Title: Causality Constraints in Conformal Field Theory

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Abstract: <p>In effective field theory, causality fixes the signs of certain interactions. I will describe how these Lorentzian constraints are encoded in the Euclidean theory, and use the conformal bootstrap to derive analogous causality constraints in CFT. Applied to spinning fields, these constraints include (some of) the Hofman-Maldacena bounds derived from conformal collider physics. I will also discuss applications to holographic theories.</p>

Causality Constraints in Conformal Field Theory

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PERIMETER ✦ FEBRUARY 2016

Causality imposes constraints on effective field theories.

Example:

$$L = (\partial\phi)^2 + \lambda(\partial\phi)^4 + \cdots$$

for $\lambda < 0$ cannot be embedded in any consistent UV theory.

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This plays a key role in the proof of the *a* theorem.

Similar constraints on higher curvature gravity are even stronger:

$$L = \sqrt{g}(R + \alpha_{GB}R^2 + \cdots) \quad \Rightarrow \alpha_{GB} = 0!$$

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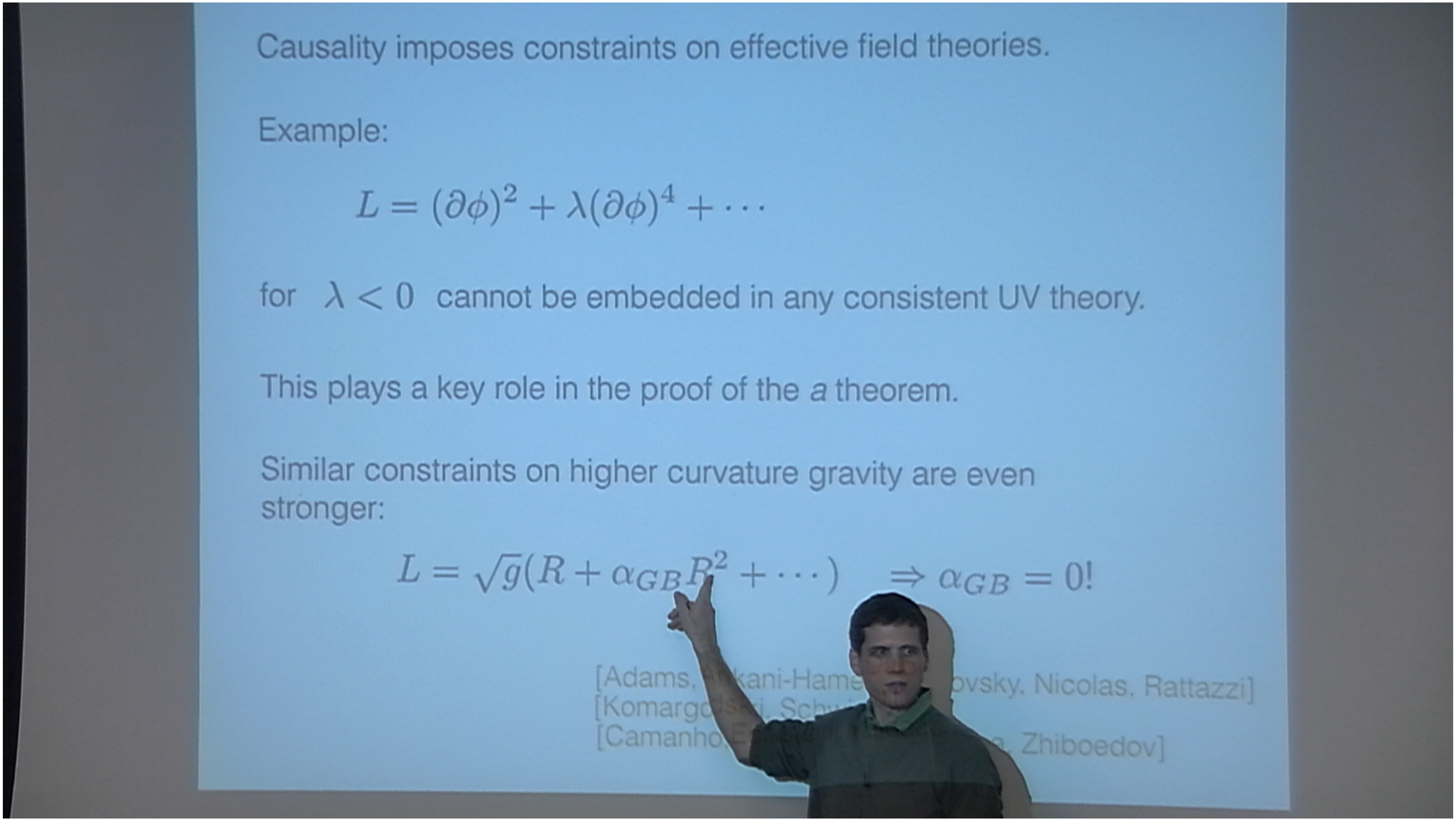

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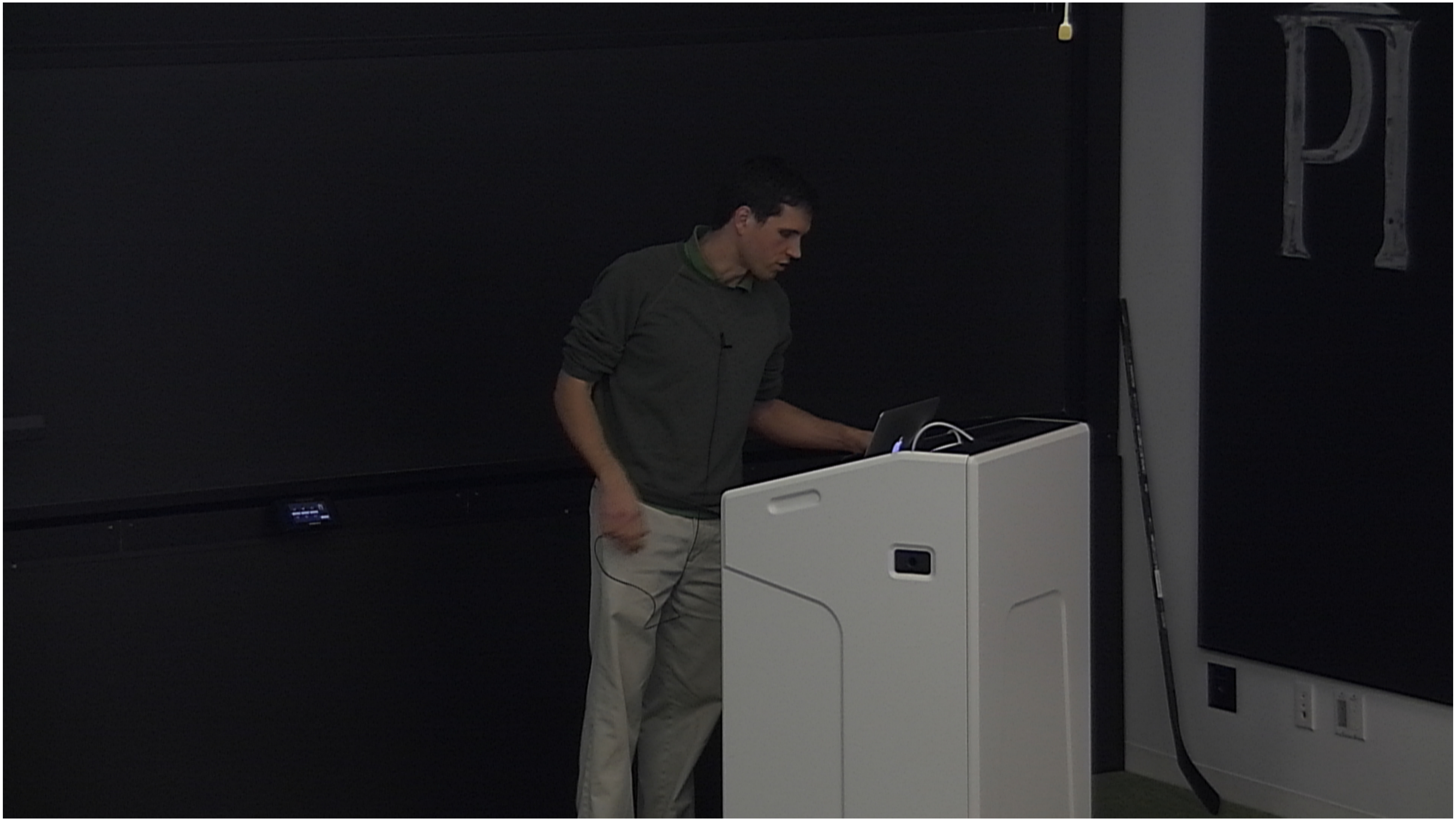


Two arguments for this type of constraint:

- Causality in non-trivial backgrounds
- Optical theorem / dispersion relations

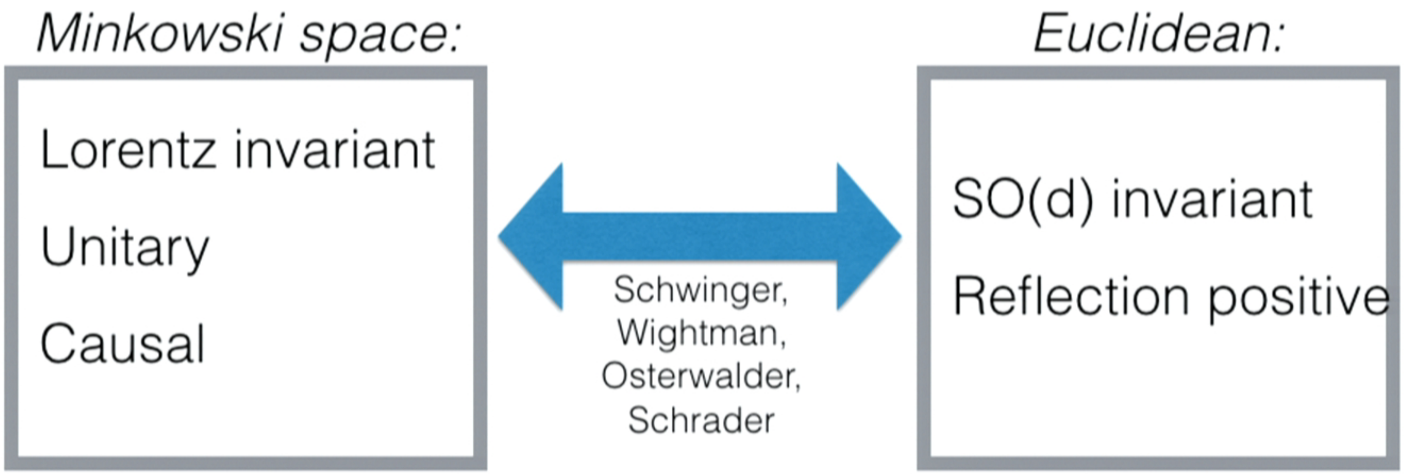
Both are indirect: to any order perturbatively around vacuum,

$$[\phi(x), \phi(y)] = 0 \quad \text{(spacelike)}$$

This must fail at some point (UV); but why and where, exactly?

Also, both arguments are inherently *Lorentzian* signature.

But good Lorentzian theories are in one-to-one correspondence with good Euclidean theories:



So where is the problem in the Euclidean theory?

But good Lorentzian theories are in one-to-one correspondence with good Euclidean theories:

Minkowski space:

- Lorentz invariant
- Unitary
- Causal

Schwinger, Wightman, Osterwalder, Schrader

Euclidean:

- SO(d) invariant
- Reflection positive

So where is the problem in the Euclidean theory?

This talk:

- Derive analogous constraints in CFT from bootstrap
  - d > 2
  - strong coupling
  - not assuming large *N*
  - but if we take *N* large, then these constraints are holographically dual to $\lambda(\partial\phi)^4$ constraints in bulk

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$$[\phi(x), \phi(y)] \neq 0$$

- We will work entirely in position space (+/-)

CFT constraints: 1509.00014 and 1601.07904 with Sachin Jain and Sandipan Kundu.

Perturbative QFT results: work in progress with Venkatesa Chandrasekaran, John Stout, and Amir Tajdini.

Holographic Motivation

Holographic CFTs show lots of universal behavior in the correlation functions:

viscosity-to-entropy $\frac{\eta}{s} = \frac{1}{4\pi}$

anomaly coefficients in <TTT>: $a = c$

etc.

These are dictated by locality in the bulk:

$$S = \int \sqrt{g}(R + \text{small})$$

Question:

How do we derive these universal facts directly from CFT?

I don't know.

But seems likely two things will be useful:

- Bootstrap (some successes in d=2)
- Causality (important in bulk argument)

Causality review

Causality:

$$\langle \Psi | [O(x),\ O(y)] | \Psi \rangle = 0 \qquad (x-y)^2 < 0\ .$$

This is a Lorentzian statement.

But bootstrap is usually formulated in terms of Euclidean correlation functions.

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So first:

How is causality encoded in Euclidean correlators?

This was understood long ago [*eg, Streater and Wightman].*

Euclidean correlators

$$G(x_1, x_2, \dots) \equiv \langle O(x_1) O(x_2) \dots \rangle$$

are:

- Permutation invariant $\quad G(x_1, x_2, \dots) = G(x_2, x_1, \dots)$
- With singularities only at coincident points
- and no branch cuts (ie, single-valued).

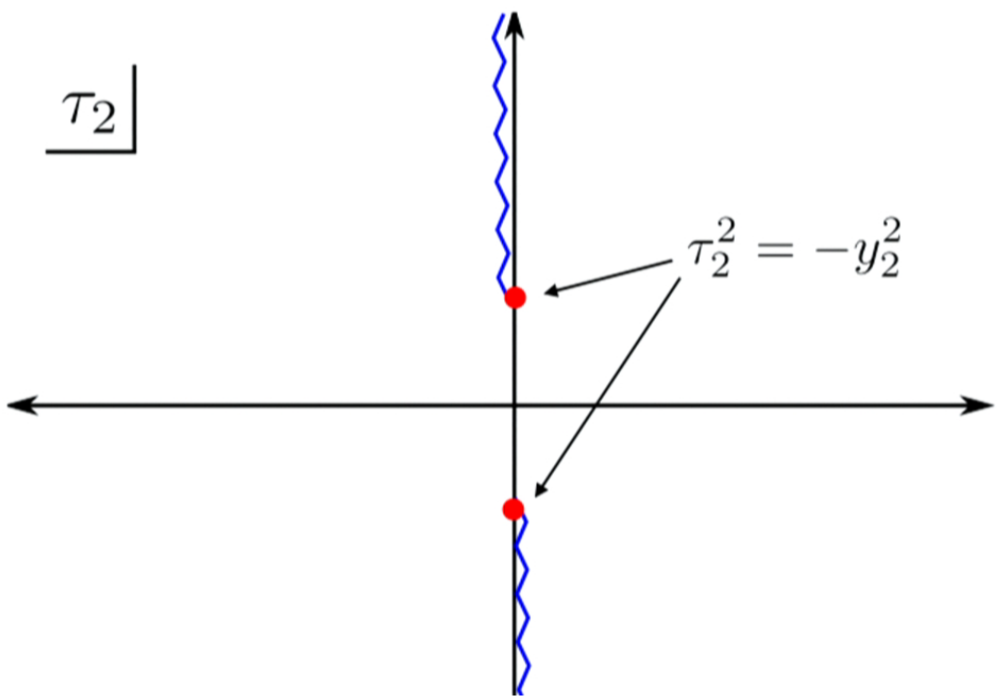
Ex: conformal scalar

$$\langle O(0,0) O(\tau_2, y_2) \rangle = (\tau_2^2 + y_2^2)^{-2\Delta}$$

But if we analytically continue to complex time: $\tau_i \in \mathbf{C}$

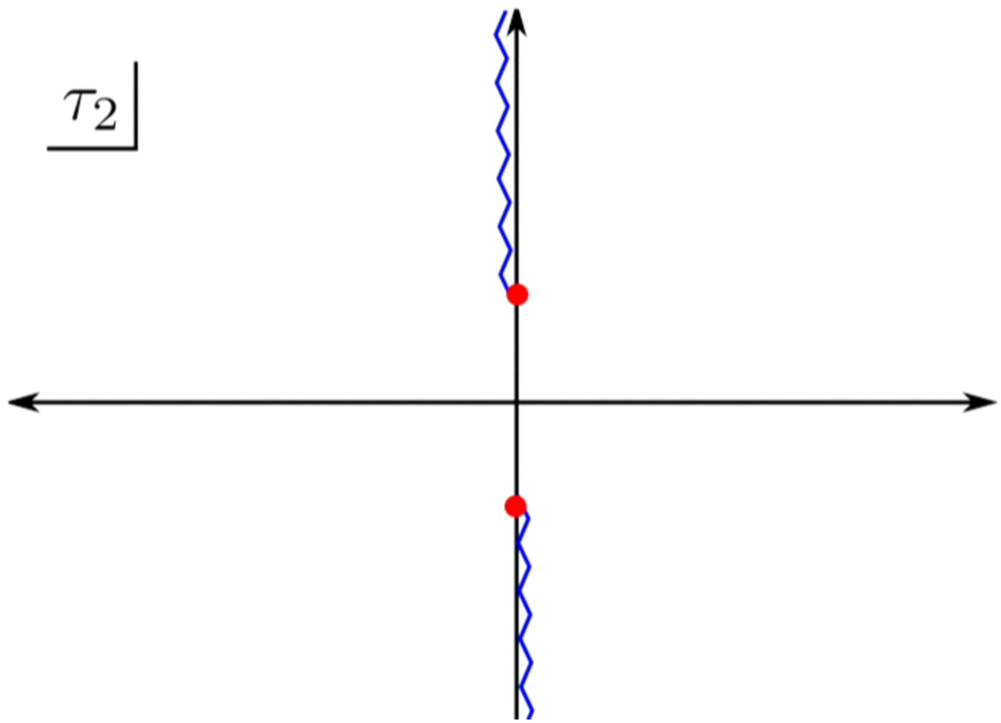
then there is an intricate structure of singularities and branch cuts.

Ex: conformal scalar 2pt function $G = (\tau_2^2 + y_2^2)^{-2\Delta}$



Therefore the analytic continuation to Lorentzian signature is ambiguous.

This ambiguity is why operators do not commute in Lorentzian QFT.



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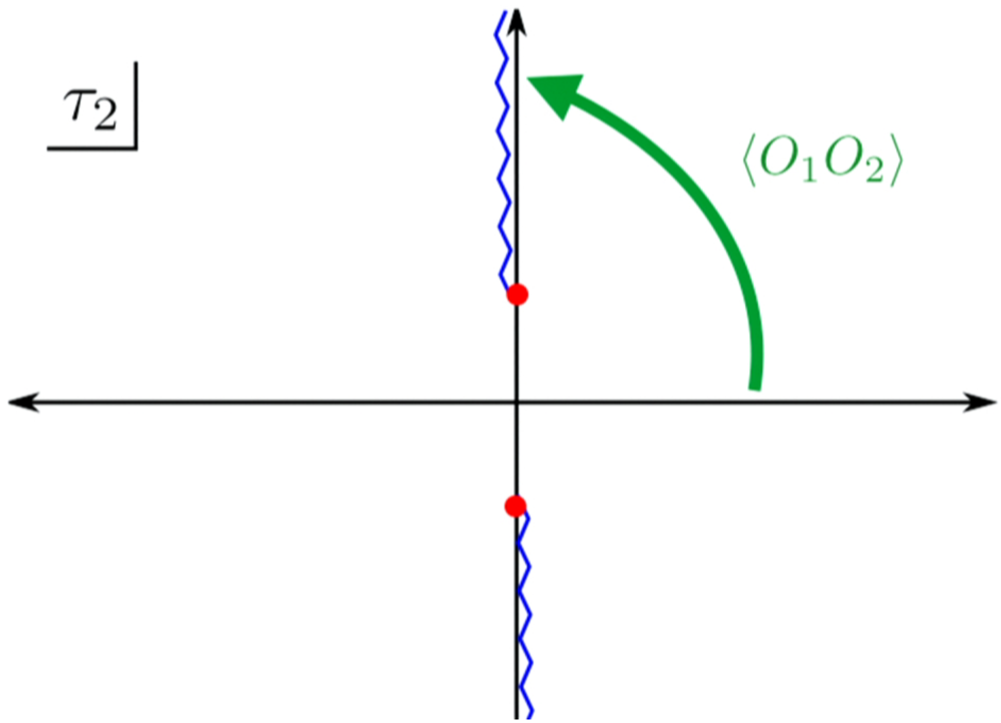

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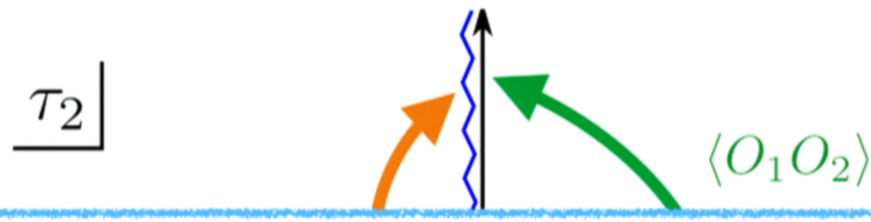
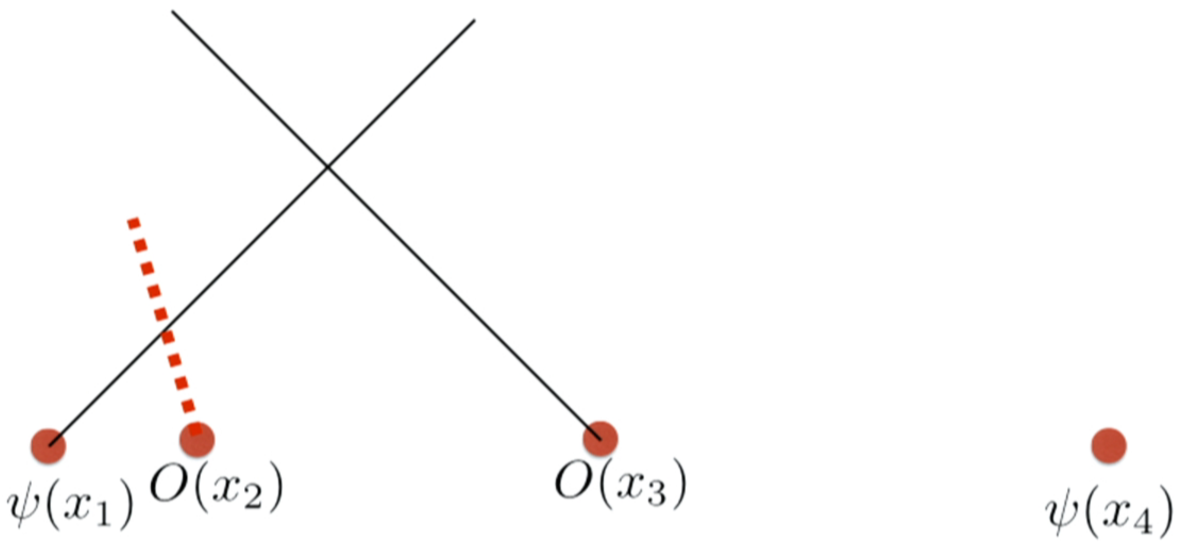


So: Commutator $\langle [O_1, O_2] \rangle$ = discontinuity across the cut.

The branch point is exactly at the Minkowski lightcone, so the 2pt function is trivially causal.

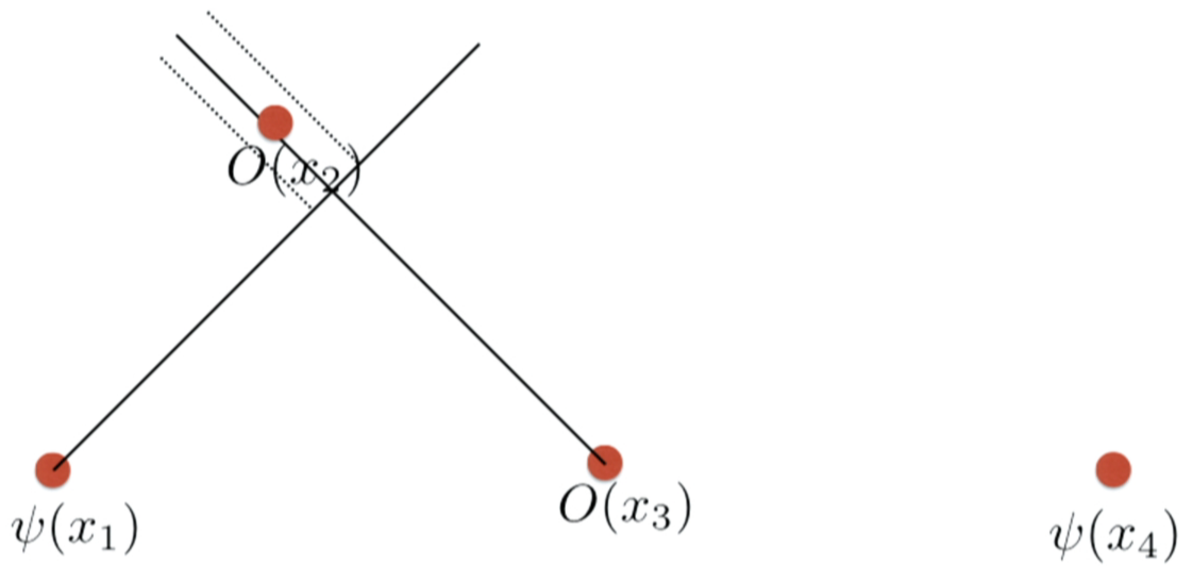
4-point functions

More generally, there is a branch cut whenever an operator passes the lightcone of another operator:

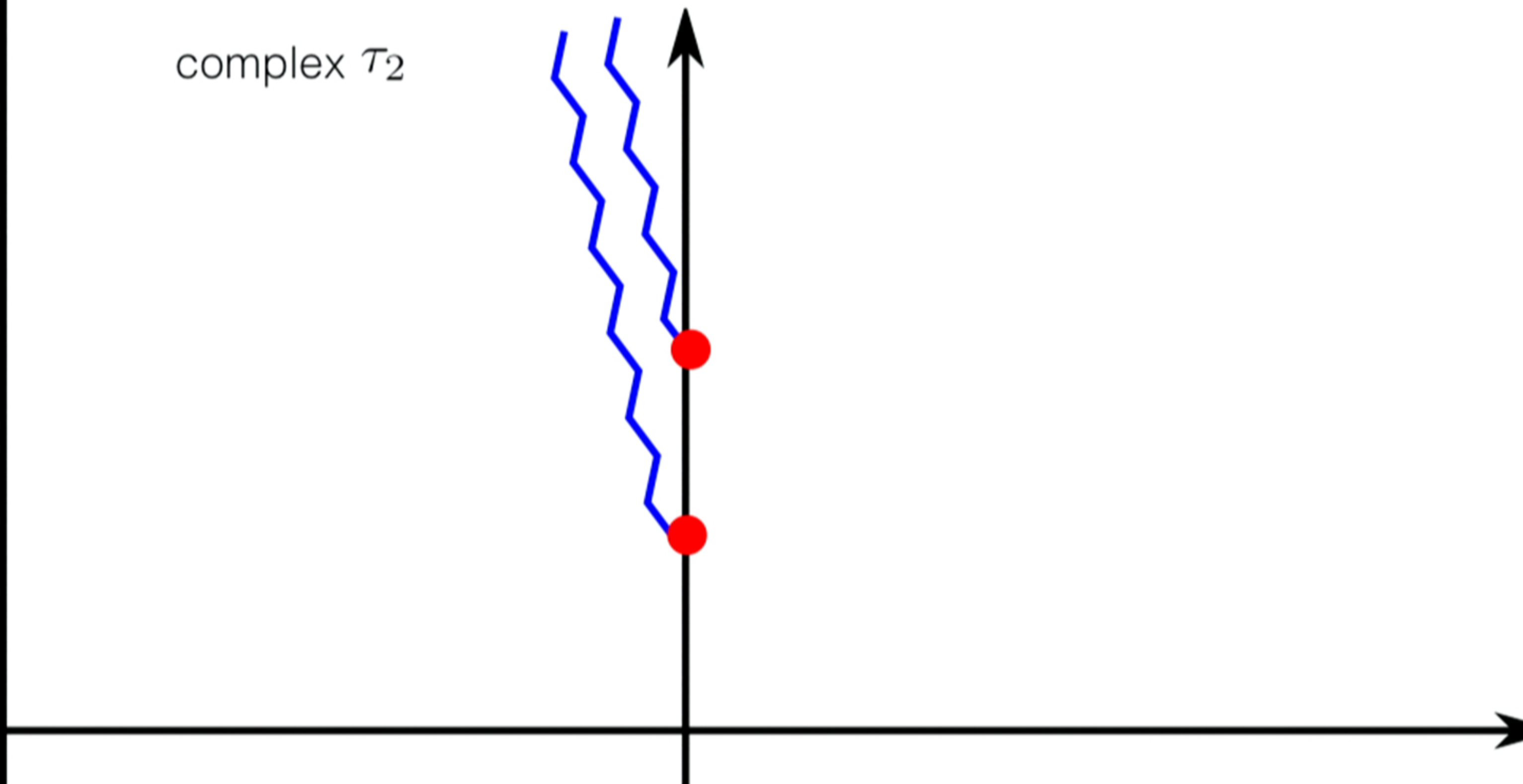


Causality is the statement that the lightcone singularity in this situation cannot appear "too soon."

This is a statement that the correlator is analytic on some region of complexified spacetime.



Same picture on the complex time plane:



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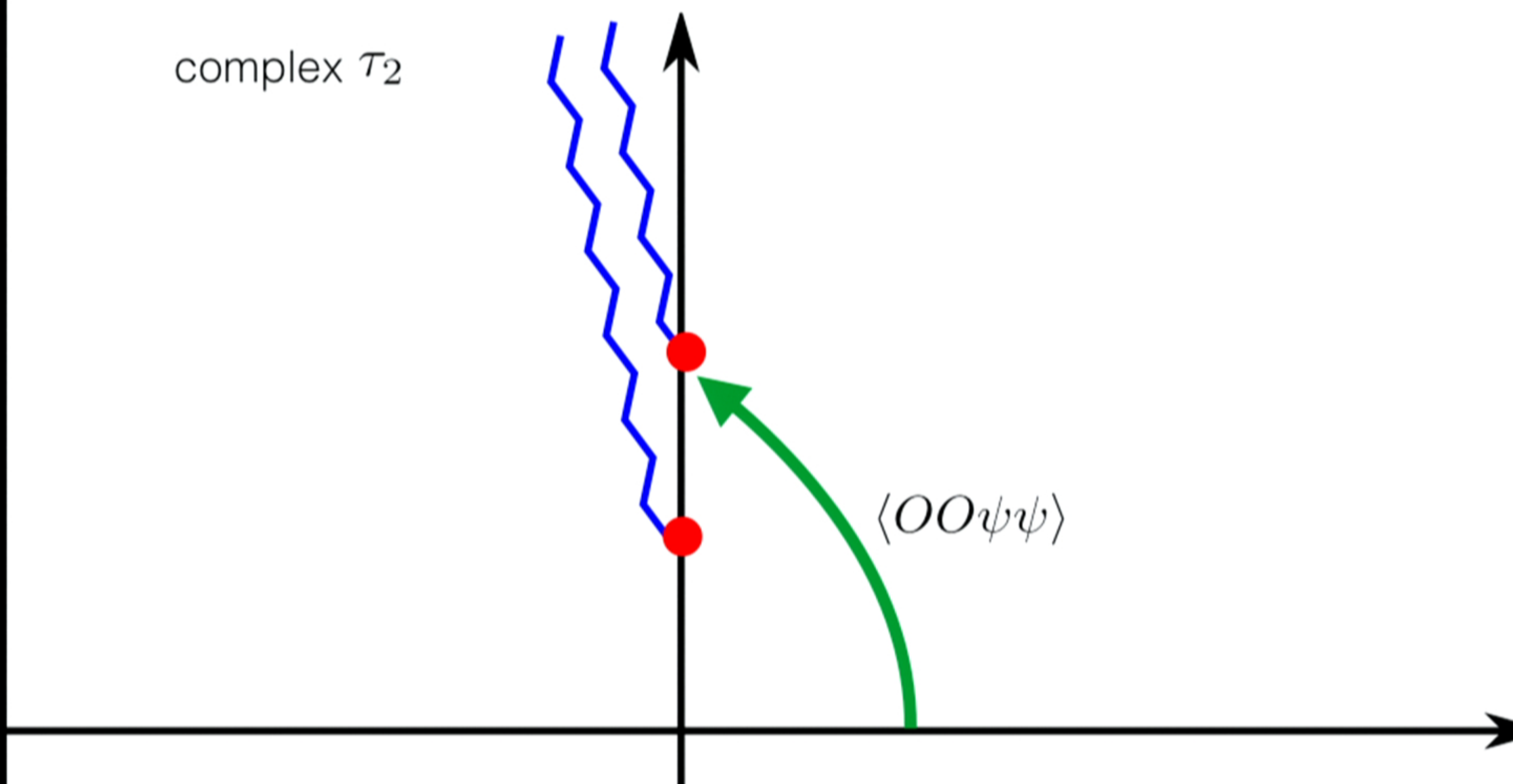

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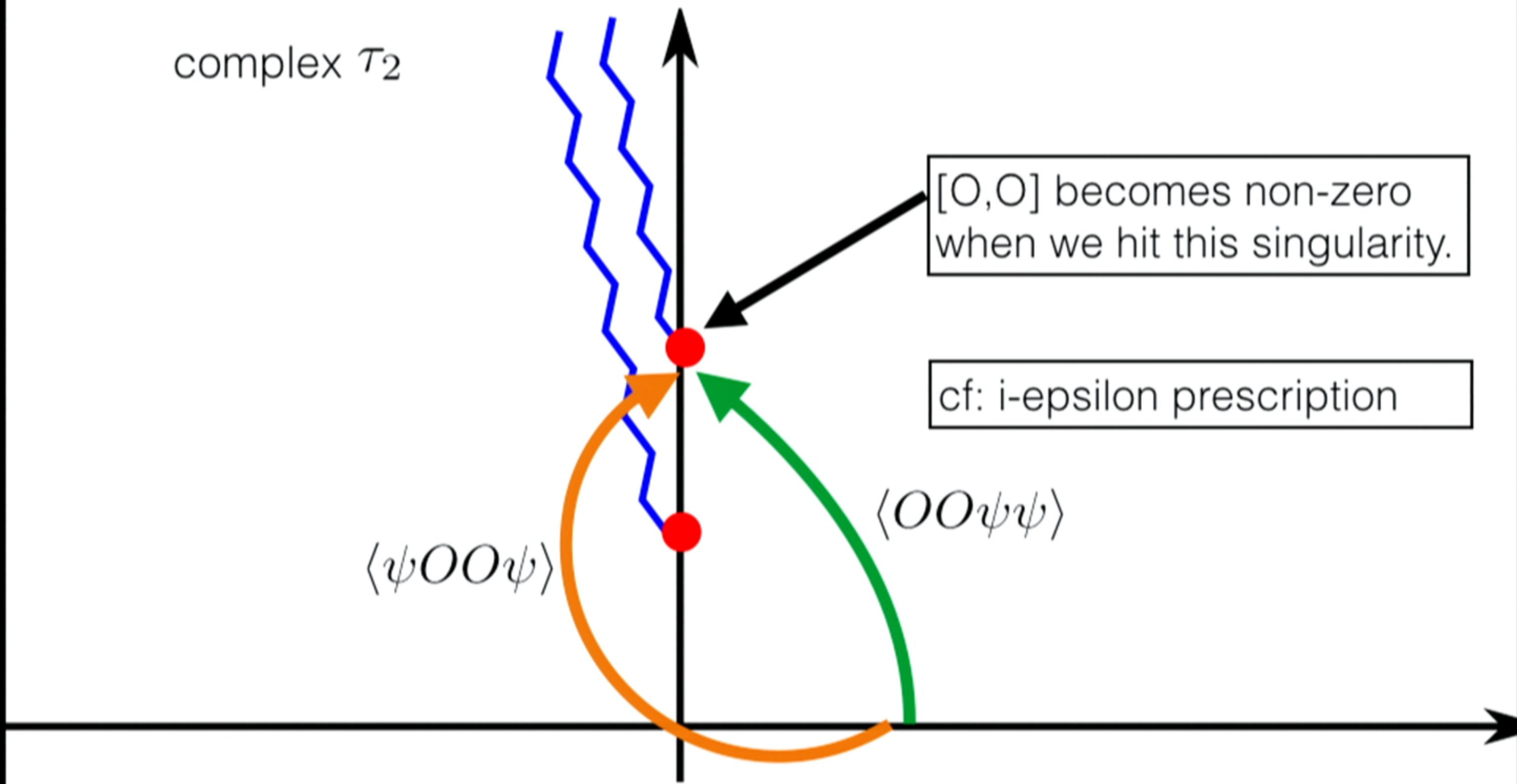


CFT

[Luscher, Mack '74]

This was for a general QFT.

In CFT, can phrase in terms of the cross ratios:

$$z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \ , \quad (1-z)(1-\bar{z}) = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

and causality is statement about where $G(z, \bar{z})$ is analytic, on a multi-sheeted ***C x C***.

(Example later.)

The key ingredient in Euclidean QFT that prevents singularities from being in the "wrong place" is reflection positivity:

Reflection-positive Euclidean theories

Unitary, causal Lorentzian theories

[Schwinger, Wightman, Osterwalder, Schrader, etc.]

We will first “rediscover” this result in CFT in a way amenable to bootstrap, then extend it to derive *low energy* constraints.

Basic idea is to use crossing symmetry of a 4pt function:

$$G(z,\bar{z}) = G(1-z, 1-\bar{z})$$

to relate UV and IR:

Reflection positivity in the UV (s-channel)

Causality of the correlator

Constraints on the IR couplings (t-channel)

Building on lightcone bootstrap,

[Komargodski, Zhiboedov;
Fitzpatrick et al; Alday et al; etc]

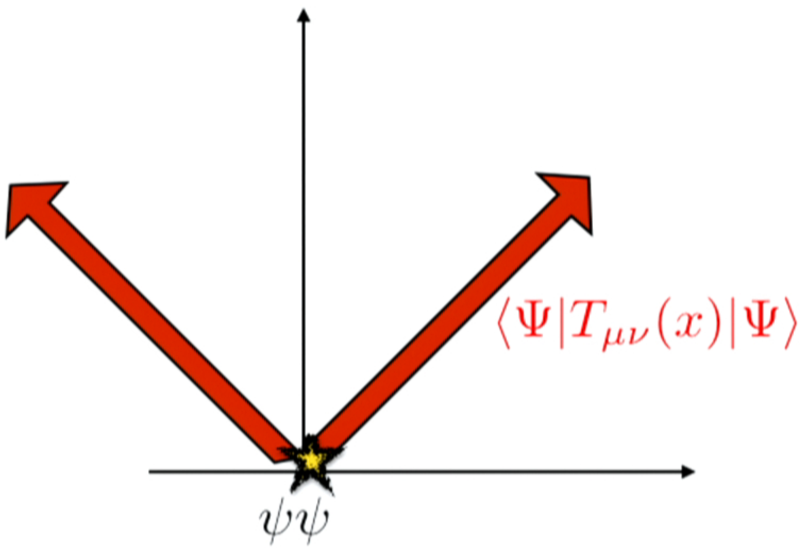
but allowing for timelike-separated operators.

The "Shockwave State" in Conformal Field Theory

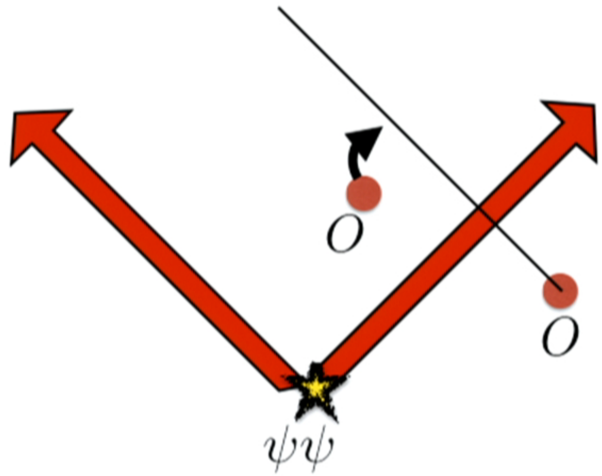
Define the "shockwave state":

$$|\Psi\rangle \equiv \psi(t = i\delta, \vec{x} = 0)|0\rangle$$

For small δ this creates a stress tensor with support on an expanding null shell:

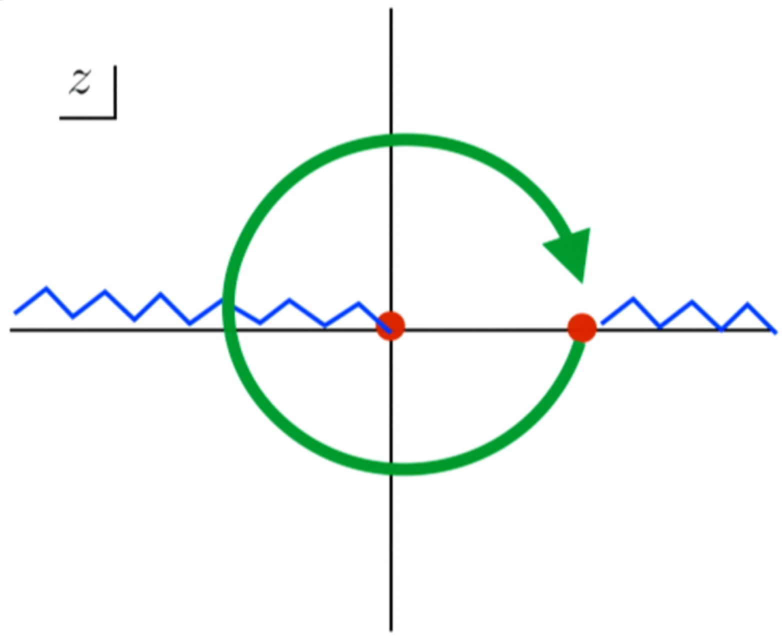


Probe the shockwave with an operator O:



The Causality Requirement:

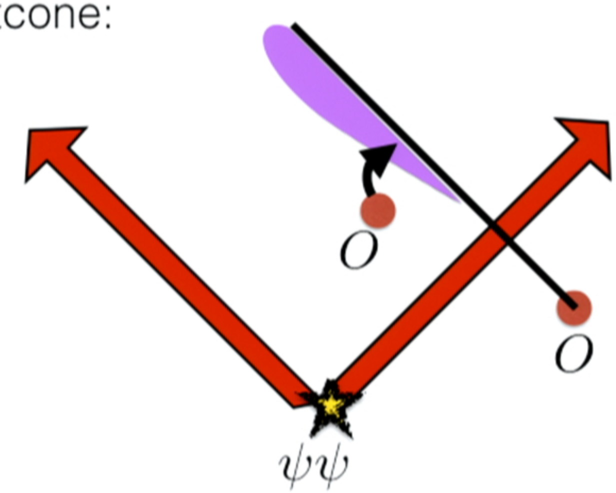
After taking *z* around zero,

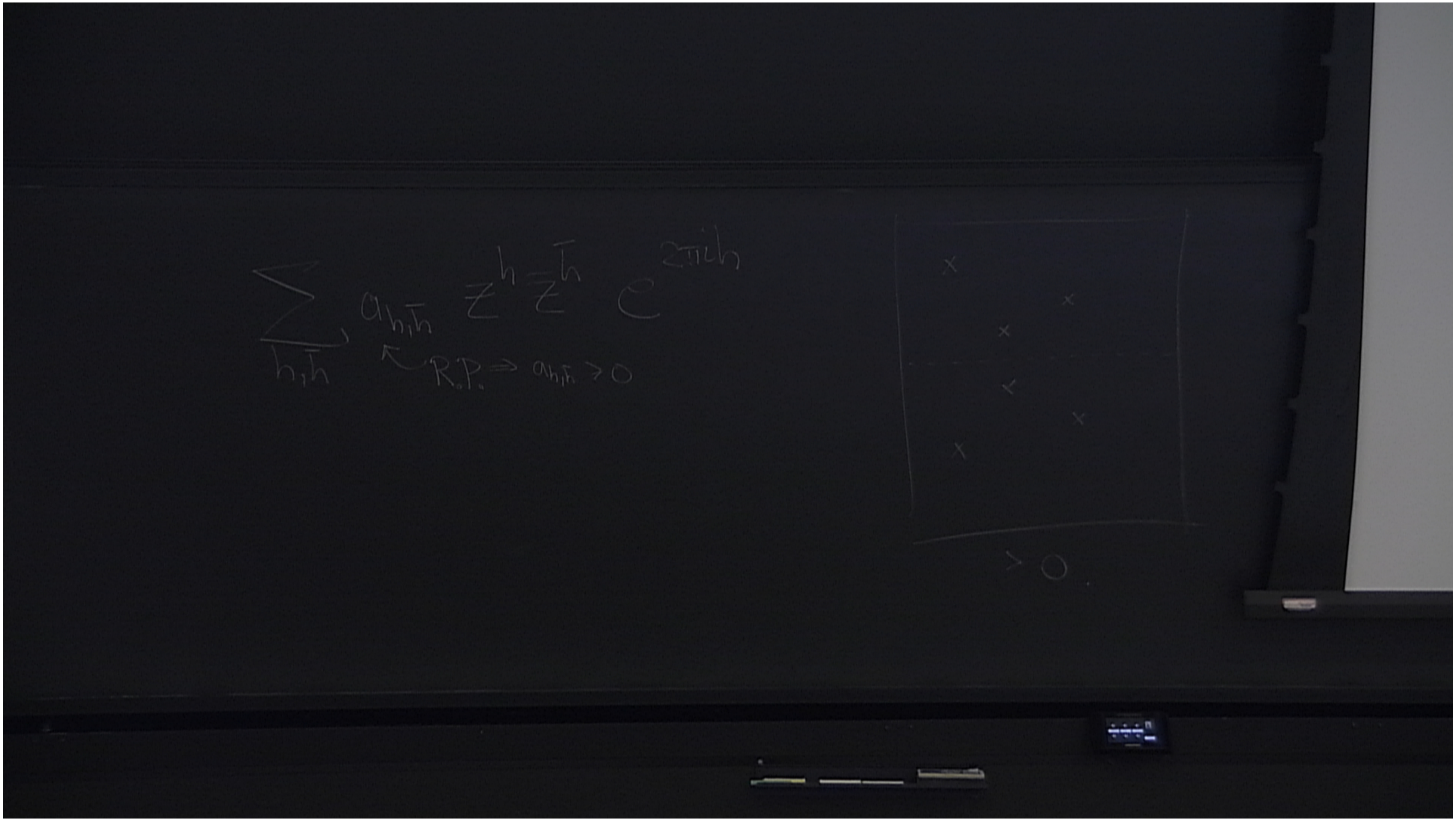


So far, we've just translated causality into a statement about a particular CFT correlator.

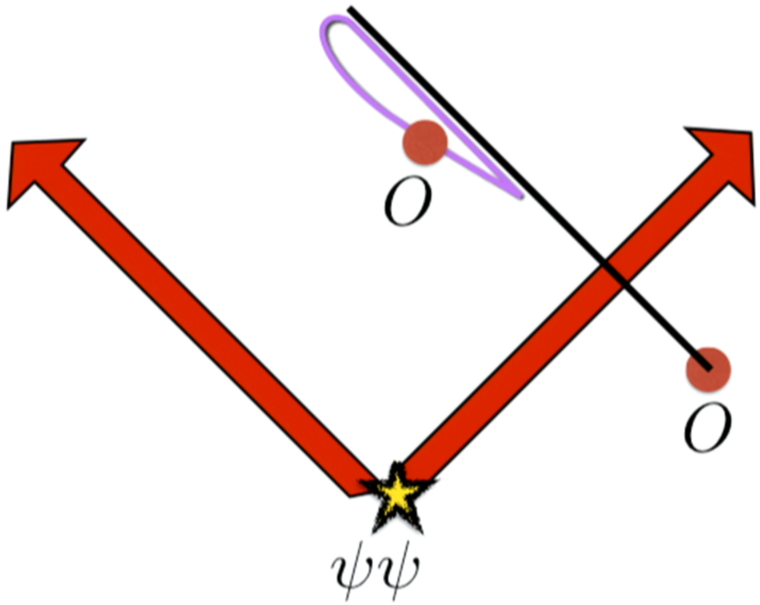
Next: analyze this correlator using the conformal block expansion.

The purple region is a complexified region of spacetime "just before" the lightcone:




Σ a_{h,h̄} z^h z̄^h̄ e^{2πih}
h,h̄
R.P. ⇒ a_{h,h̄} > 0
> 0

To derive IR constraints, integrate this analytic function on a closed contour:



And require $\oint G = 0$ ➡ Position space "dispersion relation"

When the dust settles, the sum rule + crossing symmetry implies

$$\begin{aligned}\lambda_{IR} &= \int_{UV} dx |\text{something}|^2 \\ &\geq 0\end{aligned}$$

where λ_{IR} is the coupling to the minimal-twist operator.

Typically this is the stress tensor,

$$\lambda_{IR} \sim \langle O(0)O(1)T_{--}(\infty)\rangle$$

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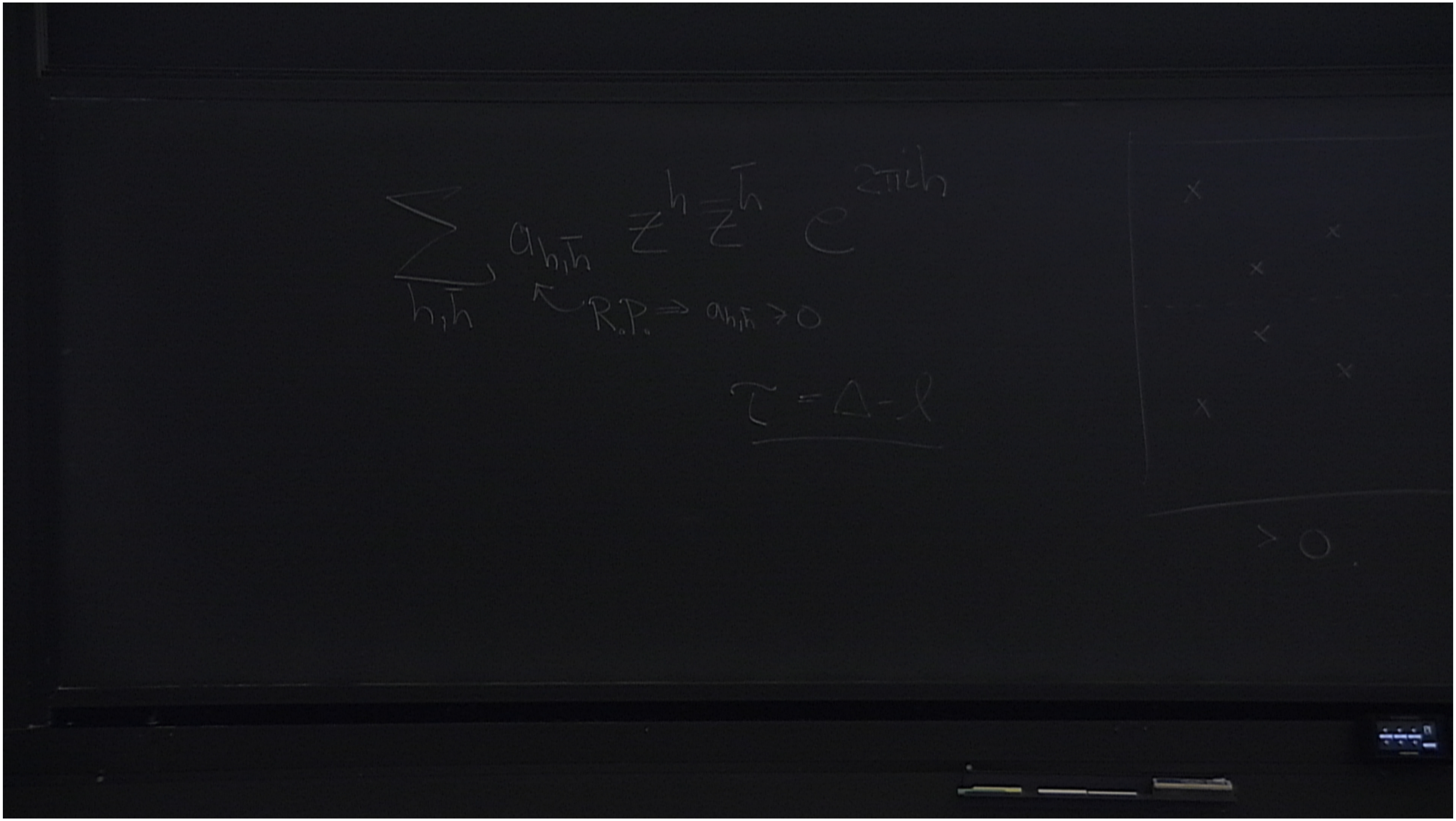
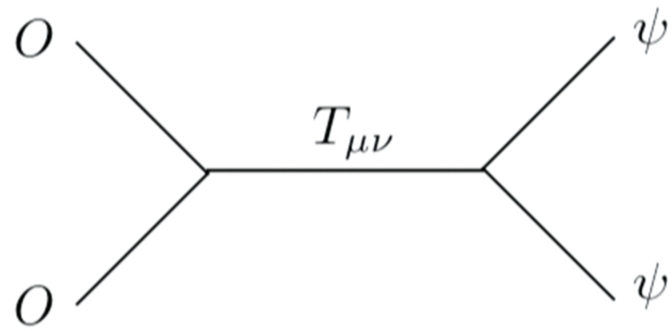

$\sum_{h,\bar{h}} a_{h,\bar{h}} z^{h} \bar{z}^{\bar{h}} e^{2\pi i h}$
R.P. $\Rightarrow a_{h,\bar{h}} \geq 0$
$\tau = \Delta - \ell$
> 0

Application #1: A trivial case

For scalar probes,



the coupling is fixed by conformal Ward identity:

$$\lambda = \frac{\Delta_O \Delta_\psi}{c} > 0$$

The causality constraint gives this obvious inequality.

Application #2: Maldacena-Zhiboedov Theorem

Similar argument ==>

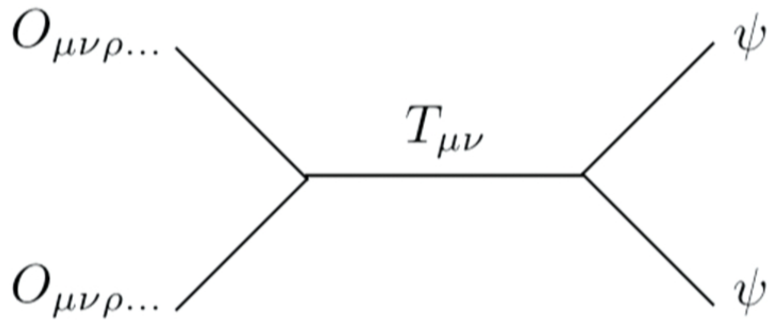
Conserved currents with spin > 2 *always* violate analyticity in the “purple region”.

Therefore if a CFT has any higher spin currents, then it has an infinite number of them.

Application #3: Probes with spin

[TH, Jain, Kundu]

For probes with spin, stress-tensor exchange is nontrivial (not fixed by Ward identity):



Same logic gives a new "null energy"-like constraint

$$\langle \varepsilon\cdot O(0)\varepsilon^*\cdot O(y^+,y^-)T_{--}(\infty)\rangle > 0 \qquad \text{for} \quad y^+ \to 0$$

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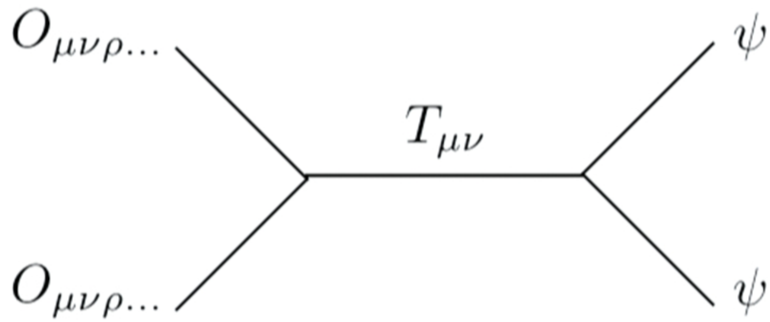

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Application #3: Probes with spin

The full 3-point function <OOT> is fixed by conformal invariance up to some numbers.

So this is a set of inequalities for those numerical couplings.

An interesting example is $\langle T_{\mu\nu} T_{\sigma\rho} T_{\gamma\delta} \rangle$

causality ==> $$\frac{13}{54} < \frac{a}{c} < \frac{31}{18}$$ (Necessary but probably not sufficient!)

cf Hofman-Maldacena energy calorimeter constraints:

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Application #3: Probes with spin

The more recent constraints

$$a = c$$

[Camanho, Edelstein, Maldacena, Zhiboedov]

are much stronger versions of Hofman-Maldacena.

To derive this from CFT is difficult, but plausibly within reach with existing techniques and enough effort:

1. Compute anomalous dimensions by lightcone bootstrap
2. Apply causality constraints

Application #4: A surprise(?) for scalars

caveat: work in progress, with potential pitfalls!

Application #5: Holographic Dual of $(\partial\phi)^2 + \lambda(\partial\phi)^4$

Consider a scalar theory in AdS with this contact interaction.

(Gravity is decoupled.)

If $\lambda < 0$, the dual CFT violates the sum rule.

Therefore we reproduce the Adams *et al.* result directly from the CFT bootstrap:

$$\lambda > 0$$

Summary:

In a causal/unitary QFT, position-space correlation functions have analogues of

- "optical theorem" positivity conditions
- "dispersion relations" relating UV <—> IR

This imposes constraints on the IR couplings of both conformal and non-conformal QFTs.

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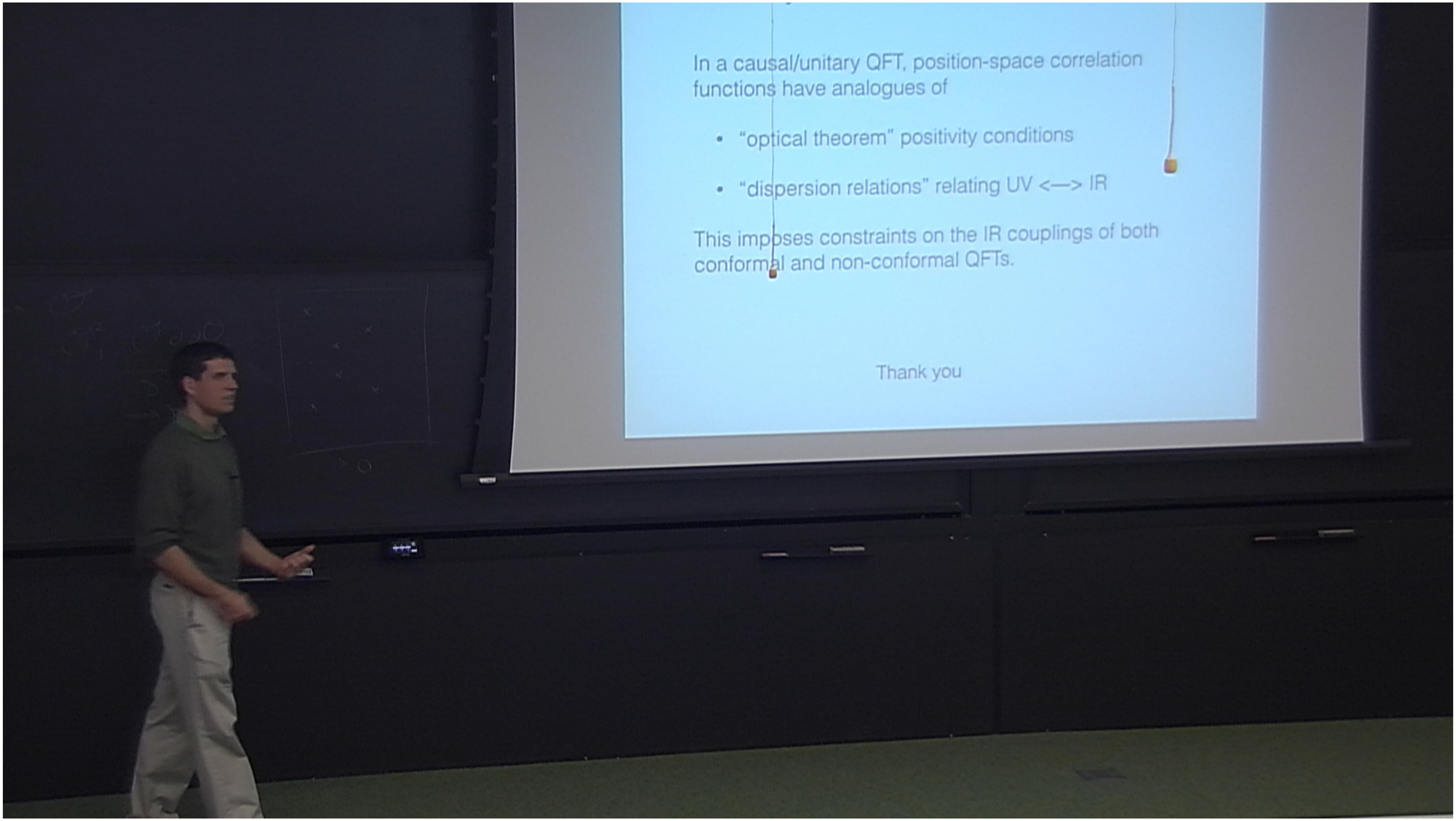
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