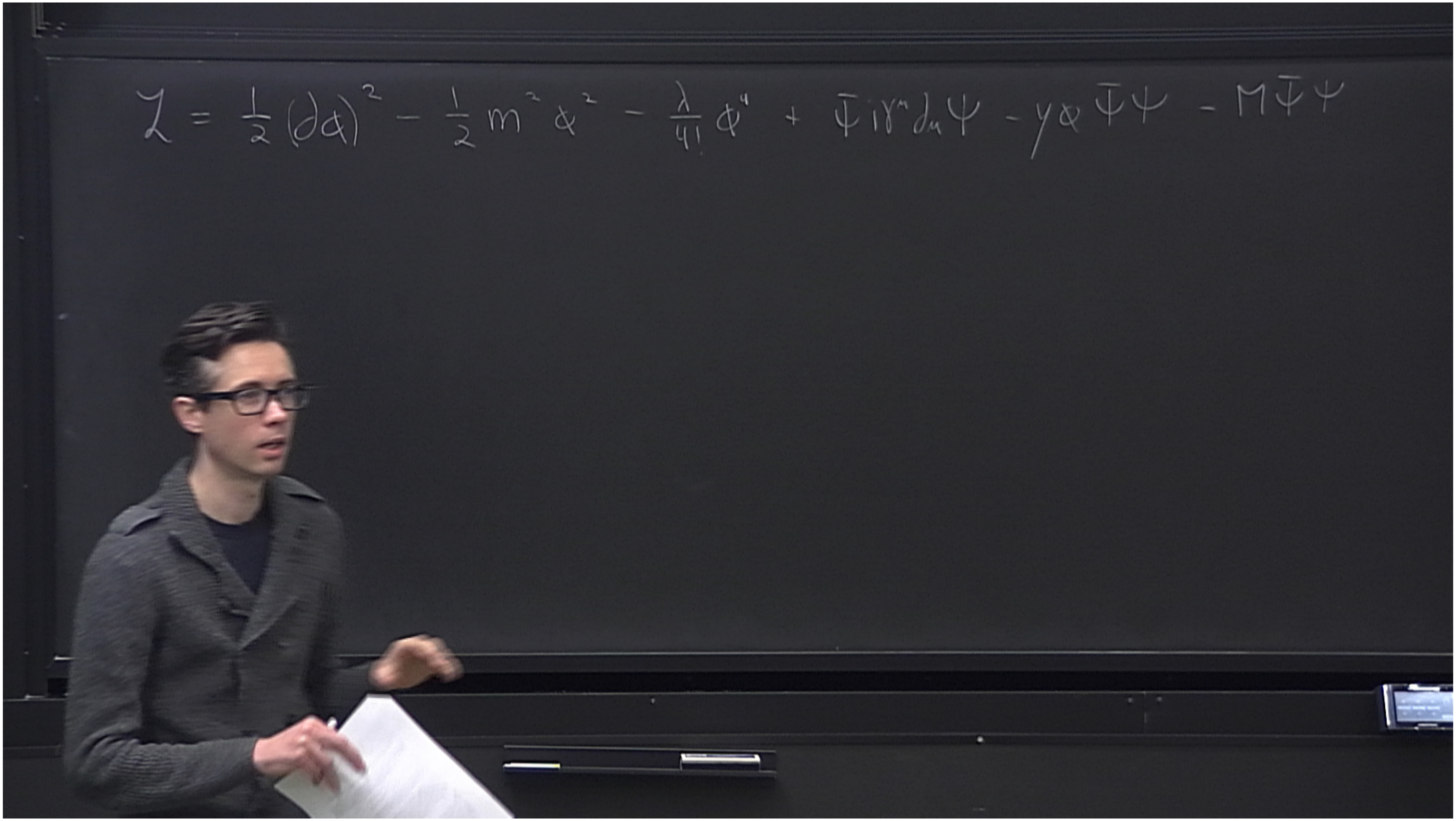
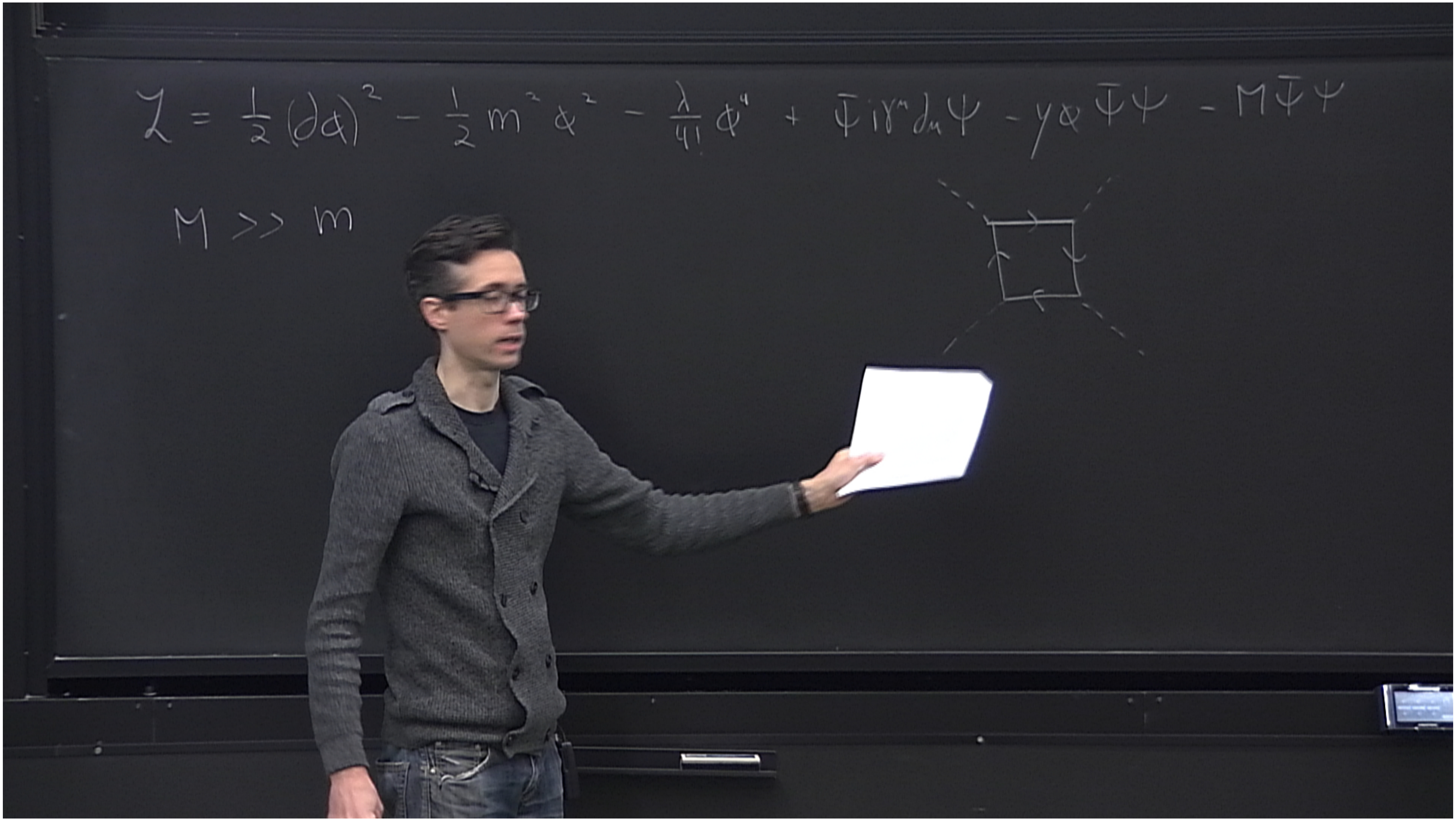
Title: PSI 2015/2016 More/Beyond Standard Model - Lecture 5

Date: Feb 26, 2016 09:00 AM

URL: http://pirsa.org/16020070

Abstract:


$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \bar{\Psi}i\gamma^\mu\partial_\mu\Psi - y\phi\bar{\Psi}\Psi - M\bar{\Psi}\Psi$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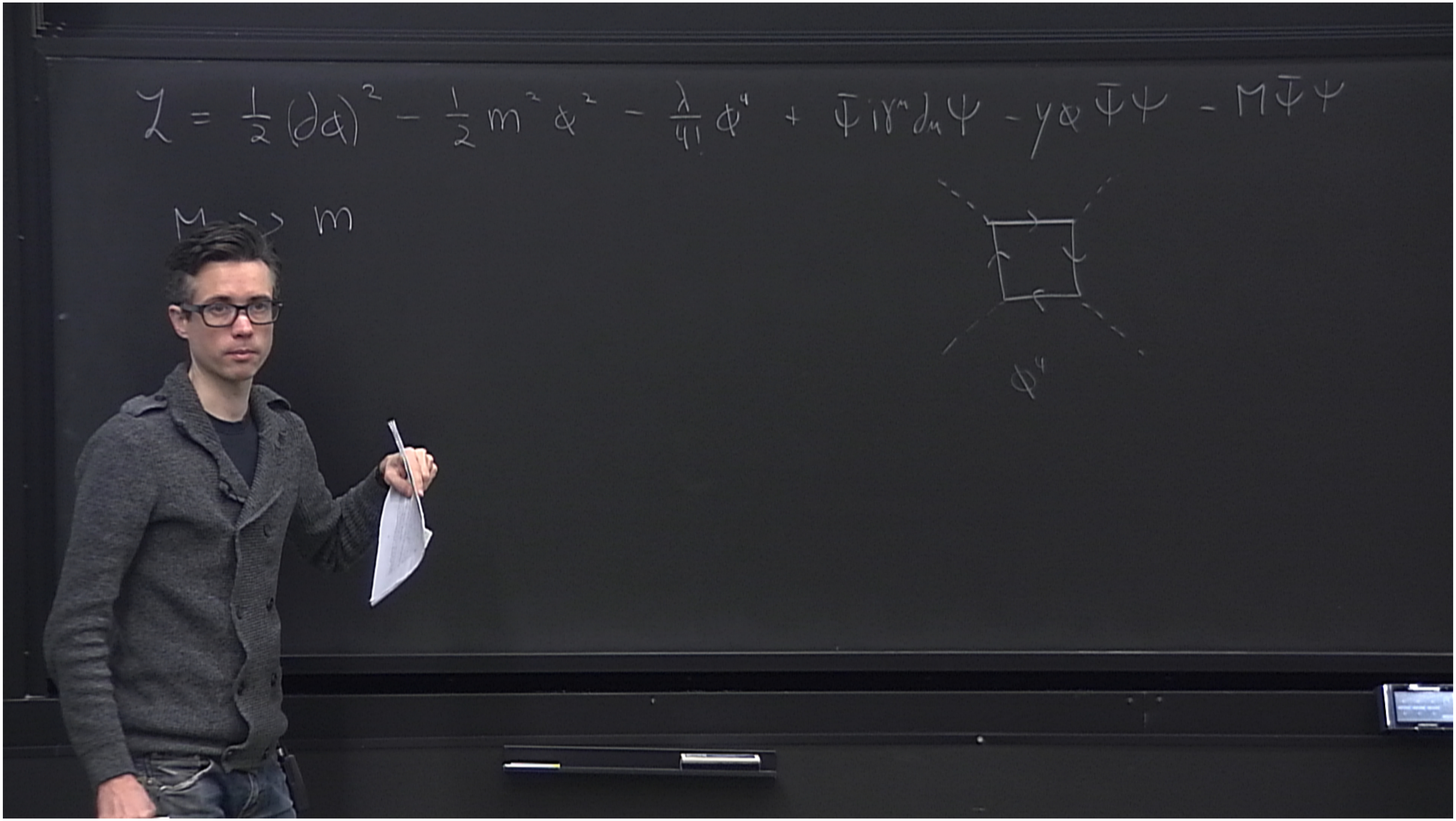
$M >> m$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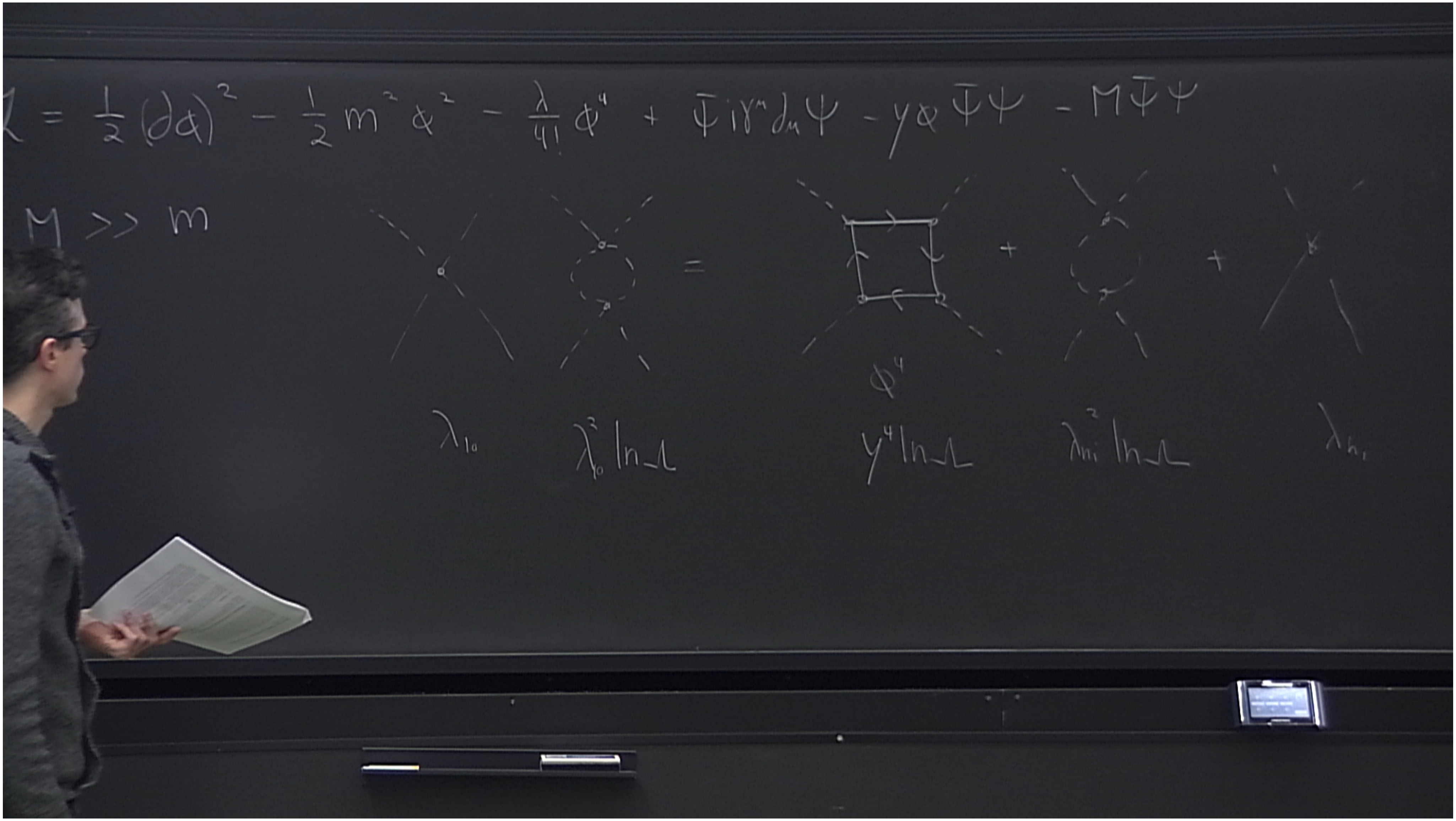
ϕ^4



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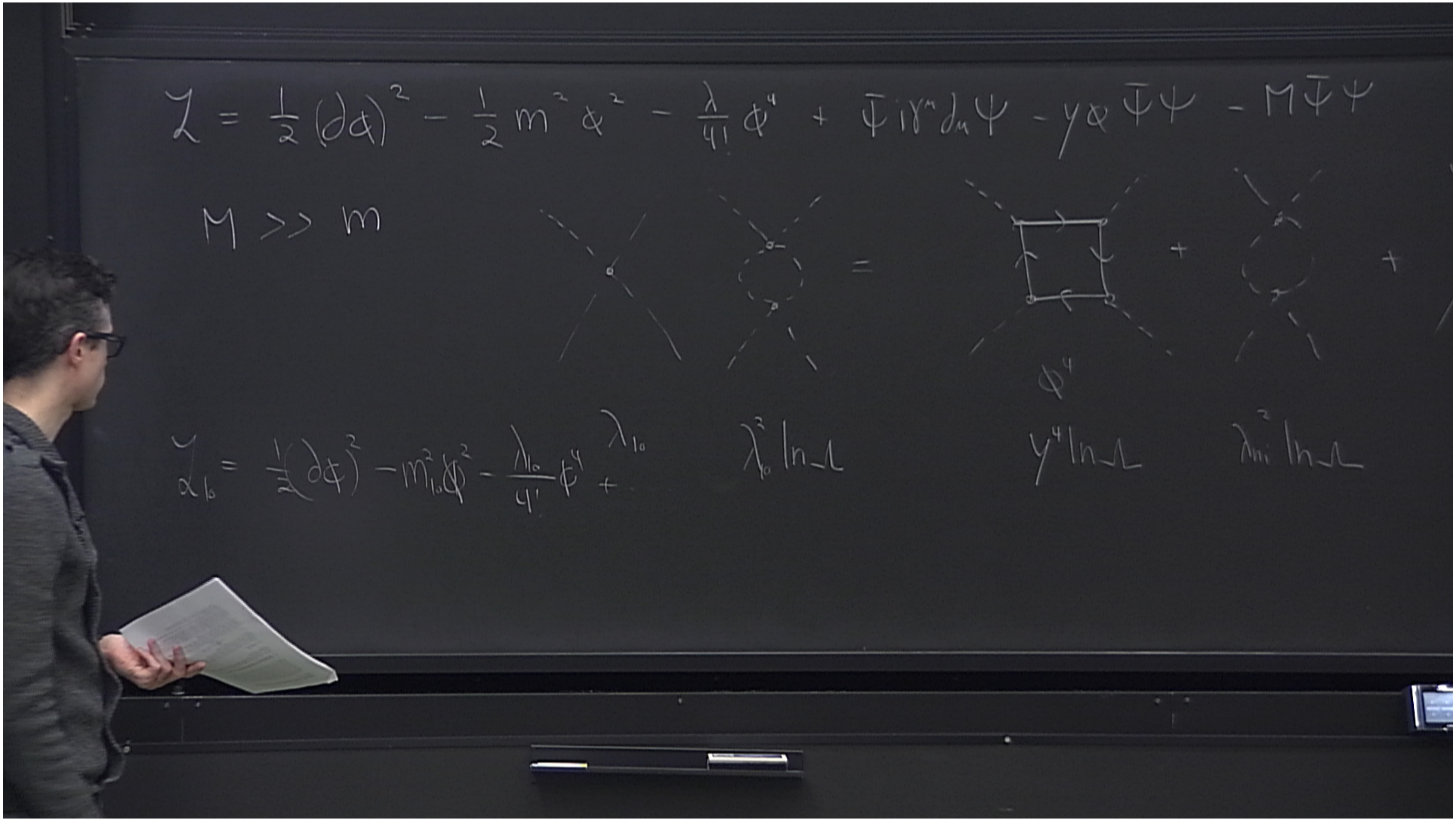
$M >> m$
ϕ^4
λ_{lo}
$\lambda_0^2 \ln\Lambda$
$y^4 \ln\Lambda$
$\lambda_{hi}^2 \ln\Lambda$
λ_{hi}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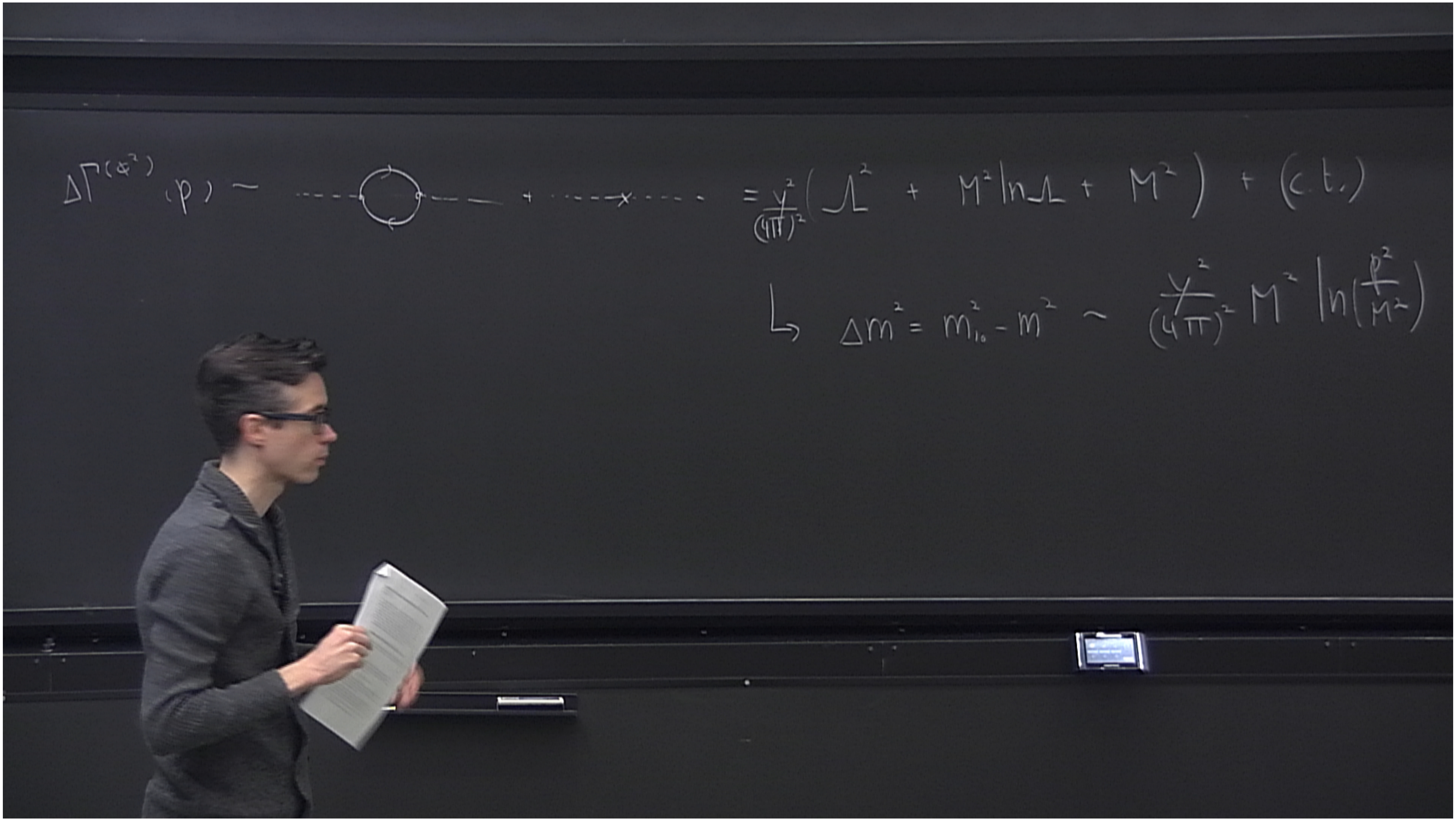
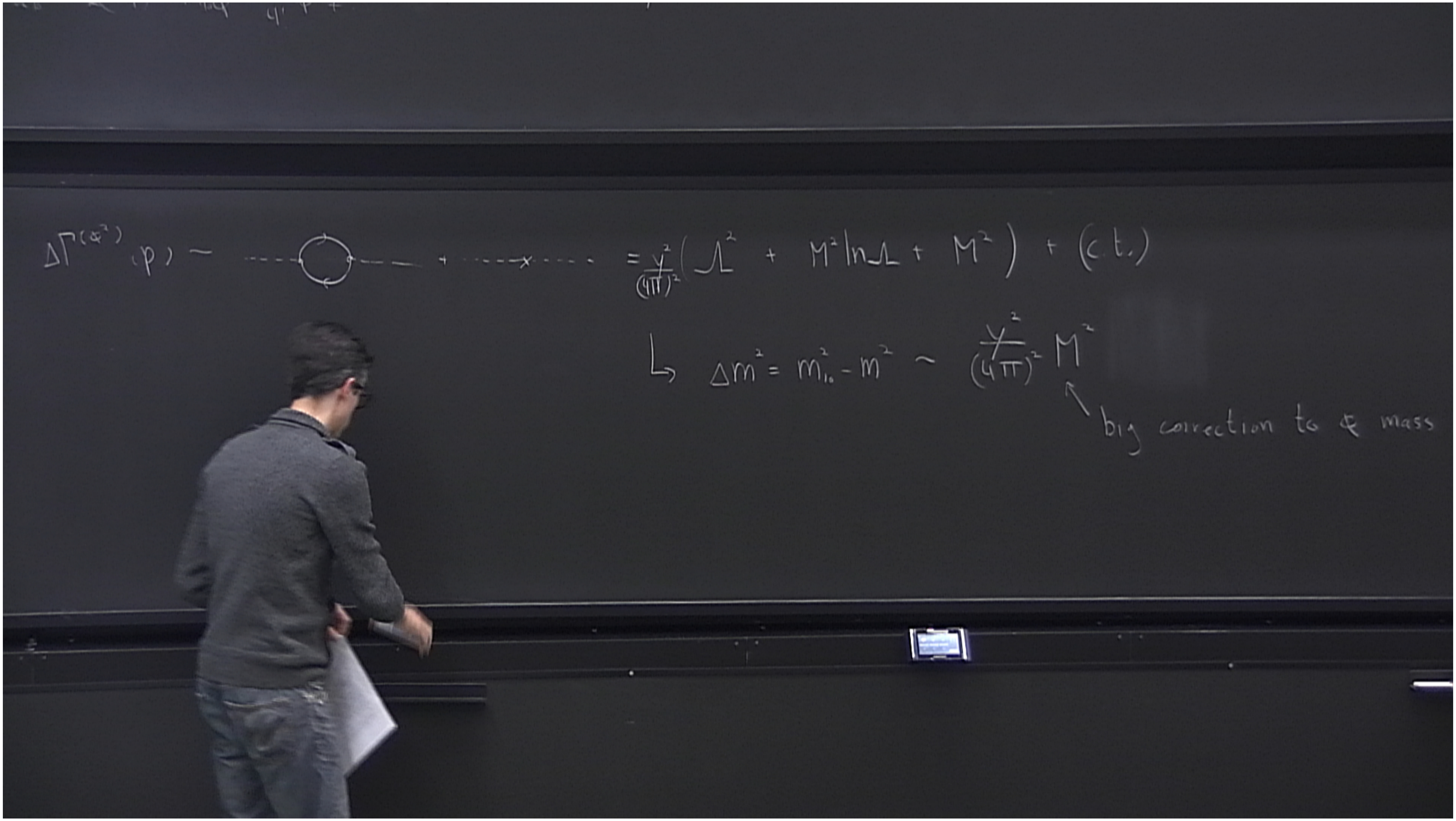
$M >> m$
$=$
$+$
$+$
ϕ^4
λ_{lo}
$\lambda_0^2 \ln\Lambda$
$y^4 \ln\Lambda$
$\lambda_{m}^2 \ln\Lambda$
$\mathcal{L}_{lo} = \frac{1}{2}(\partial\phi)^2 - m_{lo}^2\phi^2 - \frac{\lambda_{lo}}{4!}\phi^4 + \cdots$


$\Delta\Gamma^{(\phi^2)}(p) \sim$
$= \frac{y^2}{(4\pi)^2}\left(\Lambda^2 + M^2 \ln\Lambda + M^2\right) + (c.t.)$
$\hookrightarrow \Delta m^2 = m_0^2 - m^2 \sim \frac{y^2}{(4\pi)^2} M^2 \ln\left(\frac{p^2}{M^2}\right)$

$$\Lambda + M^2) + (c.t.)$$

$$m^2 \sim \frac{y^2}{(4\pi)^2} M^2 \ln\left(\frac{p^2}{M^2}\right)$$

big correction to ϕ mass


$\Delta\Gamma^{(\phi^2)}(p) \sim = \frac{y^2}{(4\pi)^2}\left(\Lambda^2 + M^2 \ln\Lambda + M^2\right) + (c.t.)$
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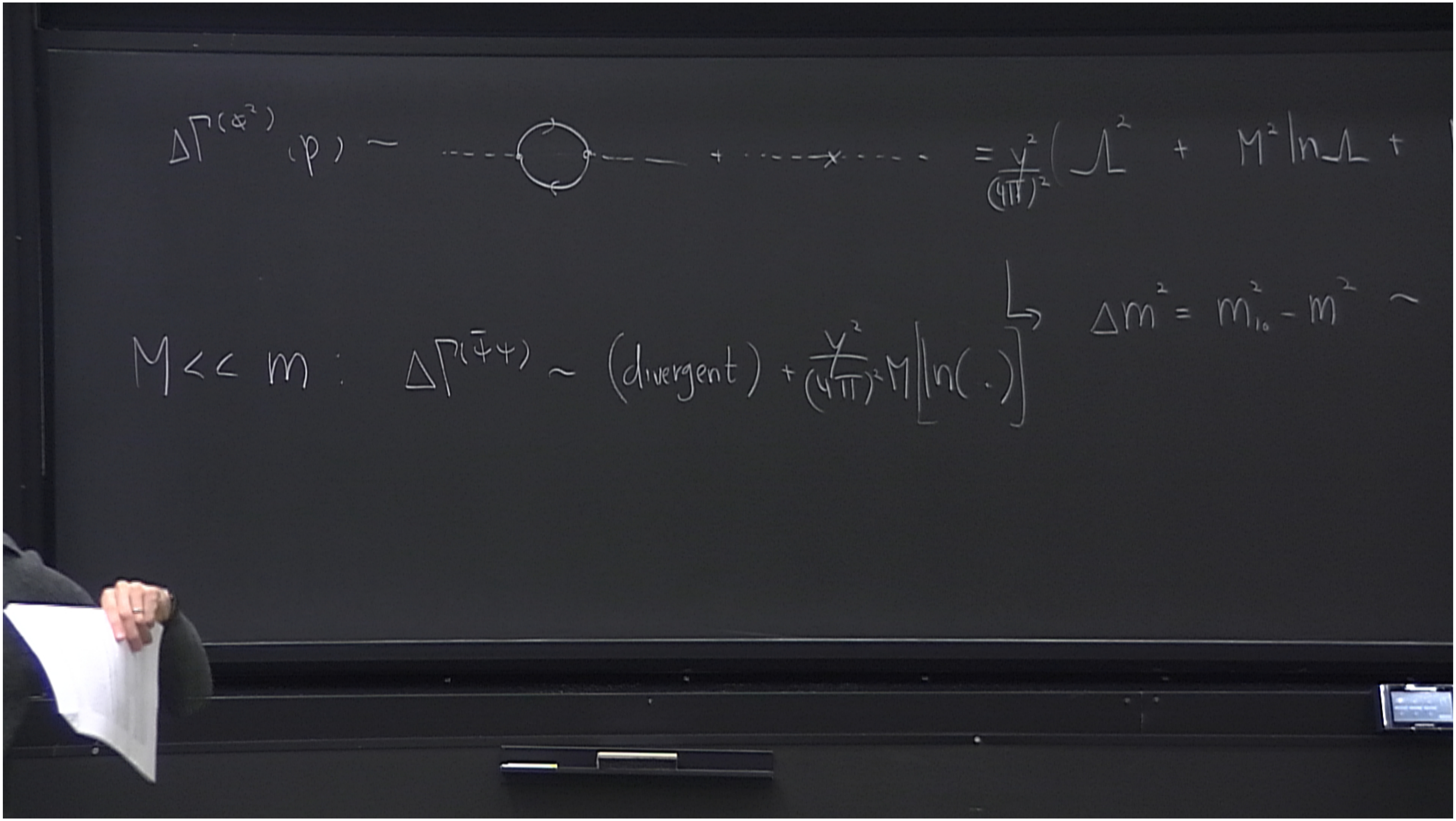
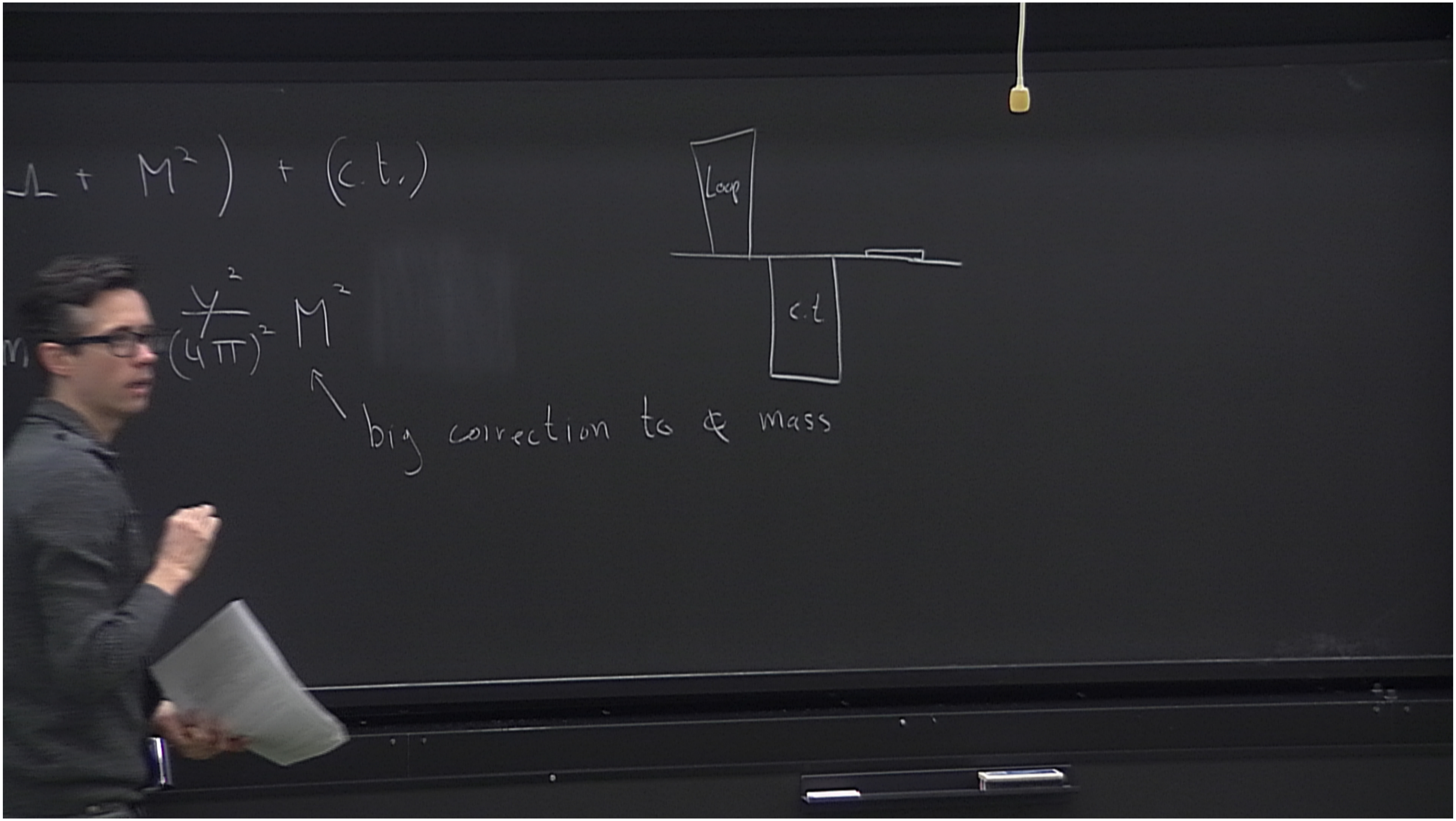
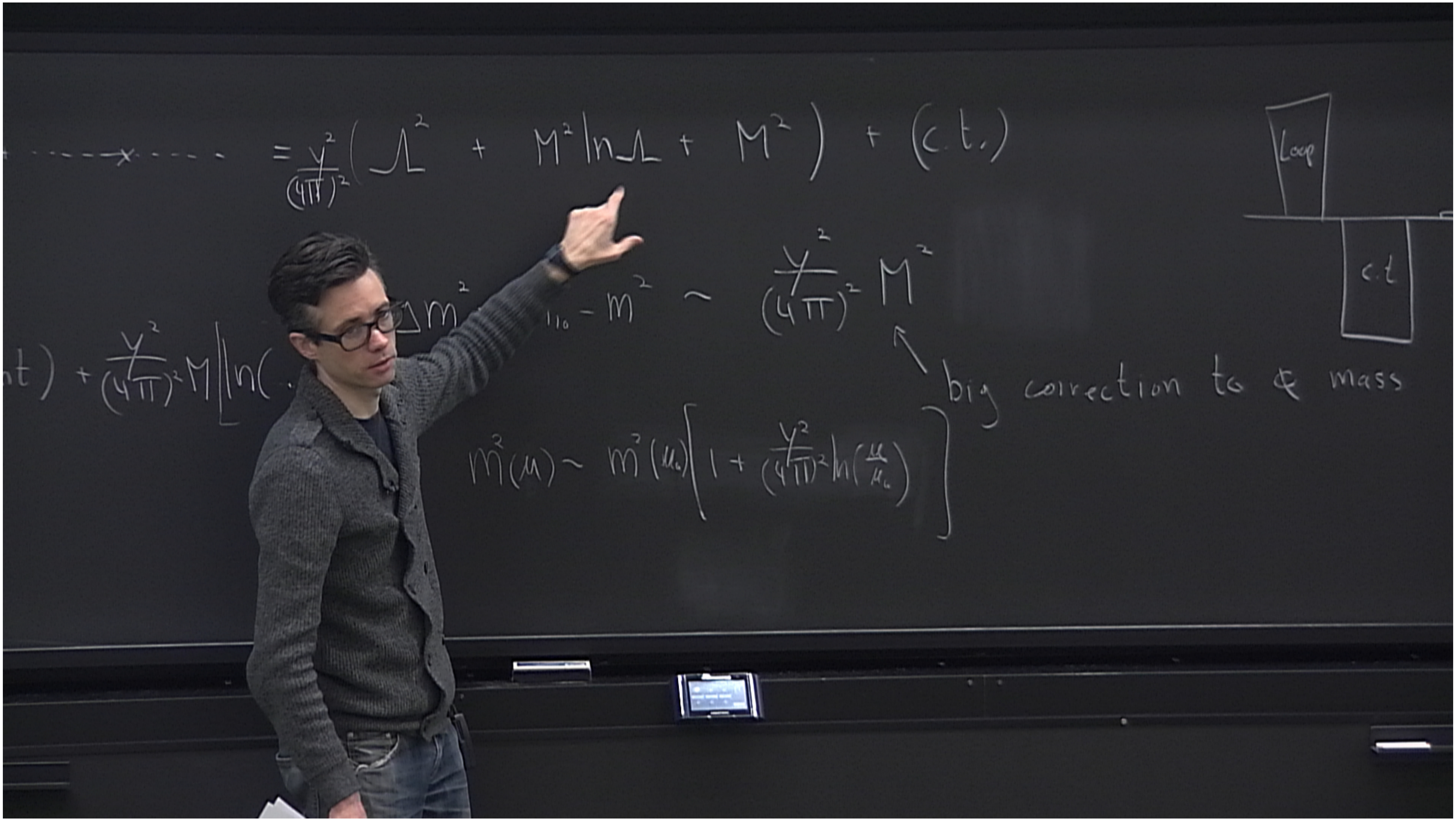
big correction to ϕ mass

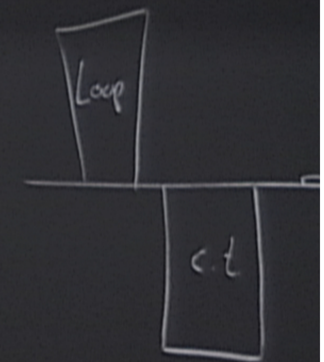

ΔΓ^(φ²)(p) ~ ---○--- + ---×--- = y²/(4π)² (Λ² + M² ln Λ +
↳ Δm² = m₀² − m² ~
M << m : ΔΓ^(ψ̄ψ) ~ (divergent) + y²/(4π)² M [ln(.)]


+ M²) + (c.t.)
Loop
c.t
big correction to φ mass



= y^2/(4π)^2 (Λ^2 + M^2 ln Λ + M^2) + (c.t.)
Loop
c.t
Δm^2 - m^2 ~ y^2/(4π)^2 M^2
big correction to φ mass
m^2(μ) ~ m^2(μ0)[1 + y^2/(4π)^2 ln(μ/μ0)]

$$= \frac{y^2}{(4\pi)^2}\left(\Lambda^2 + M^2 \ln\Lambda + M^2\right) + (c.t.)$$

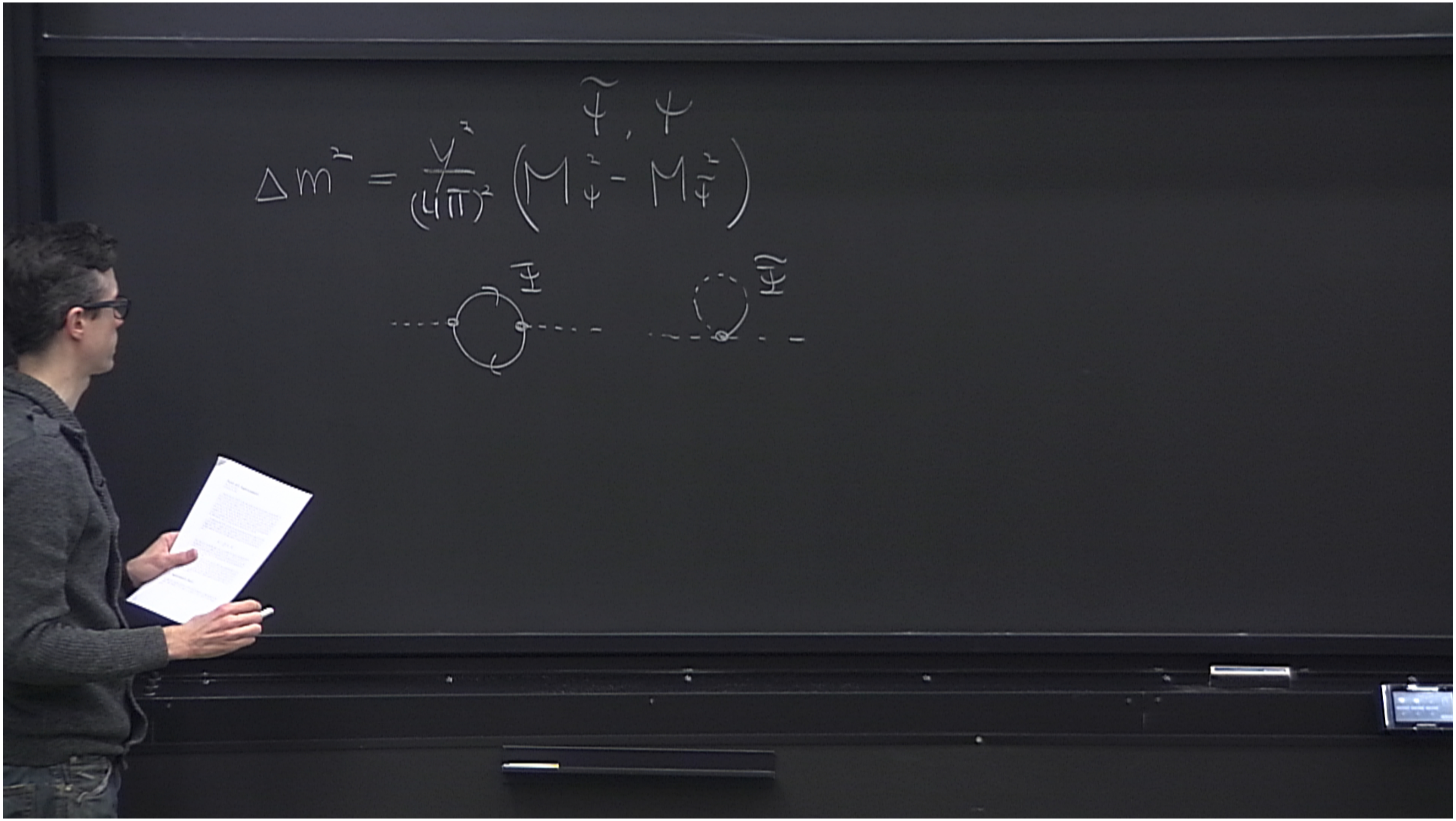


$$\hookrightarrow \Delta m^2 = m_{10}^2 - m^2 \sim \frac{y^2}{(4\pi)^2} M^2$$

$$\ldots t) + \frac{y^2}{(4\pi)^2} M \left[\ln(\,.\,)\right]$$

big correction to ϕ mass

$$m^2(\mu) \sim m^2(\mu_0)\left[1 + \frac{y^2}{(4\pi)^2}\ln\left(\frac{\mu}{\mu_0}\right)\right]$$


$\tilde{\Psi}, \Psi$
$\Delta m^2 = \frac{y^2}{(4\pi)^2}\left(M_\Psi^2 - M_{\tilde{\Psi}}^2\right)$
Ψ
$\tilde{\Psi}$

Representations of Groups

$g \in G$, $M(g) =$ linear op. (matrix)

$$M(g)\, M(h) = M(gh)$$

$$M(1) = \mathbb{I}$$

Groups

linear op. (matrix)

$M(gh)$

Lie Groups

$g(\alpha^a) \qquad \alpha^a \in \mathbb{R}^n$

$\{\alpha^a\} \rightarrow M(\alpha^a) = \exp(-i\alpha^a t^a)$

$= \mathbb{1} - i\alpha^a t^a + \frac{(-i\alpha^a t^a)^2}{2!}$

Groups

linear op. (matrix)

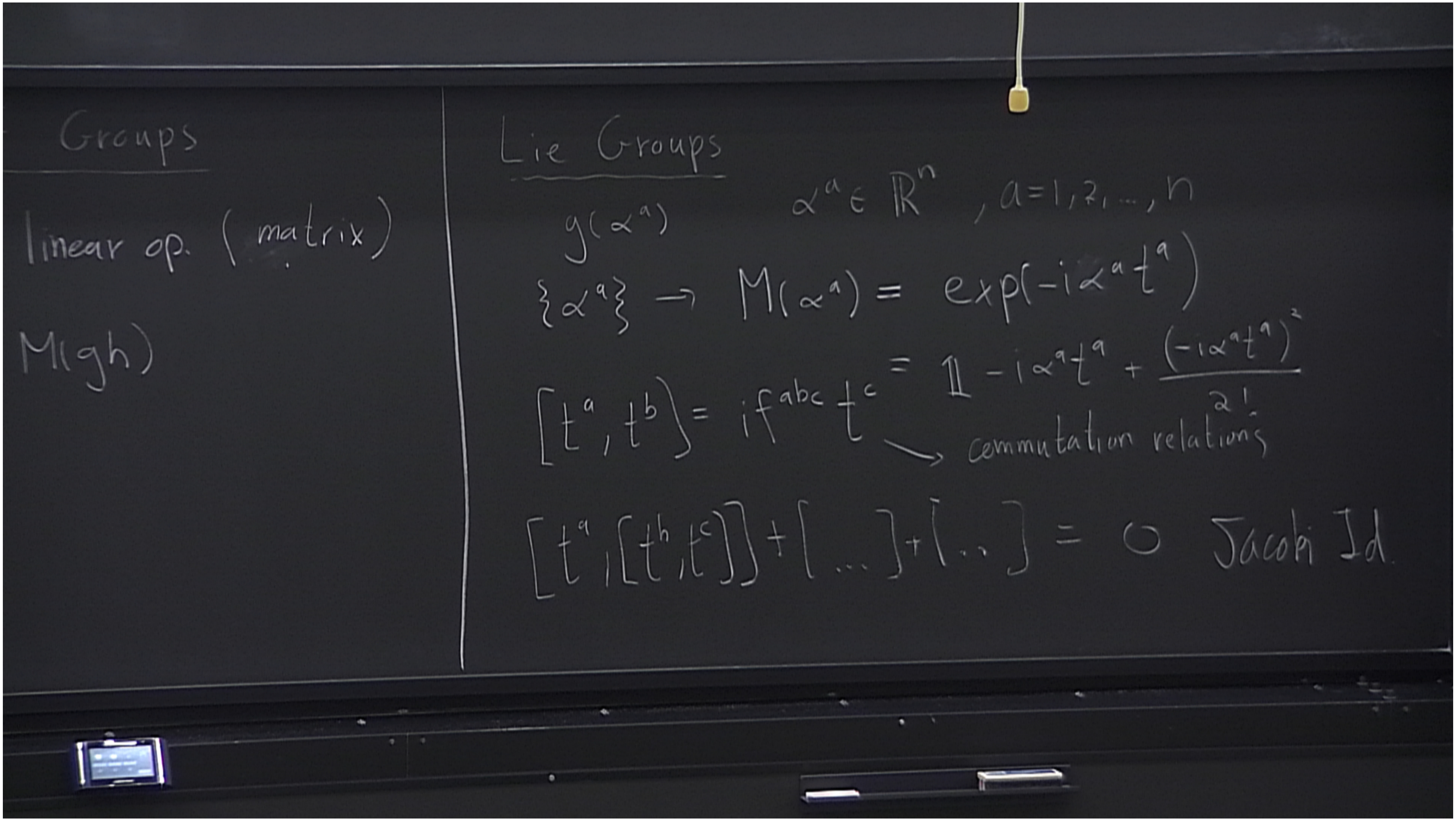
$M(gh)$

Lie Groups

$g(\alpha^a)$ $\qquad \alpha^a \in \mathbb{R}^n, \; a = 1, 2, \ldots, n$

$$\{\alpha^a\} \rightarrow M(\alpha^a) = \exp(-i\alpha^a t^a)$$

$$= \mathbb{1} - i\alpha^a t^a + \frac{(-i\alpha^a t^a)^2}{2!}$$


Groups
linear op. (matrix)
$M(gh)$
Lie Groups
$g(\alpha^a)$ $\alpha^a \in \mathbb{R}^n$, $a=1,2,\dots,n$
$\{\alpha^a\} \to M(\alpha^a) = \exp(-i\alpha^a t^a) = \mathbb{1} - i\alpha^a t^a + \frac{(-i\alpha^a t^a)^2}{2!}$
$[t^a, t^b] = i f^{abc} t^c$
commutation relations
$[t^a,[t^b,t^c]] + [\dots] + [\dots] = 0$ Jacobi Id

Poincaré Group = Lorentz + translations

$\{P^\mu, J^{\mu\nu}\}$

$J^{0i} \sim$ boosts

$J^{ij} \sim$ rotations

$J^{\mu\nu} = -J^{\nu\mu}$

$[P^\mu, P^\nu] = 0$

$[P^\mu, J^{\rho\sigma}] = (\ldots)P$

$[J^{\mu\nu}, J^{\rho\sigma}] = (\ \ )J$

slations

P

)J

$$x \to \Lambda x + a$$

$$\Lambda^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu}$$

$$\Phi_C(x) \to \left[\exp\left(-\frac{i}{2}\,\omega_{\alpha\beta}\, J^{\alpha\beta}_{j_1 j_2}\right)\right]_C{}^{D}\, \Phi_D(\Lambda^{-1}x - a)$$

slations

P

)J

$$x \to \Lambda x + a$$

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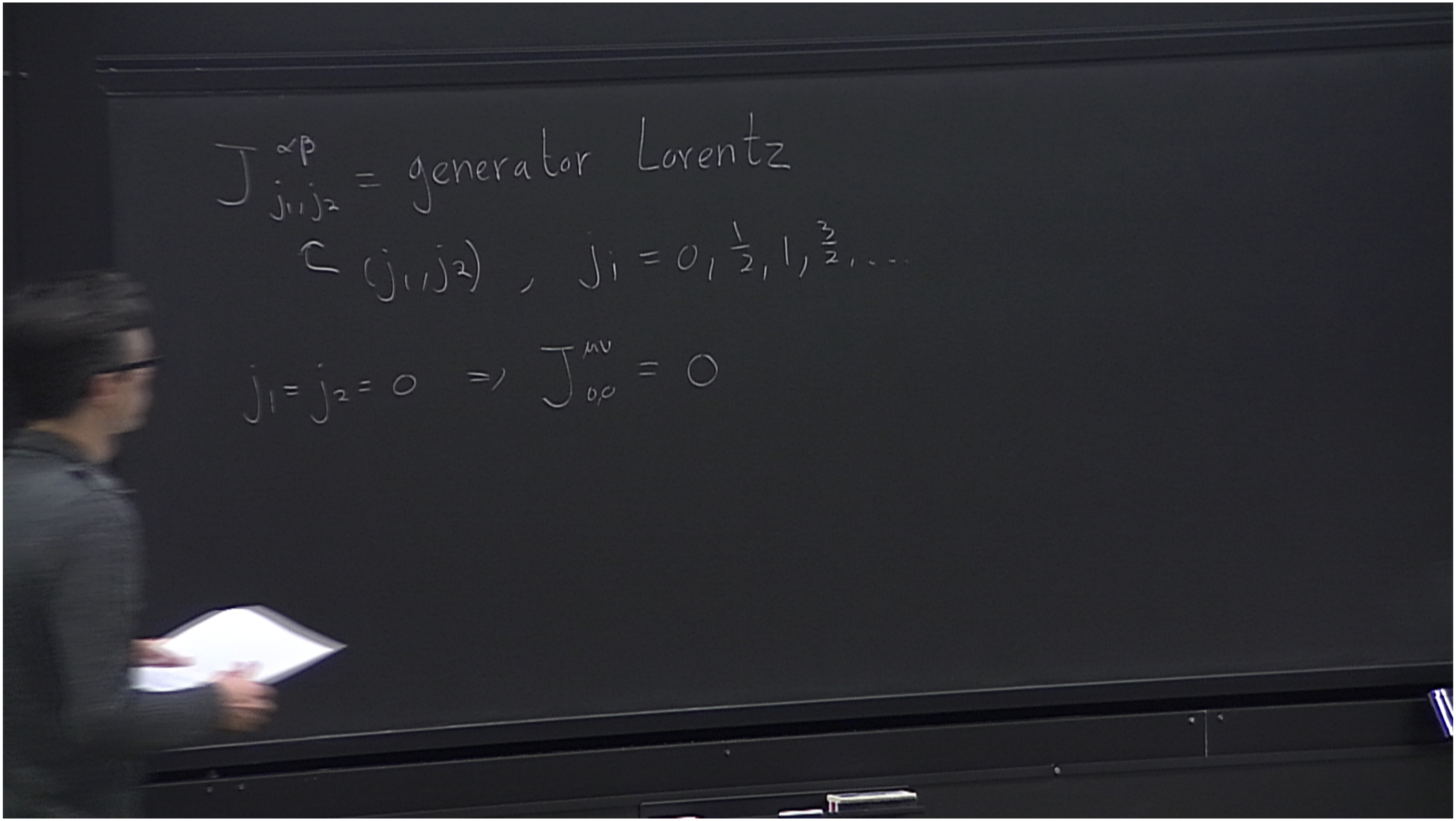
generator of (j_1, j_2) rep. of Lorentz


$J^{\alpha\beta}_{j_1,j_2}$ = generator Lorentz
$\in (j_1, j_2)$, $j_i = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$
$j_1 = j_2 = 0 \Rightarrow J^{\mu\nu}_{0,0} = 0$

$J^{\alpha\beta}_{j_1,j_2}$ = generator Lorentz

$\in (j_1, j_2)\ ,\quad j_1 = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$

$j_1 = j_2 = 0 \Rightarrow J^{\mu\nu}_{0,0} = 0 \Rightarrow s=0$ scalar field

$$\begin{cases} j_1 = \frac{1}{2}\ , & j_2 = 0 \\ j_2 = 0\ , & j_1 = \frac{1}{2} \end{cases}$$

$J^{\alpha\beta}_{j_1,j_2}$ = generator Lorentz

$\subset (j_1, j_2)$, $j_1 = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$

$j_1 = j_2 = 0 \Rightarrow J^{\mu\nu}_{0,0} = 0 \Rightarrow s=0$ scalar field

$\left\{ \begin{array}{l} j_1 = \frac{1}{2} \ , \ j_2 = 0 \\ j_2 = 0 \ , \ j_1 = \frac{1}{2} \end{array} \right.$ $\Rightarrow$ LH, RH 2-component fermions

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$j_1 = j_2 = 0 \Rightarrow J^{\mu\nu}_{0,0} = 0 \Rightarrow s=0$ scalar field ϕ

$\left\{ \begin{array}{l} j_1 = \frac{1}{2}, \; j_2 = 0 \\ j_2 = 0, \; j_1 = \frac{1}{2} \end{array} \right.$ $\Rightarrow$ LH, RH 2-component fermions ψ_L ψ_R

$(\frac{1}{2}, \frac{1}{2}) \Rightarrow s = 1$, vector representation , A^μ

Unitary rep. : $M(g^{-1}) = M^{\dagger}(g) = M^{-1}(g)$

$$P^2 = P_\mu P^\mu$$

$$W^2 = W^\mu W_\mu$$

Unitary rep. : $M(g^{-1}) = M^{\dagger}(g) = M^{-1}(g)$

$$\begin{cases} P^2 = P_\mu P^\mu \\ W^2 = W^\mu W_\mu \end{cases}, \quad W_\mu = -\epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma / 2$$

Unitary rep. : $M(g^{-1}) = M^{\dagger}(g) = M^{-1}(g)$

$$\begin{cases} P^2 = P_\mu P^\mu, & W_\mu = -\epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma / 2 \\ W^2 = W^\mu W_\mu \end{cases}$$

$P^2 \sim$ total mass of the state

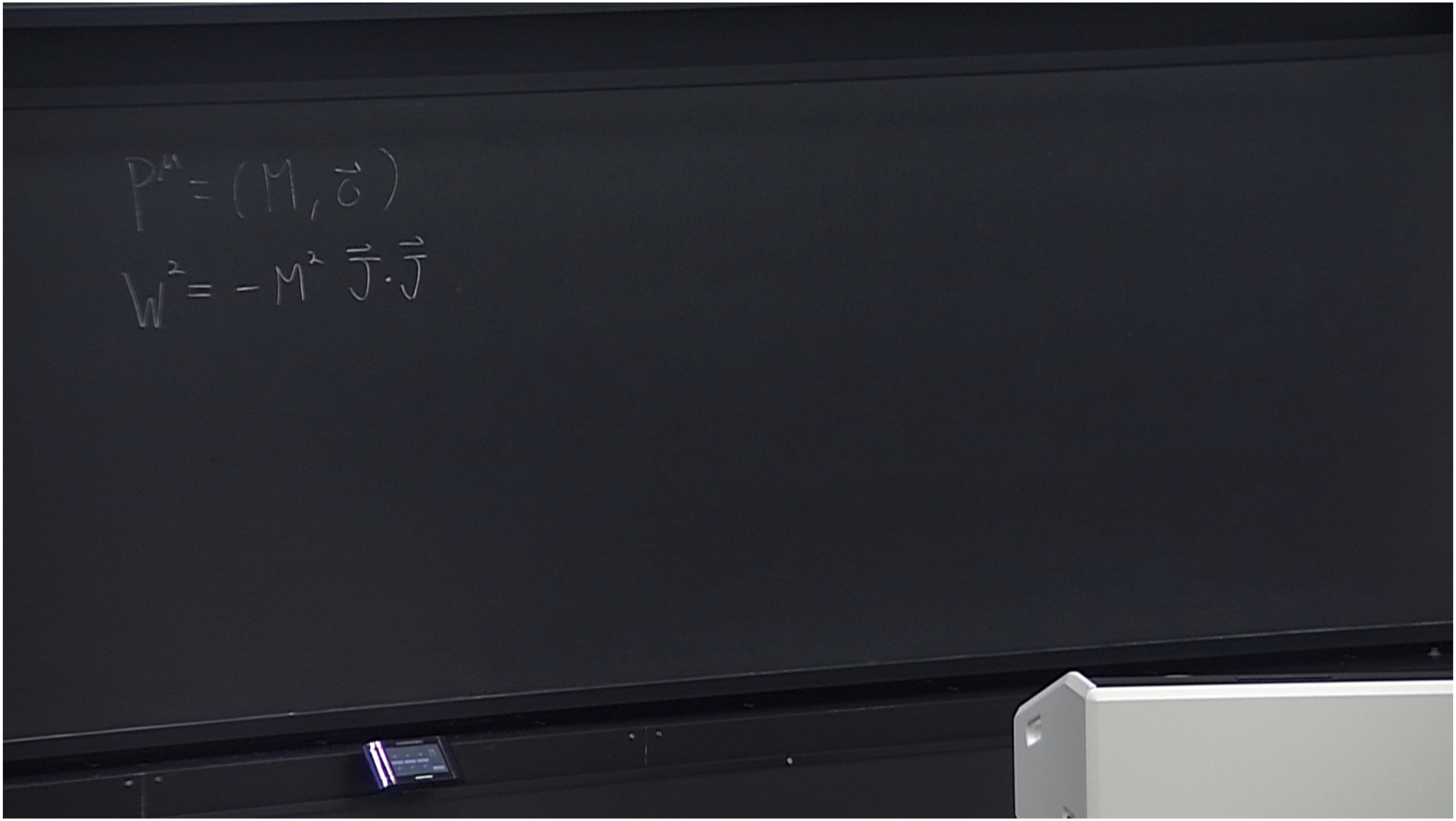
Unitary rep. : $M(g^{-1}) = M^{T}(g) = M^{-1}(g)$

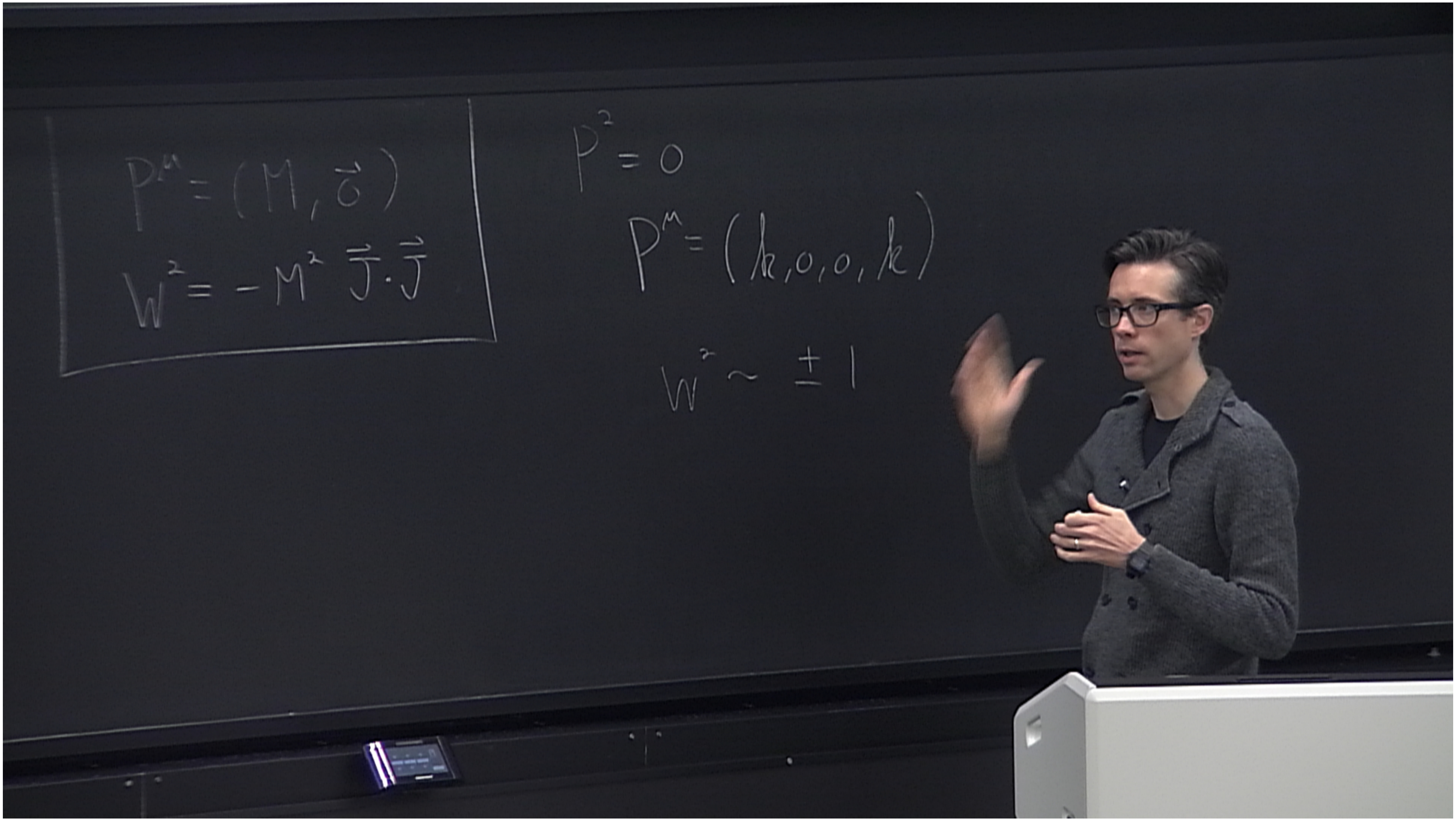
$$\begin{cases} P^2 = P_\mu P^\mu \,, \quad W_\mu = -\epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma / 2 \\ W^2 = W^\mu W_\mu \end{cases}$$

$P^2 \sim$ total mass^2 of the state

$P^\mu = (M, \vec{0}$

$W^2 = -$


$P^\mu = (M, \vec{0})$
$W^2 = -M^2\, \vec{J}\cdot\vec{J}$


P^μ = (M, 0⃗)
W² = −M² J⃗·J⃗
P² = 0
P^μ = (k, 0, 0, k)
W² ~ ±1

$$\boxed{\begin{aligned} P^\mu &= (M, \vec{0}) \\ W^2 &= -M^2\, \vec{J}\cdot\vec{J} \end{aligned}}$$

$$P^2 = 0$$

$$P^\mu = (k, 0, 0, k)$$

$$W^2 \sim \pm 1$$