Title: PSI 2015/2016 Quantum Gravity - Lecture 6

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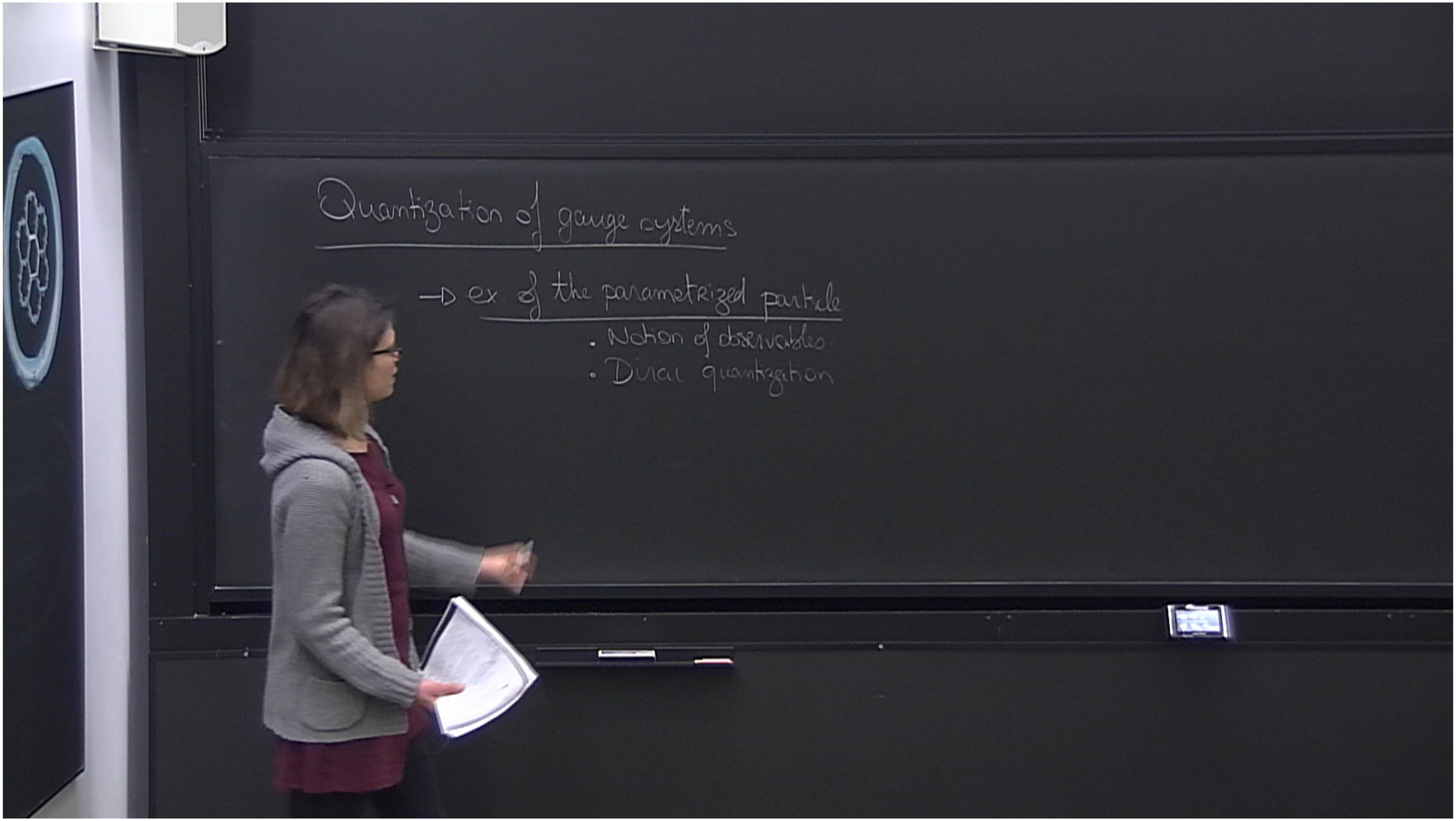
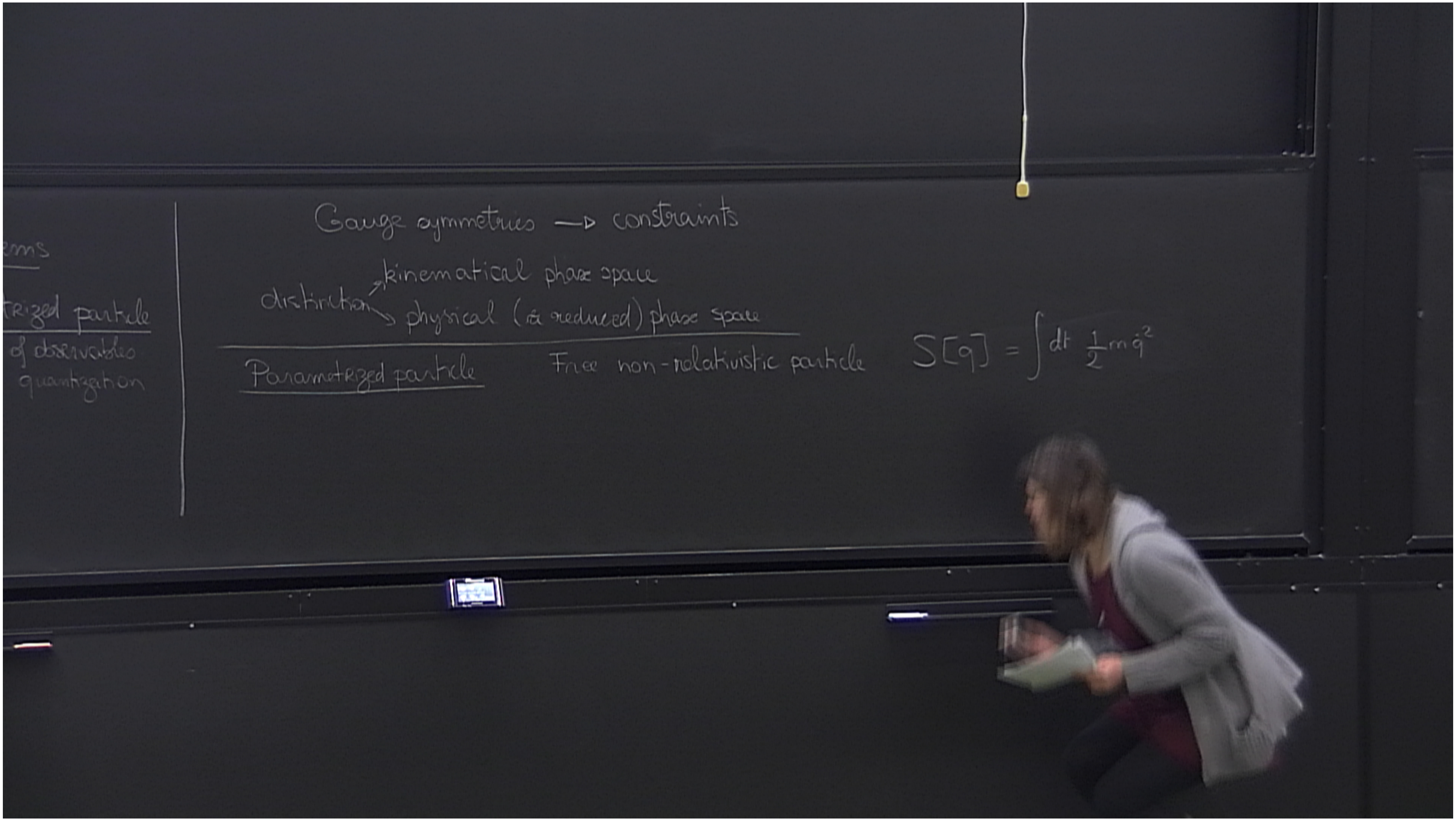
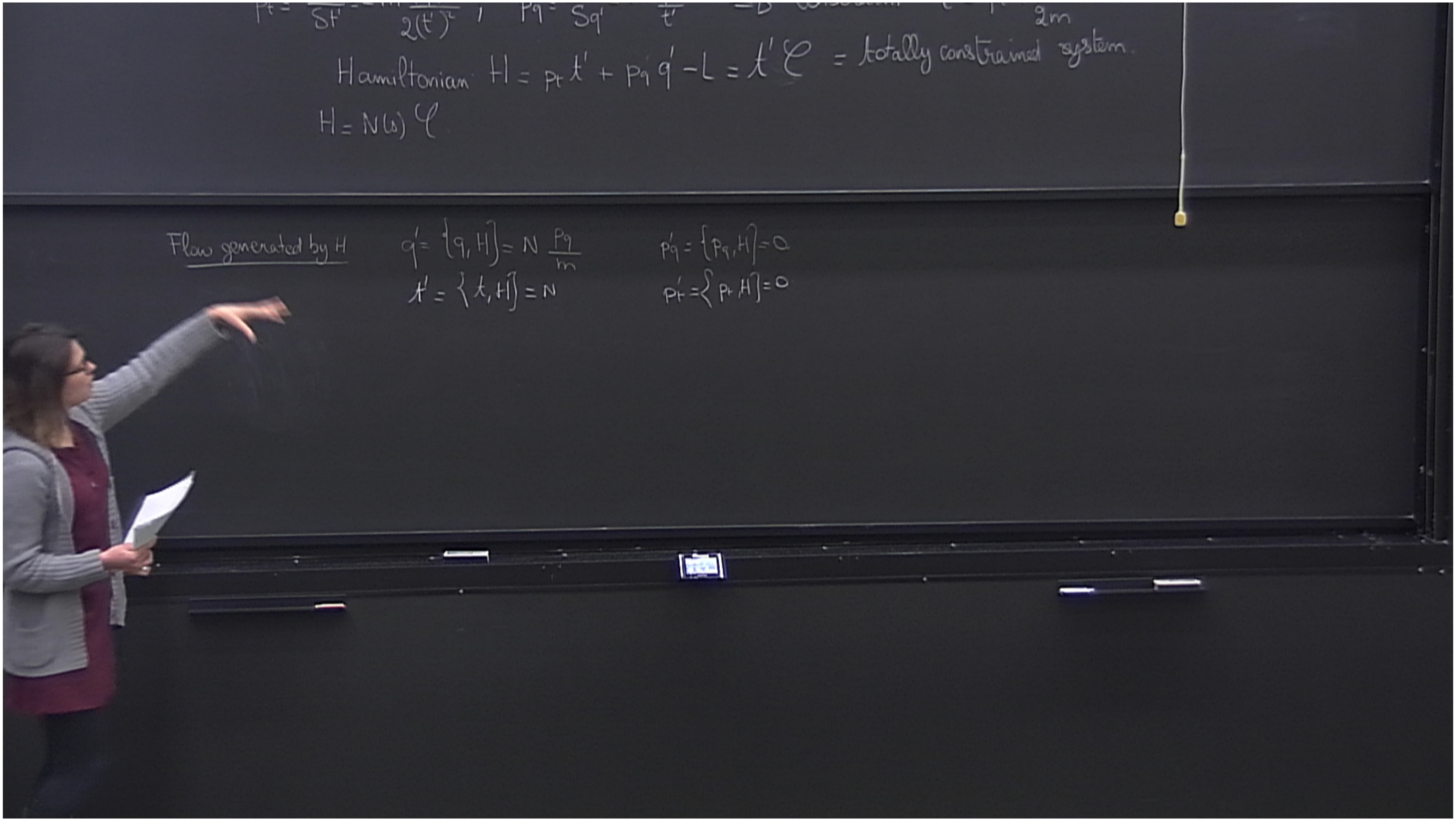
Abstract:


Quantization of gauge systems
→ ex. of the parametrized particle
• Notion of observables.
• Dirac quantization


Gauge symmetries → constraints
distribution
kinematical phase space
physical (or reduced) phase space
Parametrized particle
Free non-relativistic particle
S[q] = ∫dt ½ m q̇²
ems
trized particle
of observables
quantization


Hamiltonian $H = p_t t' + p_q q' - L = t' \mathcal{C}$ = totally constrained system.
$H = N(s) \mathcal{C}$
Flow generated by H
$q' = \{q, H\} = N \frac{p_q}{m}$
$p_q' = \{p_q, H\} = 0$
$t' = \{t, H\} = N$
$p_t' = \{p_t, H\} = 0$

$$p_t = \frac{\delta S}{\delta t'} = -m\frac{(q')^2}{2(t')^2} \; ; \quad p_q = \frac{\delta S}{\delta q'} = m\frac{q'}{t'} \quad \Rightarrow \text{Constraint} \quad \varphi = p_t + \frac{p_q^2}{2m} = 0$$

Hamiltonian $H = p_t t' + p_q q' - L = t'\varphi$ = totally constrained system.

$H = N(s)\varphi$

<u>Flow generated by H</u>

$$q' = \{q, H\} = N\frac{p_q}{m} \qquad p_q' = \{p_q, H\} = 0$$

$$t' = \{t, H\} = N \qquad p_t' = \{p_t, H\} = 0$$

A 2 physical phase space → 2 indpt Dirac observables

$$\{F, \varphi\} \underset{\text{weakly}}{\simeq} 0 \quad \text{(on the constraint surface)}$$

Construction of physical observables → (Bianca's notes + Carlo's book)

- select as many gauge fixing conditions as there are constraints

  $t = \tau$ (τ fixed parameter)

- Value of the physical observable $F_f(\tau)$ on a gauge orbit is given by the value on that point s of the Gauge orbit on which $t(s) = \tau$ holds orbit on which $t(s)=\tau$ holds.

For $f=q$ $\quad \frac{dq}{ds}=\{q,\varphi\}$ $\Rightarrow$ $q(s)=q+\frac{p_q}{m}(s-T_0)$ gauge orbit of q generated by φ.

$\frac{dt}{ds}=\{t,\varphi\}=1$ $\quad t(s)=s$ $\quad \Rightarrow t(s)=\tau=s$

$F_q^{\tau}=q(\tau)=q+\frac{p_q}{m}(\tau-T)$

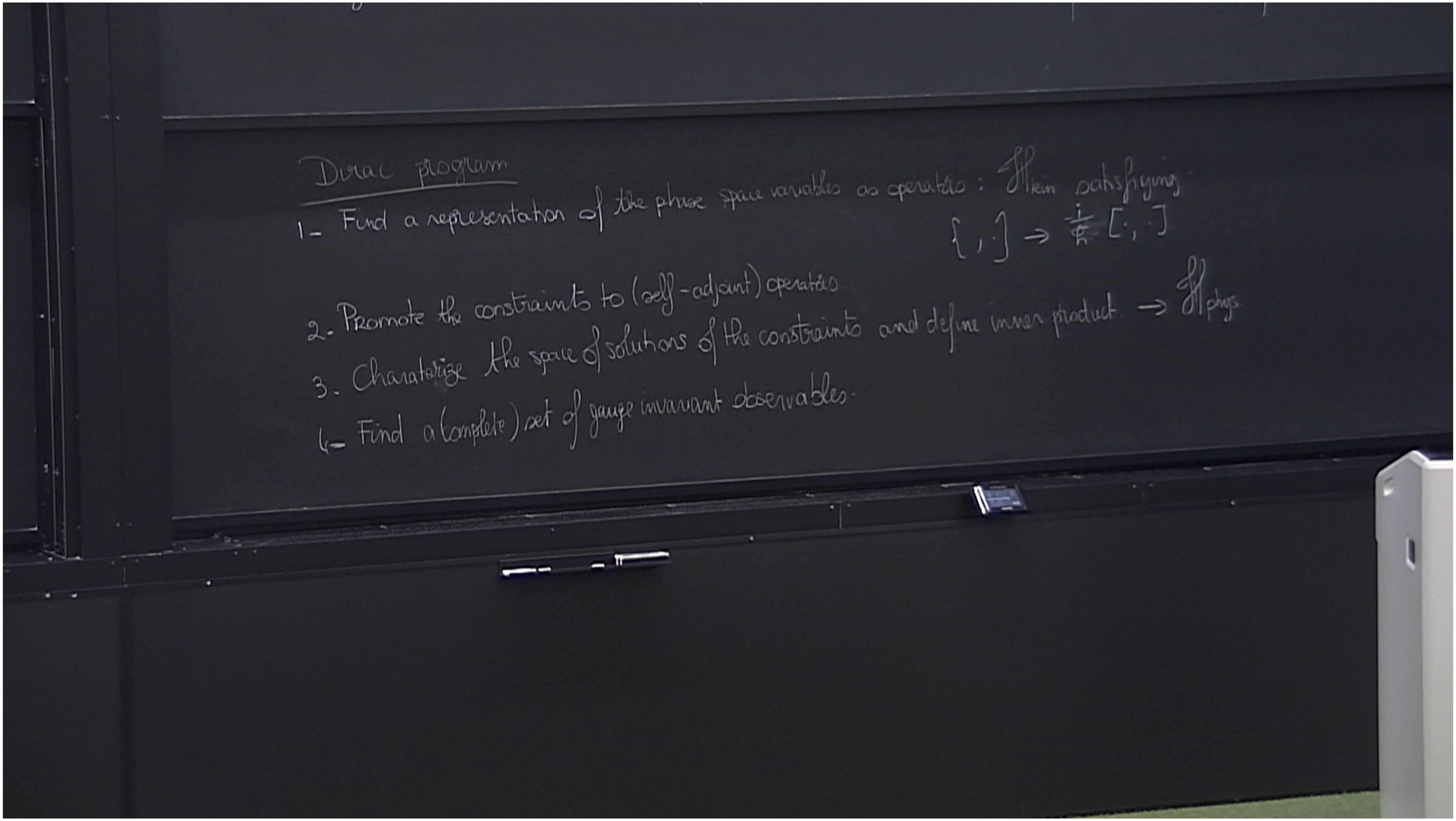
$F_t^{\tau}=\tau$

$F_{p_t}^{\tau}=(F_{p_q}^{\tau})$

For $f=p_q$

$F_{p_q}^{\tau}=p_q$

$\{F_q$


Dirac program
1 - Find a representation of the phase space variables as operators : $\mathcal{H}_{kin}$ satisfying
$\{\cdot,\cdot\} \rightarrow -\frac{i}{\hbar}[\cdot,\cdot]$
2 - Promote the constraints to (self-adjoint) operators.
3 - Characterize the space of solutions of the constraints and define inner product. $\rightarrow \mathcal{H}_{phys}$
4 - Find a (complete) set of gauge invariant observables.

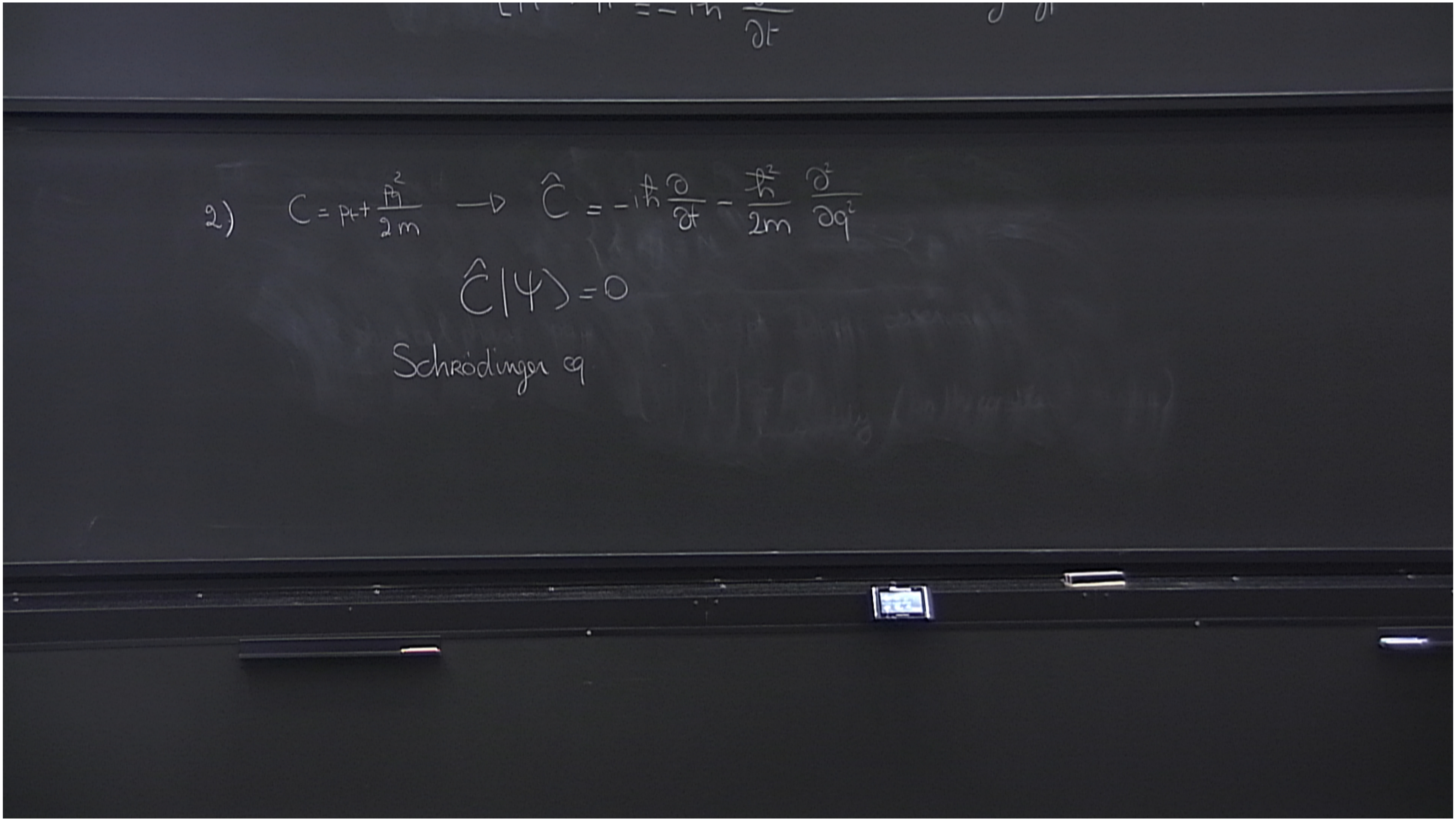
1). kinematical phase space (q, t, p_q, p_t) 4d.

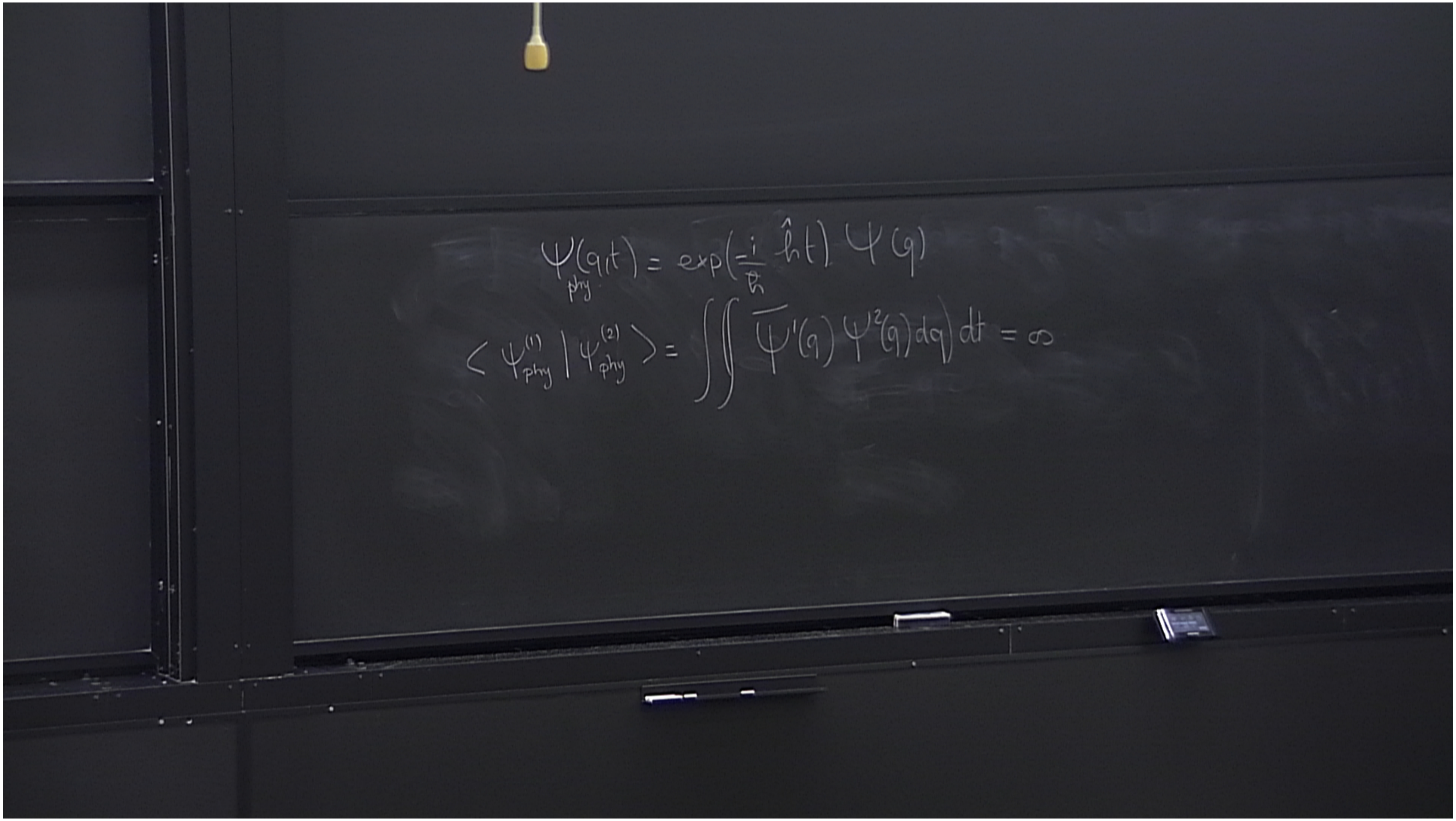
$\mathcal{H}_{kin} = \mathcal{L}^2(\mathbb{R}^2)$

$\psi(q,t)$

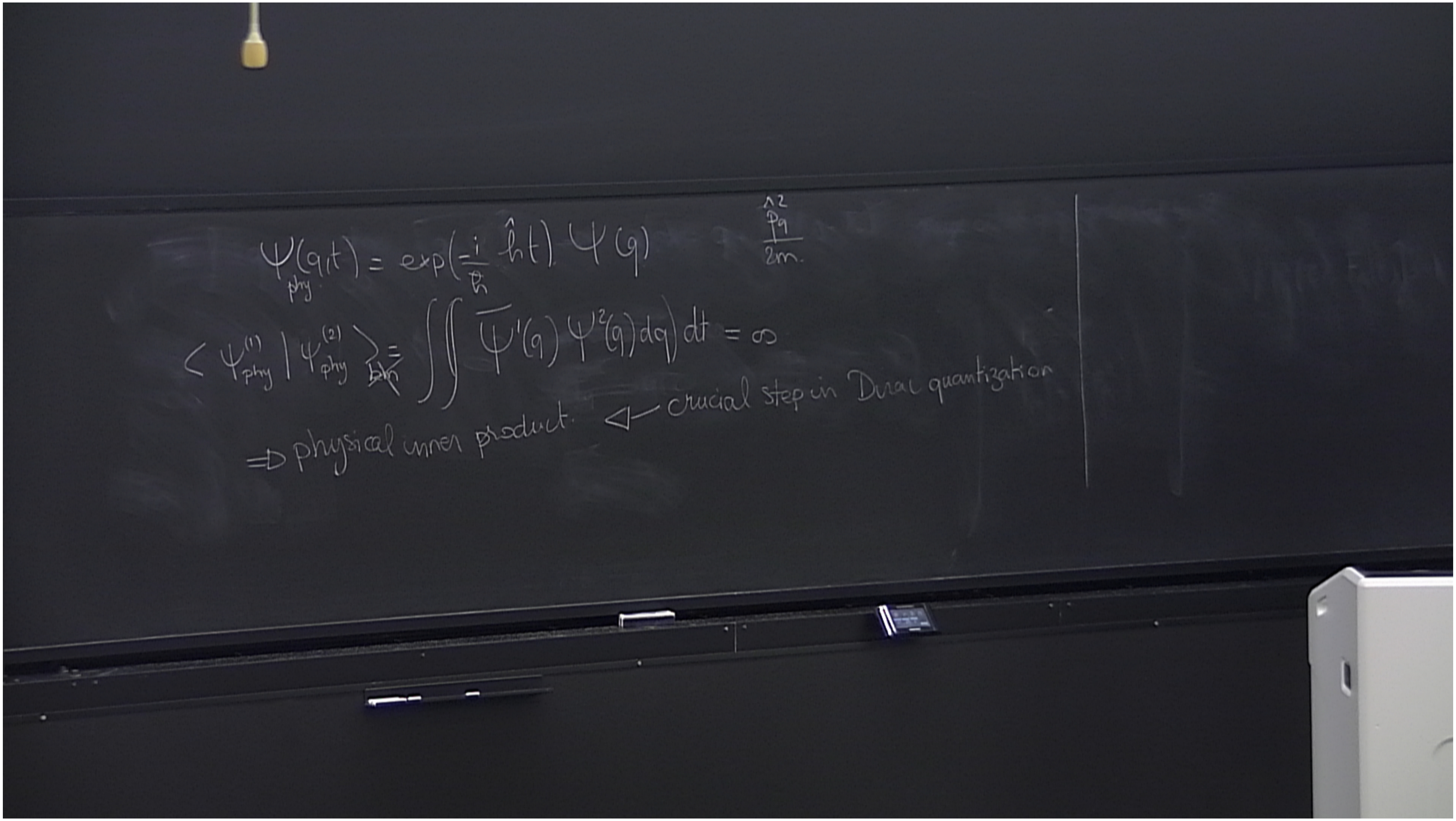
$$\left[\begin{array}{ll} q \longrightarrow \hat{q} & \hat{q}\psi = q\psi \\ t \longrightarrow \hat{t} & \hat{t}\psi = t\psi \end{array}\right]$$ configuration op. act by multiplication

$$\left[\begin{array}{l} p_q \longrightarrow \hat{p}_q = - \\ p_t \longrightarrow \hat{p}_t \end{array}\right.$$


2) $C = p_t + \frac{p^2}{2m} \longrightarrow \hat{C} = -i\hbar\frac{\partial}{\partial t} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2}$
$\hat{C}|\psi\rangle = 0$
Schrödinger eq



$\Psi_{phy}(q,t) = \exp\left(-\frac{i}{\hbar}\hat{h}t\right)\Psi(q)$
$\langle \Psi^{(1)}_{phy} | \Psi^{(2)}_{phy} \rangle = \int\left(\int \overline{\Psi^1(q)}\,\Psi^2(q)\,dq\right)dt = \infty$


$\Psi_{phy}(q,t) = \exp(\frac{-i}{\hbar} \hat{h} t) \Psi(q)$
$\frac{\hat{p}_q^2}{2m}$
$\langle \Psi^{(1)}_{phy} | \Psi^{(2)}_{phy} \rangle_{\cancel{kin}} = \int\int \overline{\Psi^1(q)} \Psi^2(q) dq) dt = \infty$
⇒ physical inner product
crucial step in Dirac quantization