in \{0, 1\}$$

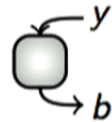
$$\rho_A = \frac{1}{2}.$$

Postquantum steering: example

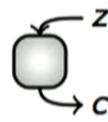
Alice

$$\sigma_{bc|yz}^A$$

Bob



Charlie



$$b, c, y, z \in \{0, 1\}$$

$$\rho_A = \frac{1}{2}.$$

- $(y, z) = (0, 0), (0, 1), (1, 0)$:

$$\sigma_{bc|yz}^A = \begin{cases} \frac{1}{4}, & \text{if } b = c, \\ 0, & \text{if } b \neq c, \end{cases}$$

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$$\sum_b \sigma_{bc|yz}^A = \frac{1}{4}, \quad \sum_c \sigma_{bc|yz}^A = \frac{1}{4}$$

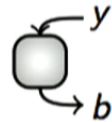
Postquantum steering: example

Alice

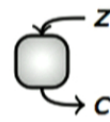
$$\bullet$$

$$\sigma_{bc|yz}^A$$

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Charlie



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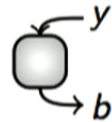
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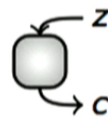
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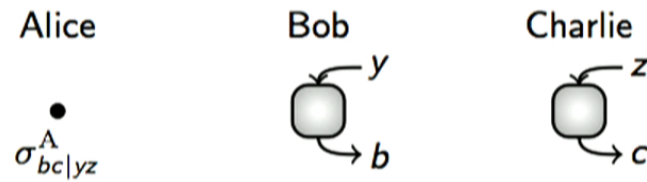
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$$p(bc|yz) = \begin{cases} \frac{1}{2}, & \text{if } b \oplus c = yz, \\ 0, & \text{otherwise.} \end{cases}$$

$$\sum_b \sigma_{bc|yz}^A = \frac{1}{4}, \quad \sum_c \sigma_{bc|yz}^A = \frac{1}{4}$$

No quantum realisation for the assemblage

Postquantum steering: example without postquantum nonlocality

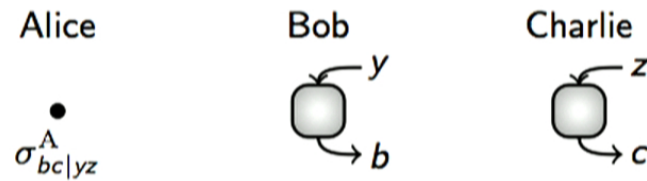


(1) Postquantum assemblage $\{\sigma_{bc|yz}^A\}_{b,y,c,z}$

(2) Quantum correlations for every measurement by Alice:

$$p(abc|xyz) = \text{tr}(M_{a|x} \otimes \sigma_{bc|yz}^A)$$

Postquantum steering: example without postquantum nonlocality



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(1) Postquantum assemblage $\sigma_{bc|yz}^A$

Steering inequality: F_{bcyz}

$$\text{tr} \sum_{bcyz} F_{bcyz} \sigma_{bc|yz}^A$$

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How to compute β_Q ? \rightarrow upper bound

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Almost quantum assemblages: $\tilde{Q} \supset Q$

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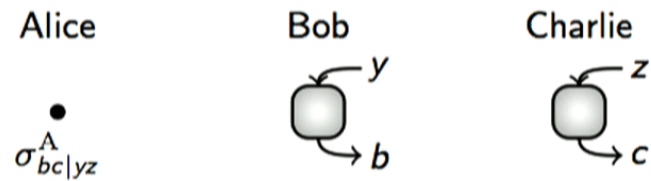
How to compute β_Q ? \rightarrow upper bound

Almost quantum assemblages: $\tilde{Q} \supset Q$

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$$\text{tr} \sum_{bcyz} F_{bcyz} \sigma_{bc|yz}^A > \beta_{\tilde{Q}} \Rightarrow \sigma_{bc|yz}^A \text{ is postquantum}$$

Example without postquantum nonlocality



(1) Postquantum assemblage $\{\sigma_{bc|yz}^A\}_{b,y,c,z}$

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$$p(abc|xyz) = \text{tr} (M_{a|x} \otimes \sigma_{bc|yz}^A)$$

(2) Quantum correlations $p(abc|xyz)$

- (i) $p(abc|xyz)$ is local
- (ii) Real qubit assemblage, local for all projective measurements
- (iii) Qutrit assemblage, local for all POVMs².

²F. Hirsch, M. T. Quintino, J. Bowles and N. Brunner, Phys. Rev. Lett, 111, 160402 (2013).

(ii) Qubit assemblage, local for all PVM

$$\Pi_{a|x}(\mu) = \mu \Pi_{a|x} + (1 - \mu) \mathbb{1}/2, \quad \sigma_{bc|yz}^{\Lambda}(\mu) = \mu \sigma_{bc|yz}^{\Lambda} + (1 - \mu) \text{tr}(\sigma_{bc|yz}^{\Lambda}) \mathbb{1}/2$$

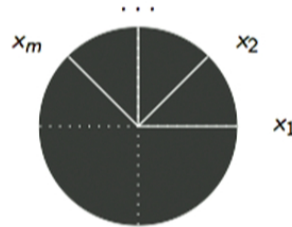
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- $p(abc|xyz) = \text{tr}_A (\Pi_{a|x}(\mu) \sigma_{bc|yz}^\Lambda) = \text{tr}_A (\Pi_{a|x} \sigma_{bc|yz}^\Lambda(\mu))$
- Noisy measurements are linear combinations of (finite number) PVMs.

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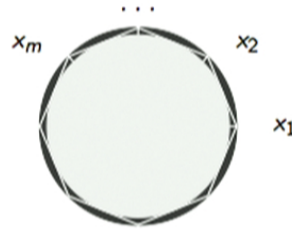


$\sigma_{bc|yz}^A$ **local for** $\{x_1, \dots, x_m\}$

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$$\Leftrightarrow \sigma_{bc|yz}^A(\mu) \text{ local } \forall \Pi_{a|x}$$

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Postquantum steering: example without postquantum nonlocality

- Four dichotomic measurements (X, Z)
- Search:

Fix $F_{bc|yz}$:

- Compute $\beta_{\tilde{Q}}$ (SDP).
- Find max violation of the inequality by the 'local' assemblages (SDP).
→ $\sigma_{bc|yz}^{\Lambda}$
- Compute $\beta^* = \text{tr} \sum_{bcyz} F_{bcyz} \sigma_{bc|yz}^*$, $\sigma_{bc|yz}^* := \sigma_{bc|yz}^{\Lambda}(\mu = \cos(\frac{\pi}{8}))$
If $\beta^* > \beta_{\tilde{Q}}$: done! , otherwise, change $F_{bc|yz}$, start over.

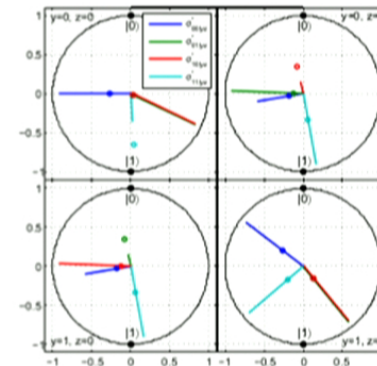
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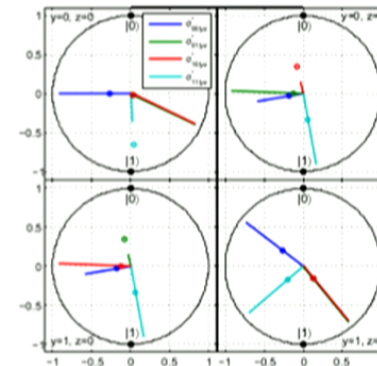
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$$\tilde{\sigma}_{bc|yz}^* = \frac{1}{3} \sigma_{bc|yz}^* + \frac{2}{3} \text{tr}(\sigma_{bc|yz}^*) |2\rangle\langle 2|$$

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Summary and open questions

- Steering beyond quantum theory → multipartite scenarios
- Genuinely new effect
→ postquantum steering $\not\Rightarrow$ postquantum nonlocality
- Fundamental difference between bipartite and multipartite scenarios

Summary and open questions

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- Genuinely new effect
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