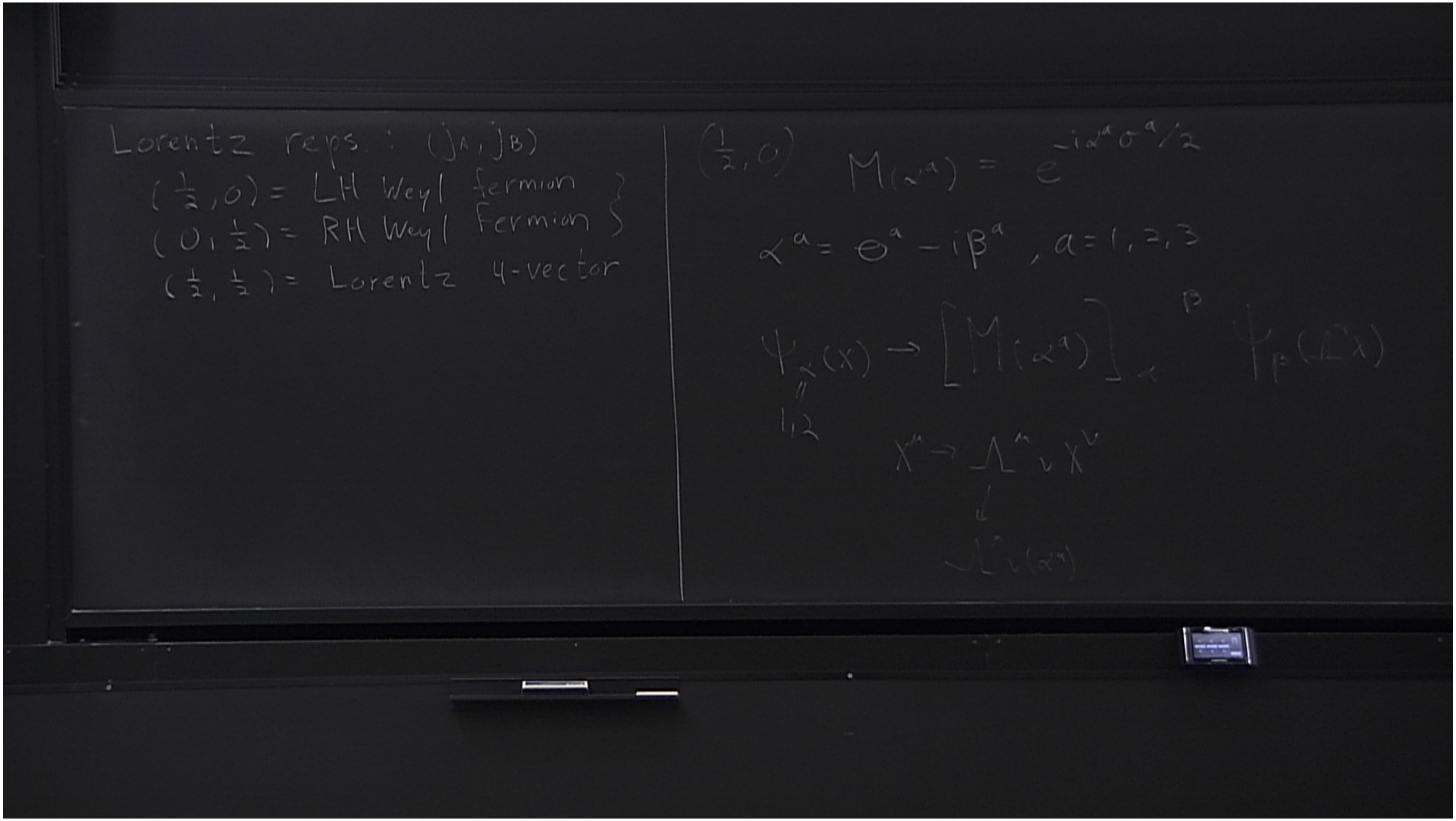
Title: PSI 2015/2016 More/Beyond Standard Model - Lecture 6

Date: Feb 29, 2016 09:00 AM

URL: http://pirsa.org/16020071

Abstract:


Lorentz reps: (j_A, j_B)
$(\frac{1}{2}, 0)$ = LH Weyl fermion
$(0, \frac{1}{2})$ = RH Weyl fermion
$(\frac{1}{2}, \frac{1}{2})$ = Lorentz 4-vector
$(\frac{1}{2}, 0)$
$M(\alpha^a) = e^{-i\alpha^a\sigma^a/2}$
$\alpha^a = \theta^a - i\beta^a$, $a = 1, 2, 3$
$\psi_\alpha(x) \to [M(\alpha^a)]_\alpha{}^\beta \, \psi_\beta(\Lambda^{-1}x)$
$\alpha = 1, 2$
$x^\mu \to \Lambda^\mu{}_\nu x^\nu$
$\Lambda^\mu{}_\nu(\alpha^a)$

$e^{-i\alpha^a\sigma^a/2}$

$a = 1,2,3$

$\ldots(\alpha^a)\Big]_\alpha^{\ \beta}\ \psi_\beta(\Lambda^{-1}x)$

$\ldots\chi^\nu$

$\ldots(\alpha^a)$

$$M^t(\alpha^a)\,\epsilon\, M(\alpha^a) = \epsilon\,, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma^2$$

$$\left[\chi^t \epsilon\, \psi\right] = \chi_\beta\, \epsilon^{\beta\alpha}\, \psi_\alpha = -\left(\epsilon^{\alpha\beta}\chi_\beta\right)\psi_\alpha$$

$$\chi^\alpha = \epsilon^{\alpha\beta}\chi_\beta$$

$$\chi\psi = \text{Lorentz invariant} = \chi^\alpha\psi_\alpha$$

$$) = e^{-i\alpha^a \sigma^a/2}$$

$$i\beta^a \ , a = 1,2,3$$

$$\left[M(\alpha^a)\right]_\alpha{}^\beta \ \psi_\beta(\Lambda^{-1}x)$$

$$\to \Lambda^\mu{}_\nu x^\nu$$

$$\Lambda^\mu{}_\nu(\alpha^a)$$

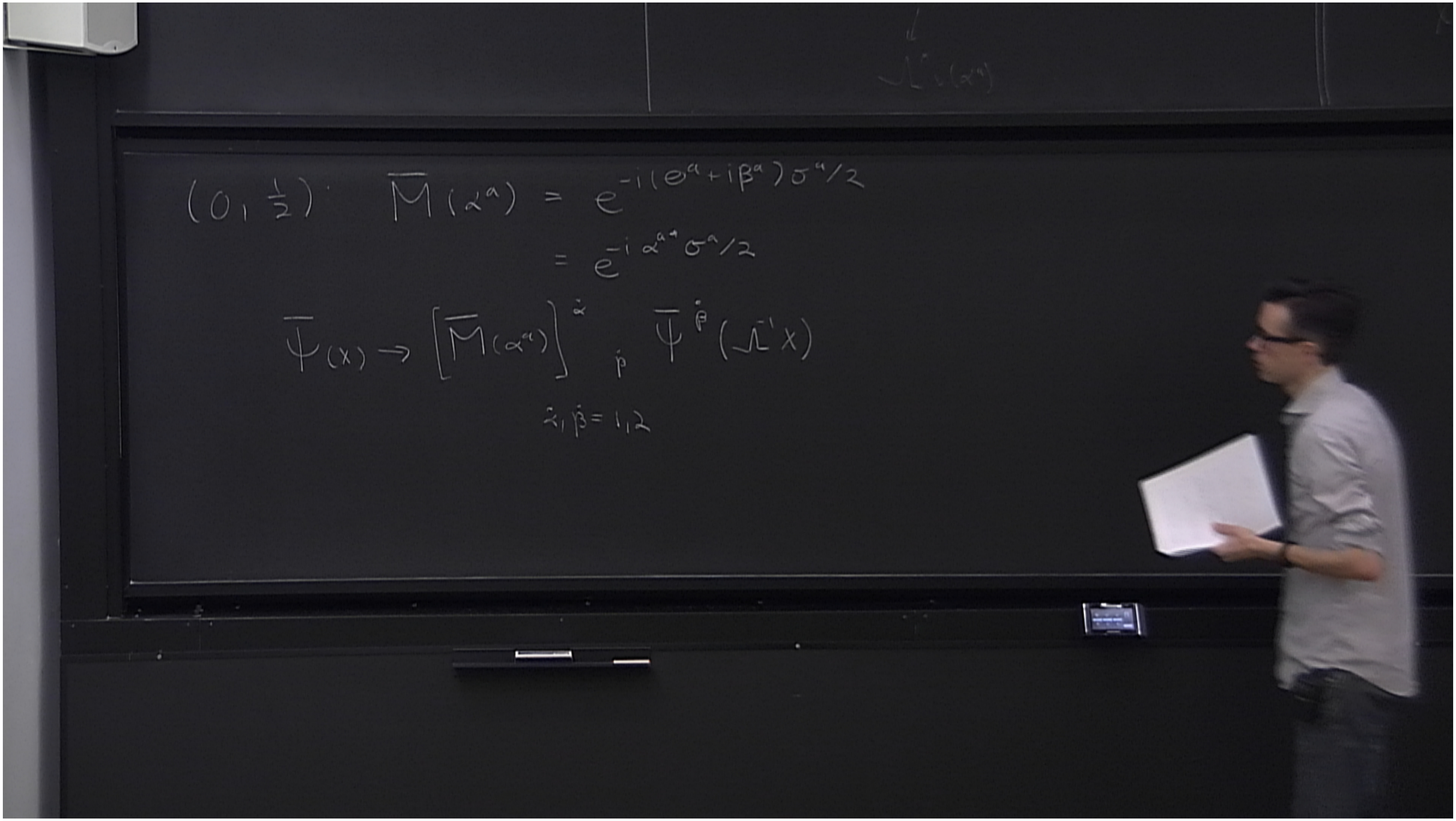
$$M^t(\alpha^a)\,\epsilon\,M(\alpha^a) = \epsilon \ , \ \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= i\sigma^2$$

$$\left[\chi^t \epsilon \psi\right] = \chi_\beta \epsilon^{\beta\alpha} \psi_\alpha = -(\epsilon^{\alpha\beta}\chi_\beta)\psi_\alpha$$

$$\chi^\alpha = \epsilon^{\alpha\beta}\chi_\beta$$

$$\chi\psi = \text{Lorentz invariant} = \chi^\alpha \psi_\alpha = \epsilon^{\alpha\beta}\chi_\beta\psi_\alpha$$


$(0, \frac{1}{2})$ $\bar{M}(\alpha^a) = e^{-i(\theta^a + i\beta^a)\sigma^a/2}$
$= e^{-i\alpha^{a*}\sigma^a/2}$
$\bar{\psi}(x) \to [\bar{M}(\alpha^a)]^{\dot{\alpha}}{}_{\dot{\beta}}\, \bar{\psi}^{\dot{\beta}}(\Lambda^{-1}x)$
$\dot{\alpha}, \dot{\beta} = 1, 2$

$$x^\mu \to \Lambda^\mu{}_\nu x^\nu$$

$$\Lambda^\mu{}_\nu(\alpha^a)$$

$\chi\psi$ = Lorentz invaria

$$(0,\tfrac{1}{2}) \qquad \bar{M}(\alpha^a) = e^{-i(\theta^a+i\beta^a)\sigma^a/2}$$

$$= e^{-i\alpha^{a*}\sigma^a/2}$$

$$\bar\psi(x) \to [\bar{M}(\alpha^a)]^{\dot\alpha}{}_{\dot\beta}\,\bar\psi^{\dot\beta}(\Lambda^{-1}x)$$

$$\dot\alpha,\dot\beta = 1,2$$

$$\bar\chi\bar\psi = \bar\chi_{\dot\alpha}\bar\psi^{\dot\alpha} = (\epsilon_{\dot\alpha\dot\beta}\bar\chi^{\dot\beta})\bar\psi^{\dot\alpha}$$

$$\bar\chi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\chi_{\dot\beta}$$

$$\epsilon^{\dot\alpha\dot\beta} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \quad \epsilon_{\dot\alpha\dot\beta} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$

$$\psi^\alpha(x) \to \epsilon^{\alpha\lambda}[M(\alpha^a)]_\lambda{}^\rho\,\psi_\rho(\Lambda^{-1}x)$$

$$\epsilon^{\alpha\lambda}\psi_\lambda(x) = \epsilon^{\alpha\lambda}[M(\alpha^a)]_\lambda{}^\rho\,\epsilon_{\rho\kappa}\psi^\kappa(\Lambda^{-1}x)$$

$$= \left[e^{+i\alpha^a(\sigma^a)^T/2}\right]^\alpha{}_\kappa\,\psi^\kappa(\Lambda^{-1}x)$$

$\chi\psi$ = Lorentz invariant $= \chi^\alpha \psi_\alpha = \epsilon^{\alpha\beta}\chi_\beta \psi_\alpha$

$$\bar\chi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\chi_{\dot\beta}$$

$$\epsilon^{\dot\alpha\dot\beta} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \quad \epsilon_{\dot\alpha\dot\beta} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$

$$\psi^\alpha(x) \to \epsilon^{\alpha\lambda}[M(\alpha^a)]_\lambda{}^\beta\, \psi_\beta(\Lambda^{-1}x)$$

$$\epsilon^{\alpha\lambda}\psi'_\lambda(x) = \epsilon^{\alpha\lambda}[M(\alpha^a)]_\lambda{}^\beta\, \epsilon_{\beta\kappa}\psi^\kappa(\Lambda^{-1}x)$$

$$= \left[e^{+i\alpha^a(\sigma^a)^t/2}\right]^\alpha{}_\kappa\, \psi^\kappa(\Lambda^{-1}x)$$

= complex conjugate of $(0,\frac{1}{2})$ $\bar M$

$\Rightarrow (\psi^\alpha)^*$ = transforms as $(0,\frac{1}{2})$

$$\bar\chi\bar\psi = \bar\chi_{\dot\alpha}\bar\psi^{\dot\alpha} = (\epsilon_{\dot\alpha\dot\beta}\bar\chi^{\dot\beta})\bar\psi^{\dot\alpha}$$

$$= \left[e^{+i\alpha^\mu(\sigma^\mu)^t/2}\right]^\alpha{}_\kappa \psi^\kappa$$

$$\chi_\alpha := \epsilon_{\alpha\beta}(\bar\chi^*)^\beta$$

$$(\sigma^\mu)_{\alpha\dot\alpha} = (\mathbb{1}, \vec\sigma)_{\alpha\dot\alpha}$$

$\mu = 0,1,2,3$

$$\psi\sigma^\mu\bar\chi := \psi^\alpha(\sigma^\mu)_{\alpha\dot\alpha}\bar\chi^{\dot\alpha} \longrightarrow \epsilon^{\alpha\lambda}(M_\lambda{}^\beta\psi_\beta)\,\sigma^\mu_{\alpha\dot\alpha}(\bar M^{\dot\alpha}{}_{\dot\beta}\bar\chi^{\dot\beta})$$

$$= -\psi_\beta\left[M^t\epsilon\sigma^\mu\bar M\right]^\beta{}_{\dot\beta}\bar\chi^{\dot\beta}$$

$$x) \to [M(\alpha^a)]^{\dot\alpha}{}_{\dot\beta}\, \psi^{\dot\beta}(\Lambda^{-1}x)$$

$\dot\alpha, \dot\beta = 1, 2$

$$= \bar\chi_{\dot\alpha} \bar\psi^{\dot\alpha} = (\epsilon_{\dot\alpha\dot\beta} \bar\chi^{\dot\beta}) \bar\psi^{\dot\alpha}$$

$$\psi^{\dot\alpha}(x) \to \epsilon^{\dot\alpha\lambda} [M(\alpha^a)]_\lambda{}^\beta\, \psi_\beta(\Lambda^{-1}x)$$

$$\epsilon^{\dot\alpha\lambda}\psi_\lambda(x) = \epsilon^{\dot\alpha\lambda}[M(\alpha^a)]_\lambda{}^\beta\, \epsilon_{\beta\kappa}\, \psi^\kappa(\Lambda^{-1}x)$$

$$= \left[e^{+i\alpha^a(\sigma^a)^t/2}\right]^{\dot\alpha}{}_\kappa\, \psi^\kappa(\Lambda^{-1}x)$$

$(0, \tfrac{1}{2})$ rep derived from the $(\tfrac{1}{2}, 0)$ ψ_α

$$\bar\psi^{\dot\alpha} = \epsilon^{\dot\alpha\beta}(\psi^*)_\beta$$

$$\epsilon_{\alpha\beta}(\bar\chi^*)^\beta$$

$$\sigma_{\alpha\dot\alpha} = (\mathbb{1}, \vec\sigma)_{\alpha\dot\alpha}$$

$,2,3$

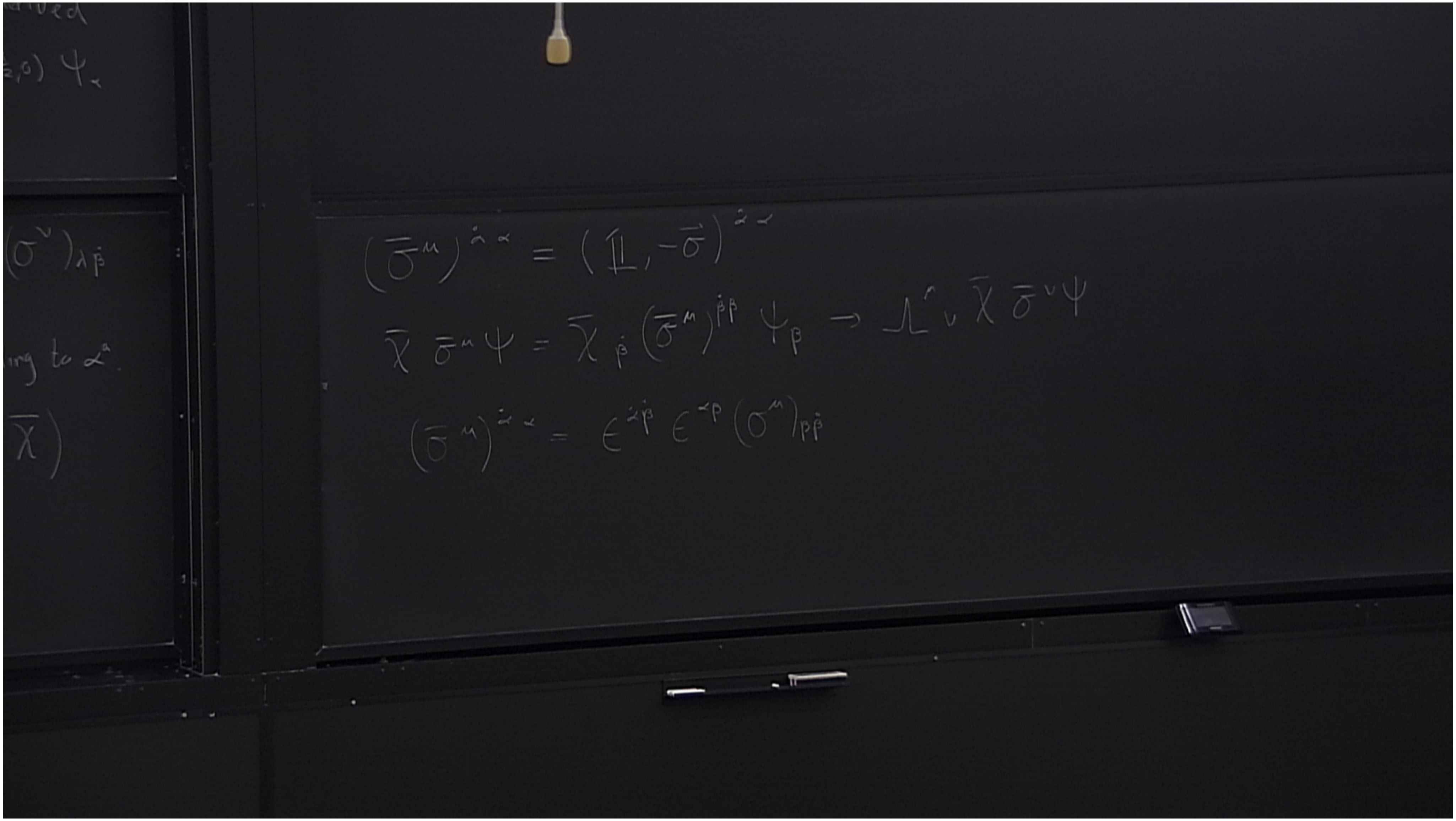
$$\psi\sigma^\mu\bar\chi := \psi^\alpha(\sigma^\mu)_{\alpha\dot\alpha}\bar\chi^{\dot\alpha} \longrightarrow \epsilon^{\alpha\lambda}(M_\lambda{}^\beta \psi_\beta)\, \sigma^\mu_{\alpha\dot\alpha} (\bar M^{\dot\alpha}{}_{\dot\beta}\, \bar\chi^{\dot\beta})$$

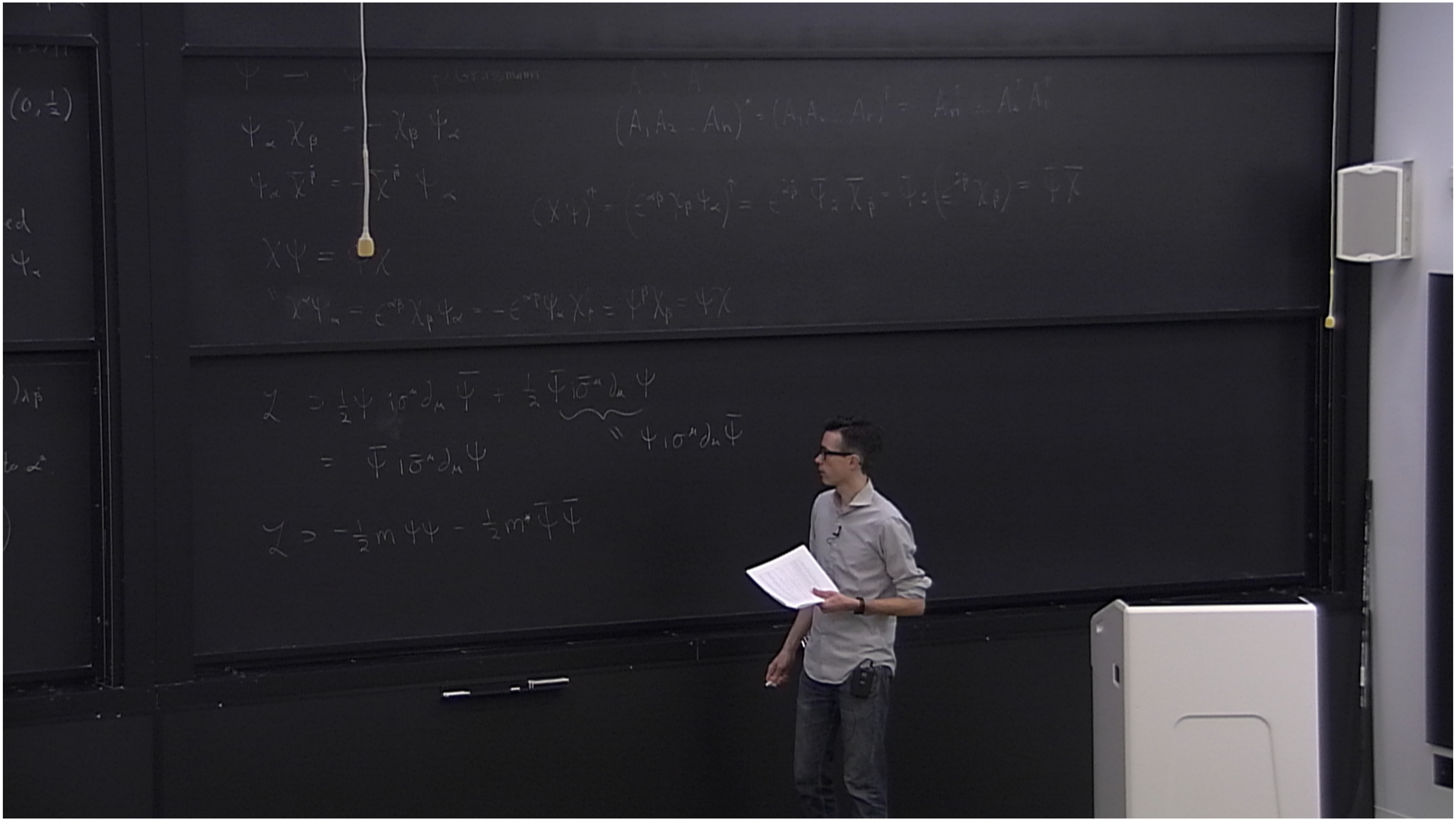
$$= -\psi_\beta \left[M^t \epsilon\, \sigma^\mu \bar M\right]^\beta{}_{\dot\beta}\, \bar\chi^{\dot\beta}$$

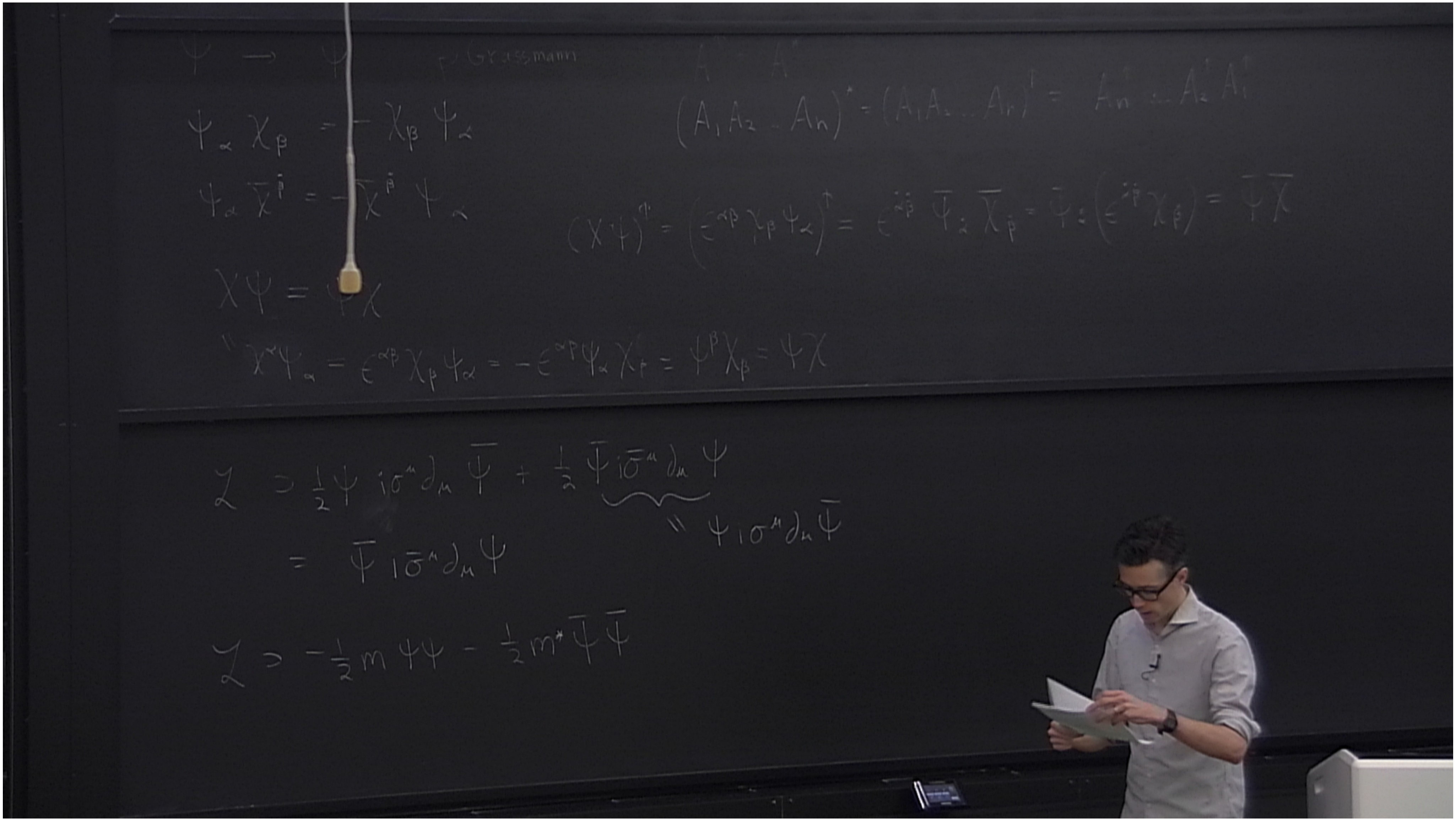
$$\left[M^t \epsilon\, \sigma^\mu \bar M\right]^\beta{}_{\dot\beta} = \epsilon^{\beta\lambda} \Lambda^\mu{}_\nu (\sigma^\nu)_{\lambda\dot\beta}$$

Λ corresponding to α^a.

$$\psi\sigma^\mu\bar\chi \to \Lambda^\mu{}_\nu (\psi\sigma^\nu\bar\chi)$$


$(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} = (\mathbb{1}, -\vec{\sigma})^{\dot{\alpha}\alpha}$
$\bar{\chi}\bar{\sigma}^\mu\psi = \bar{\chi}_{\dot{\beta}}(\bar{\sigma}^\mu)^{\dot{\beta}\beta}\psi_\beta \to \Lambda^\mu{}_\nu \bar{\chi}\bar{\sigma}^\nu\psi$
$(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}(\sigma^\mu)_{\beta\dot{\beta}}$


Grassmann
(A1 A2 ... An)† = ... = An† ... A2† A1†
ψα χβ = − χβ ψα
χψ = ψχ
L ⊃ ½ ψ iσ^μ ∂μ ψ̄ + ½ ψ̄ iσ̄^μ ∂μ ψ
= ψ̄ iσ̄^μ ∂μ ψ
L ⊃ − ½ m ψψ − ½ m* ψ̄ψ̄


Grassmann