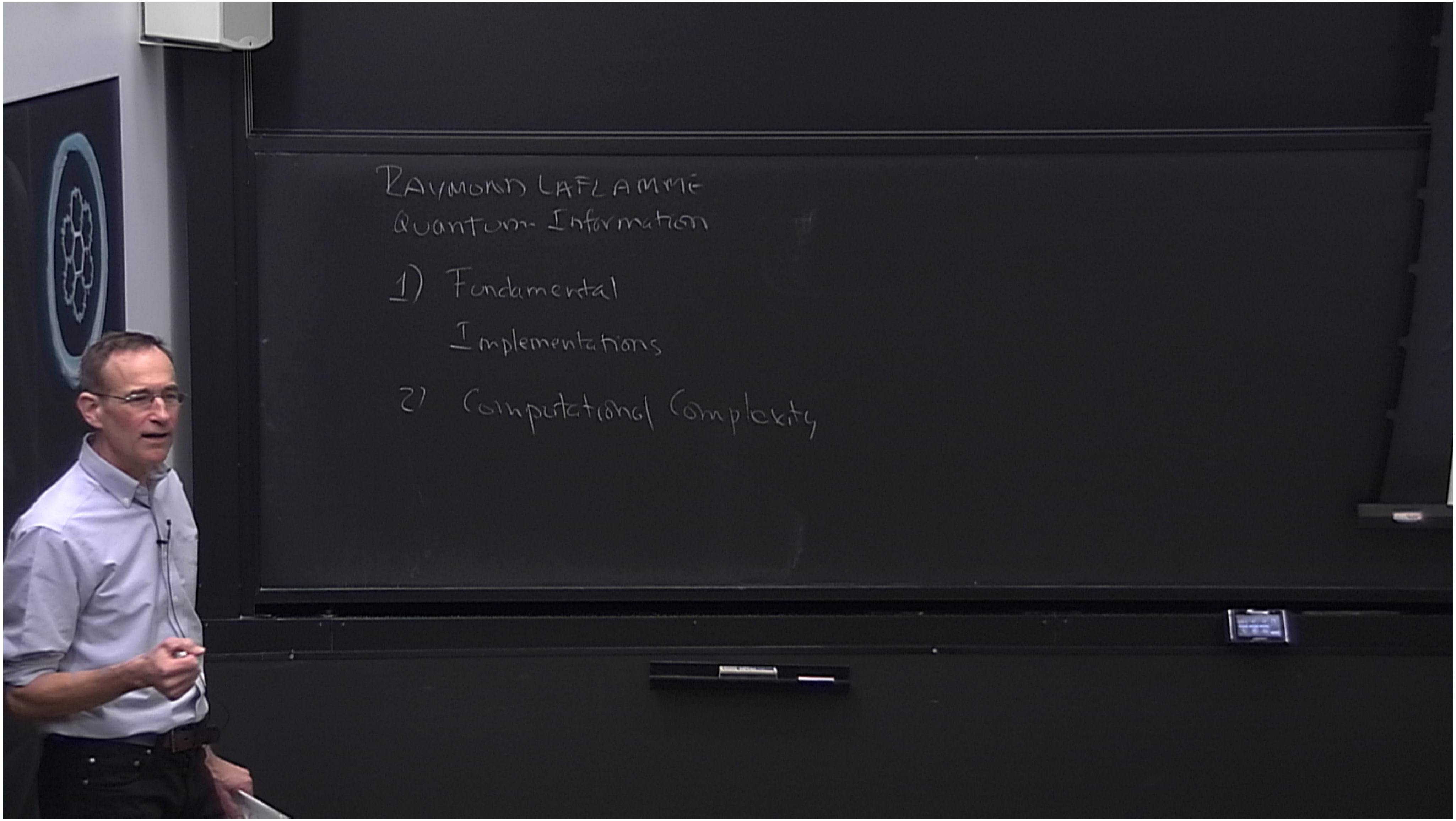
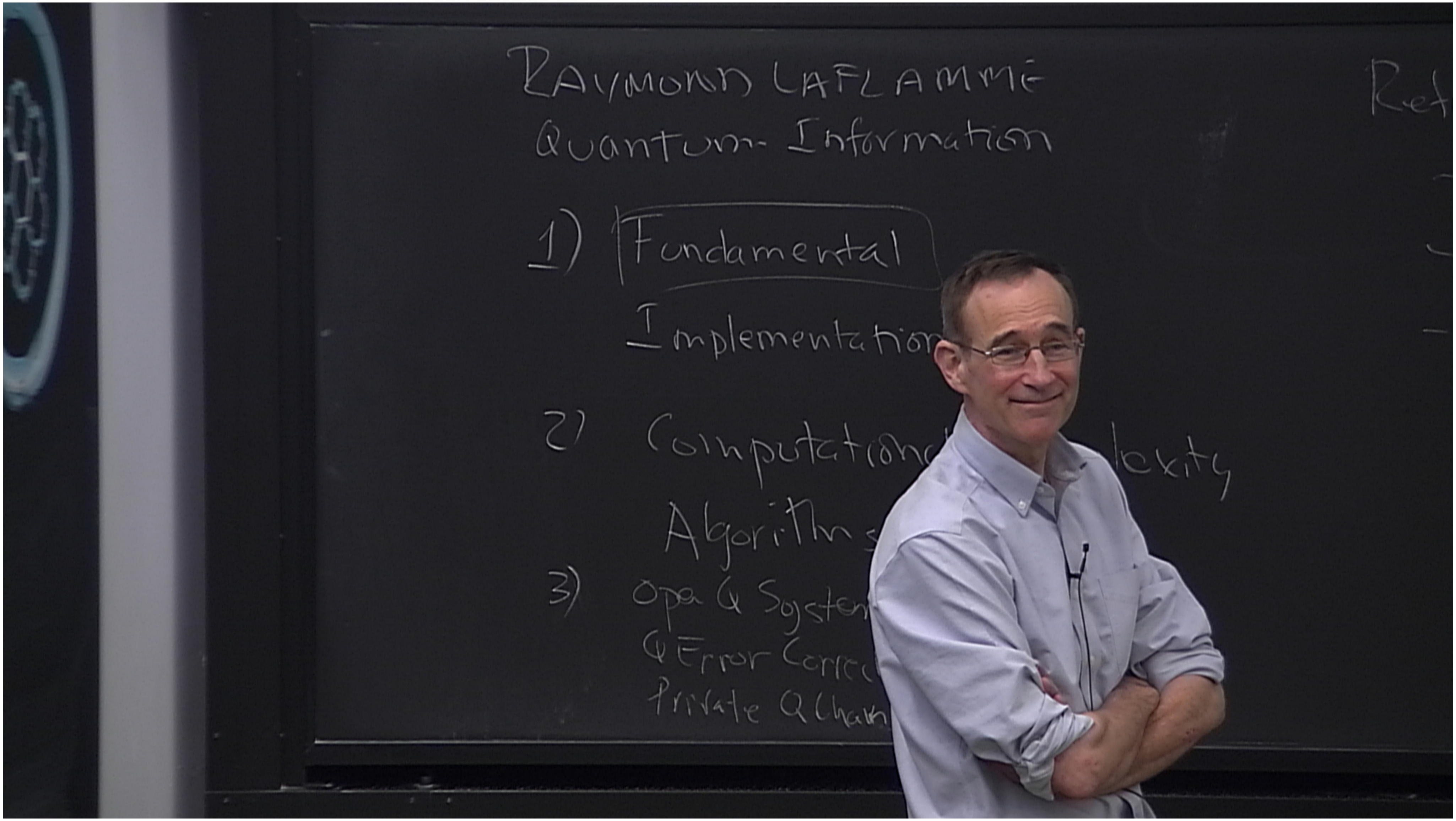
Title: PSI 2015/2016 Quantum Information - Lecture 1

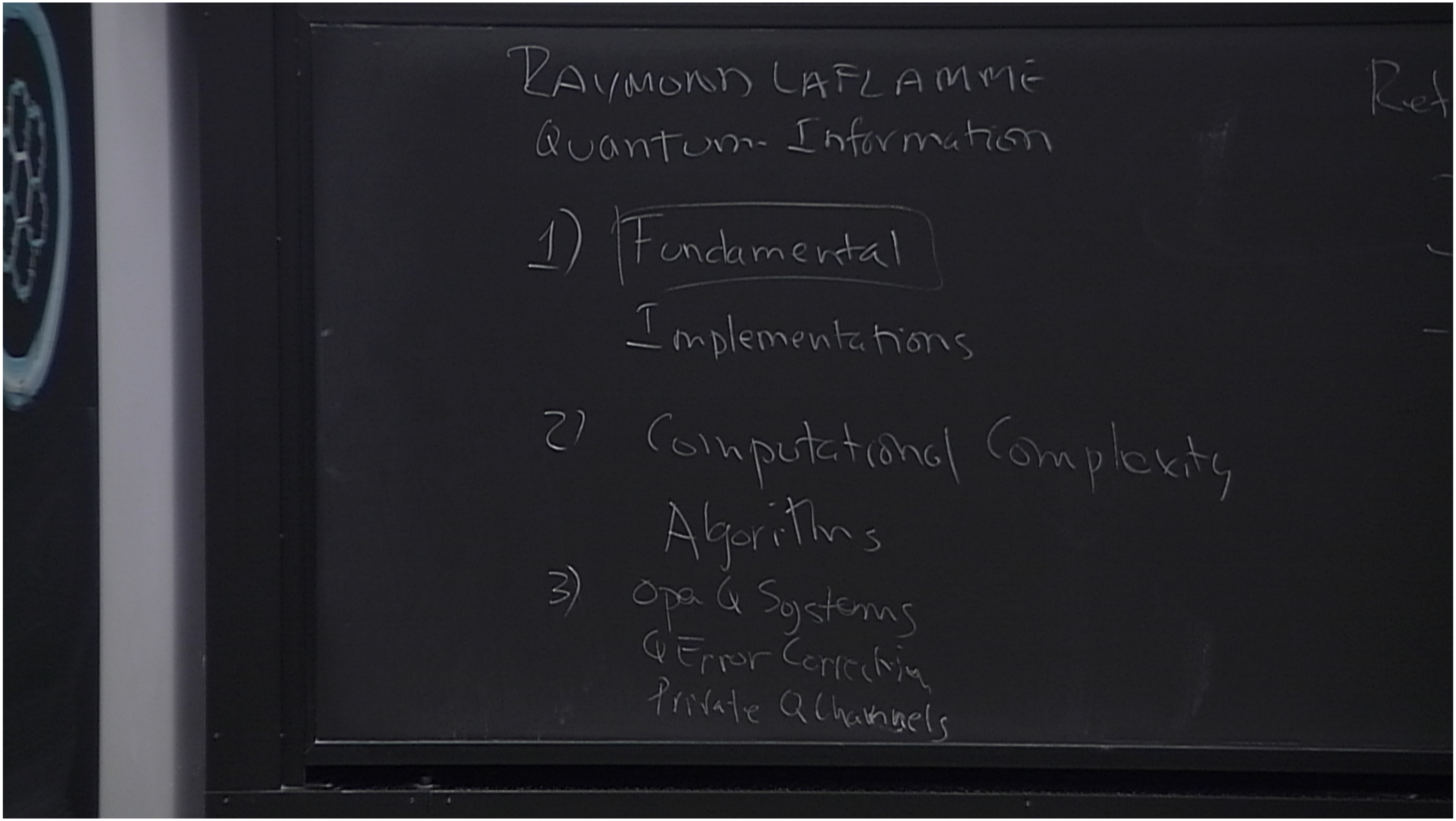
Date: Feb 22, 2016 11:30 AM

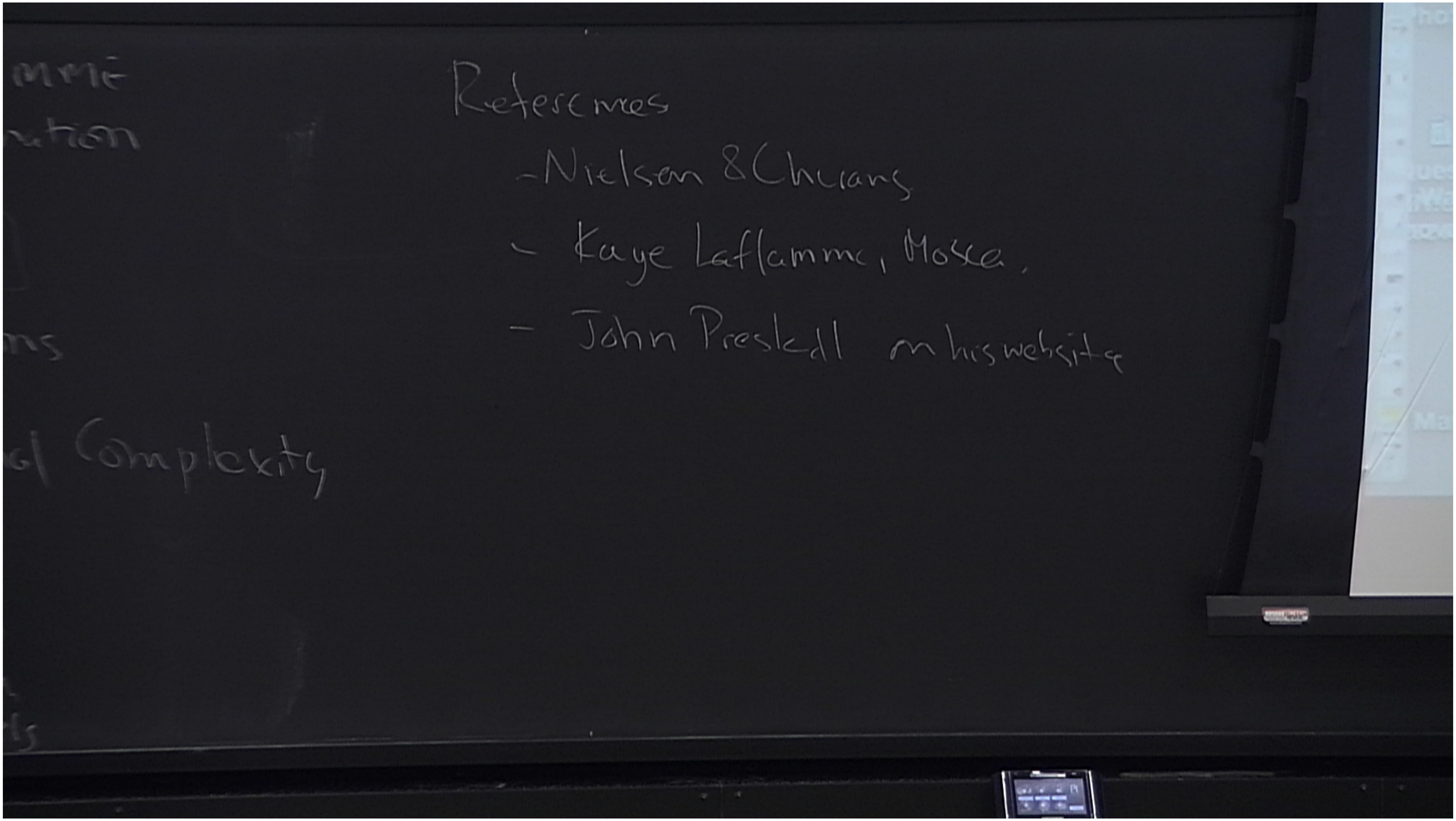
URL: http://pirsa.org/16020078

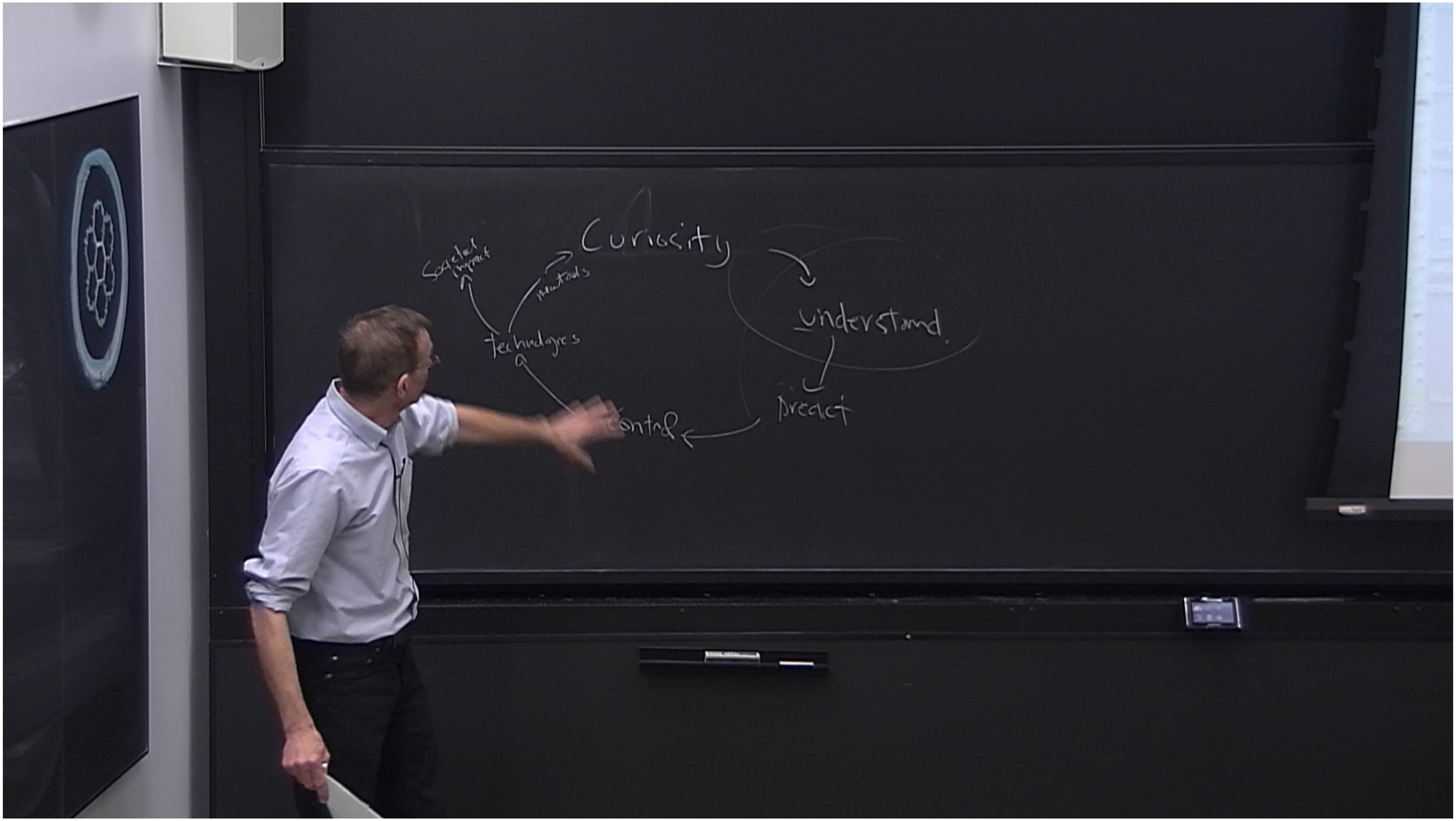
Abstract:


RAYMOND LAFLAMME
Quantum Information
1) Fundamental
Implementations
2) Computational Complexity


RAYMOND LAFLAMME
Quantum Information
1) Fundamental
Implementation
2) Computational
Algorithms
3) Open Q Systems
Q Error Correc
Private Q Chan


RAYMOND LAFLAMME
Quantum Information
1) Fundamental
Implementations
2) Computational Complexity
Algorithms
3) Open Q Systems
Q Error Correcting
Private Q Channels
Ref


References
- Nielsen & Chuang
- Kaye Laflamme, Mosca.
- John Preskill on his website
Complexity



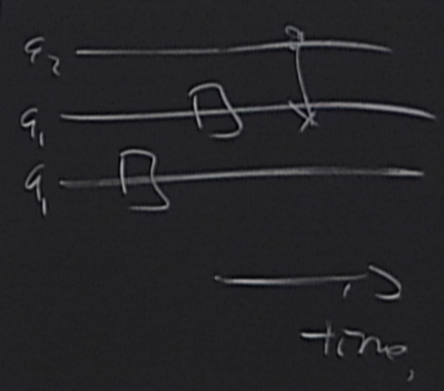
Curiosity
understand
predict
control
technologies
new tools
societal impact

the implications of QM for information processing

Success of QIS qubit

→ Algo

→ Circuit model: new language



→

Qubit?

classical bit

0

1

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Probabilistic Classical bit

0

$(1-p)$ 0

p 1

1

$$\begin{pmatrix} 1-p & \\ & p \end{pmatrix}$$

$$\rho = \frac{1}{2}\mathbb{I} + aX + bY + cZ$$

Quantum bits

↳ System who's observable are the Pauli matrices $\mathbb{I}, X, Y, Z$

—— $|1\rangle$

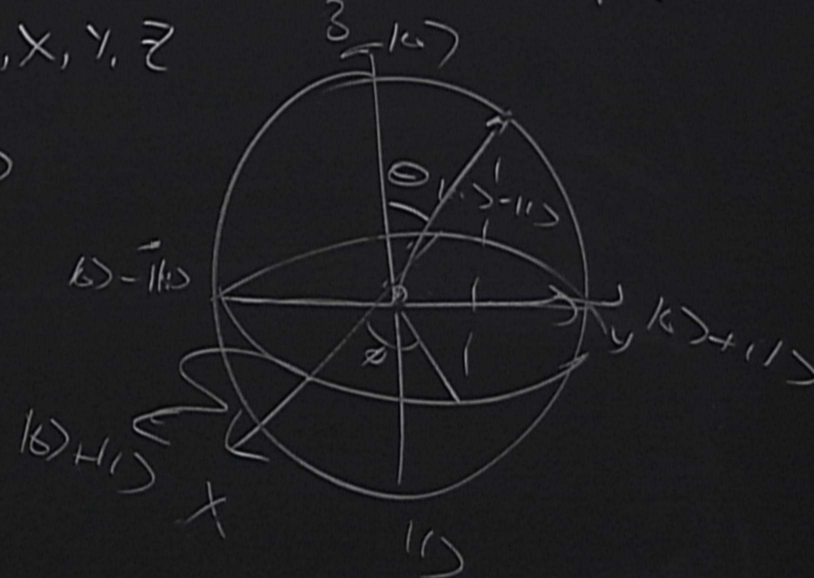
—— $|0\rangle$

$\alpha|0\rangle + \beta|1\rangle$

$|\alpha|^2 + |\beta|^2 = 1$

$\cos\theta|0\rangle + \sin\theta e^{i\phi}|1\rangle$

Bloch sphere.

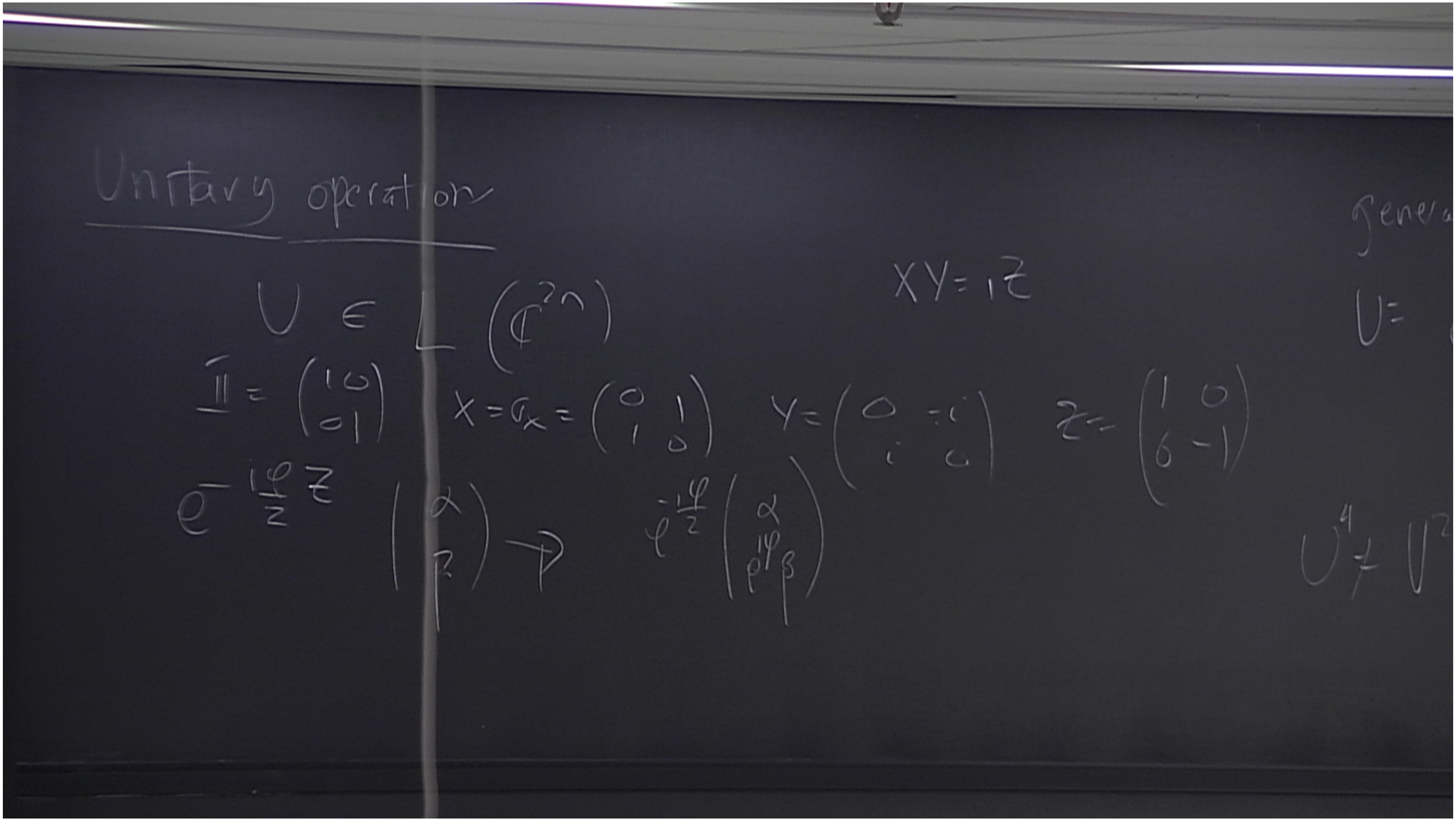


Unitary operation

$$XY = iZ$$

$$U \in L(\mathbb{C}^{2^n})$$

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad X = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$


Unitary operation
$U \in L(\mathbb{C}^{2n})$
$XY = iZ$
general
$U =$
$\mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $X = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$e^{-i\frac{\varphi}{2}Z} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \to e^{-i\frac{\varphi}{2}} \begin{pmatrix} \alpha \\ e^{i\varphi}\beta \end{pmatrix}$
$U^4 \neq U^2$

general rotation

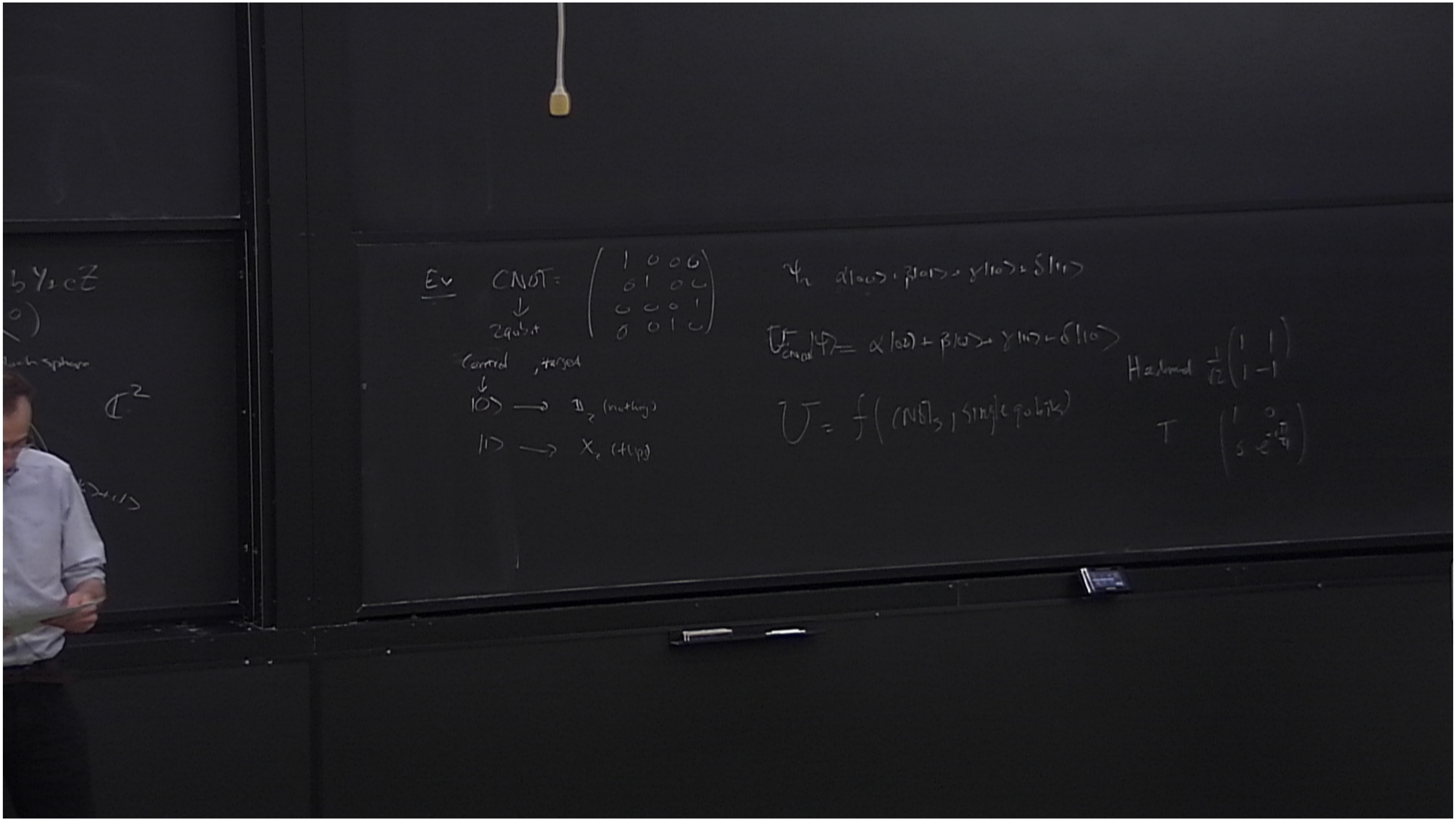
$$U = e^{-i\frac{\theta}{2}\vec{n}\cdot\vec{\sigma}} = \cos\frac{\theta}{2}\,\mathbb{1} - i\sin\frac{\theta}{2}\left(n_x X + n_y Y + n_z Z\right)$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

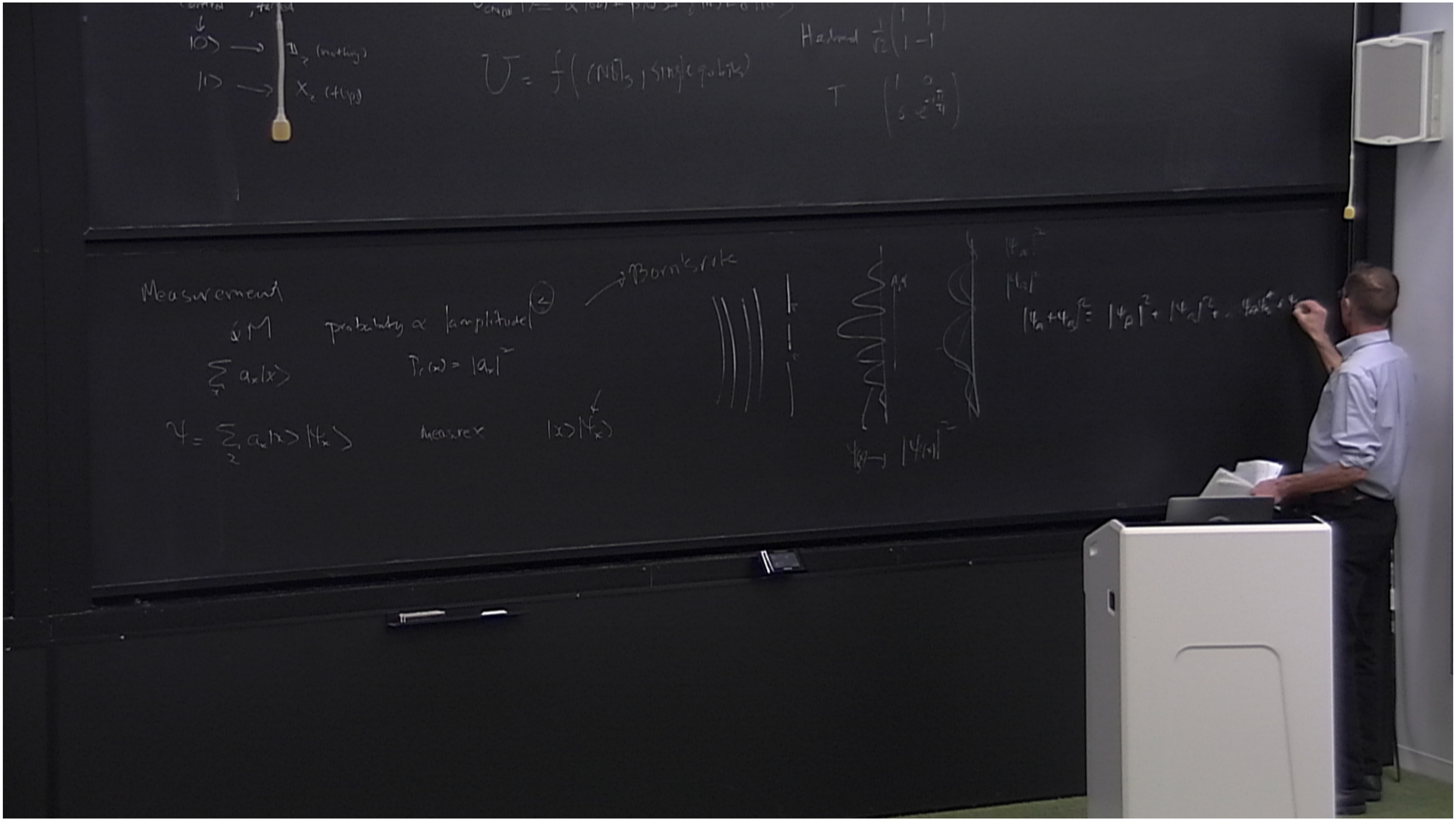
$$U|\psi\rangle = |\psi_\sigma\rangle$$

$$U^4 \neq U^2 \otimes U^2$$

$$\rho' = U\rho U^\dagger$$



Ex CNOT = $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
2qubit
Control , target
$|0\rangle \longrightarrow \mathbb{1}_2$ (nothing)
$|1\rangle \longrightarrow X_2$ (flip)
$\Psi = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
$U_{CNOT}|\Psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|11\rangle + \delta|10\rangle$
$U = f($CNOTs, single qubits$)$
Hadamard $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
T $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$


Measurement
probability ∝ |amplitude|
Born's rule
measure x
Hadamard

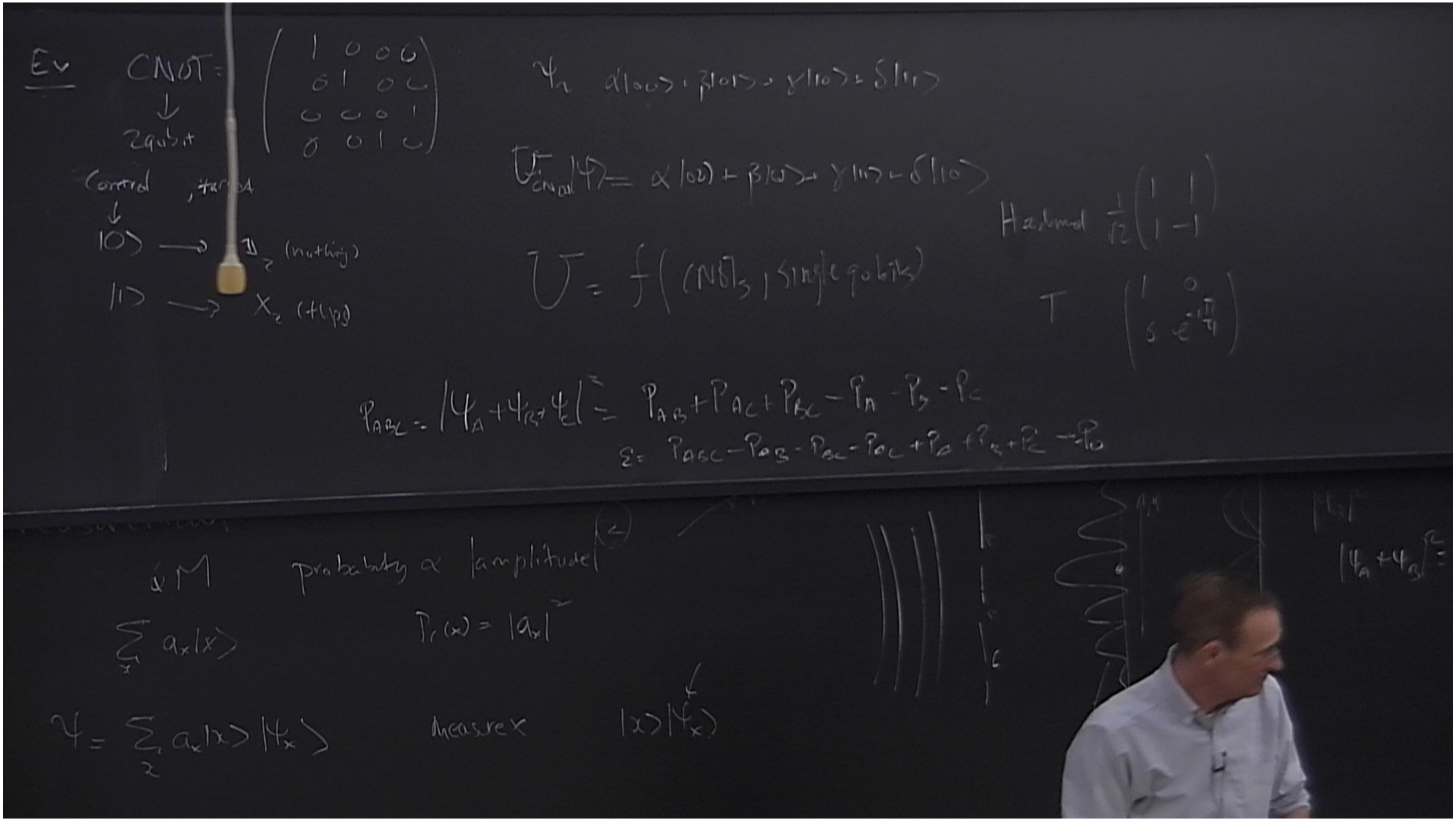
$$P_{ABC} = |\psi_A + \psi_B + \psi_C|^2 = P_{AB} + P_{AC} + P_{BC} - P_A - P_B - \ldots$$

$$\varepsilon = P_{ABC} - P_{AB} - P_{AC} - P_{BC} + P_A \ldots$$

QM probability $\propto$ $|\text{amplitude}|^2$

$\sum_x a_x |x\rangle$ $\quad\quad P_r(x) = |a_x|^2$

$\psi = \sum_x a_x |x\rangle |\psi_x\rangle$ $\quad$ measure x $\quad |x\rangle|\psi_x\rangle$


Ex
CNOT =
2 qubit
control , target
|0> → I (nothing)
|1> → X (flip)
Hadamard
probability ∝ |amplitude|^2
measure x