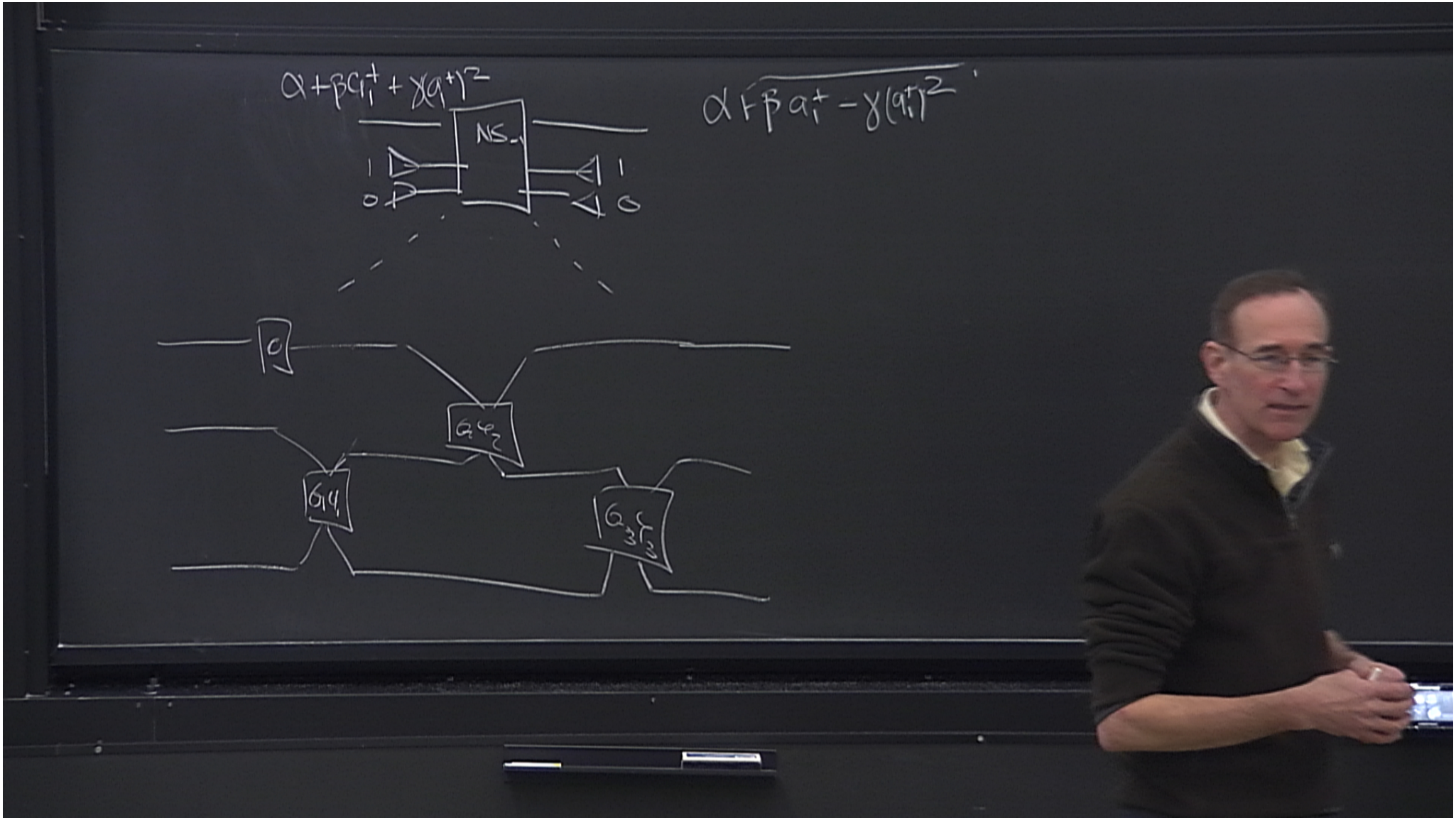
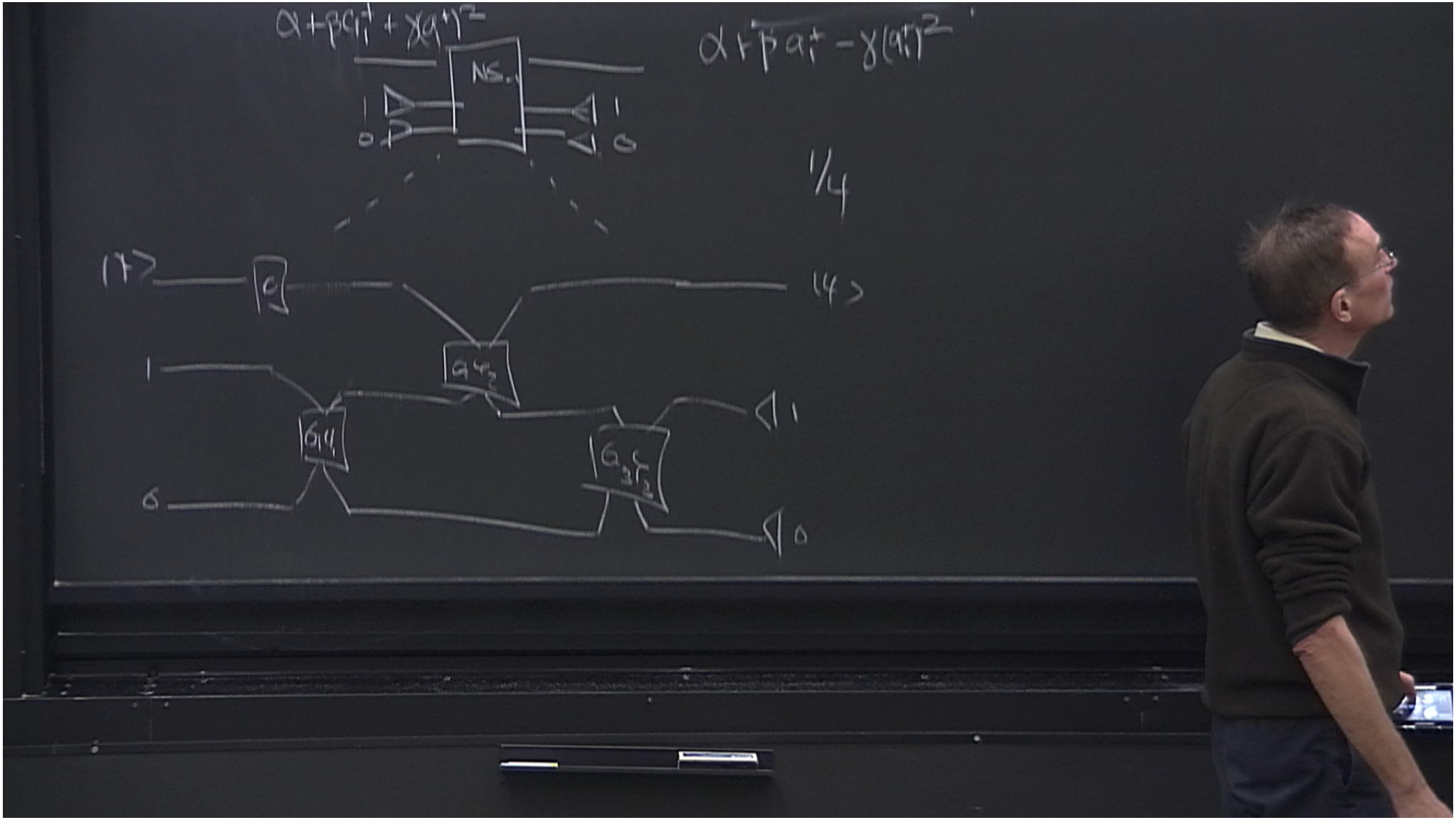
Title: PSI 2015/2016 Quantum Information - Lecture 5

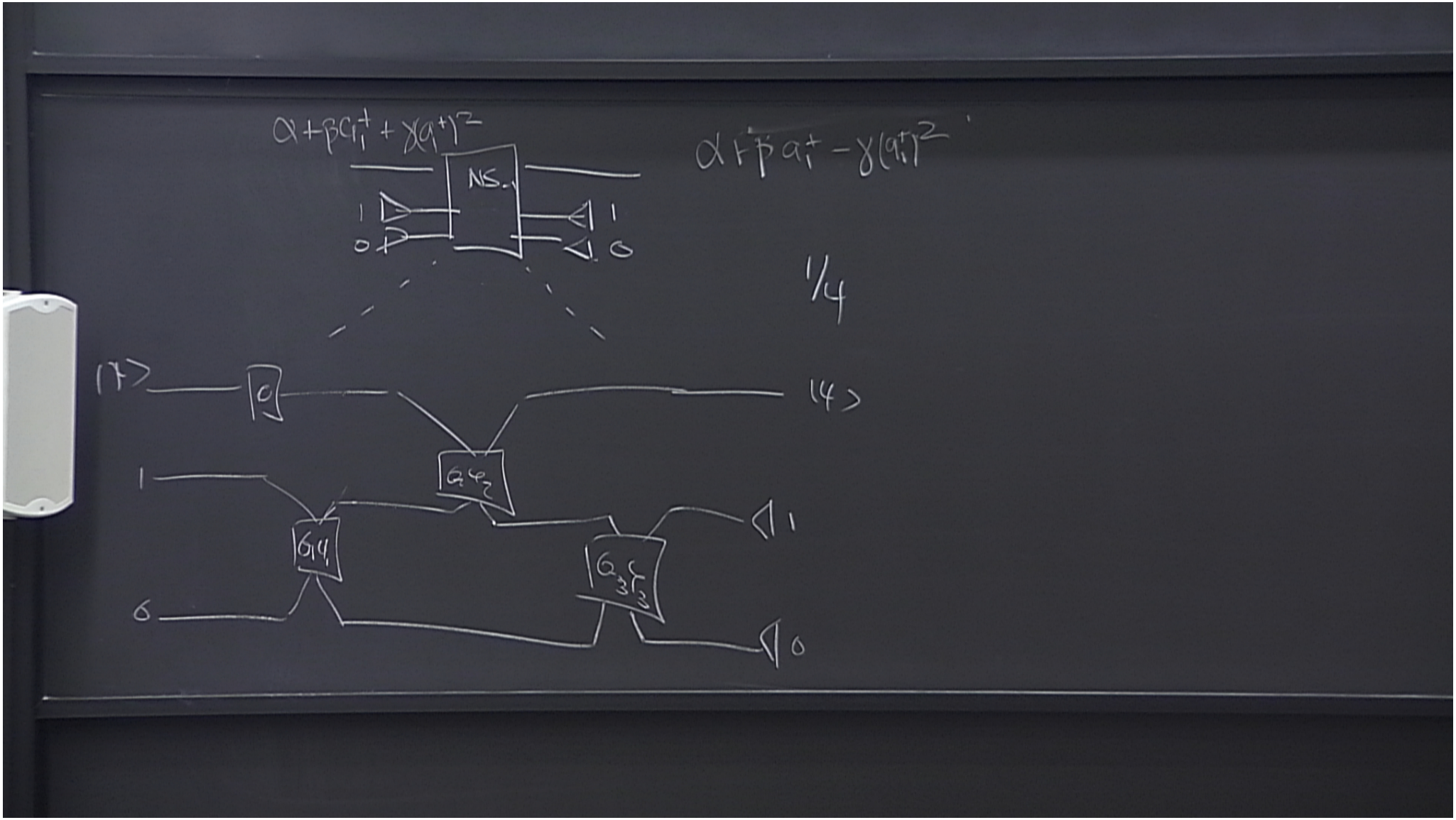
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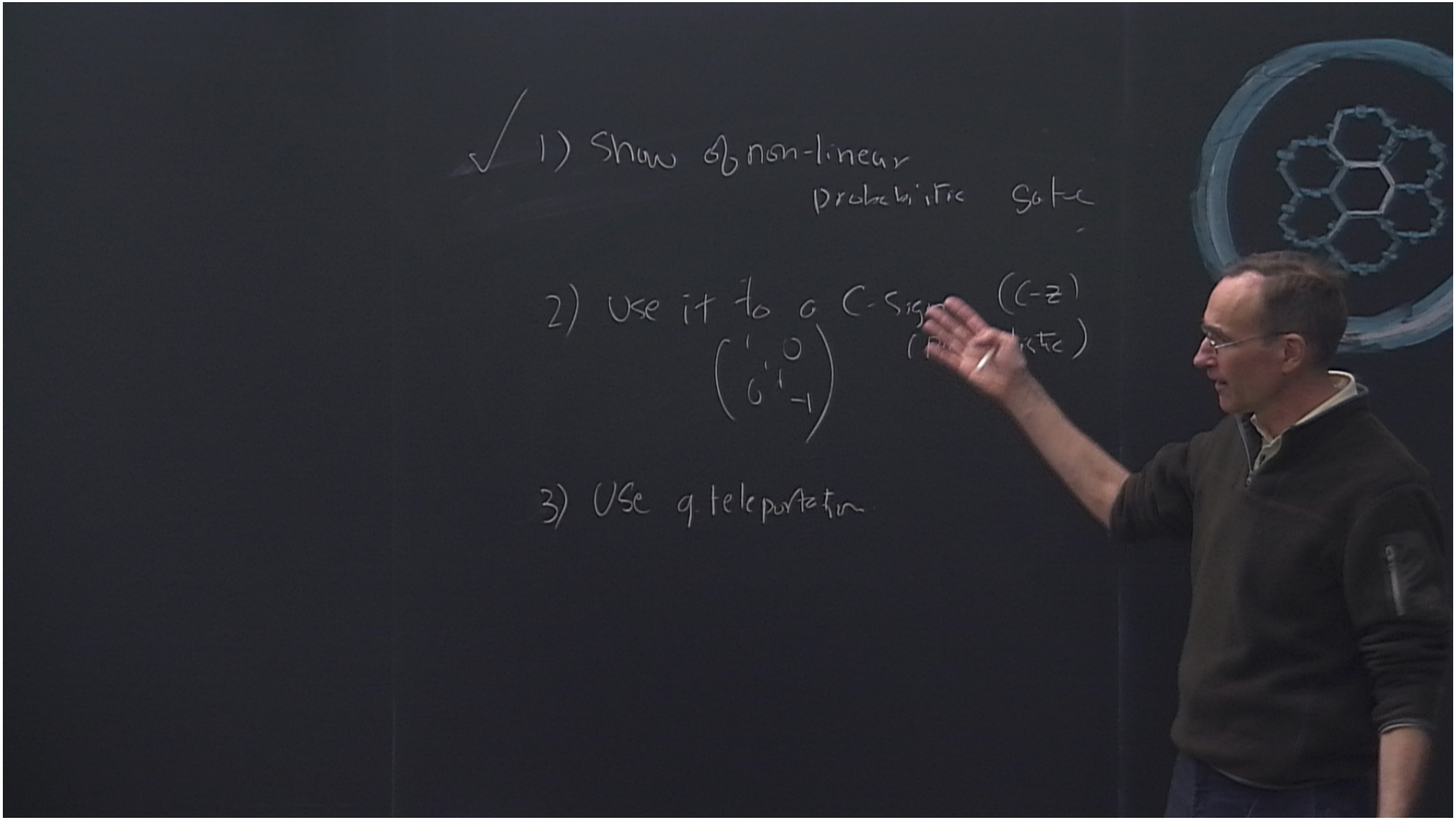
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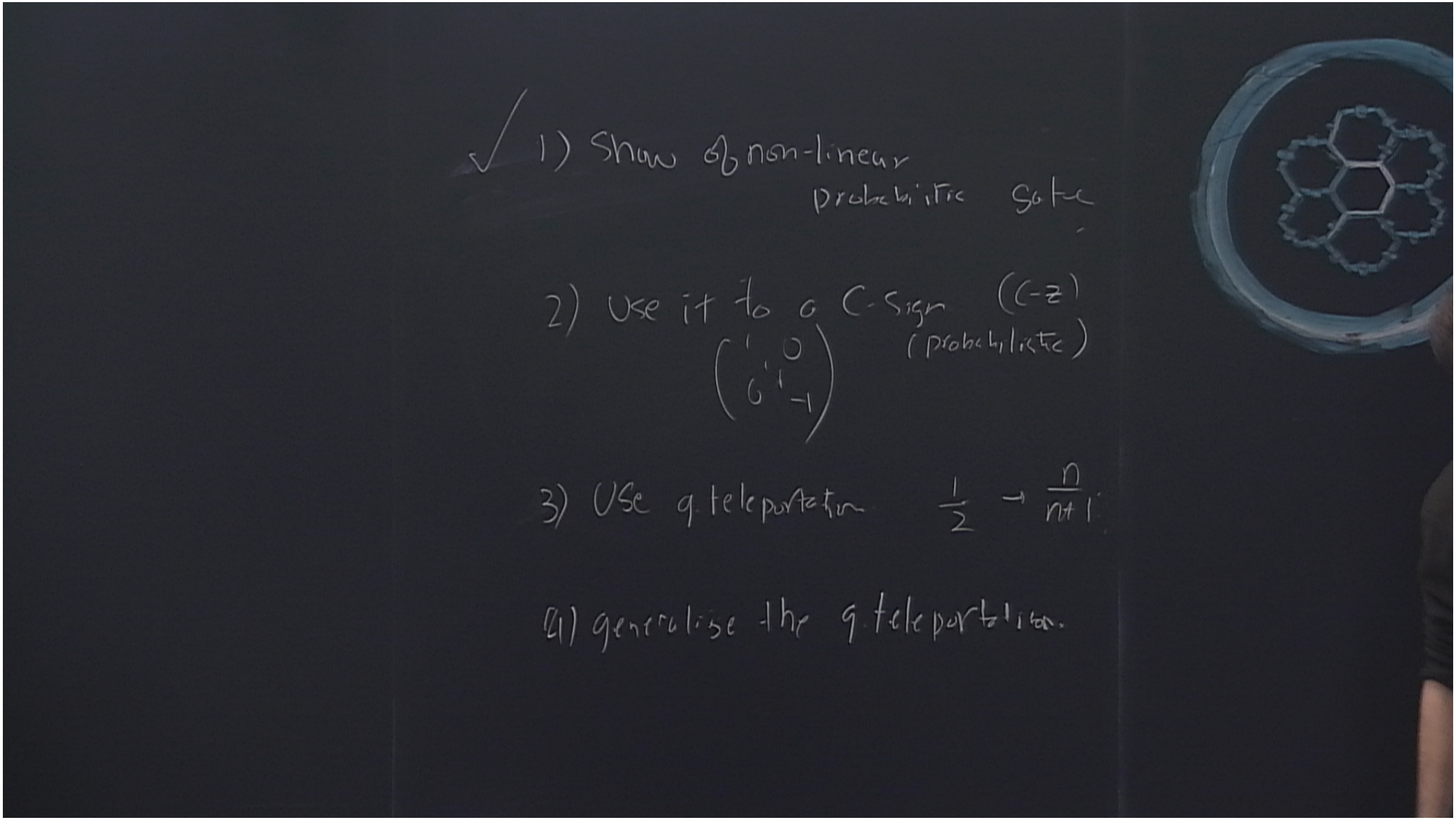
Abstract:

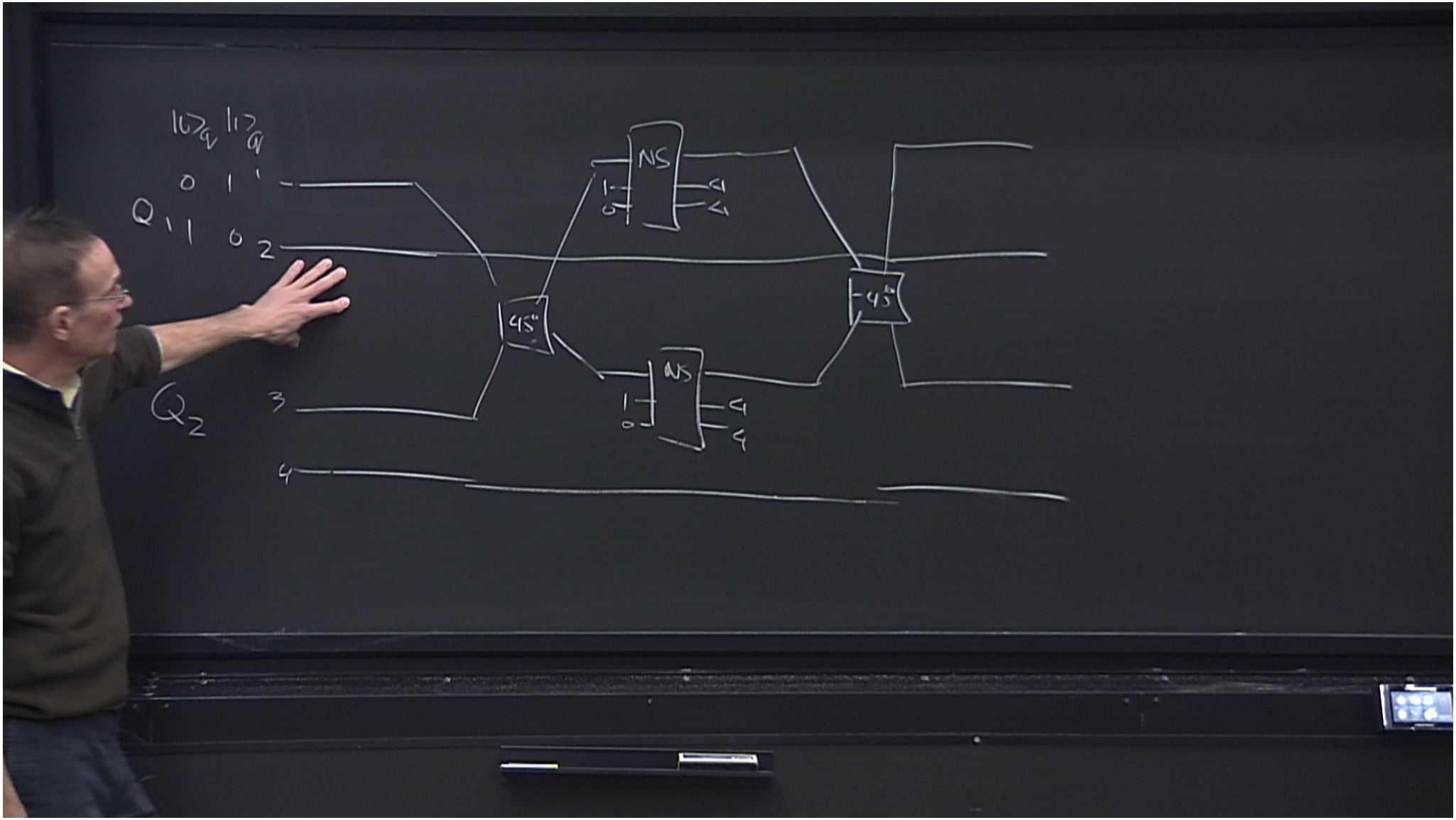

$\alpha+\beta a_i^\dagger+\gamma(a_i^\dagger)^2$
NS
$\alpha+\beta a_i^\dagger-\gamma(a_i^\dagger)^2$
G
G_4
G_2
G_3

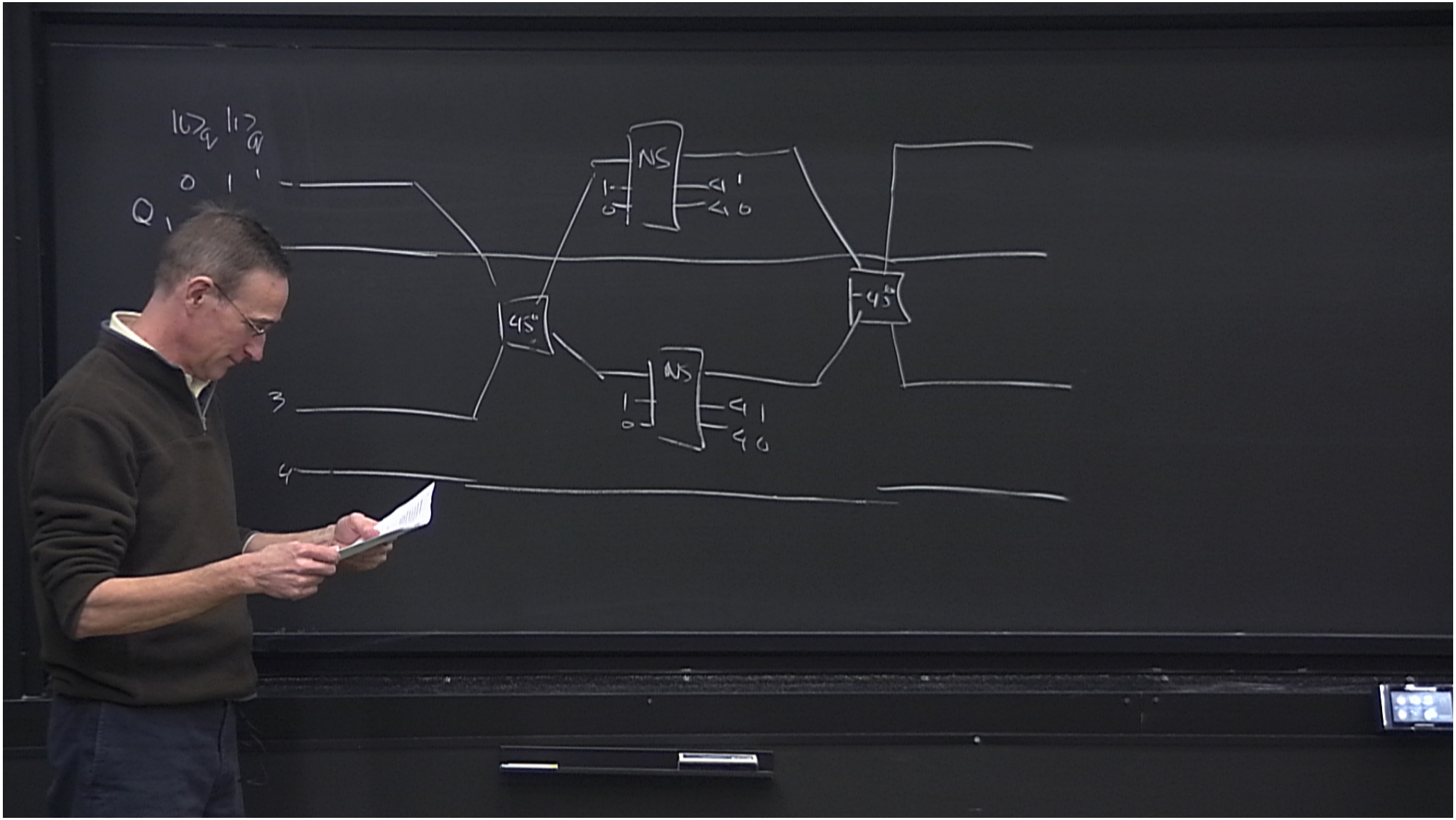

NS.
1/4
|1>
|4>
|1
|0

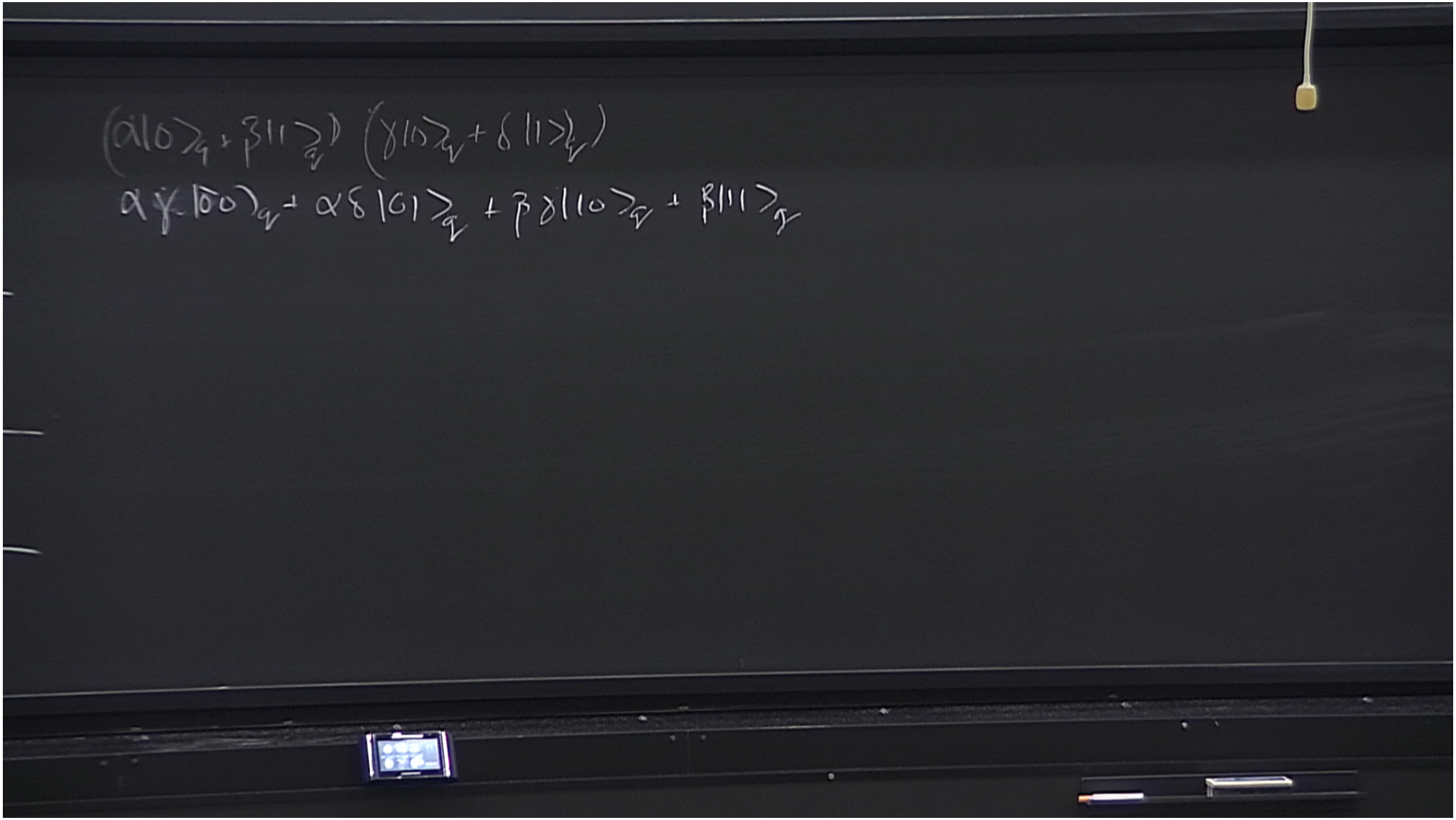

$\alpha + \beta a_1^\dagger + \gamma (a_1^\dagger)^2$
NS.
1
0
1
0
$\alpha + \beta a_1^\dagger - \gamma (a_1^\dagger)^2$
1/4
|1>
0
|4>
1
G_{42}
G_{14}
G_{33}
1
0

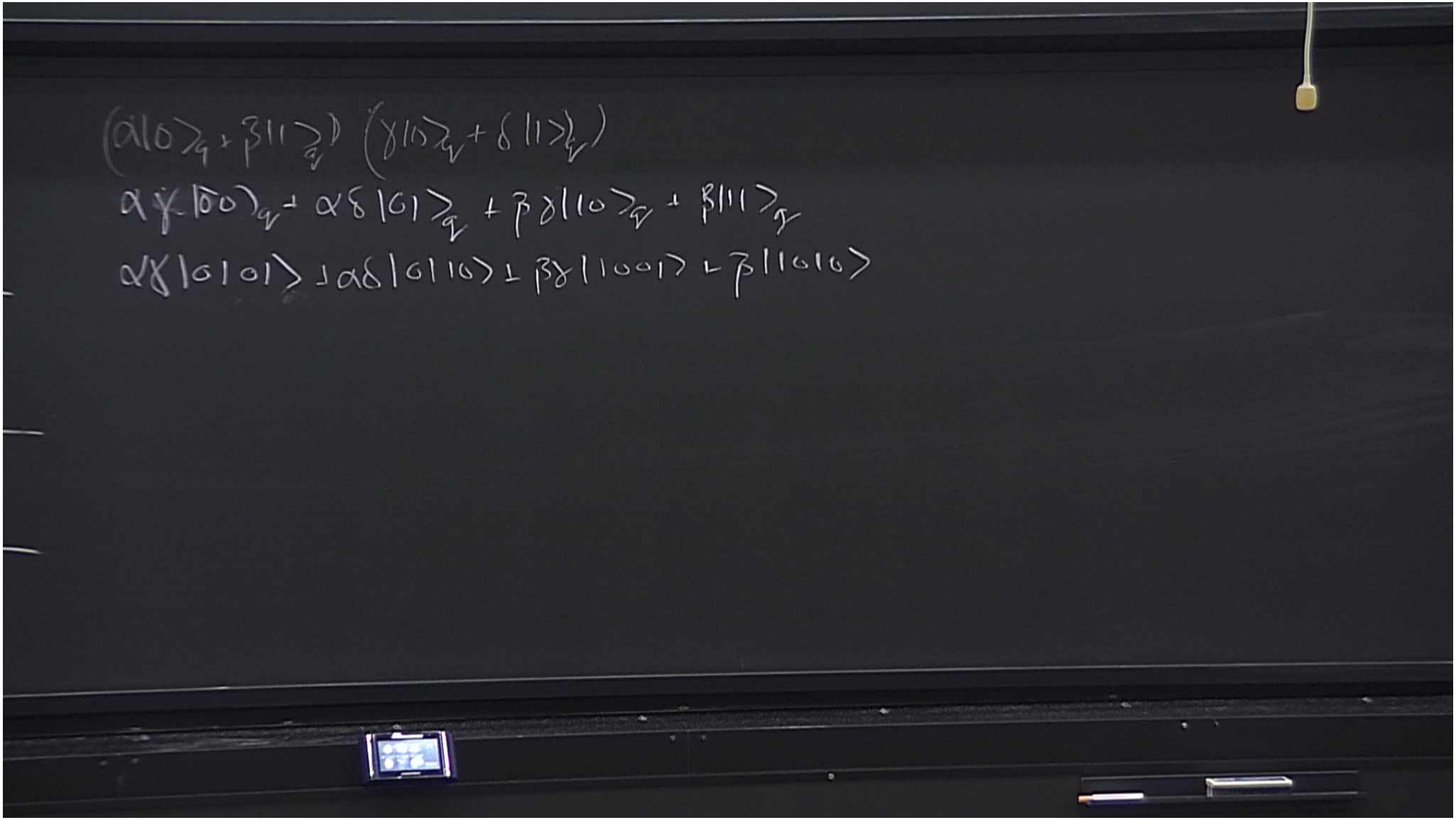

✓ 1) Show of non-linear
probabilistic Gate
2) Use it to a C-Sign ((-z)
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
3) Use q. teleportation


✓ 1) Show of non-linear
probabilistic Gate
2) Use it to a C-Sign (C-Z)
(probabilistic)
3) Use q. teleportation $\frac{1}{2} \rightarrow \frac{n}{n+1}$
4) generalise the q. teleportation.


NS
45°
NS
45°
Q2
3
4


NS
45°
NS
45°
3
4


$(\alpha|0\rangle_q + \beta|1\rangle_q)(\gamma|0\rangle_r + \delta|1\rangle_r)$
$\alpha\gamma|00\rangle_{qr} + \alpha\delta|01\rangle_{qr} + \beta\gamma|10\rangle_{qr} + \beta\delta|11\rangle_{qr}$

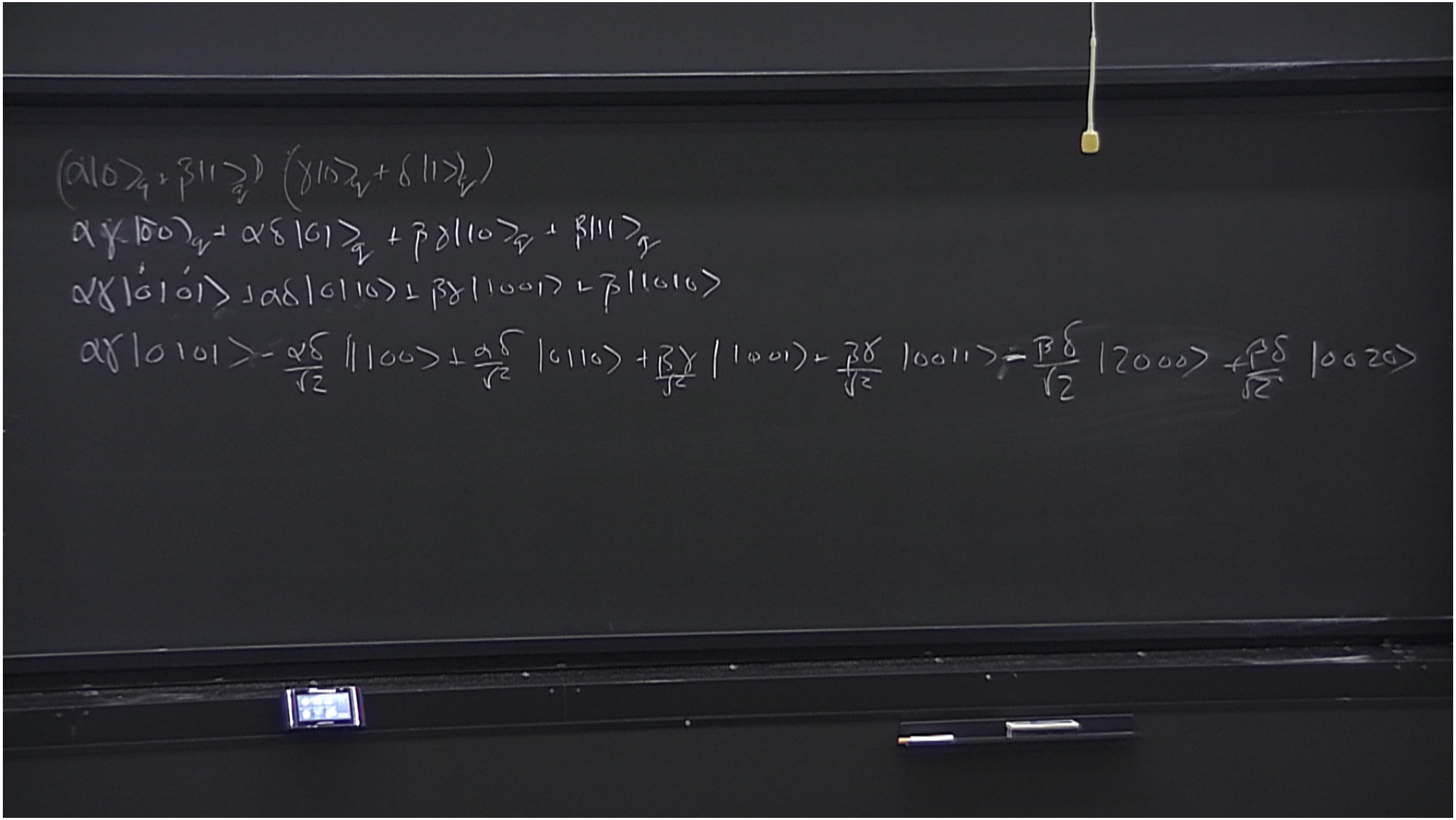

$(\alpha|0\rangle_q + \beta|1\rangle_q)(\gamma|0\rangle_q + \delta|1\rangle_q)$
$\alpha\gamma|00\rangle_q + \alpha\delta|01\rangle_q + \beta\gamma|10\rangle_q + \beta\delta|11\rangle_q$
$\alpha\gamma|0101\rangle + \alpha\delta|0110\rangle + \beta\gamma|1001\rangle + \beta\delta|1010\rangle$

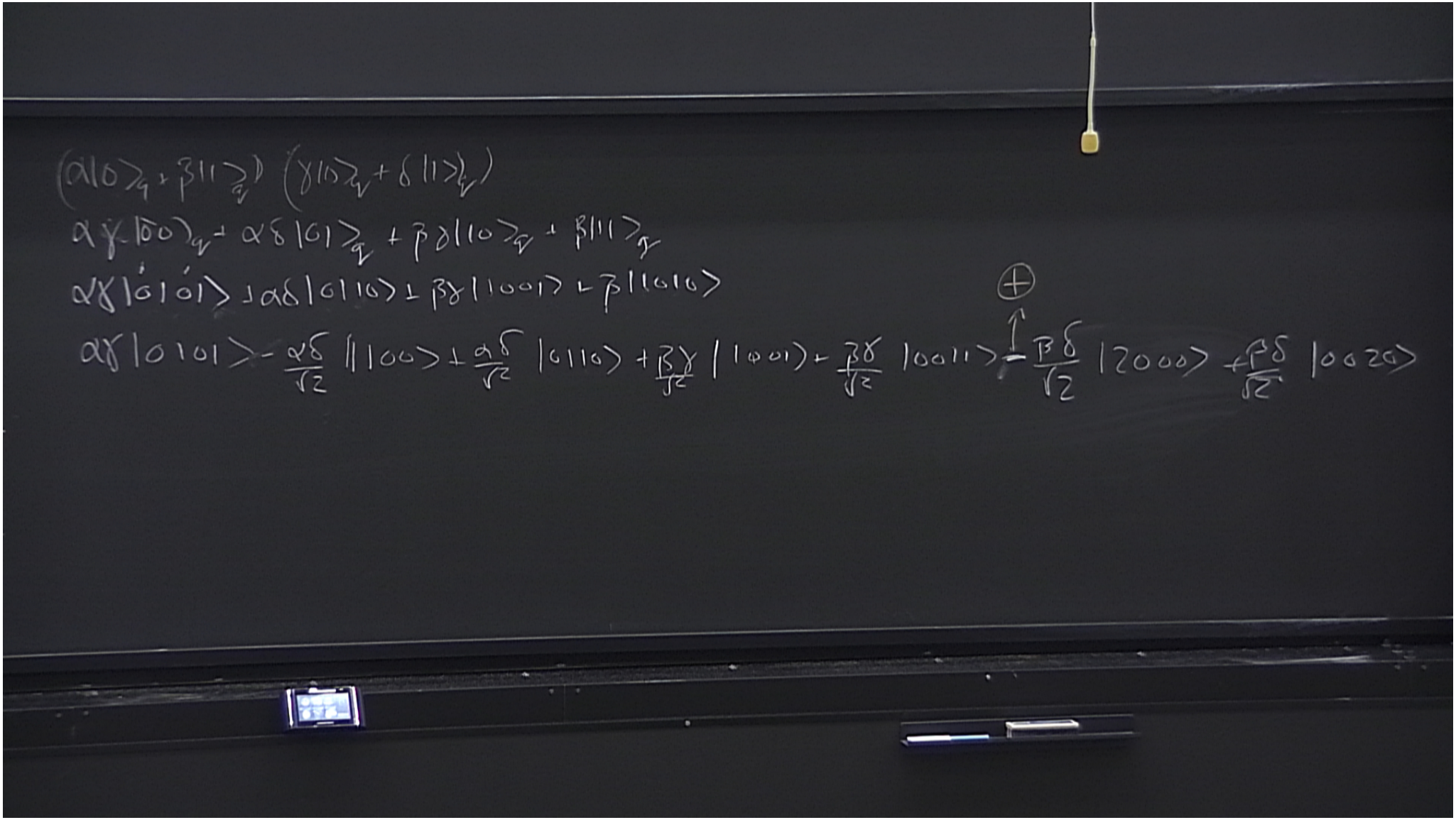
$$(\alpha|0\rangle_q + \beta|1\rangle_q)(\gamma|0\rangle_q + \delta|1\rangle_q)$$

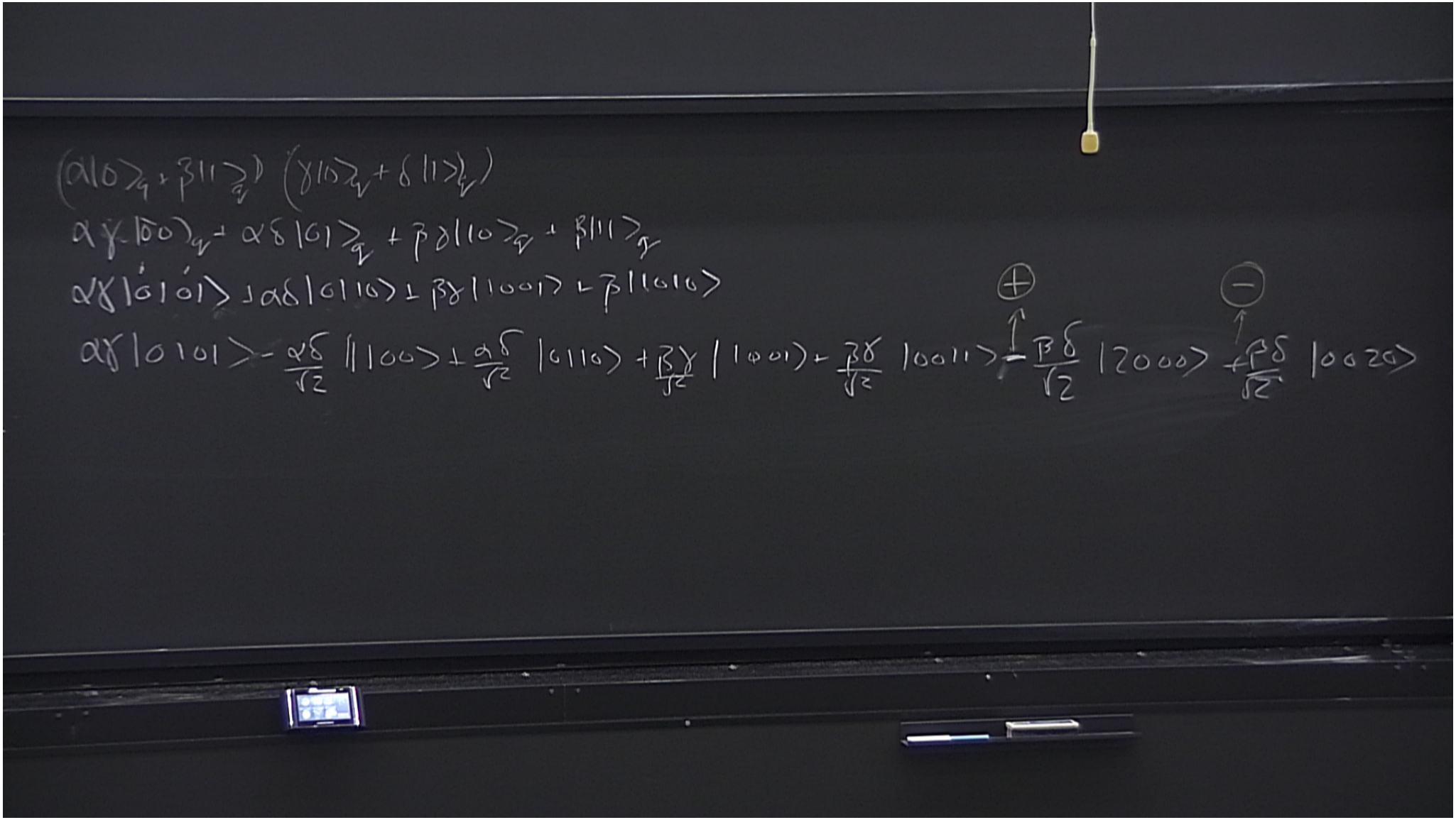
$$\alpha\gamma|00\rangle_q + \alpha\delta|01\rangle_q + \beta\gamma|10\rangle_q + \beta\delta|11\rangle_q$$

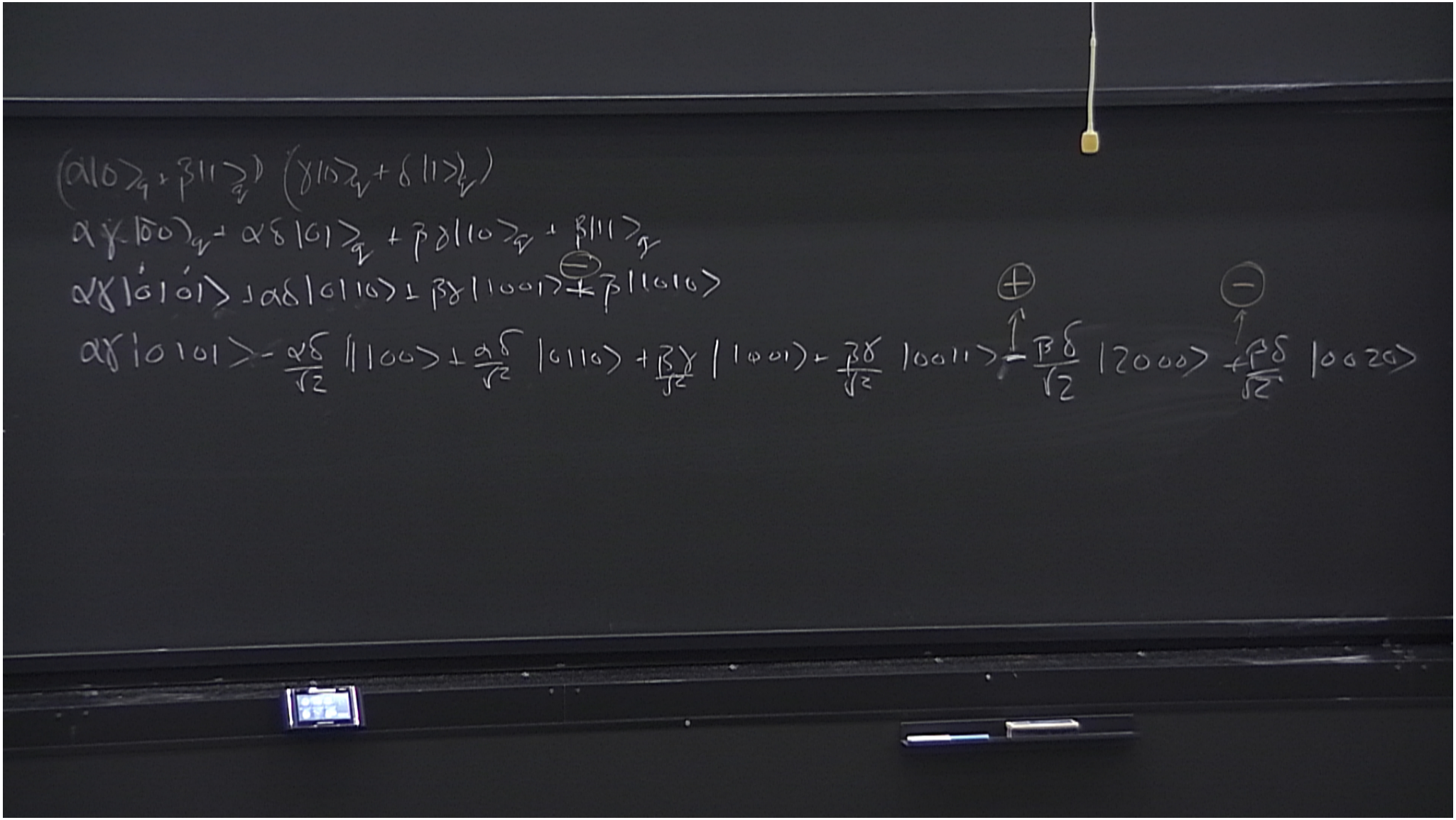
$$\alpha\gamma|0101\rangle + \alpha\delta|0110\rangle + \beta\gamma|1001\rangle + \beta\delta|1010\rangle$$

$$\alpha\gamma|0101\rangle - \frac{\alpha\delta}{\sqrt{2}}|1100\rangle + \frac{\alpha\delta}{\sqrt{2}}|0110\rangle + \frac{\beta\gamma}{\sqrt{2}}|1001\rangle + \frac{\beta\delta}{\sqrt{2}}|0011\rangle$$




$(\alpha|0\rangle_A + \beta|1\rangle_A)(\gamma|0\rangle_B + \delta|1\rangle_B)$
$\alpha\gamma|00\rangle_{AB} + \alpha\delta|01\rangle_{AB} + \beta\gamma|10\rangle_{AB} + \beta\delta|11\rangle_{AB}$
$\alpha\gamma|0101\rangle + \alpha\delta|0110\rangle + \beta\gamma|1001\rangle + \beta\delta|1010\rangle$
$\alpha\gamma|0101\rangle - \frac{\alpha\delta}{\sqrt{2}}|1100\rangle + \frac{\alpha\delta}{\sqrt{2}}|0110\rangle + \frac{\beta\gamma}{\sqrt{2}}|1001\rangle - \frac{\beta\gamma}{\sqrt{2}}|0011\rangle - \frac{\beta\delta}{\sqrt{2}}|2000\rangle + \frac{\beta\delta}{\sqrt{2}}|0020\rangle$
$\oplus$


$(\alpha|0\rangle_A + \beta|1\rangle_A)\otimes(\gamma|0\rangle_B + \delta|1\rangle_B)$
$\alpha\gamma|00\rangle_{AB} + \alpha\delta|01\rangle_{AB} + \beta\gamma|10\rangle_{AB} + \beta\delta|11\rangle_{AB}$
$\alpha\gamma|0\,0\,0\,1\rangle + \alpha\delta|0\,1\,0\,1\rangle + \beta\gamma|1\,0\,0\,1\rangle + \beta\delta|1\,0\,1\,0\rangle$
$\alpha\gamma|0101\rangle - \frac{\alpha\delta}{\sqrt{2}}|1100\rangle + \frac{\alpha\delta}{\sqrt{2}}|0110\rangle + \frac{\beta\gamma}{\sqrt{2}}|1001\rangle - \frac{\beta\gamma}{\sqrt{2}}|0011\rangle - \frac{\beta\delta}{\sqrt{2}}|2000\rangle + \frac{\beta\delta}{\sqrt{2}}|0020\rangle$
$\oplus$
$\ominus$

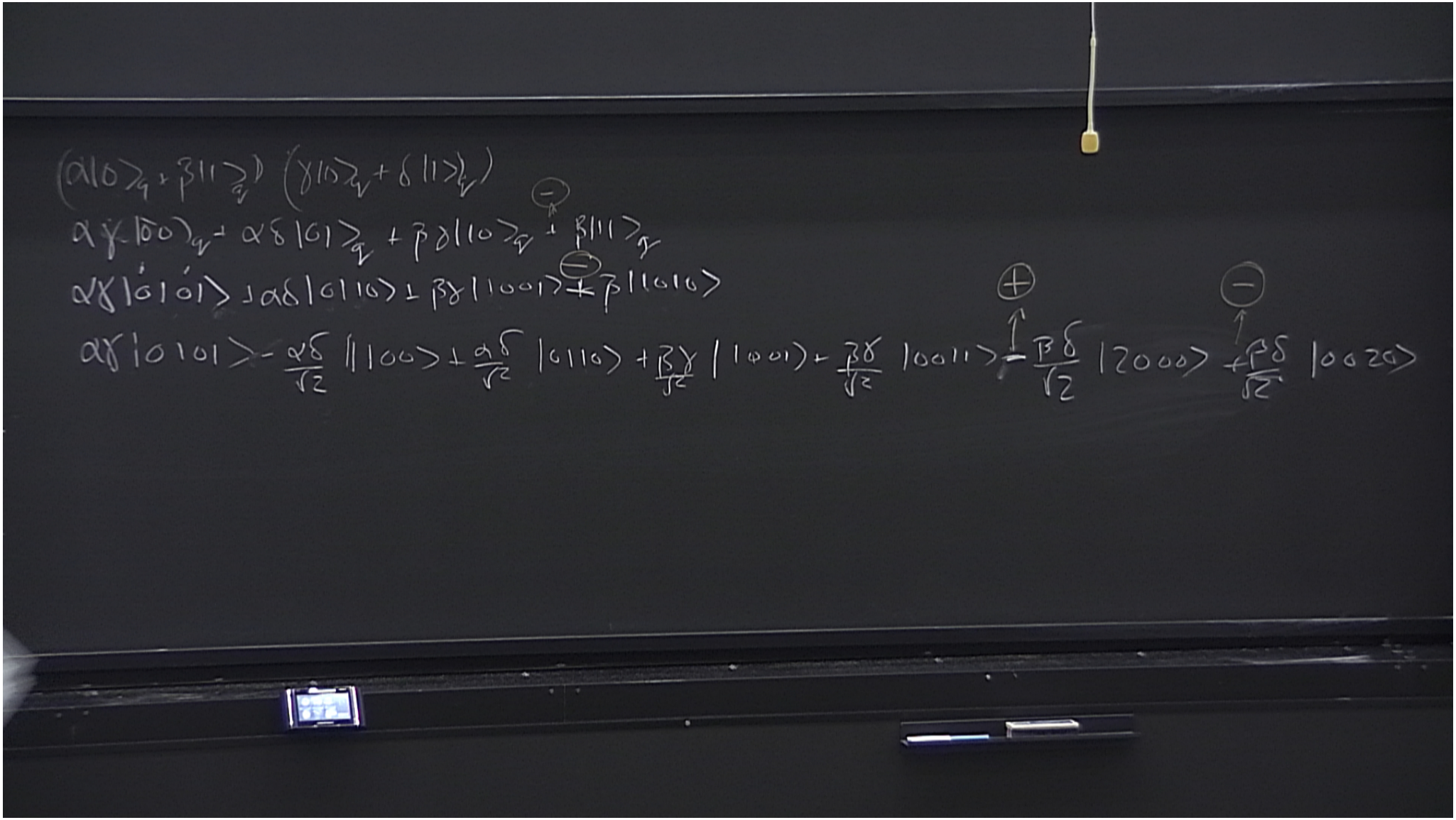


$(\alpha|0\rangle_A + \beta|1\rangle_A)(\gamma|0\rangle_B + \delta|1\rangle_B)$

$\alpha\gamma|00\rangle_{AB} + \alpha\delta|01\rangle_{AB} + \beta\gamma|10\rangle_{AB} + \beta|11\rangle_{AB}$

$\alpha\gamma|0101\rangle + \alpha\delta|0110\rangle \pm \beta\gamma|1001\rangle \pm \beta|1010\rangle$

$\alpha\gamma|0101\rangle - \frac{\alpha\delta}{\sqrt{2}}|1100\rangle + \frac{\alpha\delta}{\sqrt{2}}|0110\rangle + \frac{\beta\gamma}{\sqrt{2}}|1001\rangle - \frac{\beta\gamma}{\sqrt{2}}|0011\rangle - \frac{\beta\delta}{\sqrt{2}}|2000\rangle + \frac{\beta\delta}{\sqrt{2}}|0020\rangle$


$(\alpha|0\rangle_A + \beta|1\rangle_A)(\gamma|0\rangle_B + \delta|1\rangle_B)$
$\alpha\gamma|00\rangle_{AB} + \alpha\delta|01\rangle_{AB} + \beta\gamma|10\rangle_{AB} + \beta\delta|11\rangle_{AB}$
$\alpha\gamma|0101\rangle + \alpha\delta|0110\rangle + \beta\gamma|1001\rangle + \beta\delta|1010\rangle$
$\alpha\gamma|0101\rangle - \frac{\alpha\delta}{\sqrt{2}}|0110\rangle + \frac{\alpha\delta}{\sqrt{2}}|0110\rangle + \frac{\beta\gamma}{\sqrt{2}}|1001\rangle - \frac{\beta\gamma}{\sqrt{2}}|1001\rangle - \frac{\beta\delta}{\sqrt{2}}|2000\rangle + \frac{\beta\delta}{\sqrt{2}}|0020\rangle$

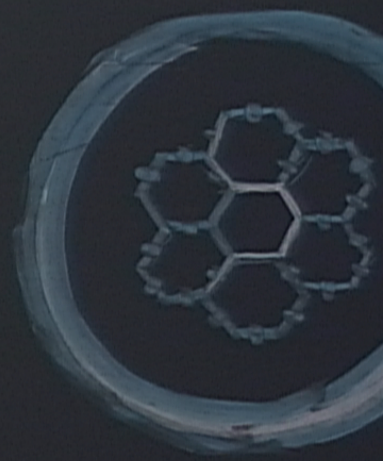
✓ 1) Show of non-linear probabilistic gate

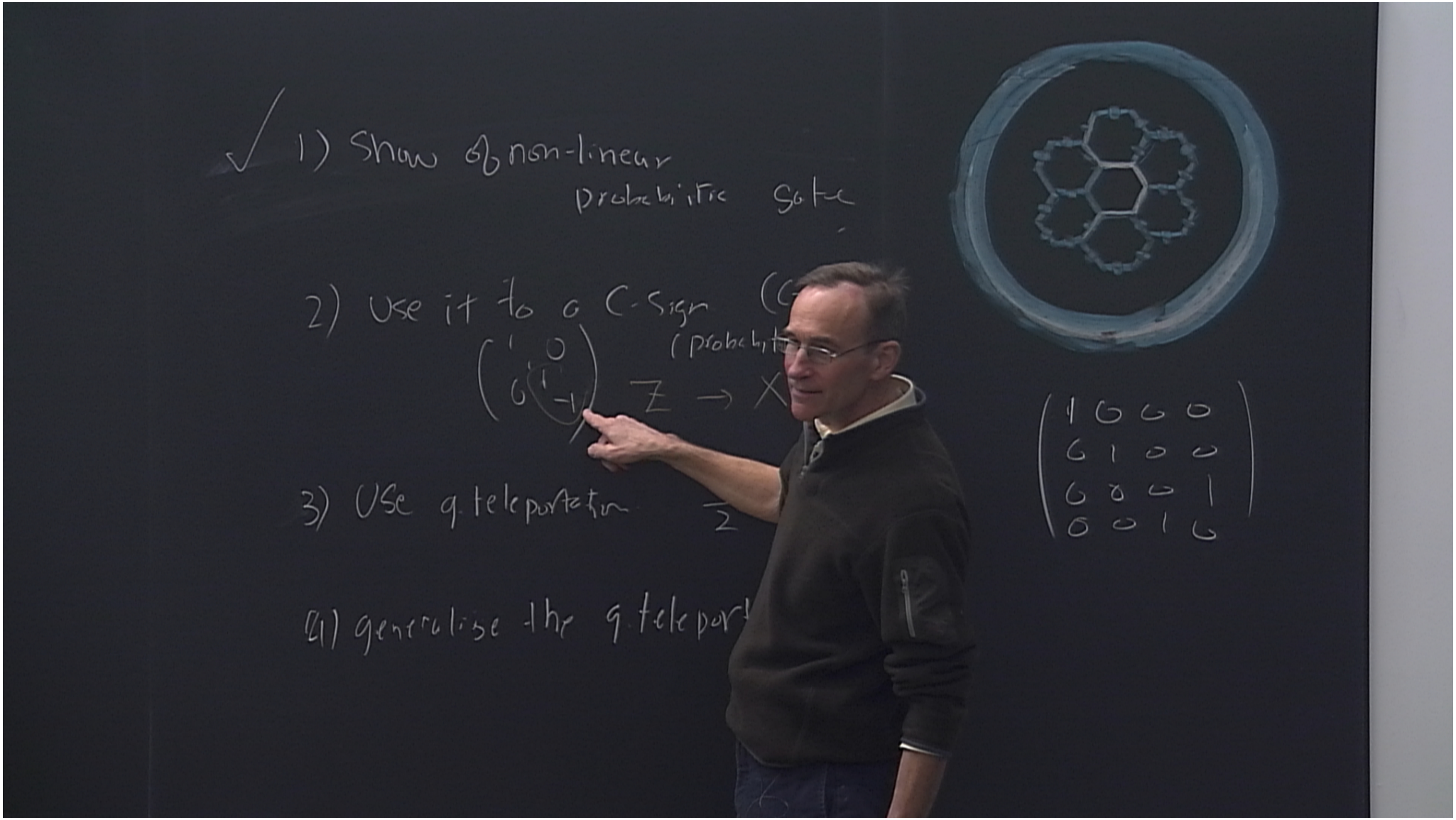
2) Use it to a C-Sign (C_Z) (probabilistic)

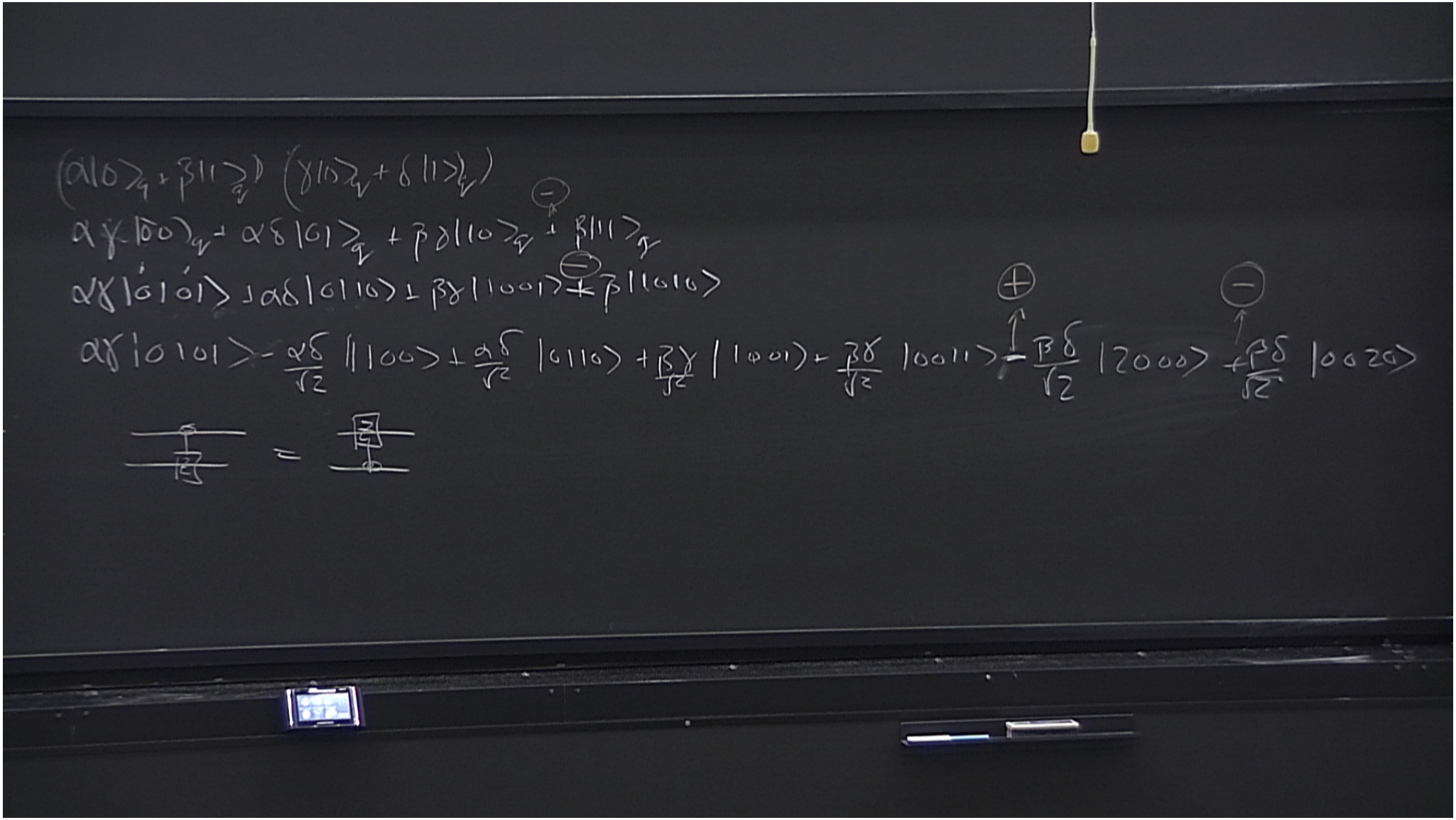
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $Z \to X$

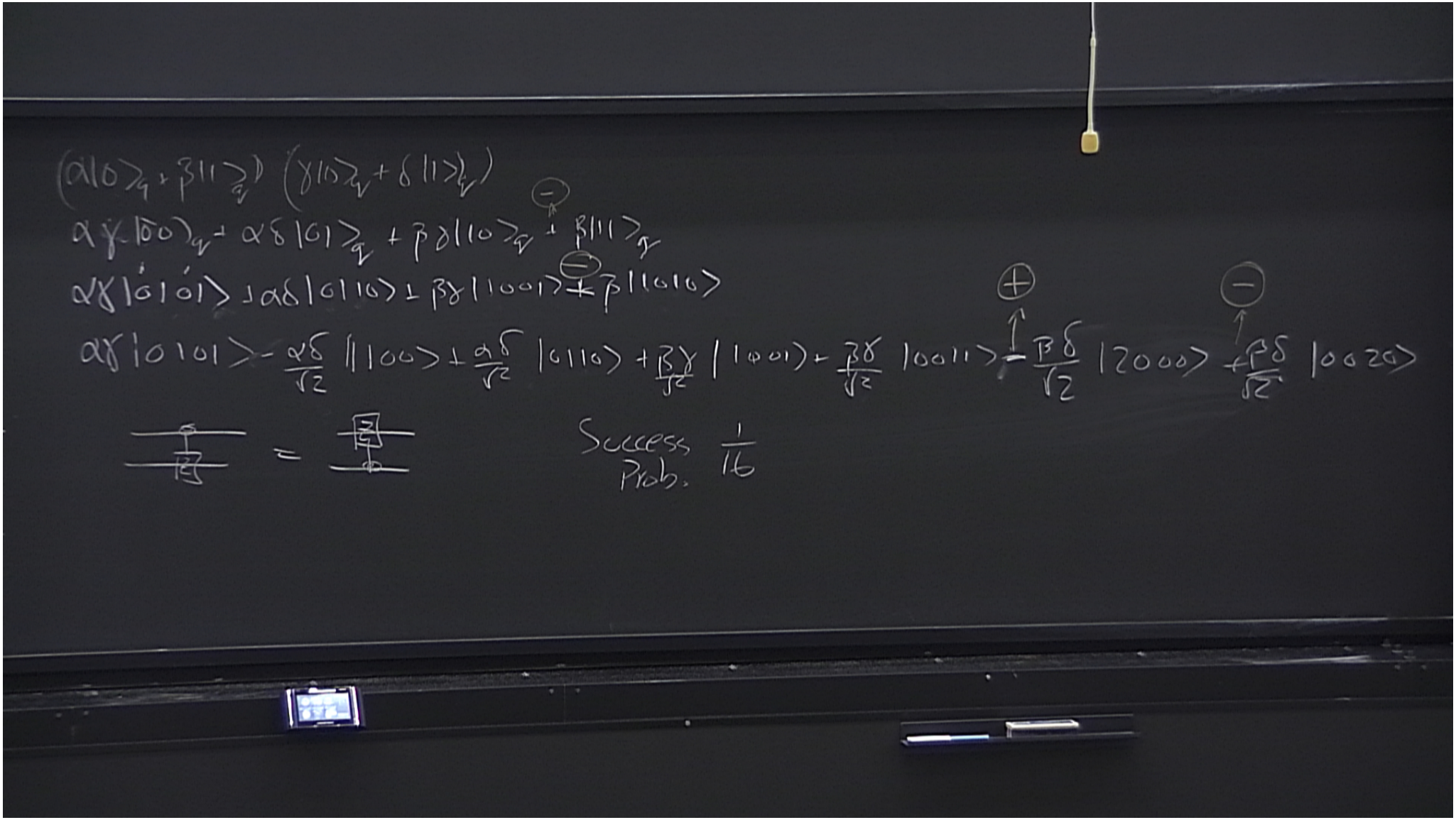
3) Use q. teleportation $\frac{1}{2} \to \frac{n}{n+1}$

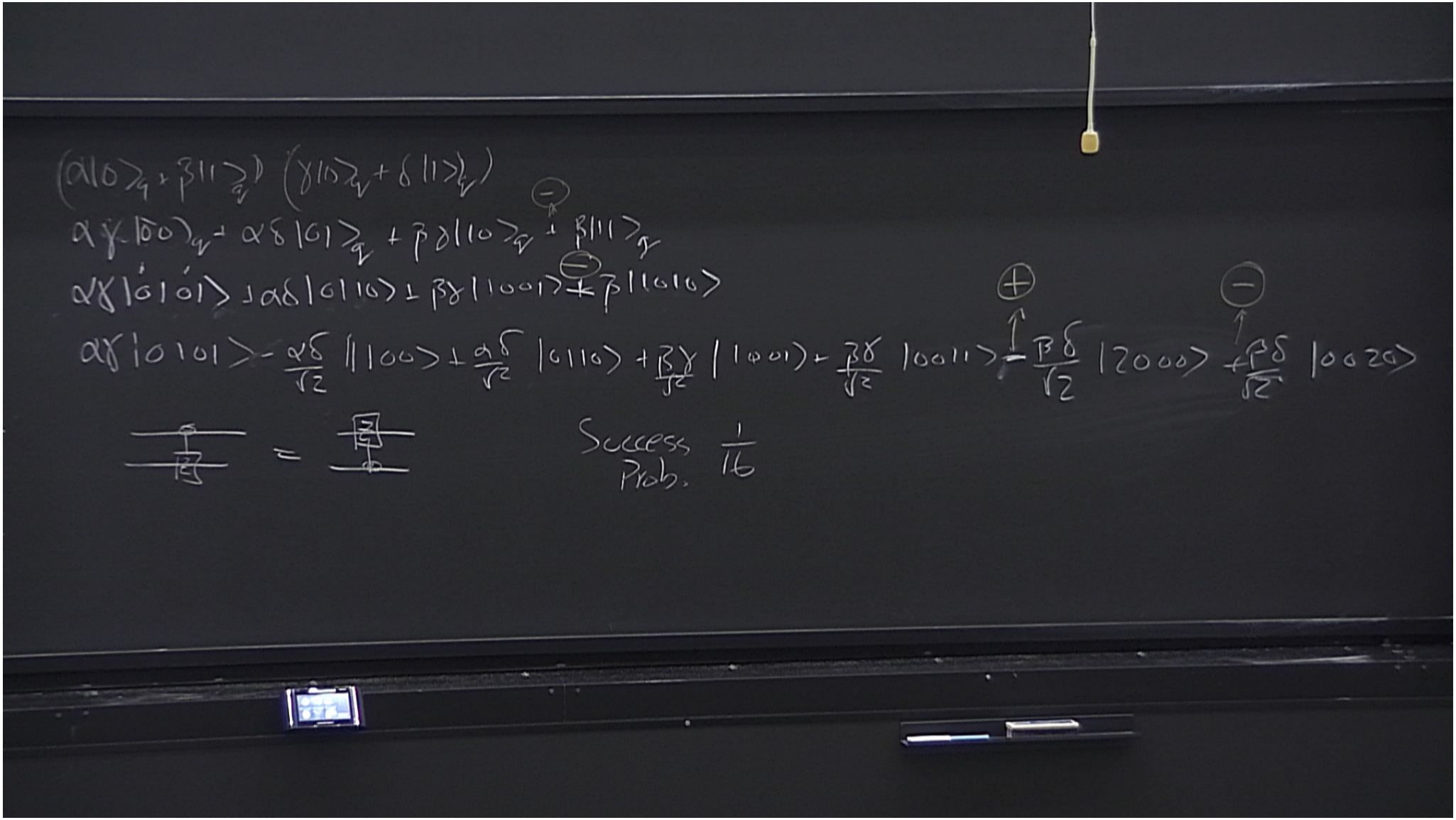
4) generalise the q. teleportation.

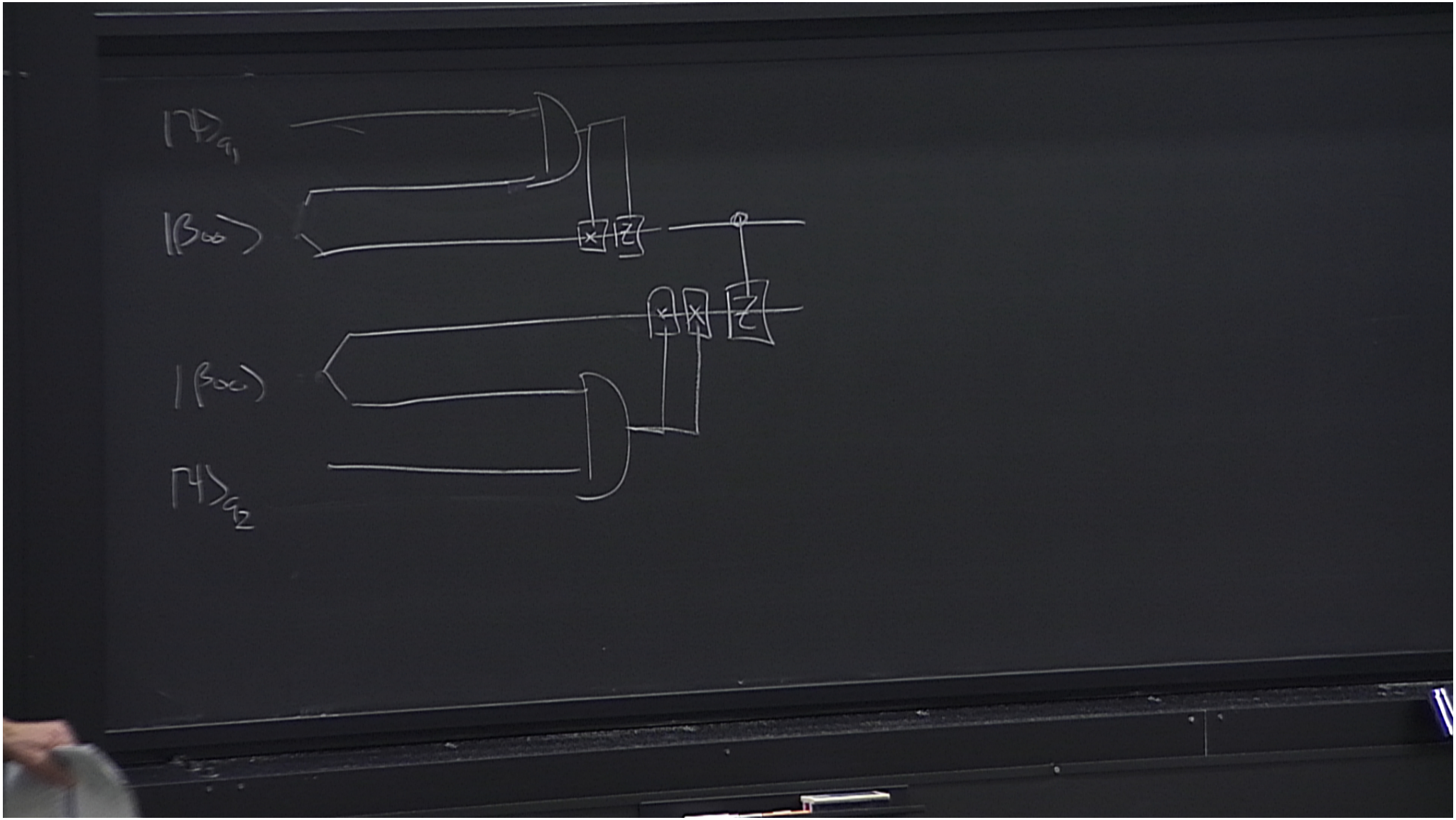


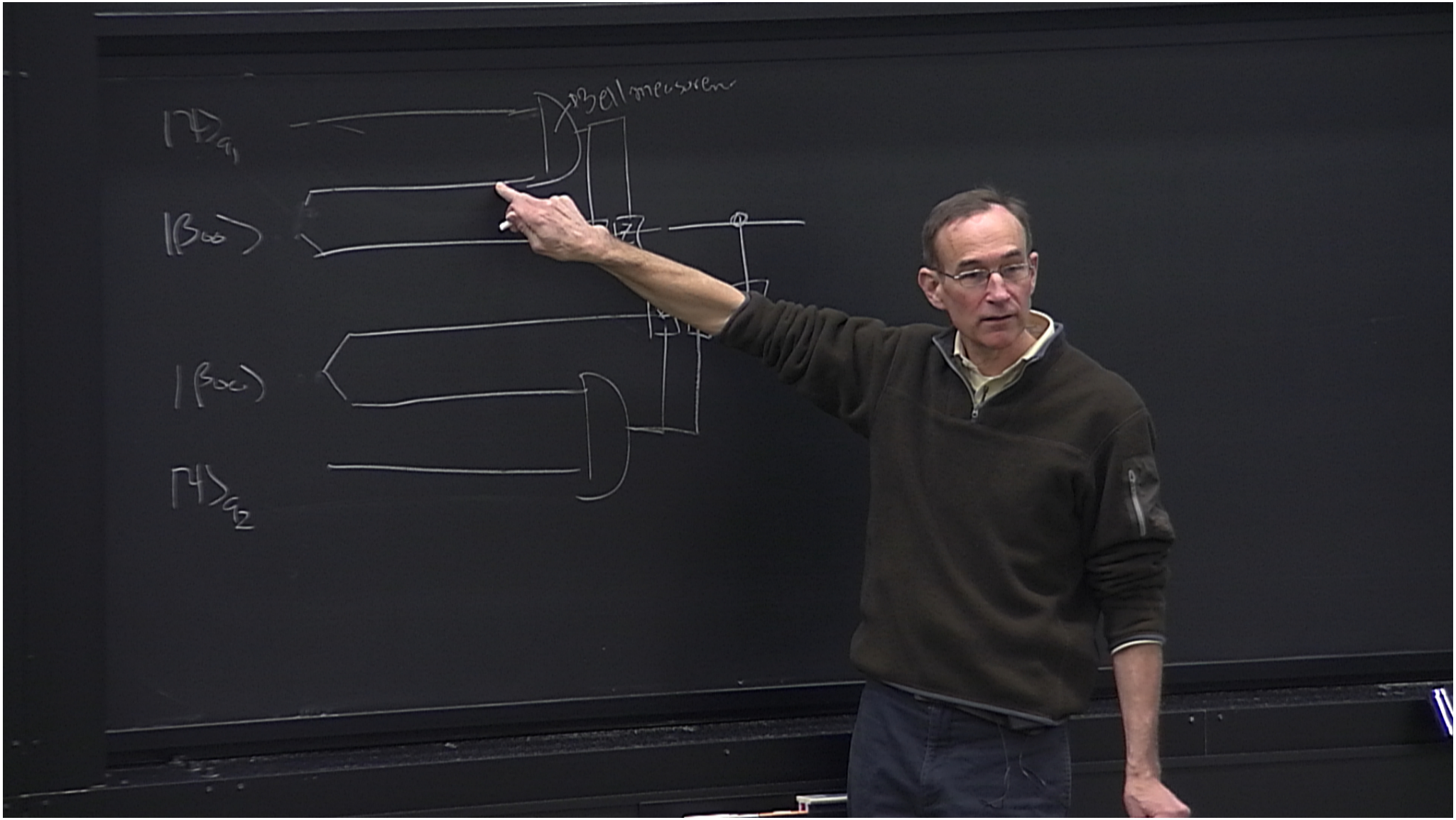

1) Show of non-linear
probabilistic Gate
2) Use it to a C-Sign
3) Use q. teleportation
4) generalise the q. teleport

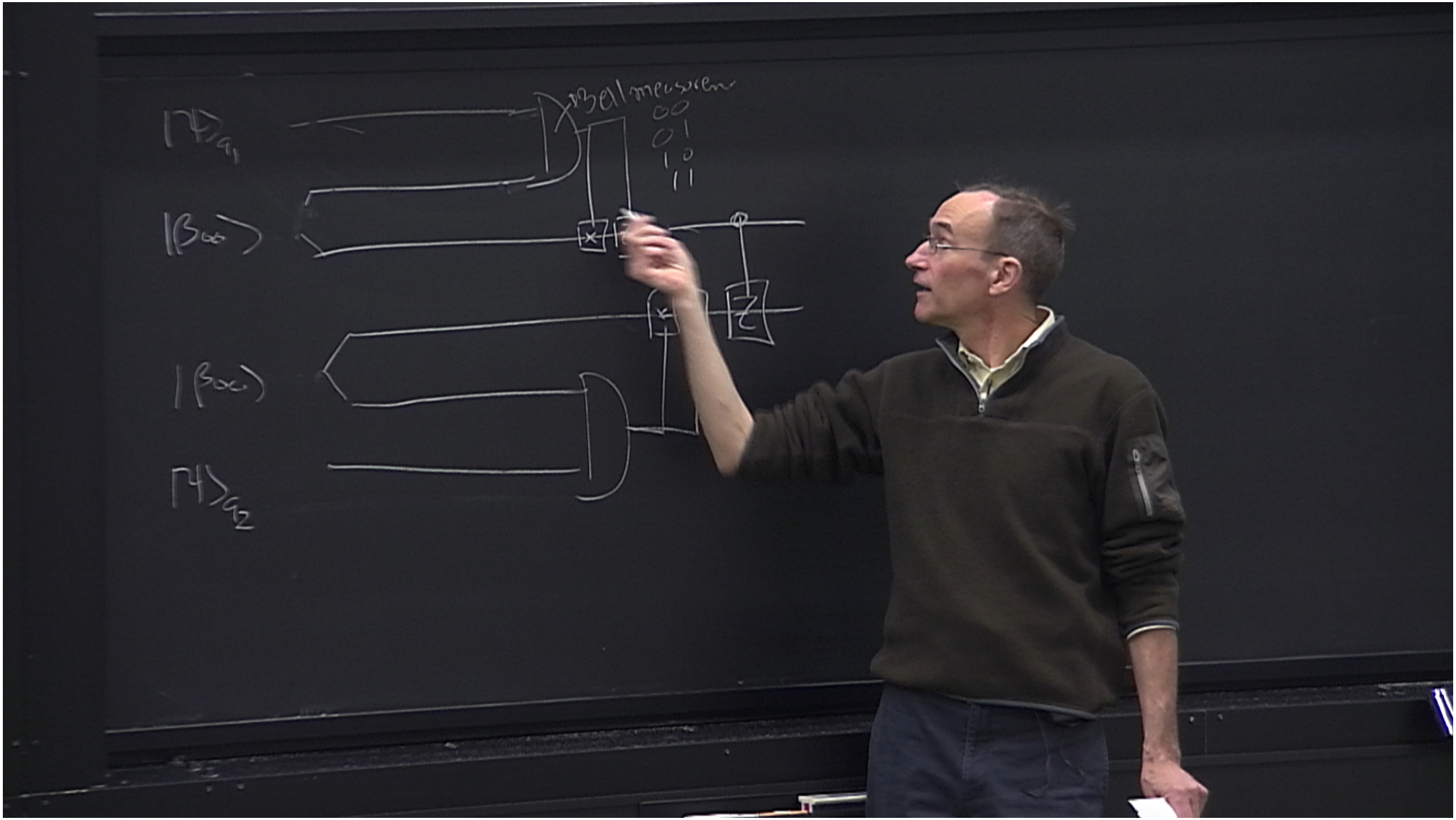

$(\alpha|10\rangle_A + \beta|11\rangle_A)(\gamma|10\rangle_B + \delta|11\rangle_B)$
$\alpha\gamma|1010\rangle + \alpha\delta|1011\rangle + \beta\gamma|1110\rangle + \beta\delta|1111\rangle$
$\alpha\gamma|1010\rangle + \alpha\delta|1011\rangle \pm \beta\gamma|1001\rangle \pm \beta\delta|1010\rangle$
$\alpha\gamma|1010\rangle - \frac{\alpha\delta}{\sqrt{2}}|1100\rangle + \frac{\alpha\delta}{\sqrt{2}}|0110\rangle + \frac{\beta\gamma}{\sqrt{2}}|1001\rangle - \frac{\beta\gamma}{\sqrt{2}}|0011\rangle - \frac{\beta\delta}{\sqrt{2}}|2000\rangle + \frac{\beta\delta}{\sqrt{2}}|0020\rangle$
=

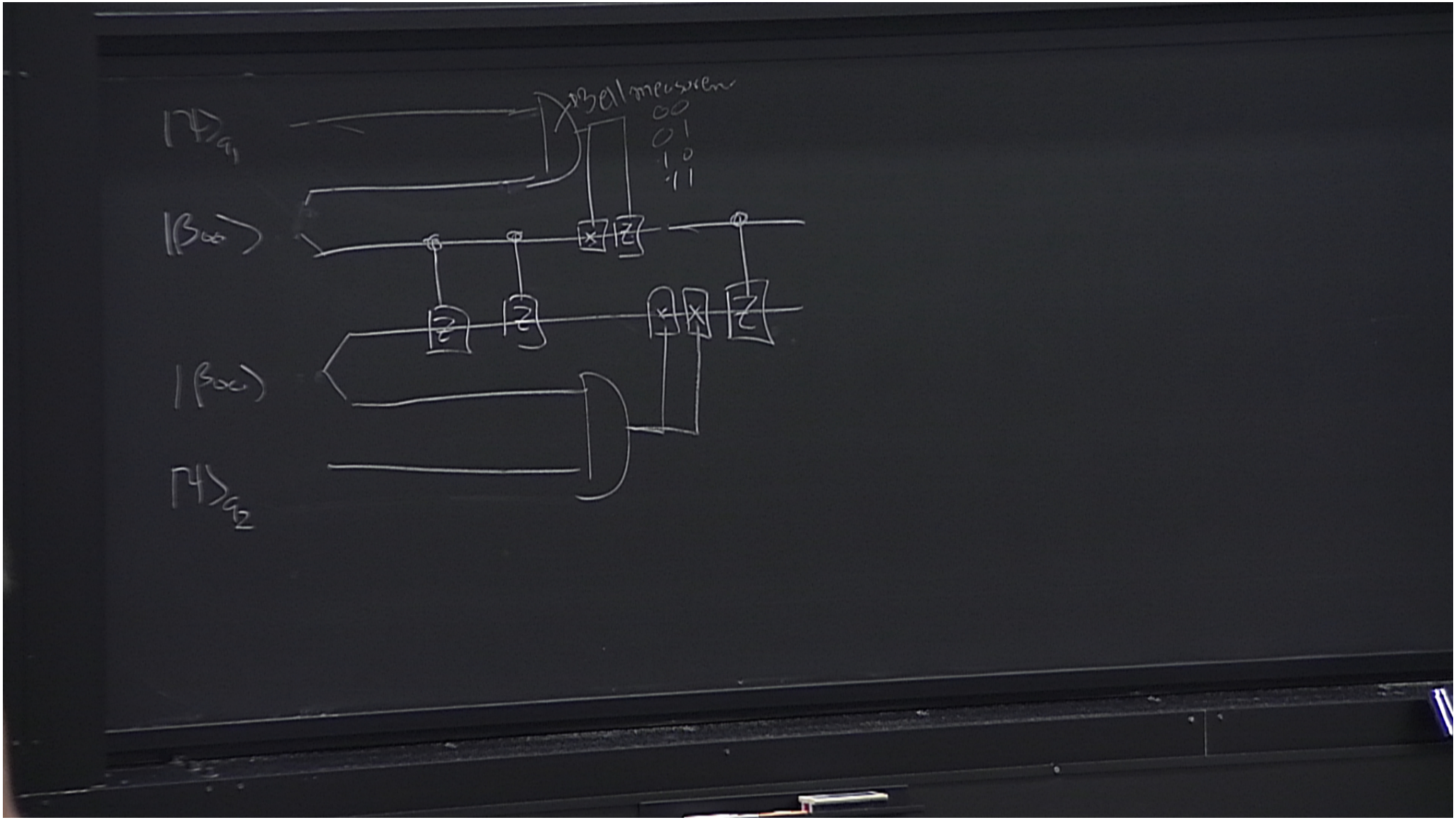

Success Prob. 1/16

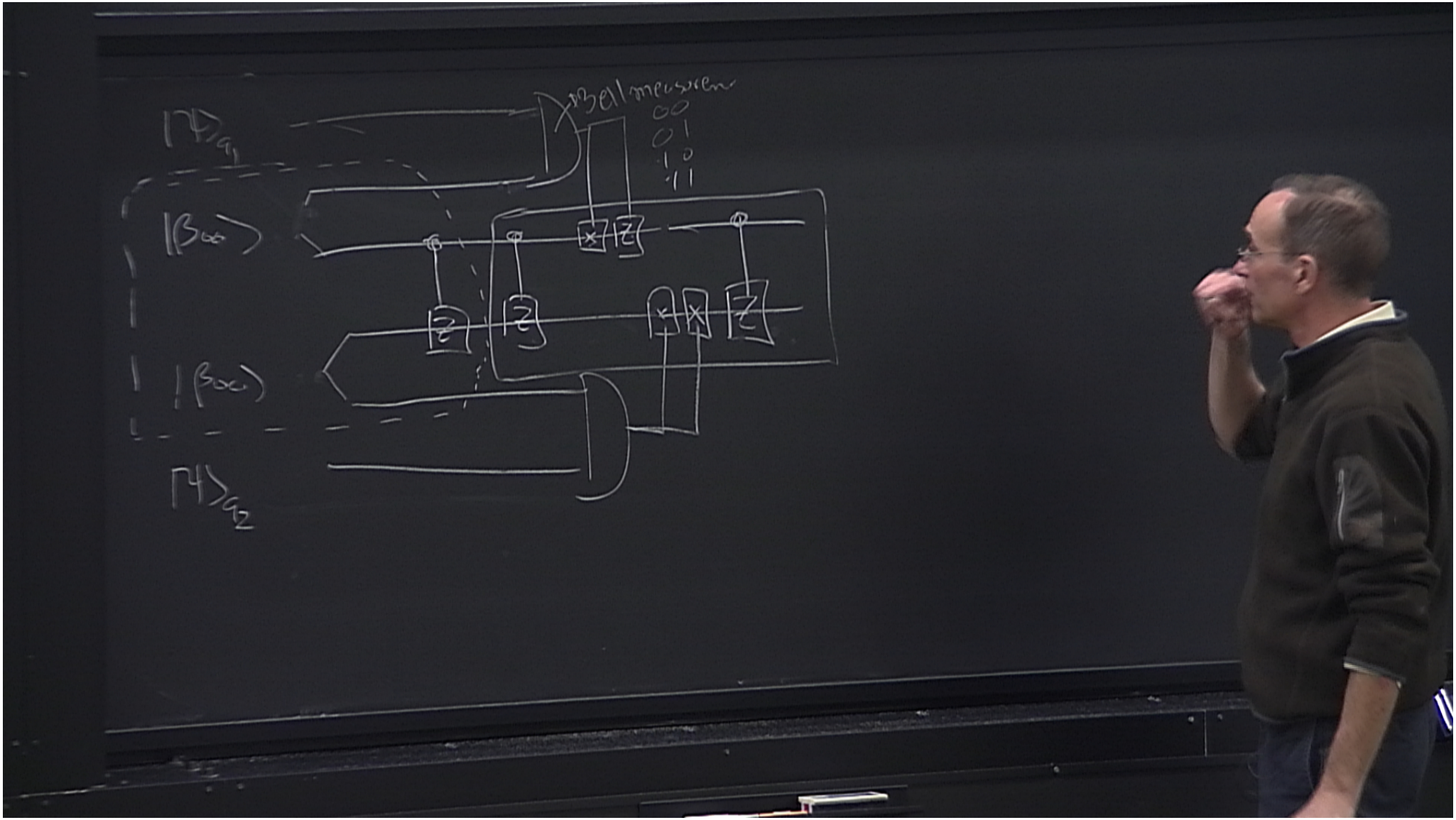

Success Prob. 1/16

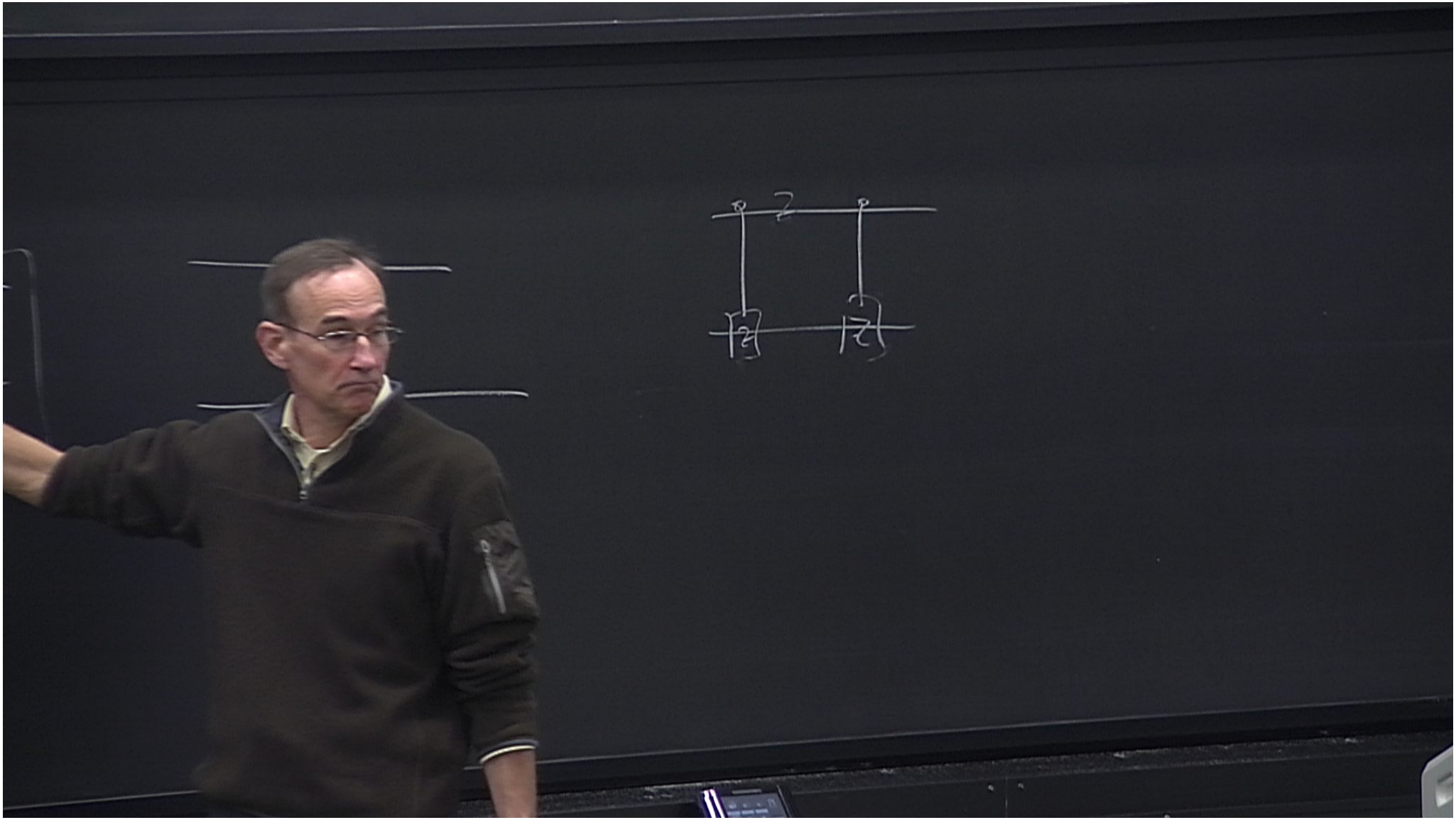

|+⟩_{q_1}
|β_{00}⟩
|β_{01}⟩
|+⟩_{q_2}

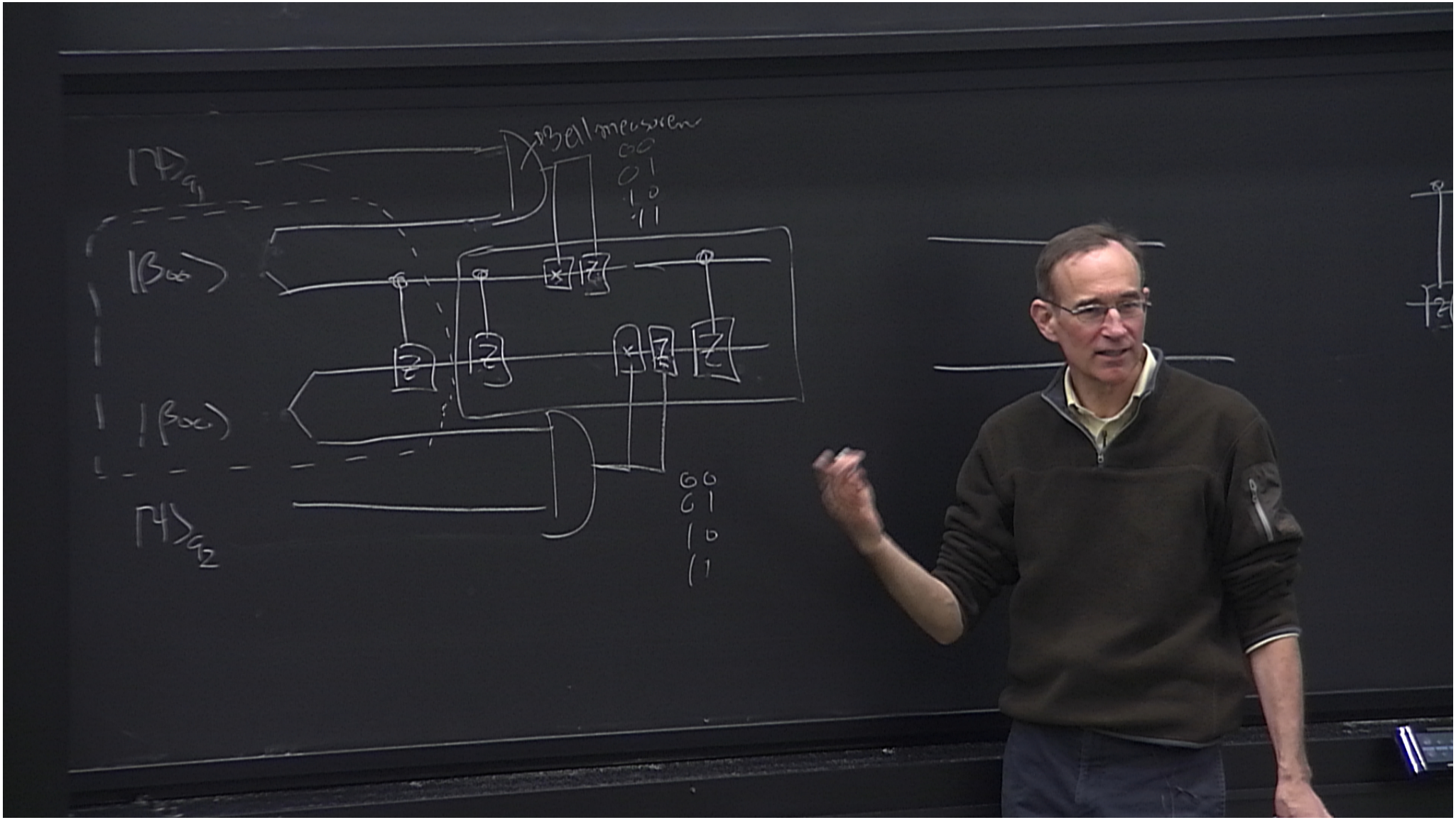

Bell measurem
|ψ⟩q1
|β00⟩
|β00⟩
|ψ⟩q2

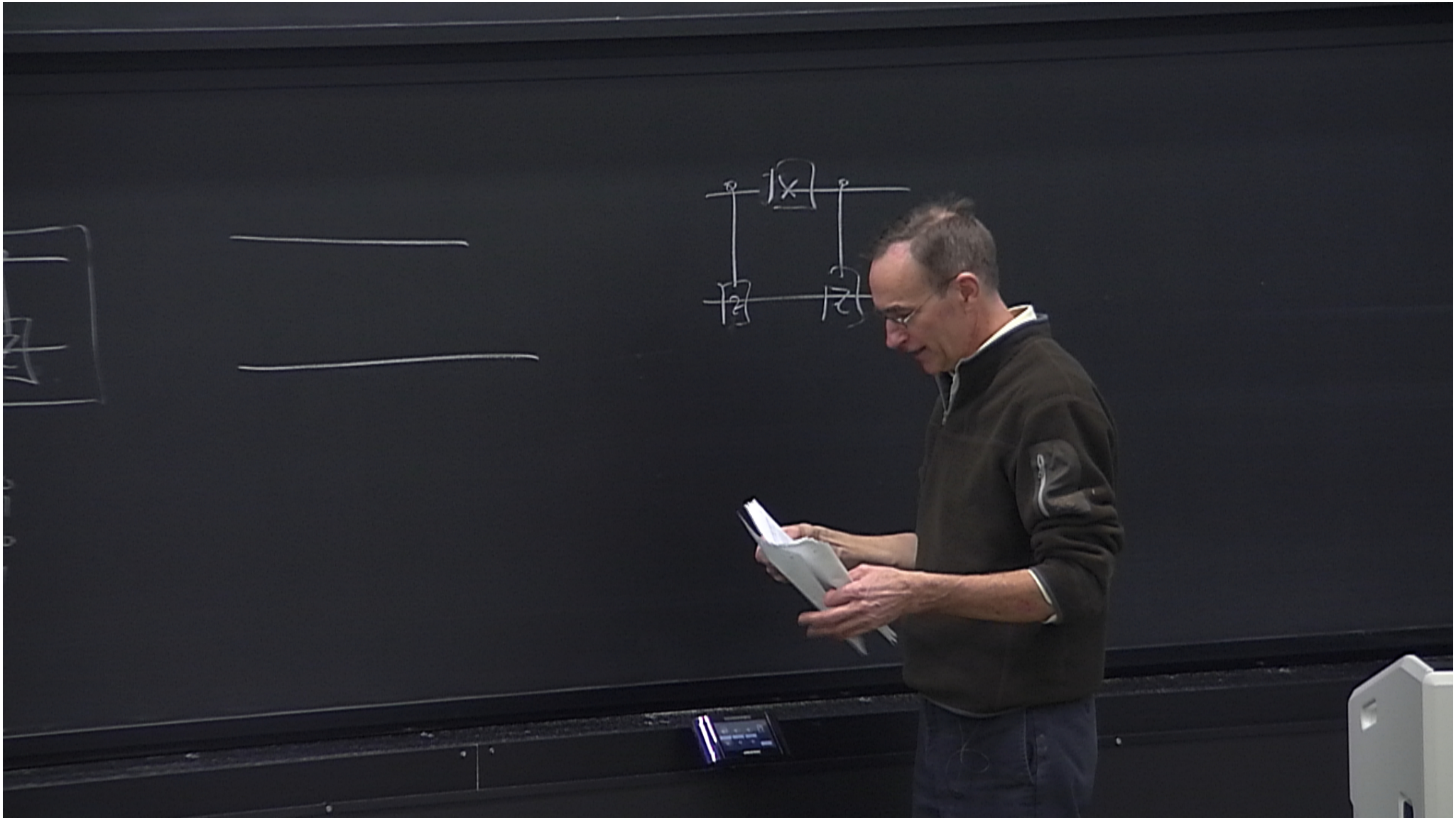

Bell measurem
00
01
10
11

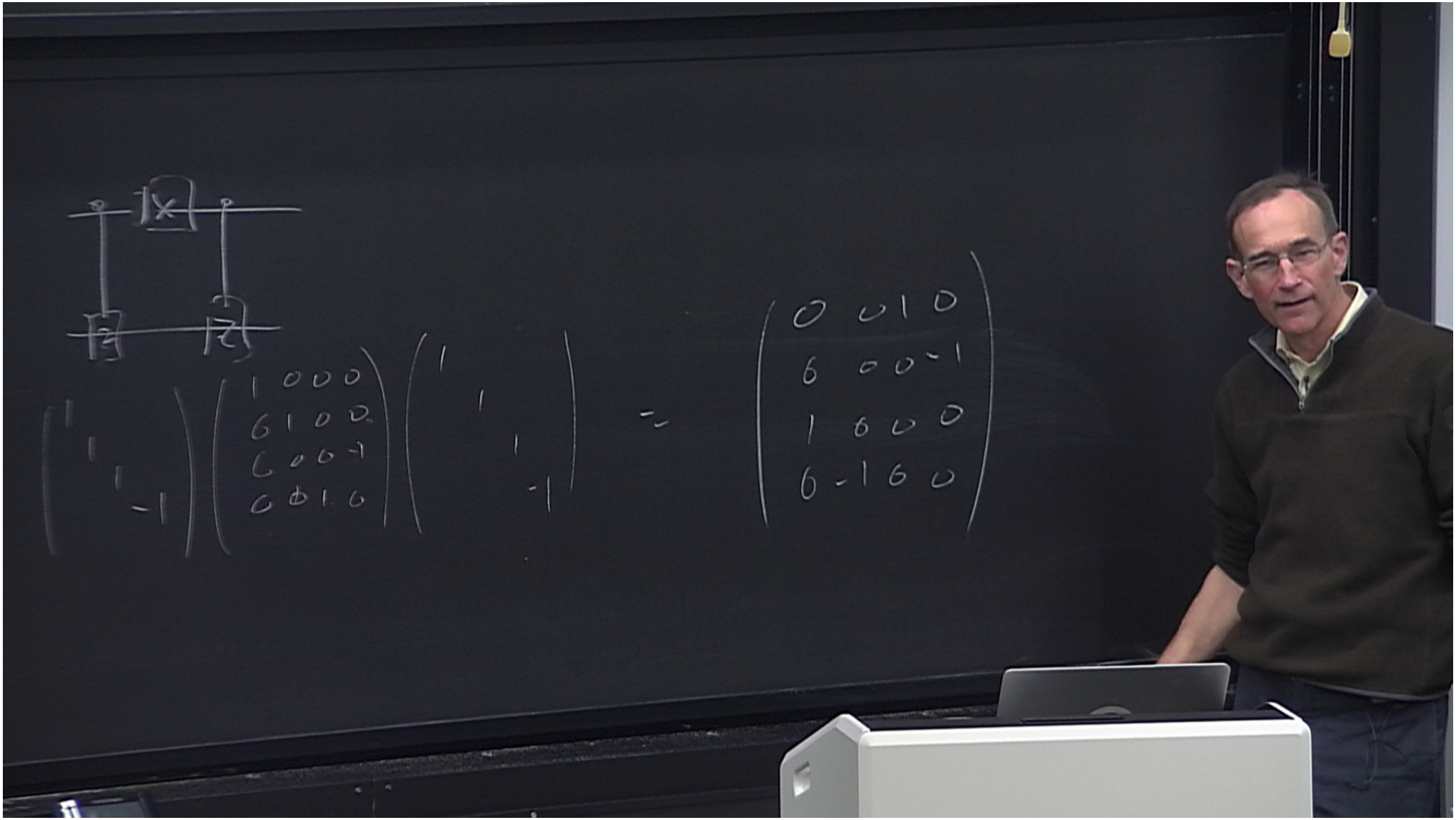

Bell measurement
00
01
10
11

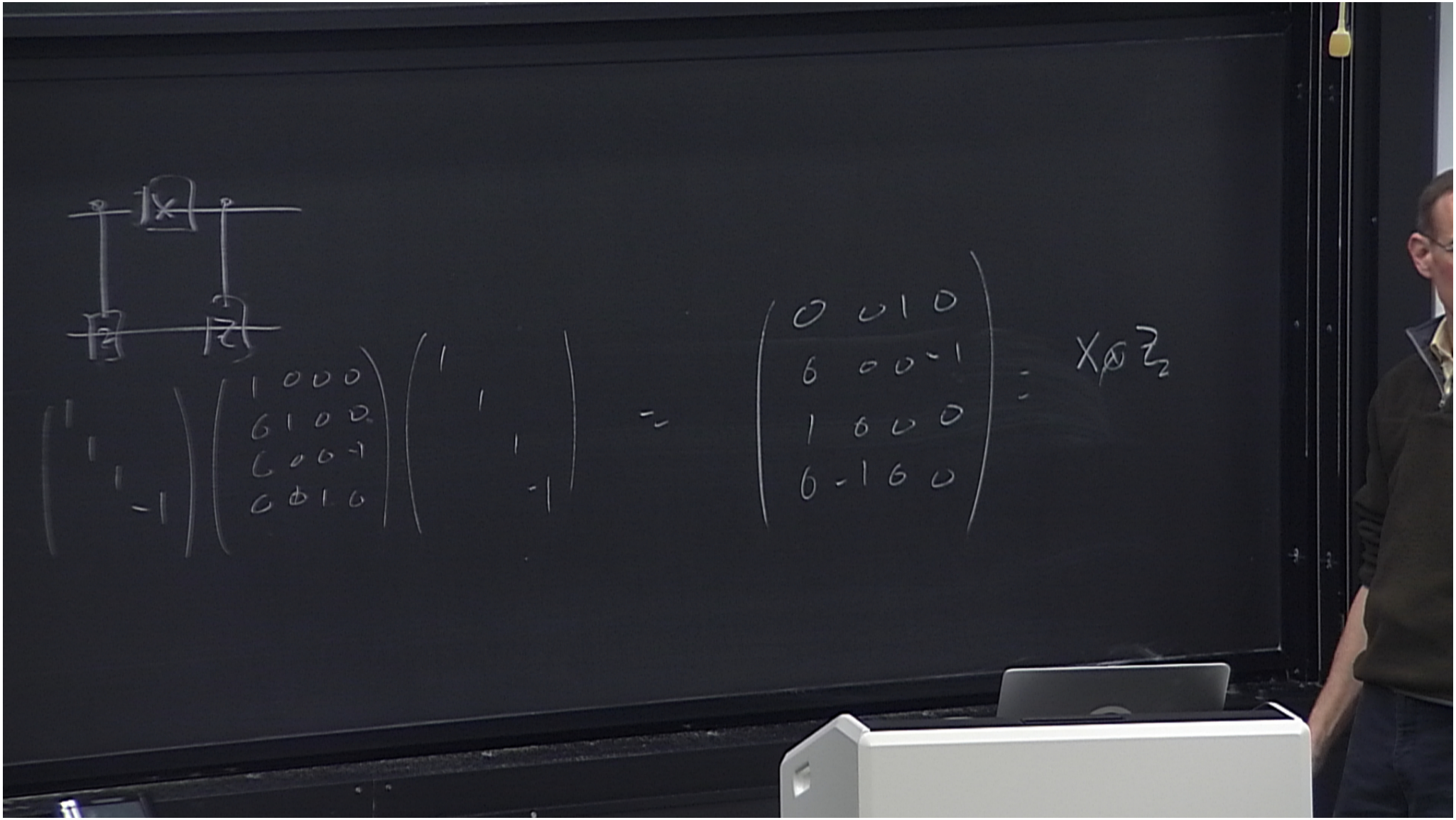

Bell measurem
00
01
10
11

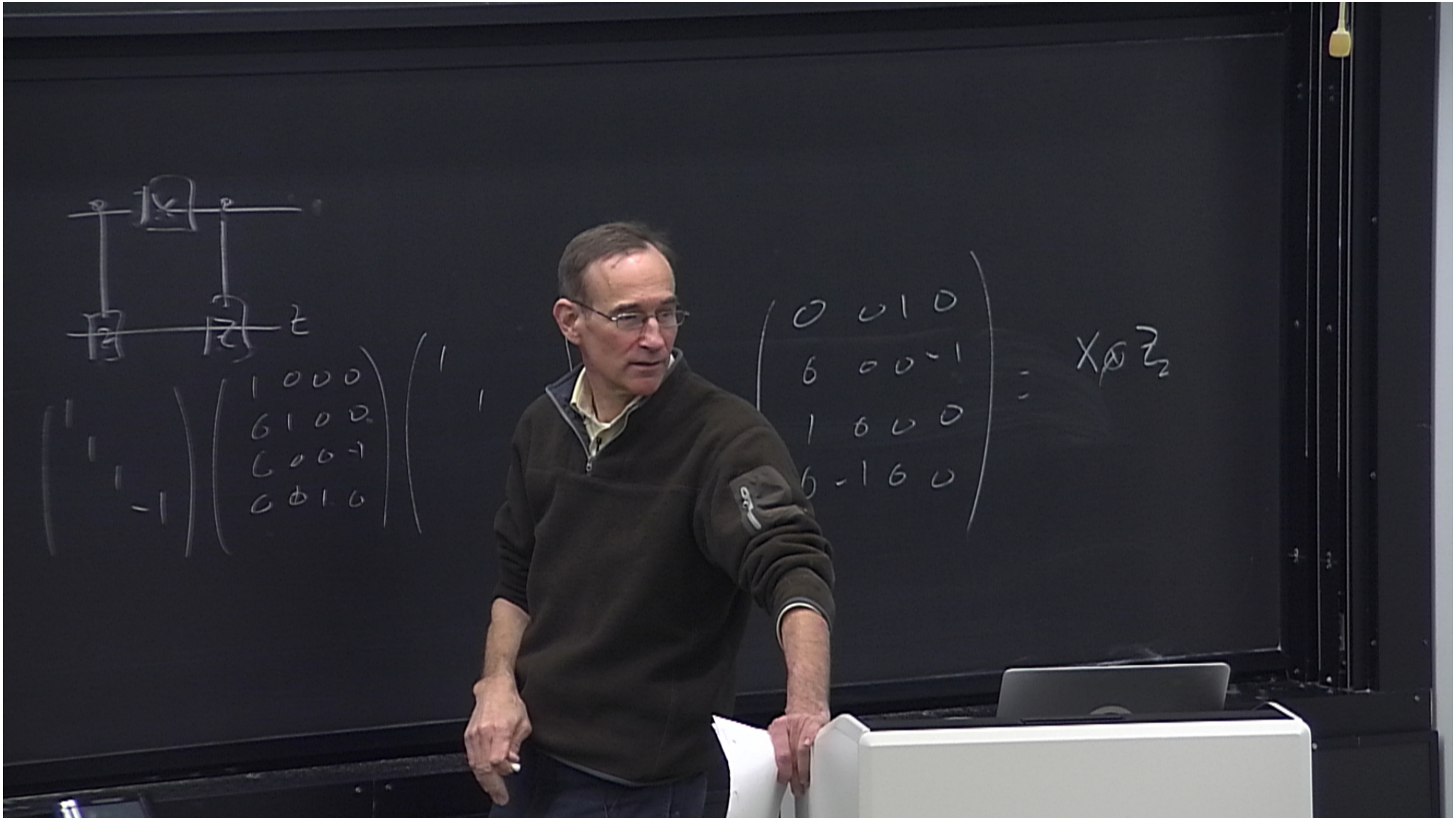

z

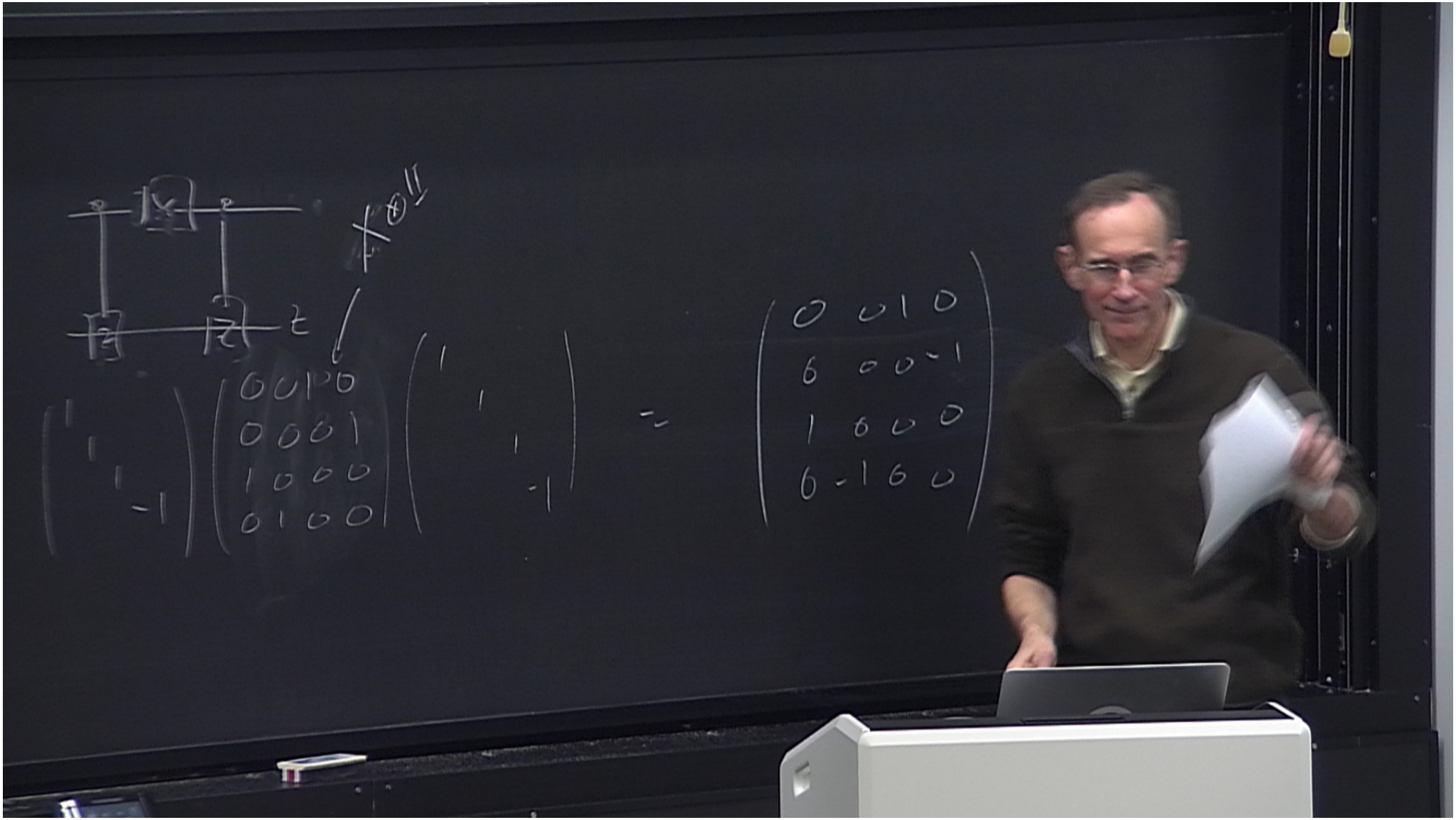

Bell measurement
00
01
10
11
00
01
10
11

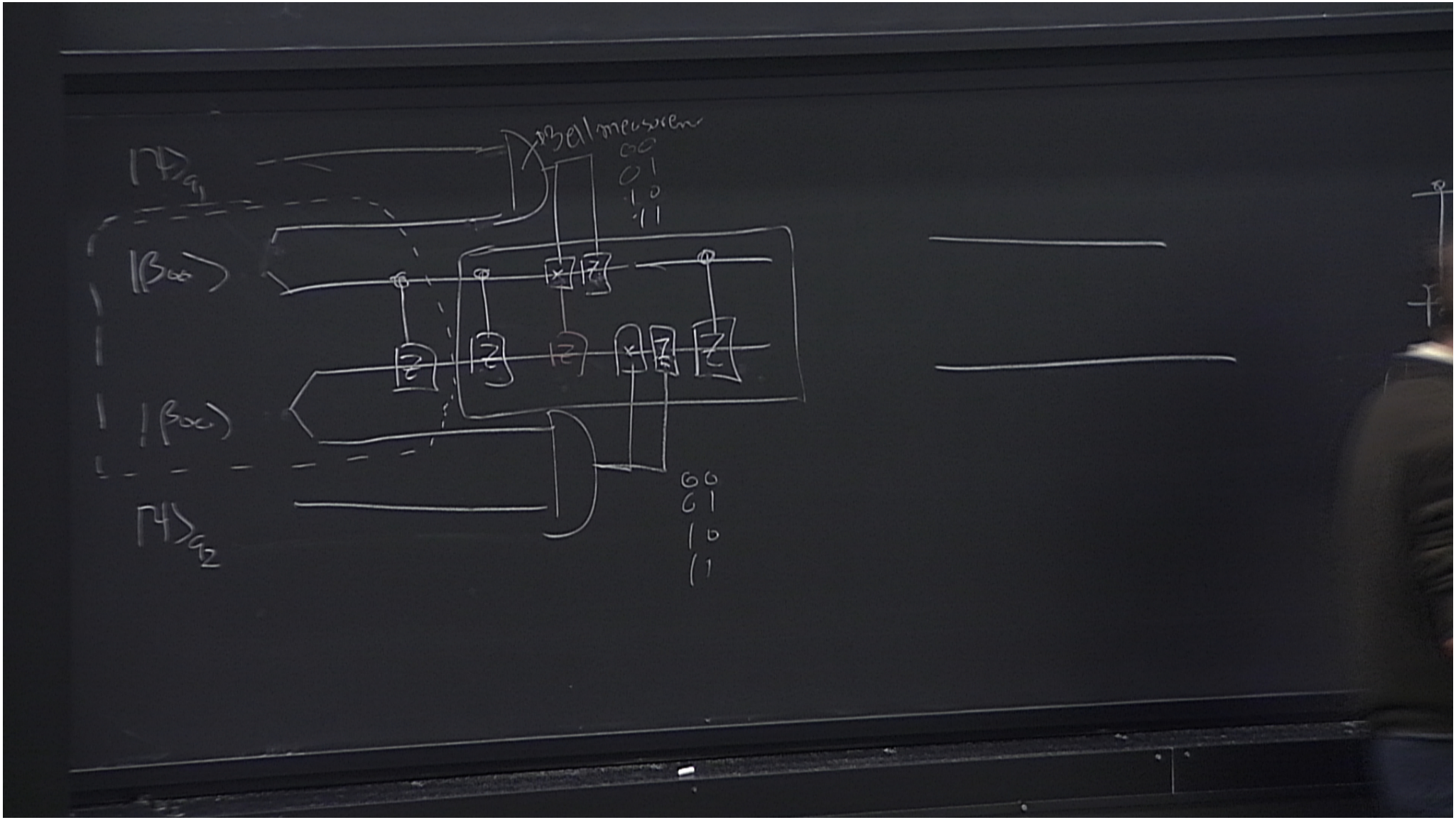


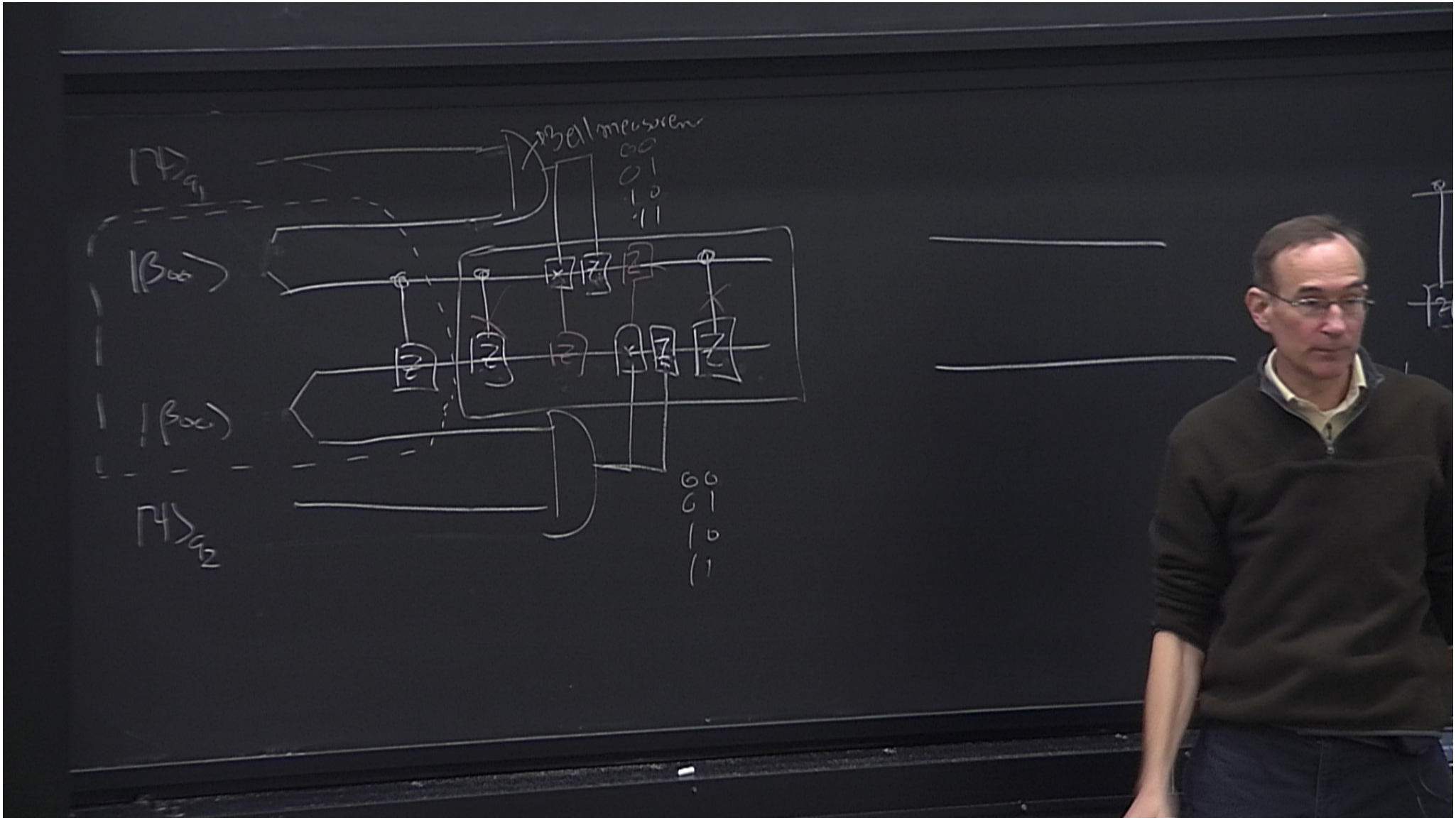


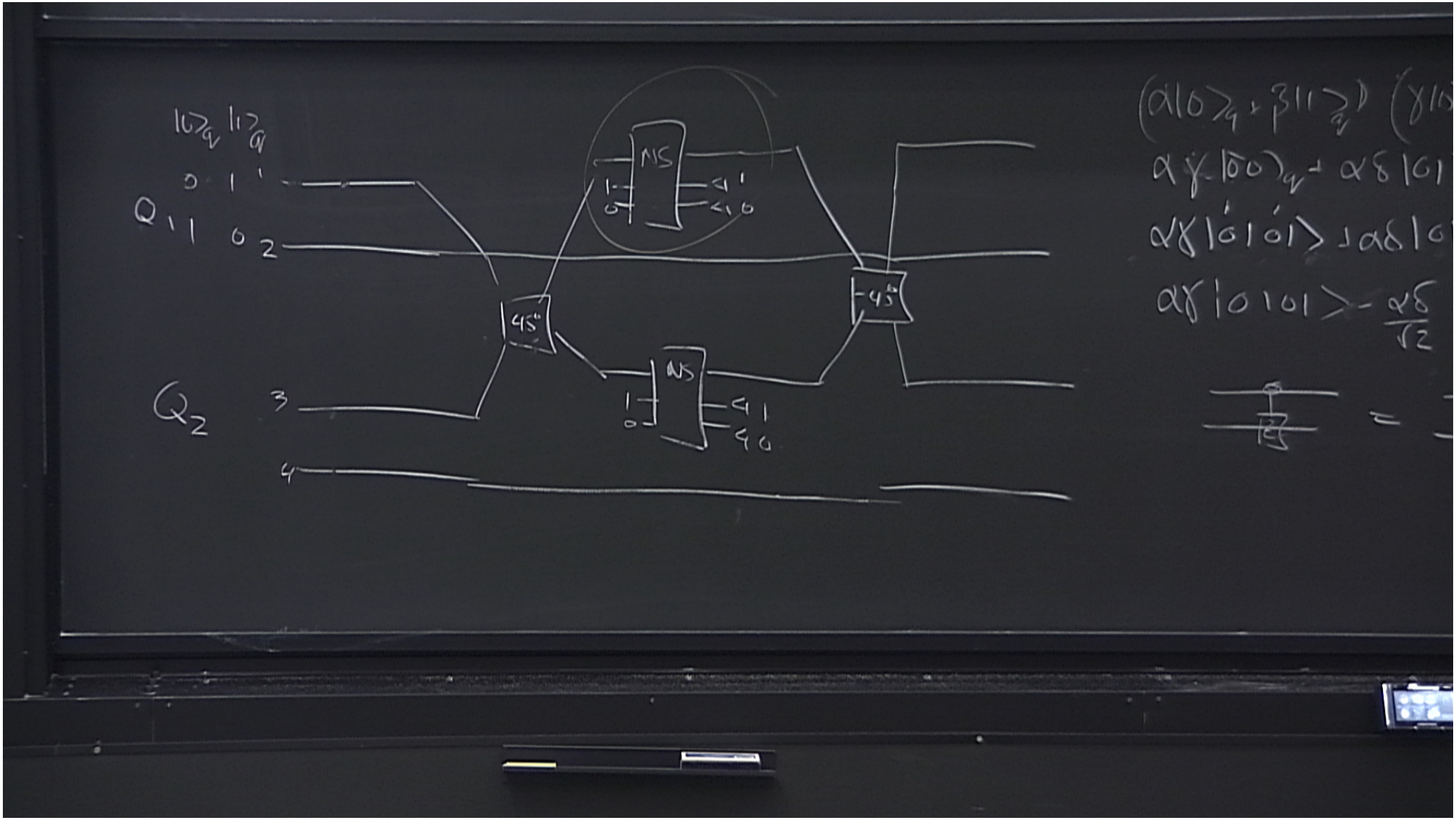

X
$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} = X \otimes Z$

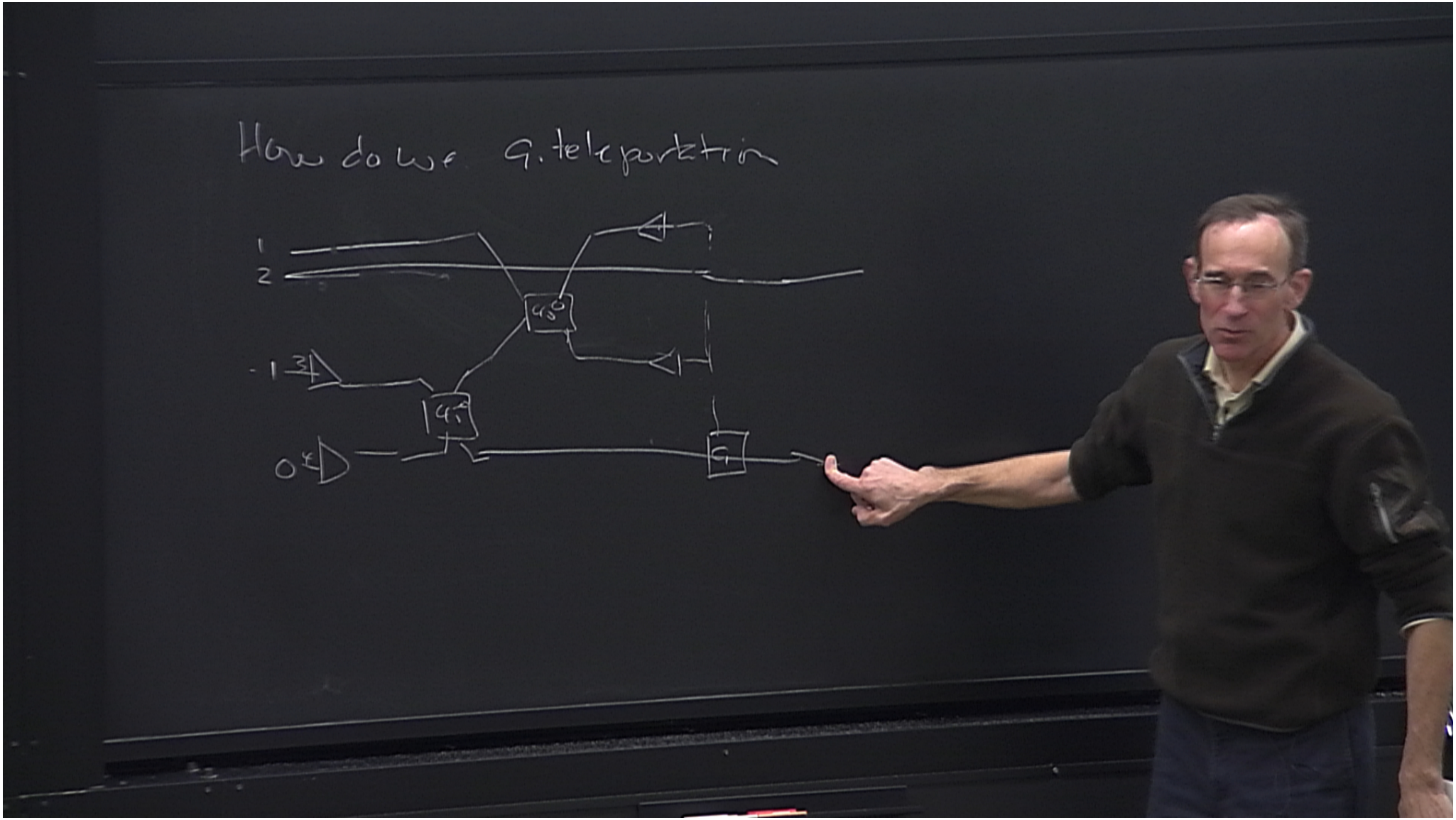

X
t
= X⊗Z₂



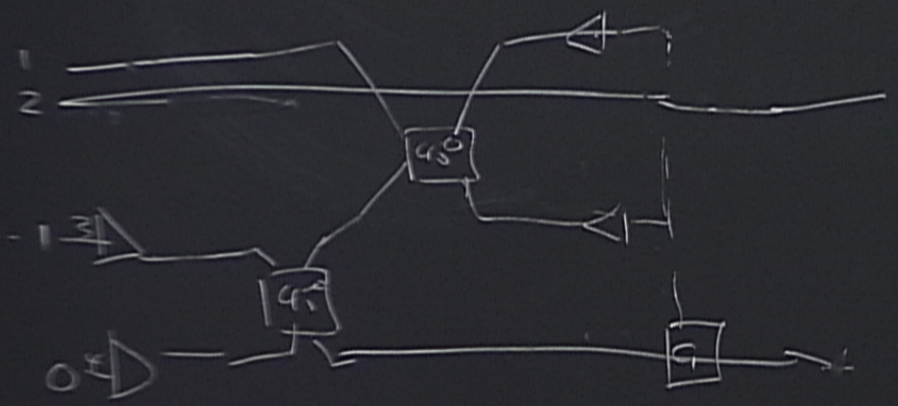

Bell measurement
00
01
10
11
00
01
10
11


Bell measurement
00
01
10
11
00
01
10
11


$|0\rangle_q$ $|1\rangle_q$
0 1
Q_1 0 2
Q_2 3
4
45°
45°
NS
NS
$(\alpha|0\rangle_q + \beta|1\rangle_q)$
$\alpha\gamma|00\rangle_q + \alpha\delta|01$
$\alpha\gamma|0\,0\,0\,1\rangle + \alpha\delta|0$
$\alpha\gamma|0\,1\,0\,1\rangle - \frac{\alpha\delta}{\sqrt{2}}$


How do we q. teleportation
1
2
3
0

How do we q. teleportation



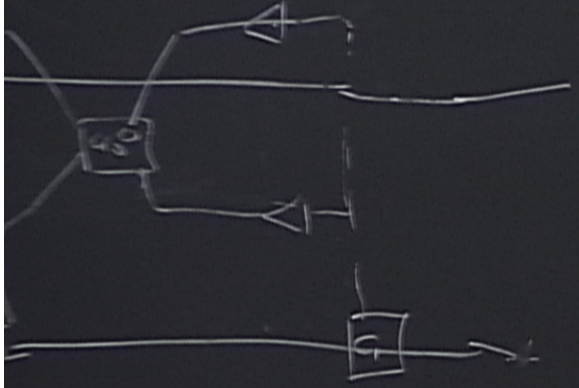
$(\alpha|01\rangle + \beta|10\rangle)|10\rangle$

$(\alpha|01\rangle + \beta|10\rangle)\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$

q. teleportation

$(\alpha|01\rangle + \beta|10\rangle)|10\rangle$

$(\alpha|01\rangle + \beta|10\rangle)\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$



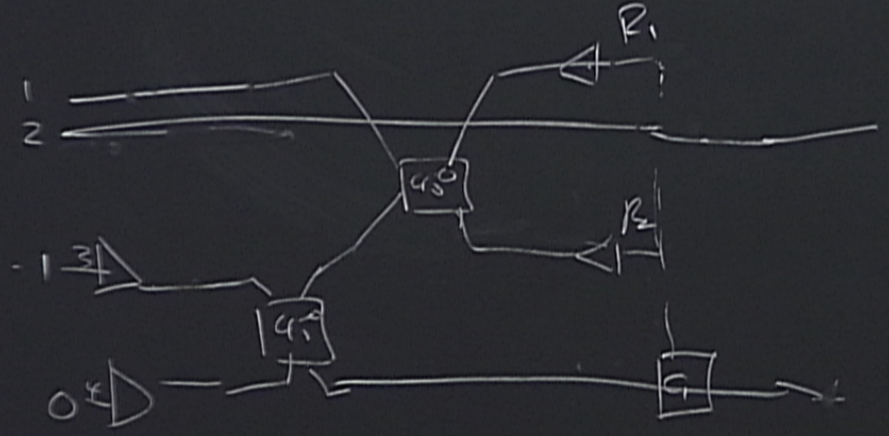
$\frac{1}{\sqrt{2}}\Big(\alpha|10101\rangle + \alpha|10110\rangle + \beta|11001\rangle + \beta|11010\rangle$

$\frac{1}{2}\Big(\sqrt{2}\alpha|10101\rangle + \alpha|11100\rangle + \alpha|10110\rangle$

$+ \beta|11001\rangle + \beta|10011\rangle$

$- \beta|12000\rangle + \beta|10020\rangle$

How do we q. teleportation



$(\alpha|01\rangle + \beta|10\rangle)|10\rangle$

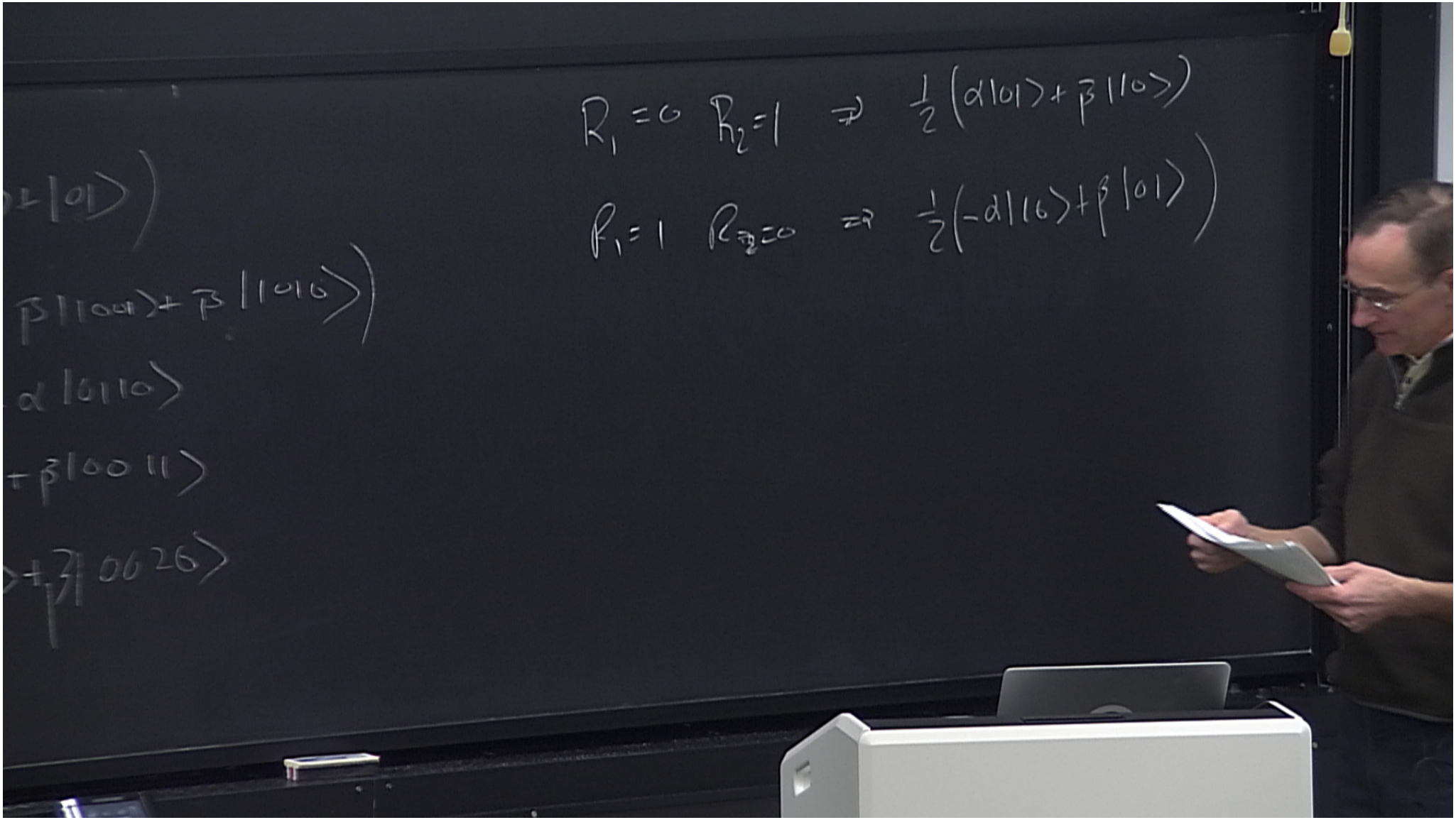
$(\alpha|01\rangle + \beta|10\rangle)\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle$

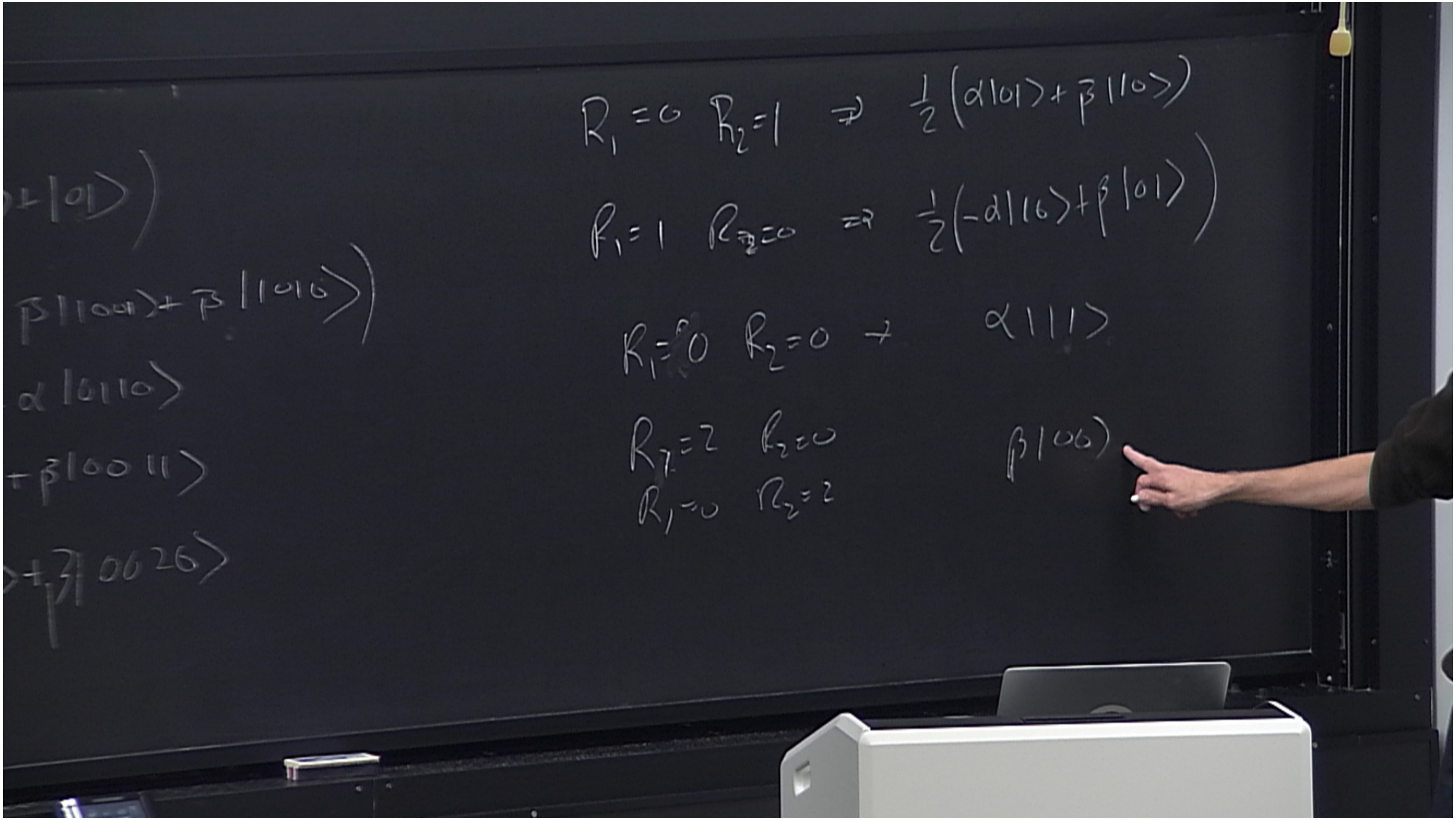
$\frac{1}{\sqrt{2}}(\alpha|10101\rangle + \alpha|0\,110\rangle + \beta|1\,0$

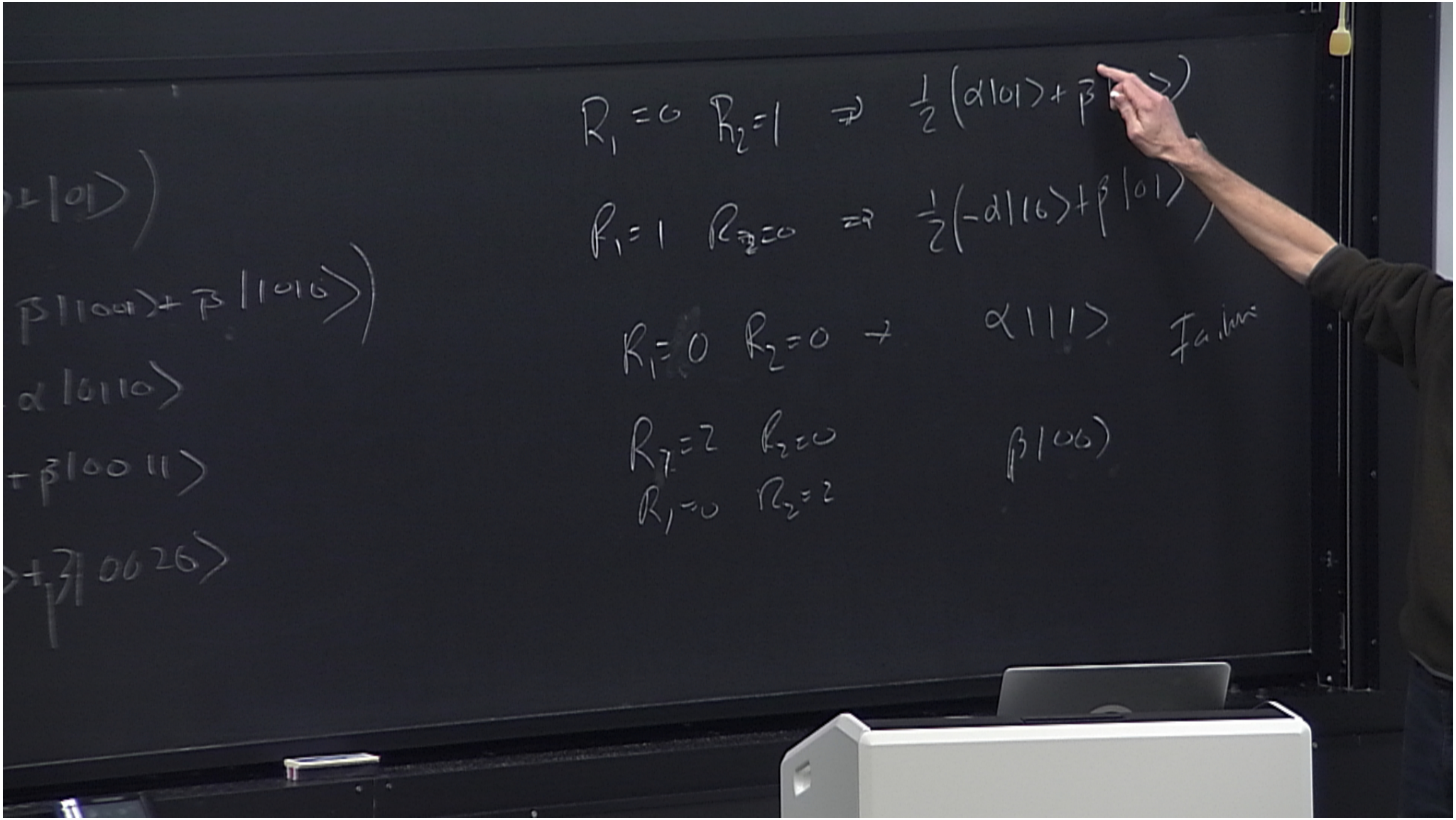
$\frac{1}{2}(\alpha|10101\rangle + \alpha|1\,100\rangle + \alpha|101$

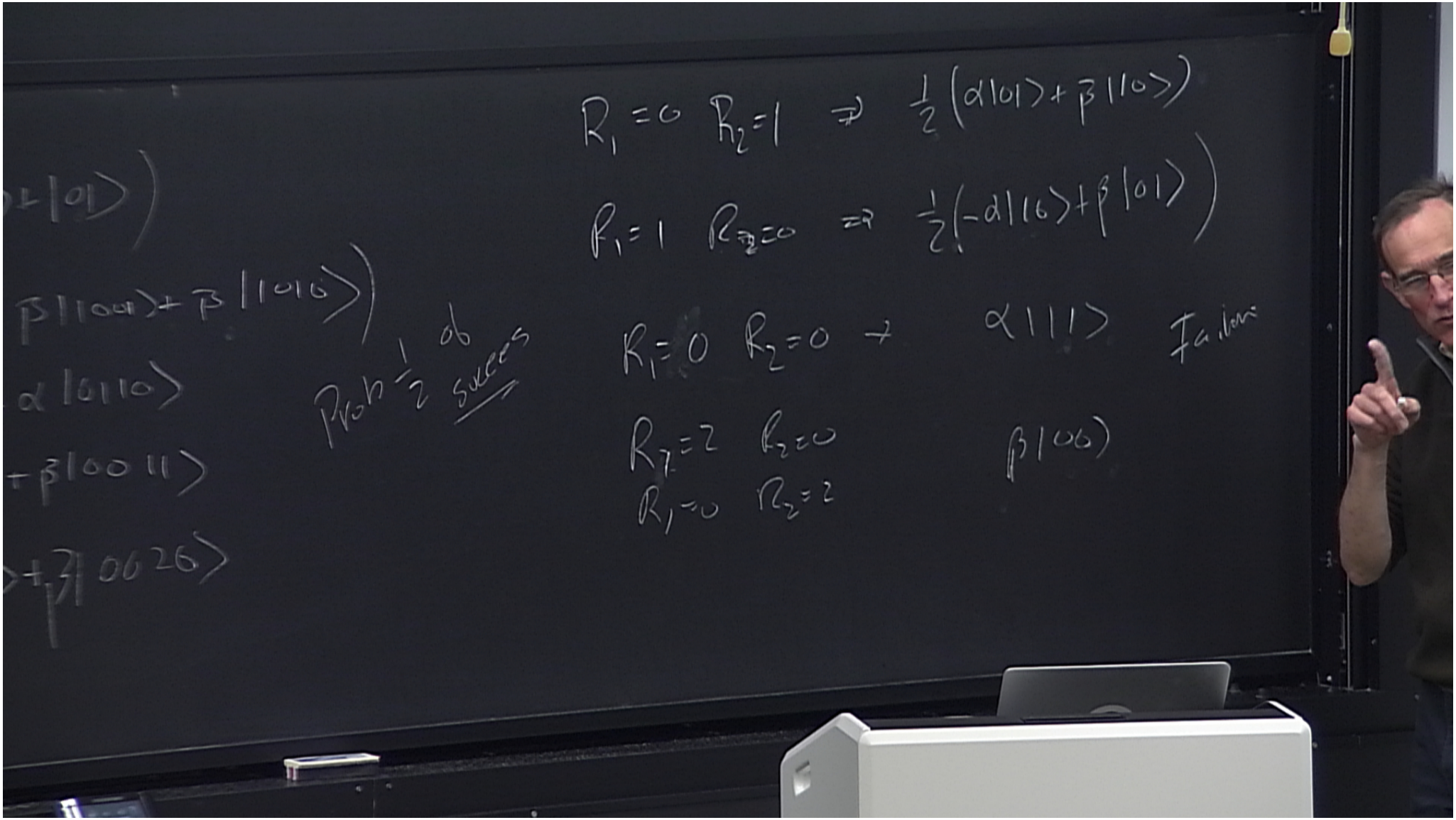
$+ \beta|1\,001\rangle + \beta|00$

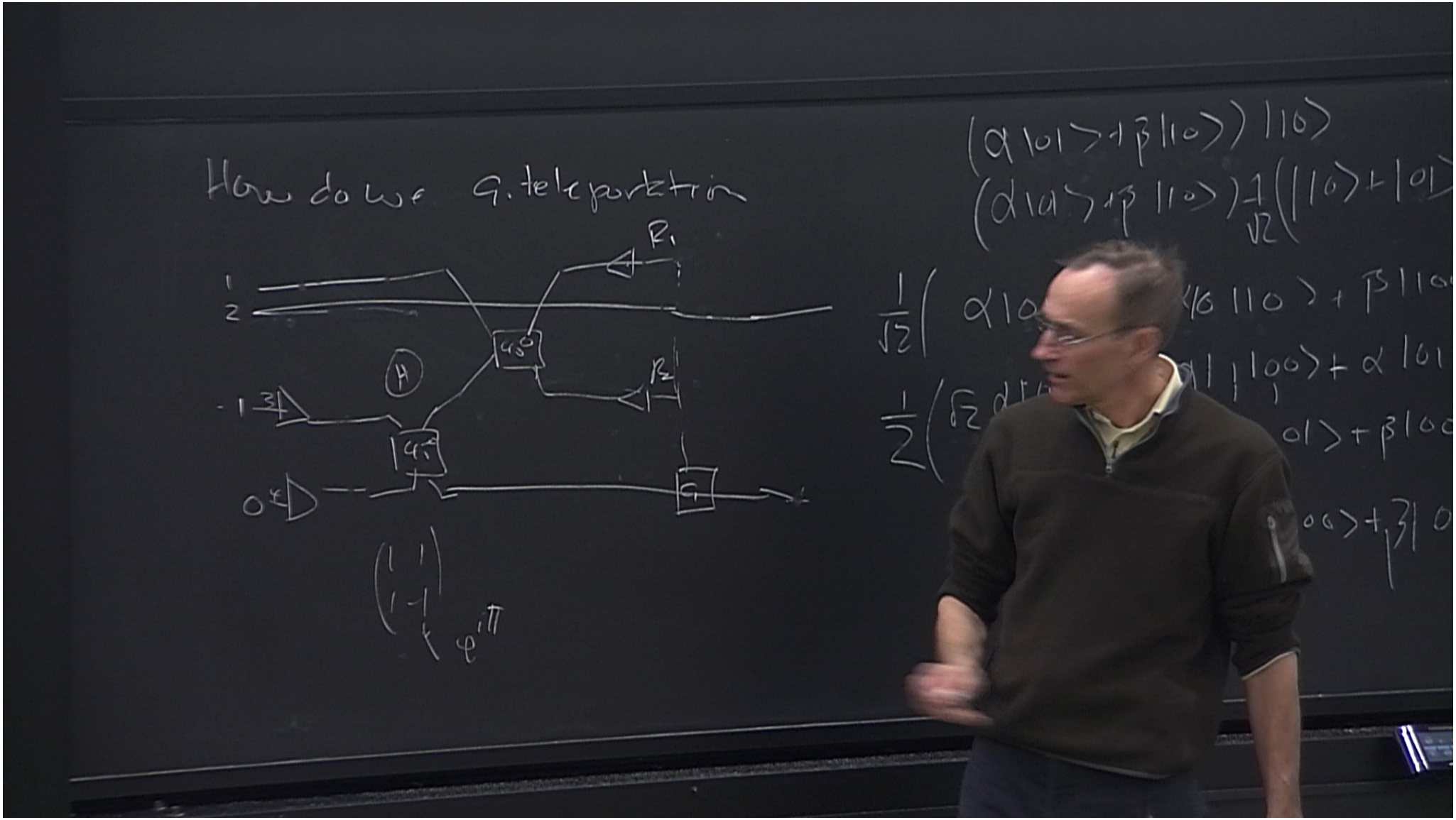
$-\beta|2\,000\rangle + \beta|0$

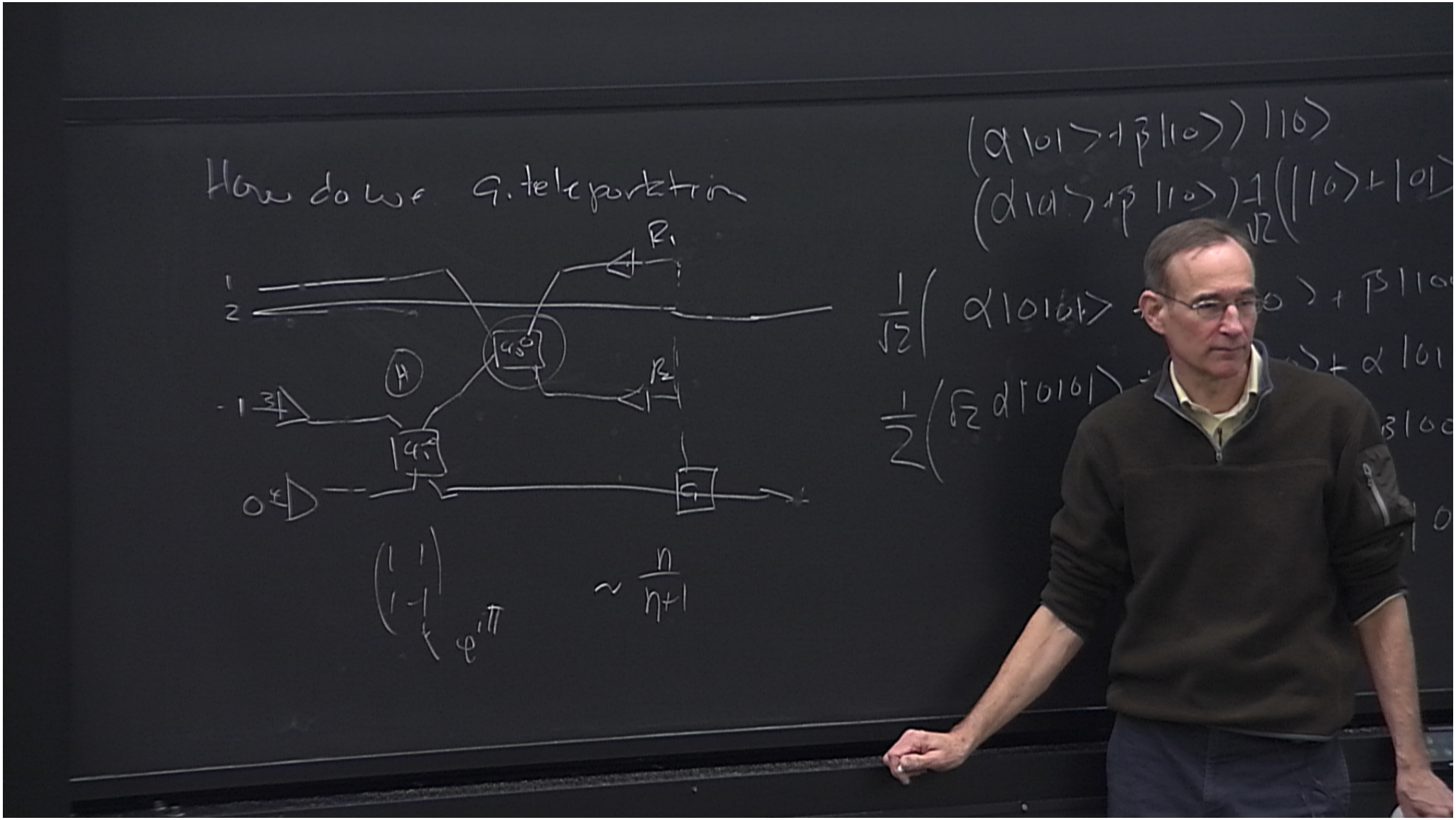

$R_1 = 0 \quad R_2 = 1 \Rightarrow \frac{1}{2}(\alpha|01\rangle + \beta|10\rangle)$
$R_1 = 1 \quad R_2 = 0 \Rightarrow \frac{1}{2}(-\alpha|10\rangle + \beta|01\rangle)$


$R_1=0 \quad R_2=1 \Rightarrow \frac{1}{2}(\alpha|01\rangle+\beta|10\rangle)$
$R_1=1 \quad R_2=0 \Rightarrow \frac{1}{2}(-\alpha|10\rangle+\beta|01\rangle)$
$R_1=0 \quad R_2=0 \Rightarrow \alpha|111\rangle$
$R_1=2 \quad R_2=0$
$R_1=0 \quad R_2=2$
$\beta|100\rangle$


$R_1 = 0 \quad R_2 = 1 \Rightarrow \frac{1}{2}(\alpha|01\rangle + \beta$
$R_1 = 1 \quad R_2 = 0 \Rightarrow \frac{1}{2}(-\alpha|10\rangle + \beta|01\rangle)$
$R_1 = 0 \quad R_2 = 0 \rightarrow \alpha|111\rangle$ Failure
$\beta|100\rangle$
$R_1 = 2 \quad R_2 = 0$
$R_1 = 0 \quad R_2 = 2$
$\alpha|0110\rangle$
$+\beta|0011\rangle$


$R_1=0$ $R_2=1$ $\Rightarrow$ $\frac{1}{2}(\alpha|01\rangle+\beta|10\rangle)$
$R_1=1$ $R_2=0$ $\Rightarrow$ $\frac{1}{2}(-\alpha|10\rangle+\beta|01\rangle)$
$R_1=0$ $R_2=0$ $\Rightarrow$ $\alpha|111\rangle$ Failure
$R_1=2$ $R_2=0$
$R_1=0$ $R_2=2$
$\beta|000\rangle$
Prob $\frac{1}{2}$ ok
success


How do we q. teleportation
R1
R2
H
1
2
3
0
$(\alpha|01\rangle+\beta|10\rangle)|10\rangle$
$(\alpha|01\rangle+\beta|10\rangle)\frac{1}{\sqrt{2}}(|10\rangle+|01\rangle$
$\frac{1}{\sqrt{2}}(\alpha|10$
$\frac{1}{2}($
$\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$
$+e^{i\pi}$


How do we q. teleportation
R1
R2
H
$\frac{1}{\sqrt{2}}$
$\frac{1}{2}$
$\sim \frac{n}{n+1}$

Bookmarks
1. Quantum Information Processing with linear optics
2. Linear Optics Quantum Compution
3. LOQC and Quantum Error Correction
4. Conclusion

For example, consider the case when $n = 2$:

$$|t_2\rangle = \frac{1}{\sqrt{3}}\Big(|0011\rangle + |1001\rangle + |1100\rangle\Big)$$

and

$$u(F_3) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2\pi i}{3}} & e^{-\frac{2\pi i}{3}} \\ 1 & e^{-\frac{2\pi i}{3}} & e^{\frac{2\pi i}{3}} \end{pmatrix}$$

Exercise: Find the linear circuit that implement this unitary transformation.

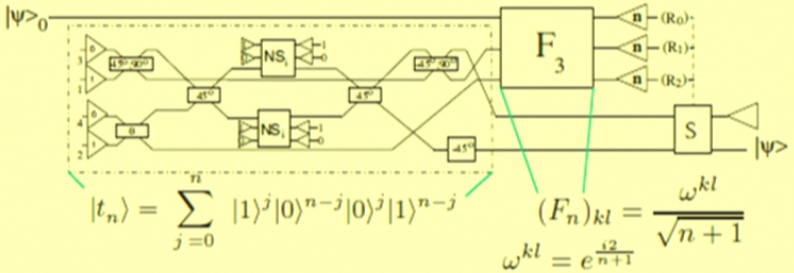
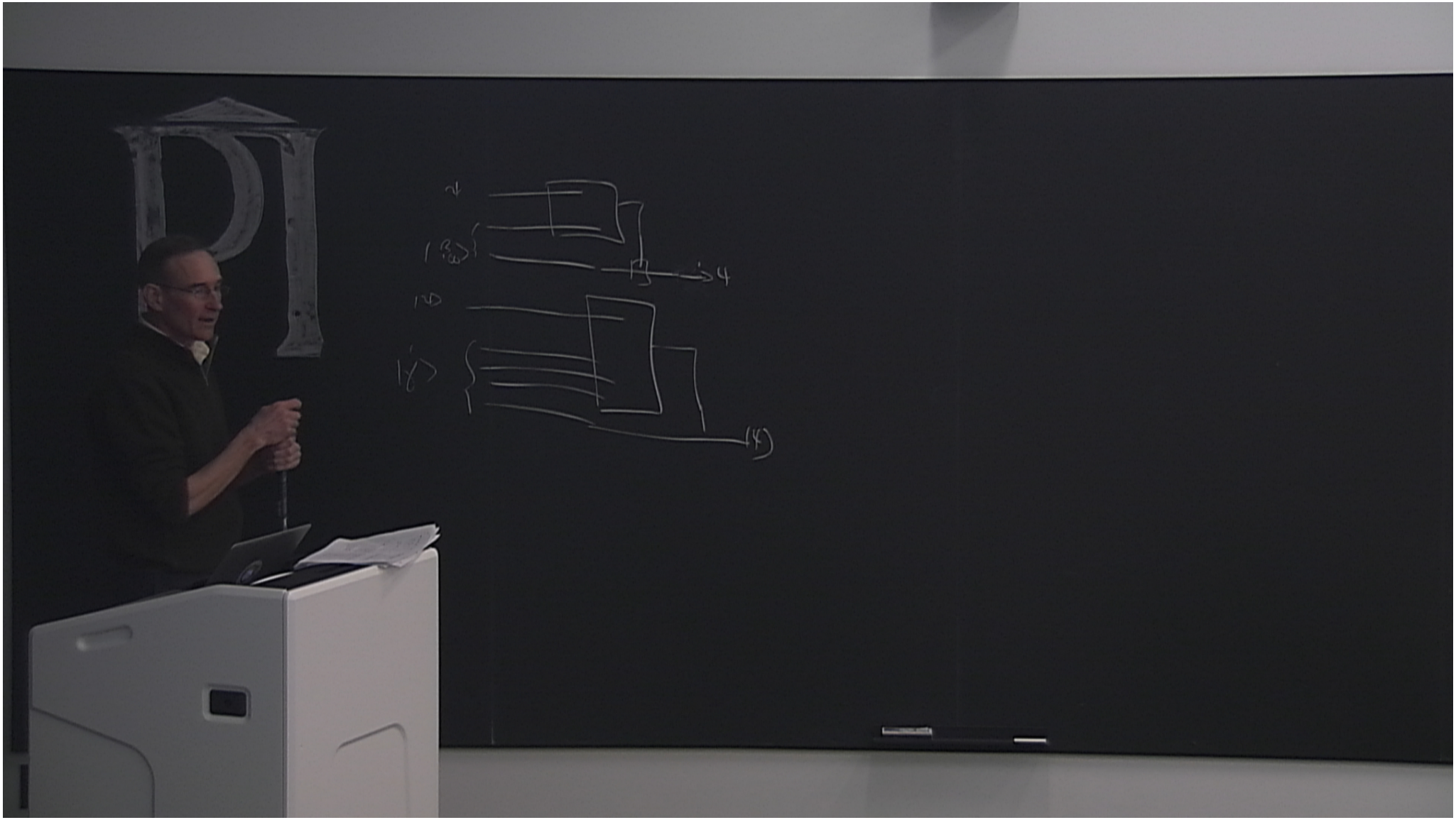
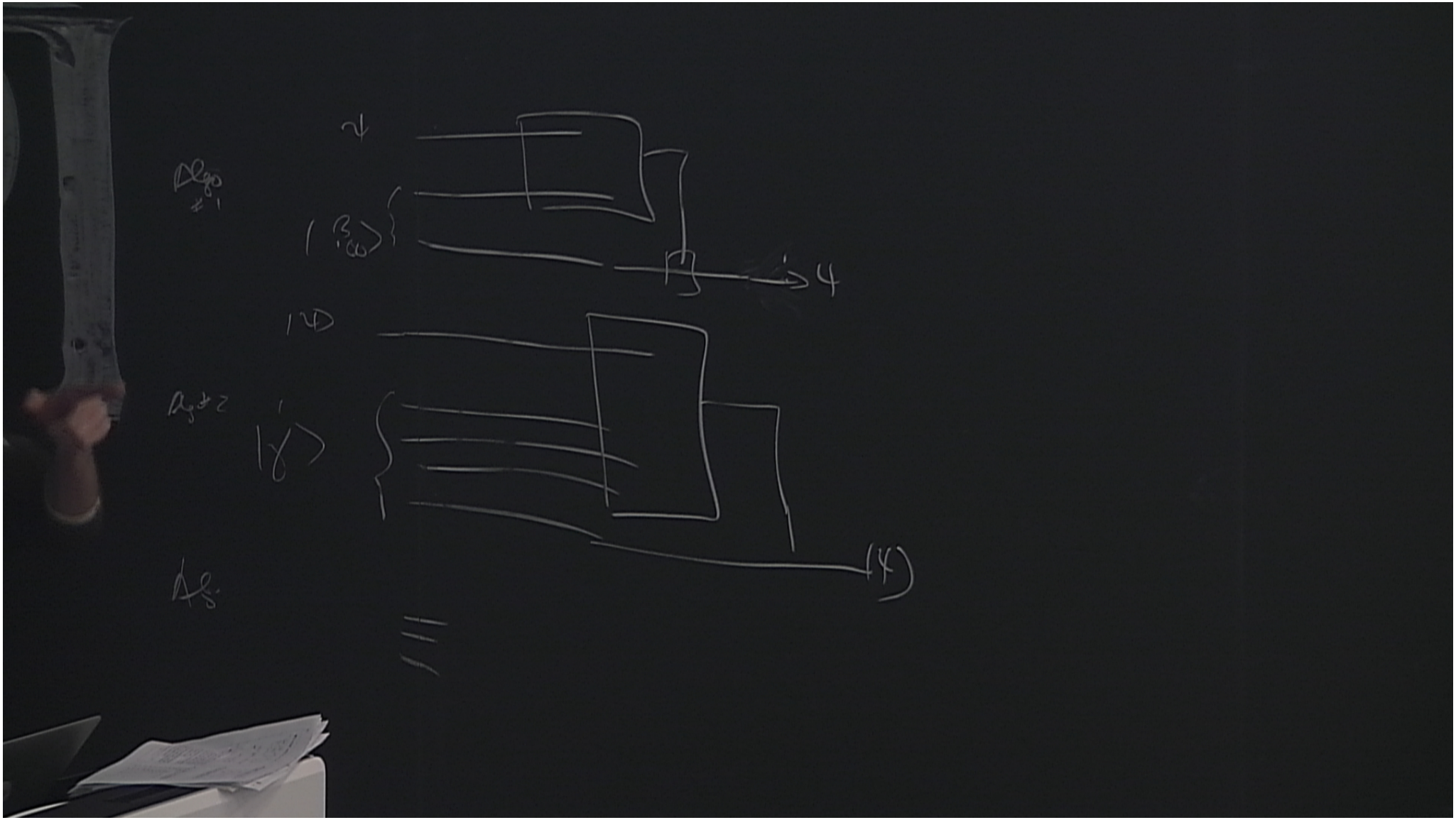
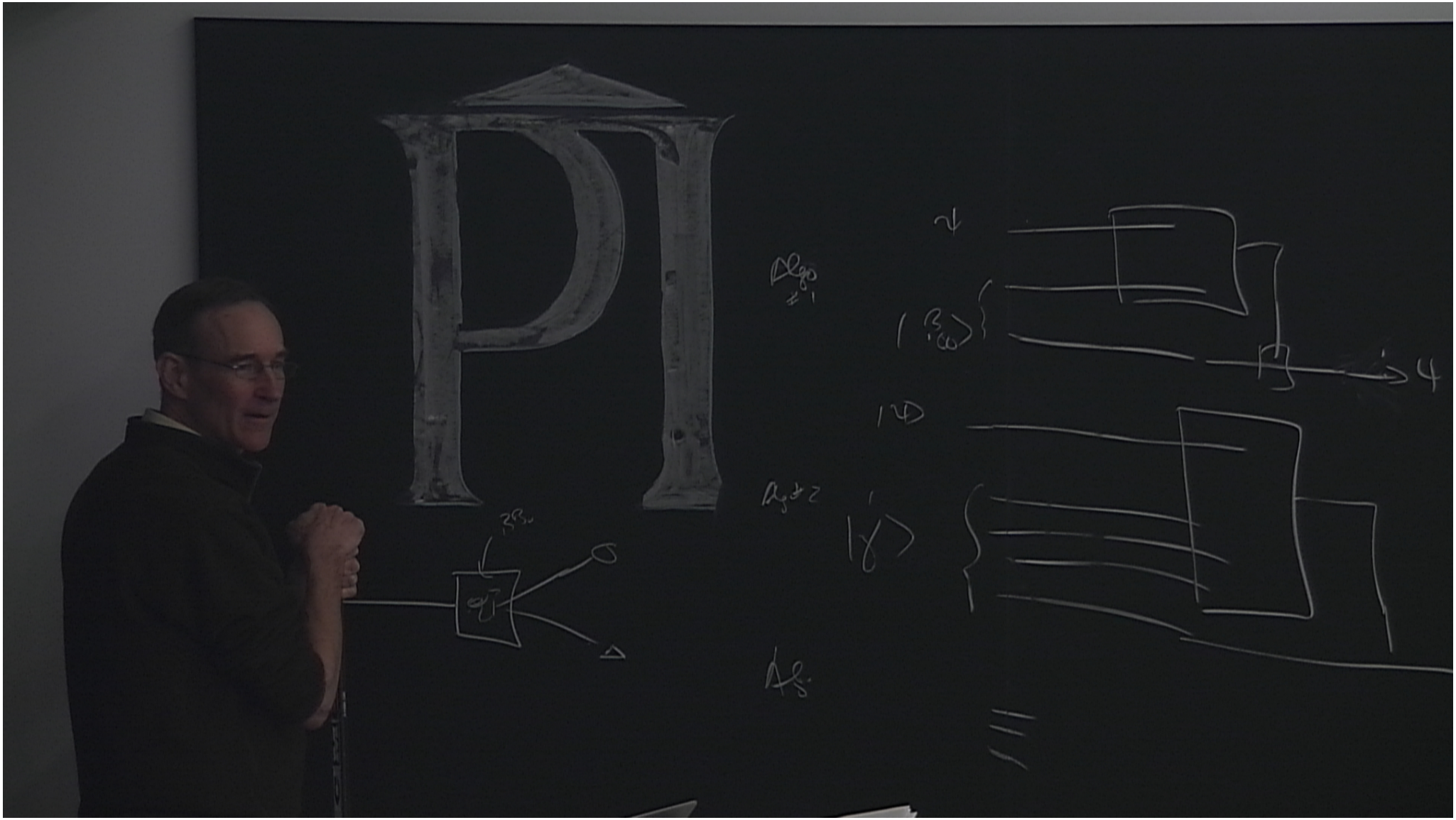
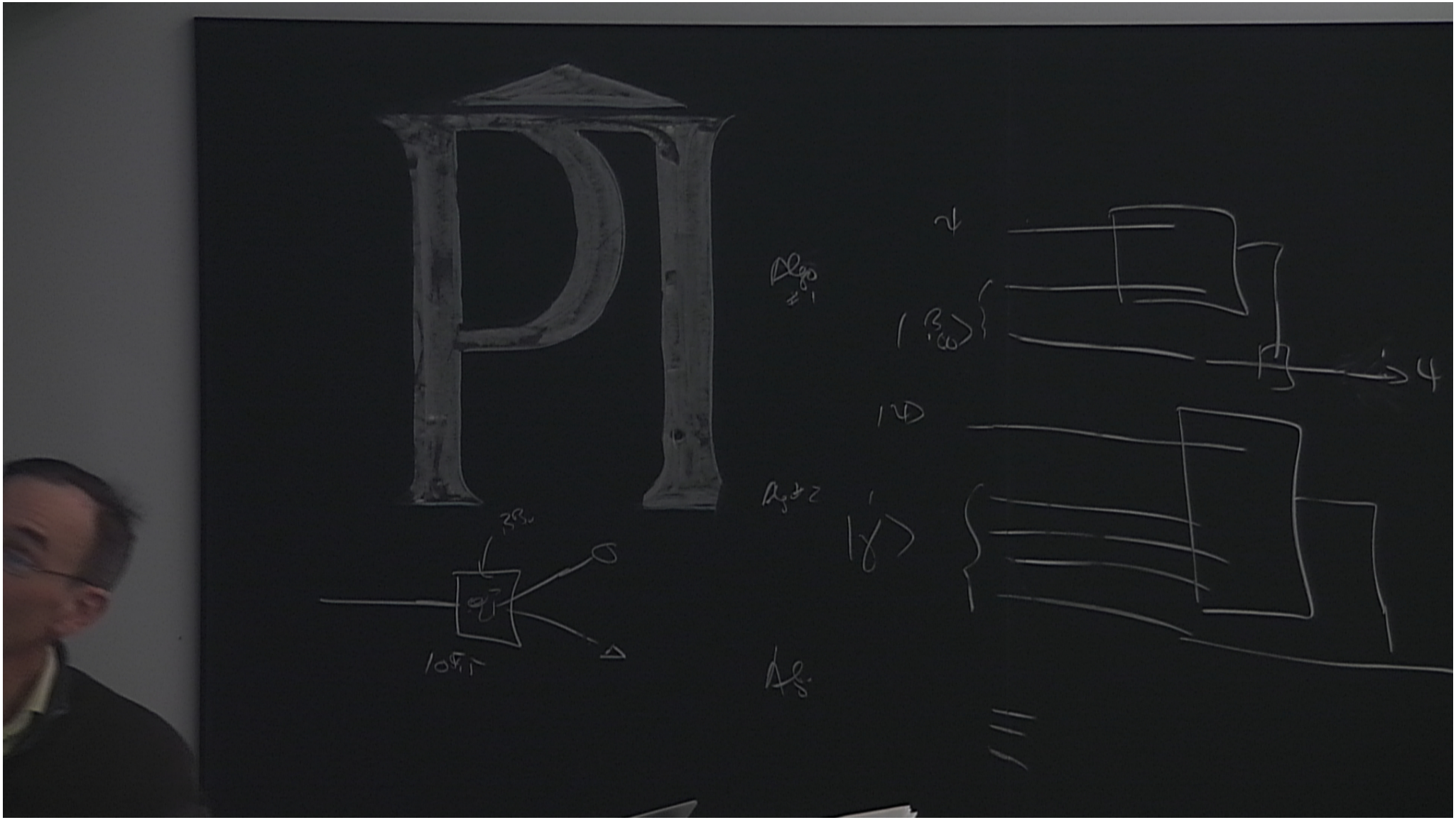


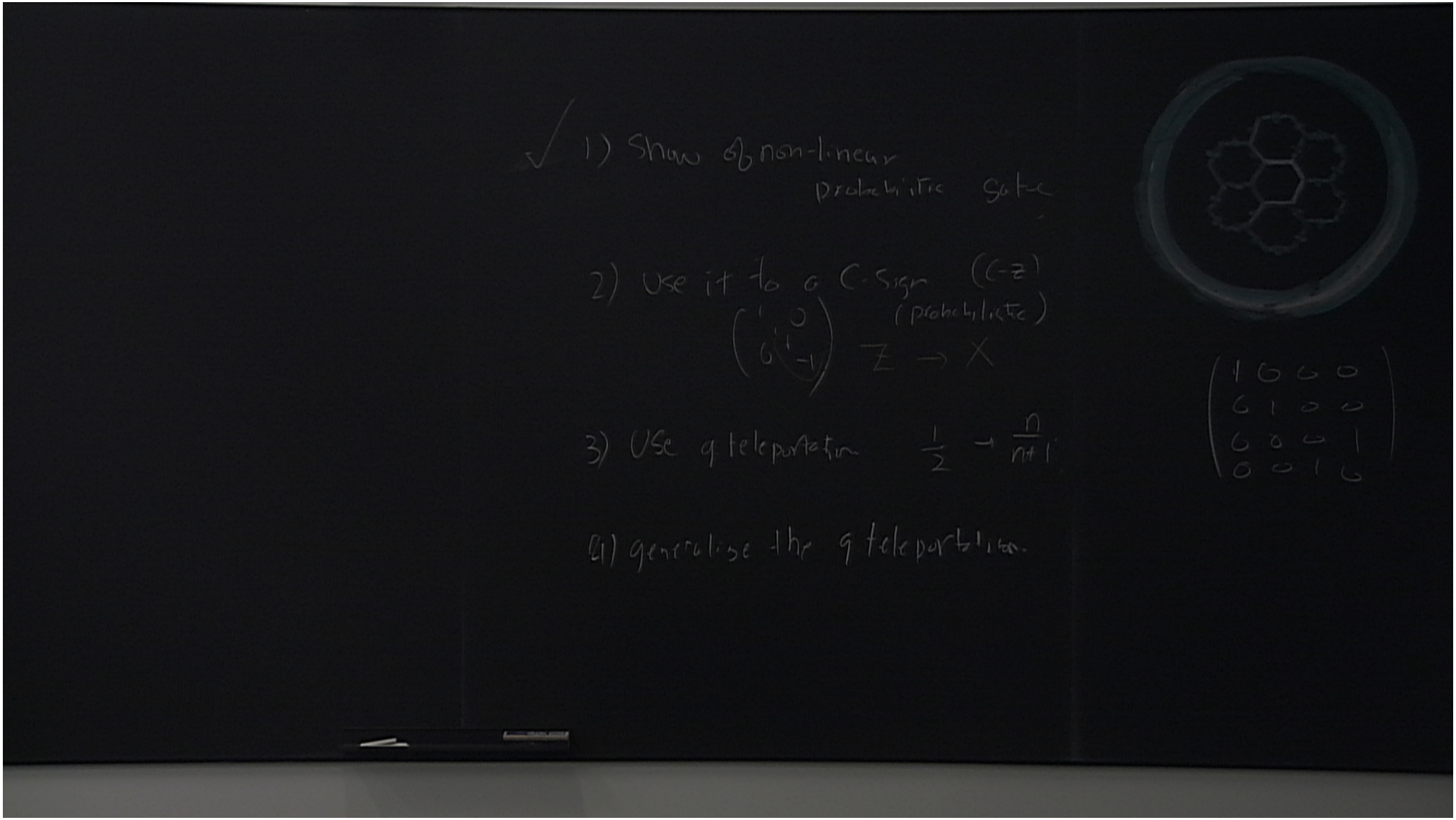
Fig. 27. – Teleportation with $|t_2\rangle$. The dashed-dotted box produces $|t_2\rangle$. The probability of success of this teleportation is 2/3.






PI


PI


✓ 1) Show of non-linear probabilistic Gate
2) Use it to a C-Sign (C-Z)
(probabilistic)
Z → X
3) Use q teleportation
4) generalise the q teleportation.