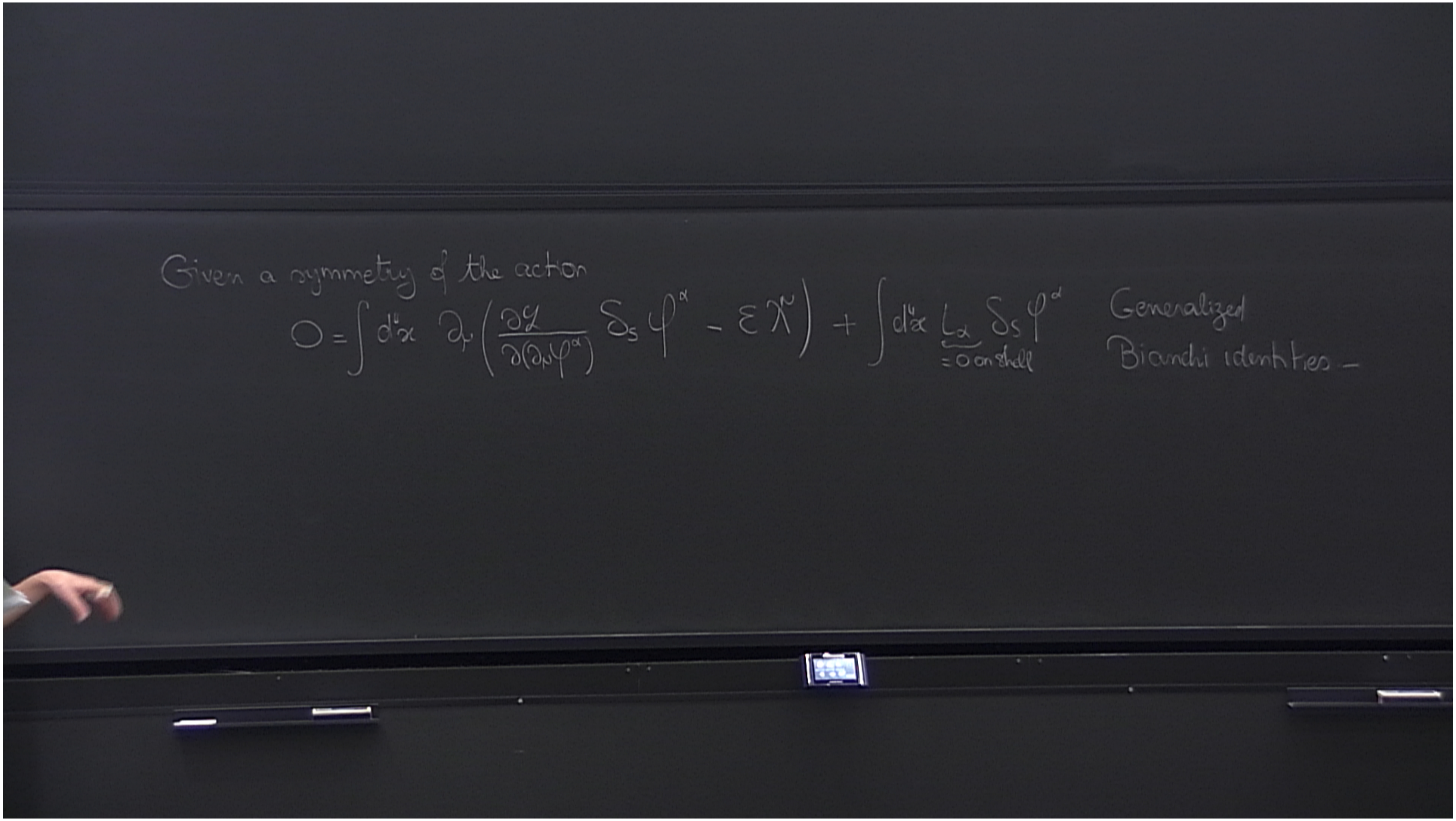
Title: PSI 2015/2016 Quantum Gravity - Lecture 4

Date: Feb 25, 2016 10:15 AM

URL: http://pirsa.org/16020075

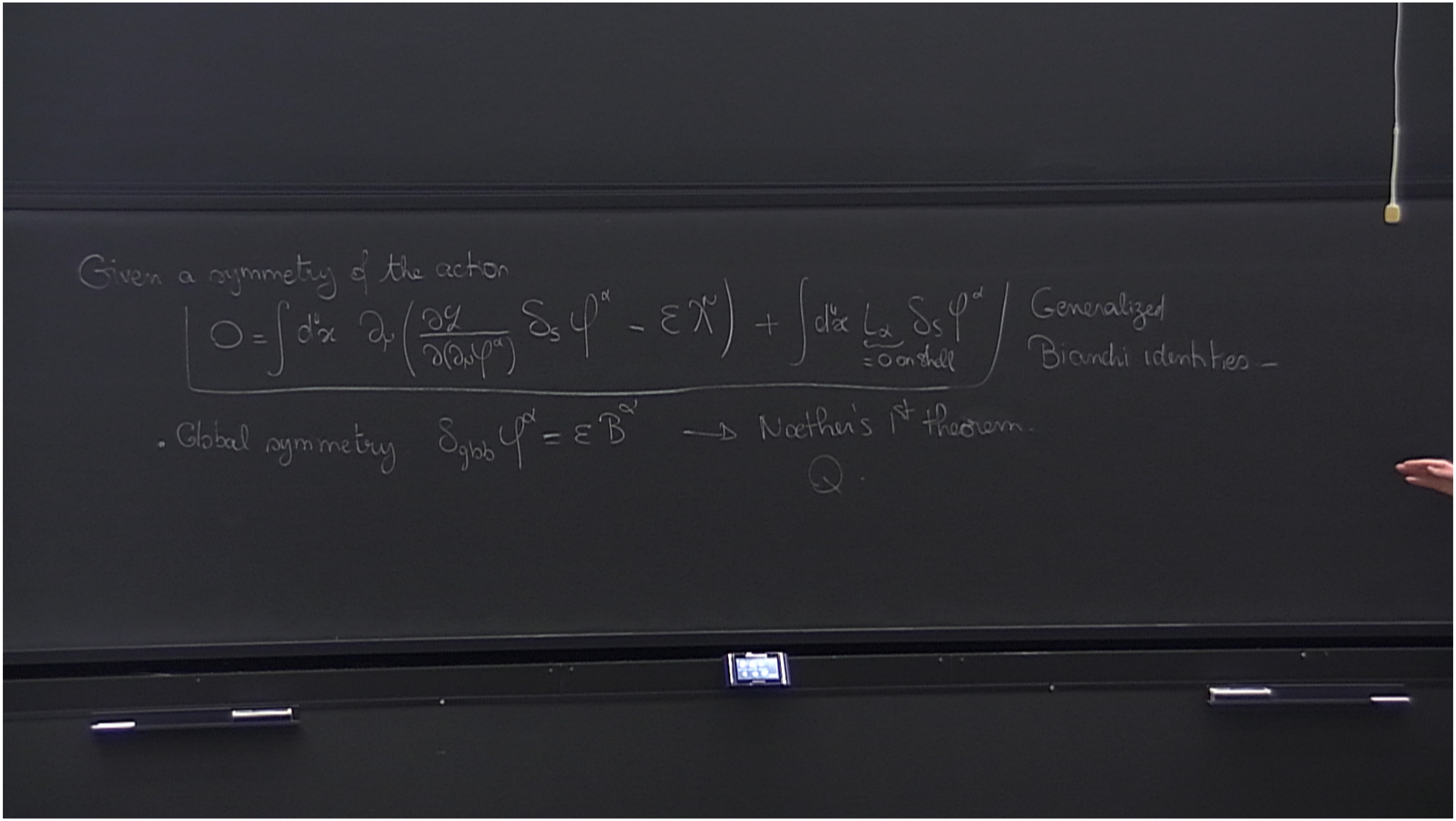
Abstract:



Given a symmetry of the action

$$0 = \int d^4x \; \partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \varphi^\alpha)} \delta_s \varphi^\alpha - \varepsilon \chi^\nu \right) + \int d^4x \; \underbrace{L_\alpha}_{=0 \text{ on shell}} \delta_s \varphi^\alpha$$

Generalized Bianchi identities –



Given a symmetry of the action

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Generalized Bianchi identities –

- Global symmetry $\delta_{glob} \varphi^\alpha = \varepsilon B^\alpha$ → Noether's 1st theorem.

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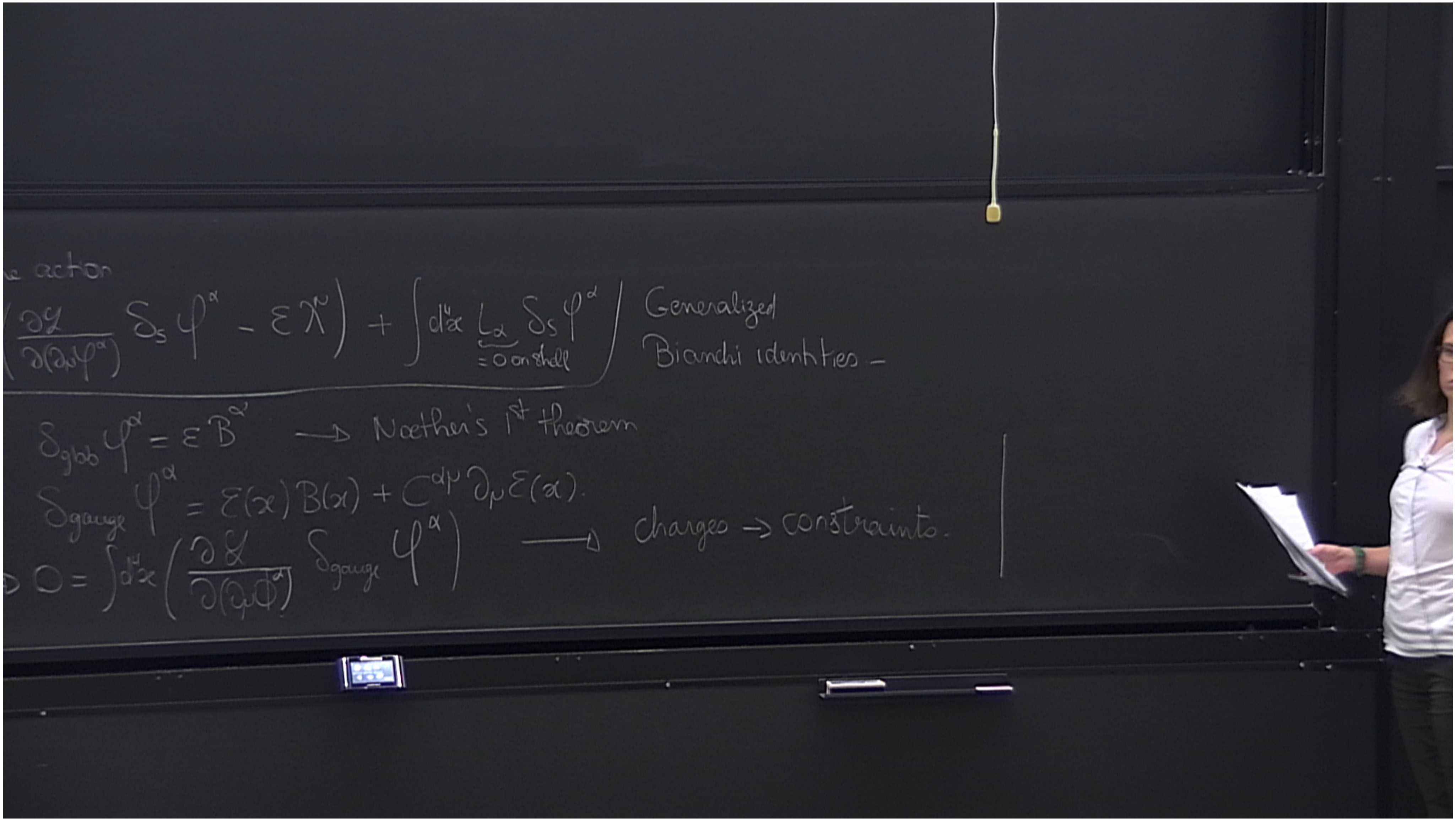


e action

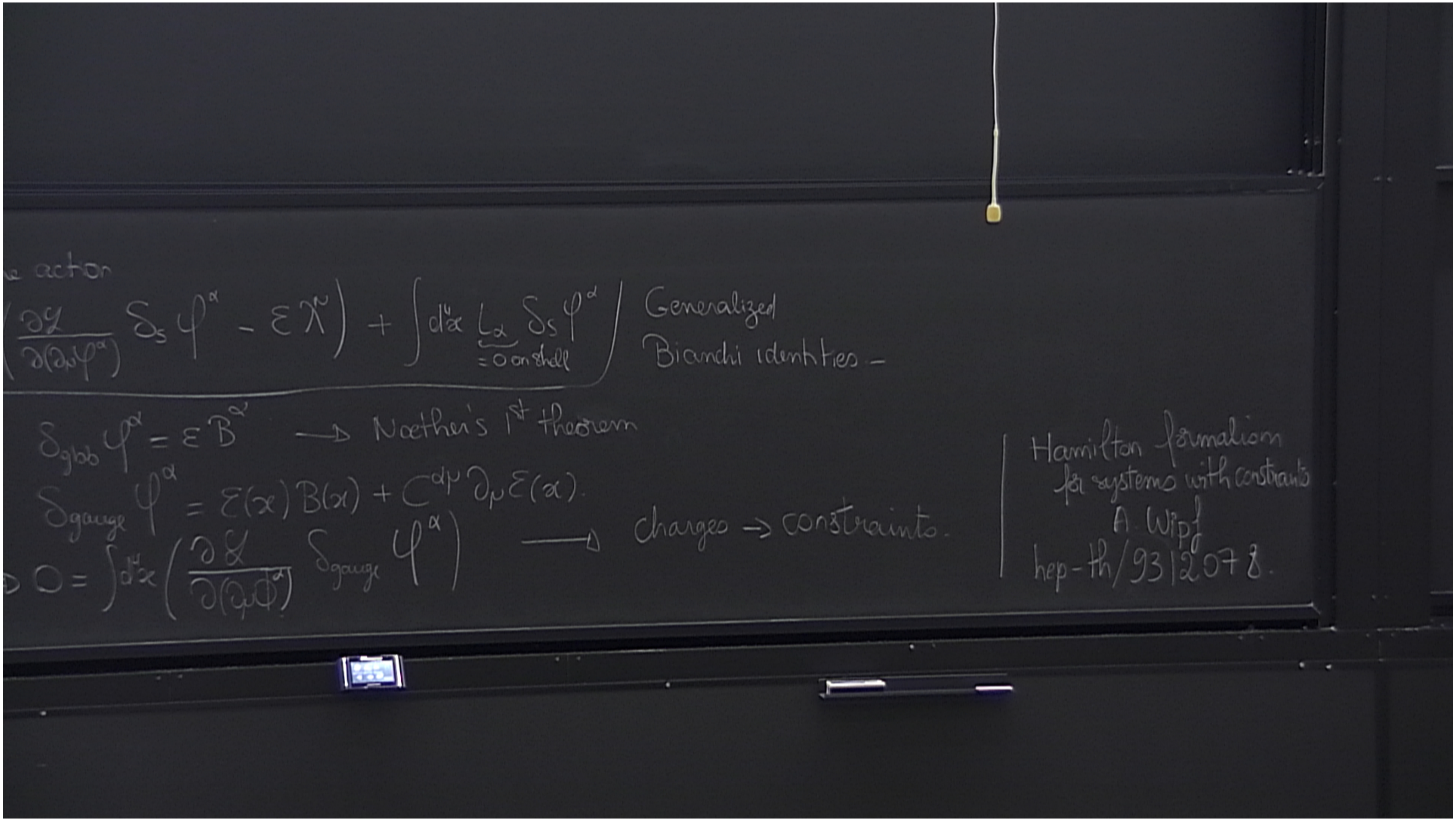
$$\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^\alpha)} \delta_s \varphi^\alpha - \varepsilon X^\mu\right) + \int d^4x \underbrace{L_\alpha}_{=0 \text{ on shell}} \delta_s \varphi^\alpha$$

Generalized Bianchi identities –

$$\delta_{glob} \varphi^\alpha = \varepsilon B^\alpha \longrightarrow \text{Noether's 1st theorem}$$

$$\delta_{gauge} \varphi^\alpha = \varepsilon(x) B(x) + C^{\alpha\mu} \partial_\mu \varepsilon(x).$$

$$\Rightarrow 0 = \int d^4x \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^\alpha)} \delta_{gauge} \varphi^\alpha\right) \longrightarrow \text{charges} \to \text{constraints.}$$



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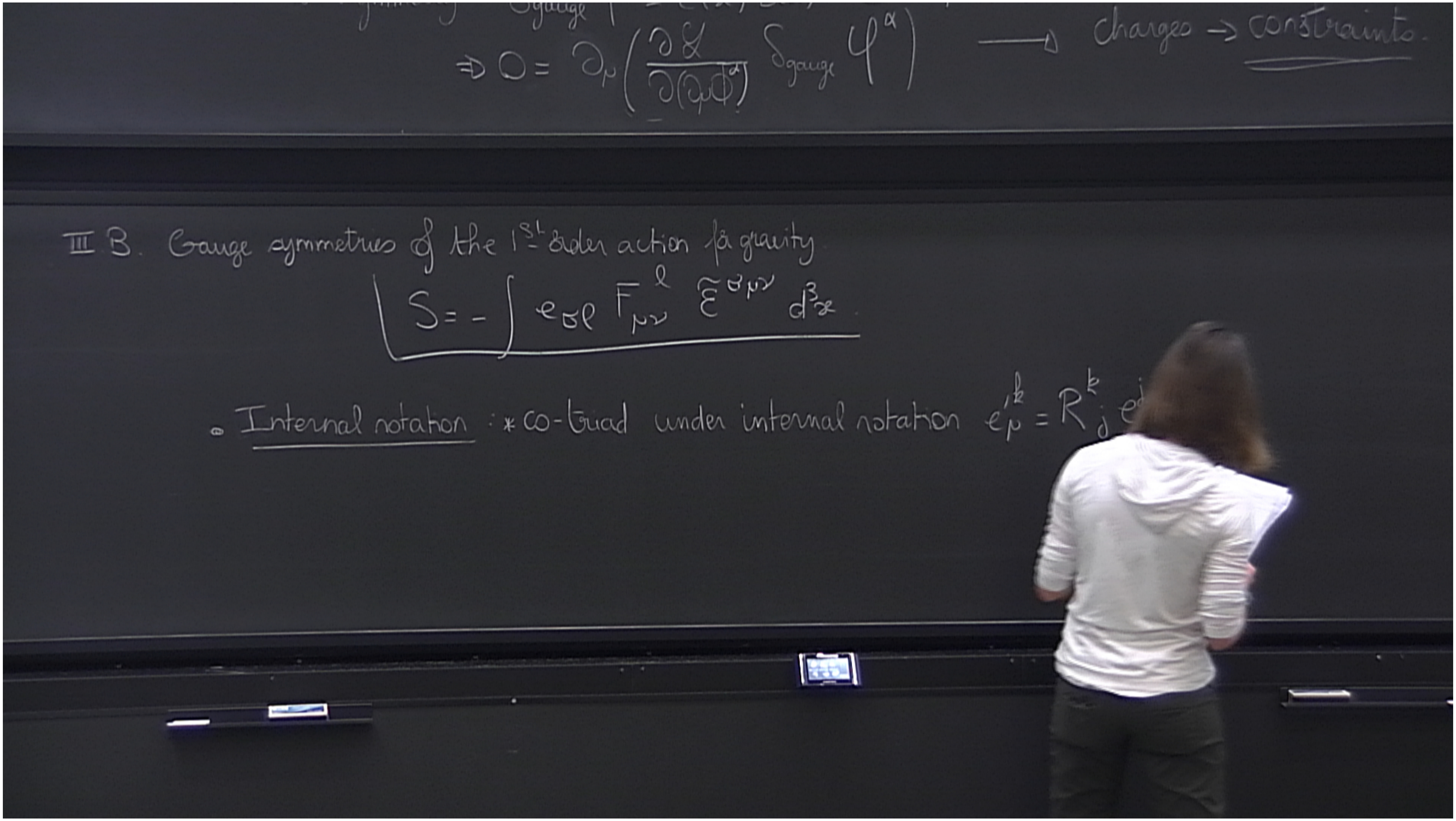
Hamilton formalism
for systems with constraints
A. Wipf
hep-th/9312078.

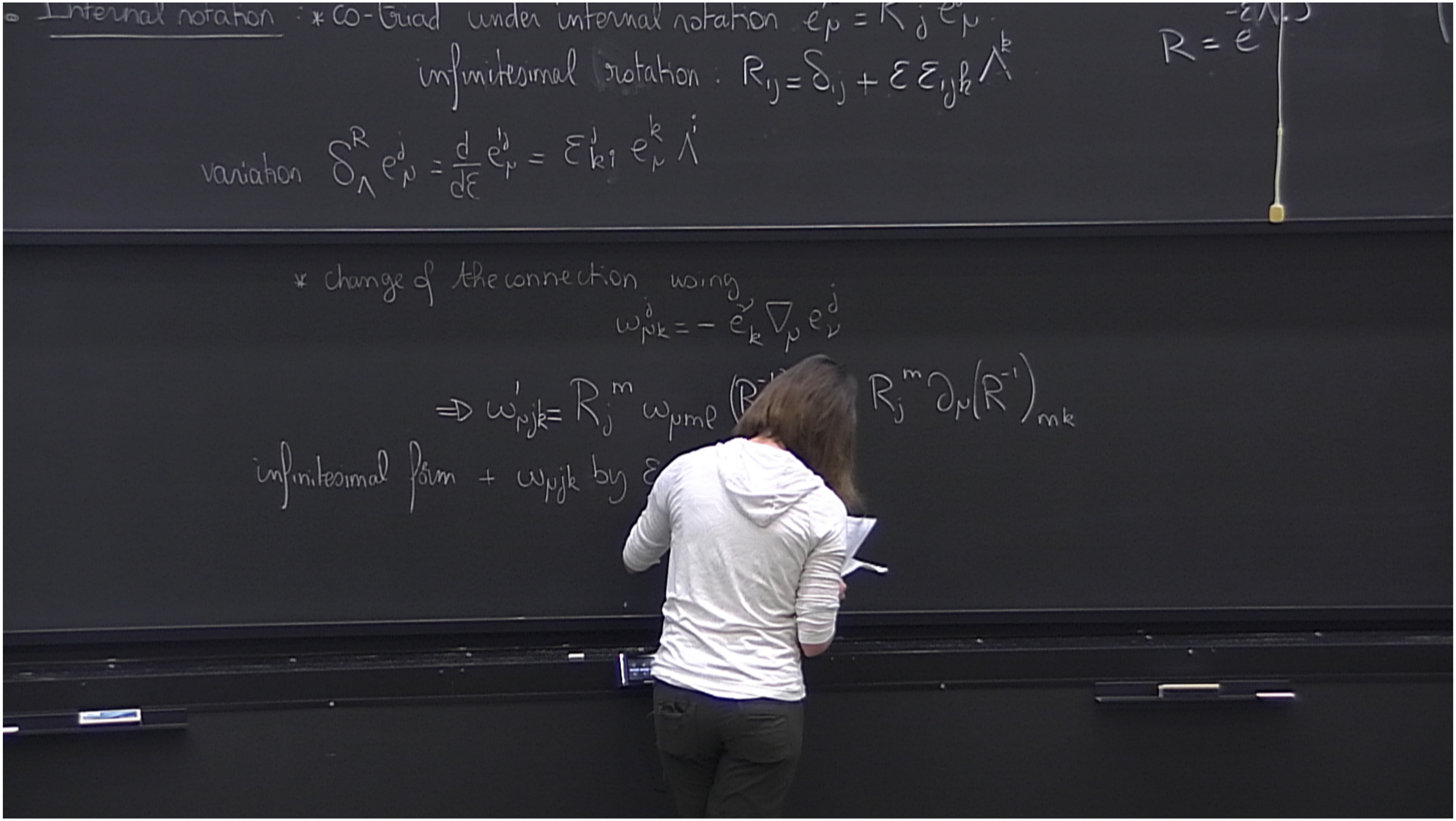


$$\Rightarrow 0 = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^\alpha)} \delta_{gauge} \varphi^\alpha \right) \longrightarrow \text{charges} \to \underline{\underline{\text{constraints}}}.$$

III B. Gauge symmetries of the 1st order action for gravity

$$\boxed{S = -\int e_{\sigma\rho} F_{\mu\nu}^{\ \ \rho} \, \tilde{\varepsilon}^{\sigma\mu\nu} \, d^3x}$$

- Internal rotation : * co-triad under internal rotation $e'^{k}_{\mu} = R^{k}{}_{j}\, e$



• Internal rotation : * co-triad under internal rotation $e'_\mu = R_j e_\mu$

$R = e^{-\varepsilon\Lambda}$

infinitesimal rotation : $R_{ij} = \delta_{ij} + \varepsilon \, \varepsilon_{ijk} \Lambda^k$

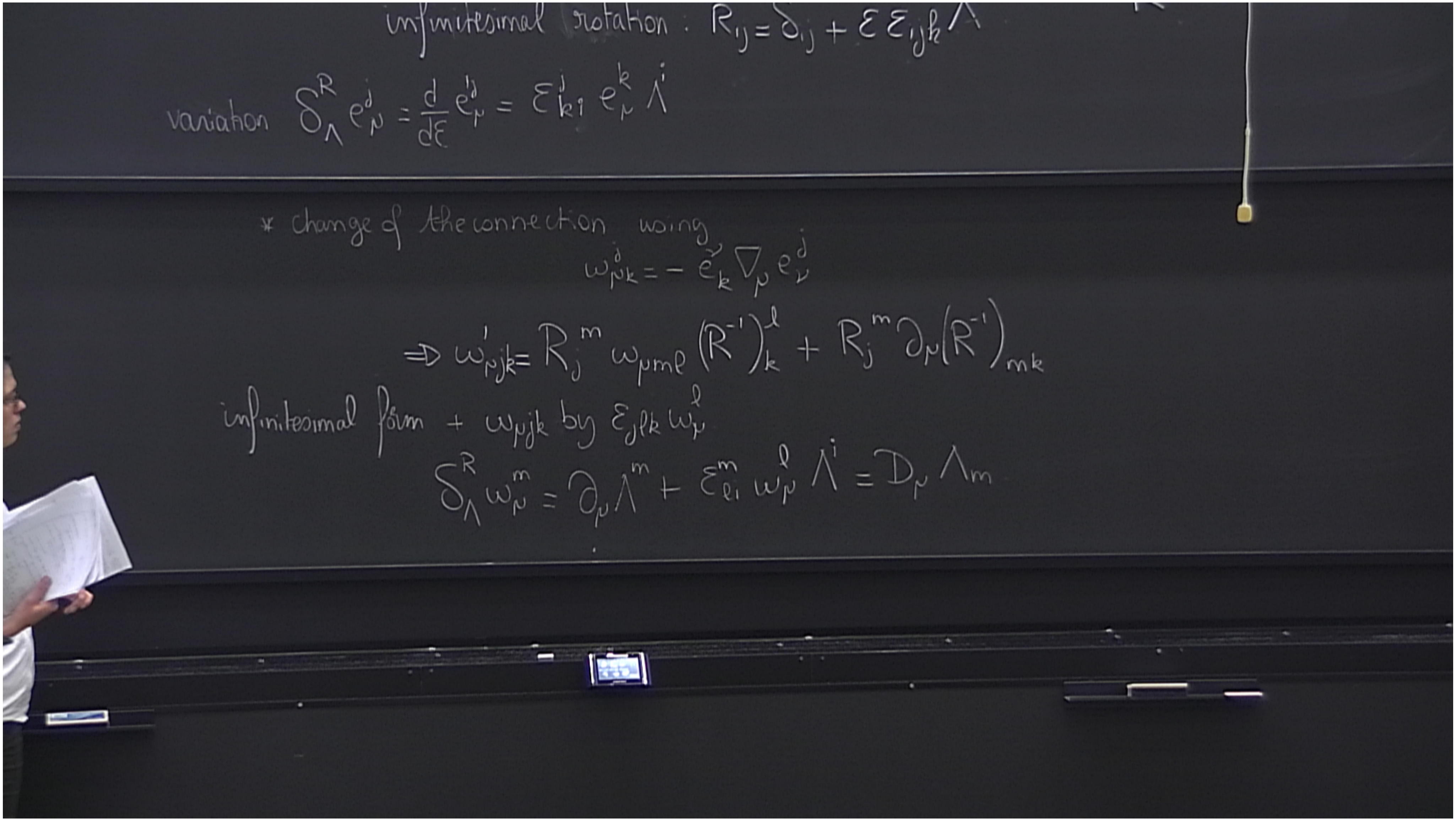
variation $\delta^R_\Lambda e^j_\nu = \frac{d}{d\varepsilon} e'^j_\nu = \varepsilon^j{}_{ki} \, e^k_\nu \, \Lambda^i$

* change of the connection using

$\omega^j_{\mu k} = - \tilde{e}^\nu_k \nabla_\mu e^j_\nu$

$\Rightarrow \omega'_{\mu jk} = R_j{}^m \, \omega_{\mu m \ell} \, (R^{-1}$ … $R_j{}^m \partial_\mu (R^{-1})_{mk}$

infinitesimal form + $\omega_{\mu jk}$ by ε



infinitesimal rotation : $R_{ij} = \delta_{ij} + \varepsilon\, \varepsilon_{ijk} \Lambda$

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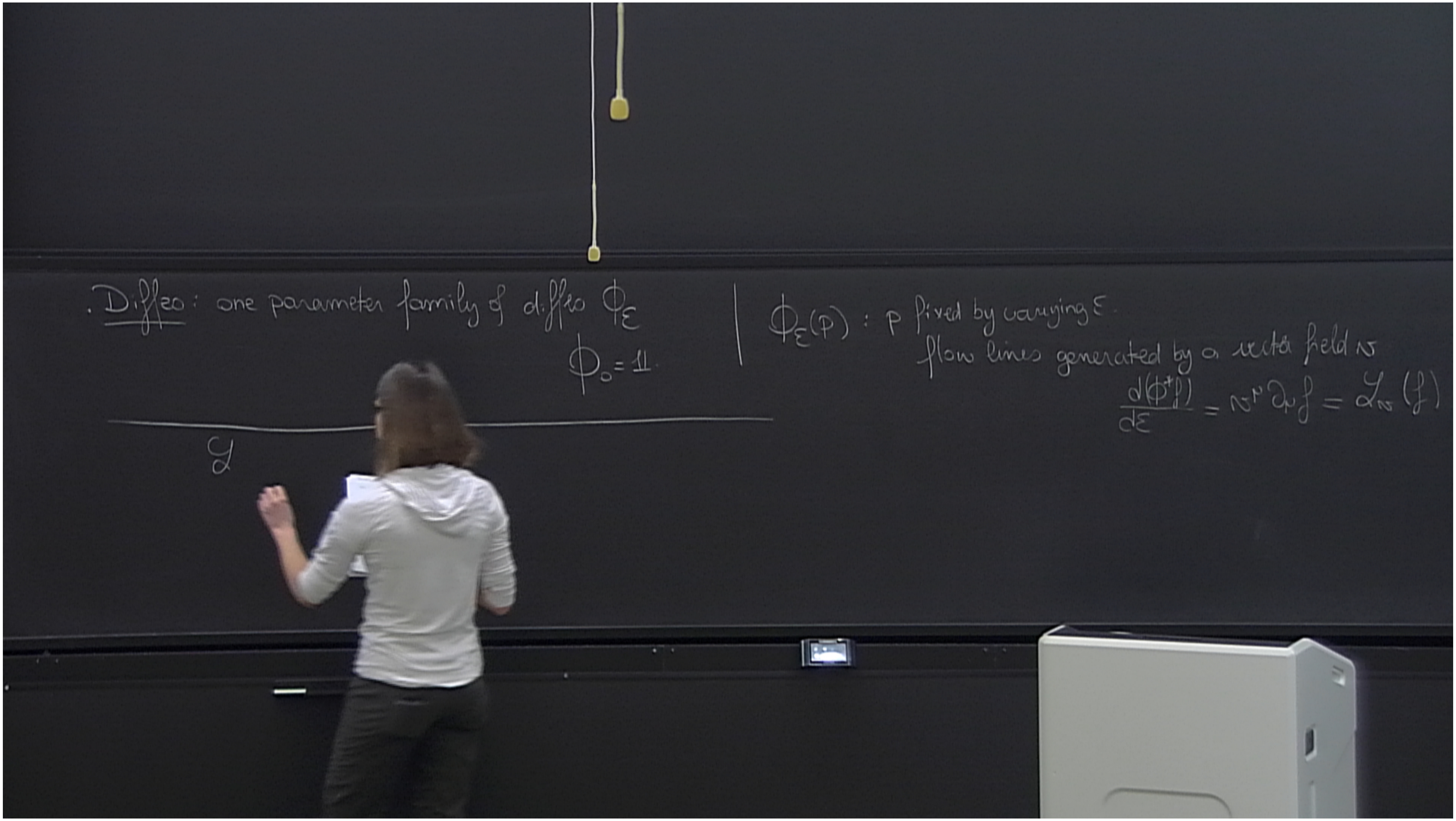
infinitesimal form + $\omega_{\mu jk}$ by $\varepsilon_{jlk}\, \omega^l_\mu$

$$\delta^R_\Lambda \omega^m_\mu = \partial_\mu \Lambda^m + \varepsilon^m_{li}\, \omega^l_\mu\, \Lambda^i = D_\mu \Lambda_m$$



. **Diffeo**: one parameter family of diffeo ϕ_ε

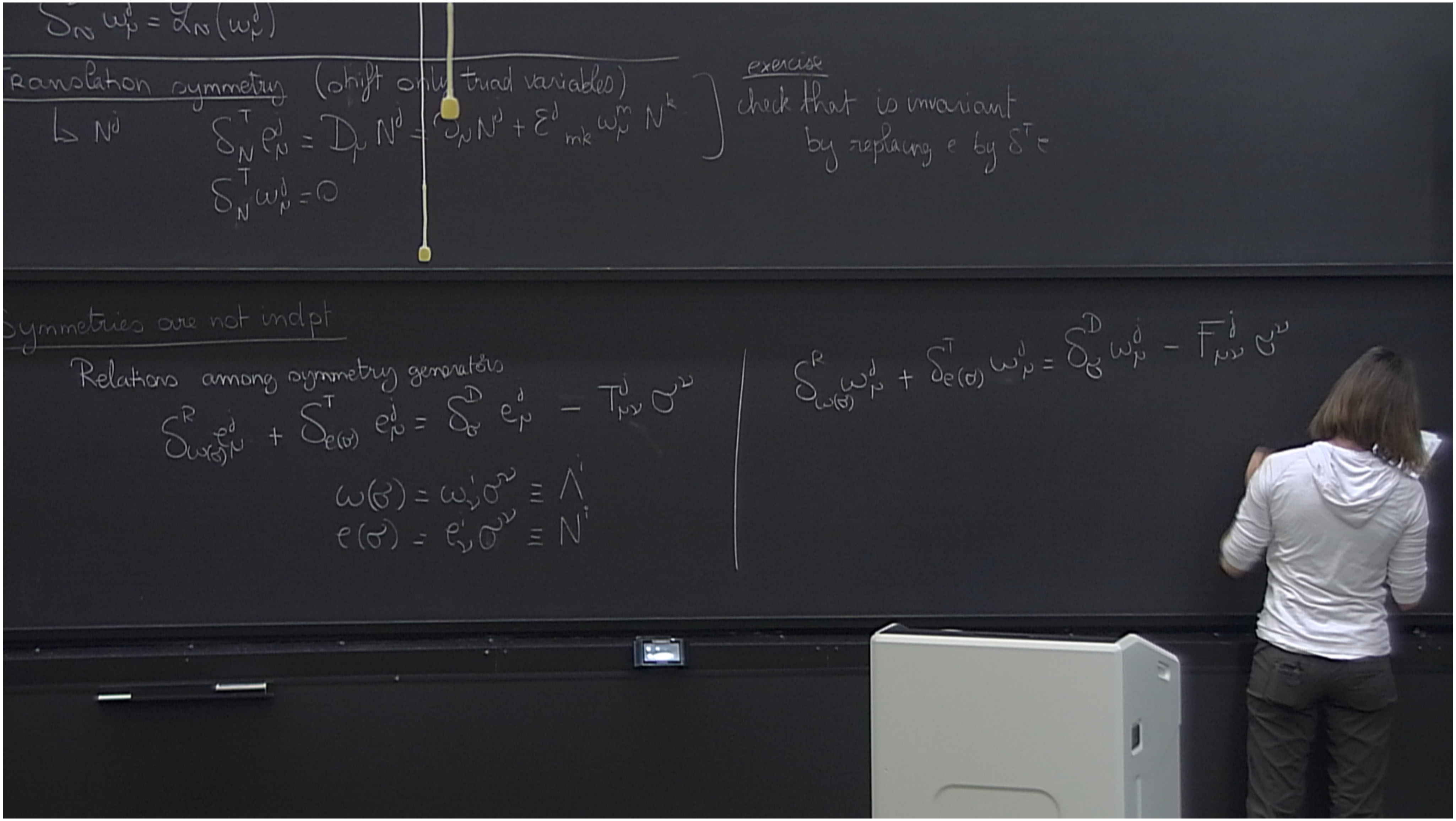
$\phi_0 = \mathbb{1}$.

$\mathcal{G}$

$\phi_\varepsilon(p)$: p fixed by varying ε.

flow lines generated by a vector field v

$$\frac{d(\phi^* f)}{d\varepsilon} = v^\mu \partial_\mu f = \mathcal{L}_v (f)$$



$\delta_N \omega^j_\mu = \mathcal{L}_N(\omega^j_\mu)$

Translation symmetry (shift only triad variables)

↳ N^j

$\delta^T_N e^j_\mu = D_\mu N^j = \partial_\mu N^j + \epsilon^j{}_{mk}\,\omega^m_\mu N^k$

$\delta^T_N \omega^j_\mu = 0$

exercise

check that is invariant
by replacing e by $\delta^T e$

Symmetries are not indpt

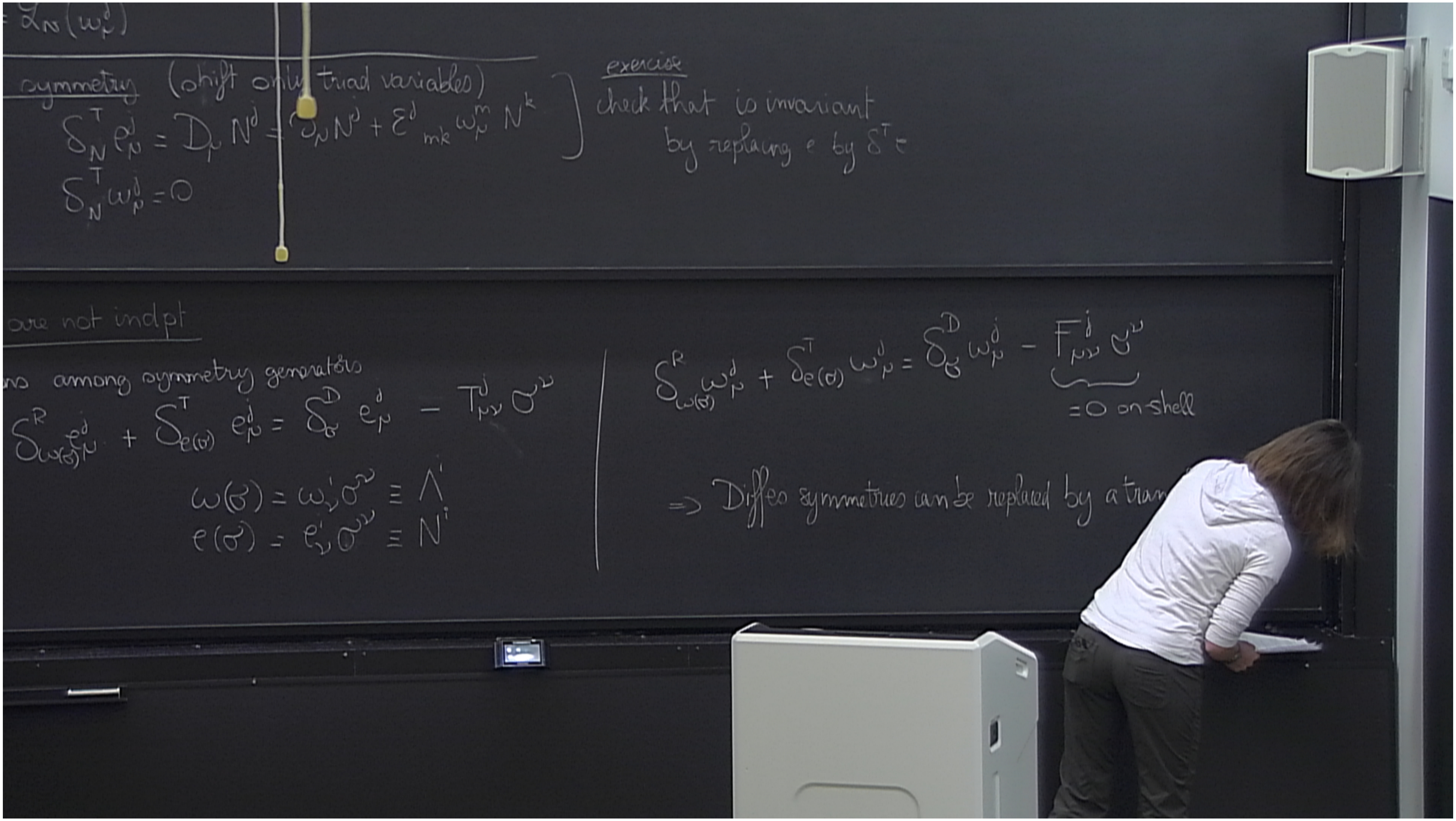
Relations among symmetry generators

$$\delta^R_{\omega(\sigma)} e^j_\mu + \delta^T_{e(\sigma)} e^j_\mu = \delta^D_\sigma e^j_\mu - T^j_{\mu\nu}\sigma^\nu$$

$$\omega(\sigma) = \omega^i_\nu \sigma^\nu \equiv \Lambda^i$$

$$e(\sigma) = e^i_\nu \sigma^\nu \equiv N^i$$

$$\delta^R_{\omega(\sigma)} \omega^j_\mu + \delta^T_{e(\sigma)} \omega^j_\mu = \delta^D_\sigma \omega^j_\mu - F^j_{\mu\nu}\sigma^\nu$$



$= \mathcal{L}_N(\omega^j_\mu)$

symmetry (shift only triad variables)

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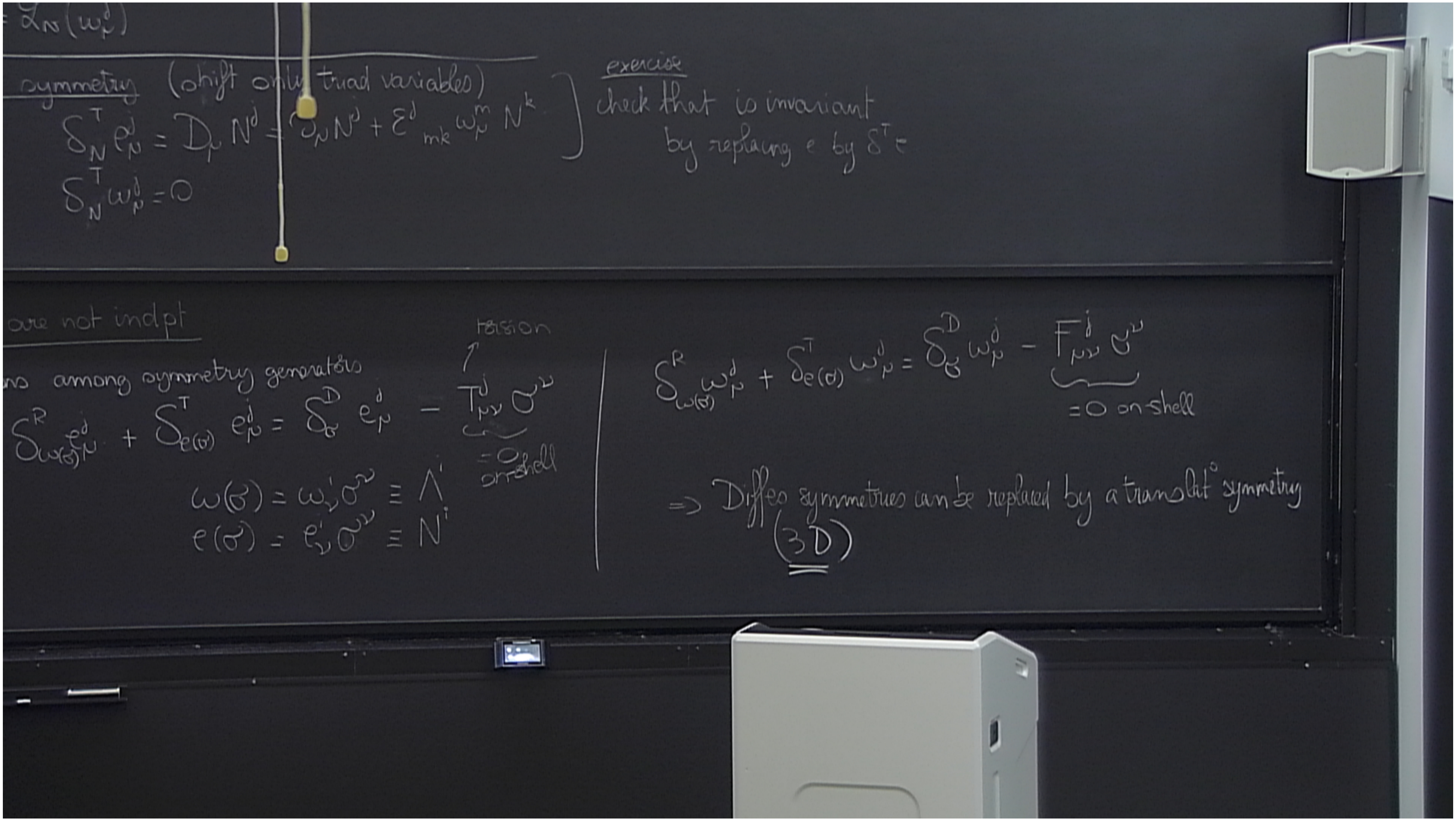
⇒ Diffeo symmetries can be replaced by a tran



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check that is invariant by replacing e by $\delta^T e$

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$$\delta^R_{\omega(\sigma)} e^j_\mu + \delta^T_{e(\sigma)} e^j_\mu = \delta^D_\sigma e^j_\mu - \underbrace{T^j_{\mu\nu}}_{=0 \text{ onshell}} \sigma^\nu$$

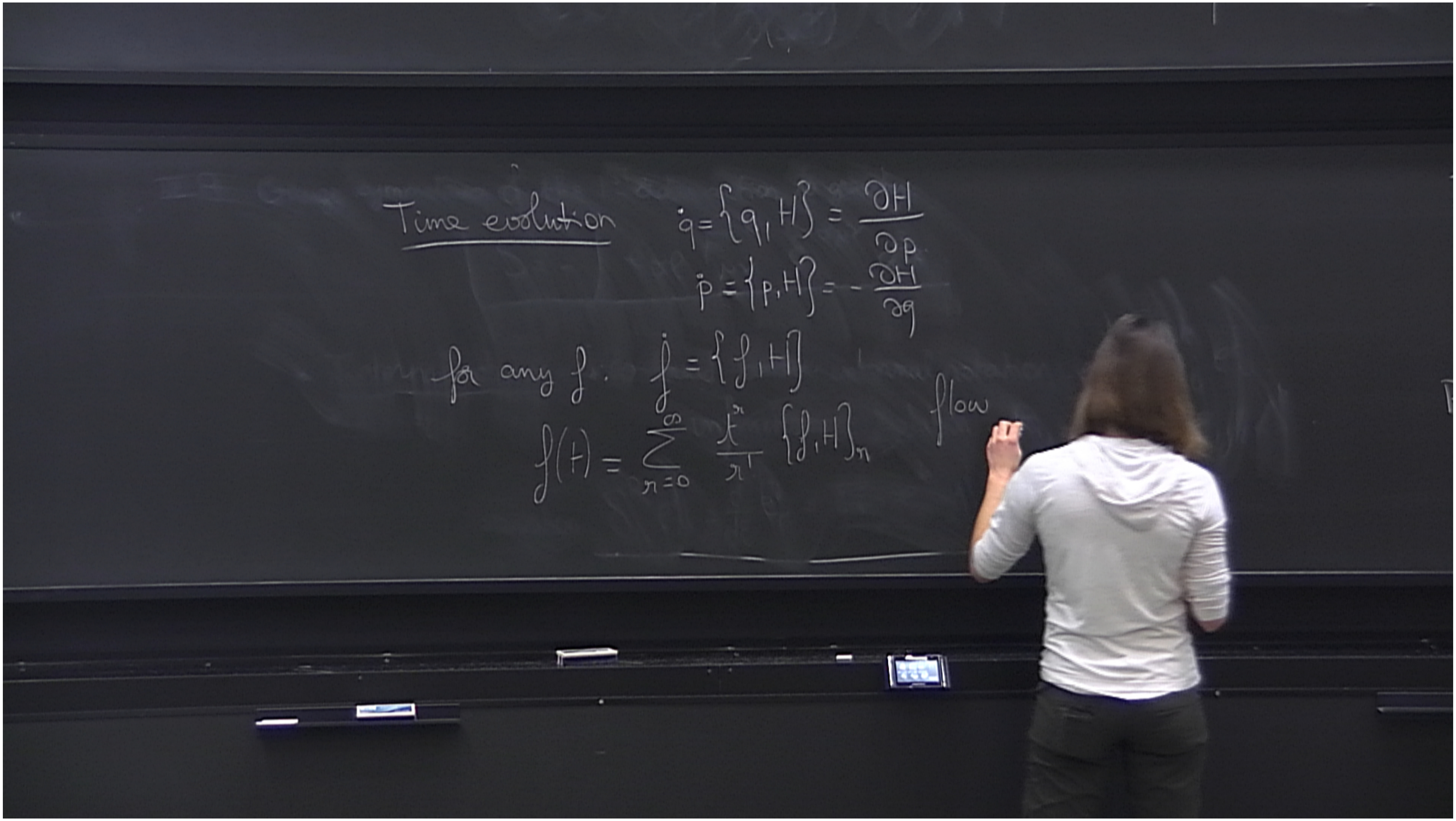
torsion

$$\omega(\sigma) = \omega^i_\nu \sigma^\nu \equiv \Lambda^i$$

$$e(\sigma) = e^i_\nu \sigma^\nu \equiv N^i$$

$$\delta^R_{\omega(\sigma)} \omega^j_\mu + \delta^T_{e(\sigma)} \omega^j_\mu = \delta^D_\sigma \omega^j_\mu - \underbrace{F^j_{\mu\nu}\sigma^\nu}_{=0 \text{ on-shell}}$$

$\Rightarrow$ Diffeo symmetries can be replaced by a translat° symmetry (3D)



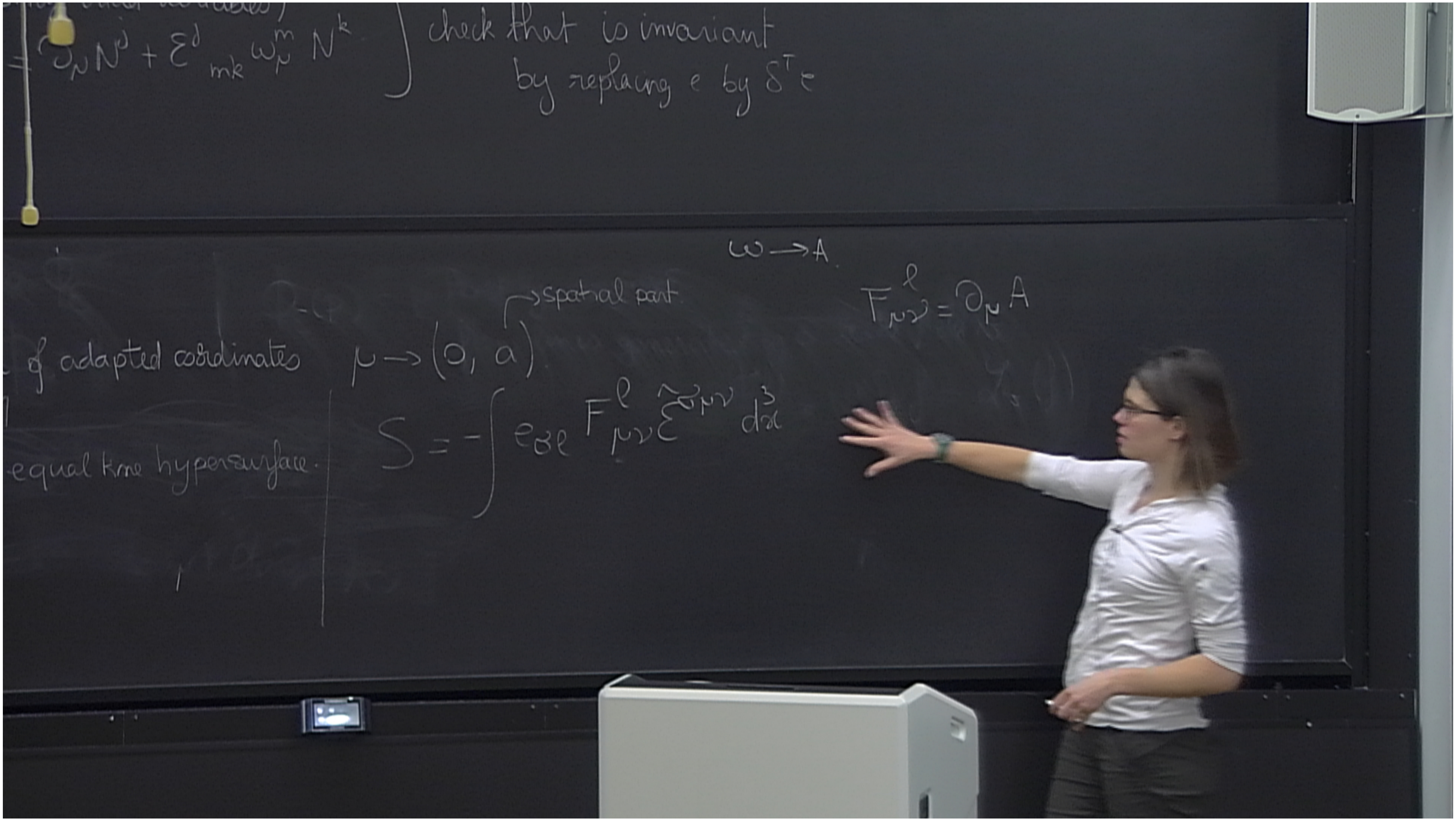
Time evolution $\dot{q} = \{q, H\} = \frac{\partial H}{\partial p}$

$\dot{p} = \{p, H\} = -\frac{\partial H}{\partial q}$

for any f: $\dot{f} = \{f, H\}$

$$f(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} \{f, H\}_n$$

flow



$= \partial_\nu N^j + \varepsilon^j{}_{mk}\, \omega^m_\nu N^k$

check that is invariant by replacing e by $\delta^T e$

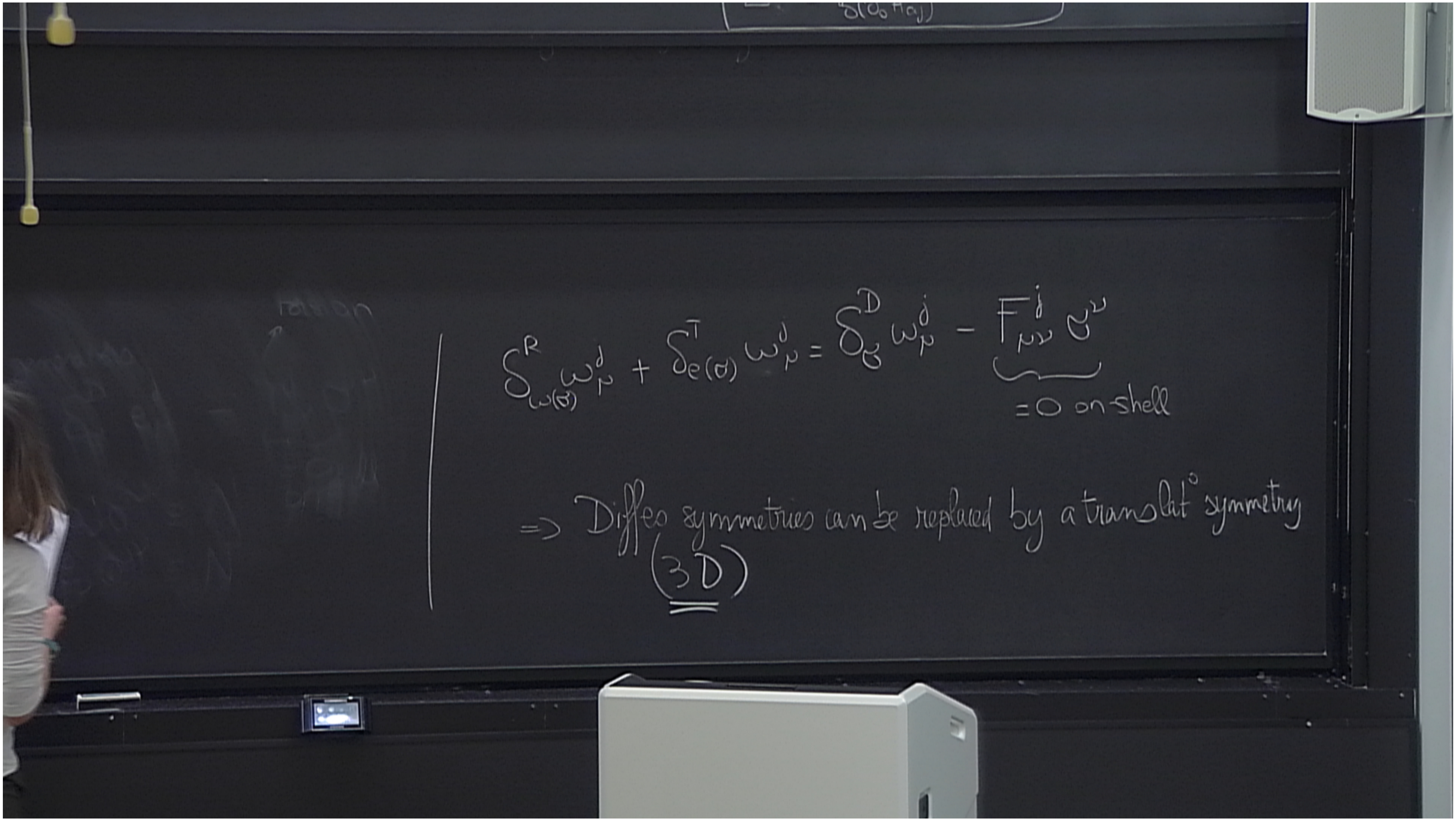
$\omega \longrightarrow A$

$F^\ell_{\mu\nu} = \partial_\mu A$

of adapted coordinates $\mu \to (0, \overset{\text{spatial part}}{a})$

equal time hypersurface.

$$S = -\int e_{\sigma\ell}\, F^\ell_{\mu\nu}\, \tilde{\varepsilon}^{\sigma\mu\nu}\, d^3x$$



$$\delta^{R}_{\omega(\xi)}\,\omega^{j}_{\mu} + \delta^{T}_{e(\xi)}\,\omega^{j}_{\mu} = \delta^{D}_{\xi}\,\omega^{j}_{\mu} - \underbrace{F^{j}_{\mu\nu}\,\xi^{\nu}}_{=0 \text{ on-shell}}$$

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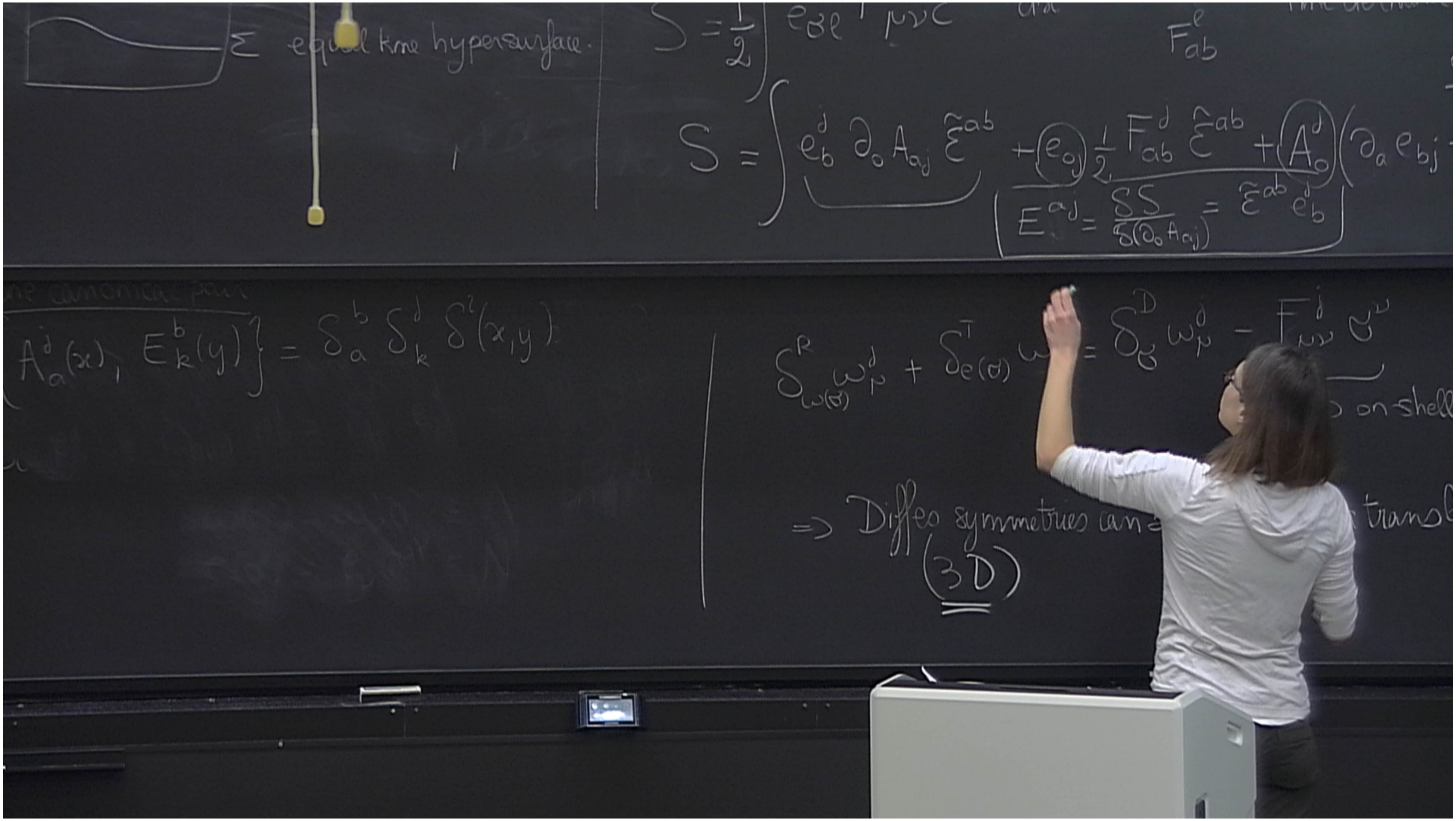
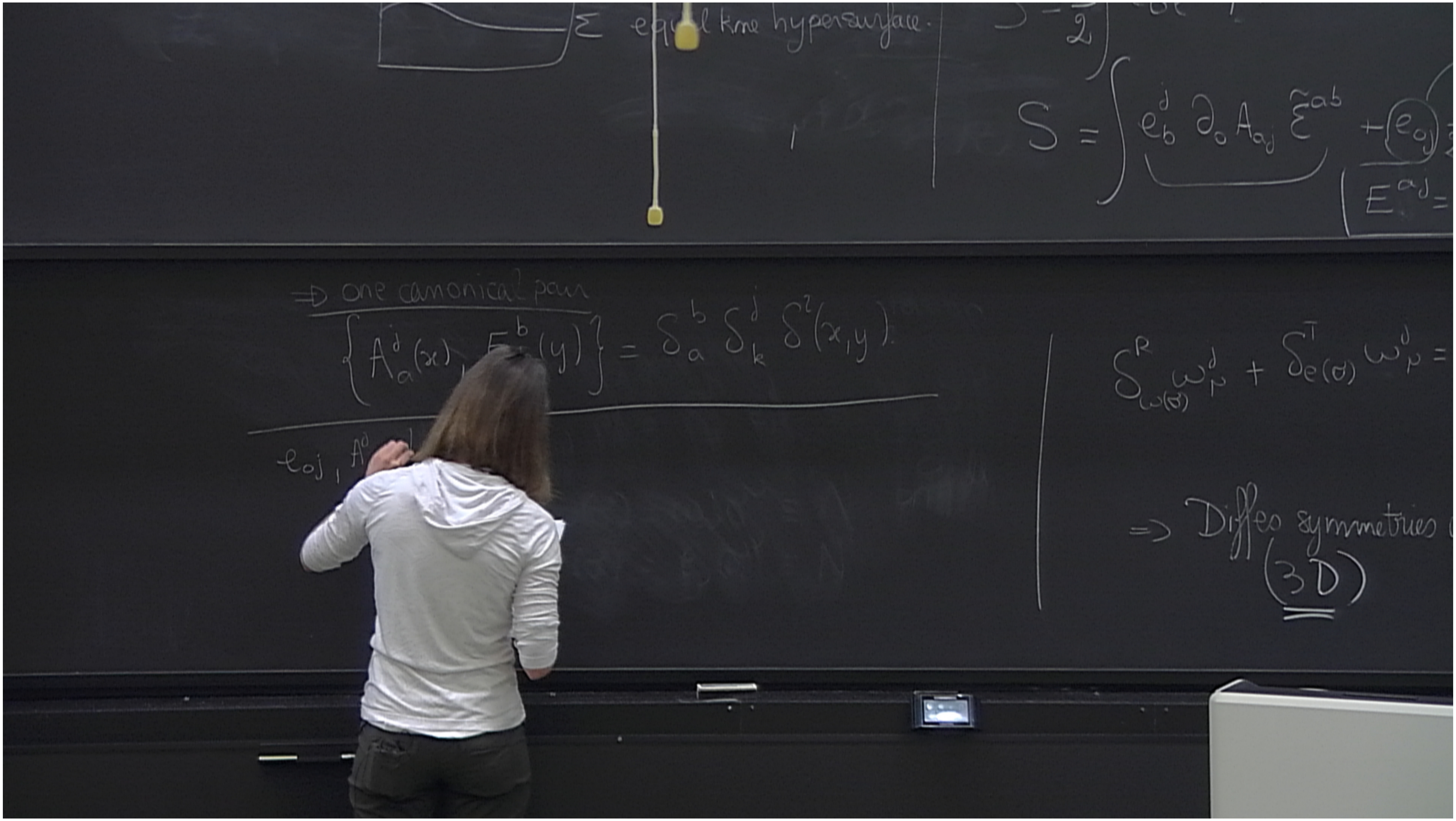


Σ equal time hypersurface.
S = 1/2 ∫ e_{σl} ε^{μνζ}
F^l_{ab}
S = ∫ e^J_b ∂_0 A_{aJ} ε̃^{ab} + e_{0J} 1/2 F^J_{ab} ε̃^{ab} + A^J_0 (∂_a e_{bJ}
E^{aJ} = δS/δ(∂_0 A_{aJ}) = ε̃^{ab} e^J_b
canonical pair
{A^J_a(x), E^b_k(y)} = δ^b_a δ^J_k δ^2(x,y)
δ^R_{ω(σ)} ω^J_μ + δ^T_{e(σ)} ω = δ^D_σ ω^J_μ − F^J_{μν} σ^ν
on-shell
⇒ Diffeo symmetries can transl
(3D)



Σ equal time hypersurface.

$$S = \int \underbrace{e^j_b \, \partial_0 A_{aj} \, \tilde{\varepsilon}^{ab}} + e_{0j}$$

$E^{aj} =$

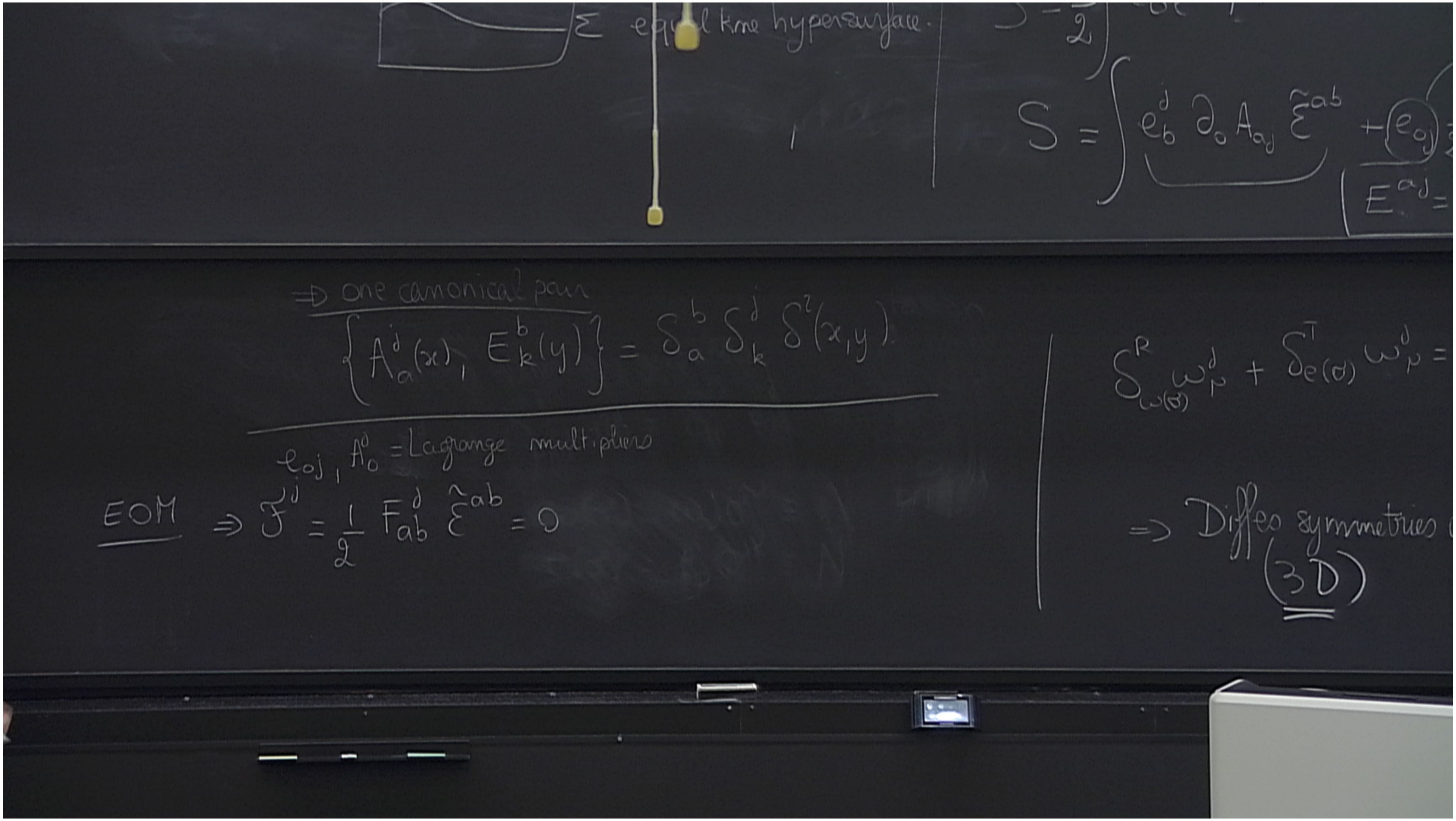
⇒ one canonical pair

$$\left\{ A^j_a(x) , E^b_k(y) \right\} = \delta_a^b \, \delta^j_k \, \delta^2(x,y).$$

e_{0j} , A^j

$$\delta^R_{\omega(\sigma)} \omega^j_\mu + \delta^T_{e(\sigma)} \omega^j_\mu =$$

⇒ Diffeo symmetries (3D)



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$E^{aj} =$

⇒ one canonical pair

$$\left\{ A^j_a(x), E^b_k(y) \right\} = \delta^b_a \, \delta^j_k \, \delta^2(x,y)$$

e_{0j}, A^j_0 = Lagrange multipliers

EOM ⇒ $\mathcal{F}^j = \frac{1}{2} F^j_{ab} \, \tilde{\varepsilon}^{ab} = 0$

$$\delta^R_{\omega(\sigma)} \omega^j_\mu + \delta^T_{e(\sigma)} \omega^j_\mu =$$

⇒ Diffeo symmetries (3D)