Title: PSI 2015/2016 Quantum Gravity - Lecture 5

Date: Feb 26, 2016 10:15 AM

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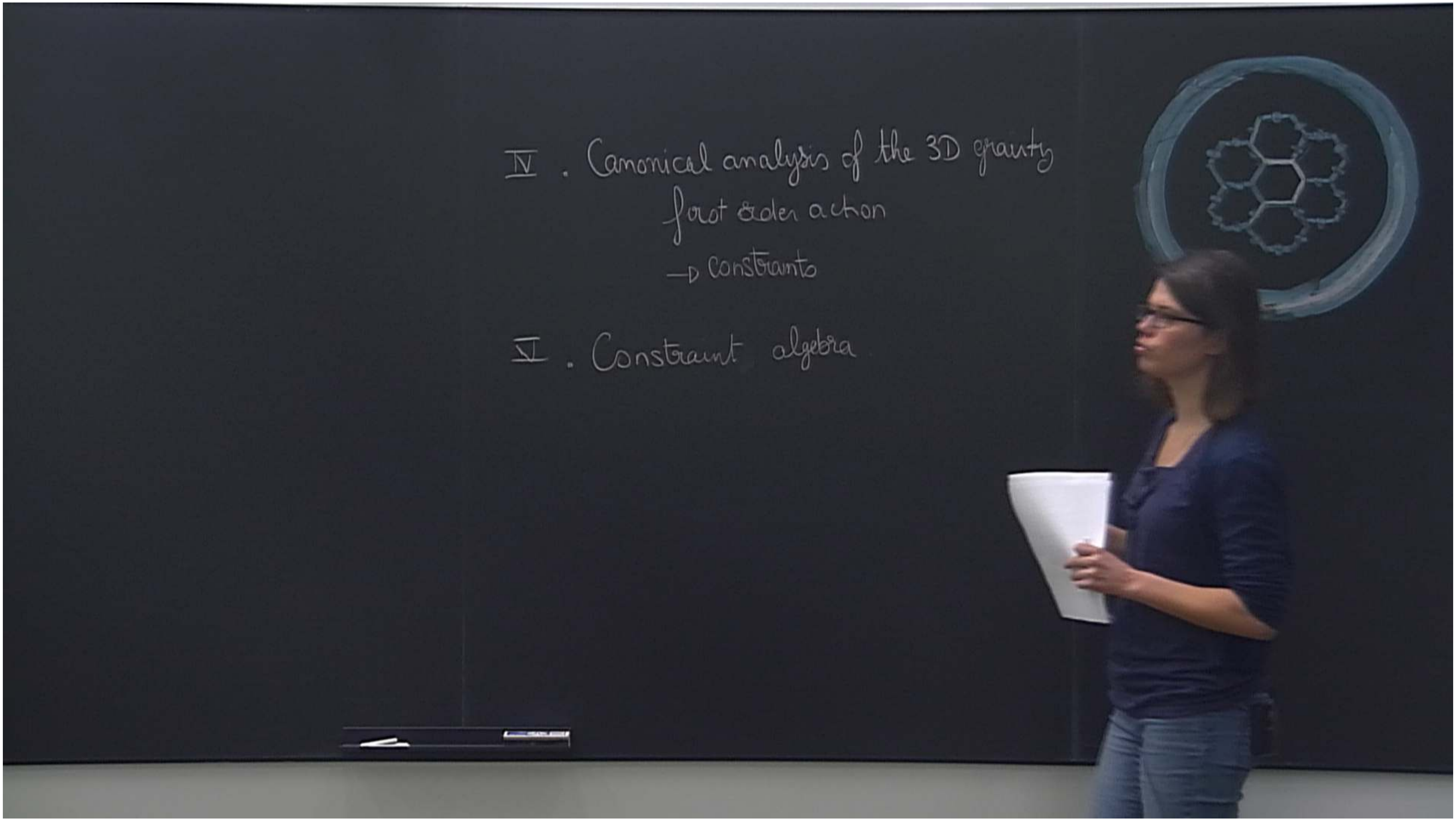
Abstract:


IV . Canonical analysis of the 3D gravity
first order action
→ constraints
V . Constraint algebra

$$S_{\text{Palatini}} = \int \left(e^j_b\, \partial_0 A_{aj}\, \tilde{\varepsilon}^{ab} + e_{0j}\, \frac{1}{2} F_{ab}{}^{j}\, \hat{\varepsilon}_{ab} + A^j_0\, \frac{1}{2} T^j_{ab}\, \tilde{\varepsilon}_{ab} \right) d^2x\, dt$$

Canonical variables $\left\{ A^j_a(x), E^b_k(y) \right\} = \delta^j_k\, \delta^b_a\, \delta(x,y)$

$$\left(\left\{ A^k_a(x), A^j_b(y) \right\} = 0, \ \left\{ E^a_k(x), E^b_j(y) \right\} = 0 \right.$$

$M = \Sigma \times \mathbb{R}.$

$S \underset{\text{Palatini}}{=} \int \Big(e^j_b\, \partial_0 A_{aj}\, \hat{\varepsilon}^{ab} + e_{0j}\, \frac{1}{2} F_{ab}{}^{j}\, \hat{\varepsilon}_{ab}$

- Canonical variables $\{ A^j_a(x),$

$\Big(\{ A^k_a(x), A^j_b(y) \} = 0, \ \{ E^a_k$

$$\ldots \hat{\mathcal{E}}_{ab} + A_0^j \frac{1}{2} T^j_{ab} \tilde{\mathcal{E}}_{ab} \Big) d^2x \, dt$$

$$\{ A_a^j(x), E_k^b(y) \} = \delta_k^j \, \delta_a^b \, \delta(x,y) \quad - \qquad E_k^b = \tilde{\varepsilon}_{bc} \, e_c^k$$

$$, \; \{ E_k^a(x), E_0^b(y) \} = 0$$

$$S \underset{\text{Palatini}}{=} \int \left(e^j_b\, \partial_0 A_{aj}\, \tilde{\varepsilon}^{ab} + \underline{e_{0j}}\, \frac{1}{2} F_{ab}^{\ j}\, \hat{\varepsilon}_{ab} + \underline{A^j_0 \frac{1}{2}}\, T^j_{ab}\, \tilde{\varepsilon}_{ab} \right) d^2x\, dt$$

- Canonical variables $\{A^j_a(x), E^b_k(y)\} = \delta^j_k\, \delta^b_a\, \delta(x,y)$ — $E^b_k = \tilde{\varepsilon}_{bc}\, e_c^{\ k}$

$\left(\{A^k_a(x), A^j_b(y)\} = 0,\ \{E^a_k(x), E^b_j(y)\} = 0 \right)$.

- e_{0j}, A^j_0 = Lagrange multipliers.

Palatini

- Canonical variables $\{A^j_a(x), E^b_k(y)\} = \delta^j_k \, \delta^b_a \, \delta(x,y)$ — $E^b_k = \varepsilon_{bc} e^k_c$

$\left(\{A^k_a(x), A^j_b(y)\} = 0, \ \{E^a_k(x), E^b_j(y)\} = 0 \right).$

- e_{0j}, A^j_0 = Lagrange multipliers.

- $\tilde{\mathcal{F}}^j = \frac{1}{2} F^j_{ab} \tilde{\varepsilon}_{ab} = \varepsilon_{ab} \left(\partial_a A^j_b + \frac{1}{2} \varepsilon^j{}_{kl} A^k_a A^l_b \right) = 0$ ⟶ Flatn

- $\mathcal{G}^i = \frac{1}{2} T^i_{ab} \tilde{\varepsilon}_{ab} = \partial_a E^a_j + \varepsilon_{jml} A^l_a E^{am}$

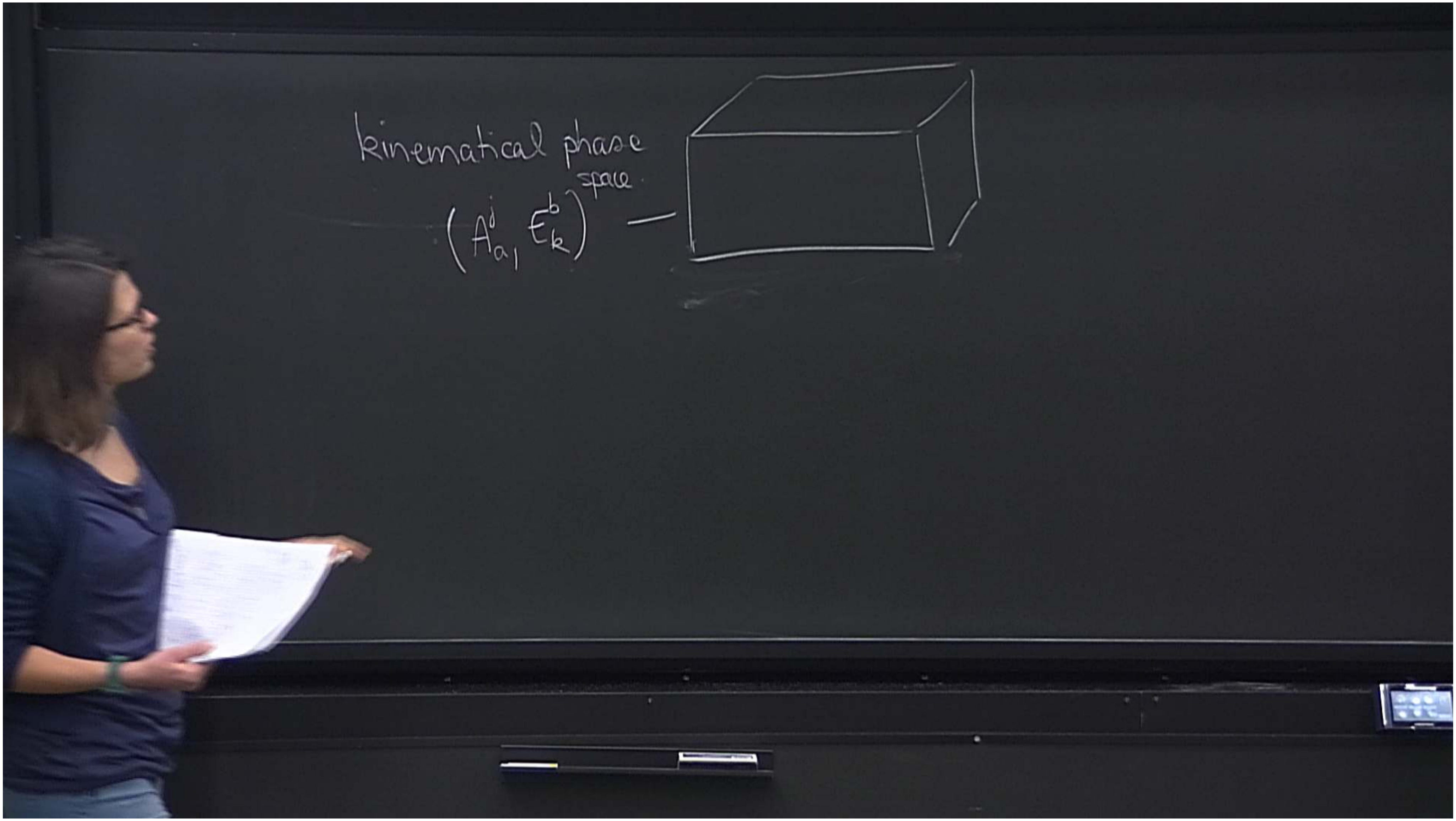
$\{A^j_a(x), E^b_k(y)\} = \delta^j_k \, \delta^b_a \, \delta(x,y)$ — $E^b_k = \varepsilon^{bc} e^k_c$

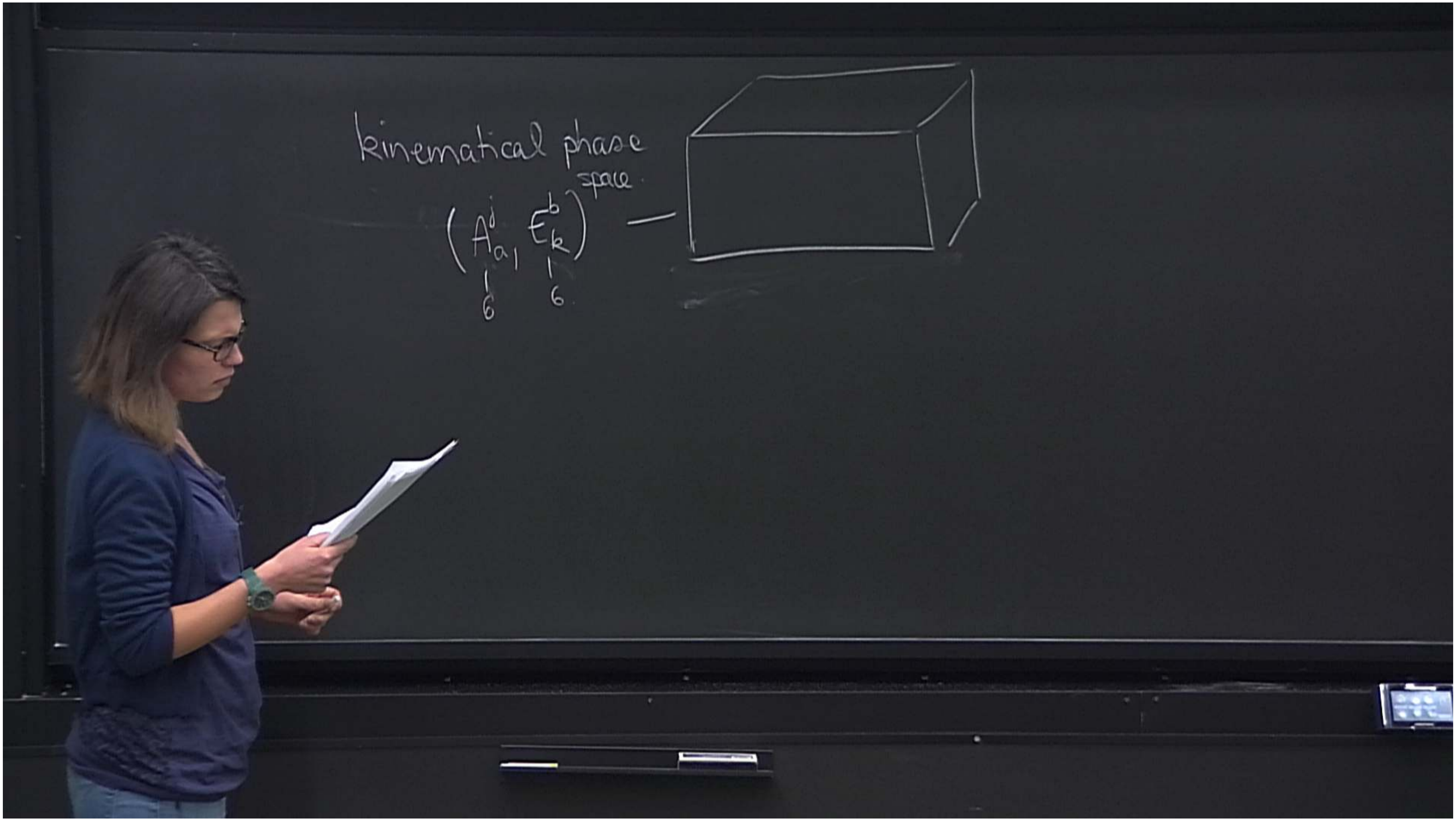
$, \{ \quad , E^b_j(y)\} = 0)$.

plie

$\partial_a A^j_b + \frac{1}{2} \varepsilon^j{}_{kl} A^k_a A^l_b) = 0$ $\longrightarrow$ Flatness constraint

$+ \varepsilon_{jml} A^l_a E^{am}$ $\longrightarrow$ Gauss constraint.

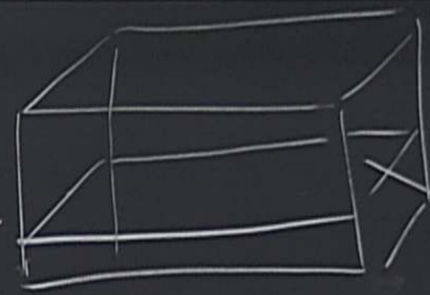

kinematical phase space
(A^j_{a}, E^b_k)


kinematical phase space
(A^i_a, E^b_k)
6
6

kinematical phase space

$$\left(A^j_a, E^b_k\right)$$

$6 \quad 6 \quad = 12$ d.o.f.



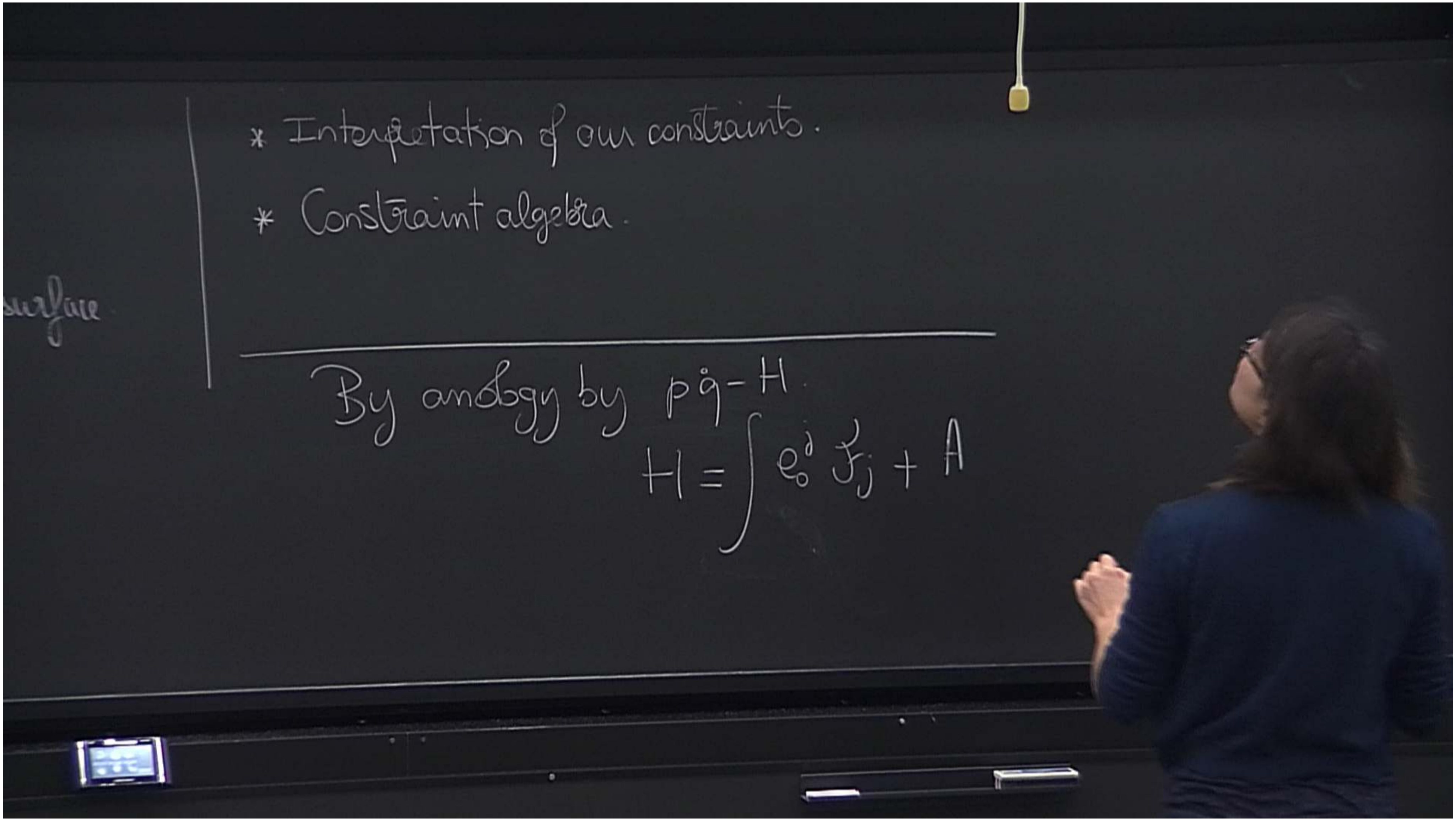
$\mathcal{G}^j = 0$, $\mathcal{V}^j = 0$

constraint hypersurface

$d = 6$

surface

* Interpretation of our constraints.
* Constraint algebra.


* Interpretation of our constraints.
* Constraint algebra.
By analogy by $p\dot{q} - H$.
$H = \int e_0^j \mathcal{F}_j + A$

surface

* Interpretation of our constraints.

* Constraint algebra.

By analogy by $p\dot{q} - H$.

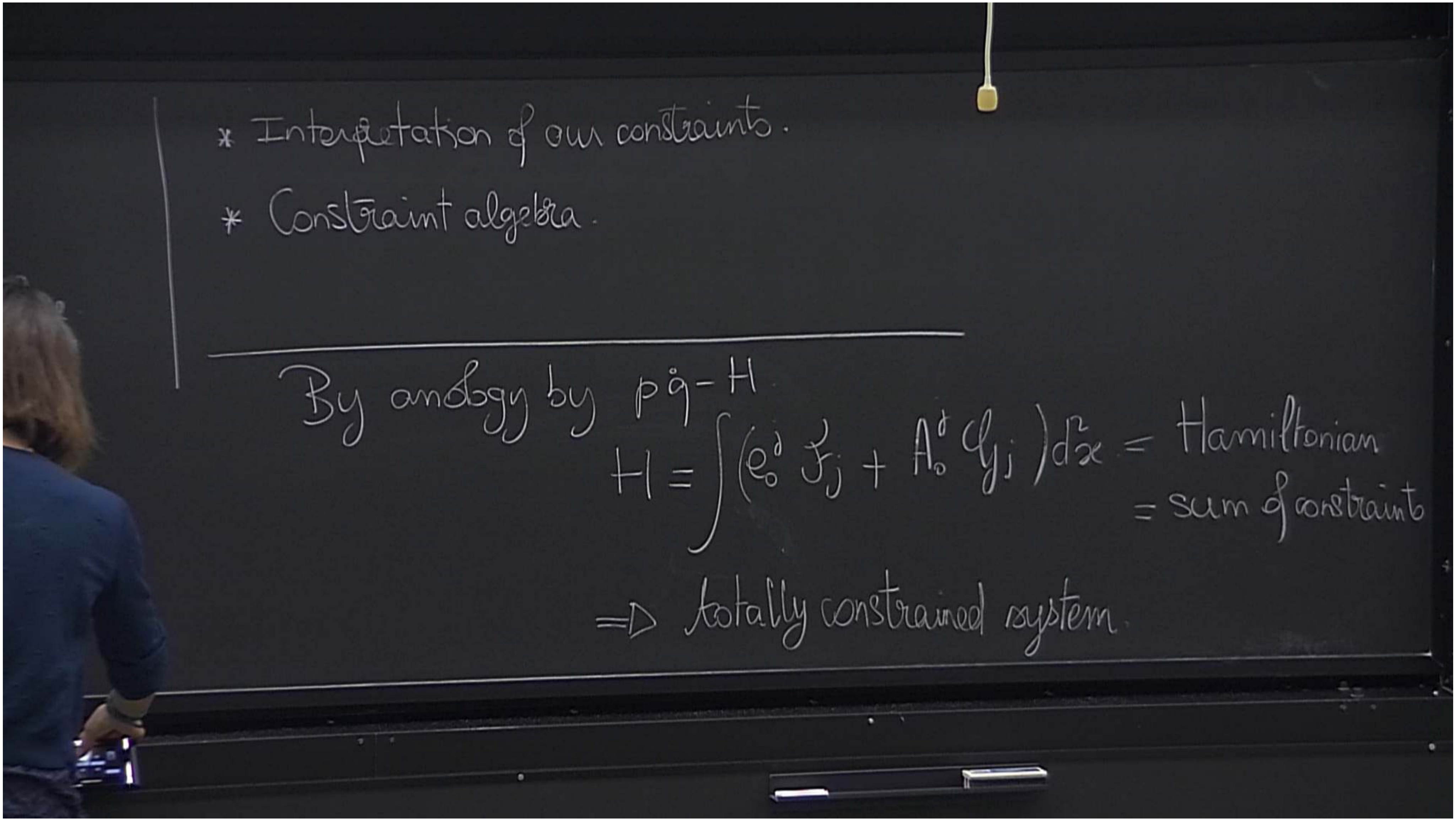
$$H = \int \left(e_0^j \mathcal{F}_j + A_0^j \mathcal{G}_j \right) d^2x$$

surface

* Interpretation of our constraints.
* Constraint algebra.

By analogy by $p\dot{q} - H$

$$H = \int \left(e_0^j \mathcal{F}_j + A_0^j \mathcal{G}_j \right) d^2x = \text{Hamiltonian}$$


* Interpretation of our constraints.
* Constraint algebra.
By analogy by $p\dot{q} - H$
$H = \int (e_0^j \mathcal{F}_j + A_0^j \mathcal{G}_j) \, d^2x$ = Hamiltonian
= sum of constraints
$\Rightarrow$ totally constrained system.

* Smeared constraints: $\mathcal{G}[\Lambda] = \int \Lambda^j \, \mathcal{G}_j \, d^3x$.

$$\mathcal{F}[N] = \int N^j \, \mathcal{F}_j \, d^2x.$$

Infinitesimal change associated to the Gauss constraints

$$\delta_\Lambda E^a_j = \left\{ E^a_j(x), \int \Lambda^k \left(\partial_b E^b_k + \varepsilon_{klm} A^l_b E^{bm}\right)(y)\, d^2y \right\}$$

Infinitesimal change associated to the Gauss constraints

$$\delta_\Lambda E^a_j = \left\{ E^a_j(x) , \int \Lambda^k \left(\partial_b E^b_k + \varepsilon_{klm} A^l_b E^{bm} \right)(y)\, d^3y \right\}$$

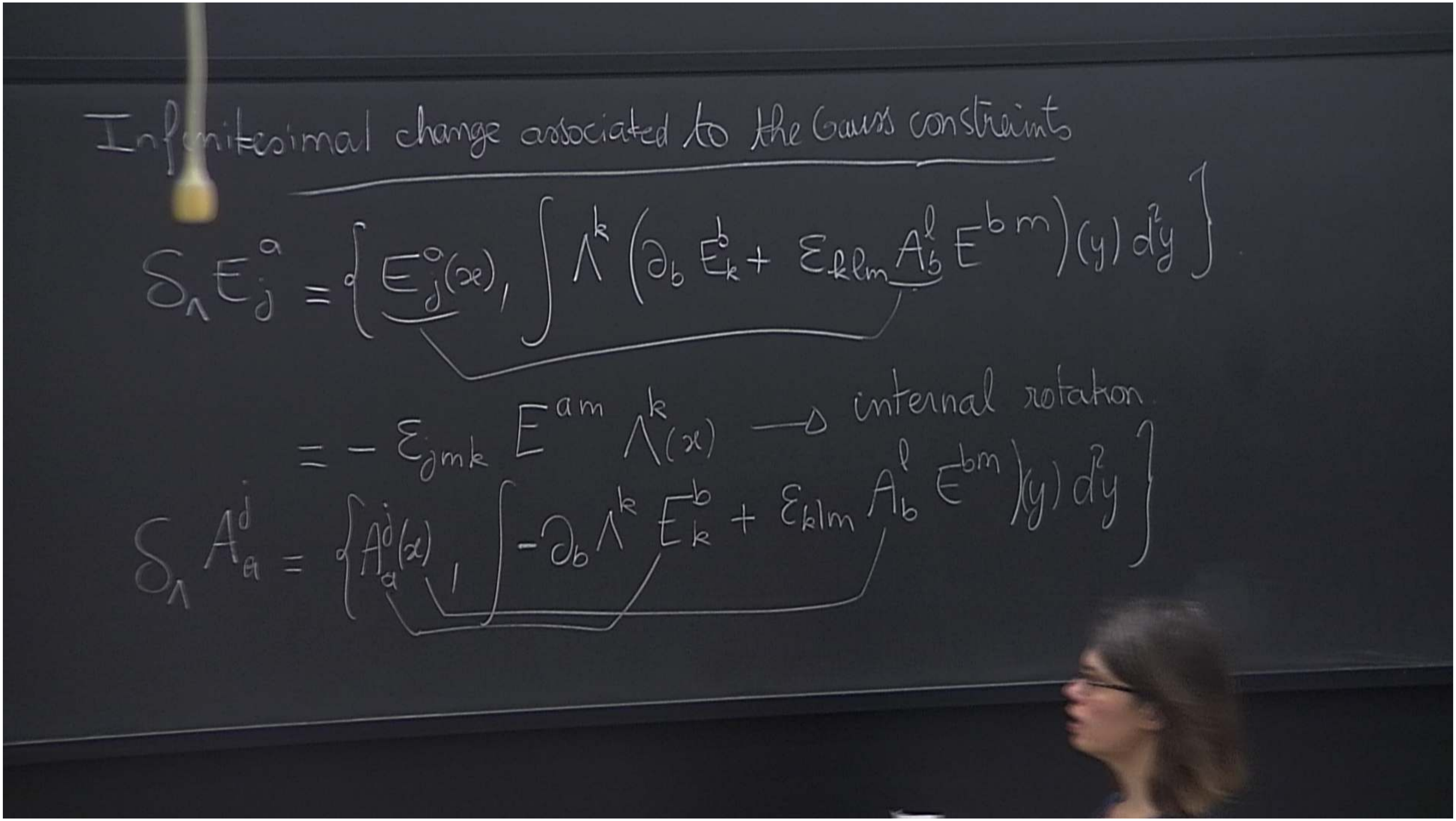
$$= - \varepsilon_{jmk} E^{am} \Lambda^k(x)$$

Infinitesimal change associated to the Gauss constraints

$$\delta_\Lambda E^a_j = \left\{ E^a_j(x) , \int \Lambda^k \left(\partial_b E^b_k + \varepsilon_{klm} A^l_b E^{bm} \right)(y)\, d^3y \right\}$$

$$= - \varepsilon_{jmk} E^{am} \Lambda^k(x) \longrightarrow \text{internal rotation}$$

$$\delta_\Lambda A^j_a = \left\{ A^j_a(x) , \int - \partial_b \Lambda^k E^b_k + \varepsilon_{klm} A^l_b E^{bm})(y)\, d^3y \right\}$$


Infinitesimal change associated to the Gauss constraints
$\delta_\Lambda E^a_j = \{ E^a_j(x), \int \Lambda^k (\partial_b E^b_k + \varepsilon_{klm} A^l_b E^{bm})(y)\, d^2y \}$
$= - \varepsilon_{jmk} E^{am} \Lambda^k(x) \longrightarrow$ internal rotation
$\delta_\Lambda A^j_a = \{ A^j_a(x), \int -\partial_b \Lambda^k E^b_k + \varepsilon_{klm} A^l_b E^{bm})(y)\, d^2y \}$

Infinitesimal change associated to the Gauss constraints

$$\delta_\Lambda E^a_j = \left\{ E^a_j(x), \int \Lambda^k \left(\partial_b E^b_k + \varepsilon_{klm} A^l_b E^{bm} \right)(y)\, d^2y \right\}$$

$$= -\varepsilon_{jmk} E^{am} \Lambda^k(x) \longrightarrow \text{internal rotation}$$

$$\delta_\Lambda A^j_a = \left\{ A^j_a(x), \int -\partial_b \Lambda^k E^b_k + \varepsilon_{klm} A^l_b E^{bm} (y)\, d^2y \right\}$$

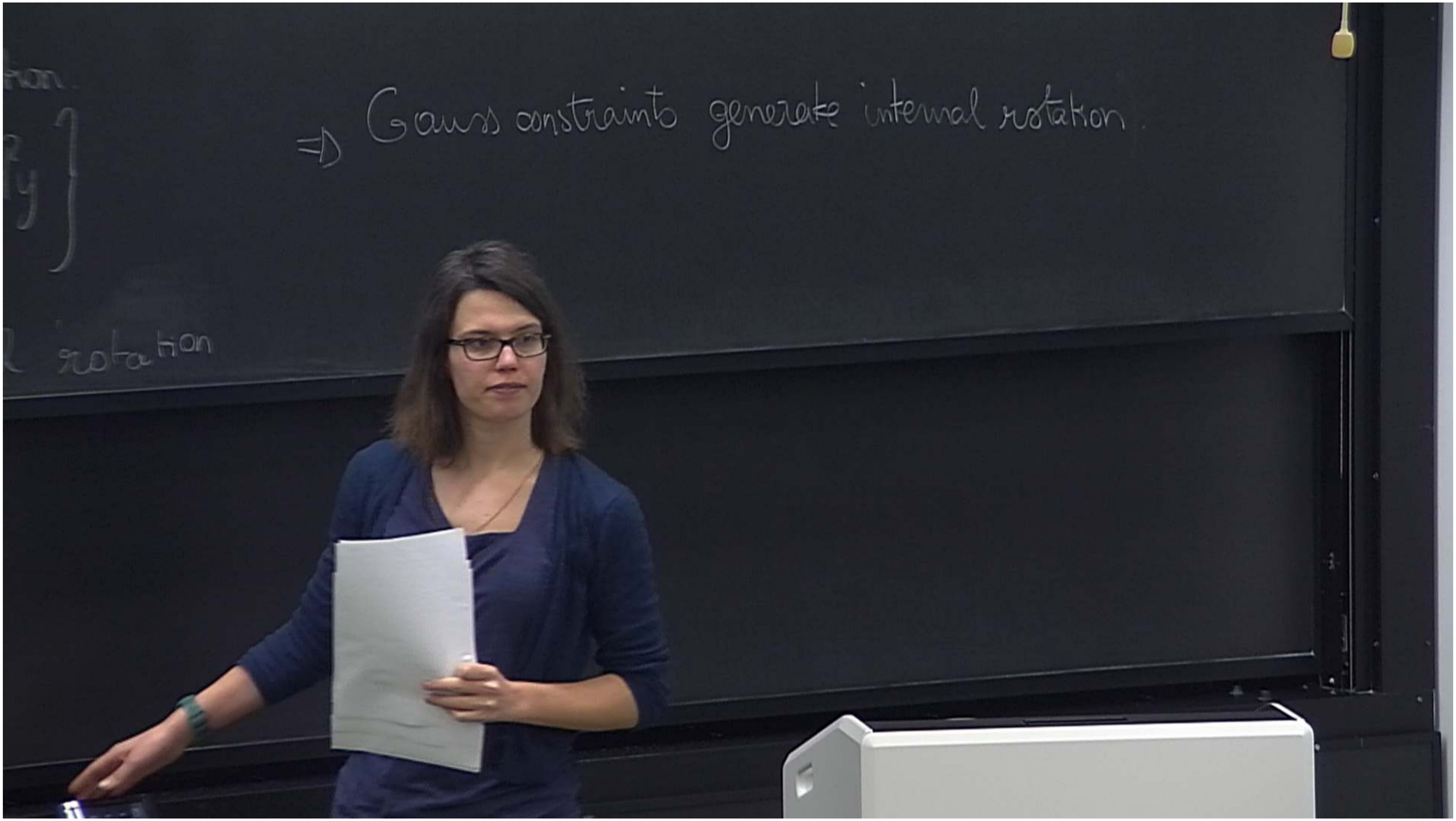
$$\delta_\Lambda A^j_a = -\left(\partial_a \Lambda^j + \varepsilon^j{}_{lm} A^l_a \Lambda^m \right)(x)$$

$$= -\varepsilon_{jmk} E^{am} \Lambda^k(x) \longrightarrow \text{internal rotation}$$

$\Rightarrow$ Gau

$$\delta_\Lambda A^j_a = \left\{ A^j_a(x) , \int \left(-\partial_b \Lambda^k E^b_k + \varepsilon_{klm} A^l_b E^{bm} \right)(y)\, d^2y \right\}$$

$$\delta_\Lambda A^j_a = -\left(\partial_a \Lambda^j + \varepsilon^j{}_{lm} A^l_a \Lambda^m \right)(x) \longrightarrow \text{internal rotation}$$


⇒ Gauss constraints generate internal rotation
rotation

Flatness constraints

$$\delta_N E^a_j(x) = \tilde{\varepsilon}^{ca}\left(\partial_c N^j + \varepsilon_{jmk} A^m_c E^k\right).$$

Flatness constraints

$$\delta_N E^a_j(x) = \tilde{\varepsilon}^{ca}\left(\partial_c N^j + \varepsilon_{jmk} A^m_c E^k\right).$$

$$\delta_N A^j_a(x) = 0.$$

$$\delta_\Lambda A^j_a = -\left(\partial_a \Lambda^j + \varepsilon^j{}_{lm} A^l_a \Lambda^m\right)(x) \longrightarrow \text{internal rotation}$$

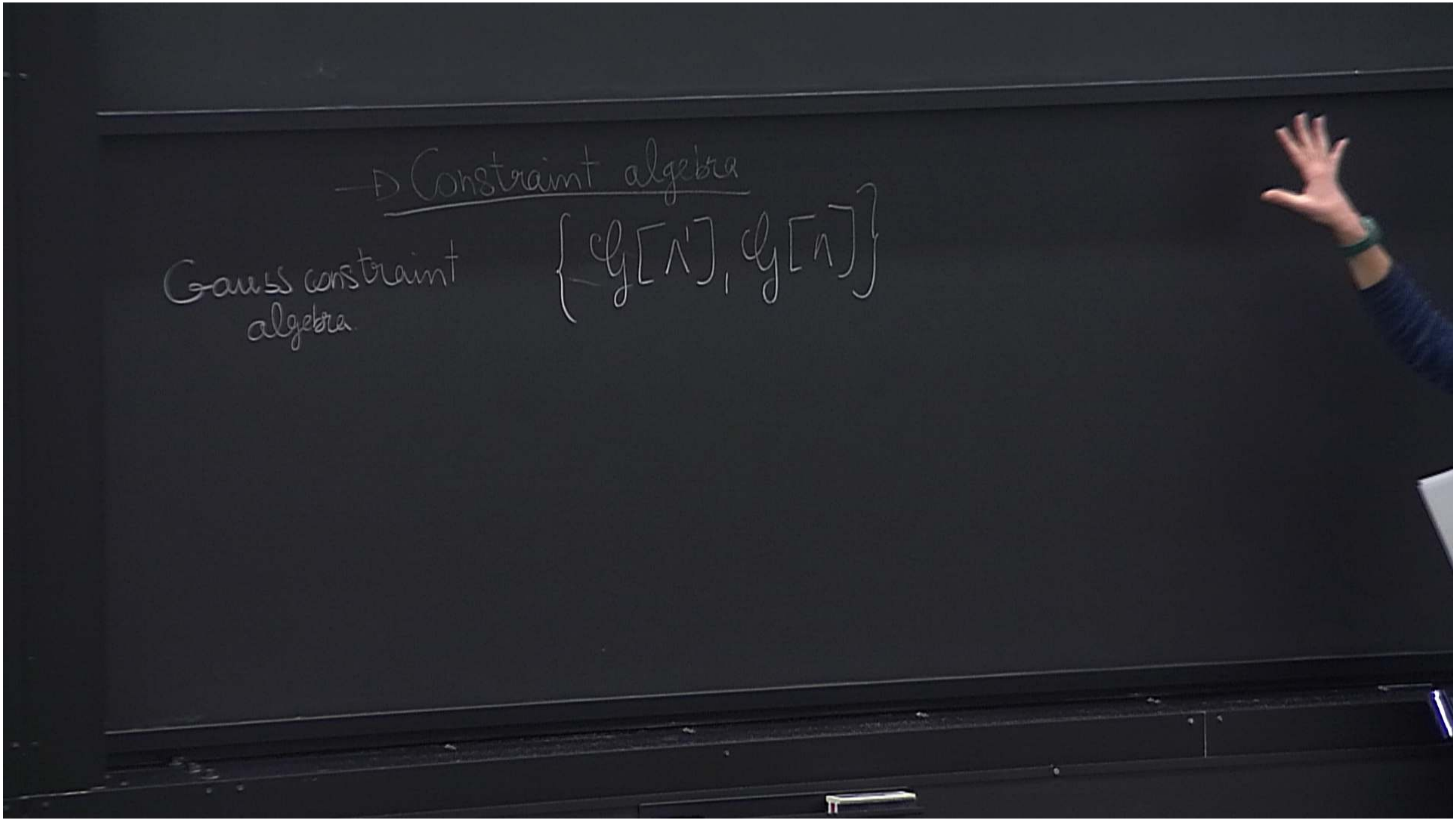
Flatness constraints

$$\delta_N E^a_j(x) = \tilde{\varepsilon}^{ca}\left(\partial_c N^j + \varepsilon_{jmk} A^m_c N^k\right).$$

Flatness constraints generate translation symmetry.

$$\delta_N A^j_a(x) = 0.$$

On shell diffeo = combination translation + rotation.


→ Constraint algebra
Gauss constraint algebra
$\{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\}$

raint algebra

$$\{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} \overset{\text{IBP}}{\overset{\checkmark}{=}} \Big\{ \int \Big(-E^b_k\, \partial_b (\Lambda')^k + \varepsilon_{klm} (\Lambda')^k A^l_b\, E^b_m \Big)(y)\, d^3y\, ,$$

raint algebra

$$\{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} \overset{\text{IBP}}{\underset{\checkmark}{=}} \left\{ \int \left(-E^b_k \partial_b (\Lambda')^k + \varepsilon_{klm} (\Lambda')^k A^l_b E^b_m \right)(y)\, d^3y,\ \mathcal{G}[\Lambda] \right\}$$

Constraint algebra

① IBP

$$\{\mathcal{G}[\Lambda'],\mathcal{G}[\Lambda]\} \overset{\checkmark}{=} \left\{\int\left(-E^b_k\,\partial_b(\Lambda')^k + \varepsilon_{klm}(\Lambda')^k A^l_b E^b_m\right)(y)\,d^3y,\ \mathcal{G}[\Lambda]\right\}$$

② Use action $\mathcal{G}[\Lambda]$ on A, E

$$= \int \partial_b(\Lambda')^k\,\varepsilon_{kml}\,E^{bm}\Lambda^l - \varepsilon_{klm}(\Lambda')^k\left(\partial_b\Lambda^l + \varepsilon^{lqp}A_b\right.$$

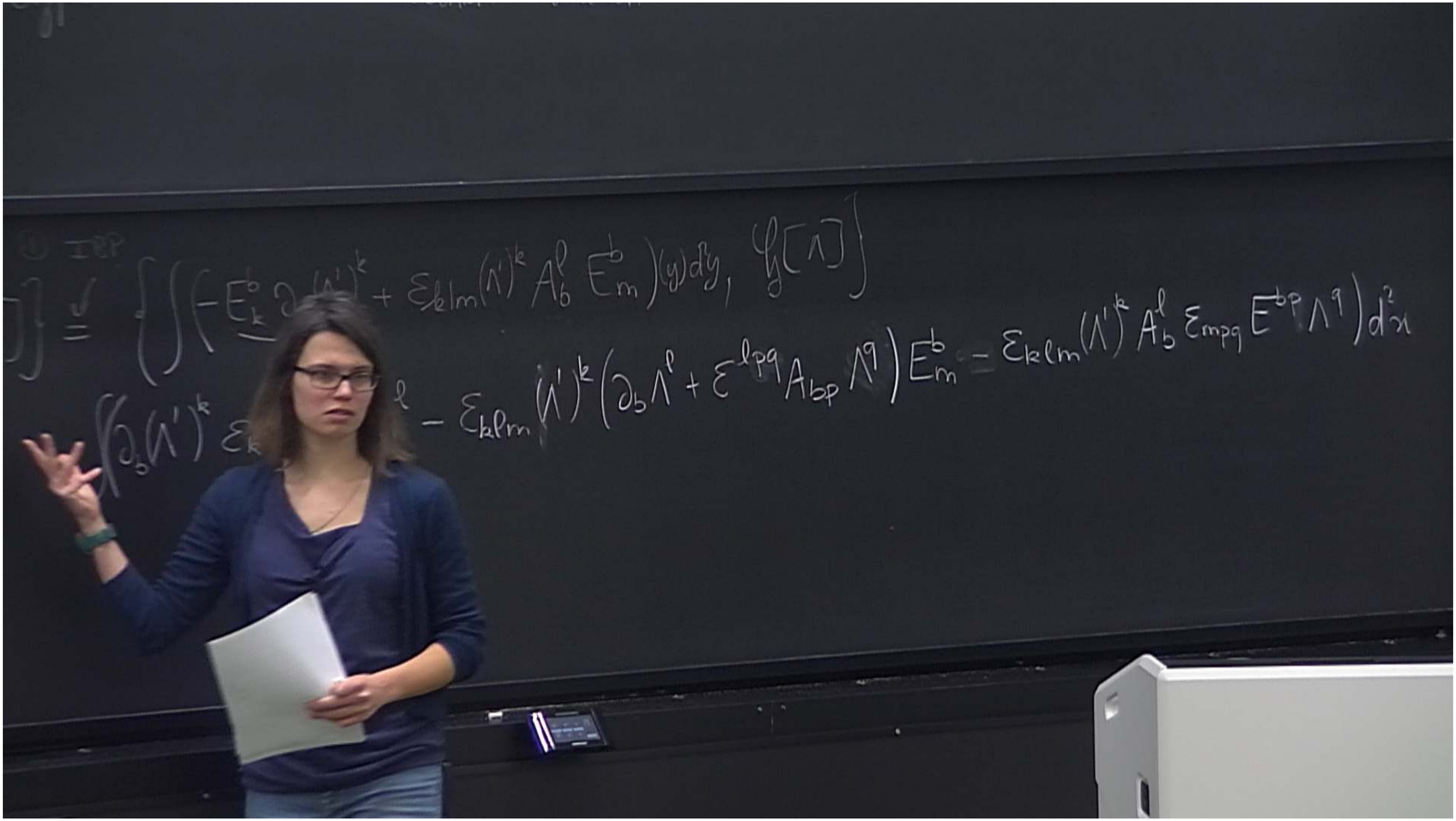
Constraint algebra

① IBP

$$\{\mathcal{G}[\Lambda'],\mathcal{G}[\Lambda]\} \overset{\checkmark}{=} \left\{\int\left(-\underline{E^b_k\,\partial_b(\Lambda')^k} + \varepsilon_{klm}(\Lambda')^k A^l_b\, E^b_m\right)(y)\,d^3y,\ \mathcal{G}[\Lambda]\right\}$$

② Use action $\mathcal{G}[\Lambda]$ on A, E

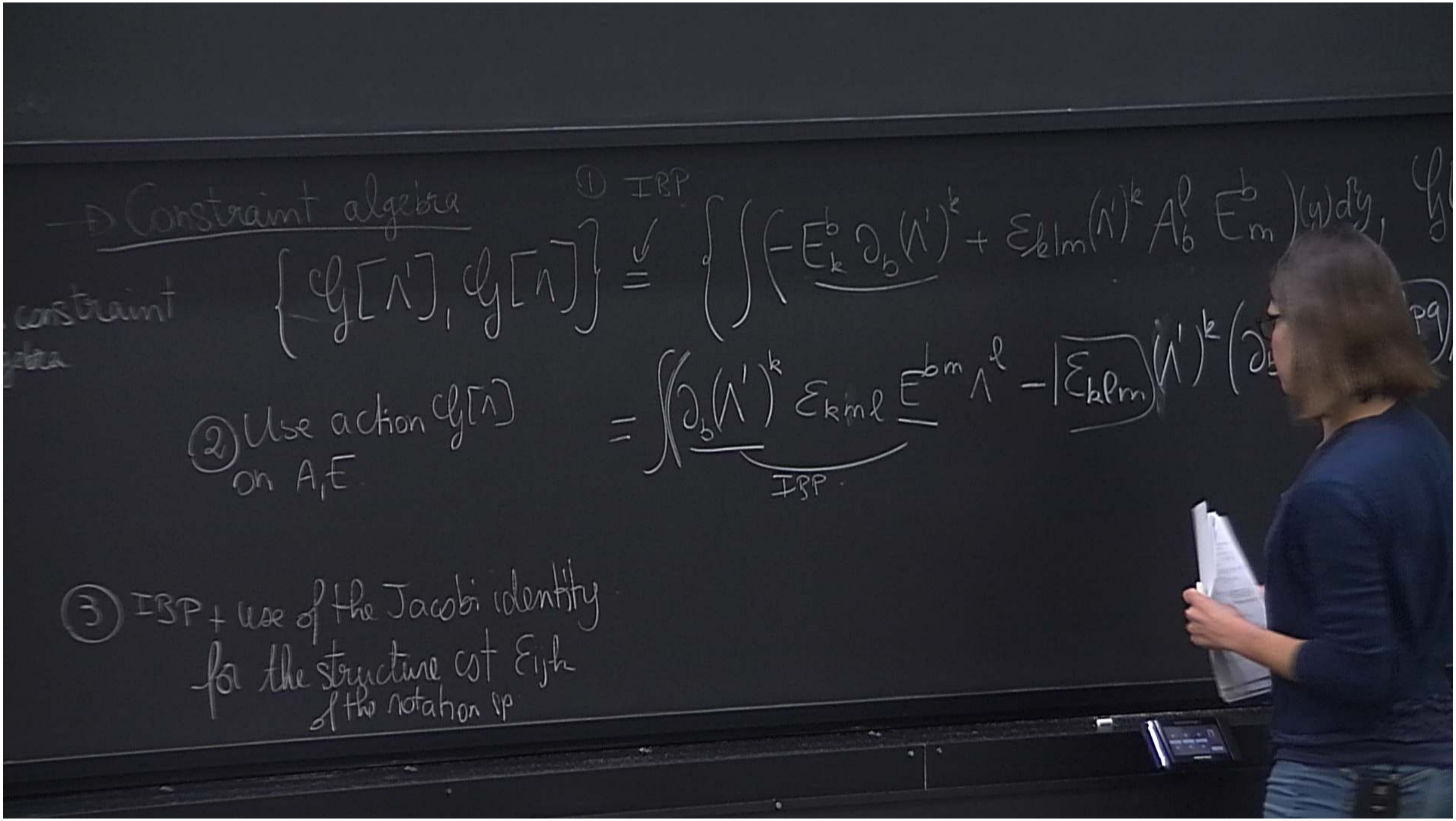
$$= \int \partial_b(\Lambda')^k\,\varepsilon_{kml}\,E^{bm}\Lambda^l - \varepsilon_{klm}(\Lambda')^k\left(\partial_b\Lambda^l + \varepsilon^{lpq}A_{bp}\,E\right.$$



① IBP

$$\}\ \overset{?}{=}\ \left\{\int\left(-\underbrace{E^b_k\,\partial_b(\Lambda')^k}\ +\ \varepsilon_{klm}(\Lambda')^k A^l_b\,E^b_m\right)(y)\,d^2y,\ \mathcal{G}_y[\Lambda]\right\}$$

$$=\int\Big(\underbrace{\underline{\partial_b}(\Lambda')^k\,\varepsilon_{kml}\,\underline{E}^{bm}}_{\text{IBP}}\,\Lambda^l-\boxed{\varepsilon_{klm}}(\Lambda')^k\left(\partial_b\Lambda^l+\boxed{\varepsilon^{lpq}}A_{bp}\Lambda^q\right)E^b_m-\boxed{\varepsilon_{klm}}(\Lambda')^k A^l_b\,\boxed{\varepsilon_{mpq}}\,E^{bp}\Lambda^q\Big)\,d^2x$$


Constraint algebra
constraint
① IBP
② Use action G[Λ] on A, E
IBP
③ IBP + use of the Jacobi identity
for the structure cst ε_ijk
of the rotation gp

constraint

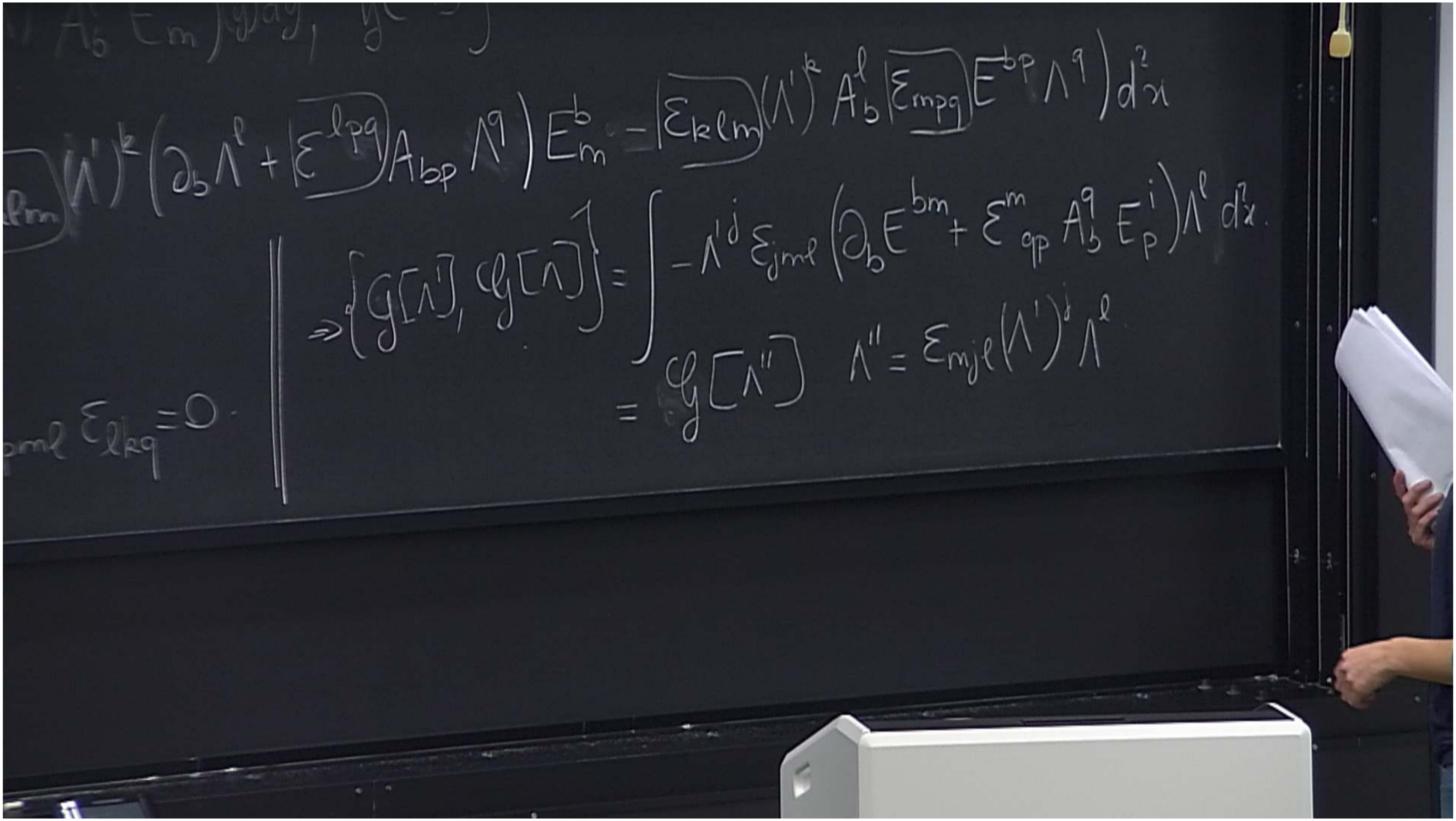
$$\{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} \stackrel{?}{=} \int \left(-E^b_k \partial_b (\Lambda') + \varepsilon_{klm} (\Lambda') A_b E_m \right)(y)\, dy$$

② Use action $\mathcal{G}[\Lambda]$ on A, E

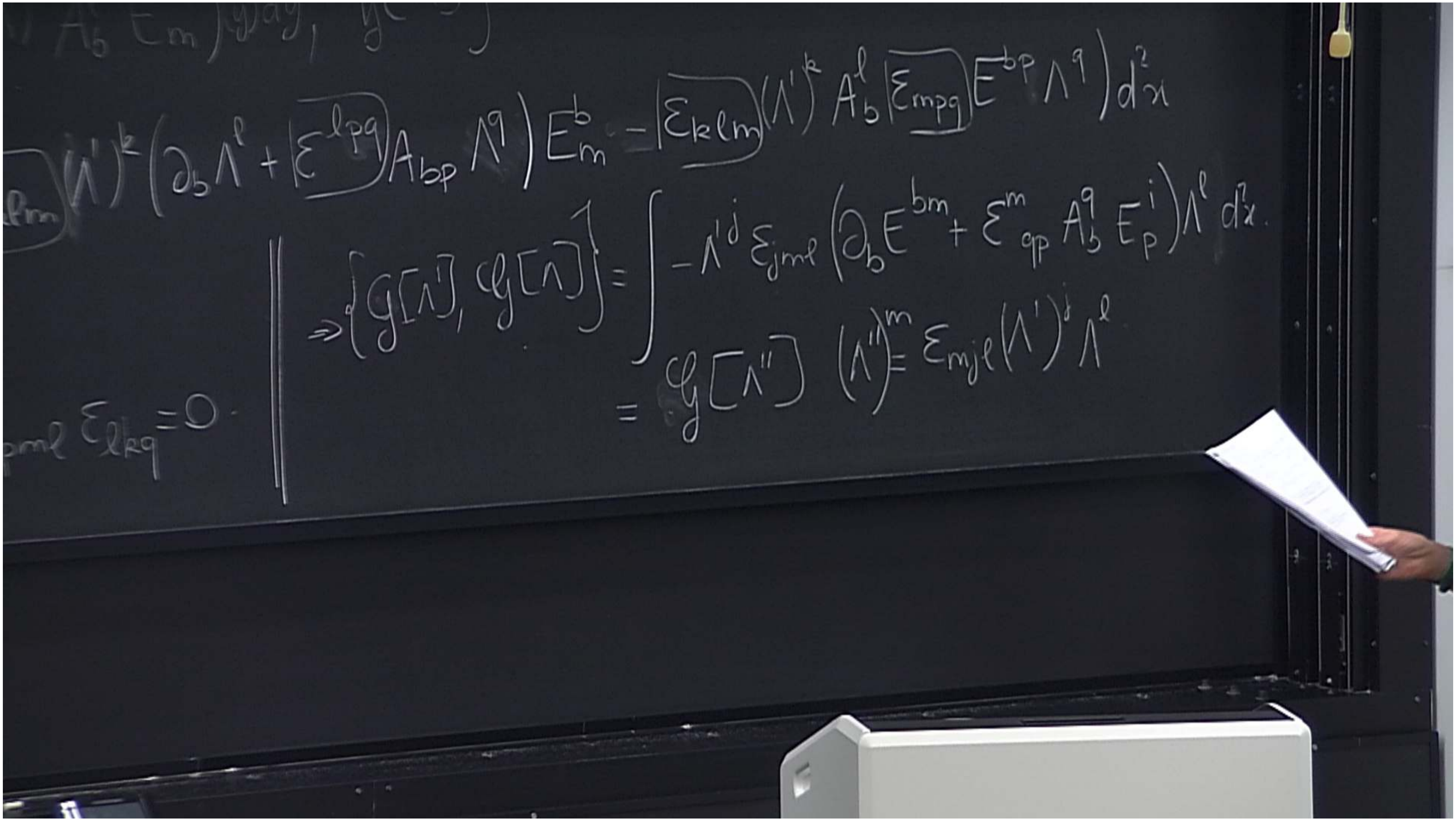
$$= \int \big(\underbrace{\partial_b (\Lambda')^k \varepsilon_{kml} E^{bm}}_{\text{IBP}} \Lambda^l - \boxed{\varepsilon_{klm}} (\Lambda')^k (\partial_b \Lambda^l + \boxed{\varepsilon^{lpq}}$$

③ IBP + use of the Jacobi identity for the structure cst ε_{ijk} of the rotation gp

$$\Rightarrow \varepsilon_{klm} \varepsilon_{lpq} + \varepsilon_{kpl} \varepsilon_{lmq} + \varepsilon_{pml} \varepsilon_{lkq} = 0.$$



$\varepsilon_{klm}(\Lambda')^k(\partial_b \Lambda^l + \varepsilon^{lpq} A_{bp} \Lambda^q) E^b_m - \varepsilon_{klm}(\Lambda')^k A^l_b \varepsilon_{mpq} E^{bp} \Lambda^q) d^2x$
$\Rightarrow \{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} = \int -\Lambda'^j \varepsilon_{jml}(\partial_b E^{bm} + \varepsilon^m{}_{qp} A^q_b E^i_p)\Lambda^l d^2x$
$= \mathcal{G}[\Lambda'']$
$\Lambda'' = \varepsilon_{mjl}(\Lambda')^j \Lambda^l$
$\varepsilon_{lkq} = 0.$


$(\Lambda')^k\left(\partial_b\Lambda^\ell + \varepsilon^{\ell pq} A_{bp}\Lambda^q\right)E^b_m - \varepsilon_{k\ell m}(\Lambda')^k A^\ell_b\, \varepsilon_{mpq} E^{bp}\Lambda^q\big)\, d^2x$
$\Rightarrow \{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} = \int -\Lambda'^j \varepsilon_{jm\ell}\left(\partial_b E^{bm} + \varepsilon^m{}_{qp} A^q_b E^i_p\right)\Lambda^\ell\, d^2x.$
$= \mathcal{G}[\Lambda''] \quad (\Lambda'')^m = \varepsilon_{mj\ell}(\Lambda')^j\Lambda^\ell$
$\varepsilon_{\ell kq} = 0.$

Λ, Λ'

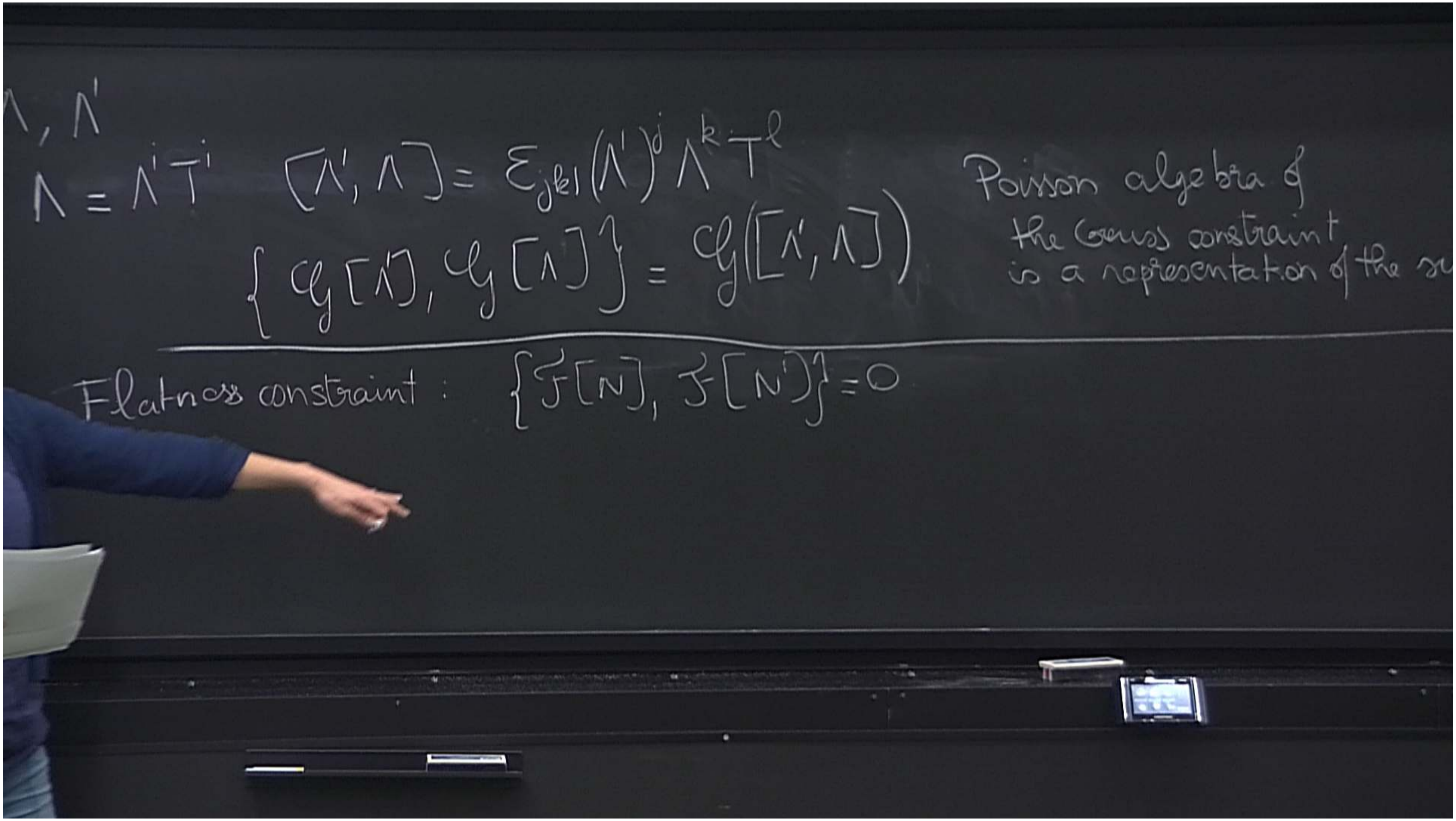
$\Lambda = \Lambda^i T^i \qquad [\Lambda', \Lambda] = \varepsilon_{jkl} (\Lambda')^j \Lambda^k T^l$ Poisson alg

$\{ \mathcal{G}[\Lambda'], \mathcal{G}[\Lambda] \} = \mathcal{G}([\Lambda', \Lambda])$

$$, \Lambda] = \varepsilon_{jkl} (\Lambda')^j \Lambda^k T^l$$

$$[\Lambda'], \mathcal{G}[\Lambda] \} = \mathcal{G}([\Lambda', \Lambda])$$

Poisson algebra of
the Gauss constraint
is a representation of the su(2) lie algebra.


Λ, Λ'
$\Lambda = \Lambda^i T^i$
$[\Lambda', \Lambda] = \varepsilon_{jkl} (\Lambda')^j \Lambda^k T^l$
$\{ \mathcal{G}[\Lambda'], \mathcal{G}[\Lambda] \} = \mathcal{G}([\Lambda', \Lambda])$
Poisson algebra of the Gauss constraint is a representation of the su
Flatness constraint : $\{ \mathcal{F}[N], \mathcal{F}[N'] \} = 0$

Λ, Λ'

$\Lambda = \Lambda^i T^i \qquad [\Lambda', \Lambda] = \varepsilon_{jkl} (\Lambda')^j \Lambda^k T^l$

$\{ \mathcal{G}[\Lambda'], \mathcal{G}[\Lambda] \} = \mathcal{G}([\Lambda', \Lambda])$

Poisson algebra of the Gauss constraint is a representation of the su

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- Flatness constraint: $\{ \mathcal{F}[N], \mathcal{F}[N'] \} = 0$
- $\{ F[N], \mathcal{G}[\Lambda] \} = \mathcal{F}[[N, \Lambda]]$

Λ, Λ'

$\Lambda = \Lambda^i T^i \qquad [\Lambda', \Lambda] = \varepsilon_{jkl} (\Lambda')^j \Lambda^k T^l$

$\{ \mathcal{G}[\Lambda'], \mathcal{G}[\Lambda] \} = \mathcal{G}([\Lambda', \Lambda])$

Poisson algebra of the Gauss constraint is a representation of the su

- Flatness constraint: $\{ \mathcal{F}[N], \mathcal{F}[N'] \} = 0$
- $\{ F[N], \mathcal{G}[\Lambda] \} = \mathcal{F}[[N, \Lambda]] = -\int N^j \varepsilon_{jmk} \mathcal{F}^m \Lambda^k d^3x$

Λ, Λ'

$\Lambda = \Lambda^i T^i \quad [\Lambda', \Lambda] = \varepsilon_{jkl} (\Lambda')^j \Lambda^k T^l$

$$\{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} = \mathcal{G}([\Lambda', \Lambda])$$

Poisson algebra of the Gauss constraint is a representation of the su(2) lie algebra

– Flatness constraint : $\{\mathcal{F}[N], \mathcal{F}[N']\} = 0$

$$- \{F[N], \mathcal{G}[\Lambda]\} = \mathcal{F}[[N, \Lambda]] = -\int N^j \varepsilon_{jmk} \mathcal{F}^m \Lambda^k d^2x$$

algebra of
auss constraint
representation of the su(2) lie algebra.

x

Constraint algebra.

$$\{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} = \mathcal{G}([\Lambda', \Lambda])$$

$$\{\vec{\mathcal{F}}[N], \vec{\mathcal{F}}[N']\} = 0$$

$$\{\vec{\mathcal{F}}[N], \mathcal{G}[\Lambda]\} = \vec{\mathcal{F}}([N, \Lambda])$$

algebra of
auss constraint
representation of the $su(2)$ lie algebra.

x

Constraint algebra

$$\{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} = \mathcal{G}([\Lambda', \Lambda])$$

$$\{\vec{\mathcal{F}}[N], \vec{\mathcal{F}}[N']\} = 0$$

$$\{\vec{\mathcal{F}}[N], \mathcal{G}[\Lambda]\} = \vec{\mathcal{F}}([N, \Lambda])$$

<u>Closed</u> algebra.

algebra of

...auss constraint

representation of the su(2) lie algebra.

*

...face.

$\{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} = \mathcal{G}([\Lambda', \Lambda])$

$\{\vec{F}[N], \vec{F}[N]\} = 0$

$\{\vec{F}[N], \mathcal{G}[\Lambda]\} = \vec{F}([N, \Lambda])$

Closed algebra.

$\rightarrow 1^{st}$

* Interpretation of our constraints.
* Constraint algebra.

algebra of

...ass constraint

representation of the su(2) lie algebra.

$$\{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} = \mathcal{G}([\Lambda, \Lambda'])$$

$$\{\vec{\mathcal{F}}[N], \vec{\mathcal{F}}[N']\} = 0$$

$$\{\vec{\mathcal{F}}[N], \mathcal{G}[\Lambda]\} = \vec{\mathcal{F}}([N, \Lambda])$$

Closed algebra.

$\rightarrow$ 1st class constraint.

* Interpretation of our constraints.

* Constraint algebra.

...face.

algebra of

...ss constraint

representation of the $su(2)$ lie algebra.

$$\{\mathcal{G}[\Lambda'], \mathcal{G}[\Lambda]\} = \mathcal{G}([\Lambda', \Lambda])$$

$$\{\vec{\mathcal{F}}[N], \vec{\mathcal{F}}[N]\} = 0$$

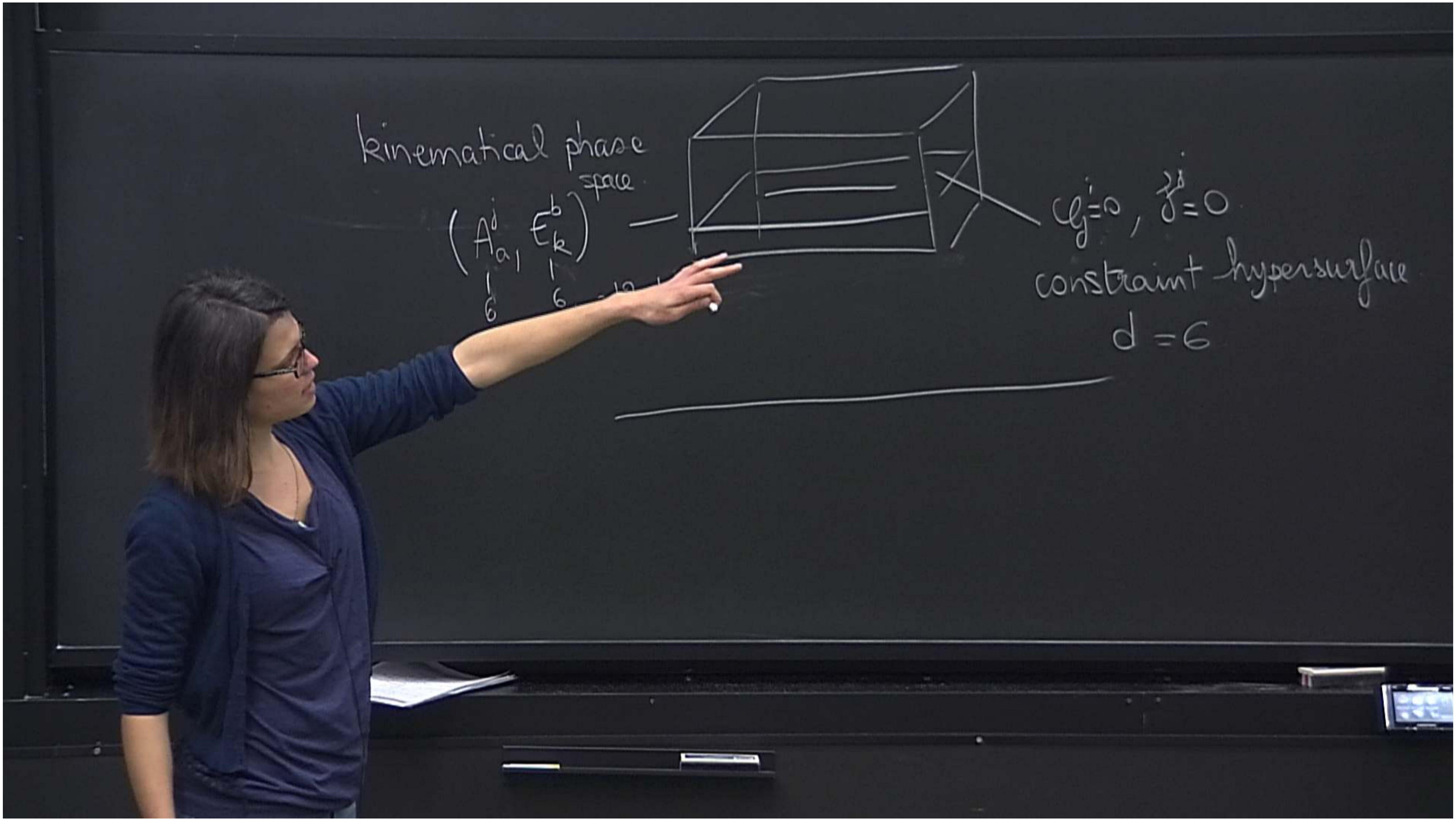
$$\{\vec{\mathcal{F}}[N], \mathcal{G}[\Lambda]\} = \vec{\mathcal{F}}([N, \Lambda])$$

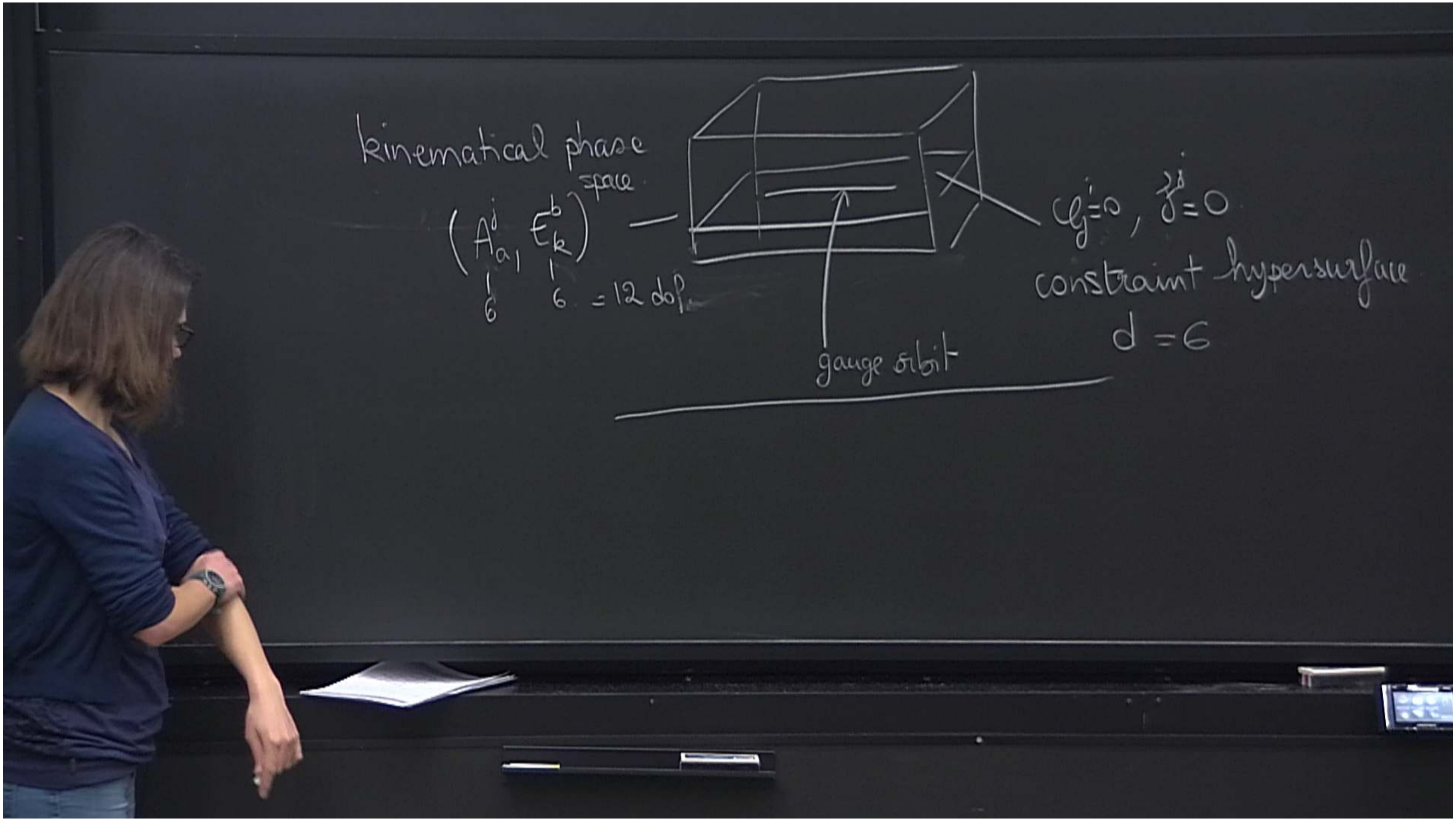
Closed algebra.

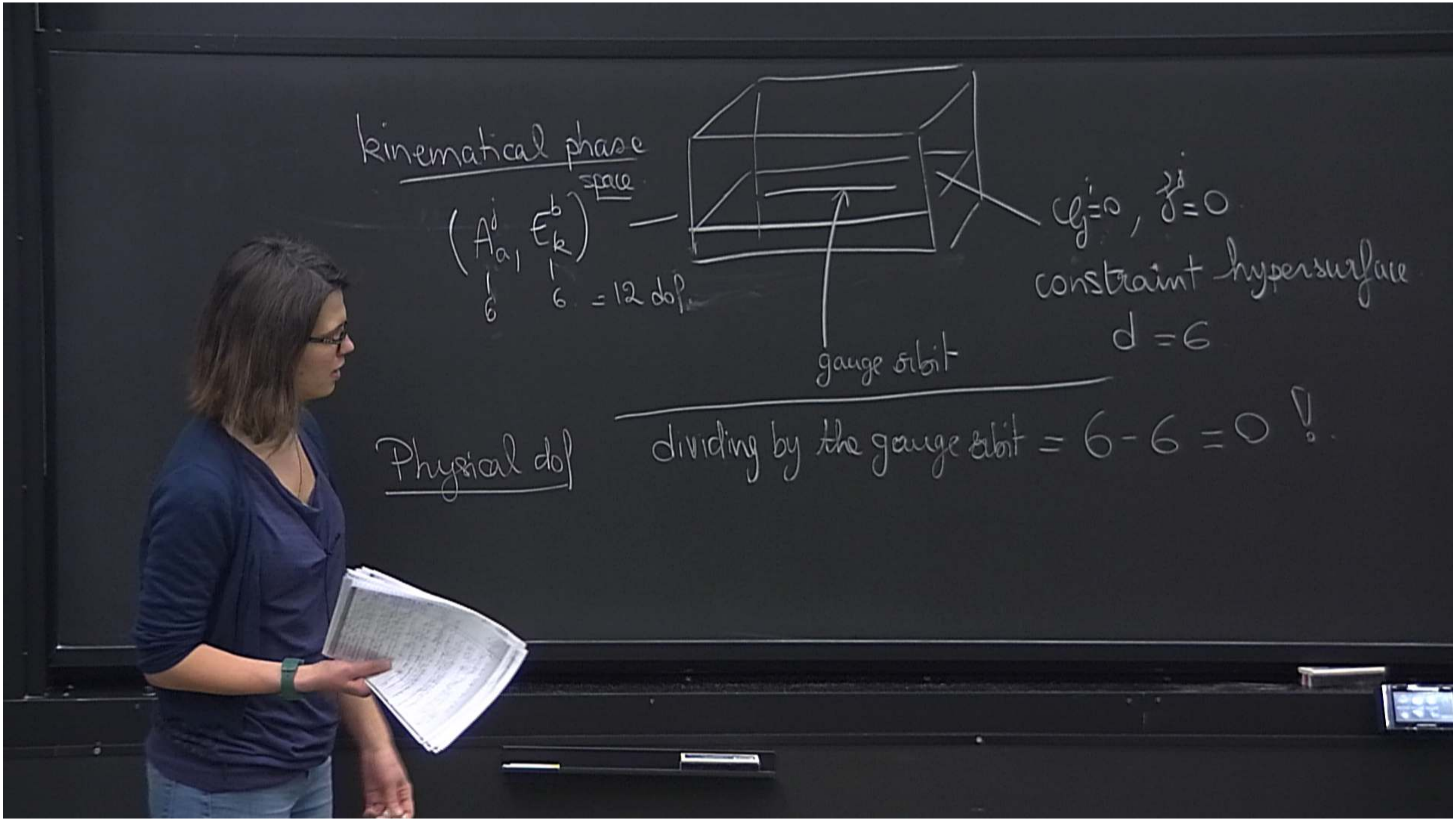
$\mathbb{R}^3 \rtimes so(3)$.

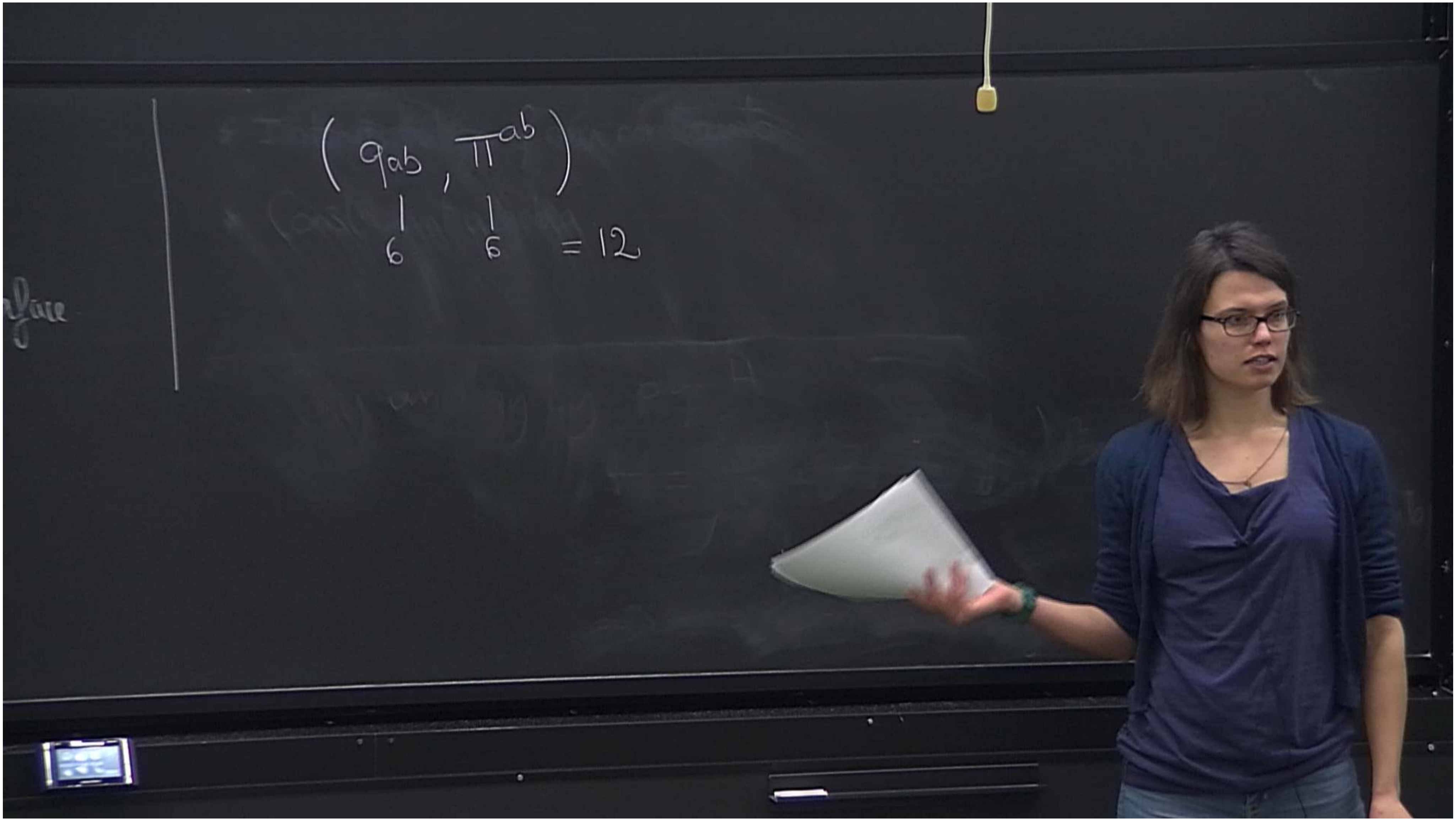
$\rightarrow$ 1st class constraint.

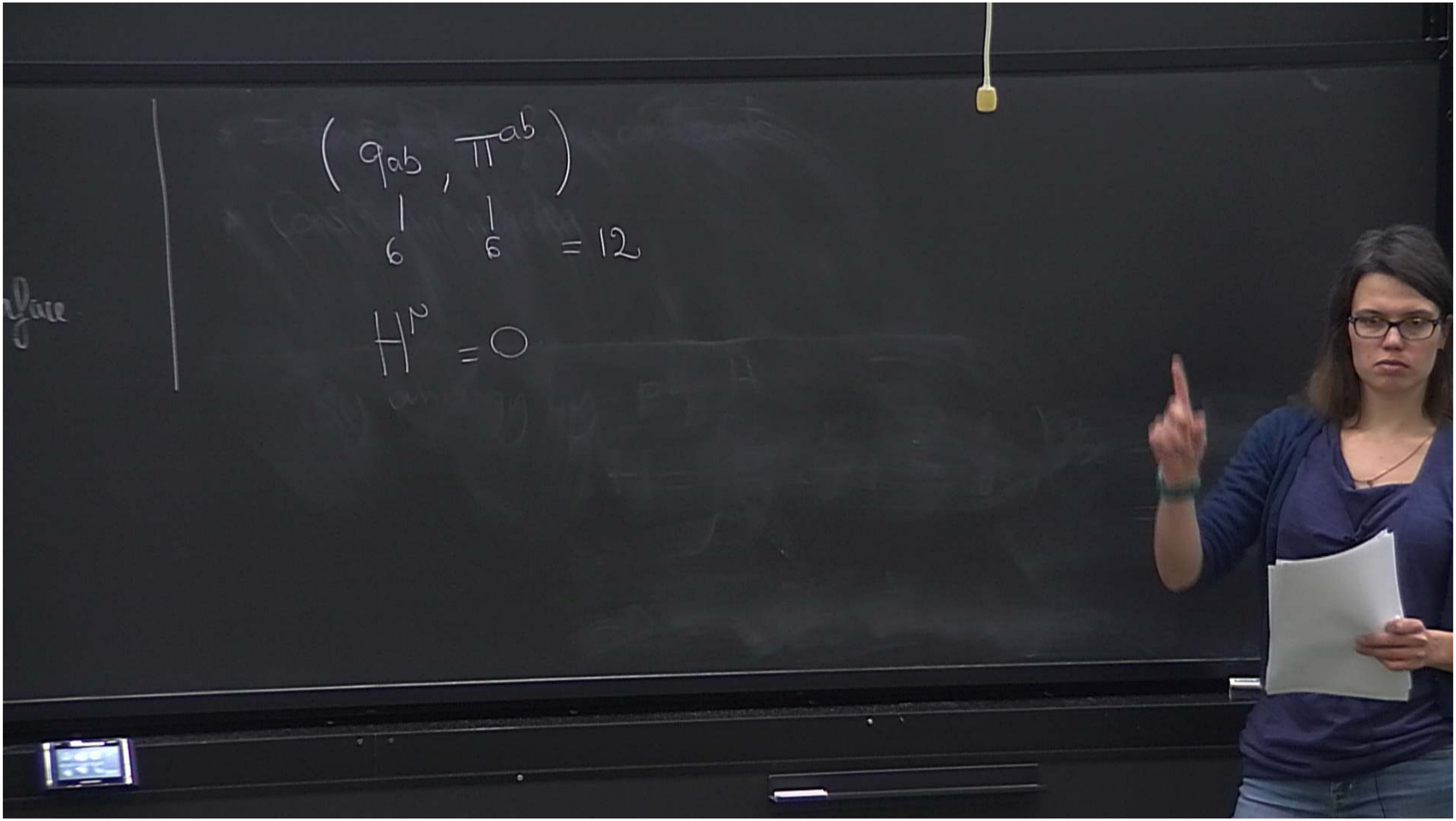
* Interpretation of our constraints.
* Constraint algebra.


kinematical phase space
constraint hypersurface
d = 6


kinematical phase space
(A^j_a, E^b_k)
6 6 = 12 dof
$G_j = 0$, $J^j = 0$
constraint hypersurface
d = 6
gauge orbit


kinematical phase space
(A^j_a, E^b_k)
6 6 = 12 dof
$G^j=0$, $V^j=0$
constraint hypersurface
d = 6
gauge orbit
Physical dof
dividing by the gauge orbit = 6 - 6 = 0 !


(q_{ab}, π^{ab})
$6 \quad 6 = 12$



(q_{ab}, π^{ab})
6 6 = 12
$H^{\nu} = 0$

...rface

(q_{ab}, π^{ab})

$6 \quad 6 \quad = 12$

$H^\nu = 0 \leftarrow 4$

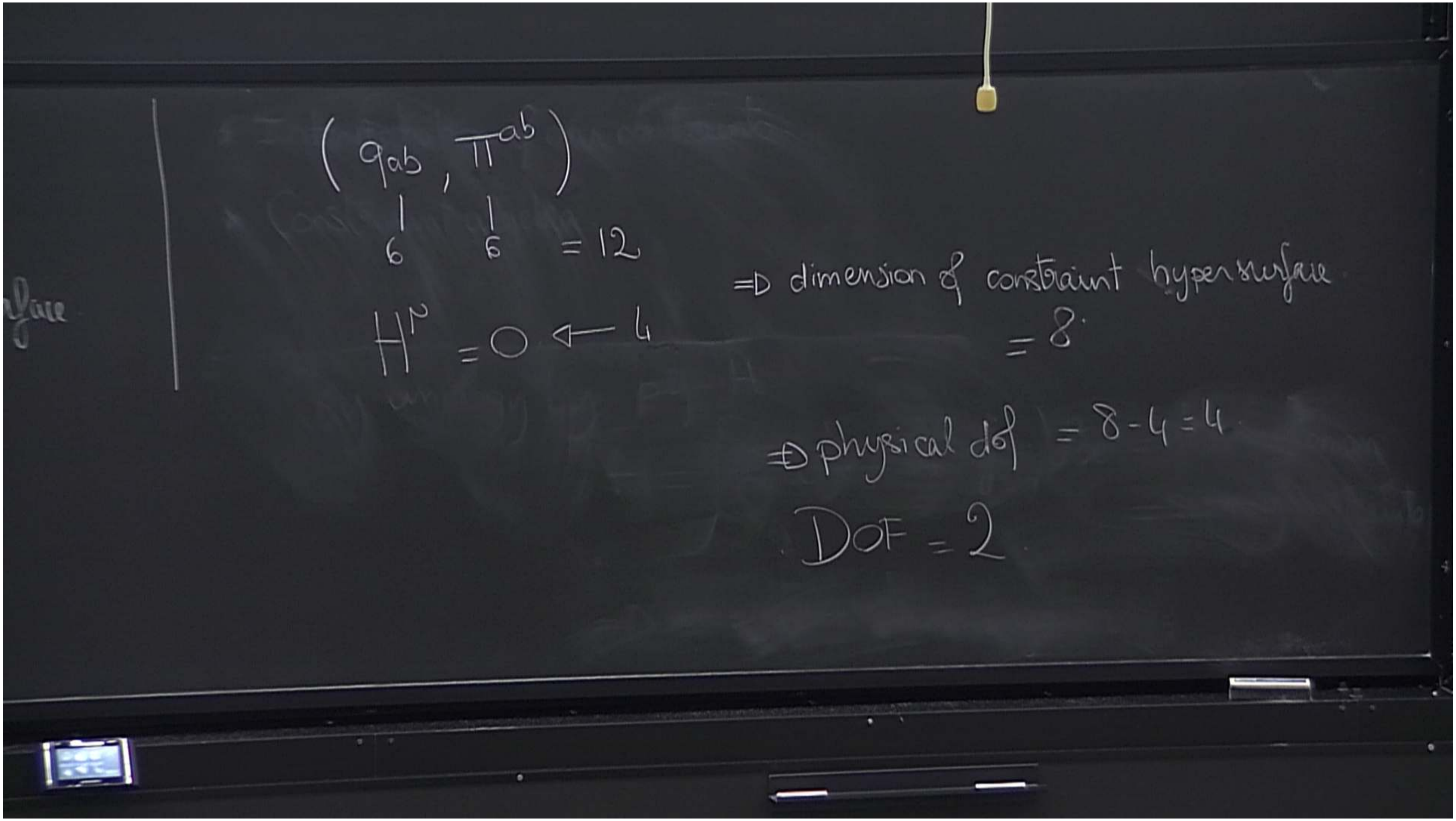
$\Rightarrow$ dimension of constraint hypersurface $= 8$.

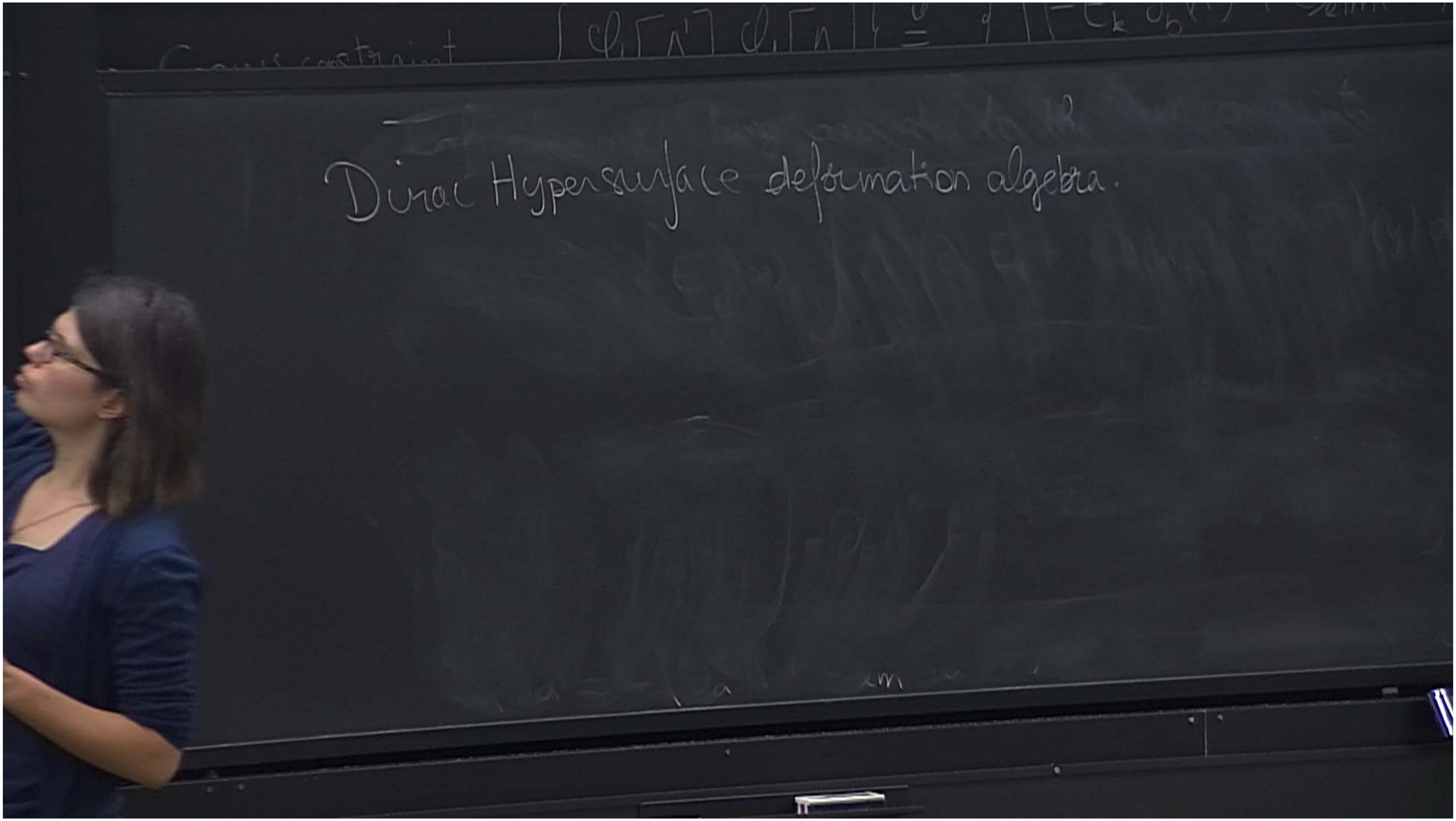
$$\left(\underset{\substack{| \\ 6}}{q_{ab}} \, , \, \underset{\substack{| \\ 6}}{\pi^{ab}} \right) \quad = 12$$

$$H^{\mu} = 0 \leftarrow 4$$

$\Rightarrow$ dimension of constraint hypersurface $= 8$.

$\Rightarrow$ physical dof $= 8 - 4 = 4$.


(q_{ab}, π^{ab})
$6 \quad 6 \quad = 12$
$H^{\nu} = 0 \leftarrow 4$
⇒ dimension of constraint hypersurface
$= 8$
⇒ physical dof $= 8 - 4 = 4$
DOF = 2

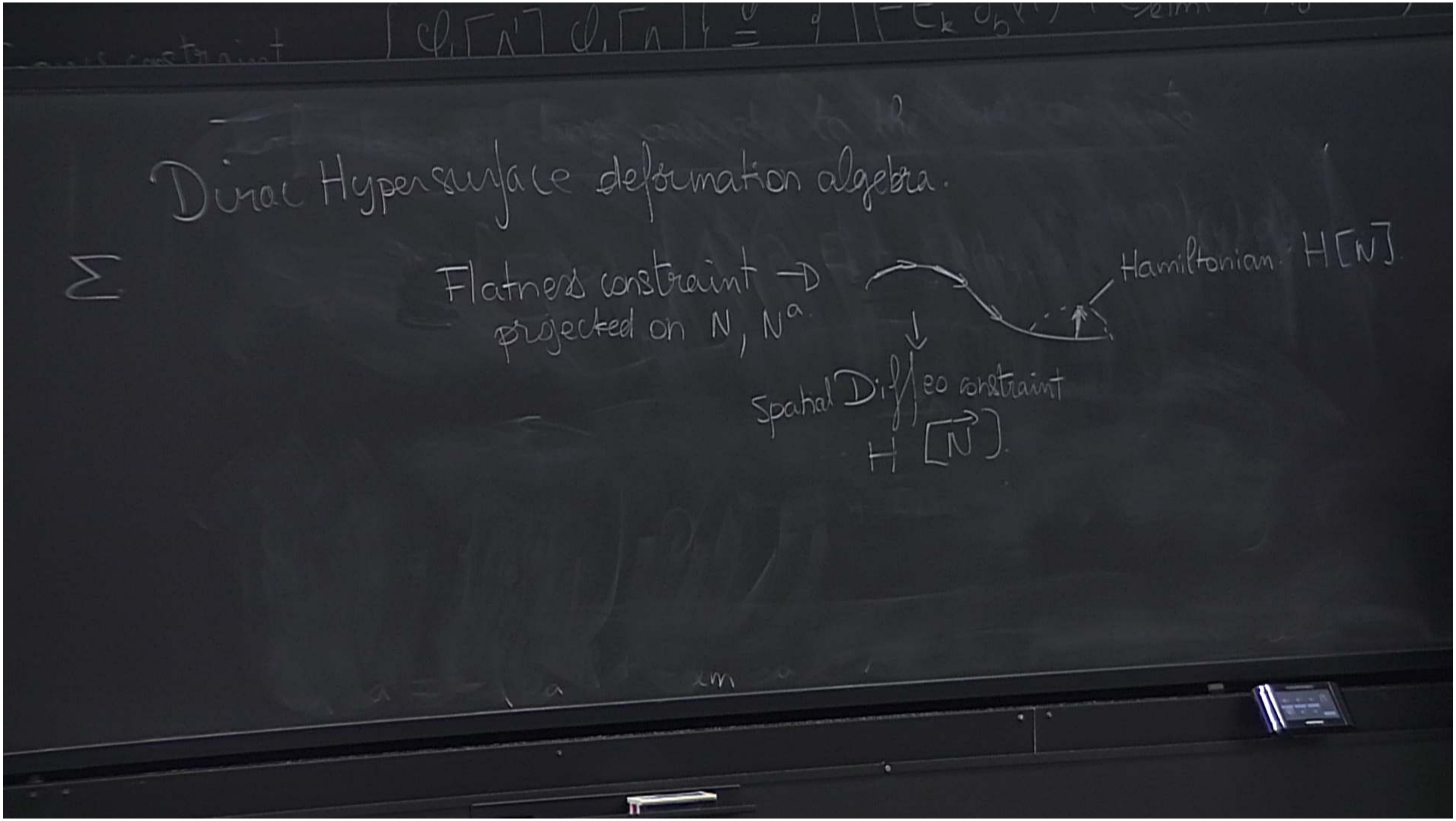

Dirac Hypersurface deformation algebra.

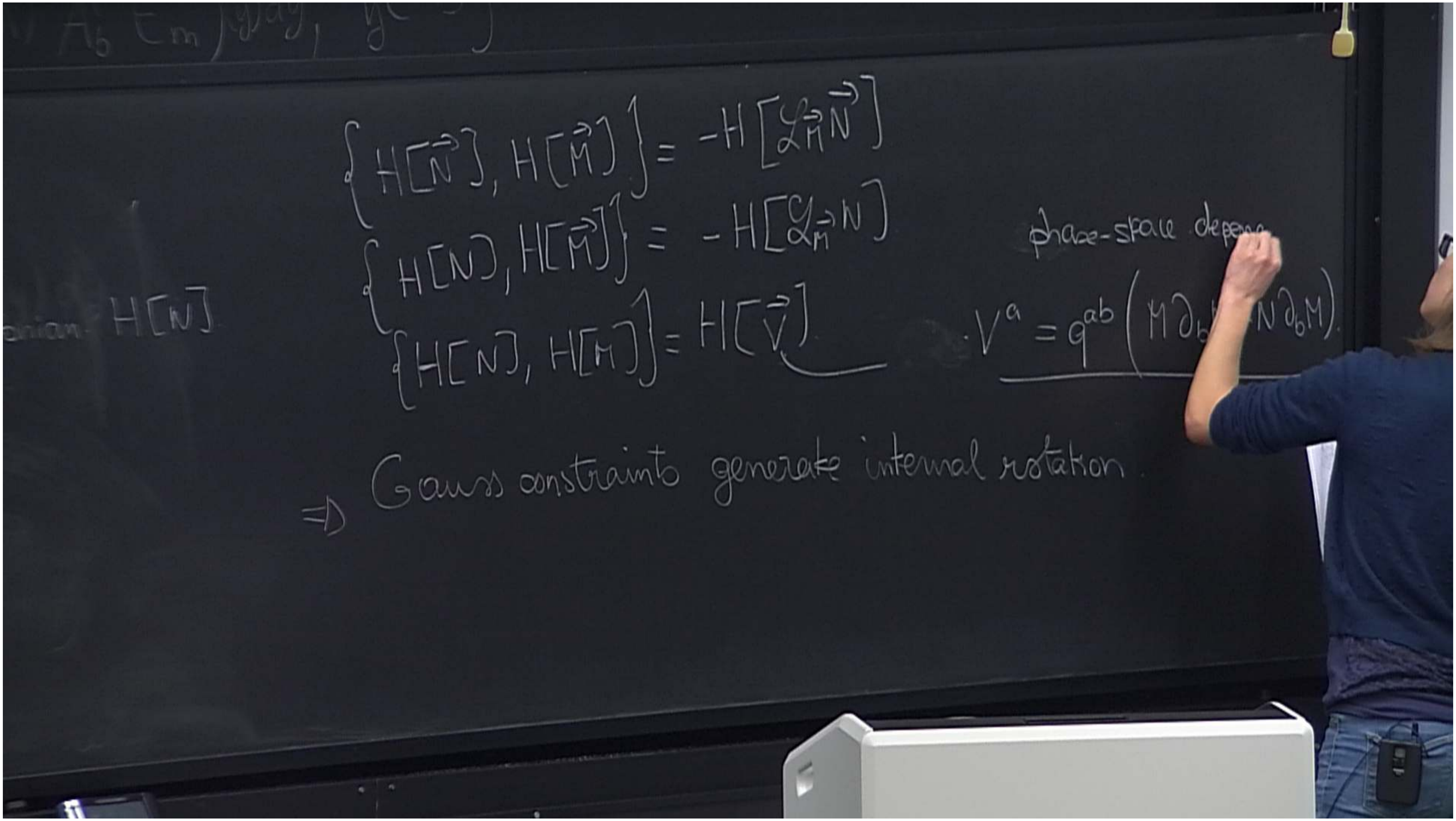
Dirac Hypersurface deformation algebra.

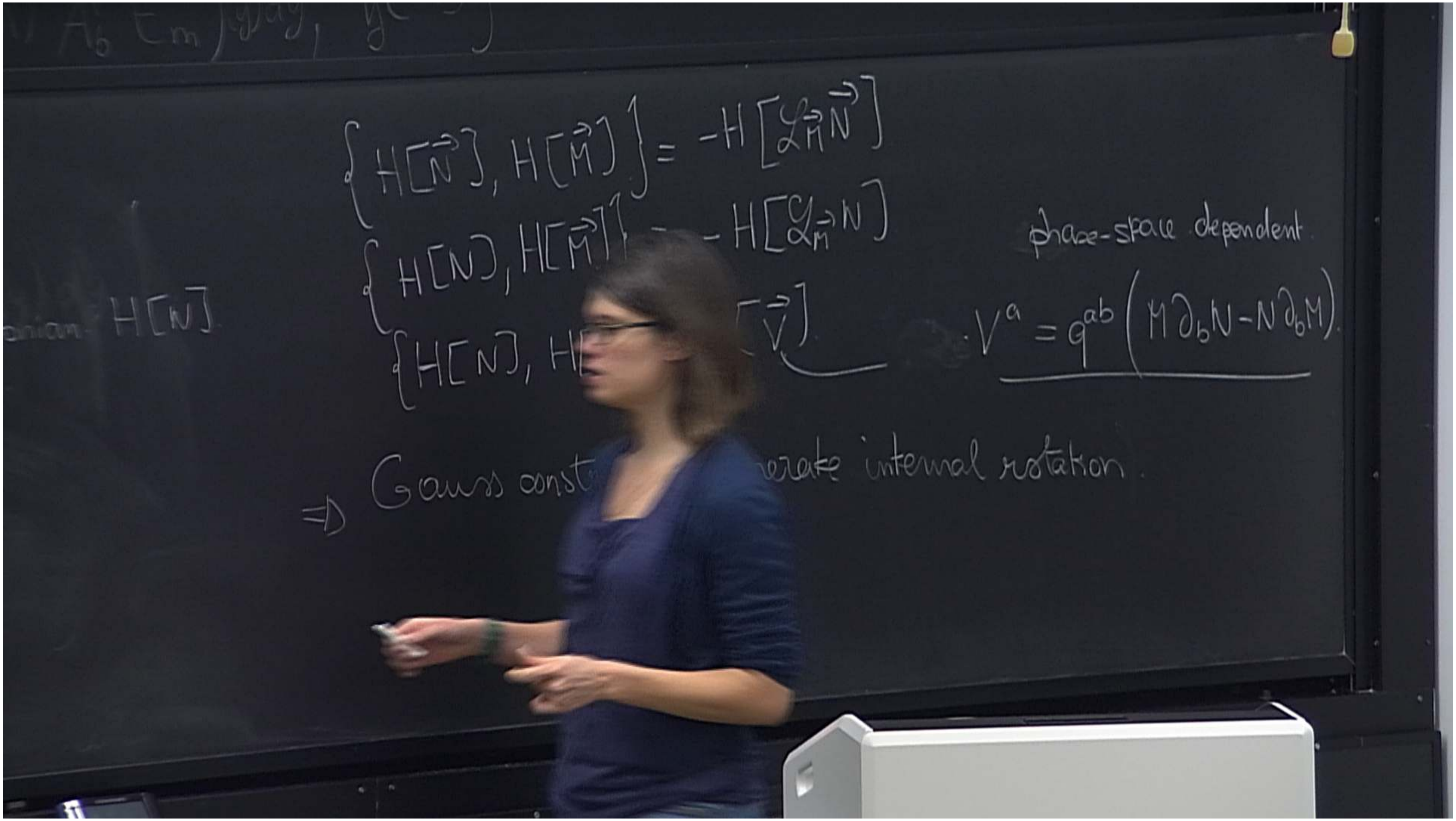
Σ

Flatness constraint $\rightarrow$
projected on N, N^a

Spatial Diffeo constraint


Dirac Hypersurface deformation algebra.
Σ
Flatness constraint →
projected on N, N^a.
Hamiltonian H[N]
Spatial Diffeo constraint
H[N⃗]


$\{H[\vec{N}], H[\vec{M}]\} = -H[\mathcal{L}_{\vec{M}}\vec{N}]$
$\{H[N], H[\vec{M}]\} = -H[\mathcal{L}_{\vec{M}}N]$
$\{H[N], H[M]\} = H[\vec{V}]$
phase-space depen
$V^a = q^{ab}(M\partial_b$
$N\partial_b M)$
$\Rightarrow$ Gauss constraints generate internal rotation


{H[N⃗], H[M⃗]} = −H[𝓛_M⃗ N⃗]
{H[N], H[M⃗]}
− H[𝓛_M⃗ N]
{H[N], H
[V⃗]
phase-space dependent
V^a = q^ab (M∂_b N − N∂_b M)
H[N]
Gauss const
erate internal rotation