

Title: PSI 2015/2016 Quantum Information - Lecture 2

Date: Feb 23, 2016 11:30 AM

URL: <http://pirsa.org/16020079>

Abstract:

Tuesday. KLM chap 485

Wed. Nielsen & Chung. Section 7.4

Thursday. Myers/Latman
arxiv quant-ph 0512104

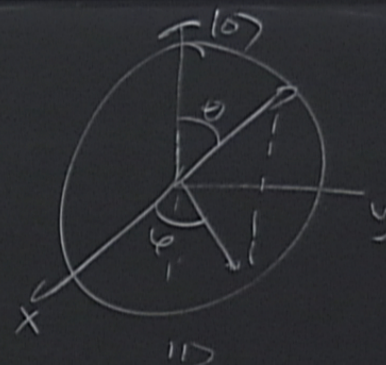
Friday. N&L. 7.7

Tuesday. KLM chap 4&5

{ Wed. Nielsen & Chung - Section 7.4
Thursday. Myers/Latamnia
arxiv quant-ph 0512104
Friday. N&L. 7.7

$$|4\rangle \in \mathcal{H}^2$$

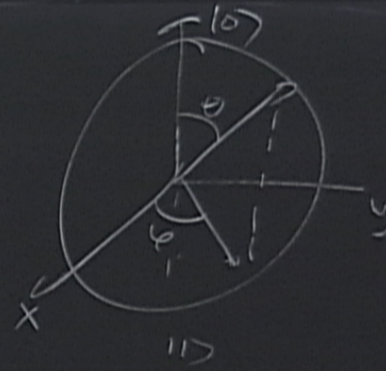
$$e^{-i\eta} \left(\cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\varphi} |1\rangle \right)$$



$$|4\rangle \in \mathcal{H}^2$$

$$e^{-i\eta} \left(\cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\varphi} |1\rangle \right)$$

$$\rho = \frac{1}{2} \mathbb{I} + \sin \theta \cos \varphi X \\ + \sin \theta \sin \varphi Y \\ + \cos \theta Z$$

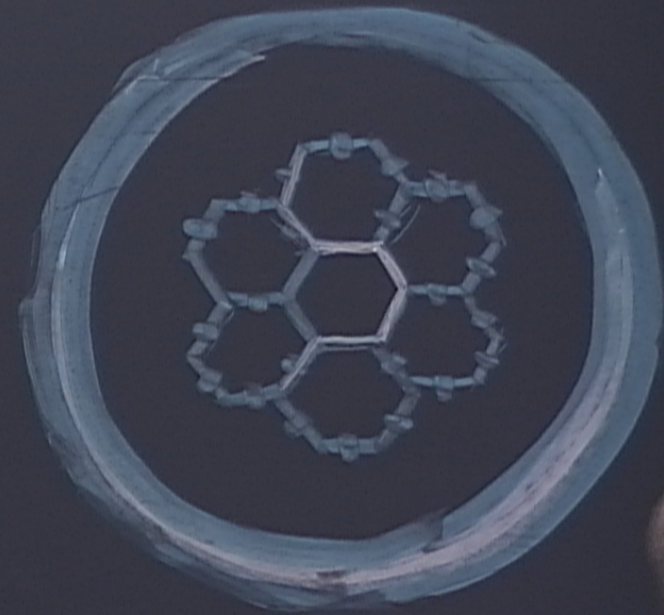


Tuesday. KLM chap 485

Wed. Nielsen & Chuang - Section 7.4

Thursday. Myers/Latham
arXiv quant-ph 0512104

Friday. N&L 7.7



Sorkin Mod Phys Lett A 9 319 (1994)

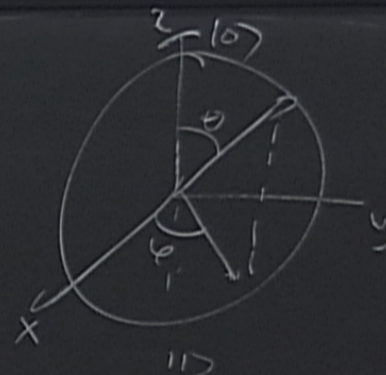
Sinha et al. Science 329, 418 (2010)

$$|1\rangle \in \mathbb{C}^2$$

$$e^{-i\frac{\theta}{2}} \left(\cos\frac{\theta}{2} |0\rangle + \sin\frac{\theta}{2} e^{i\phi} |1\rangle \right)$$

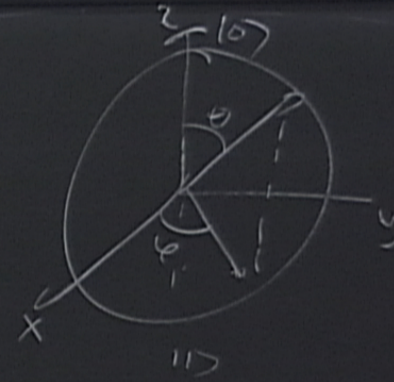
$$\begin{aligned} &+ \sin\theta \cos\phi X \\ &+ \sin\theta \sin\phi Y \\ &+ \cos\theta Z \end{aligned}$$

$\frac{1}{2}$ qubit gate U_2



$$|\psi\rangle \in \mathbb{C}^2$$

$$e^{-i\theta} \left(\cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\phi} |1\rangle \right)$$



$$\rho = \frac{1}{2} \mathbb{I} + \sin \theta \cos \phi X + \sin \theta \sin \phi Y + \cos \theta Z$$

$\frac{1}{2} \text{ Pauli gate } U_z$

Measurement

$$|\psi\rangle = \sum_x a_x |x\rangle$$

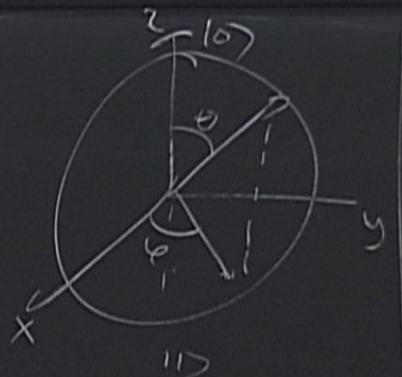
measure x

get $|x\rangle$ with prob. $|a_x|^2$

$$\left(\cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\varphi} |1\rangle \right)$$

$\theta \cos \varphi$ X
 $\sin \varphi$ Y
 $\cos \frac{\theta}{2}$ Z

↓ qubit gate U_2



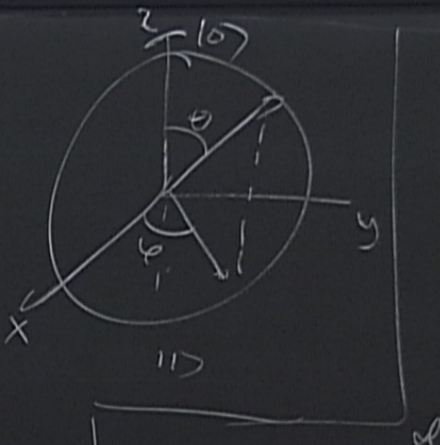
$$\psi = \alpha |0\rangle + \beta |1\rangle$$

$\sum_x a_x |x\rangle$ measure X
 get $|x\rangle$ with prob. $|a_x|^2$

$$\left(\cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\phi} |1\rangle \right)$$

$\theta \cos \phi$ X
 $\sin \phi$ Y
 $\cos \frac{\theta}{2}$ Z

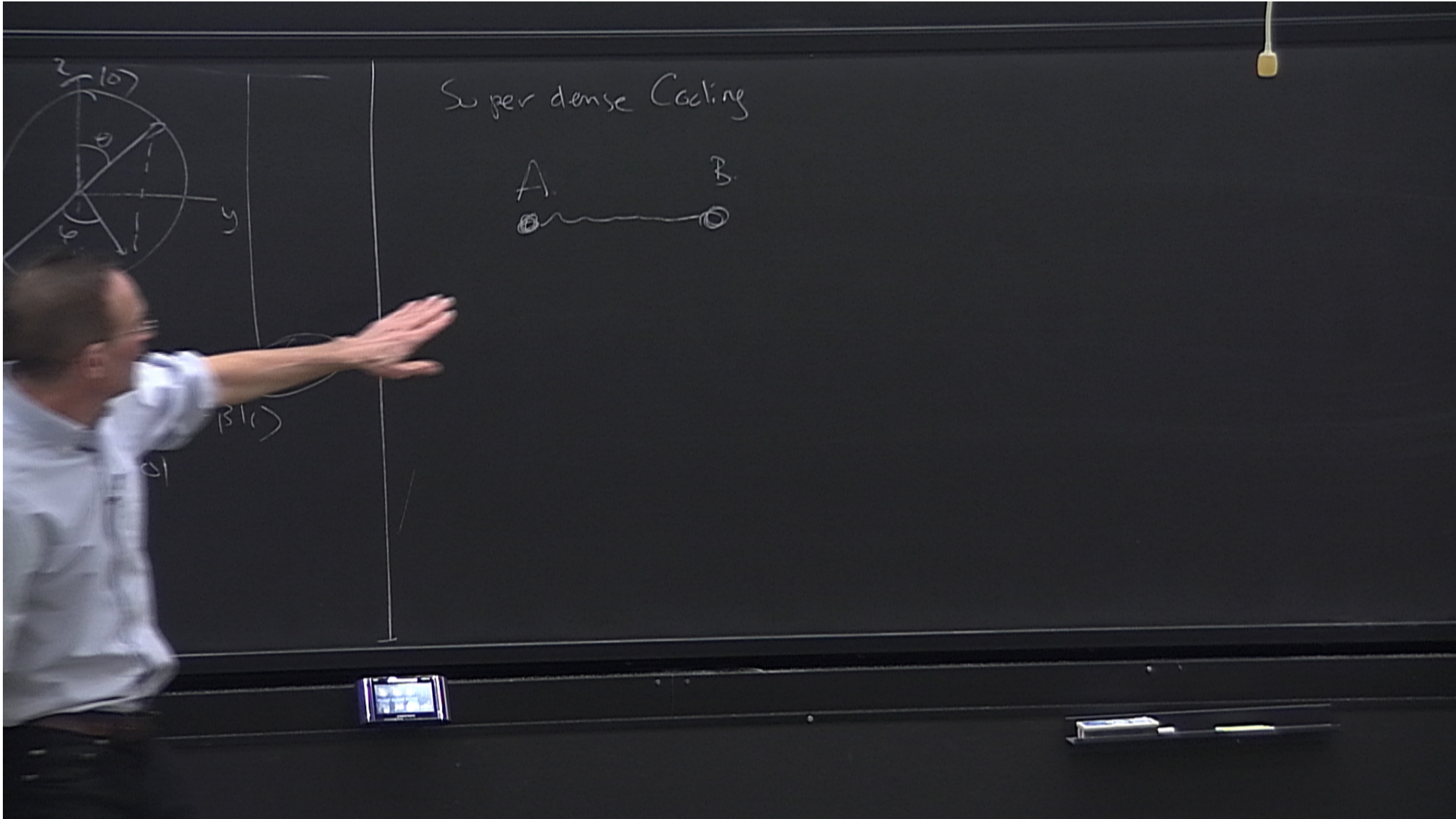
1 qubit gate U_2

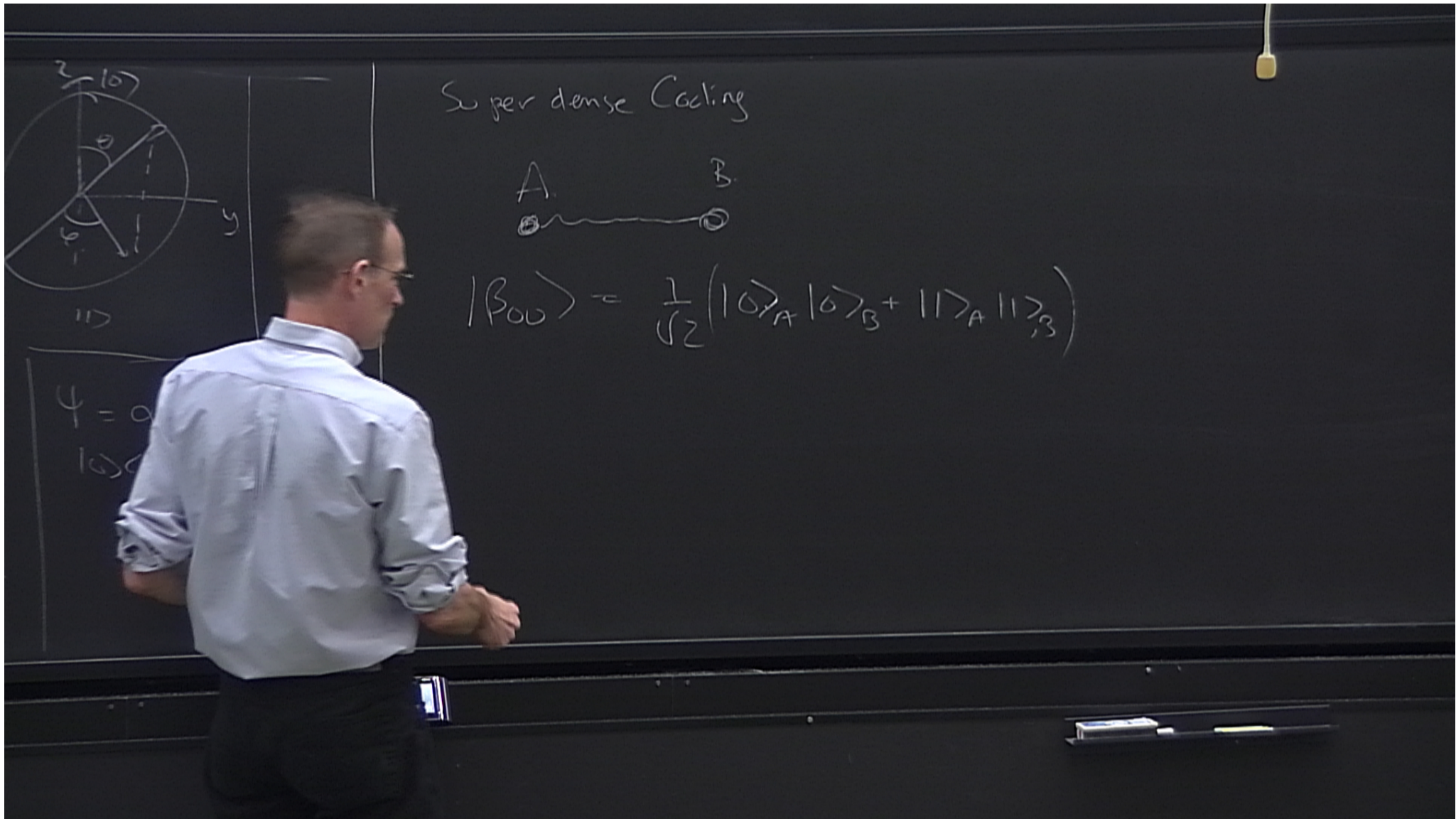


$$\psi = \alpha |0\rangle + \beta |1\rangle$$

$$|0\rangle\langle 0|$$

$\sum_x a_x |x\rangle$ measure X
 get $|x\rangle$ with prob. $|a_x|^2$





Super dense Cooling

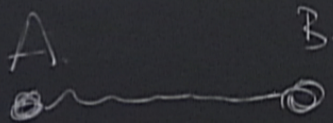
A. B.
○————○

$$|B_{00}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$$

$\psi = 0$

$|0\rangle$

Super dense Coding



$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$$

$$I \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = |\beta_{00}\rangle$$

$$X \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle) = |\beta_{01}\rangle$$

$$Z \otimes I$$

Super dense Coding



$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$$

$$I \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = |\beta_{00}\rangle$$

$$X \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle) = |\beta_{01}\rangle$$

$$Z \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) = |\beta_{10}\rangle$$

$$Z \otimes X |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (-|10\rangle + |01\rangle) = |\beta_{11}\rangle$$

Super dense Coding



$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$$

$$I \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = |\beta_{00}\rangle$$

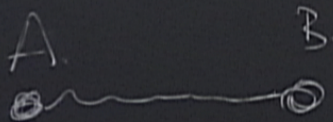
$$X \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle) = |\beta_{01}\rangle$$

$$Z \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) = |\beta_{10}\rangle$$

$$Z \otimes X |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (-|10\rangle + |01\rangle) = |\beta_{11}\rangle$$

orthogonal

Super dense Coding



$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$$

$$I \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = |\beta_{00}\rangle$$

$$X \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle) = |\beta_{01}\rangle$$

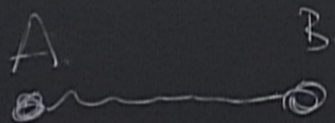
$$Z \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) = |\beta_{10}\rangle$$

$$X \otimes Z |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (-|10\rangle + |01\rangle) = |\beta_{11}\rangle$$

orthogonal

full spin 0

Super dense Coding



$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$$

$$I \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = |\beta_{00}\rangle$$

$$X \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle) = |\beta_{01}\rangle$$

$$Z \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) = |\beta_{10}\rangle$$

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orthogonal

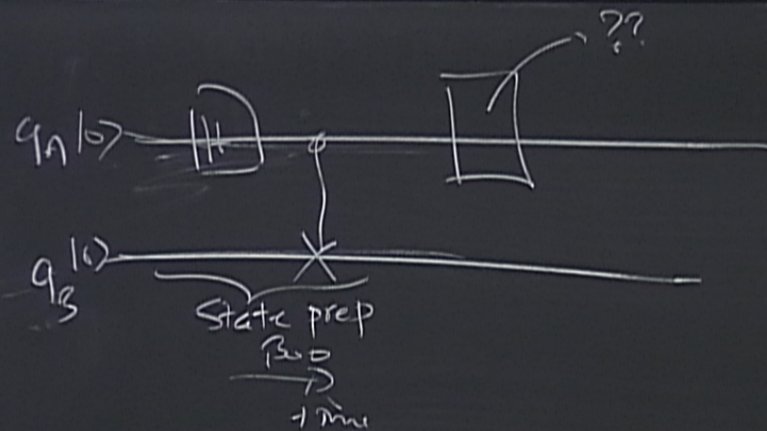
Bob measures

$$|\beta_{00}\rangle \langle \beta_{00}|$$

$$|\beta_{01}\rangle \langle \beta_{01}|$$

$$|\beta_{10}\rangle \langle \beta_{10}|$$

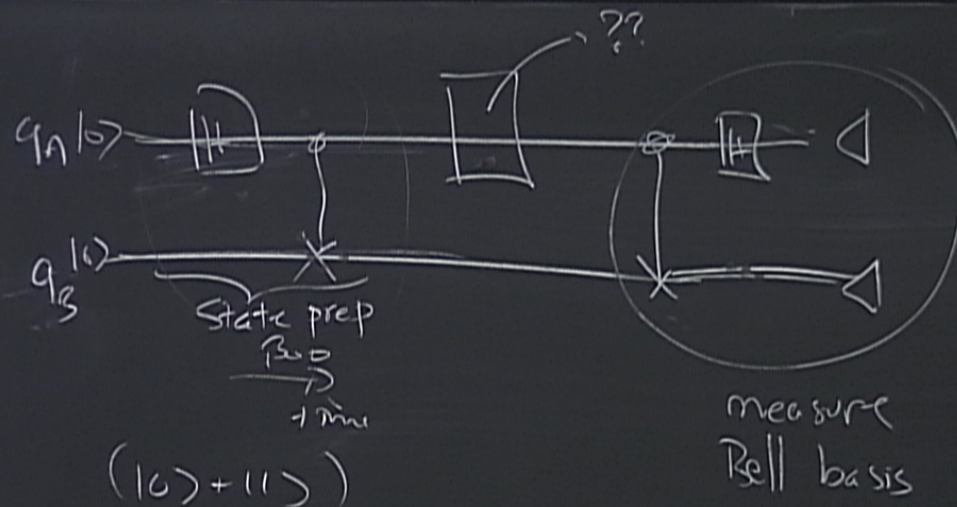
$$|\beta_{11}\rangle \langle \beta_{11}|$$



$$(|0\rangle + |1\rangle)$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

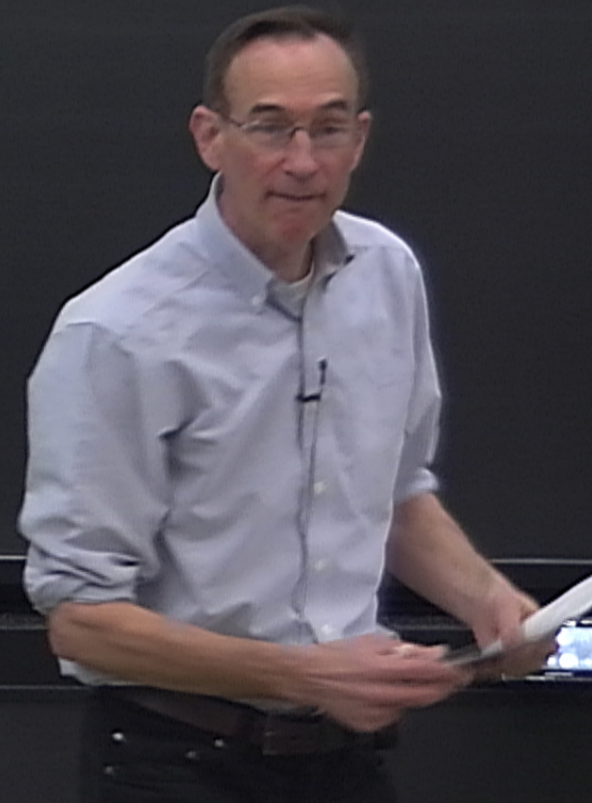




$$(|0\rangle + |1\rangle)$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

measure
Bell basis





$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$$

$$I \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = |\beta_{00}\rangle$$

$$X \otimes I |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle) = |\beta_{01}\rangle$$

$$I \otimes X |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle) = |\beta_{10}\rangle$$

$$X \otimes X |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (-|10\rangle + |01\rangle) = |\beta_{11}\rangle$$

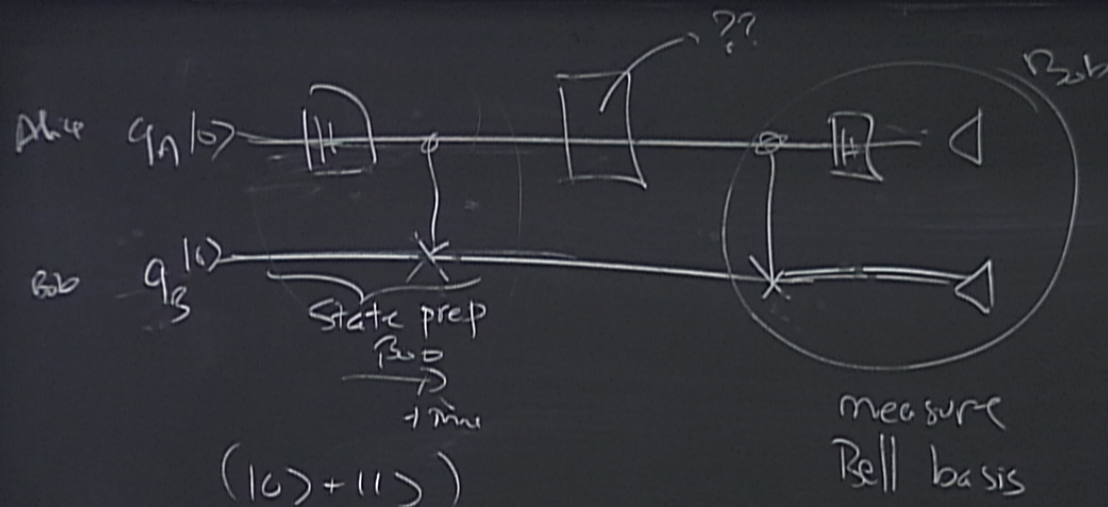
orthogonal

$$|\beta_{00}\rangle \langle \beta_{00}|$$

$$|\beta_{01}\rangle \langle \beta_{01}|$$

$$|\beta_{10}\rangle \langle \beta_{10}|$$

$$|\beta_{11}\rangle \langle \beta_{11}|$$

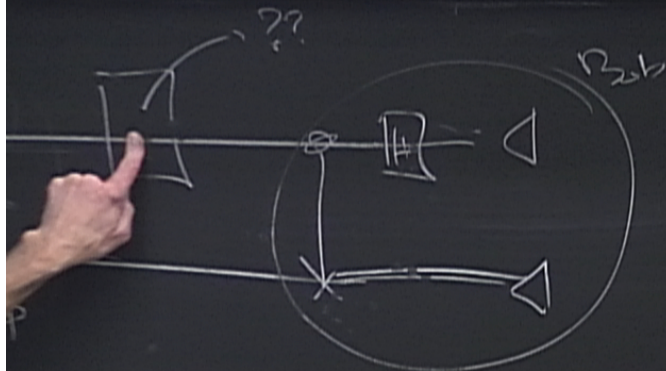


$$(|0\rangle + |1\rangle)$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$U X U^\dagger$$

$$U U^\dagger = I$$



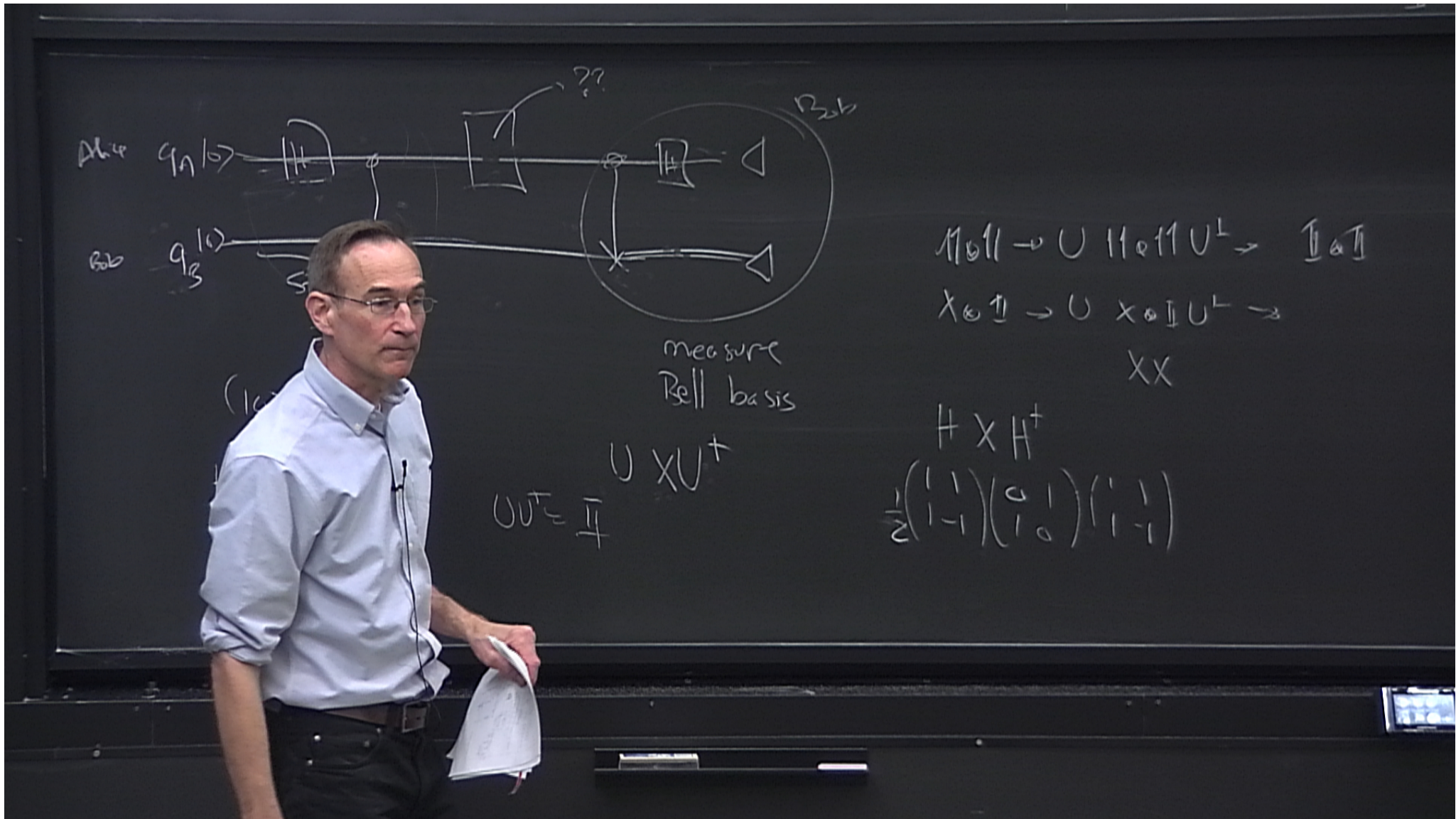
measure
Bell basis

$$|10\rangle \rightarrow U |10\rangle U^\dagger \rightarrow |10\rangle$$

$$X \otimes I \rightarrow U X \otimes I U^\dagger \rightarrow$$

$$U X U^\dagger$$

$$U U^\dagger = I$$



State prob

measure Bell basis

measure

Bell basis

XU^+

$11011 \rightarrow U 11011 U^L \rightarrow 11011$

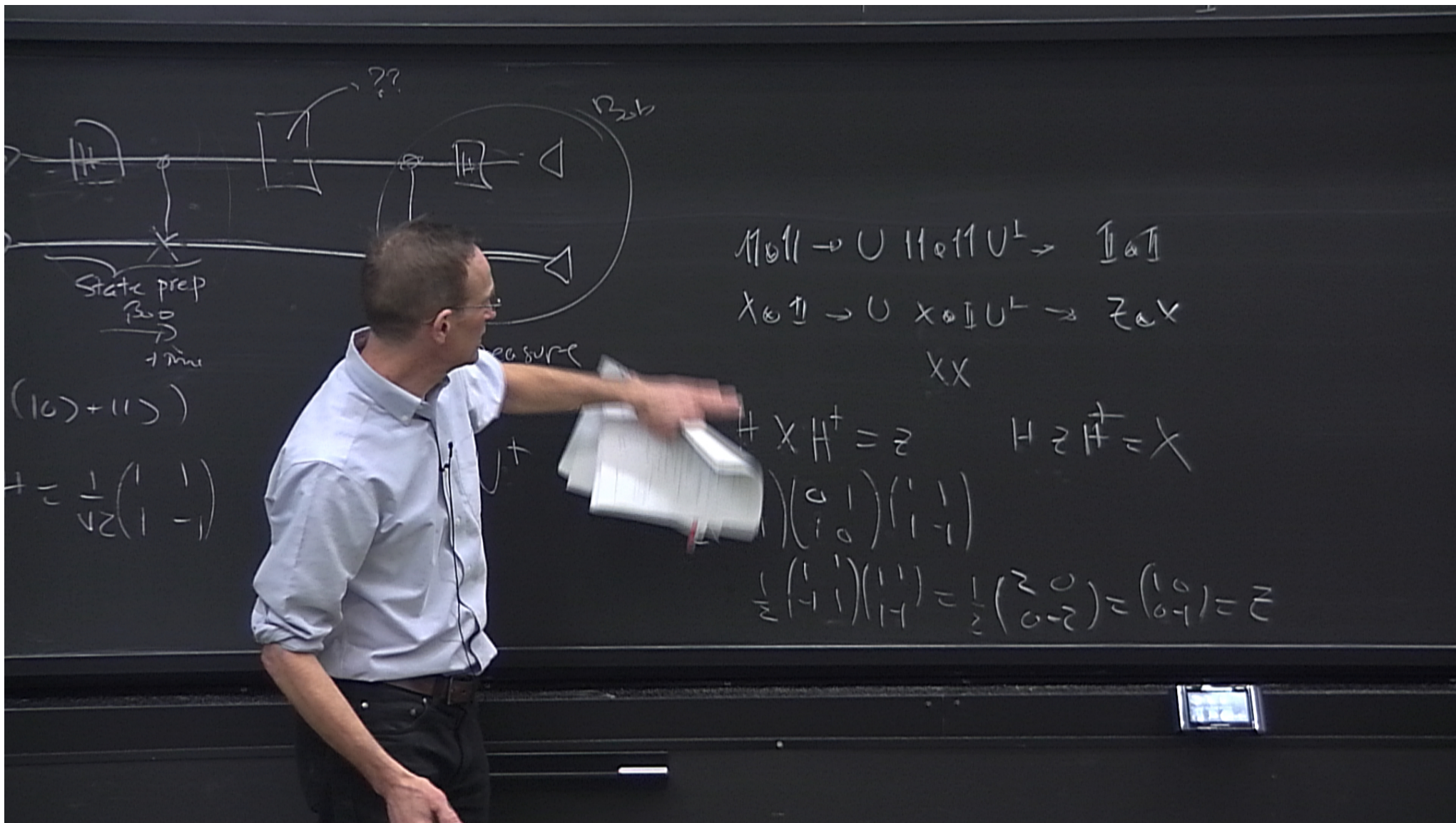
$X011 \rightarrow U X011 U^L \rightarrow$

XX

$H X H^+ = Z \quad H Z H^+ = X$

$\frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$\frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Z$



$$11011 \rightarrow U 11011 U^T \rightarrow \mathbb{I} \otimes \mathbb{I}$$

$$X \otimes \mathbb{I} \rightarrow U X \otimes \mathbb{I} U^T \rightarrow Z \otimes X$$

$$Z \otimes \mathbb{I} \rightarrow \quad \rightarrow X \otimes \mathbb{I}$$

$$\widetilde{Z} X \otimes \mathbb{I} \rightarrow$$

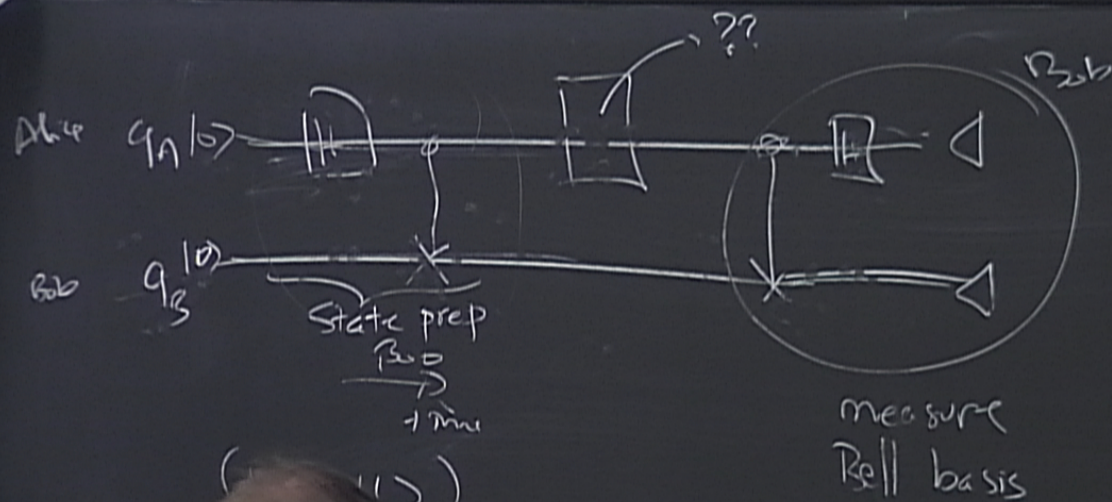
$$X \otimes X$$

$\widetilde{X}$

$$\begin{array}{c} U \quad Z \quad X \quad U^T \\ \quad \quad \quad \wedge \\ \quad \quad \quad U^T U \\ U Z U^T \quad U X U^T \end{array}$$

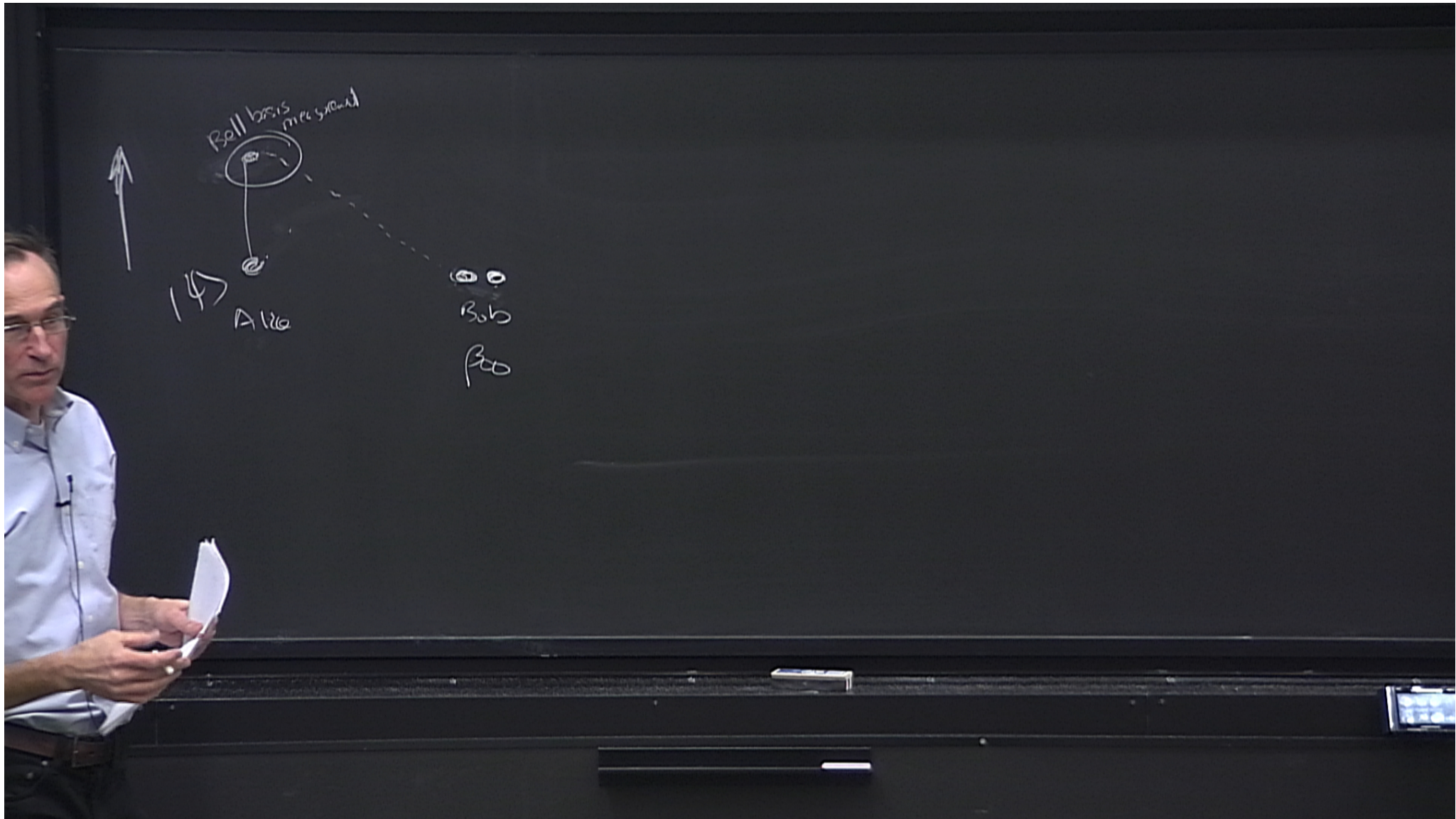
$$\begin{array}{l}
 11011 \rightarrow U 11011 U^\dagger \rightarrow \mathbb{I} \otimes \mathbb{I} \rightarrow |00\rangle \\
 X \otimes \mathbb{I} \rightarrow U X \otimes \mathbb{I} U^\dagger \rightarrow Z \otimes X \rightarrow |01\rangle \\
 Z \otimes \mathbb{I} \rightarrow \quad \quad \quad \rightarrow X \otimes \mathbb{I} \rightarrow |10\rangle \\
 \overline{Z} X \otimes \mathbb{I} \rightarrow \quad \quad \quad \rightarrow X \otimes X \rightarrow |11\rangle \\
 \overline{X}
 \end{array}$$

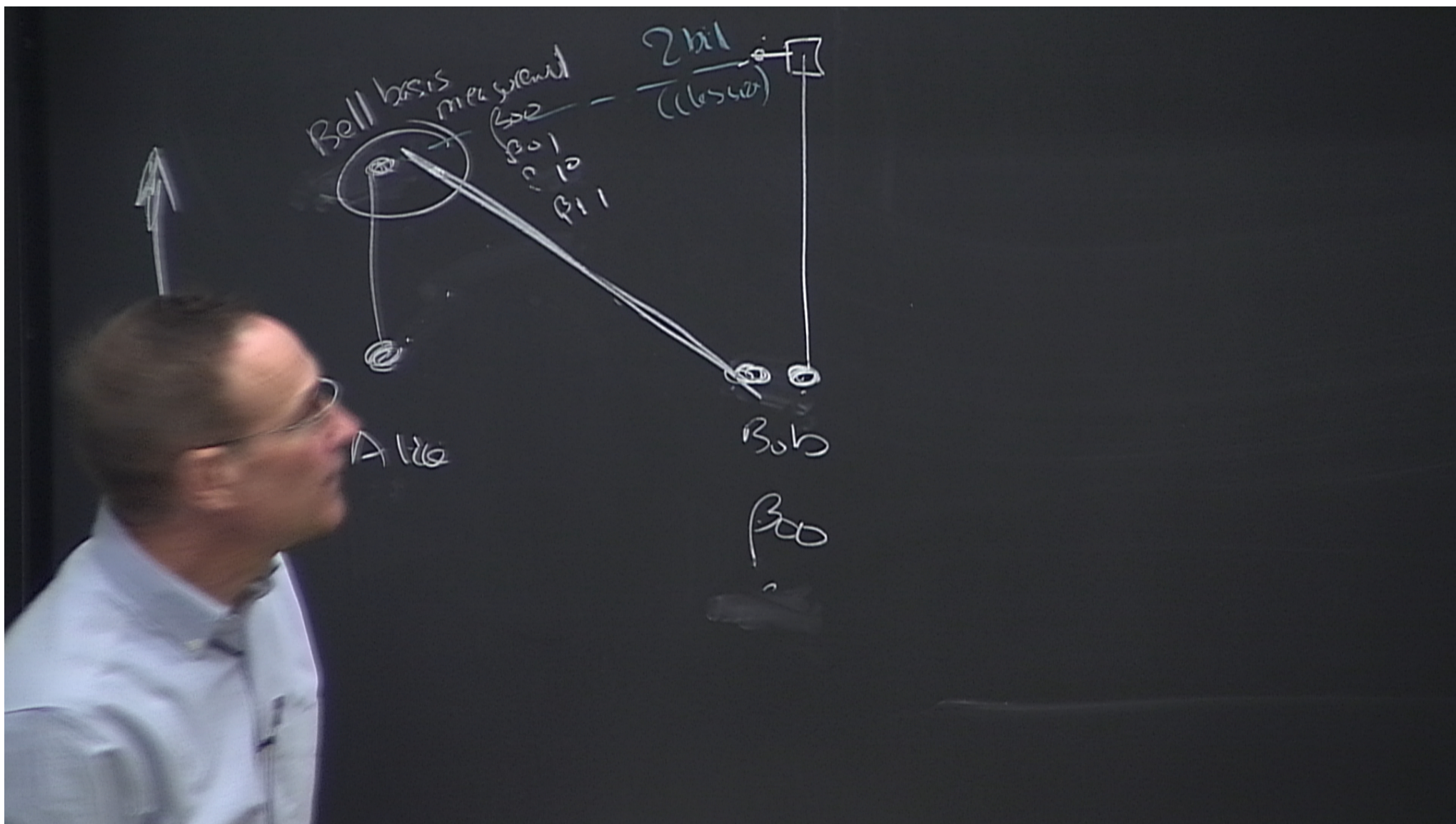
$$\begin{array}{c}
 U \quad Z \quad X \quad U^\dagger \\
 \quad \quad \quad \wedge \\
 \quad \quad \quad U^\dagger U \\
 U Z U^\dagger \quad U X U^\dagger
 \end{array}$$

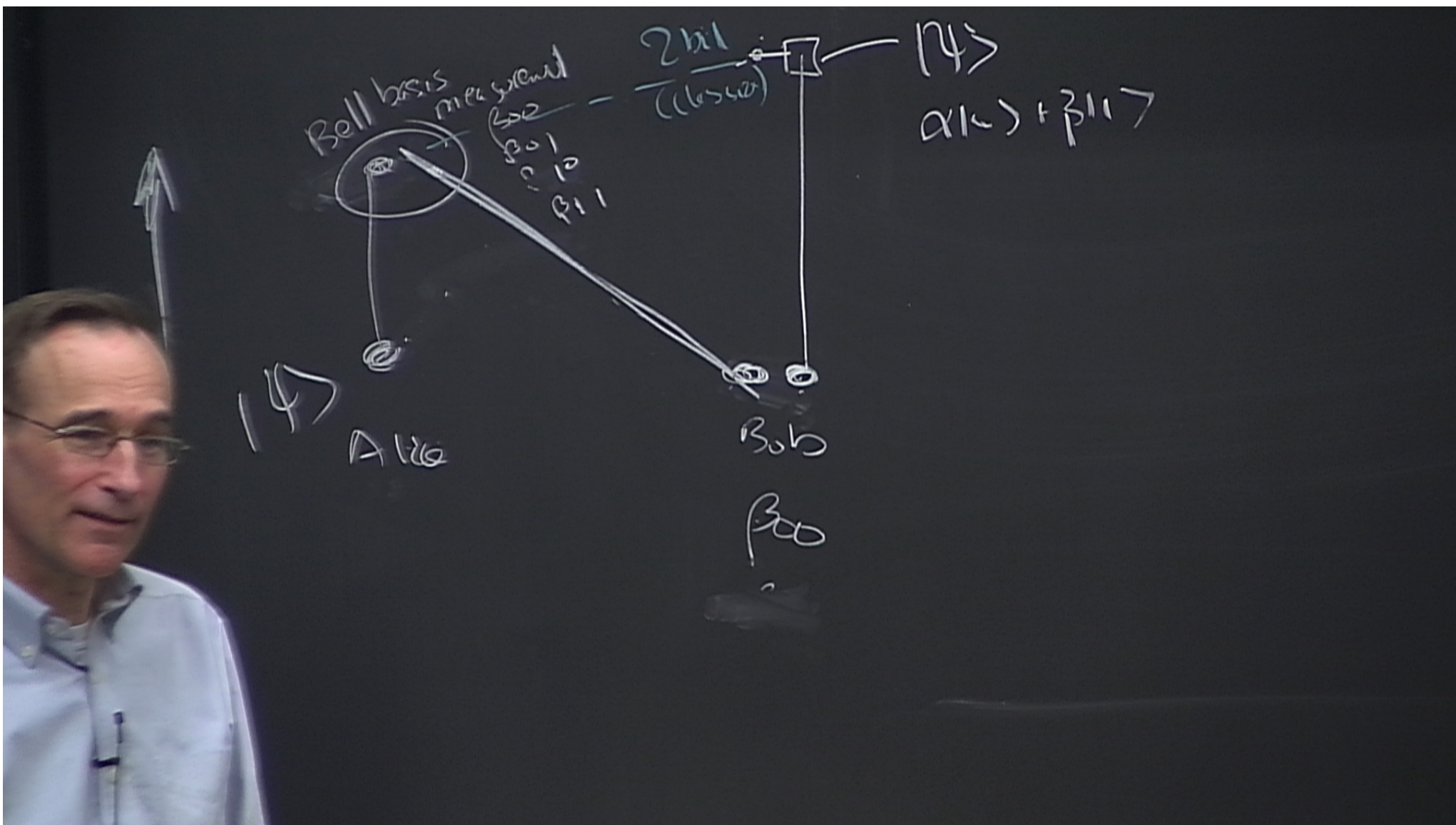


$$\begin{aligned}
 |00\rangle &\rightarrow U |00\rangle U^\dagger \rightarrow |00\rangle \rightarrow \\
 |01\rangle &\rightarrow U |01\rangle U^\dagger \rightarrow |01\rangle \rightarrow \\
 |10\rangle &\rightarrow U |10\rangle U^\dagger \rightarrow |10\rangle \rightarrow \\
 |11\rangle &\rightarrow U |11\rangle U^\dagger \rightarrow |11\rangle \rightarrow
 \end{aligned}$$

$$U X U^\dagger = X$$







$$\begin{aligned}
 |4\rangle |\beta_{00}\rangle &= \frac{1}{\sqrt{2}} (\alpha |10\rangle + \beta |11\rangle) (|00\rangle + |11\rangle) \\
 &= \frac{1}{\sqrt{2}} (\alpha |1000\rangle + \alpha |1011\rangle + \beta |1100\rangle + \beta |1111\rangle) \\
 &= \frac{1}{\sqrt{2}} (\alpha (|\beta_{00}\rangle + |\beta_{01}\rangle) |10\rangle + \alpha (|\beta_{01}\rangle + |\beta_{11}\rangle) |11\rangle + \beta (|\beta_{01}\rangle - |\beta_{11}\rangle) |10\rangle + \beta (|\beta_{00}\rangle - |\beta_{10}\rangle) |11\rangle)
 \end{aligned}$$

$$|4\rangle |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (\alpha |10\rangle + \beta |11\rangle) (|00\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}} (\alpha |1000\rangle + \alpha |0111\rangle + \beta |1100\rangle + \beta |1111\rangle)$$

$$\frac{1}{\sqrt{2}} (\alpha (|\beta_{00}\rangle + |\beta_{01}\rangle) |10\rangle + \alpha (|\beta_{01}\rangle + |\beta_{11}\rangle) |11\rangle + \beta (|\beta_{01}\rangle - |\beta_{11}\rangle) |00\rangle + \beta (|\beta_{00}\rangle - |\beta_{10}\rangle) |01\rangle)$$

$$\frac{1}{\sqrt{2}} (|\beta_{00}\rangle (\alpha |10\rangle + \beta |11\rangle) + |\beta_{01}\rangle (\alpha |11\rangle + \beta |10\rangle) + |\beta_{10}\rangle (\alpha |10\rangle - \beta |11\rangle) + |\beta_{11}\rangle (\alpha |11\rangle - \beta |10\rangle))$$

$$|4\rangle |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (\alpha |10\rangle + \beta |11\rangle) (|00\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}} (\alpha |1000\rangle + \alpha |1011\rangle + \beta |1100\rangle + \beta |1111\rangle)$$

$$= \frac{1}{\sqrt{2}} (\alpha (|\beta_{00}\rangle + |\beta_{01}\rangle) |10\rangle + \alpha (|\beta_{01}\rangle + |\beta_{11}\rangle) |11\rangle + \beta (|\beta_{01}\rangle - |\beta_{11}\rangle) |10\rangle + \beta (|\beta_{00}\rangle - |\beta_{10}\rangle) |11\rangle)$$

$$= \frac{1}{\sqrt{2}} (\underbrace{|\beta_{00}\rangle (\alpha |10\rangle + \beta |11\rangle)}_{\mathbb{I} |4\rangle} + \underbrace{|\beta_{01}\rangle (\alpha |11\rangle + \beta |10\rangle)}_{X |4\rangle} + \underbrace{|\beta_{10}\rangle (\alpha |10\rangle - \beta |11\rangle)}_{Z |4\rangle} + \underbrace{|\beta_{11}\rangle (\alpha |11\rangle - \beta |10\rangle)}_{XZ |4\rangle})$$

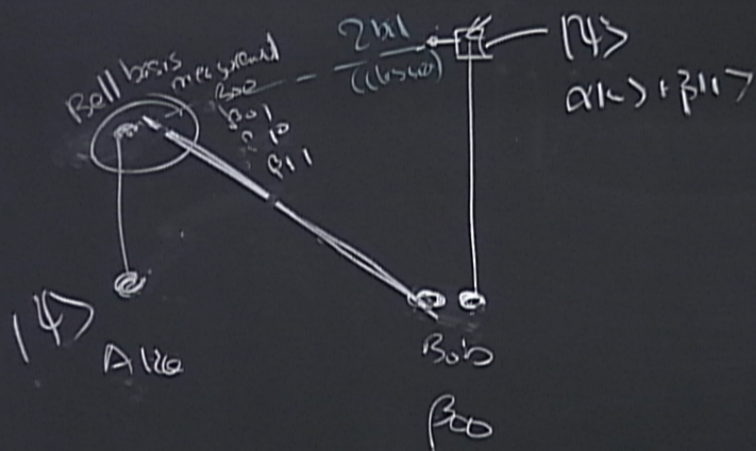
$$|4\rangle |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (\alpha |10\rangle + \beta |11\rangle) (|00\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}} (\alpha |1000\rangle + \alpha |1011\rangle + \beta |1100\rangle + \beta |1111\rangle)$$

$$= \frac{1}{\sqrt{2}} (\alpha (|\beta_{00}\rangle + |\beta_{01}\rangle) |10\rangle + \alpha (|\beta_{01}\rangle + |\beta_{11}\rangle) |11\rangle + \beta (|\beta_{01}\rangle - |\beta_{11}\rangle) |10\rangle + \beta (|\beta_{00}\rangle - |\beta_{10}\rangle) |11\rangle)$$

$$= \frac{1}{\sqrt{2}} (\underbrace{\beta_{00}}_{\mathbb{I} |4\rangle} (\alpha |10\rangle + \beta |11\rangle) + \underbrace{\beta_{01}}_{X |4\rangle} (\alpha |11\rangle + \beta |10\rangle) + \underbrace{\beta_{10}}_{Z |4\rangle} (\alpha |10\rangle - \beta |11\rangle) + \underbrace{\beta_{11}}_{XZ |4\rangle} (\alpha |11\rangle - \beta |10\rangle))$$

$\otimes X$



$$\begin{aligned}
 |\psi\rangle |\beta_{00}\rangle &= \frac{1}{\sqrt{2}} (\alpha|0\rangle + \beta|1\rangle) \left(\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \right) \\
 &= \frac{1}{\sqrt{2}} (\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle) \\
 &= \frac{1}{\sqrt{2}} (\alpha(|\beta_{00}\rangle + |\beta_{01}\rangle) + \beta(|\beta_{10}\rangle + |\beta_{11}\rangle)) \\
 &= \frac{1}{\sqrt{2}} |\beta_{00}\rangle (\alpha|0\rangle + \beta|1\rangle) + \frac{1}{\sqrt{2}} |\beta_{10}\rangle (\alpha|0\rangle + \beta|1\rangle) \\
 &\quad + \frac{1}{\sqrt{2}} |\beta_{01}\rangle (\alpha|0\rangle + \beta|1\rangle) + \frac{1}{\sqrt{2}} |\beta_{11}\rangle (\alpha|0\rangle + \beta|1\rangle)
 \end{aligned}$$

$\mathbb{I} |\psi\rangle$

$$|\psi\rangle |\beta_{00}\rangle = \frac{1}{\sqrt{2}} (\alpha|0\rangle + \beta|1\rangle) (|00\rangle + |11\rangle)$$

$$= \frac{1}{\sqrt{2}} (\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle)$$

$$= \frac{1}{\sqrt{2}} (\alpha(|\beta_{00}\rangle + |\beta_{01}\rangle) |0\rangle + \alpha(|\beta_{01}\rangle + |\beta_{11}\rangle) |1\rangle + \beta(|\beta_{00}\rangle - |\beta_{11}\rangle) |0\rangle + \beta(|\beta_{01}\rangle - |\beta_{11}\rangle) |1\rangle)$$

$$= \frac{1}{\sqrt{2}} (\underbrace{|\beta_{00}\rangle (\alpha|0\rangle + \beta|1\rangle)}_{\mathbb{I} |\psi\rangle} + \underbrace{|\beta_{01}\rangle (\alpha|1\rangle + \beta|0\rangle)}_{X |\psi\rangle} + \underbrace{|\beta_{10}\rangle (\alpha|0\rangle - \beta|1\rangle)}_{Z |\psi\rangle} + \underbrace{|\beta_{11}\rangle (\alpha|1\rangle - \beta|0\rangle)}_{XZ |\psi\rangle})$$

$$\text{tr}_{12} (|\psi\rangle_{123} \langle\psi|_{123})$$

$$= \frac{1}{2} \mathbb{I}$$

No cloning theorem.

$$|\psi\rangle |\phi\rangle \xrightarrow{U} |\psi\rangle |\psi\rangle$$

No cloning theorem.

$$|\psi\rangle|0\rangle \xrightarrow{U} |\psi\rangle|\psi\rangle$$

$$|\psi\rangle|0\rangle$$

$$|\psi\rangle|\psi\rangle$$

$$\langle 0|\langle\psi||\psi\rangle|0\rangle$$

No cloning theorem.

$$|\psi\rangle|\phi\rangle \xrightarrow{U} |\psi\rangle|\psi\rangle$$

$$|\psi\rangle|0\rangle$$

$$|\psi\rangle|\psi\rangle$$

$$\langle 0 | \langle \psi | \underbrace{U^\dagger U}_{=1} | \psi \rangle | 0 \rangle = \langle \psi | \langle \psi | | \psi \rangle | \psi \rangle$$

No cloning theorem.

$$|\psi\rangle|\phi\rangle \xrightarrow{U} |\psi\rangle|\psi\rangle$$

$$\langle\psi|\psi\rangle = c$$

$$|\psi\rangle|0\rangle$$

$$|\psi\rangle|\psi\rangle$$

$$\langle 0 | \underbrace{\langle\psi|}_{U^\dagger} |\psi\rangle |0\rangle = \langle\psi| \langle\psi| |\psi\rangle |\psi\rangle = c^2$$

No cloning theorem.

$$|\psi\rangle|b\rangle \xrightarrow{U} |\psi'\rangle|\psi\rangle$$

$$\langle\psi|\psi\rangle = c$$

$$|\psi\rangle|0\rangle$$

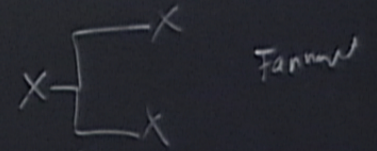
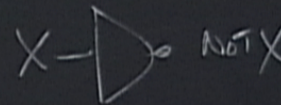
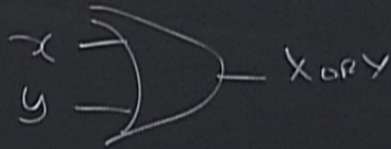
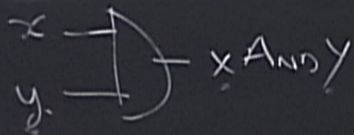
$$|\psi\rangle|\psi\rangle$$

$$c=0$$

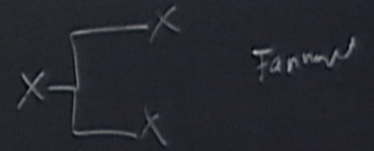
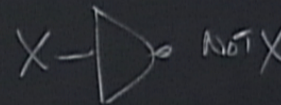
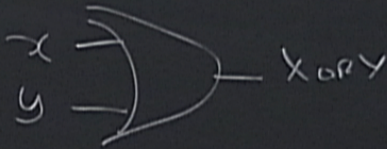
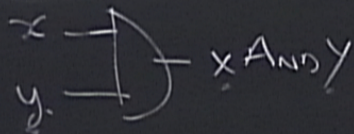
$$c=1$$

$$\langle 0 | \underbrace{\langle \psi |}_{U^\dagger} U | \psi \rangle | 0 \rangle = \langle \psi | \langle \psi | | \psi \rangle | \psi \rangle = c^2$$

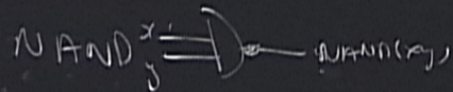
Circuits, Reversible, and universality



Circuits, Reversible, and Universality



$$f: \{0,1\}^n \rightarrow \{0,1\}^m$$



FANOUT

